4D symplectic tracking in the final doublet

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4D symplectic integrator for the particle tracking through the final quadrupole of the SuperB final focus.

- investigate and correct multipole components;
- study of the aberrations;
- study of the dynamical aperture.

Tracking method

Generalized leapfrog method * 1

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Relativistic hamiltonian:

$$\mathcal{H} = c\sqrt{m^2c^2 + (p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2}$$
(1)

We need to separate the square root in the exponential:

$$e^{-h:\mathcal{H}:}$$
 (2)

$$\mathcal{H}' = \frac{(p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2}{2M'}$$
(3)

where

$$M' = \sqrt{m^2c^2 + (p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2}$$
 gives the same Hamilton equations of (1)

^{*}E. Chacon-Golcher, F. Neri. A symplectic integrator with arbitrary vector and scalar potential, Physics Letters A 372 (2008)

Tracking method

Generalized leapfrog method 2

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Generalized leapfrog method

$$e^{-h:\mathcal{H}':} = e^{-\frac{h}{2}:\frac{(p_{X}-eA_{X})^{2}}{2M'}:} e^{-\frac{h}{2}:\frac{(p_{Y}-eA_{Y})^{2}}{2M'}:} e^{-h:\frac{(p_{Z}-eA_{Z})^{2}}{2M'}:} e^{-h:\frac{(p_{Z}-eA_{Z})^{2}}{2M'}:} e^{-\frac{h}{2}:\frac{(p_{X}-eA_{X})^{2}}{2M'}:} + O(h^{3})$$
(4)

the x component can be calculated:

$$x^{f} = x^{i} + \frac{p_{X}^{i} - eA_{X}(x^{i}, y^{i}, z^{i})}{M'} \frac{h}{2} \qquad y^{f} = y^{i} \qquad z^{f} = z^{i}$$
 (5)

$$p_{X}^{f} = p_{X}^{i} + eA_{X}(x^{f}, y^{f}, z^{f}) - eA_{X}(x^{i}, y^{i}, z^{i})$$
(6)

$$p_y^f = p_y^i + \int_{x^i}^{x^f} e \frac{\partial}{\partial y^i} A_x(x^i, y^i, z^i) dx'$$
 (7)

$$p_z^f = p_z^i + \int_{x^i}^{x^f} e^{\frac{\partial}{\partial z^i}} A_x(x^i, y^i, z^i) dx'$$
 (8)

integrals of equations (7) and (8) are evaluated with a second order gaussian quadrature. 4 D > 4 P > 4 B > 4 B > B 9 Q P

Integration

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I obtain the vector potential and its derivatives integrating the Biot-Savart law.

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \oint_{l'} \frac{I d\vec{l'}}{|\vec{r} - \vec{r'}|} \tag{9}$$

The numerical integral is done with an 11^{th} order gaussian method. Every coil was divided in 12 arcs.

Test magnetic field

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Test

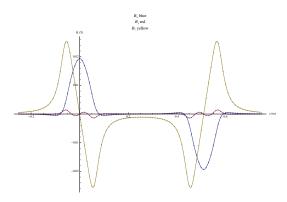
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Magnetic field on the magnet axis calculated with ${\sf C}$ program and with Mathematica



The order of the gaussian integral and the number of intervals are choosen to have an accuracy of 10^{-6} on fields values.

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The rest of the algorithm on a single time step Δt is $O(\Delta t^3)$. With n time steps, the rest is $O(n^{-2})$.

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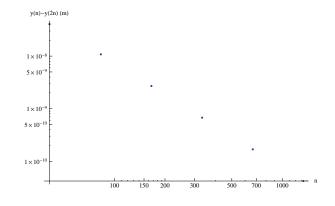
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Conclusions

With the time step $\Delta t = 4 \, \mathrm{ps}$:

- execution time: about $50 \, \mathrm{s}/particle$ (from $15 \, \mathrm{cm}$ before the magnet to the focus point).
- ullet accuracy of results: $< 1\,\mathrm{nm}$ on three coordinates

With this code, the simulation of all the final focus will be possible.

Magnet model

internal coils

$$x_1 = r_1 \cos(\phi);$$

•
$$y_1 = -r_1 \operatorname{sen}(\phi);$$

•
$$z_1 = -A_1 \sin(2\phi) - \frac{\phi \Delta z}{2\pi} + N\Delta z$$
; • $z_2 = -A_2 \sin(2\phi) + \frac{\phi \Delta z}{2\pi}$;

external coils

$$x_2 = r_2 \cos(\phi);$$

•
$$y_2 = r_2 \operatorname{sen}(\phi);$$

•
$$z_2 = -A_2 \sin(2\phi) + \frac{\phi \Delta z}{2\pi}$$

$$0 < \phi < 2N\pi$$

$$\Delta z = 6.4 \,\mathrm{mm}$$

$$N = 47$$

$$I = 2500 \,\mathrm{A}$$

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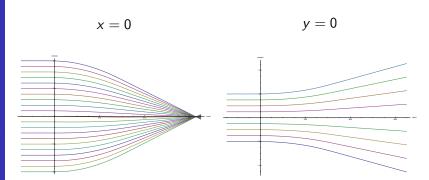
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Tracking

4D symplectic tracking in the final doublet Focus point is not in the magnet axis.

$$F_{\rm v} \simeq -18\,\mu{
m m}$$

(10)

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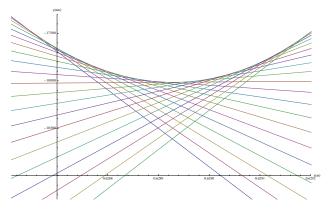
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Tracking

sextupole component

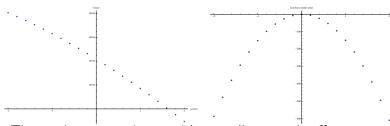
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The focus is not a point: there is a sextupole component.

The distance f between the principal plane and the focus:

$$f_j = \frac{y'^n_j}{y'_j} \tag{11}$$

sextupole effect

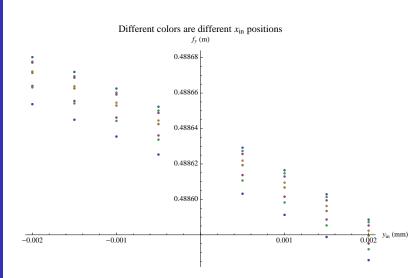


The results are consistent with a small sextupole effect.

Tracking focus for different x positions

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Conclusions

The code will be used to simulate the QD0 doublet, to investigate the value of the multipole components, and to simulate all the final focus magnets.