Heavy Flavour:
Theoretical Predictions in the precision Era

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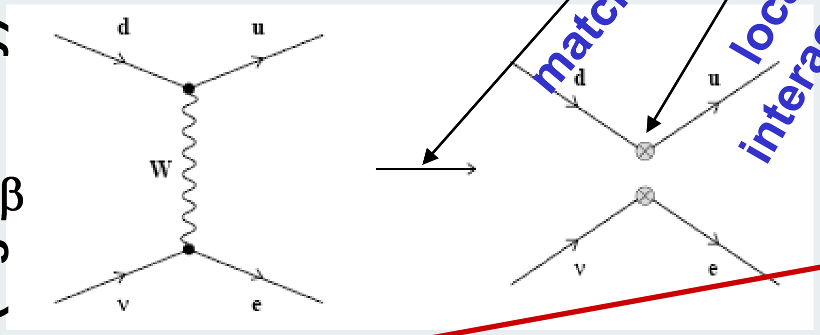
Flavour Physics:

- precision tests of the Standard Model (SM)
- search for New Physics (NP)

Theoretical Approach to Weak Decays of Hadrons
(including the effects of strong interactions)

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_i^{CKM} C_i(\mu) Q_i(\mu)$$

(e.g. β-decay)



$C_i(\mu)$: Wilson coefficients (short-distance)
$Q_i(\mu)$: local operators (long-distance)

- perturbative QCD corrections
[up to NNLO (e.g. $b \to s\gamma$, $b \to s\, l^+ l^-$)
+Renormalization Group
to resum large $\ln^n (M_W/\mu)$]
- affected by NP

- non-perturbative QCD contributions
[main source of theoretical uncertainty]

Wilson coefficients: Theoretical Status (within the SM)

C_i: short-distance ⇒ perturbative calculations

M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 [arXiv:hep-ph/9808385].

M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl. Phys. B **639** (2002) 389 [arXiv:hep-ph/0202106].

M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308** (2003) 031 [arXiv:hep-ph/0308029].

G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].

In the '90s basically the NLO QCD corrections to all relevant decays and transitions have been calculated (for $\Delta\Gamma_{d,s}$ in 2003)

In the last decade, NNLO QCD corrections to several flavour processes have been calculated, e.g. $B_s \to X_s \gamma$, $B \to X_s l^+ l^-$

M. Misiak *et al.*, Phys. Rev. Lett. **98** (2007) 022002 [arXiv:hep-ph/0609232].

H. H. Asatryan, H. M. Asatrian, C. Greub and M. Walker, Phys. Rev. D **66** (2002) 034009 [arXiv:hep-ph/0204341].

H. M. Asatrian, K. Bieri, C. Greub and A. Hovhannisyan, Phys. Rev. D **66** (2002) 094013 [arXiv:hep-ph/0209006].

P. Gambino, M. Gorbahn and U. Haisch, Nucl. Phys. B **673** (2003) 238 [arXiv:hep-ph/0306079].

C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, arXiv:hep-ph/0312090.

A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B **648** (2003) 254 [arXiv:hep-ph/0208088].

A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, arXiv:hep-ph/0312128.

M. Beneke, T. Feldmann and D. Seidel, NLO," Eur. Phys. J. C **41** (2005) 173 [arXiv:hep-ph/0412400].

Hadronic Matrix Elements: Theoretical Tools

$\langle Q_i \rangle$: long-distance → **Non-perturbative methods**

<u>The primary role of Lattice QCD</u>:

- **True theory (QCD) simulated on a finite and discrete space-time.**
- **Physical results require continuum and infinite volume limits, and extrapolations to the physical masses.**
- **Recent (~10 years) simulations are unquenched (N_f=2, 2+1).**
- **Accuracy has significantly improved in the last years**

Further improvements are expected thanks to increasing computational power, improved algorithms and theoretical approaches
→larger volumes, finer lattices, lower masses, N_f=2+1+1…

HEAVY FLAVOUR PHYSICS ON THE LATTICE

Collaboration	Quark action	N_f	a [fm]	$(M_\pi)^{min}$ [MeV]	Observables
MILC + FNAL, HPQCD,…	Improved staggered	2+1	$\geq$ 0.045	230	$f_{D(s)}$, $D \to \pi/K\, l\nu$, $f_{B(s)}$, $B_{B(s)}$, $B \to D/\pi\, l\nu$
ETMC	Twisted mass	2 2+1+1	$\geq$ 0.054	260	$f_{D(s)}$, $D \to \pi/K\, l\nu$, $f_{B(s)}$

Let's have a look at the status of the lattice results…

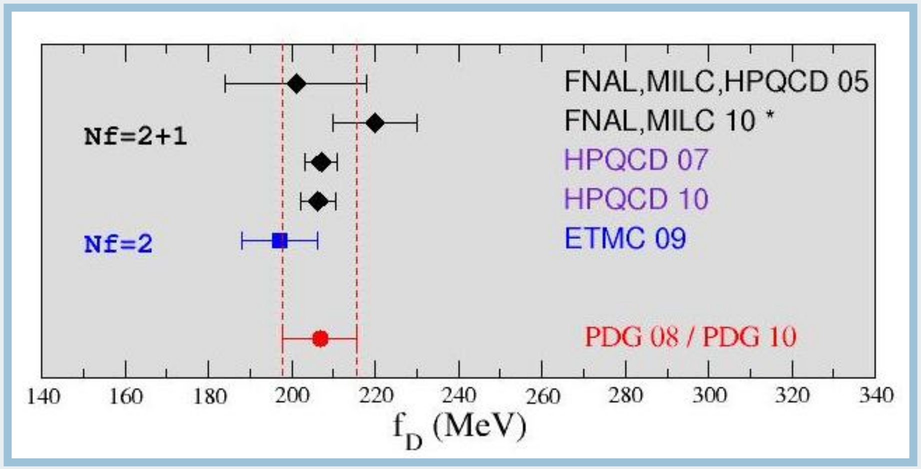
Leptonic $D_{(s)}$ mesons decays: f_D, f_{Ds}

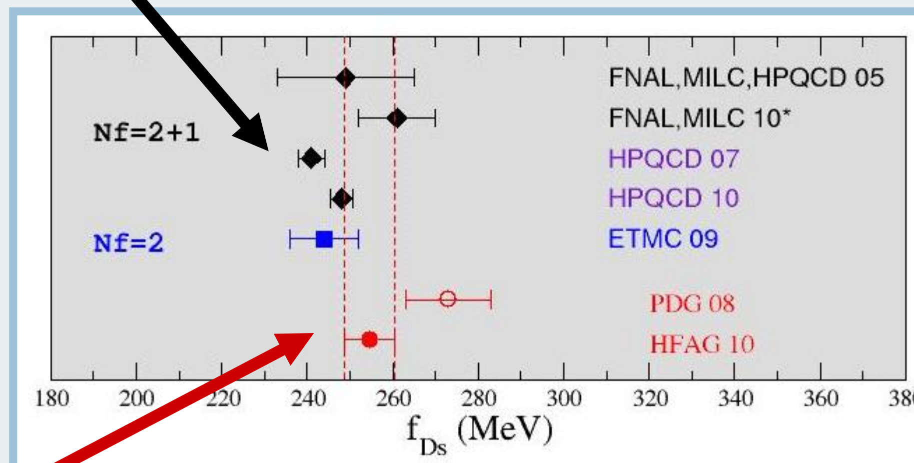
The 2008 summer puzzle

f_{Ds} = 241   3 MeV   HPQCD 07

f_{Ds} = 273   10 MeV   PDG 08

3 sigma deviation





2010: the puzzle solution

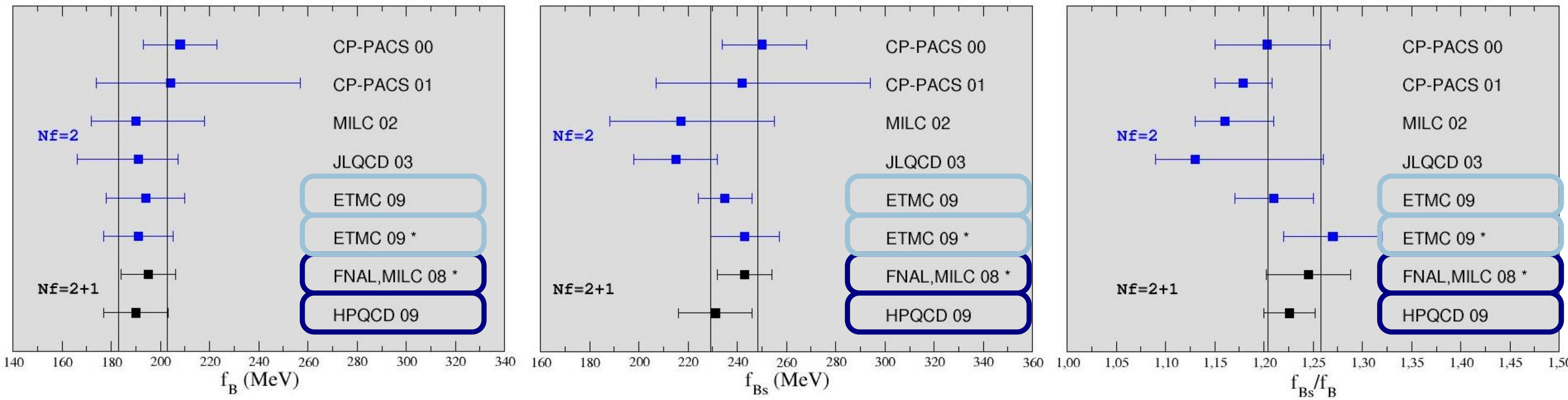
f_{Ds} = 248.0   2.5 MeV   HPQCD 10   + 3% (2.3 σ)

f_{Ds} = 254.6   5.9 MeV   HFAG 10   - 7% (1.8 σ)

(CLEO-c, Belle, BaBar)

1 sigma agreement

B-mesons decay constants f_B, f_{Bs} and B-$\bar{B}$ mixing, $B_{Bd/s}$



f_{Bs} = 238.8   9.5 MeV

f_B = 192.8   9.9 MeV

4-5%

f_{Bs}/f_B = 1.231   0.027

2%

Combining with the only modern calculation **HPQCD [0902.1815]:**

$\hat{B}_{Bd} = 1.26 \pm 0.11, \quad \hat{B}_{Bs} = 1.33 \pm 0.06$

$f_{Bs}\sqrt{\hat{B}_{Bs}}$ = 275   13 MeV

5%

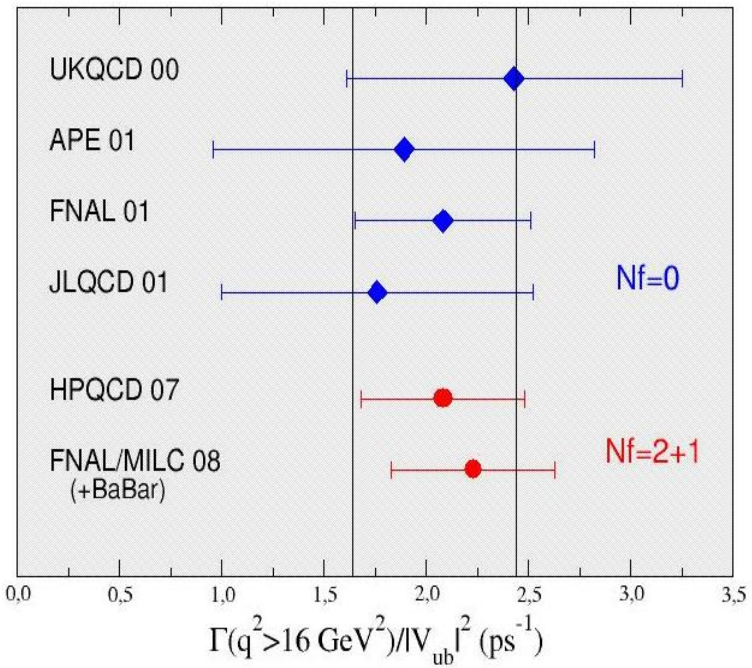
$\xi = 1.243 \pm 0.028$

2%

Exclusive vs Inclusive V_{ub}

THEORETICALLY CLEAN

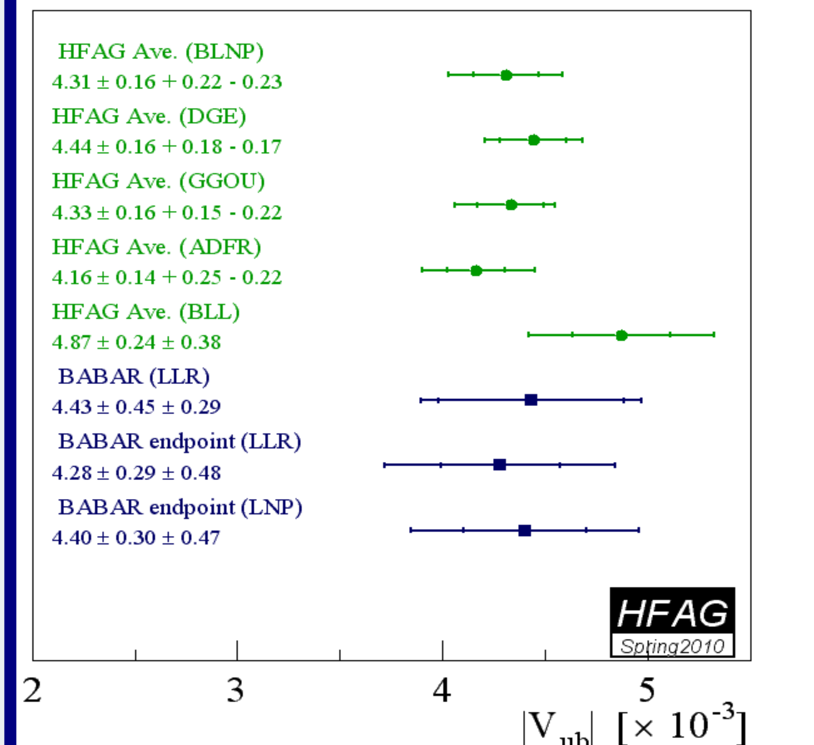
but more lattice calculations are certainly desired



$|V_{ub}|_{excl.} = (35.0 \pm 4.0)\ 10^{-4}$

IMPORTANT LONG DISTANCE CONTRIBUTIONS.

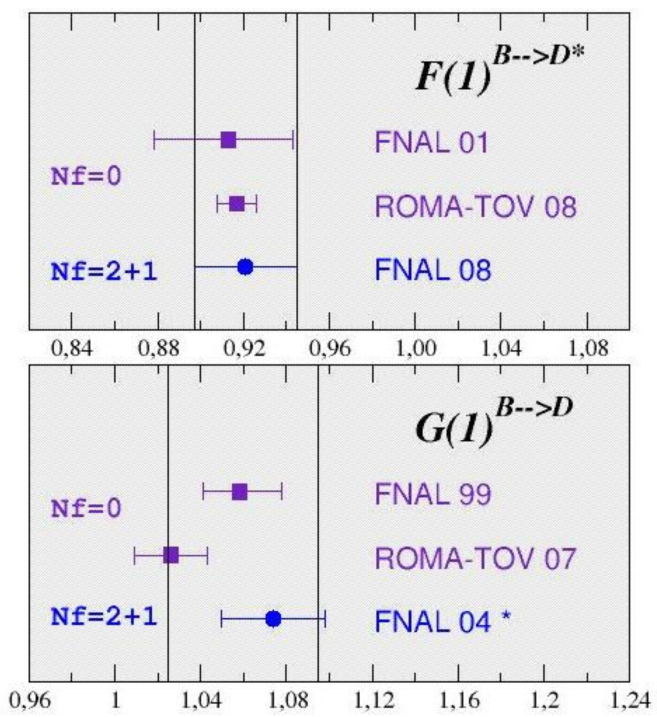
The results have some model dependence

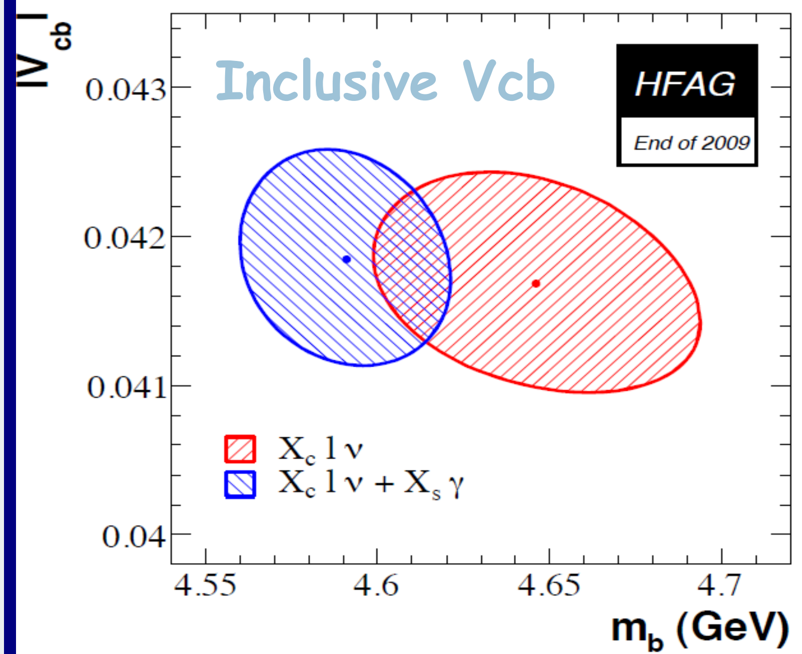


$|V_{ub}|_{incl.} = (42.0 \pm 1.5 \pm 5.0)\ 10^{-4}$

$|V_{ub}|_{SM\text{-}Fit} = (35.5 \pm 1.4)\ 10^{-4}$

Exclusive vs Inclusive V_{cb}





$|V_{cb}|_{excl.} = (39.0 \pm 0.9)\ 10^{-3}$

$|V_{cb}|_{incl.} = (41.7 \pm 0.7\ )\ 10^{-3}$

UTfit $|V_{cb}|_{SM\text{-}Fit} = (42.7 \pm 1.0)\ 10^{-3}$

The role of B-physics in the UTA

The UTA within the Standard Model

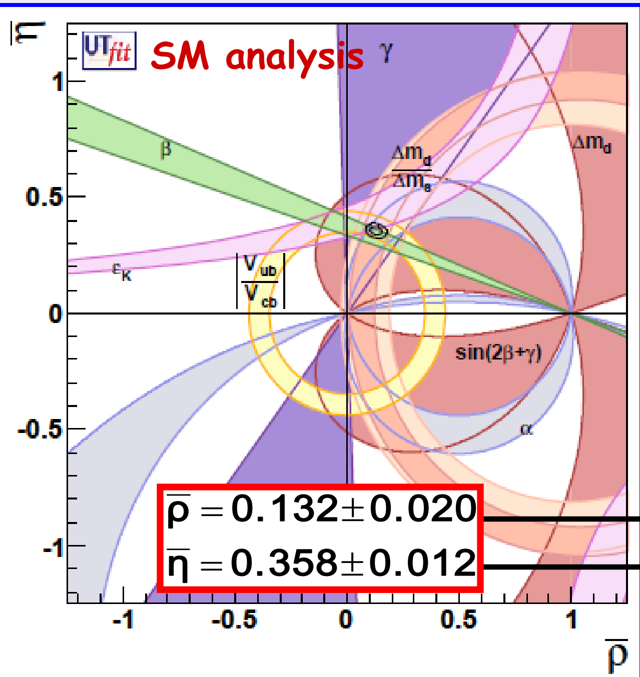
The experimental constraints:

$$\varepsilon_K, \Delta m_d, \left|\frac{\Delta m_s}{\Delta m_d}\right|, \left|\frac{V_{ub}}{V_{cb}}\right|$$

Involving a b quark

$$\sin 2\beta, \cos 2\beta, \alpha, \gamma(2\beta+\gamma)$$

overconstrain the CKM parameters consistently



$\bar{\rho}$: ~16%

$\bar{\eta}$: ~ 3%

The UTA has established that the CKM matrix is the dominant source of flavour mixing and CP violation

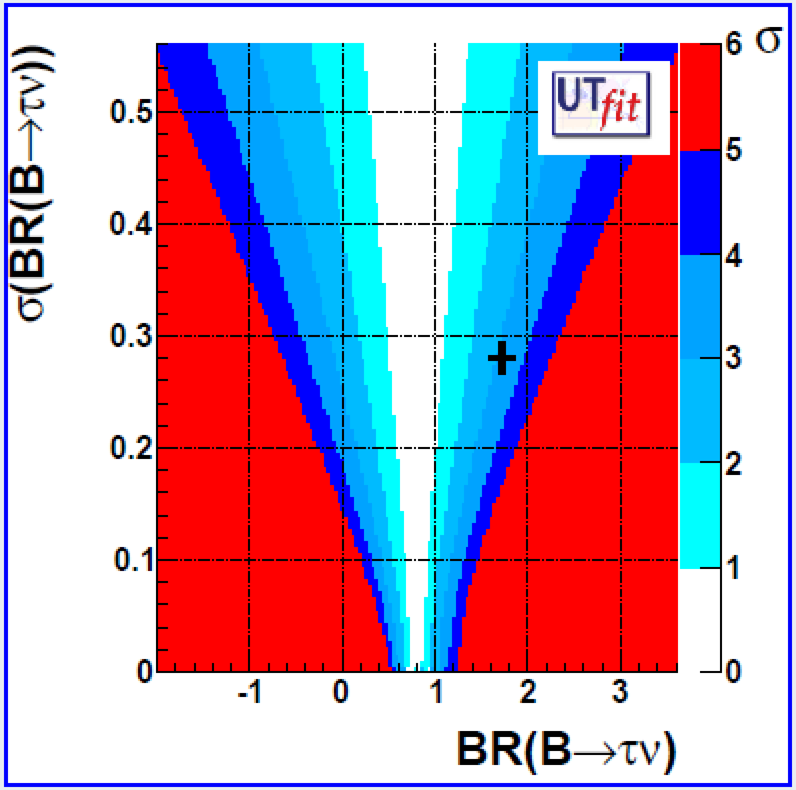
From a closer look

From the UTA (excluding its exp. constraint)

	Prediction	Measurement	Pull
$\sin 2\beta$	0.771±0.036	0.654±0.026	2.6 ←
γ	69.6°±3.1°	74°±11°	<1
α	85.4°±3.7°	91.4°±6.1°	<1
$\|V_{cb}\| \cdot 10^3$	42.69±0.99	40.83±0.45	+1.6
$\|V_{ub}\| \cdot 10^3$	3.55±0.14	3.76±0.20	<1
$BR(B \to \tau\nu) \cdot 10^4$	0.805±0.071	1.72±0.28	-3.2 ←

UTfit

$BR(B \to \tau \nu)_{SM} = (0.805\ \ 0.071) \cdot 10^{-4}$
[UTfit, update of 0908.3470]
turns out to be smaller by ~3.2 σ than the experimental value
$BR(B \to \tau \nu)_{exp} = (1.72 \pm 0.28) \cdot 10^{-4}$



The experimental state of the art

BaBar Semileptonic tag (0912.2453)
BaBar Hadronic tag (0708.2260, 1008.0104)

Belle Semileptonic tag (1006.4201)
Belle Hadronic tag (hep-ex/0604018)

$$BR(B \to \tau\nu) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

- **$BR(B \to \tau \nu)_{exp}$ prefers a large value for $|V_{ub}|$** (f_B under control and improved by the UTA)
- **But a shift in the central value of $|V_{ub}|$ would not solve the β tension**

→ **the debate on V_{ub} (excl. vs incl, various models…) is not enough to explain all**

The UTA beyond the Standard Model

UTfit

Model-independent UTA: bounds on deviations from the SM (+CKM)

- Parametrize generic NP in $\Delta F=2$ processes, in all sectors
- Use all available experimental info
- Fit simultaneously the CKM and NP parameters

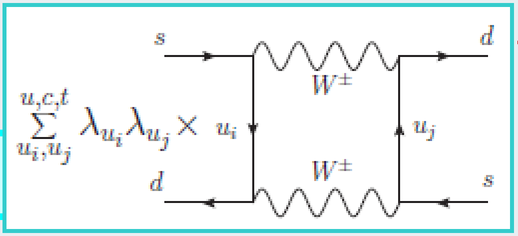
From this (NP) analysis:

$$\bar\rho = 0.135 \pm 0.040$$
$$\bar\eta = 0.374 \pm 0.026$$

In good agreement with the results from the SM analysis

$$\bar\rho = 0.132 \pm 0.020$$
$$\bar\eta = 0.358 \pm 0.012$$

NP contributions in the mixing amplitudes:



$$H^{\Delta F=2} = m + \frac{i}{2}\Gamma \qquad A = m_{12} = \langle M | m | \bar M \rangle \qquad \Gamma_{12} = \langle M | \Gamma | \bar M \rangle$$

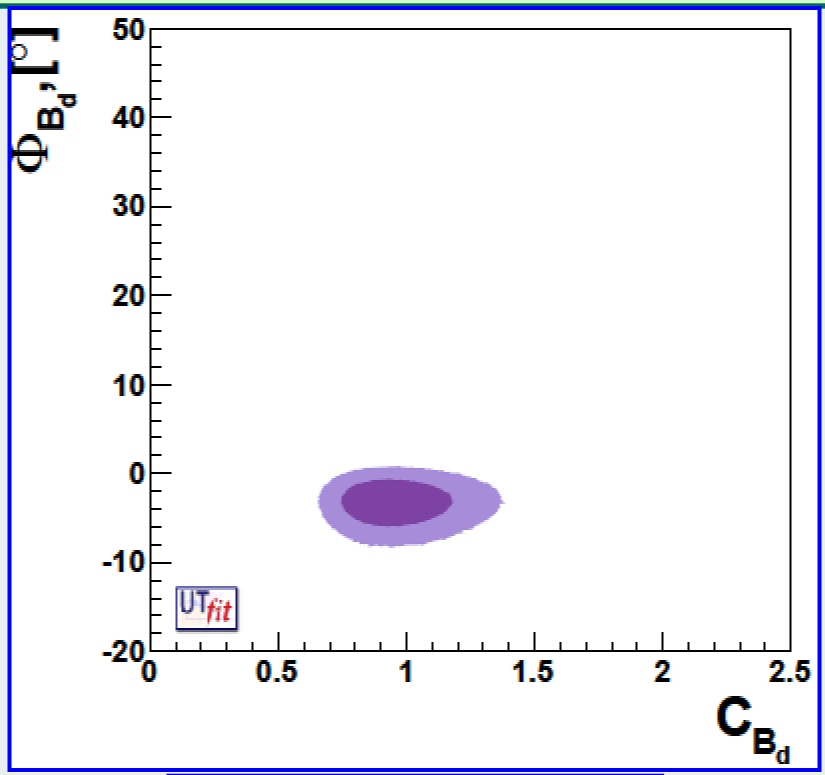
K mixing amplitude (2 real parameters):

$$\mathrm{Re}\, A^K = C_{\Delta m_K} \mathrm{Re}\, A_K^{SM} \qquad \mathrm{Im}\, A_K = C_{\varepsilon_K} \mathrm{Im}\, A_K^{SM}$$

B_d and B_s mixing amplitudes (2+2 real parameters):

$$A_q e^{2i\phi_q} = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$

For B_d-$\bar{B}_d$ mixing,
the mixing phase ϕ_{Bd} is found
1.8 σ away from the SM expectation
(reflecting the tension in $\sin 2\beta$)



$$C_{B_d} = 0.95 \pm 0.14$$
$$[0.70, 1.27] \leftrightarrow 95\%$$
$$\Phi_{B_d} = (-3.1 \pm 1.7)^\circ$$
$$[-7.0, 0.1]^\circ \leftrightarrow 95\%$$

Results for the B_s mixing amplitude:

UTfit

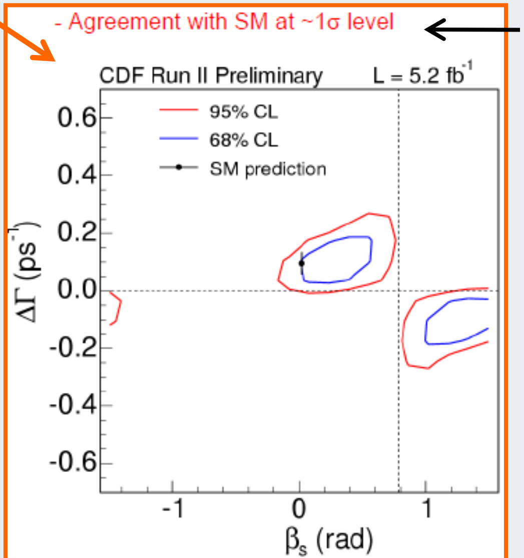
In 2009, by combining CDF and DØ results for ϕ_{Bs}:

UTfit: 2.9σ (update of 0803.0659)
HFAG: 2.2σ (0808.1297)
CKMfitter: 2.5σ (0810.3139)
Tevatron B w.g.: 2.1σ (http://tevbwg.fnal.gov)

More than 2σ deviation for every statistical approach!

In 2010, two surprising news:

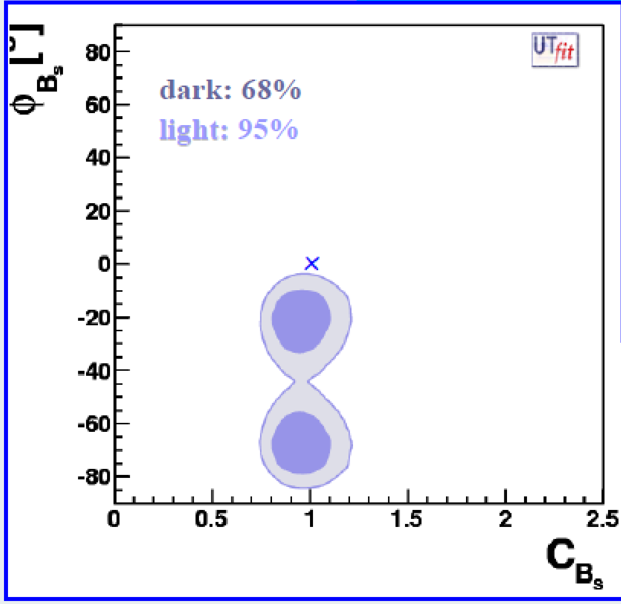
The 2010 CDF measurement reduces the significance of the deviation.
The likelihood is not yet available.



Before it was $1.8\ \sigma$

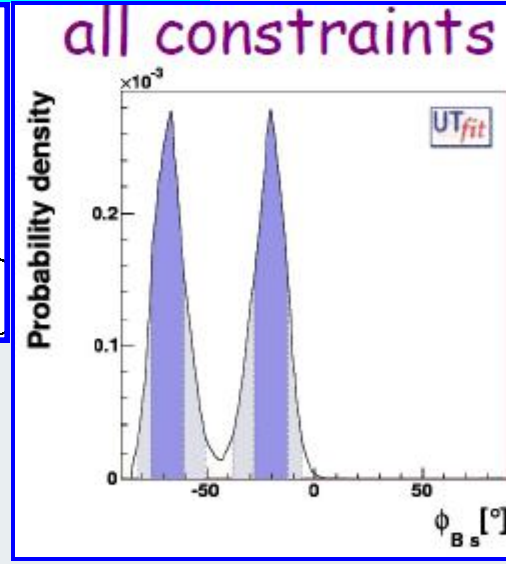
The DØ measurement of $a_{\mu\mu}$ points to large β_s but also to large $\Delta\Gamma_s$ requiring a non-standard Γ_{12} ?!?!?
If confirmed, two (UNLIKELY) explanations:
- Huge (tree-level-like) NP contributions in Γ_{12} (a factor 2.5: why only in Γ_{12}??)
- Bad failure of the OPE in Γ_{12} (while in Γ_{11} (b-hadron lifetimes) works well)

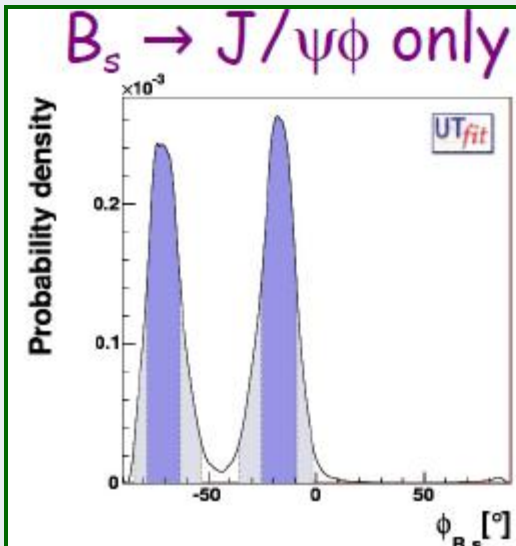
Updated Results including NEW DØ results
(new CDF results are not yet available)

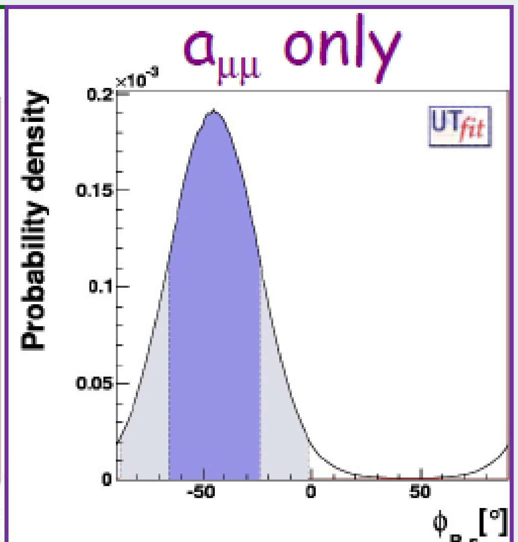


$$C_{B_s} = 0.95 \pm 0.10$$
$$([0.78, 1.16] \leftrightarrow 95\%)$$
$$\phi_{B_s} = (-20 \pm 8)^\circ \cup (-68 \pm 8)^\circ$$
$$([-38, -6]^\circ \cup [-81, -51]^\circ \leftrightarrow 95\%)$$

Deviation from the SM at 3.1σ







$a_{\mu\mu}$ and $B_s \to J/\Psi\, \phi$ point to large but different values of ϕ_{Bs} (N.B. the UTA beyond the SM allows for NP in loops only, i.e. tree-level NP in Γ_{12} is not allowed)

Further confirmations from experiments are looked forward!

(They are golden modes for theorists)

Theorist's Golden Modes (in heavy flavour)

Suppression within the SM ⇒ **Sensitivity to NP**

- **FCNCs forbidden at tree-level in the SM**
(radiative and rare decays: $b \to (s,d)\,\gamma$, $b \to (s,d)\,l^+l^-$, $b \to s\nu\bar{\nu}$, $B_{d,s} \to l^+l^-$,…)

- **CKM-, helicity-suppression**
(semileptonic CP-asymmetry: A^s_{SL}…, t-dep. CP-asymmetries: $A_{CP}(B \to K^*\gamma)$, and CP-asymmetries in D^0-$\bar{D}^0$ system)

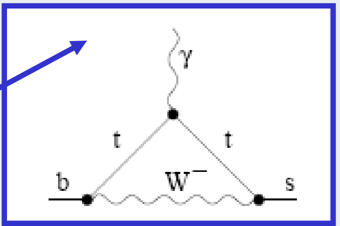
Small hadronic uncertainties ⇒ **Theoretically clean**

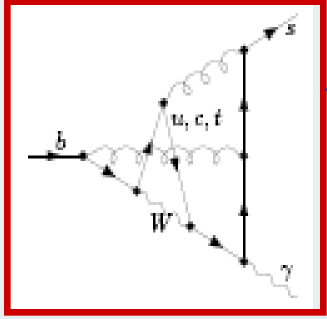
- **At most one hadron in the final state**
(leptonic and semileptonic decays: $B_{d,s} \to l^+ l^-$, $b \to (s,d)\,l^+l^-$, $b \to s\nu\bar{\nu}$,…)
- **Smearing of bound-effects in the final state**
(Inclusive quantities: lifetimes, ΔM_q, $\Delta\Gamma_q/\Gamma_q$, A^q_{SL}, ϕ_s,…)
- **Suppression/cancellation of some hadronic uncertainties**
(clean dominant contributions, peculiar ratios/correlations: $A_{CP}(B \to J_\psi K_S)$, $\Delta M_s/\Delta M_d$,…)

Some Golden Modes from a closer look

$b \to s\gamma$ (inclusive)

- highly sensitive to NP (loop FCNC)
- theoretically well under control
- with a small experimental error





$$Br(B \to X_s\gamma)^{exp}_{E_\gamma > 1.6GeV} = (3.55 \pm 0.24 \pm 0.09)10^{-4}$$

[HFAG, 2010]

NNLO QCD correct. to Wilson coefficients
+ RG resummation of large logs $\ln(M_W^2/m_b^2)$
[M.Misiak et al., hep-ph/0609232]

$$Br(B \to X_s\gamma)^{SM}_{E_\gamma > 1.6GeV} = (3.15 \pm 0.23)10^{-4}$$

Benzke, Lee, Neubert, Paz (1003.5012):
there are $1/m_b$ effects beyond OPE
(photon conversion into light partons),
which imply a 4% irreducible uncertainty)

$b \to s\gamma$ (exclusive)

Theoretical predictions require QCD factorization:

$Br(B \to K^*\gamma)$ is theoretically cleaner than $Br(B \to \rho\gamma)$,
where $O(\Lambda_{QCD}/m_b)$ corrections turn out to be relevant

Interesting exclusive observables
are the t-dep. CP-asymmetries $A_{CP}(B \to V\gamma)$:
- they are (helicity) suppressed within the SM ~O(1%)
- their observation would be a clear signal of NP

$B_q \to l^+l^-$

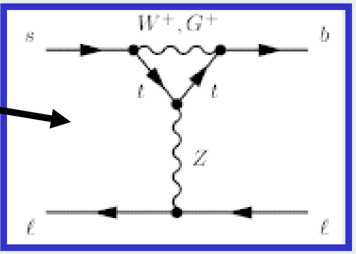
The $\mu^+\mu^-$ modes are experimentally the best:
- e^+e^- is m_e^2/m_μ^2 suppressed
- $\tau^+\tau^-$ has at least other two missing ν from decaying τ's

Upper bounds have been recently improved

$Br(B_d \to \mu^+\mu^-) < 7.6 \cdot 10^{-9}$
$Br(B_s \to \mu^+\mu^-) < 4.3 \cdot 10^{-8}$

(95% C.L)
[CDF, summer 2010]

- Highly sensitive to NP
  (loop FCNC: Z-penguin dominated)
- Theoretically clean (purely leptonic)



$\Rightarrow$ $Q_A = (\bar{b}_L \gamma^\mu q_L)\,(\bar{l}\gamma^\mu\gamma^5 l)$

Wilson coeff. at NLO in QCD
[Misiak, J.Urban hep-ph/9901278,
G.Buchalla, A.J.Buras hep-ph/9901288]

$Br(B_d \to \mu^+\mu^-)^{SM} = (1.0 \pm 0.1) \cdot 10^{-10}$
$Br(B_s \to \mu^+\mu^-)^{SM} = (3.2 \pm 0.2) \cdot 10^{-9}$

There is still a lot of room for NP

$b \to (s,d)\ l^+l^-$

- Highly sensitive to NP (loop FCNC)
- Main SM contribution from em dipole operator (Q_7^γ),
  and ew penguin operators (Q^9, Q^{10})
- Close to the charm threshold (long-distance) $c\bar{c}$ resonances appear

Inclusive: $B \to X_s\, l^+ l^-$

- Wilson coefficients at NNLO in QCD
  [C.Bobeth et al., hep-ph/0312090, M.Gorban, U. Haisch, hep-ph/0411071]
- HQE at $O(\Lambda^2_{QCD}/m_c^2)$, $O(\Lambda^2_{QCD}/m_b^2)$, $O(\Lambda^3_{QCD}/m_b^3)$
- QED corrections
- bremmstrahlung effects

[T.Huber at al., hep-ph/0512066]

$$Br(\bar{B} \to X_s \mu^+\mu^-)^{SM} = (1.59 \pm 0.11)\cdot 10^{-6}$$
$$Br(\bar{B} \to X_s e^+e^-)^{SM} = (1.64 \pm 0.11)\cdot 10^{-6}$$

$1\,GeV^2 < m^2_{ll} < 6\,GeV^2$

Sensitive to the interference of the Wilson coefficients C_7 and C_9

The forward-backward asymmetry (A_{FB}) is sensitive to C_7C_{10} and C_9C_{10}

Exclusive: $B \to K^*\, l^+ l^-$

The most interesting observables are:

- $d\Gamma^2 / (dq^2\, d\cos\theta_l)$ → extraction of C_9/C_7 and C_{10}/C_7 sensitive to NP in $C_{9,10}$
- A_{FB} and its zero q_0^2: main source of uncertainty → hadronic inputs
- A_I (isospin asymmetry between neutral and charged B): Small within the
- Muon to electron ratio: $R_H \equiv \int_{q_1}^{q_2} dq^2 \frac{d\Gamma(B \to H\mu^+\mu^-)}{dq^2} \Big/ \int_{q_1}^{q_2} dq^2 \frac{d\Gamma(B \to He^+e^-)}{dq^2}, \qquad H = \{K, K^*\}$

Inclusive: $B \to X_s\, \nu \bar{\nu}$

- Highly sensitive to NP
  (loop FCNC: Z-penguin and box dominated)
- Theoretically very clean
- It could provide the cleanest determination of $|V_{td}/V_{ts}|$

…and D-Physics…

W.r.t. B-Physics, **long-distance** contributions can be **important.**
Within the OPE: the expansion parameter Λ_{QCD}/m_c is not as small as Λ_{QCD}/m_b and $\alpha_s(m_c)>\alpha_s(m_b)$

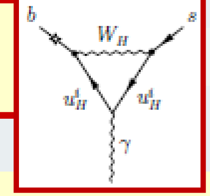
$$x_D = \frac{\Delta M_D}{\bar{\Gamma}}, \qquad y_D = \frac{\Delta \Gamma_D}{2\bar{\Gamma}}$$

$x_D \sim y_D \sim$ (0.5-1)% (BaBar+Belle)
Is it compatible with the SM?
Difficult answer due to large uncertainties
(long-distance contributions)

…but, there is a corner to search for NP: CP-violation

- No sign of CP-violation has been observed yet
- Within the SM it is expected to be very small (M_{12}^D, Γ_{12}^D real to a good approx.)
  (mixing induced CP-asymmetry (rough estimate):~ $x_D \sin \phi_f$) → $\arg\frac{q}{p}$
- A measured value larger than **1%** would be a NP signal

Beyond the SM



within MFV

$$H_{eff}^{NP} = \frac{G_F}{\sqrt{2}} \bullet \left[V_i^{CKM} \left(C_i^{NEW}(\mu)\, Q_i(\mu) + C_i^{MFV}(\mu)\, Q_i^{MFV}(\mu) \right) + V_i^{non\text{-}MFV} C_i^{non\text{-}MFV}(\mu)\, Q_i^{non\text{-}MFV}(\mu) \right]$$

beyond MFV

Minimal Flavour Violation (MFV): supported by the great success of the SM in Flavour Physics!

the SM Yukawa couplings are the only *building-blocks* of flavour violation

[G.D´Ambrosio et al., hep-ph/0207036], [A.J. Buras et al., hep-ph/0007085]

MFV models can also contain flavour blind CP-violating phases (FBPs) which make the C^i complex, with significant implications for phenomenology (interplay between FBPs and the CKM phase, e.g. in Flavour Blind MSSM)

Beyond MFV:
New sources of flavour violation ($V_i^{non\text{-}MFV}$) can appear

New operators (Q_i^{MFV}, $Q_i^{non\text{-}MFV}$) can appear

Some Appealing NP Models

(where extensive analyses have been performed, with Wilson coefficients calculated at LO in QCD)

MFV

- MSSM with MFV
- Appelquist-Cheng-Dobrescu (ACD) Model
- Littlest Higgs model without T-parity
- …

MFV+FBPs

- Flavour blind MSSM (FBMSSM)
- 2Higgs doublet model with FBPs ($2HDM_{\overline{MFV}}$)
- …

Beyond MFV

- SUSY flavour models (δLL, AC, RVV2, AKM)
- SUSY-GUTs ($SSU(5)_{RN}$, SO(10) CMM)
- SUSY with effective MFV
- Littlest Higgs with T-Parity (LHT)
- Z'-models
- Randall-Sundrum model with custodial protection
- SM with a 4th generation (SM4)
- Composite Higgs models
- RHMFV
- Left-right symmetric models
- …

DNA of Flavour Physics

by Andrzej Buras **(1012.1447)**

★★★ Large NP effects

★★ Small-moderate NP effects

★ Invisible NP effects

SUSY models

	AC	RVV2	AKM	δLL	FBMSSM	$SSU(5)_{\mathrm{RN}}$
$D^0 - \bar{D}^0$	★★★	★	★	★	★	★
ϵ_K	★	★★★	★★★	★	★	★★★
$S_{\psi\phi}$	★★★	★★★	★★★	★	★	★★★
$S_{\phi K_S}$	★★★	★★	★	★★★	★★★	★★
$A_{\mathrm{CP}}(B \to X_s\gamma)$	★	★	★	★★★	★★★	★
$A_{7,8}(K^*\mu^+\mu^-)$	★	★	★	★★★	★★★	★
$B_s \to \mu^+\mu^-$	★★★	★★★	★★★	★★★	★★★	★★★

Non-SUSY models

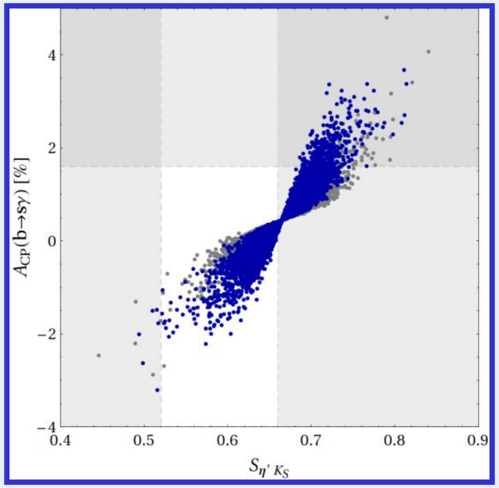
	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	★	★	★★		
$A_{\mathrm{CP}}(B \to X_s\gamma)$	★		★		
$A_{7,8}(K^*\mu^+\mu^-)$	★★	★	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★

Even more important are correlations

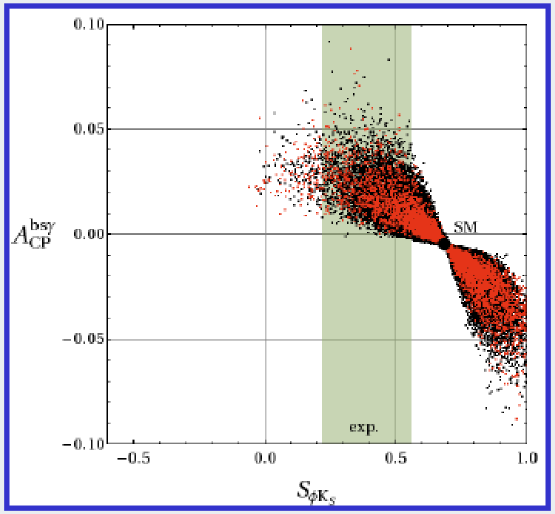
- less sensitive to model parameters
- useful to discriminate different models

Some examples:

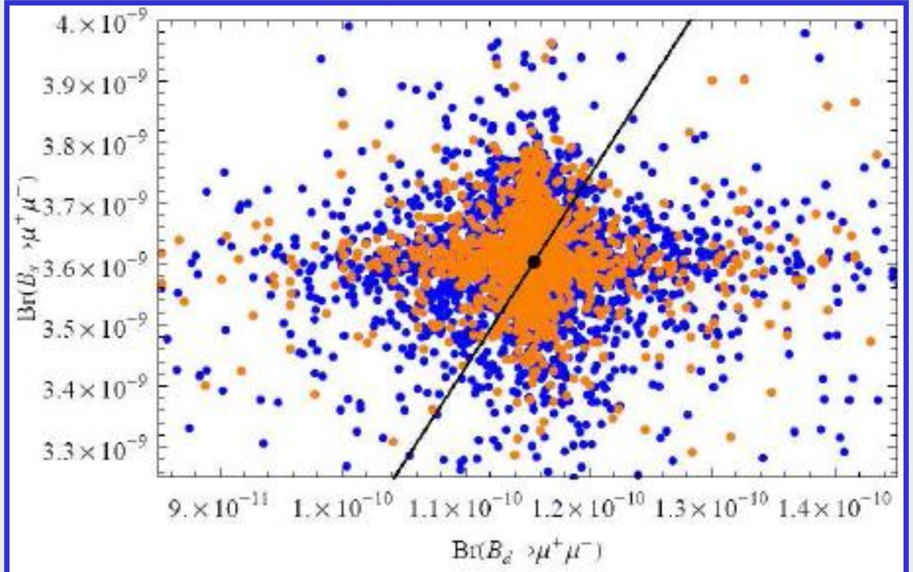
SUSY with Effective MFV
R. Barbieri et al. (1102.0726)



Flavor Blind MSSM
W. Almannshofer et al. (0808.0707)

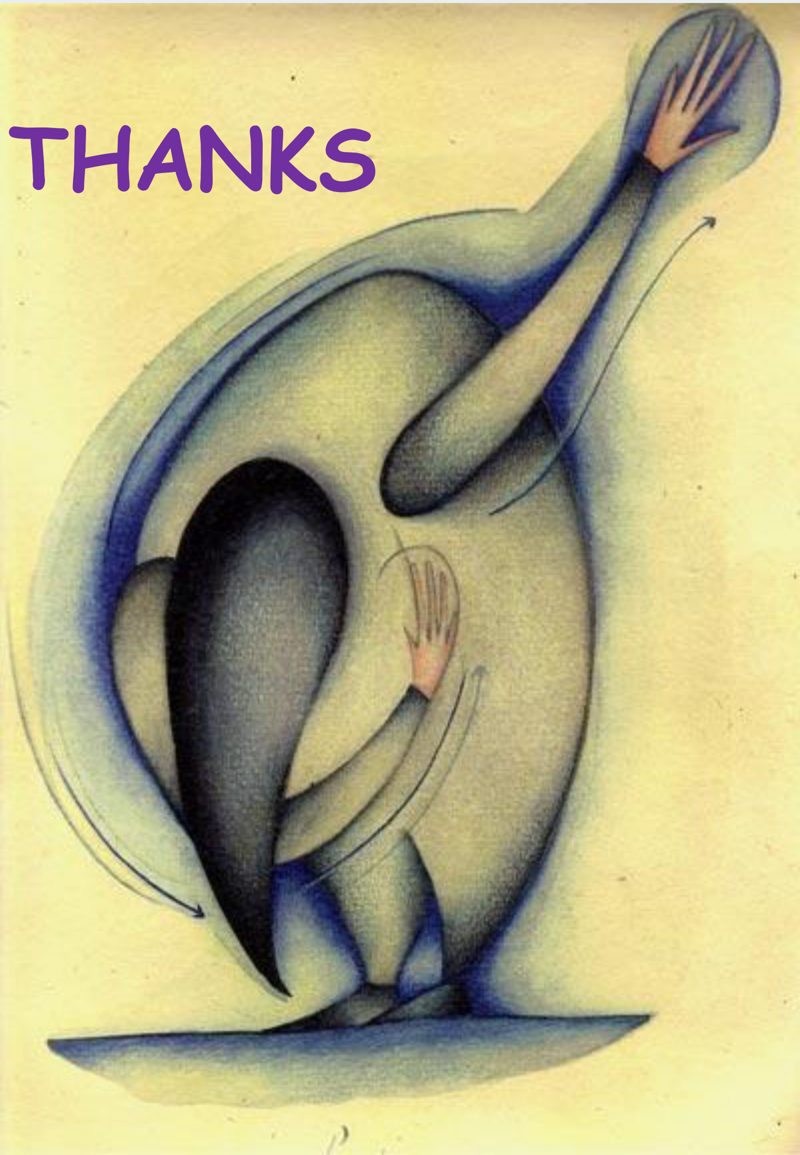


Randall Sundrum with custodial protection
M. Blanke et al. (0812.3803)



and many others…

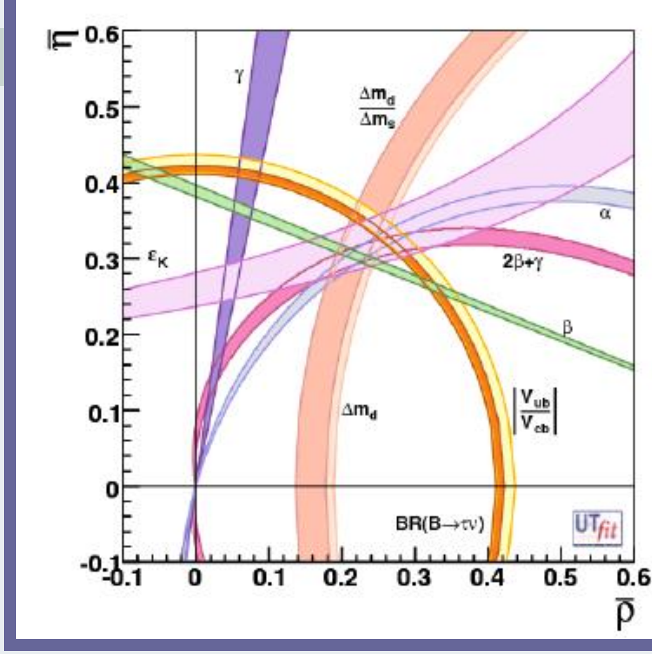
We are looking forward (and getting closer to) the experimental answers!



BACKUP

A look at the future

by Vittorio Lubicz



V.Lubicz @
Villa Mondragone
Monte Porzio Catone - Italy
13 - 15 November 2006

Hadronic matrix element	Lattice error in 2006	Lattice error in 2009	6 TFlop Year [2009]	60 TFlop Year [2011 LHCb]	1-10 PFlop Year [2015 SuperB]
$f_+^{K\pi}(0)$	0.9%	**0.5%**	0.7%	0.4%	**< 0.1%**
$\hat{B}_K$	11%	**5%**	5%	3%	**1%**
f_B	14%	**5%**	3.5 - 4.5%	2.5 - 4.0%	**1 – 1.5%**
$f_{Bs}B_{Bs}^{1/2}$	13%	**5%**	4 - 5%	3 - 4%	**1 – 1.5%**
ξ	5%	**2%**	3%	1.5 - 2 %	**0.5 – 0.8 %**
$\mathcal{F}_{B\to D/D^*l\nu}$	4%	**2%**	2%	1.2%	**0.5%**
$f_+^{B\pi}$,...	11%	**11%**	5.5 - 6.5%	4 - 5%	**2 – 3%**
$T_1^{B\to K^*/\rho}$	13%	**13%**	----	----	**3 – 4%**