



SAPIENZA
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and



INDIRECT SEARCH OF EXOTIC MESONS: $B \rightarrow J/\psi + \text{All}$

CHIARA SABELLI

Based on **arXiv:1104.1781** with
TJ Burns, F Piccinini, AD Polosa and V Prospero

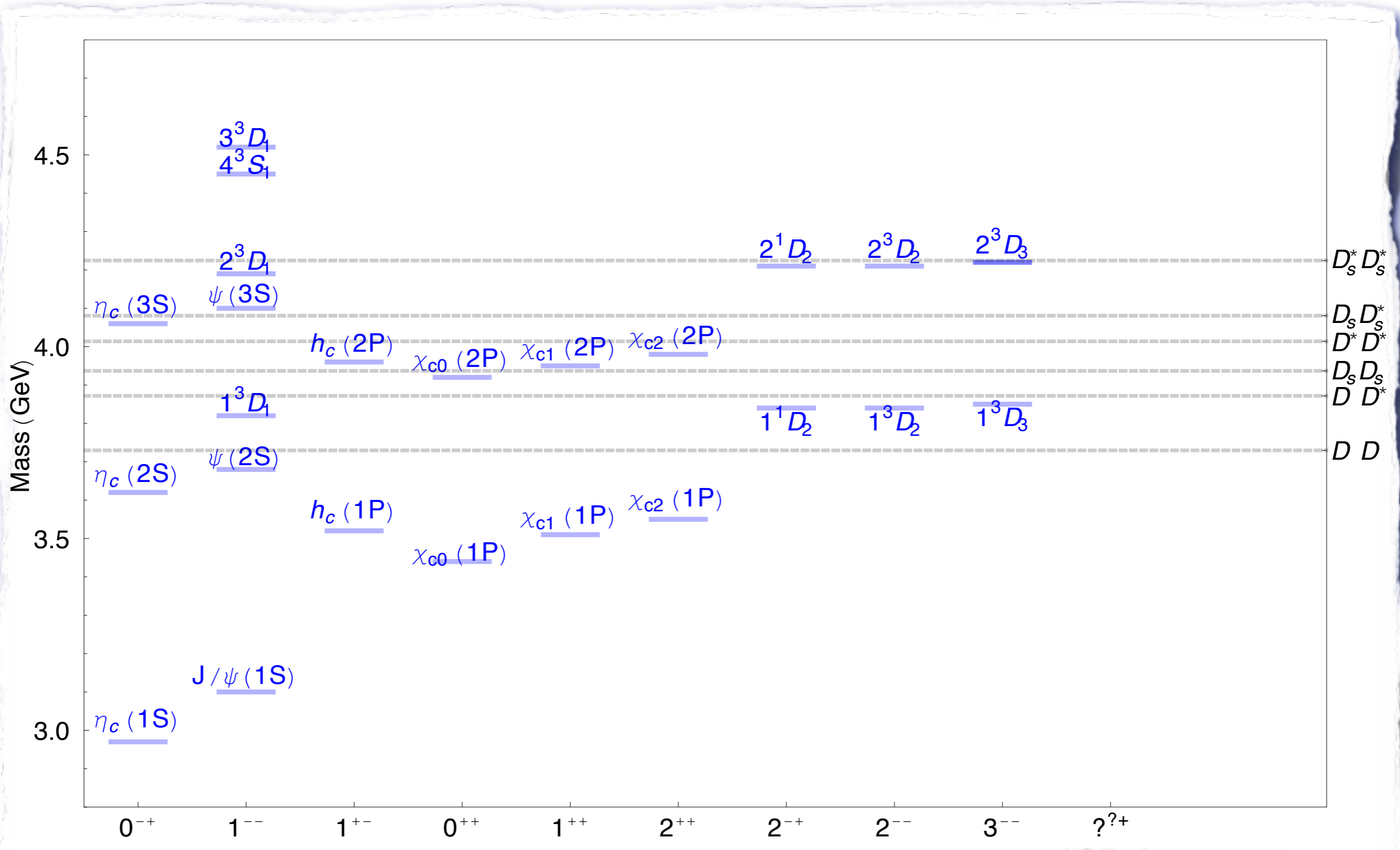
IFAE 2011 - Perugia, 27 Aprile 2011

Outline

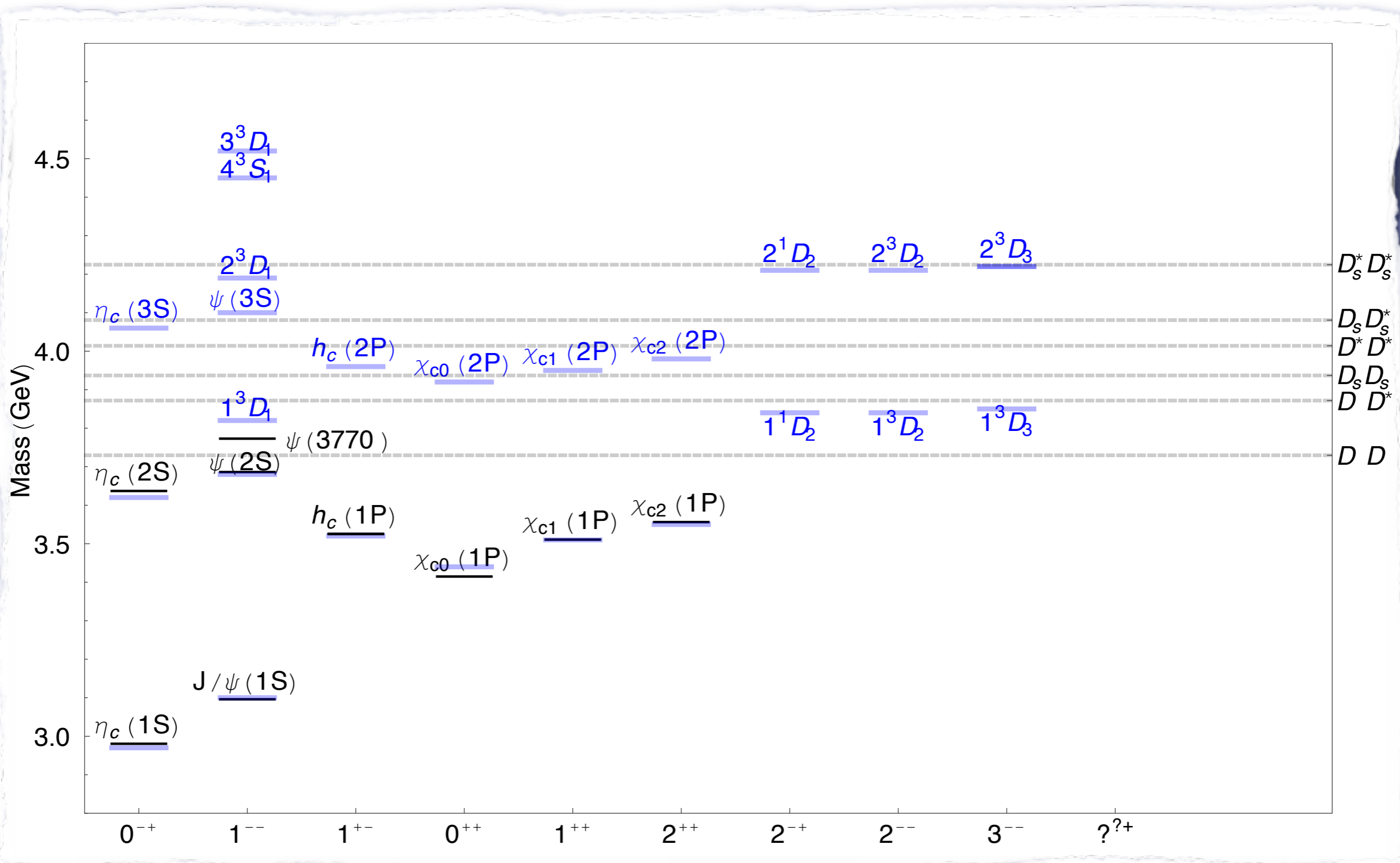
- ▶ Standard **Charmonium** and **Charmonium-like** resonances (**XYZ** mesons).
- ▶ **Exotic mesons**: molecules, tetraquarks, hybrids, hadrocharmonium.
- ▶ **Inclusive production of J/ ψ in B decays**: state of the art and **update**.
 - ▶ Two body modes (color singlet).
 - ▶ Non resonant multi body modes (color octet) NRQCD.
 - ▶ Contribution from XYZ mesons.
- ▶ **Results**.

Predictions for cc^* states

:: Godfrey, Isgur, Phys. Rev. D 32, 189–231 (1985) ::

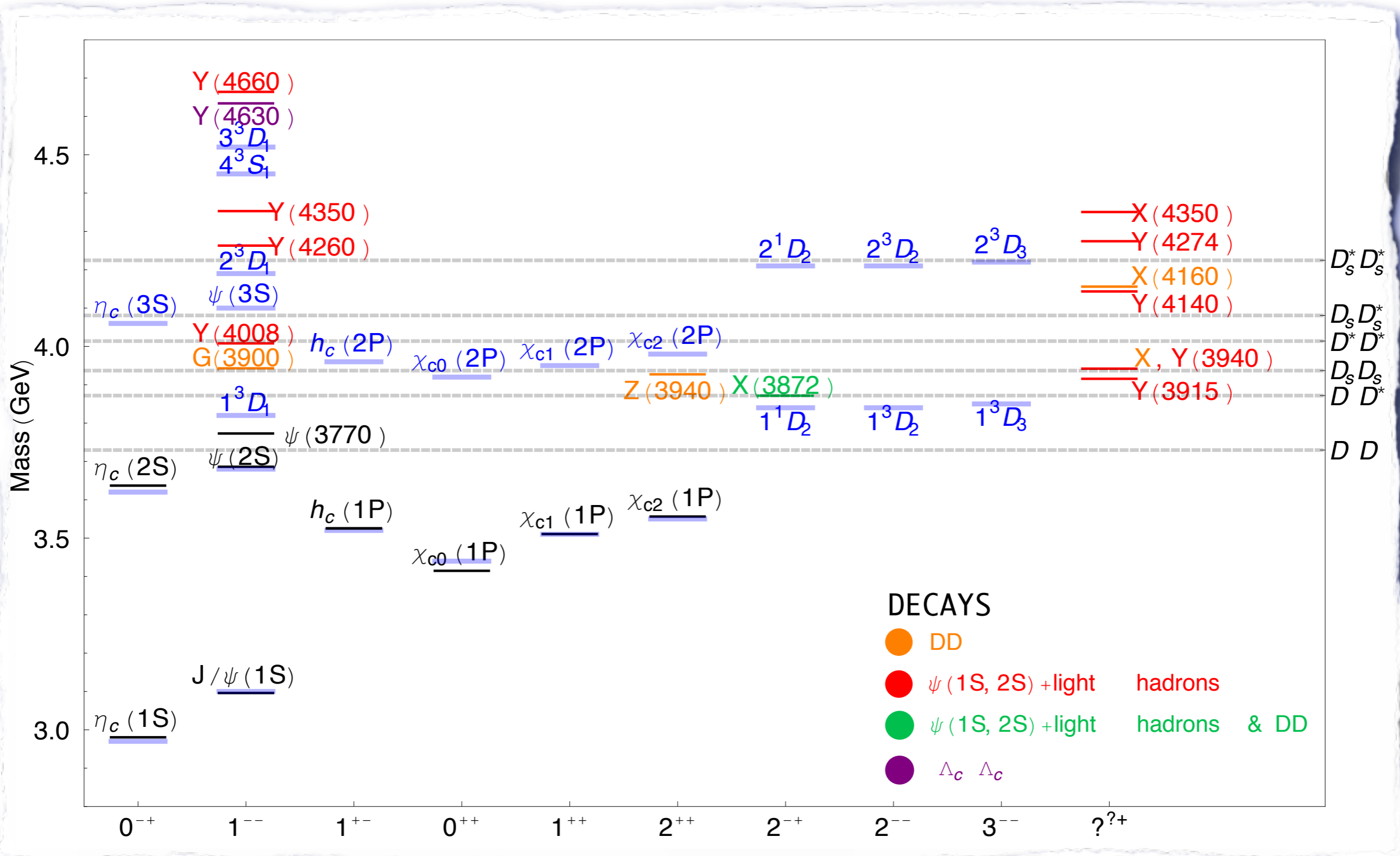


Data for cc^* states



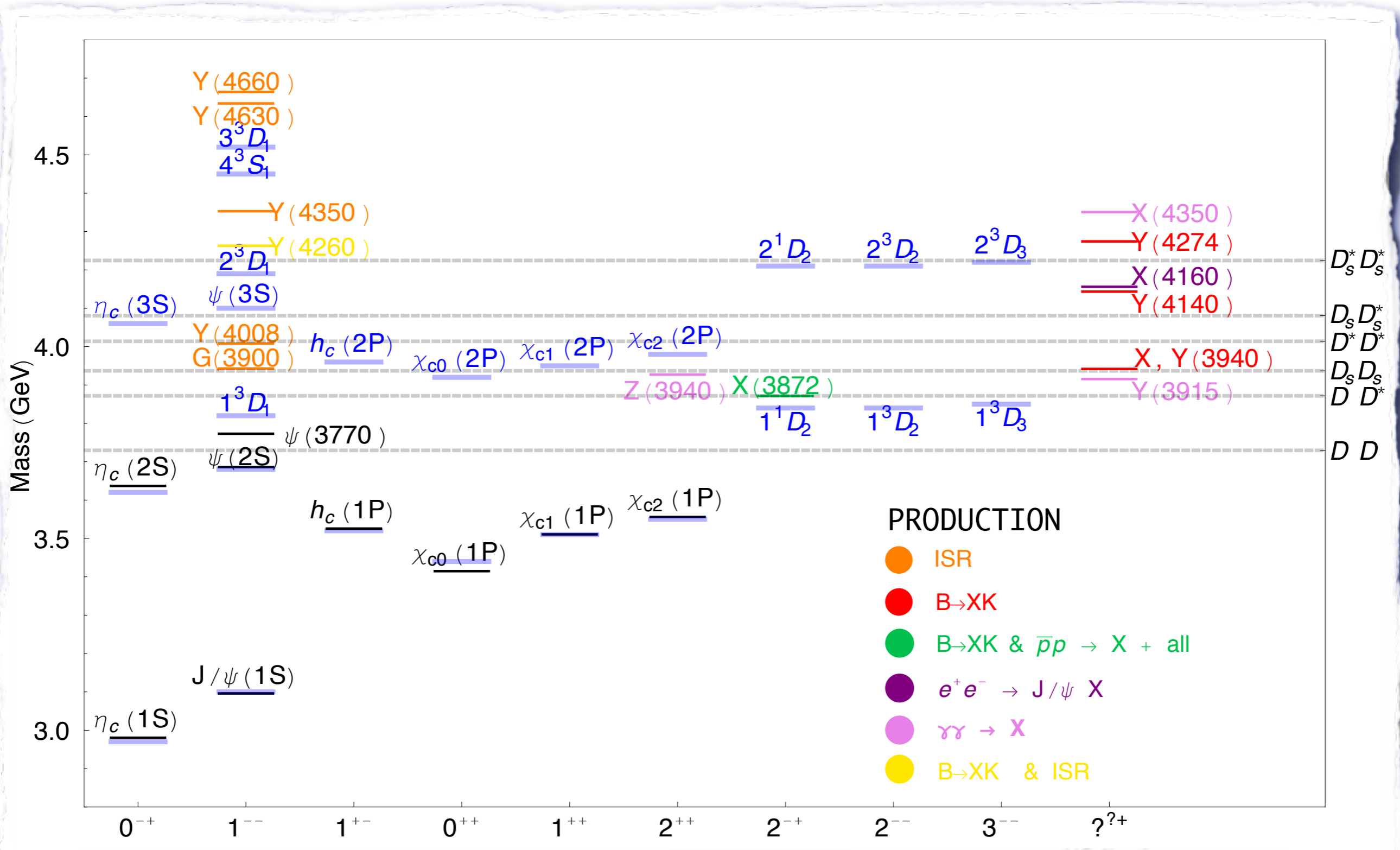
Unexpected (I)

:: Heavy Quarkonium Working Group, Eur.Phys.J. C71 (2011) 1534 ::



Unexpected (2)

:: Heavy Quarkonium Working Group, Eur.Phys.J. C71 (2011) 1534 ::



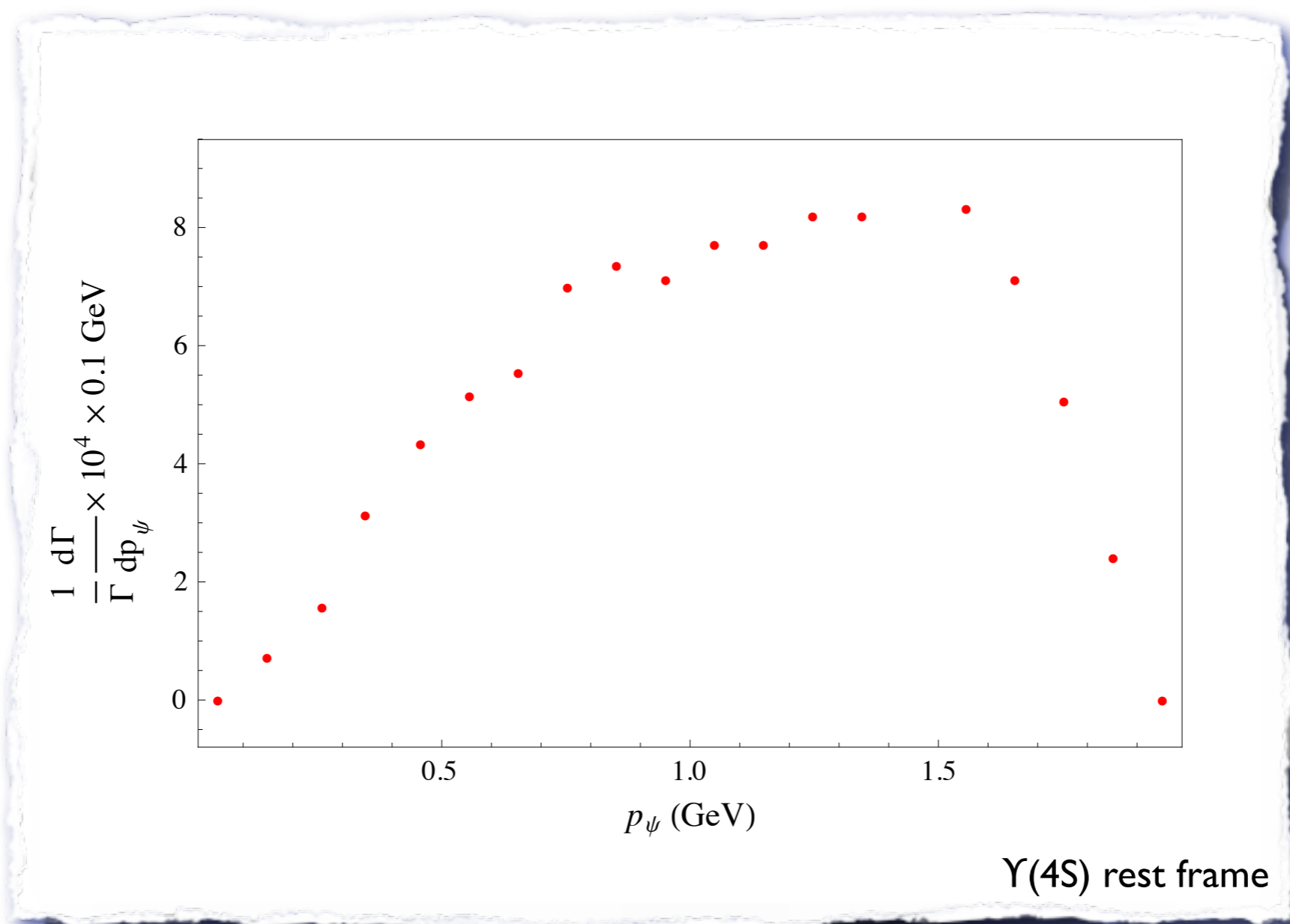
Are they exotic hadrons?

- ▶ Many **exotic candidates** have been identified among the so-called **XYZ particles**.
- ▶ Exotic means non qq^* or qqq structures ... **what else?**
 - ▶ **Strongly interacting clusters of hadrons: molecules**
[Voloshin; Tornqvist; Close; Braaten; Swanson...]
 - ▶ **Tetraquark mesons, Pentaquarks, ...**
[Maiani, Piccinini, Polosa, Riquer ...]
 - ▶ **Hybrids**
[Close, Kou&Pene, ...]
 - ▶ **Hadrocharmonium**
[Voloshin]
- ▶ XYZ can be **revealed directly** (peaks in invariant mass distributions).
- ▶ XYZ can be revealed **INdirectly** as **intermediate states** in a number of processes: e.g. heavy ions collision, inclusive B decays.

B \rightarrow J/ ψ + All

:: BaBar, Phys. Rev. D67,032002 ::

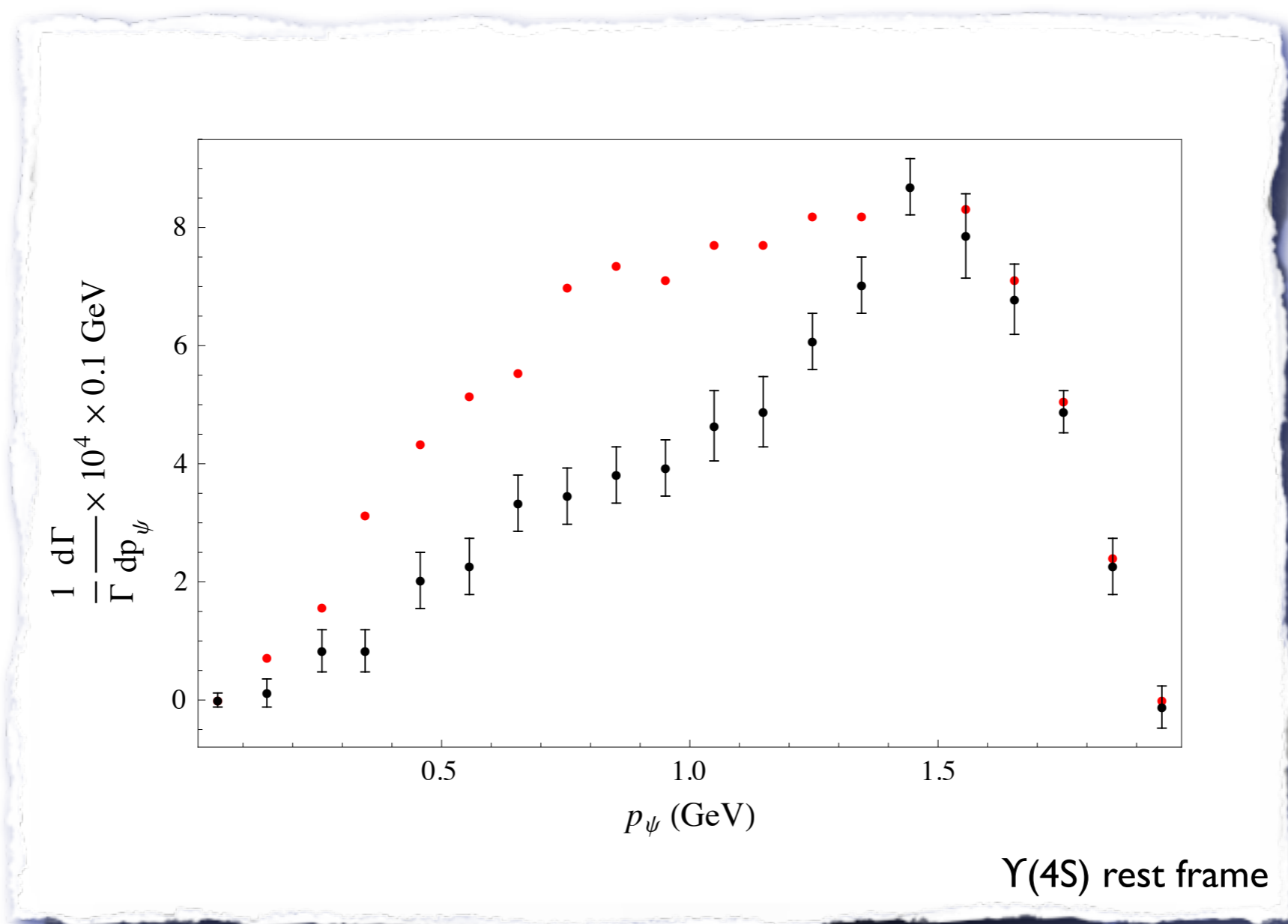
- ▶ In e^+e^- collisions at $\sqrt{s} \sim m_{\Upsilon(4S)}$ with 20.3 fb^{-1} they measure: **B \rightarrow J/ ψ + All**.
- ▶ In **B \rightarrow J/ ψ + All** there is a feed-down from **$\chi_{c1,2} \rightarrow \text{J}/\psi \gamma$** and **$\psi(2S) \rightarrow \text{J}/\psi \pi^+ \pi^-$** .



B \rightarrow J/ ψ + All : direct

:: BaBar, Phys. Rev. D67,032002 ::

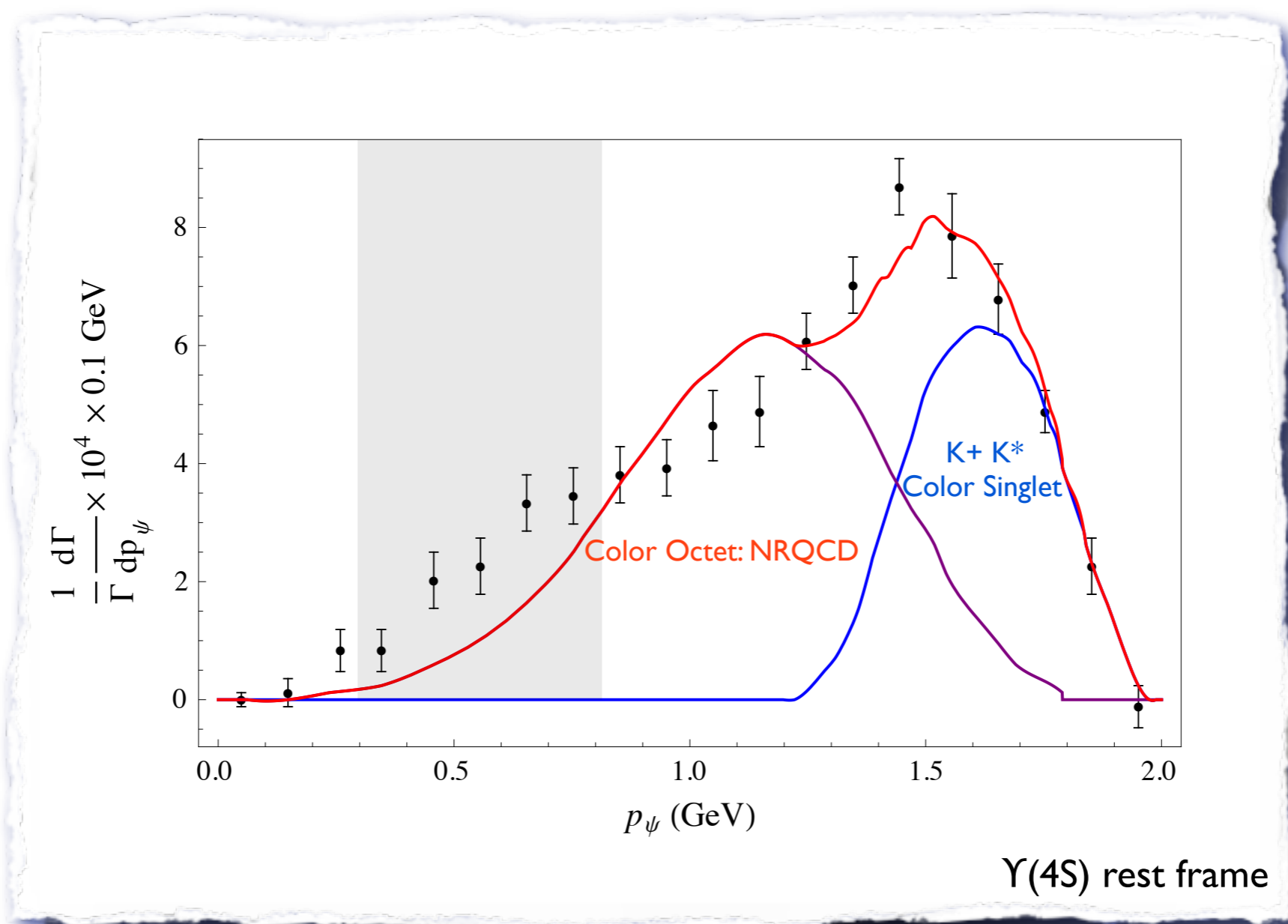
- Subtracting the feed-down from $\chi_{c1,2} \rightarrow J/\psi \gamma$ and $\psi(2S) \rightarrow J/\psi \pi^+ \pi^-$, they obtain the p^* decay distribution of J/ ψ produced directly in B decays.



B \rightarrow J/ ψ + All : direct

:: BaBar, Phys. Rev. D67,032002 ::

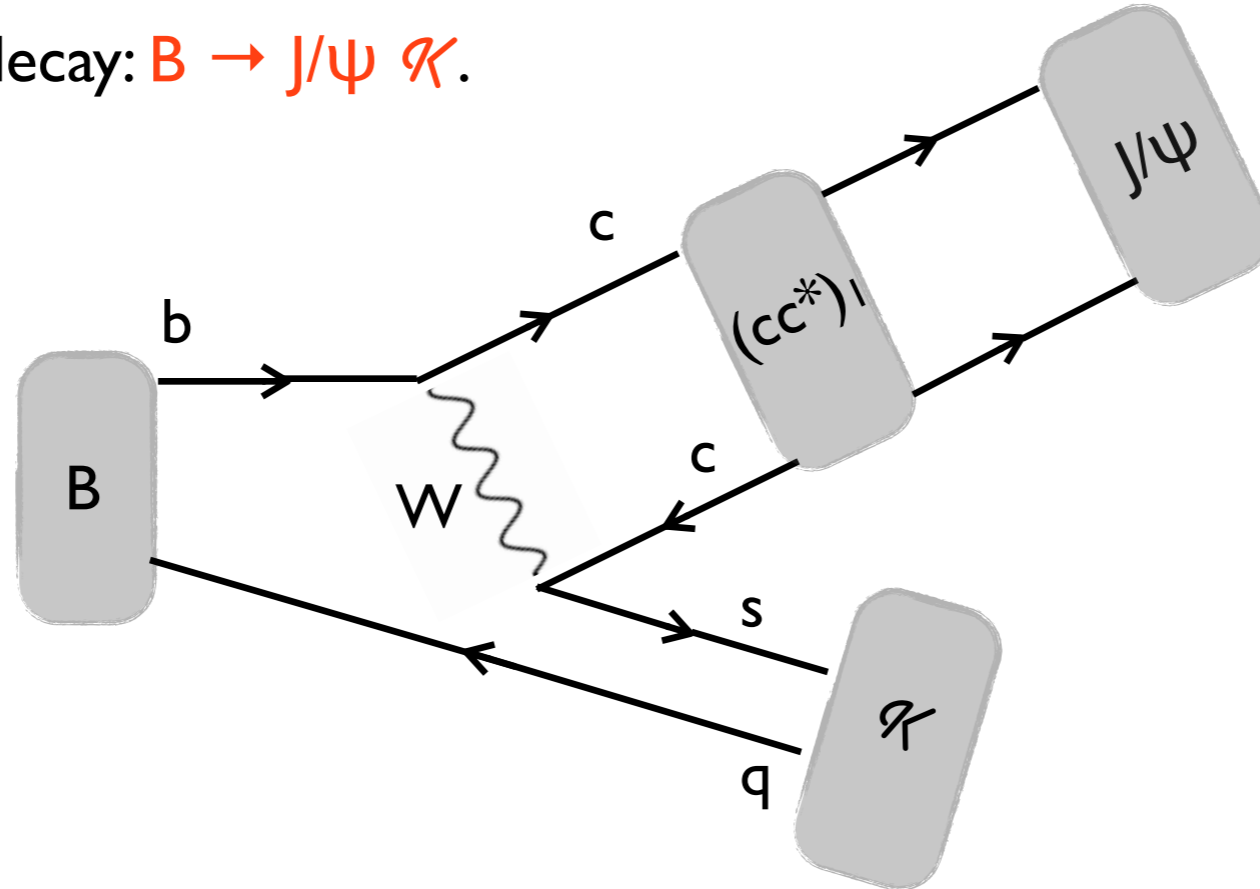
- ▶ Subtracting the feed-down from $\chi_{c1,2} \rightarrow J/\psi \gamma$ and $\psi(2S) \rightarrow J/\psi \pi^+ \pi^-$, they obtain the p^* decay distribution of J/ ψ produced directly in B decays.
- ▶ Theoretical predictions reveal an excess at low p_ψ .



B → J/ψ + K

:: K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010) ::

- ▶ If the cc* pair is produced in **color singlet** configuration one has a two body decay: **B → J/ψ K**.



B⁺ DECAY MODES

Fraction (Γ_i/Γ)

Charmonium modes

$J/\psi(1S)K^+$	$(1.007 \pm 0.035) \times 10^{-3}$
$J/\psi(1S)K^*(892)^+$	$(1.43 \pm 0.08) \times 10^{-3}$
$J/\psi(1S)K(1270)^+$	$(1.8 \pm 0.5) \times 10^{-3}$

B → J/ψ + K

PHYSICAL REVIEW D 83, 032005 (2011)

Study of the $K^+ \pi^+ \pi^-$ final state in $B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$ and $B^+ \rightarrow \psi' K^+ \pi^+ \pi^-$

(The Belle Collaboration)

J_1	Submode	Decay fraction
	Nonresonant $K^+ \pi^+ \pi^-$	$0.152 \pm 0.013 \pm 0.028$
1^+	$K_1(1270) \rightarrow K^*(892)\pi$	$0.232 \pm 0.017 \pm 0.058$
	$K_1(1270) \rightarrow K\rho$	$0.383 \pm 0.016 \pm 0.036$
	$K_1(1270) \rightarrow K\omega$	$0.0045 \pm 0.0017 \pm 0.0014$
	$K_1(1270) \rightarrow K_0^*(1430)\pi$	$0.0157 \pm 0.0052 \pm 0.0049$
	$K_1(1400) \rightarrow K^*(892)\pi$	$0.223 \pm 0.026 \pm 0.036$
1^-	$K^*(1410) \rightarrow K^*(892)\pi$	$0.047 \pm 0.016 \pm 0.015$
	$K_2^*(1430) \rightarrow K^*(892)\pi$	$0.088 \pm 0.011 \pm 0.011$
2^+	$K_2^*(1430) \rightarrow K\rho$	0.0233 (fixed)
	$K_2^*(1430) \rightarrow K\omega$	0.00036 (fixed)
	$K_2^*(1980) \rightarrow K^*(892)\pi$	$0.0739 \pm 0.0073 \pm 0.0095$
	$K_2^*(1980) \rightarrow K\rho$	$0.0613 \pm 0.0058 \pm 0.0059$
	$K(1600) \rightarrow K^*(892)\pi$	$0.0187 \pm 0.0058 \pm 0.0050$
2^-	$K(1600) \rightarrow K\rho$	$0.0424 \pm 0.0062 \pm 0.0110$
	$K_2(1770) \rightarrow K^*(892)\pi$	$0.0164 \pm 0.0055 \pm 0.0061$
	$K_2(1770) \rightarrow K_2^*(1430)\pi$	$0.0100 \pm 0.0028 \pm 0.0020$
	$K_2(1770) \rightarrow Kf_2(1270)$	$0.0124 \pm 0.0033 \pm 0.0022$
	$K_2(1770) \rightarrow Kf_0(980)$	$0.0034 \pm 0.0017 \pm 0.0011$



$$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi \rightarrow \mathcal{R}_i J/\psi \rightarrow J/\psi K^+ \pi^+ \pi^-) = \mathcal{B}_{\text{tot}} f_i^j$$

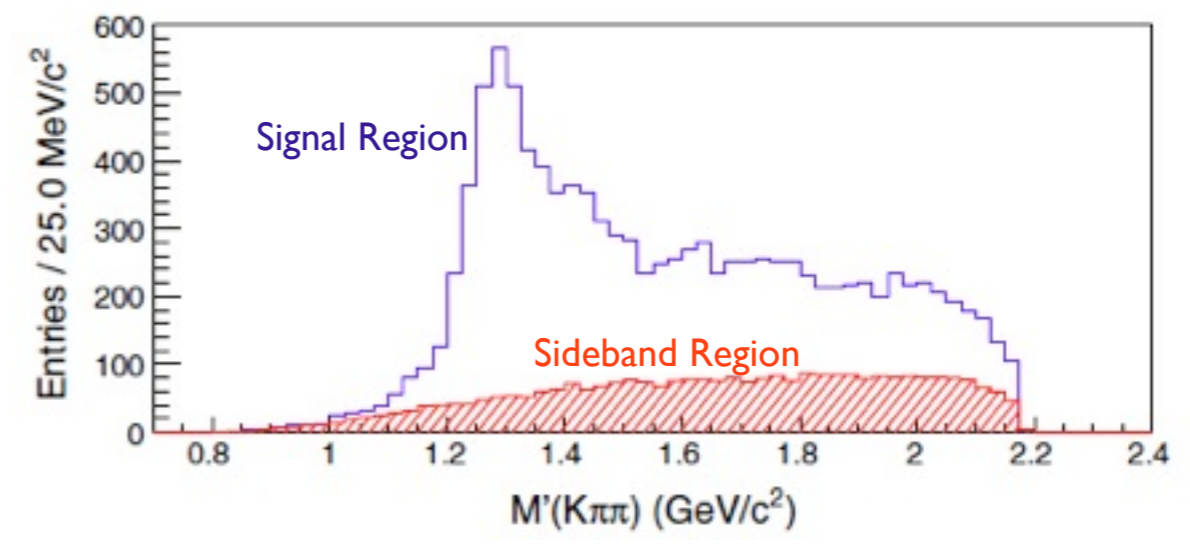


TABLE V. Fitted parameters of the signal function for $B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$, along with the corresponding decay fractions.

B → J/ψ + K

:: Burns, Piccinini, Polosa, Prospero, Sabelli, arXiv:1104.1781::

► From the fractions we compute the **two body branching ratios**

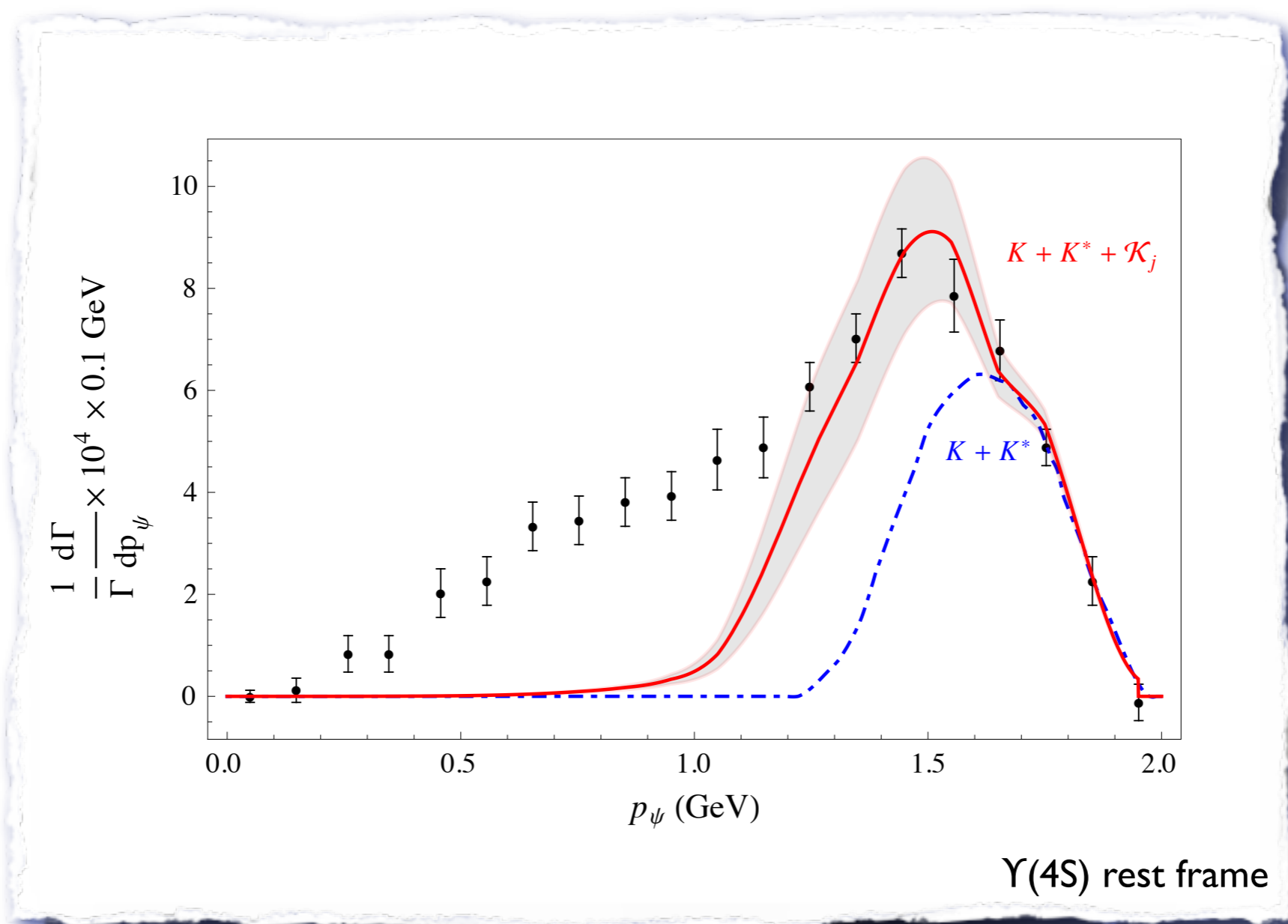
K_j	m_{K_j} (GeV)	Γ_{K_j} (GeV)	$\mathcal{B}(B^+ \rightarrow K_j J/\psi) \times 10^5$
K	0.494	–	95.0 ± 3.6
K^*	0.892	0.050	137.0 ± 7.8
$K_1(1270)$	1.270	0.090	144.0 ± 29.3
$K_1(1400)$	1.403	0.174	25.1 ± 5.7
$K^*(1410)$	1.414	0.232	$> 5.1 \pm 2.4$ and $< 11.8 \pm 5.7$
$K_2^*(1430)$	1.430	0.100	40.2 ± 24.0
$K_2(1600)$	1.605	0.115	$> 8.4 \pm 2.9$
$K_2(1770)$	1.773	0.186	$> 4.4 \pm 1.5$
$K_2(1980)$	1.973	0.373	$> 15.2 \pm 2.5$

*
*

* <http://hfag.phys.ntu.edu.tw/b2charm/index.html>

B \rightarrow J/ ψ + \mathcal{K}

- Two body contributions accounts for the high p_ψ region:
we found good agreement for $p_\psi > 1.2$ GeV.



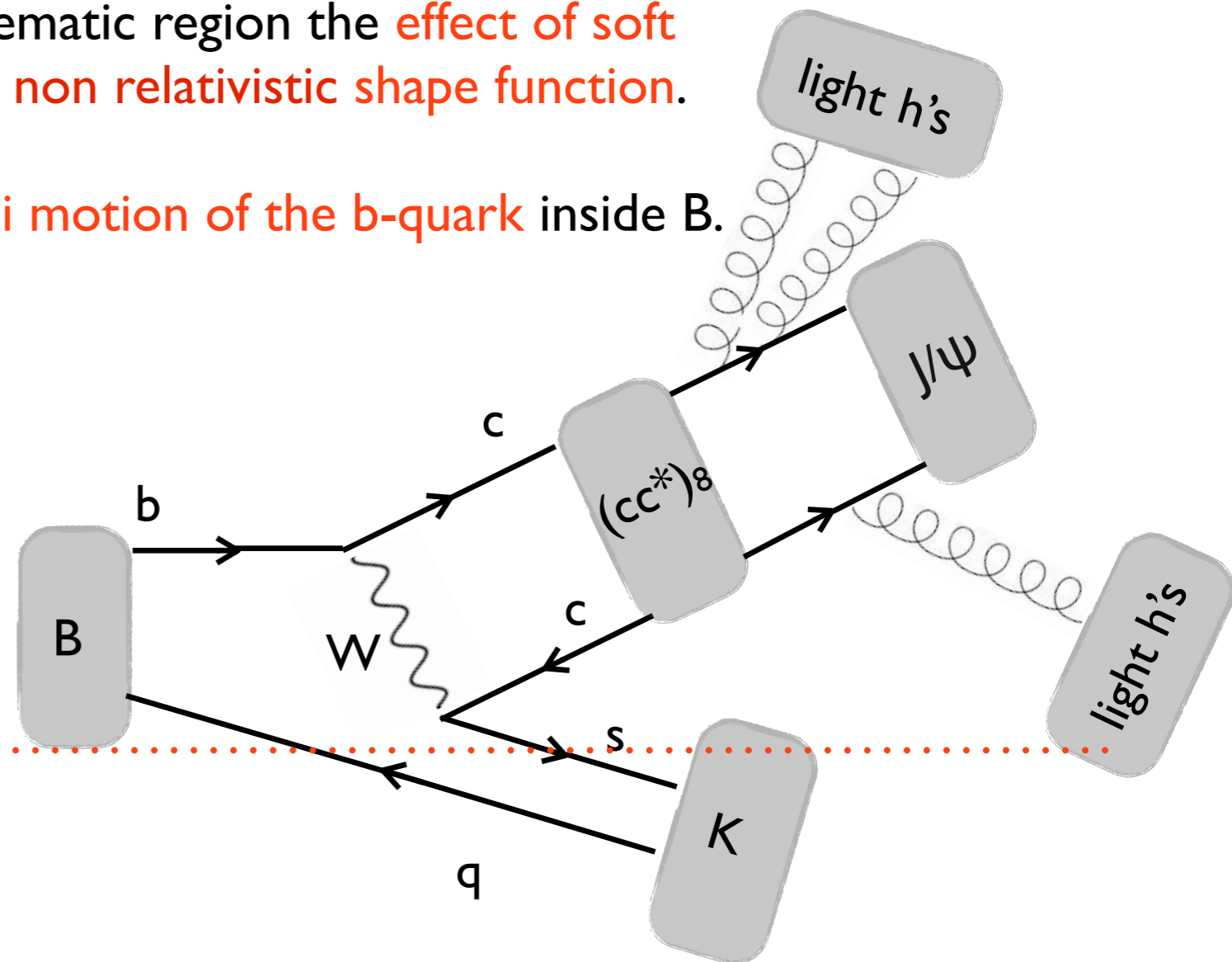
Color Octet Contribution

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- ▶ If the cc^* pair is produced in **color octet** configuration one has a **multi body** decay.
- ▶ **NRQCD matrix elements** describe the fragmentation $(cc^*)_8 \rightarrow J/\psi$.
- ▶ Near the extreme endpoint of the kinematic region the **effect of soft gluon emission** can be modelled with a **non relativistic shape function**.
- ▶ **ACCMM model** accounts for the **Fermi motion of the b-quark** inside B.

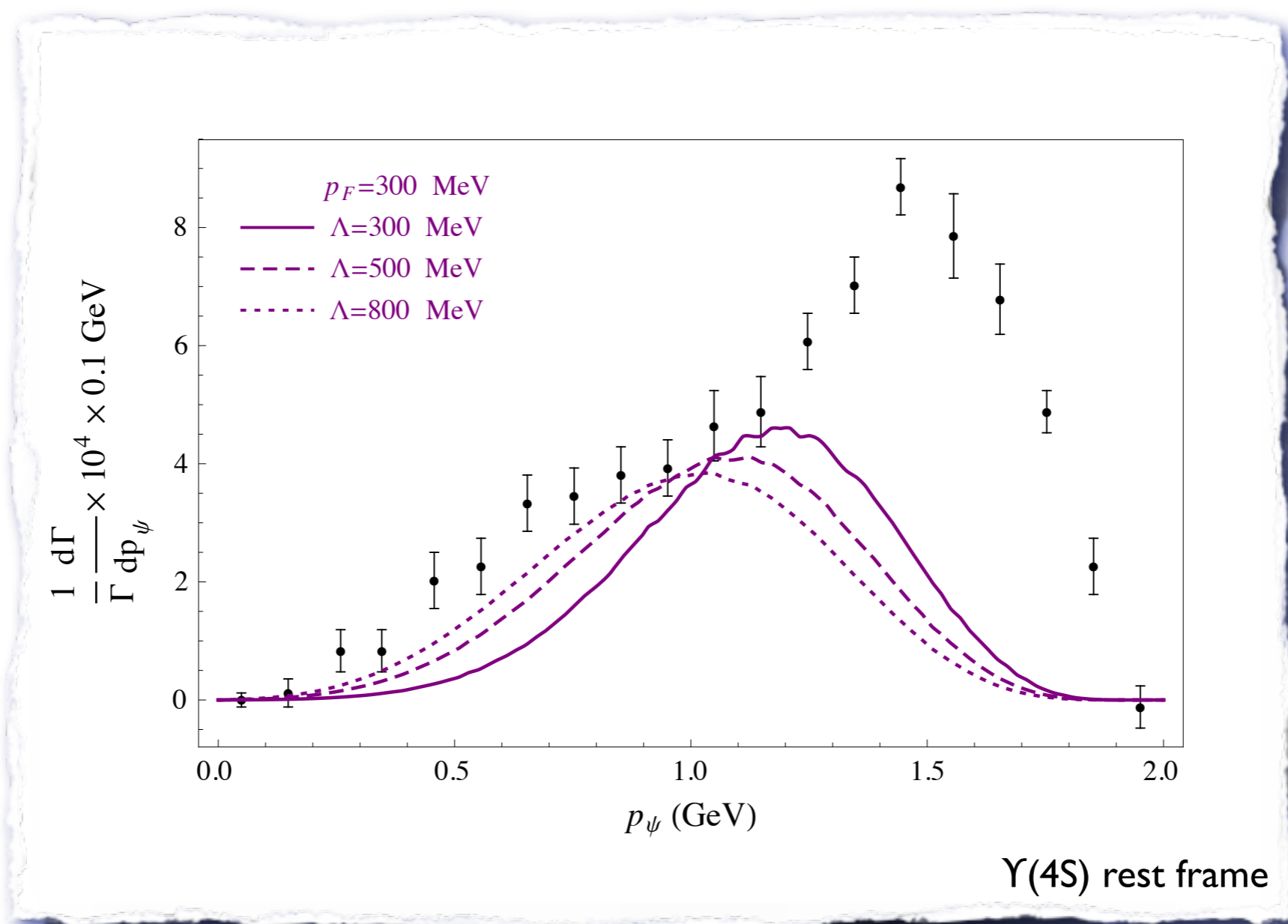
Hypothesis:

no interaction between the hard part and soft part of the process.

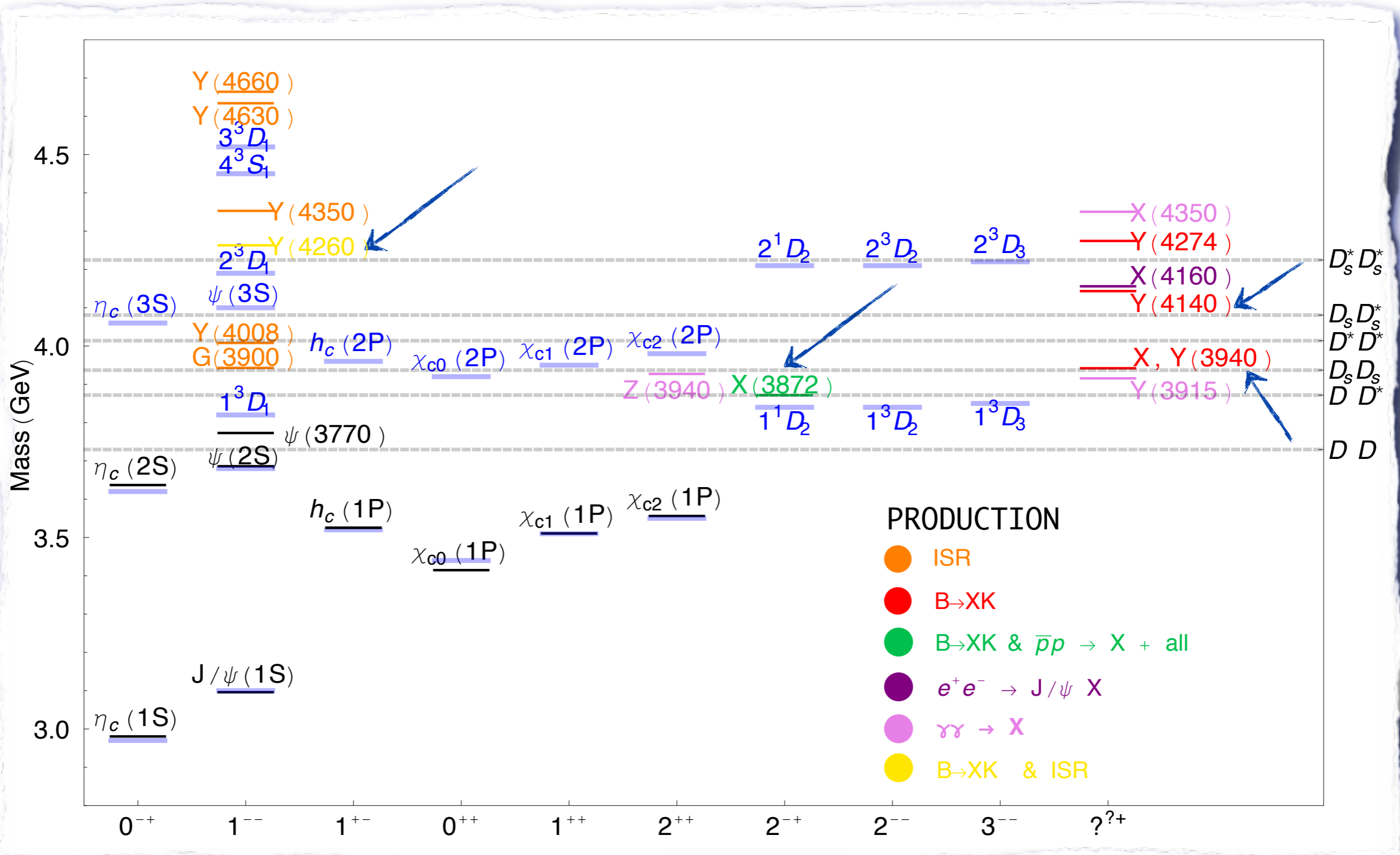


Color Octet Contribution

- **Two** main **parameters** to model the color octet contribution:
- $\Lambda_{\text{QCD}} \in [200, 450] \text{ MeV}$: the characteristic energy-momentum scale of the soft gluons;
 - $p_F \in [300, 450] \text{ MeV}$: Fermi momentum of the b-quark inside the B-meson.



Which XYZ contribute to $B \rightarrow J/\psi + \text{All}$?



B → K X → K J/ψ + light hadrons

► **B → K(500) X → K(500) J/ψ + light hadrons** branching ratios are known:

\mathcal{X}_j	$m_{\mathcal{X}_j}$ (GeV)	$\Gamma_{\mathcal{X}_j}$ (GeV)	Final State	$\mathcal{B}(B \rightarrow K \mathcal{X}_j \rightarrow K J/\psi + \text{light hadrons}) \times 10^5$
X(3872)	3.872	0.003	$J/\psi \rho \rightarrow J/\psi \pi^+ \pi^-$	0.72 ± 0.22 [A]
			$J/\psi \omega$	0.6 ± 0.3 [B]
Y(3940)	3.940	0.087	$J/\psi \omega$	3.70 ± 1.14 [C]
Y(4140)	4.140	0.012	$J/\psi \phi$	0.9 ± 0.4 [D]
Y(4260)	4.260	0.095	$J/\psi f_0 \rightarrow J/\psi \pi^+ \pi^-$	2.00 ± 0.73 [C]

[A] B. Aubert et al. (BABAR), Phys. Rev. **D77**, 111101 (2008), 0803.2838.
 [B] P. del Amo Sanchez et al. (BABAR), Phys. Rev. **D82**, 011101 (2010), 1005.5190.
 [C] <http://hfag.phys.ntu.edu.tw/b2charm/index.html>.
 [D] K. Yi and f. t. C. collaboration, PoS EPS-HEP **2009**, 2009:085,2009 (2009), 0910.3163.

► For heavy kaons K we deduce the coupling **B-K X** from the **B-K(500) X** one:

Spin 0 K $\langle \mathcal{X}(\epsilon, p) \mathcal{K}(q) | B(P) \rangle = g \epsilon \cdot q$

Spin 1 K $\langle \mathcal{X}(\epsilon, p) \mathcal{K}(\eta, q) | B(P) \rangle = g' \epsilon \cdot \eta$

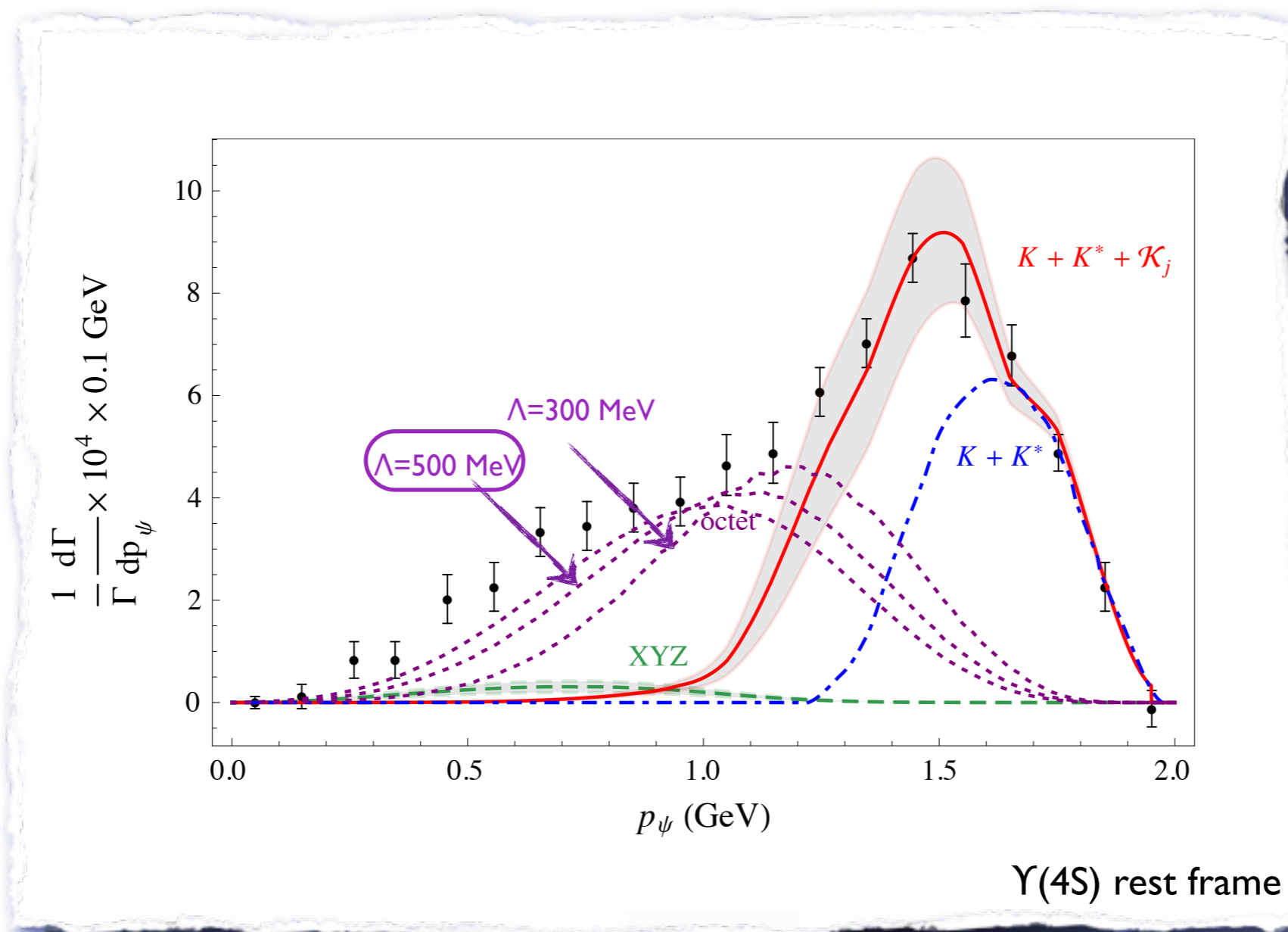
$$g' = \Lambda g$$

Λ some mass scale

We assume
 $\Lambda = m_{K(j=1)}$
 taking all X to be Spin 1 states.

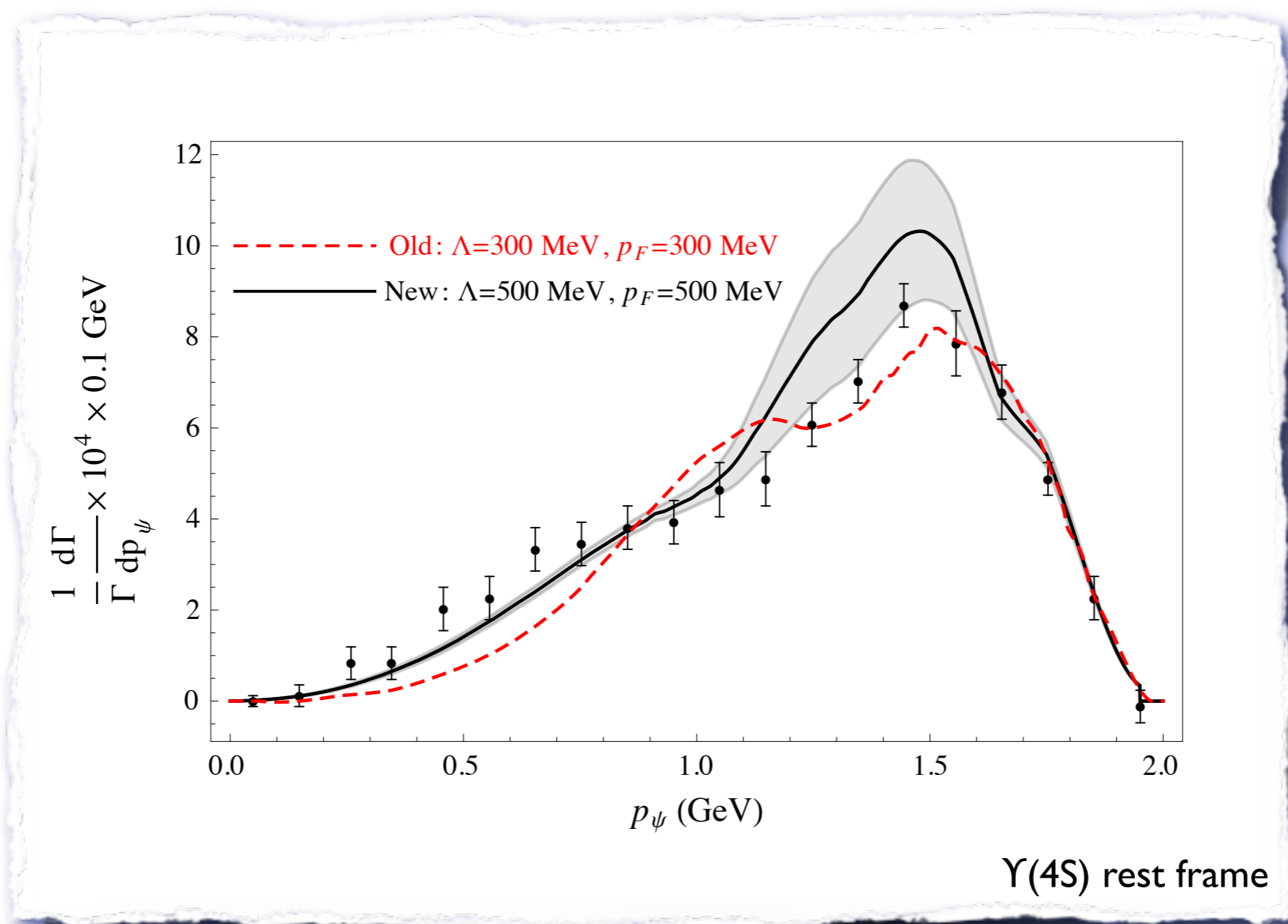
Results (I)

- ▶ We simulate the decay $B \rightarrow \mathcal{K} X \rightarrow \mathcal{K} J/\psi + \text{light hadrons}$ taking into account the **partial decay wave**.
- ▶ We fit the sum of all the contributions to data using as a **free parameter the overall normalization of the color octet component**.



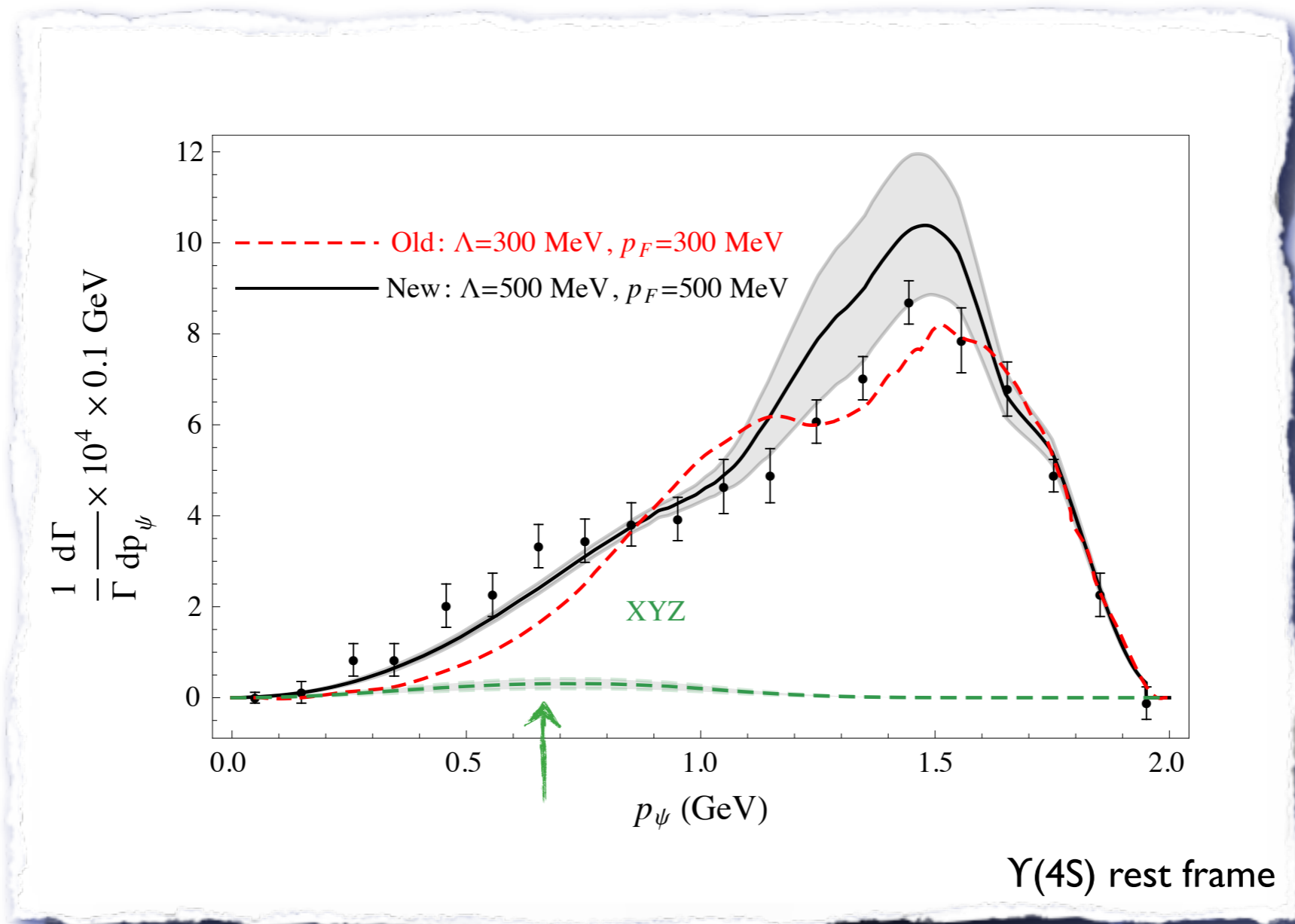
Results (2)

- The best fit in the allowed region for the two parameters (Λ_{QCD} , p_F) is obtained choosing: $\Lambda_{\text{QCD}} = 500 \text{ MeV}$ and $p_F = 500 \text{ MeV}$.



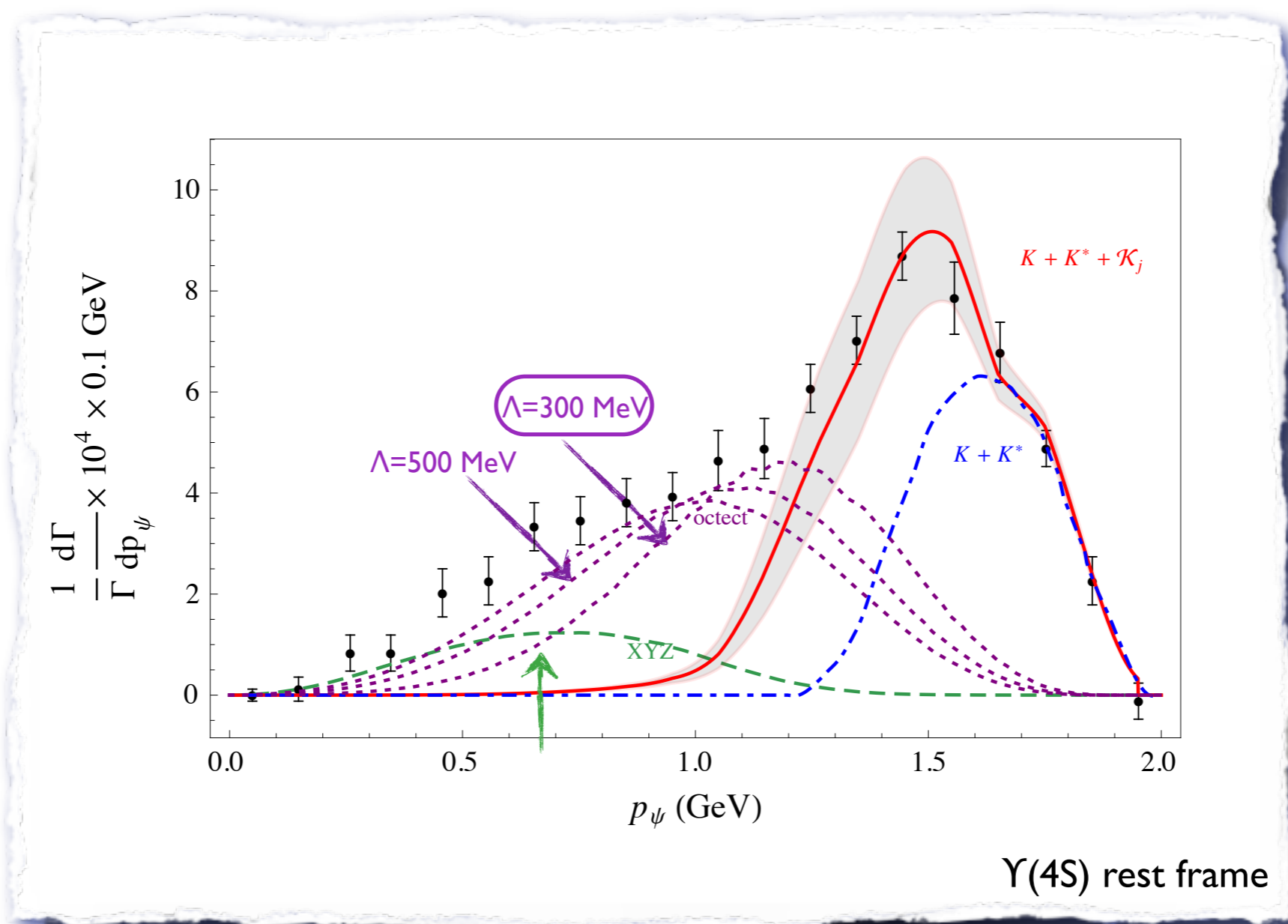
Results (2)

- The best fit in the allowed region for the two parameters (Λ_{QCD} , p_F) is obtained choosing: $\Lambda_{\text{QCD}} = 500 \text{ MeV}$ and $p_F = 500 \text{ MeV}$.



Results (3)

- If the branching ratio due to XYZ turns out to be larger than the one measured (more XYZ states!) the best fit could be obtained with more reasonable parameters for the color octet component.



Back Up

B → J/ψ + All

:: BaBar, Phys. Rev. D67,032002 ::

▶ In e^+e^- collisions at $\sqrt{s} \sim m_{\Upsilon(4S)}$ with 20.3 fb^{-1} they measure:

$B \rightarrow J/\psi + \text{All}, B \rightarrow \psi(2S) + \text{All}, B \rightarrow \chi_{c1,2} + \text{All}.$

▶ $J/\psi \rightarrow e^+e^-, \mu^+\mu^-$

$\psi(2S) \rightarrow e^+e^-, \mu^+\mu^-$ and $J/\psi \pi^+\pi^-$

$\chi_{c1,2} \rightarrow J/\psi \gamma$

▶ $\mathcal{B}(J/\psi \rightarrow e^+e^-) = 5.94\%$

$\mathcal{B}(J/\psi \rightarrow \mu^+\mu^-) = 5.93\%$

▶ $\mathcal{B}(\psi(2S) \rightarrow e^+e^-) = 0.765\%$

$\mathcal{B}(\psi(2S) \rightarrow \mu^+\mu^-) = 0.760\%$

$\mathcal{B}(\psi(2S) \rightarrow J/\psi \pi^+\pi^-) = 33.1\%$

▶ $\mathcal{B}(\chi_{c1} \rightarrow J/\psi \gamma) = 34.1\%$

$\mathcal{B}(\chi_{c2} \rightarrow J/\psi \gamma) = 19.4\%$

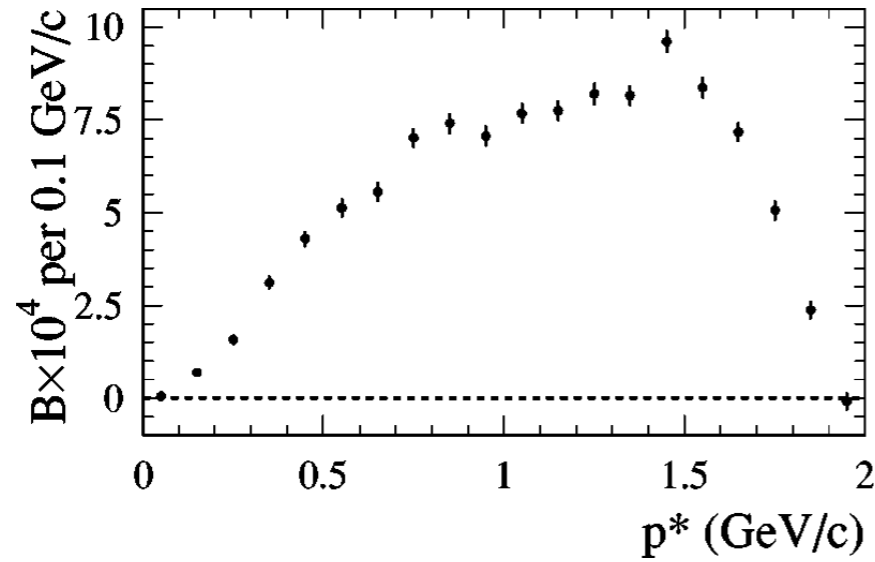
TABLE II. Summary of B branching fractions (percent) to charmonium mesons with statistical and systematic uncertainties. The direct branching fraction is also listed, where appropriate. The last column contains the world average values [15].

<i>Meson</i>	<i>Value</i>	<i>Stat</i>	<i>Sys</i>	<i>World Average</i>
J/ψ	1.057	± 0.012	± 0.040	1.15 ± 0.06
J/ψ direct	0.740	± 0.023	± 0.043	0.80 ± 0.08
χ_{c1}	0.367	± 0.035	± 0.044	0.36 ± 0.05
χ_{c1} direct	0.341	± 0.035	± 0.042	0.33 ± 0.05
χ_{c2}	0.210	± 0.045	± 0.031	0.07 ± 0.04
χ_{c2} direct	0.190	± 0.045	± 0.029	-
$\psi(2S)$	0.297	± 0.020	± 0.020	0.35 ± 0.05

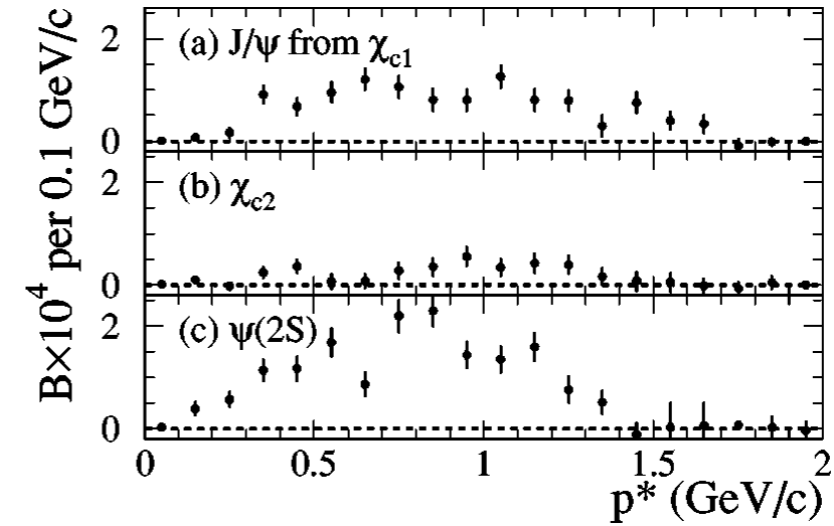
B → J/ψ + All

:: BaBar, Phys. Rev. D67,032002 ::

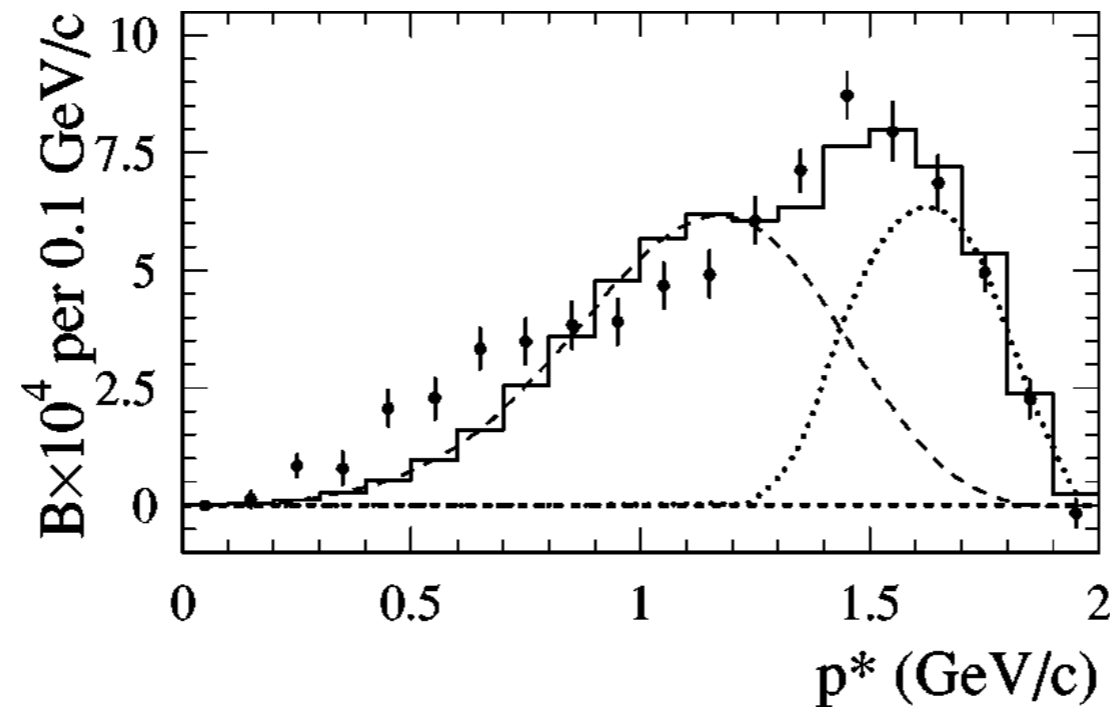
Total contribution



Feed-down contribution



Direct contribution



B → J/ψ + K

:: Belle, Phys. Rev. D83,032005 ::

- ▶ 535 × 10⁶ BB* events (492 fb⁻¹) from e⁺e⁻ collisions at √s ~ m_{Υ(4S)}
- ▶ ΔE = E*(B) - E_{beam} and M_{bc} = √(E_{beam}² - P*²(B))
 Signal region: -3σ_{ΔE} < ΔE - μ_{ΔE} < 3σ_{ΔE}
 Sideband region: -0.13 GeV < ΔE - μ_{ΔE} < -0.05 GeV
 & +0.05 GeV < ΔE - μ_{ΔE} < +0.13 GeV
- ▶ ℳ(B → J/ψ K⁺π⁺π⁻) = (7.16 ± 0.10 ± 0.60) × 10⁻⁴
 ℳ(B → ψ(2S) K⁺π⁺π⁻) = (4.31 ± 0.20 ± 0.50) × 10⁻⁴
- ▶ The PDF is p(x, a), with x = M²(Kππ), M²(Kπ), M²(ππ) and a = fit parameters

$$p(\vec{x}; \vec{a}) = n_B \frac{p_B(\vec{x})}{\int p_B(\vec{x}) d^3x} + n_S \frac{p_S(\vec{x}; \vec{a})}{\int p_S(\vec{x}; \vec{a}) d^3x}$$

Background modelled from sideband region

Signal

$$p_S(\vec{x}; \vec{a}) = \varepsilon(\vec{x}) \phi(\vec{x}) s(\vec{x}; \vec{a})$$

$$s(\vec{x}; \vec{a}) \equiv s(\vec{x}; a_k)$$

$$= |a_{nr} A_{nr}(\vec{x})|^2 + \sum_{J_1} \left| \sum_{J_2} a_{J_1 J_2} A_{J_1 J_2}(\vec{x}) \right|^2$$

B → J/ψ + K

:: Belle, Phys. Rev. D83,032005 ::

- ▶ The $K^+\pi^+\pi^-$ final state is modelled as a **non resonant signal** plus a **superposition of initial state resonances R_1** .
The latter are assumed to decay through intermediate state resonances R_2

$$R_1 \rightarrow a R_2 \text{ and } R_2 \rightarrow bc$$

Signal

$$p_S(\vec{x}; \vec{a}) = \varepsilon(\vec{x}) \phi(\vec{x}) s(\vec{x}; \vec{a})$$

$$s(\vec{x}; \vec{a}) \equiv s(\vec{x}; a_k)$$

$$= |a_{nr} A_{nr}(\vec{x})|^2 + \sum_{J_1} \left| \sum_{J_2} a_{J_1 J_2} A_{J_1 J_2}(\vec{x}) \right|^2$$

complex coefficients

- ▶ Since the components of the signal function are not individually normalized, a decay fraction is calculated as

$$f_k = \frac{\int \phi(\vec{x}) |a_k A_k(\vec{x})|^2 d^3x}{\int \phi(\vec{x}) s(\vec{x}; \vec{a}) d^3x}$$

B → J/ψ + K

:: Burns, Piccinini, Polosa, Prospero, Sabelli, arXiv:1104.1781::

► Belle measures

$$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi \rightarrow R_i J/\psi \rightarrow J/\psi K^+ \pi^+ \pi^-) = \mathcal{B}_{\text{tot}} f_i^j$$

where the intermediate resonant states are

$$\mathcal{R}_i = K\rho, K\omega, K^*\pi, K_0^*(1430)\pi, K_2^*(1430)\pi \text{ and } K f_{0,2}$$

► To extract **two body branching ratios** one needs to take into account **isospin multiplicity**

$$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi \rightarrow R_i J/\psi \rightarrow J/\psi K^+ \pi^+ \pi^-) = \mathcal{I}_i \times \mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi) \times \mathcal{B}(\mathcal{K}_j \rightarrow R_i) \times \mathcal{B}(R_i \rightarrow K\pi\pi)$$

where the isospin factors are

$$\mathcal{I}(K\rho) = 1/3, \mathcal{I}(K^*\pi) = \mathcal{I}(K_0^*(1430)\pi) = 4/9, \mathcal{I}(K\omega) = 1, \mathcal{I}(K f_0) = \mathcal{I}(K f_2) = 2/3$$

so that

$$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi) = \frac{\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi \rightarrow R_i J/\psi \rightarrow J/\psi K^+ \pi^+ \pi^-)}{\mathcal{I}_i \times \mathcal{B}(\mathcal{K}_j \rightarrow R_i) \times \mathcal{B}(R_i \rightarrow K\pi\pi)} = \frac{\mathcal{B}_{\text{tot}} f_i^j}{\mathcal{I}_i \times \mathcal{B}(\mathcal{K}_j \rightarrow R_i) \times \mathcal{B}(R_i \rightarrow K\pi\pi)}$$

B → J/ψ + K

:: Burns, Piccinini, Polosa, Prospero, Sabelli, arXiv:1104.1781::

- **Interference effects** among different heavy kaons \mathcal{K}_j have been **neglected**, so that one needs to **rescale the two body branching ratios by some factor**.

$$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi \rightarrow \mathcal{R}_i J/\psi \rightarrow J/\psi K^+ \pi^+ \pi^-) = \mathcal{B}_{\text{tot}} \tilde{f}_i^j$$

$$\tilde{f}_i^j = C \times \left(1 - \frac{\Gamma_j}{m_j} \right) f_i^j \quad \mathcal{B}_{\text{tot}} = (71.6 \pm 1 \pm 6) \times 10^{-5}$$

\mathcal{K}_j	$m_{\mathcal{K}_j}$ (GeV)	$\Gamma_{\mathcal{K}_j}$ (GeV)	$\mathcal{B}(B^+ \rightarrow \mathcal{K}_j J/\psi) \times 10^5$	
K	0.494	–	95.0 ± 3.6	*
K^*	0.892	0.050	137.0 ± 7.8	*
$K_1(1270)$	1.270	0.090	144.0 ± 29.3	
$K_1(1400)$	1.403	0.174	25.1 ± 5.7	
$K^*(1410)$	1.414	0.232	$> 5.1 \pm 2.4$ and $< 11.8 \pm 5.7$	
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* <http://hfag.phys.ntu.edu.tw/b2charm/index.html>

B → K X → K J/ψ + light hadrons

► **B → K(500) X → K(500) J/ψ + light hadrons** branching ratios are known:

\mathcal{X}_j	$m_{\mathcal{X}_j}$ (GeV)	$\Gamma_{\mathcal{X}_j}$ (GeV)	Final State	$\mathcal{B}(B \rightarrow K \mathcal{X}_j \rightarrow K J/\psi + \text{light hadrons}) \times 10^5$
X(3872)	3.872	0.003	$J/\psi \rho \rightarrow J/\psi \pi^+ \pi^-$	0.72 ± 0.22 [A]
			$J/\psi \omega$	0.6 ± 0.3 [B]
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Y(4140)	4.140	0.012	$J/\psi \phi$	0.9 ± 0.4 [D]
Y(4260)	4.260	0.095	$J/\psi f_0 \rightarrow J/\psi \pi^+ \pi^-$	2.00 ± 0.73 [C]

[A] B. Aubert et al. (BABAR), Phys. Rev. **D77**, 111101 (2008), 0803.2838.

[B] P. del Amo Sanchez et al. (BABAR), Phys. Rev. **D82**, 011101 (2010), 1005.5190.

[C] <http://hfag.phys.ntu.edu.tw/b2charm/index.html>.

[D] K. Yi and f. t. C. collaboration, PoS EPS-HEP **2009**, 2009:085,2009 (2009), 0910.3163.

► For heavy kaons \mathcal{K} we deduce the coupling **B-K X** from the **B-K(500) X** one:

Spin 0 $\mathcal{K} \quad \langle \mathcal{X}(\epsilon, p) \mathcal{K}(q) | B(P) \rangle = g \epsilon \cdot q$

Spin 1 $\mathcal{K} \quad \langle \mathcal{X}(\epsilon, p) \mathcal{K}(\eta, q) | B(P) \rangle = g' \epsilon \cdot \eta$

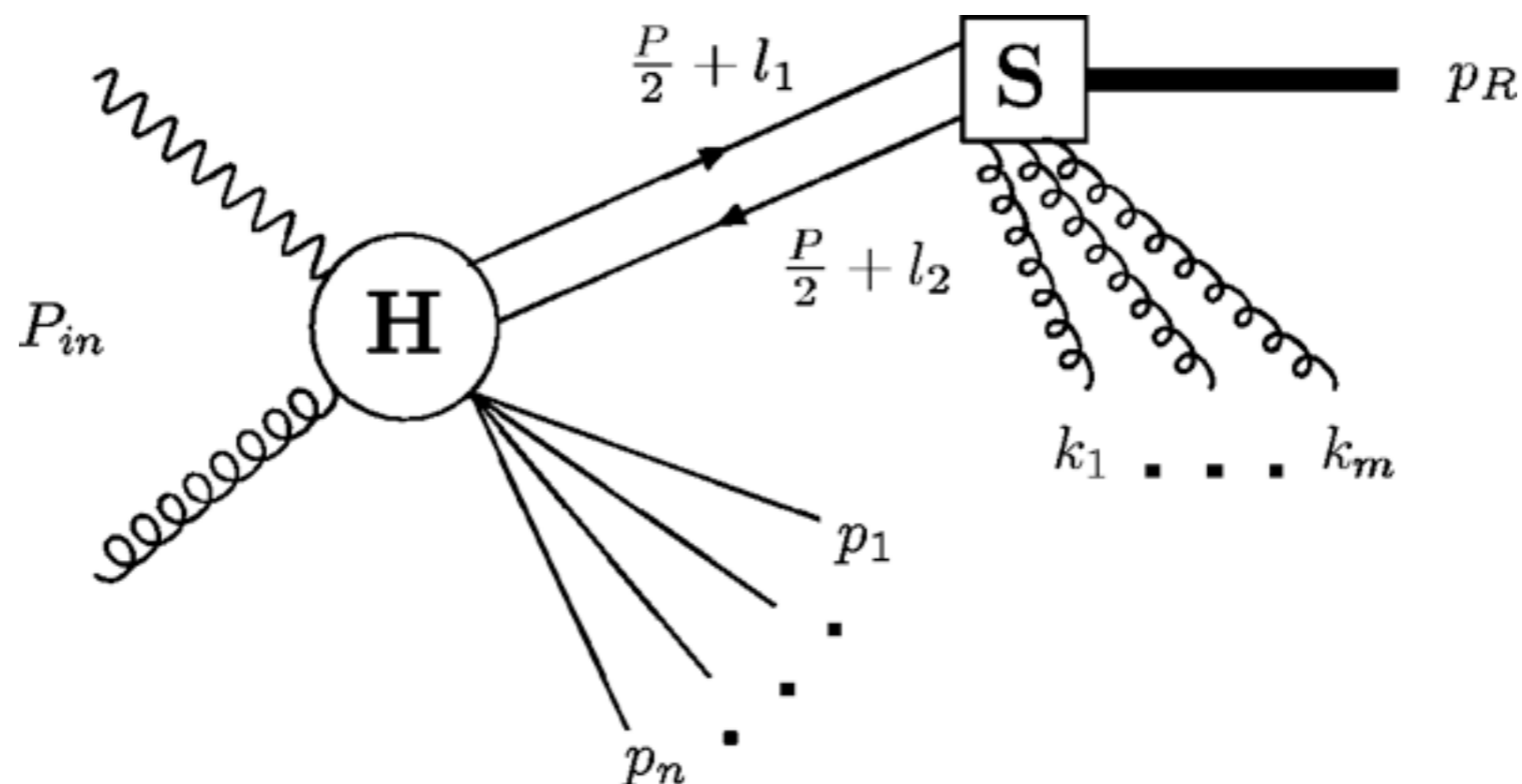
$$g' = \Lambda g$$

Λ some mass scale

From $\mathcal{B}(B \rightarrow K^* X(3872)) \times \mathcal{B}(X(3872) \rightarrow J/\psi \pi^+ \pi^-) < 0.34 \times 10^{-5}$
 we deduce $\Lambda > 600 \text{ MeV} \approx m_{K^*}$ and thus we assume $\Lambda = m_{K^*}$,
 taking all \mathcal{X} to be Spin 1 states.

Color Octet Contribution

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::



- ▶ At leading order in the non-relativistic expansion the cc^* pair has to be produced in a color singlet 3S_1 state.
- ▶ At relative order $v^4 \approx 1/15$ in the non-relativistic expansion, J/ ψ can also be produced through cc^* in $^1S_0^{(8)}, ^3P_J^{(8)}, ^3S_1^{(8)}$ color octet states
- ▶ The short-distance structure of the $\Delta B=1$ weak effective Hamiltonian favors the production of color octet cc^* pairs in the $b \rightarrow cc^* q$ transition

Factorization

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

► The hard and soft part of the process can be factorized

$$(2\pi)^3 2p_R^0 \frac{d\sigma}{d^3p_R} \equiv \sum_n \int \frac{d^4l}{(2\pi)^4} \hat{\sigma}(c\bar{c}[n])(l) F_n(l)$$

$$\text{flux} \times \int d\text{PS}[p_i] (2\pi)^4 \delta\left(P+l+\sum_i p_i - P_{in}\right) \times H_n(P_{in}, P, l, p_i)$$

hard process cross section

$$F_n(l) = \int \frac{dk^2}{2\pi} \frac{d^3k}{(2\pi)^3 2k^0} (2\pi)^4 \times \delta(p_R+k-P-l) \Phi_n(k; p_R, P)$$

shape function

► The distribution can be written as an integral over the energy and invariant mass of the soft radiated system:

$$(2\pi)^3 2p_R^0 \frac{d\sigma}{d^3p_R} = \sum_n \int_0^{\alpha\beta} \frac{dk^2}{2\pi} \int_{(\alpha^2+k^2)/(2\alpha)}^{(\beta^2+k^2)/(2\beta)} dk_0 \times \text{flux} \times \bar{H}_n(P_{in}, P, l, p_X) \frac{1}{4\pi(\beta-\alpha)} \Phi_n(k; p_R, P)$$

Shape Function

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- ▶ The color octet configurations which contribute to B \rightarrow J/ ψ + All are

$$n = {}^1S_0^{(8)}, {}^3P_0^{(8)}, {}^3S_1^{(8)}$$

- ▶ The shape function is related to the NRQCD matrix elements as

$$\begin{aligned} \int \frac{d^4l}{(2\pi)^4} F_n(l) &= \frac{1}{(2\pi)^3} \int_0^\infty dk^2 \int_{\sqrt{k^2}}^\infty dk_0 \\ &\times \sqrt{k_0^2 - k^2} \Phi_n(k; p_R, P) \\ &= \langle \mathcal{O}_n^{J/\psi} \rangle, \end{aligned}$$

- ▶ An ansatz for the shape function is

$$\Phi_n(k; p_R, P) = a_n |k|^{b_n} \exp(-k_0^2 / \Lambda_n^2) k^2 \exp(-k^2 / \Lambda_n^2)$$

which is exact in the Coulombic limit. The exponential cutoff reflects the expectations that the typical energy and invariant mass of the radiated system is of order $\Lambda_n \approx m_c v^2$

Shape Function

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

► The decay distribution in the rest frame of the $c\bar{c}^*$ pair is

$$\frac{d\hat{\Gamma}}{d\hat{E}_R} = \frac{|\hat{p}_R|}{4\pi^2} \sum_n \int_0^{\alpha\beta} \frac{dk^2}{2\pi} \int_{(\alpha^2+k^2)/(2\alpha)}^{(\beta^2+k^2)/(2\beta)} dk_0$$

$$\times \frac{1}{2m_b} H_n(m_b, M_{c\bar{c}}(k)) \frac{M_R}{8\pi m_b |\hat{p}_R|} \Phi_n(k)$$

where

$$M_{c\bar{c}}^2(k) = (p+l)^2 = (p_R+k)^2 = M_R^2 + 2M_R k_0 + k^2$$

► To normalize the shape function one uses

$$\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle = (0.5-1.0) \times 10^{-2} \text{ GeV}^3$$

$$M_k^{J/\psi}(^1S_0^{(8)}, ^3P_0^{(8)}) = \langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle + \frac{k}{m_c^2} \langle \mathcal{O}_8^{J/\psi}(^3P_0) \rangle \quad M_{3.1}^{J/\psi}(^1S_0^{(8)}, ^3P_0^{(8)}) = (1.0-2.0) \times 10^{-2} \text{ GeV}^3$$

Fermi Motion

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- ▶ The b quark is moving inside the B meson at rest with a momentum p according to some distribution with a width of few hundred MeV. The cloud of gluons and light quarks is treated as spectator.

$$\Phi_{\text{ACM}}(p) = \frac{4}{\sqrt{\pi} p_F^3} \exp(-p^2/p_F^2)$$

- ▶ One needs thus to consider a floating b-mass

$$m_b^2(p) = M_B^2 + m_{sp}^2 - 2M_B \sqrt{m_{sp}^2 + p^2}$$

- ▶ To obtain the distribution in the B rest frame

$$\frac{d\Gamma}{dE_R} = \int_{\max\{0, p_-\}}^{p_+} dp p^2 \Phi_{\text{ACM}}(p) \frac{m_b^2(p)}{2p E_b(p)} \times \int_{\hat{E}_R^{\min}(p)}^{\hat{E}_R^{\max}(p)} \frac{d\hat{E}_R}{\hat{E}_R} \frac{d\hat{\Gamma}}{d\hat{E}_R}$$