# TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTION and FRAGMENTATION FUNCTIONS: A SHORT THEORY OVERVIEW

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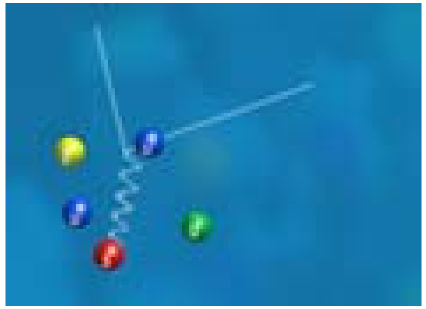
**Probing Strangeness in Hard Processes**

**INFN, Laboratori Nazionali di Frascati – October 18-21, 2010**



# Outline

- **Early TMD parton model approaches for unpolarized processes**
- **Azimuthal and transverse single spin asymmetries**
  - Sivers and Collins effects in pp collisions and SIDIS
- **Generalized parton model: a leading twist TMD approach**
  - Full inclusion of spin and intrinsic parton motion ($k_\perp$) effects
- **Theoretical developments**
  - **Inclusion of QCD initial/final state interactions**
  - **TMD color gauge invariant approach**
  - Factorization, universality and process dependence of TMDs
- **Open points, future developments**

# TMD approaches for unpolarized inclusive processes

Early attempts of including intrinsic parton motion effects date back right to the formulation of parton model by Feynman and collabs.

Aiming at improve phenomenological applications of parton model to:

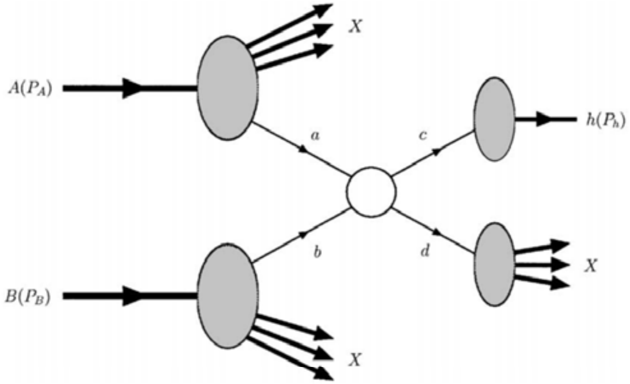
- unpolarized cross sections for inclusive single particle production in high-energy hadronic collisions
- transverse momentum distribution of the lepton pair in Drell-Yan processes

LO (and even NLO) QCD calculations in a collinear factorization scheme were unable to explain many of experimental data available

Standard QCD collinear factorization scheme:

intrinsic parton motion integrated over up to the large energy scale in the process (giving raise to the evolution with scale of the soft parton distribution and fragmentation functions) and neglected in hard partonic processes.

# Inclusive single particle production in a QCD-inspired collinear parton model




$$\frac{E_h\, d\sigma^{AB\to h+X}}{d^3\boldsymbol{p}_h} = \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\, f_{a/A}(x_a,Q^2)\, f_{b/B}(x_b,Q^2)\, \frac{d\hat\sigma^{ab\to cd}}{d\hat t}\, \frac{s}{\pi z^2}\, \delta(\hat s+\hat t+\hat u)\, D_{h/c}(z,Q^2)$$

# ...and with inclusion of intrinsic parton motion

$$\frac{E_h\, d\sigma^{AB\to h+X}}{d^3\boldsymbol{p}_h} \propto \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\; d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^2\boldsymbol{k}_{\perp h}\, J(z,h_{\perp h})$$

$$f_{a/A}(x_a,k_{\perp a};Q^2)\, f_{b/B}(x_b,k_{\perp b};Q^2)\, \frac{d\hat\sigma^{ab\to cd}}{d\hat t}\, \delta(\hat s+\hat t+\hat u)\, D_{h/c}(z,k_{\perp h};Q^2)$$

# TMD enhancement mechanisms for (un)polarized cross sections [forward production region]

- In the intermediate-large $p_T$ region ($\sim$ 1–4 GeV), inclusion of $k_\perp$ effects in hard processes raises the dominant t-channel partonic cross sections

  [$\propto 1/t^2$] since, for large positive $x_F$ , $t(k_\perp \neq 0) < t(k_\perp = 0)$;

  (troubles with factorization and universality?)

- Intrinsic $k_\perp$ effects shift (reduce) the average value of the light-cone momentum fractions ($x_a$, $x_b$, z) within the convolution integrals for the cross sections $d\sigma(AB \to C+X)$.

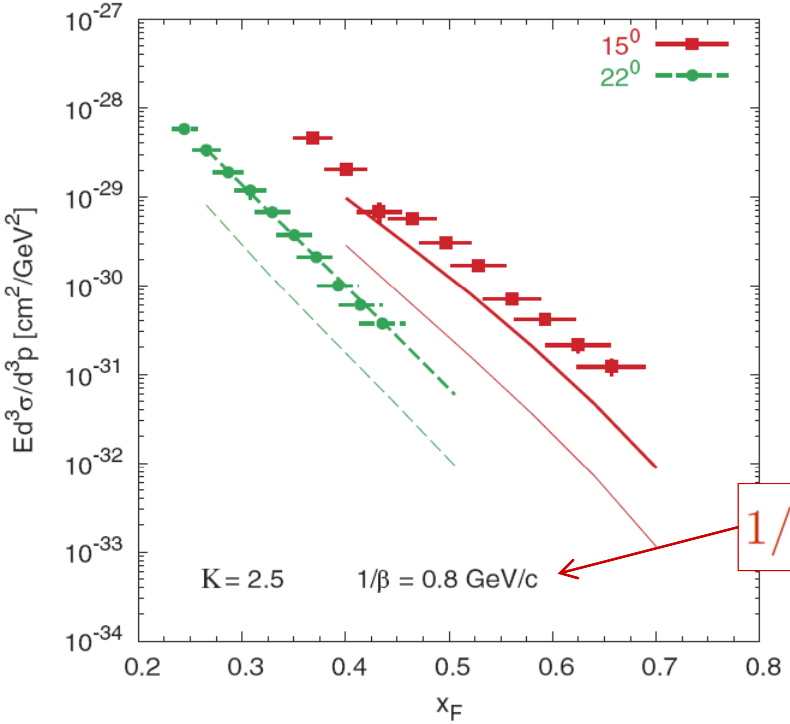
- In the forward region (large positive Feynman x), since typically min($x_a$, z) > $x_F$, even a small shift may have a substantial effect on the cross section in a phase-space region of $x_a/z$ where the PDFs/FFs vary (decrease) very rapidly.

$$\text{e.g. for } x \to 1 \qquad f_{a/p}(x) \sim (1-x)^\alpha \qquad \alpha \sim 3-5$$

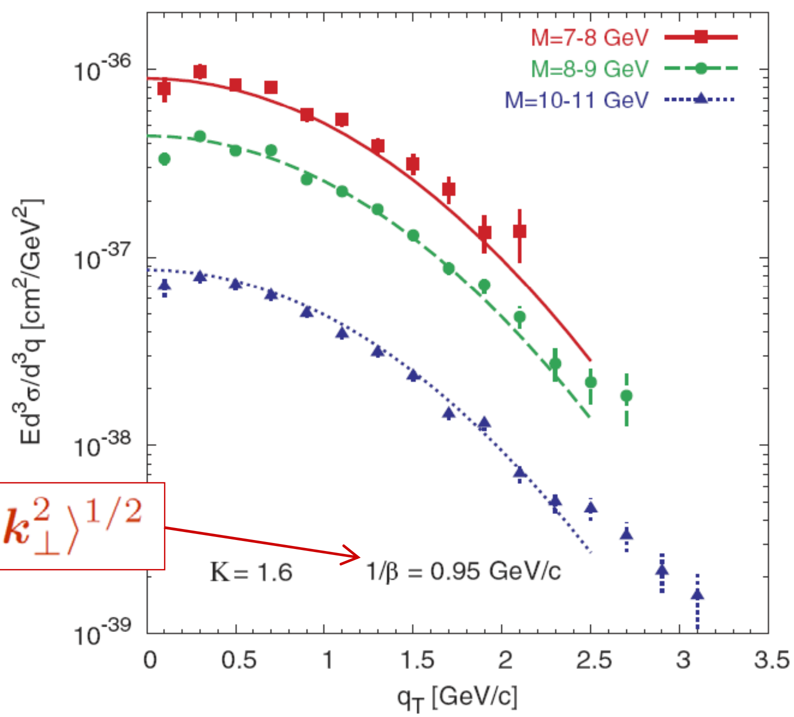
- Warning: At large Feynman x convolution integrals run over more and more limited phase-space regions where even unpolarized PDFs and FFs are not very well constrained by available fits.

# Some results (just an example)
## U. D'Alesio, FM [PRD70 (2004)]

$\pi^0$ production in pp collisions, $\sqrt{s}=23$ GeV




DY process, $\sqrt{s}\simeq 40$ GeV, $x_F = 0.1$




$$1/\beta = \langle \boldsymbol{k}_\perp^2 \rangle^{1/2}$$

**LO calculation: requires anyway K-factors**

**best-fit effective $k_\perp$ depends on (increases with) c.m. energy**

**unable to explain small-scattering angle results**

**factorization and universality breaking?**

# Transverse spin physics

It was an early common belief that transverse spin effects should play a negligible role in high-energy hadronic reactions

In fact, there are several transv. spin effects contradicting this prejudice:

quark transversity distribution

transverse single spin asymmetries (SSAs)
transverse hyperon polarization in unpol. hadronic collisions

spin-spin correlations in pp elastic scattering

Azimuthal asymmetries in (un)polarized SIDIS, Drell-Yan and $e^+e^- \to h_1 h_2\, X$ processes

# Single spin asymmetries in high-energy inclusive and semi-inclusive particle production

## Theoretical expectactions (circa 1980)

### Transverse single spin asymmetries (SSA), $A_N$

**In the collinear approach the hadronic SSA arises from the analogous partonic SSA**

**This is related to the imaginary part of interference terms between two elementary scattering amplitudes, off-diagonal (helicity-flip) in the helicity indices of the transversely polarized parton**

$$|\uparrow,\downarrow\rangle_y = (1/\sqrt{2})(|+\rangle \pm i|-\rangle) \Longrightarrow |\langle\ |...|\uparrow\rangle|^2 - |\langle\ |...|\downarrow\rangle|^2 \propto \mathrm{Im}\langle\ |...|\pm\rangle\langle\ |...|\mp\rangle^*$$

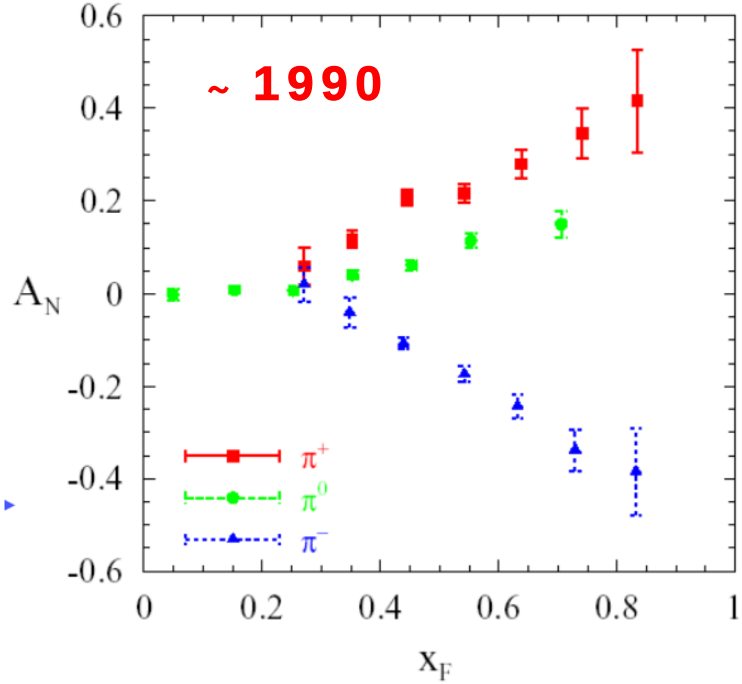
**At tree level, all helicity amplitudes are real**
**Imaginary parts can only arise through higher-order (loop) contributions**

**The QCD massless q–g coupling preserves helicity at all perturbative orders**
**Helicity flip contributions in the amplitudes must be prop. to (powers of) $m_q / E_q$**

$$\hat{a}_N = \frac{d\hat{\sigma}^{a^\uparrow b\to cd} - d\hat{\sigma}^{a^\downarrow b\to cd}}{d\hat{\sigma}^{a^\uparrow b\to cd} + d\hat{\sigma}^{a^\downarrow b\to cd}} \propto \alpha_s(\hat{s})\,\frac{m_q}{\hat{s}} \sim \alpha_s(p_T)\,\frac{m_q}{p_T}$$

**Kane, Pumplin, Repko PRL 41 (1978)**

# Unexpected experimental results for pion SSAs in pp collisions

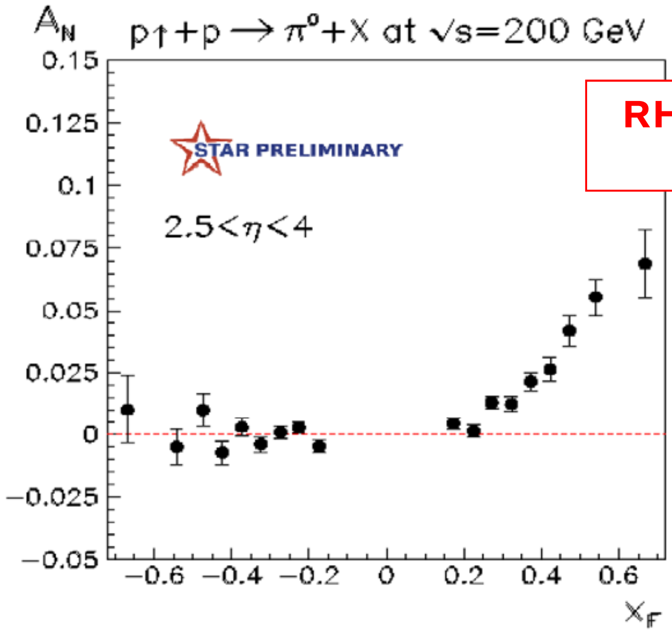



Fermilab E704: $A_N(p^\uparrow p \to \pi X)$

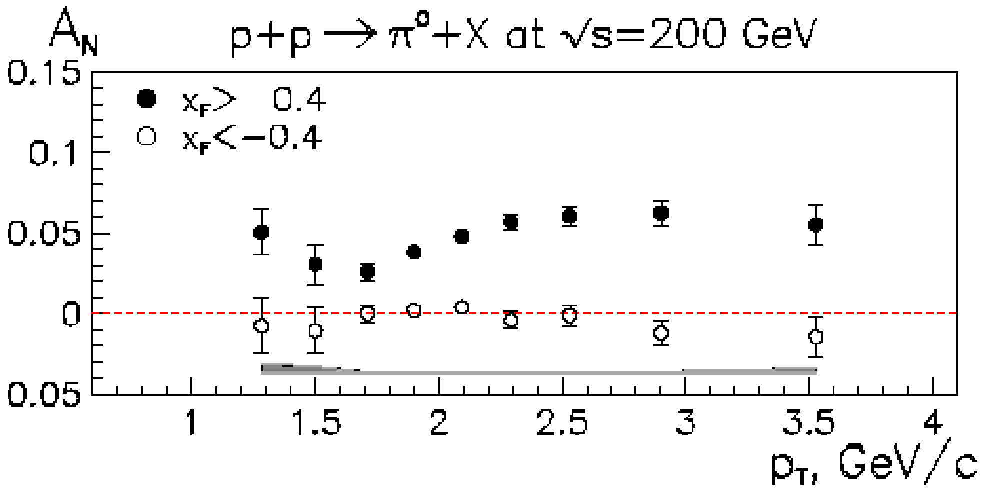
$\sqrt{s} = 20$ GeV, $0.7 < p_T < 2.0$ GeV

$$\boxed{x_F = 2p_L/\sqrt{s}}$$

**See talk by A. Martin**




**RHIC-STAR 2008**



# Transverse SSAs – a challenge for theory

## 1991

D. Sivers suggests that the huge hadronic SSAs observed at large positive Feynman x and intermediate $p_T$, could be related to:

A transverse momentum dependent (TMD) PDF with a significant asymmetry in the azimuthal distribution of (un)polarized partons around the direction of motion of the transv. polarized parent hadron

**(The leading-twist TMD Sivers distribution function)**

$$\Delta\hat{f}_{a/A^\uparrow}(x,\boldsymbol{k}_\perp) \equiv \hat{f}_{a/A^\uparrow}(x,\boldsymbol{k}_\perp) - \hat{f}_{a/A^\downarrow}(x,\boldsymbol{k}_\perp) = \hat{f}_{a/A^\uparrow}(x,\boldsymbol{k}_\perp) - \hat{f}_{a/A^\uparrow}(x,-\boldsymbol{k}_\perp)$$

**Chiral –even (no quark helicity-flip required) naively Time Reversal odd (T-odd)**

An azimuthally coherent [higher-twist in ($k_\perp/p_T$)] dependence of the unpol. partonic cross sections from intrinsic parton momenta ($k_\perp$), relevant at the intermediate-large hadronic $p_T$ observed in the experiments ($p_T \sim$1-4 GeV)
[The same effect advocated in order to enhance unpolarized cross sections]

# Transverse SSAs – a challenge for theory

## 1993

J. Collins proposes an alternative mechanism, acting in the fragmentation process, for SSAs in semi-inclusive DIS with a transversely polarized proton target, $lp^{\uparrow} \longrightarrow l' \, h \, X$.

This mechanism involves:

The TMD transversity distribution in the initial polarized hadron (opening an alternative way to measure it!)

The partonic transverse polarization transfer from the initial polarized parton to the final fragmenting parton

An azimuthal asymmetry around the jet axis in the distribution of the observed (un)polarized hadrons (the Collins effect)

$$\Delta\hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) \equiv \hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) - \hat{D}_{C/q^{\downarrow}}(z,\boldsymbol{k}_{\perp}) = \hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) - \hat{D}_{C/q^{\uparrow}}(z,-\boldsymbol{k}_{\perp})$$

**Chiral-odd, naively T-odd**

# Transverse spin physics

A new class of leading twist (dominant terms in a 1/Q power expansion, with Q the large energy scale) spin and transverse momentum dependent (TMD) partonic distribution and fragmentation functions play a fundamental role in this game

Polarized TMD distributions are intimately related to:

parton orbital motion inside hadrons

hadron structure (tomography) in the impact parameter space

generalized parton distributions and Deeply Virtual Compton Scattering

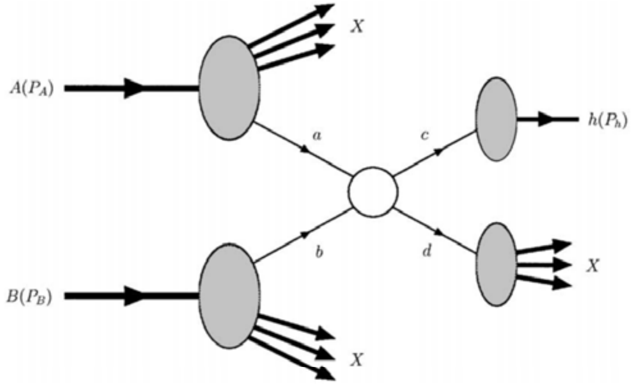
Wigner quantum phase-space distributions for partons

light-cone hadronic wave functions

[ violation of QCD helicity selection rules in exclusive hadronic decays]

The study of transversely polarized observables in different kinematical situations and in different processes will hopefully clarify in much more detail the dynamical structure of hadrons

# Inclusive single particle production in QCD-inspired collinear parton model




$$\frac{E_h \, d\sigma^{AB\to h+X}}{d^3\boldsymbol{p}_h} = \sum_{a,b,c,d} \int dx_a \, dx_b \, dz \, f_{a/A}(x_a, Q^2) \, f_{b/B}(x_b, Q^2) \, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}} \, \frac{s}{\pi z^2} \, \delta(\hat{s}+\hat{t}+\hat{u}) \, D_{h/c}(z, Q^2)$$

# ...and with inclusion of intrinsic parton motion

$$\frac{E_h \, d\sigma^{AB\to h+X}}{d^3\boldsymbol{p}_h} \propto \sum_{a,b,c,d} \int dx_a \, dx_b \, dz \, d^2\boldsymbol{k}_{\perp a} \, d^2\boldsymbol{k}_{\perp b} \, d^2\boldsymbol{k}_{\perp h} \, J(z, h_{\perp h})$$

$$f_{a/A}(x_a, k_{\perp a}; Q^2) \, f_{b/B}(x_b, k_{\perp b}; Q^2) \, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}} \, \delta(\hat{s}+\hat{t}+\hat{u}) \, D_{h/c}(z, k_{\perp h}; Q^2)$$

# Generalized parton model in the helicity formalism: Polarized leading-twist TMD PDFs and FFs

Consider as an example the doubly polarized invariant differential cross section for inclusive single particle production, $A(S_A)\ B(S_B) \longrightarrow C + X$:

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$
$$\times\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,$$

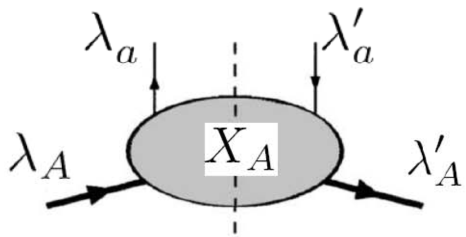
$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) = \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \not\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A} \equiv \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

$$\rho^{A,S_A}_{\lambda_A,\lambda'_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_Z & P^A_X - \mathrm{i}P^A_Y \\ P^A_X + \mathrm{i}P^A_Y & 1-P^A_Z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_L & P^A_T e^{-\mathrm{i}\phi_{S_A}} \\ P^A_T e^{\mathrm{i}\phi_{S_A}} & 1-P^A_L \end{pmatrix}_{A,S_A}$$

Helicity Density Matrix of hadron A

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) \equiv \not\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,\boldsymbol{k}_{\perp a})\hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A}(x_a,\boldsymbol{k}_{\perp a})$$

**Related to the leading twist hadronic correlator**



## Rotational invariance

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})\exp[i(\lambda_A-\lambda'_A)\phi_a]$$

## Parity invariance of strong interactions

$$F^{-\lambda_a,-\lambda'_a}_{-\lambda_A,-\lambda'_A} = (-1)^{2(S_A-s_a)}(-1)^{(\lambda_A-\lambda_a)+(\lambda'_A-\lambda'_a)}F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

Some parity relations are different for quarks ($s_q$=1/2) and gluons ($s_g$=1)

## Angular momentum conservation in the forward direction ($k_T \to 0$)

$$F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a}) \sim \left(\frac{k_{\perp a}}{M}\right)^{|\lambda_A-\lambda_a-(\lambda'_A-\lambda'_a)|}\tilde{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})$$

## We end up with 8 independent leading twist TMD functions (e.g. for quarks)

$\boxed{F^{++}_{++} \pm F^{--}_{++}}$ $\Rightarrow$ Obviously purely real quantities
Unpolarized and longitudinally polarized PDFs, $f_1$ and $g_{1L}$

$\boxed{F^{+-}_{+-} \pm F^{-+}_{+-}}$ $\Rightarrow$ Purely real(imaginary) quantities for quarks(gluons)
Contributions to the quark TMD transversity DF,: $h_1$, $h_{1T}^{\perp}$

$\boxed{F^{++}_{+-} \pm F^{--}_{+-}}$ $\Rightarrow$ Sivers function and $g_{1T}^{\perp}$ PDF

$\boxed{F^{+-}_{++} \pm F^{+-}_{--}}$ $\Rightarrow$ Boer-Mulders function and $h_{1L}^{\perp}$ PDF

These last 4 complex functions are not independent; can also be written in terms of the real and imaginary parts of two of them

## TMD fragmentation functions into unpol. hadrons (leading twist)

Introducing the soft, nonperturbative helicity fragmentation amplitudes for the process c $\to$ C + X, the following properties hold for their products:

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum\!\!\!\!\!\!\!\int_{X,\lambda_X} \hat{D}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C})\, \hat{D}^*_{\lambda'_C,\lambda_X;\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})\, e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$\hat{D}^{C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum_{\lambda_C,\lambda_C} \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$D^{C}_{-\lambda_c,-\lambda'_c}(z,k_{\perp C}) = (-1)^{2s_c}(-1)^{\lambda_c+\lambda'_c} D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})$$

### Quark sector

$$\hat{D}^{C/q}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/q}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/q}(z,k_{\perp C})$$

$$2\,\mathrm{Im}\, D^{C/q}_{+-}(z,k_{\perp C}) \equiv \Delta^N \hat{D}_{C/q^\uparrow}(z,k_{\perp C})$$

Collins function

### Gluon sector

$$\hat{D}^{C/g}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/g}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/g}(z,k_{\perp C})$$

$$2\,\mathrm{Re}\, D^{C/g}_{+-}(z,k_{\perp C}) \equiv \Delta^N \hat{D}_{C/T_1^g}(z,k_{\perp C})$$

**Notice: for spin 1/2 hadrons ($\Lambda$s,...) in analogy with the PDF case there are again 8 independent TMD FFs [including the Collins and polarizing FFs]**

## The most phenomenologically relevant for SSAs are

### Sivers distribution function (chiral-even, naively T-odd)

$$\boxed{\Delta \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\downarrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\uparrow}(x, -\boldsymbol{k}_\perp)}$$

**Transverse SSAs in polarized AB $\to$ C+X, SIDIS, DY processes**

### Boer-Mulders function (chiral odd, naively T-odd)

$$\boxed{\Delta \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) \equiv \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) - \hat{f}_{a^\downarrow/A}(x, \boldsymbol{k}_\perp) = \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) - \hat{f}_{a^\uparrow/A}(x, -\boldsymbol{k}_\perp)}$$

**Azimuthal Asy.s in unpolarized AB $\to$ C+X, SIDIS, DY processes**

### Collins fragmentation function (chiral-odd, naively T-odd)

$$\boxed{\Delta \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) \equiv \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{C/q^\downarrow}(z, \boldsymbol{k}_\perp) = \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{C/q^\uparrow}(z, -\boldsymbol{k}_\perp)}$$

**Azimuthal and transv. Asy.s in (un)polarized AB $\to$ h+X, SIDIS, DY, $e^+e^- \to h_1\, h_2\, X$**

### "Polarizing" fragmentation function (chiral even, naively T-odd)

$$\boxed{\Delta \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) \equiv \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{\Lambda^\downarrow/q}(z, \boldsymbol{k}_\perp) = \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{\Lambda^\uparrow/q}(z, -\boldsymbol{k}_\perp)}$$

**Transv. hyperon polarization in unpolarized AB $\to$ C+X, SIDIS processes**

# Generalized TMD parton model approach

## Advantages

Clear physical meaning

Partonic (probabilistic) interpretation

Full $k_\perp$ kinematics

....

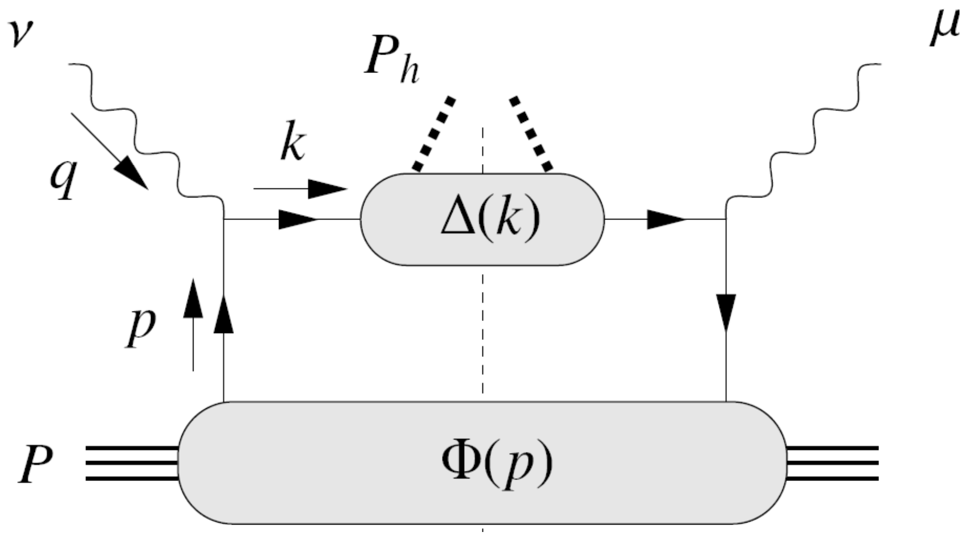
## Problems

Factorization assumed

Mechanism for T-odd PDFs not clarified

...

# TMD Color gauge invariant approach




The hadron tensor in semi-inclusive deep-inelastic scattering (SIDIS)

$$\Phi_{ij}(p;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi} \langle P,S|\,\overline{\psi}_j(0)\,\psi_i(\xi)\,|P,S\rangle$$

$$\Delta_{ij}(k;P_h,S_h) = \sum\!\!\!\!\!\!\int_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{-ik\cdot\xi} \langle 0|\,\psi_i(0)\,|P_h,S_h;X\rangle\langle P_h,S_h;X|\,\overline{\psi}_j(\xi)\,|0\rangle$$

**Mulders, Boer, Pijlman, Bomhof**

Hadronic correlators as given in previous slide are not invariant under local $SU_c(3)$ gauge transformations, since involve bilocal products of quark fields;
We need to connect the quark fields by means of a path-ordered gauge link (Wilson line)

$$\mathcal{U}^{C}_{[\zeta;\xi]} = \mathcal{P}\exp\left[-ig\int_C d\eta \cdot A^a(\eta)\, T^a\right]$$

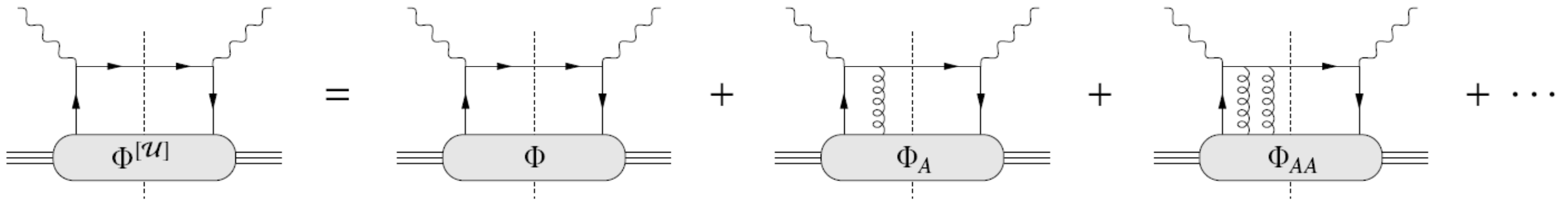
C : integration path with endpoints ζ and ξ;

$\mathcal{P}$ denotes path-ordering: parametrize the integration path using

$\eta^\mu(s)$ with $s \in [0, 1]$ and $\eta^\mu(0) = \zeta^\mu$ and $\eta^\mu(1) = \xi^\mu$

The properly invariant light-front ($\xi \cdot n$) definition of the correlator is therefore

$$\Phi^{[\mathcal{U}]}_{ij}(x,p_T) = \int \frac{d(\xi\cdot P)\, d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S\,|\,\overline{\psi}_j(0)\, \mathcal{U}_{[0;\xi]}\, \psi_i(\xi)\,|P,S\,\rangle\,\rfloor_{\rm LF}$$



Parametrize the (color gauge invariant) hadronic correlator imposing general hermiticity, parity and time-reversal property. One ends up with the following parameterization (leading-twist only)

$$\begin{aligned}\Phi^{[\mathcal{U}]}(x,p_T;P,S) = \tfrac{1}{2}\Big\{ & f_1(x,p_T^2)\,\not{P} + \tfrac{1}{2}\,h_{1T}(x,p_T^2)\,\gamma_5[\not{S}_T,\not{P}] \\ & + S_L\,g_{1L}(x,p_T^2)\,\gamma_5\not{P} + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,g_{1T}(x,p_T^2)\,\gamma_5\not{P} \\ & + S_L\,h_{1L}^{\perp}(x,p_T^2)\,\gamma_5\frac{[\not{p}_T,\not{P}]}{2M} + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,h_{1T}^{\perp}(x,p_T^2)\,\gamma_5\frac{[\not{p}_T,\not{P}]}{2M} \\ & + i h_1^{\perp}(x,p_T^2)\,\frac{[\not{p}_T,\not{P}]}{2M} - \frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp}(x,p_T^2)\,\not{P}\Big\}\,.\end{aligned}$$

In the collinear (transverse momentum integrated) case this parameterization of the correlator reads

$$\Phi(x;P,S) = \tfrac{1}{2}\left\{ f_1(x)\,\not{P} + S_L\,g_1(x)\,\gamma_5\not{P} + \tfrac{1}{2}\,h_1(x)\,\gamma_5[\not{S}_T,\not{P}] \right\}$$

Analogously, one can define a gauge invariant gluon correlator

$$\begin{aligned}\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu}(x,p_T) = \frac{n_\rho\,n_\sigma}{(p\cdot n)^2}\int\frac{d(\xi\cdot P)\,d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi} \\ \times\,\langle P,S|\;\mathrm{Tr}\big[\,F^{\mu\rho}(0)\,\mathcal{U}_{[0;\xi]}\,F^{\nu\sigma}(\xi)\,\mathcal{U}'_{[\xi;0]}\,\big]\,|P,S\rangle\,\big]_{\mathrm{LF}}\end{aligned}$$

It was a common belief that gauge fields should vanish at light-cone infinity and transverse Wilson lines could be ignored

In the axial gauge the Wilson lines $U^n$ should reduce to simple unit operators

**Gauge links could be effectively omitted $\Rightarrow$ vanishing of naively T-odd distributions**

In 2002 Brodsky, Hwang and Schmidt have shown by explicit calculation within a spectator diquark model of one-gluon FSI's in semi-inclusive DIS, that transverse single spin asymmetries can be nonvanishing at leading twist

Soon after Collins proved that this result is related to the proper treatment of gauge links and that

**time-reversal invariance does not imply the vanishing of the Sivers function**

Instead, the Sivers function should have opposite signs in SIDIS and Drell-Yan processes

$$\boxed{f_{1T}^{\perp}(x,p_T^2)\rfloor_{\text{Drell-Yan}} = -f_{1T}^{\perp}(x,p_T^2)\rfloor_{\text{SIDIS}}}$$

Gluonic initial and final state interactions lead to future pointing Wilson lines in SIDIS, while they give rise to past pointing Wilson lines in Drell-Yan processes

Wilson lines and the effects of gluonic initial and final state interactions are process dependent

In general universality properties of the naively T-odd distributions can be spoiled.
For SIDIS and DY processes a simple generalized form of universality seems to hold.

Collins and Metz have shown that inclusion of gauge links preserves universality for the Collins fragmentation function.
Very important also from a phenomenological point of view

For basic hadronic processes, SIDIS, Drell-Yan, $e^+e^-$ annihilation, the hard processes are (at tree level) simple e.m. vertices
$\Rightarrow$ only future/past pointing Wilson lines occur

For processes involving hadrons both in the initial/final states the situation is much more involved: even simple generalized universality properties can be lost
Different partonic processes can give rise to different gauge-link factors

# Theoretical information on TMD functions

Simple positivity bounds, e.g.

$$\left|\frac{\Delta^N f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp)}{2\,f_{q/p}(x,\boldsymbol{k}_\perp)}\right| = \left|\frac{f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) - f_{q/p^\downarrow}(x,\boldsymbol{k}_\perp)}{f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) + f_{q/p^\downarrow}(x,\boldsymbol{k}_\perp)}\right| \le 1$$

Soffer bound for the ($k_\perp$ dependent) transversity distribution

Generalized positivity bounds for $k_\perp$ moments of TMD distribution and fragmentation functions [Bacchetta, Boglione, Henneman, Mulders, PRL 85 (2000)]

$$|h_1| \le \frac{1}{2}(f_1 + g_{1L}) \le f_1$$

$$|h_{1T}^{\perp(1)}| \le \frac{1}{2}(f_1 - g_{1L}) \le f_1$$

$$\boxed{f_{1T}^{\perp(1)}(x) \equiv \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{2M^2} f_{1T}^\perp(x, |\boldsymbol{k}_\perp|)}$$

$$\begin{aligned}(g_{1T}^{(1)})^2 + (f_{1T}^{\perp(1)})^2 &\le \frac{\boldsymbol{p}_T^2}{4M^2}(f_1 + g_{1L})(f_1 - g_{1L}) \\ &\le \frac{\boldsymbol{p}_T^2}{4M^2} f_1^2, \\ (h_{1L}^{\perp(1)})^2 + (h_1^{\perp(1)})^2 &\le \frac{\boldsymbol{p}_T^2}{4M^2}(f_1 + g_{1L})(f_1 - g_{1L}) \\ &\le \frac{\boldsymbol{p}_T^2}{4M^2} f_1^2.\end{aligned}$$

# Theoretical information on TMD functions

Burkardt sum rule for the Sivers distribution

$$\langle \boldsymbol{k}_\perp \rangle = \sum_{a=q,\bar{q},g} \langle \boldsymbol{k}_\perp \rangle_a = \int \mathrm{d}x \int \mathrm{d}^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp \sum_{a=q,\bar{q},g} \Delta \hat{f}_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) = 0$$

Intuitively expected, the non trivial fact is the proof of its validity in presence of final state interactions that might spoil the simple partonic interpretation

[Burkardt PRD 69 (2004)]

Fulfilled by some model calculations of Sivers function and with good approximation by available parameterizations

Schäfer – Teryaev sum rule for the Collins function

[ Schäfer Teryaev PRD 61 (2000)]

$$\sum_h \int \mathrm{d}z\, z H_1^{\perp(1)q}(z) = \sum_h \int \mathrm{d}z \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{|\boldsymbol{k}_\perp|}{4M_h} \Delta^N D_{h/q^\uparrow}(z, |\boldsymbol{k}_\perp|) = 0$$

# Theoretical information on TMD PDFs and FFs: models

Spectator model for the nucleon with scalar and axial-vector diquarks
single gluon rescattering to generate T-odd distributions (Sivers, Boer-Mulders)
point-like, dipolar, Gaussian nucleon-quark-diquark vertex form factors
MIT bag model wave functions for quarks
[Gamberg, Goldstein; Bacchetta, Metz, Mulders, Conti, Radici, Burkardt,...]

Instanton liquid model for QCD vacuum + MIT bag model [Cherednikov et al.]

Light-cone constituent quark model
[Boffi, Efremov, Pasquini, Schweitzer]

Covariant parton model: relations among TMD and collinear PDFs
[Efremov, Schweitzer, Teryaev, Zavada; D'Alesio, Leader, FM]

Generalized Wandzura-Wilczek relations
[Mulders et al; Metz et al]

Large $N_c$ QCD and TMD parton distributions
[Pobylitsa]

**...and parameterizations [see talk by S. Melis]**

## Summary of interesting accessible processes and effects involved

| Process | Twist | Sivers | Collins | B-M | Pol FF | Theor. status | Discrim. power |
|---|---|---|---|---|---|---|---|
| SIDIS | 2 | ● | ● | ● | ● | * * * * | * * * * |
| Drell-Yan | 2 | ● | | ● | | * * * * | * * * * |
| $e^+e^- \to h_1 h_2\ X$ | 2 | | ● | | ● | * * * * | * * * * |
| $AB \to h\ X$ | 3 | ● | ● | ● | ● | * * | * * |
| $AB \to \gamma\ X$ | 3 | ● | | ● | | * * * | * * * * |
| $AB \to h_1 h_2 X$ | 2 | ● | ● | ● | ● | * * * | * * * |
| $AB \to j\ X$ | 3 | ● | | ● | | * * | * * * * |
| $AB \to j(h) X$ | 3 | ● | ● | ● | ● | * * * | * * * * |
| $AB \to j\ \gamma\ X$ | 2 | ● | | ● | | * * * | * * * * |

# Open points, outlook

- **Factorization for inclusive particle production in hadronic collisions in the TMD approach**
  [Mulders et al, Vogelsang, Yuan, Collins, Qiu…]
- **Evolution with scale of TMD distribution and fragmentation functions**
  [Henneman, Boer, Mulders; Ceccopieri, Trentadue; Cherednikov, Stefanis]
- **Soft factors from soft-gluon radiation (spin independent?)**
  [Ji, Ma, Yuan; Collins, Metz; formal aspects of factorization]
- **Potential suppression of azimuthal asymmetries due to Sudakov factors**
  [D. Boer]
- **Parton off-shellness , fully unintegrated parton correlation functions**
  [Watt, Martin, Riskin; Linnyk, Leupold, Mosel; Collins, Rogers, Stasto]
- **Experimental tests of universality (breaking?) for TMD PDFs and FFs**
- **Improved parameterizations and phenomenological constraints**

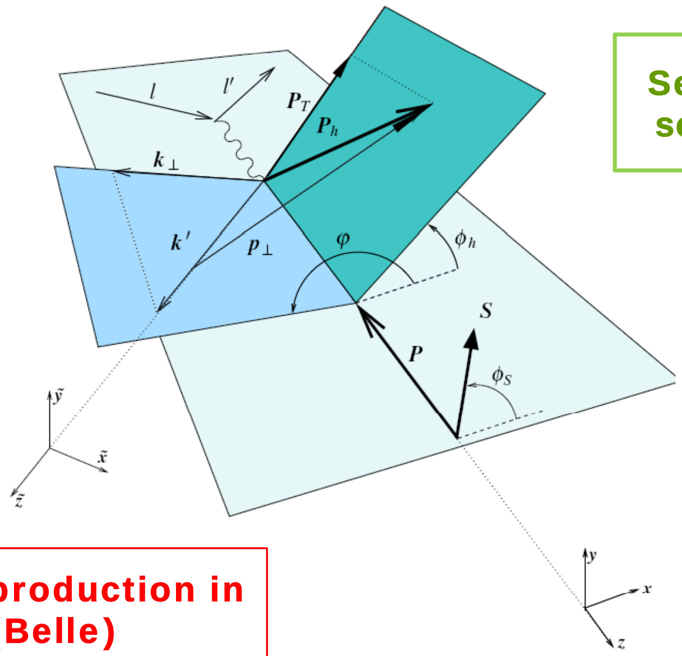
# Useful references on spin physics and TMDs

## [ reversed chronological order ]

- V. Barone, F. Bradamante, A. Martin, Prog. Part. Nucl. Phys. 65, in press (2010)
- U. D'Alesio, FM, Prog. Part. Nucl. Phys. 61, 394 (2008)
- V. Barone, P. Ratcliffe, Transverse Spin Physics, World Scientific (2003)
- V. Barone, A. Drago. P. Ratcliffe, Phys. Rep. 359, 1 (2002)
- E. Leader, Spin in Particle Physics, Cambridge UP (2001)

# Backup slides

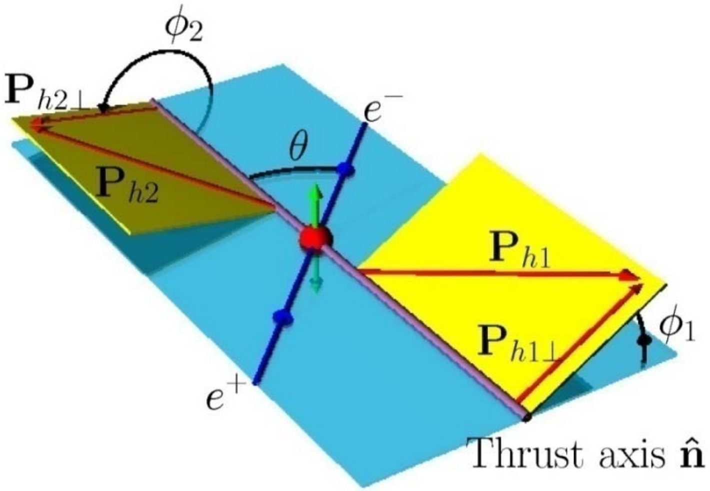
# Spin / azimuthal asymmetries in (un)polarized inclusive hadron production



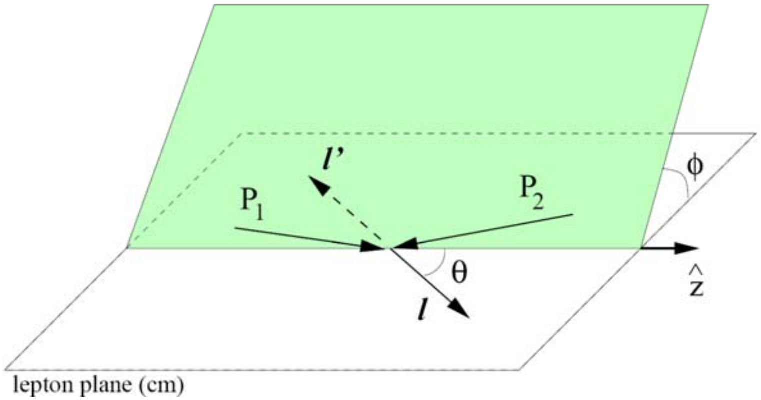

Semi-inclusive deeply inelastic scattering (SIDIS), $lp \to l'h+X$

HERMES-DESY
COMPASS-CERN
JLAB

inclusive hadron pair production in $e^+e^-$ collisions (Belle)




Fermilab
COMPASS-CERN (pol)
RHIC (pol)
PAX-GSI (antiprotons)



# Jaffe, Ji – Amsterdam group notation

f, g, h : unpolarized, longitudinally pol., transversely pol. quark

Subscript 1: leading twist distribution
Subscript L: longitudinally polarized hadron
Subscript T: transversely polarized hadron

Apex $\perp$ : presence of transverse momenta with uncontracted Lorentz indices

Uncontracted Lorentz indices

Transversely polarized quark

$$h_{1L}^{\perp}$$

Longitudinally polarized proton

Leading twist distribution

# TMD fragmentation functions – spin 1/2 hadrons, LT

$$\rho^{C}_{\lambda_C,\lambda'_C}\,\hat{D}_{C/c,s_c}(z,\boldsymbol{k}_{\perp C}) = \sum_{\lambda_c,\lambda'_c} \hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\rho^{c,s_c}_{\lambda_c,\lambda'_c}$$

$$D^{-\lambda_C,-\lambda'_C}_{-\lambda_c,-\lambda'_c}(z,k_{\perp C}) = (-1)^{2(s_c-S_C)}\,(-1)^{(\lambda_c+\lambda'_c)-(\lambda_C+\lambda'_C)}\,D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})$$

Summary of quark and gluons
helicity fragmentation amplitudes for spin 1/2 hadrons

$$\hat{D}^{C}_{c}(z,k_{\perp C}) = D^{++}_{++} + D^{++}_{--}$$

unpolarized FF

$$\Delta^N \hat{D}_{C/c^\uparrow}(z,\boldsymbol{k}_{\perp C}) = 4\,\mathrm{Im}D^{++}_{+-}\,\sin(\phi_{s_c}-\phi^H_C)$$

Collins FF

$$\Delta^N \hat{D}_{C/T^c_1}(z,\boldsymbol{k}_{\perp C}) = -4\,P^c_{lin}\mathrm{Re}D^{++}_{+-}\,\cos(2\phi_{l_c}-2\phi^H_C)$$

$$\Delta\hat{D}^{C/c}_{S_{Z_C}/s_z}(z,k_{\perp C}) = D^{++}_{++} - D^{++}_{--}$$

$$\Delta\hat{D}^{C/c}_{S_{Z_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = 2\,\mathrm{Re}D^{++}_{+-}\,\cos(\phi_{s_c}-\phi^H_C)$$

$$\Delta\hat{D}_{S_{Z_C}/c,s_z}(z,\boldsymbol{k}_{\perp C}) = -2\,P^c_{lin}\mathrm{Im}(D^{++}_{+-})\,\sin(2\phi_{l_c}-2\phi^H_C)$$

$$\Delta\hat{D}^{C/c}_{S_{X_C}/s_z}(z,k_{\perp C}) = 2\,\mathrm{Re}D^{+-}_{++}$$

$$\Delta\hat{D}^{C/c}_{S_{X_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = \left(D^{+-}_{+-} + D^{-+}_{+-}\right)\,\cos(\phi_{s_c}-\phi^H_C)$$

$$\Delta\hat{D}_{S_{X_C}/P_{lin}}(z,\boldsymbol{k}_{\perp C}) = -\left(\mathrm{Im}D^{+-}_{+-} + \mathrm{Im}D^{-+}_{+-}\right)\,\sin(2\phi_{l_c}-2\phi^H_C)$$

$$\Delta\hat{D}^{C/c}_{S_{Y_C}/c}(z,k_{\perp C}) = -2\,\mathrm{Im}D^{+-}_{++}$$

Polarizing FF

$$\Delta^-\hat{D}^{C/c}_{S_{Y_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = \left(D^{+-}_{+-} - D^{-+}_{+-}\right)\,\sin(\phi_{s_c}-\phi^H_C)$$

$$\Delta^-\hat{D}_{S_{Y_C}/P_{lin}}(z,\boldsymbol{k}_{\perp C}) = \left(\mathrm{Im}D^{+-}_{+-} - \mathrm{Im}D^{-+}_{+-}\right)\cos(2\phi_{l_c}-2\phi^H_C)$$

In a pQCD scheme, without proper gauge links time-reversal invariance would imply the vanishing of the Sivers and Boer-Mulders (all T-odd) distribution functions

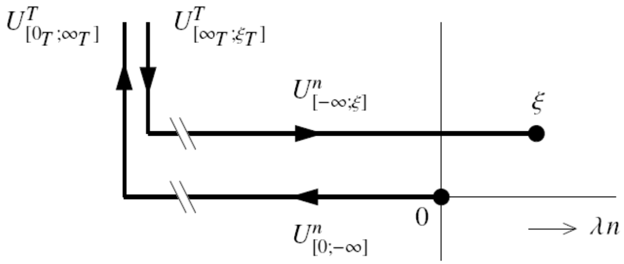
The same argument does not apply to the hadronic correlator for the fragmentation process, due to the explicit appearance of hadronic final state interactions

Historically, the Collins mechanism in the fragmentation process was introduced as an alternative way to explain large SSAs and to measure the transversity distribution in SIDIS

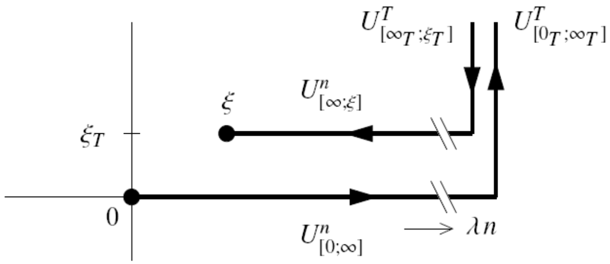
Wilson lines arise from resummation of gluonic initial and final state interactions between the hadronic remnants and the current (active) quarks. They may depend on the hard elementary scattering processes associated with the hadronic process considered

**Gluonic initial and final state interactions lead to future pointing Wilson lines in SIDIS, while they give rise to past pointing Wilson lines in Drell-Yan processes**

# A pictorial representation



past pointing Wilson line $\mathcal{U}^{[-]}$



future pointing Wilson line $\mathcal{U}^{[+]}$

$$\mathcal{U}^{[\pm]}_{[0;\xi]} = U^{n}_{[0;\pm\infty]}\, U^{T}_{[0_T;\infty_T]}\, U^{T}_{[\infty_T;\xi_T]}\, U^{n}_{[\pm\infty;\xi]}$$

$$U^{n}_{[\xi;\xi+\eta]} = \mathcal{P}\exp\left[-ig\int_0^{\eta\cdot P/n\cdot P} d\lambda\; n\cdot A^a(\xi+\lambda n)\,T^a\right] \qquad U^{T}_{[\xi;\xi+\eta]} = \mathcal{P}\exp\left[-ig\int_{\xi_T}^{\xi_T+\eta_T} d\zeta_T\;\cdot A^a_T(\zeta)\,T^a\right]$$

It was an early common belief that gauge fields should vanish at light-cone infinity and transverse Wilson lines could be ignored

In the axial gauge the Wilson lines $U^n$ should reduce to simple unit operators

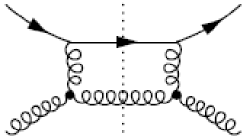
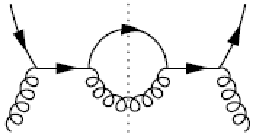
**Gauge links could be effectively omitted $\Rightarrow$ vanishing of naively T-odd distributions**

In 2002 Brodsky, Hwang and Schmidt have shown by explicit calculation within a spectator diquark model of one-gluon FSI in semi-inclusive DIS that transverse single spin asymmetries can be nonvanishing at leading twist [Sivers effect]

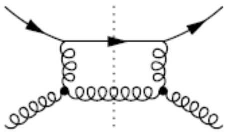
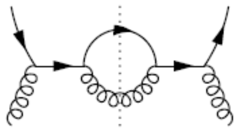
Soon after Collins has proven that this result is related to the proper treatment of gauge links and that

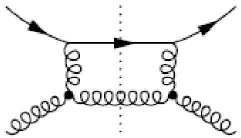
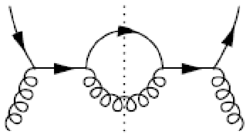
**time-reversal invariance does not imply the vanishing of the Sivers function**

## Gluonic pole hard scattering cross sections (e.g. $qg \longrightarrow qg$)

$$\frac{d\hat{\sigma}_{qg\to qg}}{d\hat{t}} =$$

$$+$$

$$+ \cdots$$

$$= -\frac{4\pi\alpha_S^2 T_F^2}{\hat{s}^2}\,\frac{\hat{s}^2+\hat{u}^2}{2\,\hat{s}\hat{u}}\left\{\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right\}$$

$$\frac{d\hat{\sigma}_{[q]g\to qg}}{d\hat{t}} = \Phi^{[D_1]}(x,p_T)\otimes$$

$$+\,\Phi^{[D_2]}(x,p_T)\otimes$$

$$+ \cdots$$

$$= C_G^{[D_1]}(q_i)$$

$$+\,C_G^{[D_2]}(q_i)$$

$$+ \cdots$$

**gauge invariance preserved!**

$$= -\frac{4\pi\alpha_S^2 T_F^2}{\hat{s}^2}\,\frac{\hat{s}^2+\hat{u}^2}{2\,\hat{s}\hat{u}}\left\{\frac{\hat{s}^2}{\hat{t}^2} - \frac{N^2+1}{N^2-1}\left\{\frac{\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right\}\right\},$$

## An alternative procedure: invariants products of physically independent momenta: the case of $A(S_A) \to a(s_a) + X$

There are at most 4 independent (unit) momenta:
2 real vectors ($\mathbf{p}_A$ and $\mathbf{k}_{\perp a}$) and 2 polarization (pseudo)vectors ($\mathbf{S}_A$ and $\mathbf{s}_a$)

Evaluate all nontrivial independent scalar combinations of their dot and cross products: to each of them associate a leading-twist independent distribution

A) Single spin, hadron transversely polarized, parton unpolarized

$$(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{S}}_A$$

B) Single spin, hadron unpolarized, parton transversely polarized

$$(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{s}}_a$$

C) Double spin, both hadron and parton (transv. and/or long.) polarized

$$\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{S}}_A \, (PS) \qquad \hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{s}}_a \, (PS) \qquad \hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{S}}_A \, (PS) \qquad \hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{s}}_a \, (PS) \qquad \hat{\boldsymbol{S}}_A \cdot \hat{\boldsymbol{s}}_a \, (S)$$

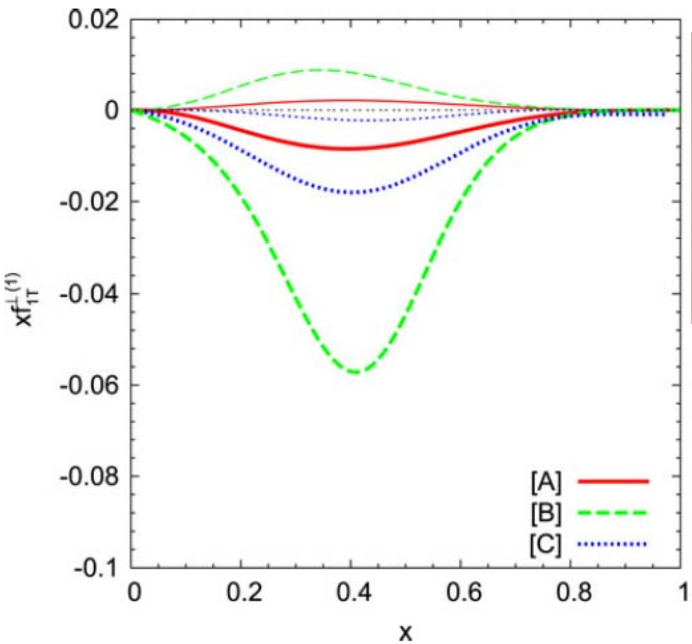
Notice: combinations of cross products (V and PV) are not independent:

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) = (\boldsymbol{a} \cdot \boldsymbol{c})(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{c})$$

$$
\begin{aligned}
&\Phi_{a/A}(x_a, \hat{\boldsymbol{p}}_A, \hat{\boldsymbol{k}}_{\perp a}; \boldsymbol{P}^A, \boldsymbol{P}^a) \\
&\quad = \frac{1}{2}\Big\{ f_{a/A}(x_a, k_{\perp a}) \cdot 1 \\
&\quad + \Delta^N f_{a^\uparrow/A}(x_a, k_{\perp a})\, (\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{s}}_a \\
&\quad + \frac{1}{2}\Delta^N f_{a/A^\uparrow}(x_a, k_{\perp a})\, (\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{S}}_A \\
&\quad + \Delta^- f_{s_y/S_T}(x_a, k_{\perp a}) \left[ \hat{\boldsymbol{S}}_A \cdot \hat{\boldsymbol{s}}_a - (\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{s}}_a) - (\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{s}}_a) \right] \\
&\quad + \Delta f_{s_z/S_Z}(x_a, k_{\perp a})\, (\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{s}}_a) \\
&\quad + \Delta f_{s_x/S_Z}(x_a, k_{\perp a})\, (\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{s}}_a) \\
&\quad + \Delta f_{s_z/S_T}(x_a, k_{\perp a})\, (\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{p}}_A \cdot \hat{\boldsymbol{s}}_a) \\
&\quad + \Delta f_{s_x/S_T}(x_a, k_{\perp a})\, (\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{S}}_A)(\hat{\boldsymbol{k}}_{\perp a} \cdot \hat{\boldsymbol{s}}_a) \Big\}
\end{aligned}
$$

# Theoretical information on TMD PDFs and FFs: models

TMD distributions:
spectator model for the nucleon with scalar and axial-vector diquarks [ B]
single gluon rescattering to generate T-odd distributions (Sivers, Boer-Mulders)
Different choices for the nucleon-quark-diquark form factor (point-like, dipolar, Gaussian)
MIT bag model wave functions for quarks [A]
Instanton liquid model for QCD vacuum + MIT bag model [C]




Sivers function (first $k_T$ moment)
[A] Yuan PLB 575 (2003)
[B] Bacchetta, Schäfer, Yang PLB 578 (2004)
[C] Cherednikov et al PLB 642 (2006)
Thik lines: u quark
Thin lines: d quark

Gamberg, Goldstein et al
Lu, Ma
Brodsky, Yuan

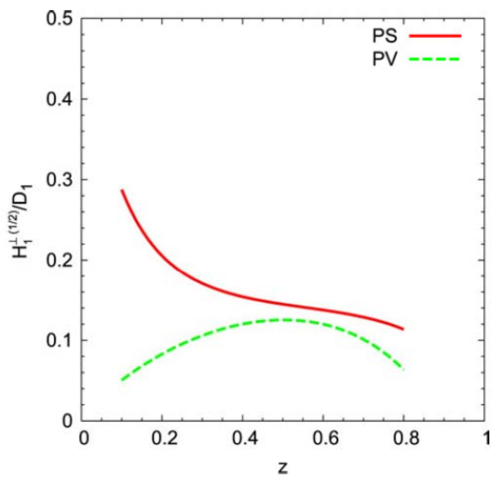
# Theoretical information on TMD PDFs and FFs: models

## Boer-Mulders function

All models predict a negative BM function for both u and d quarks
[Gamberg  Goldstein - Bacchetta Schäfer, Yang]
Same conclusion reached relating it with the Fourier transform of chirally odd GPDs
[Burkardt, Hannafious ,PLB 658 (2008)]

## Collins fragmentation function

Different calculations  modelling the fragmentation process
at tree level with insertion of one-loop corrections
pseudoscalar/pseudovector pion-quark couplings with pion/gluon loops
[Bacchetta Kundu Metz Mulders -  Gamberg Goldstein Oganessyan]




Amrath, Bacchetta, Metz
PRD 71 (2005)

# Useful references on spin physics
## [ reversed chronological order ]


- V. Barone, F. Bradamante, A. Martin, Prog. Part. Nucl. Phys. 65, in press (2010)
- U. D'Alesio, FM, Prog. Part. Nucl. Phys. 61, 394 (2008)
- V. Vogelsang, J. Phys. G 34, S149 (2007)
- S.D. Bass, Rev. Mod. Phys. 77 1257 (2005)
- V. Barone, P. Ratcliffe, Transverse Spin Physics, World Scientific (2003)
- V. Barone, A. Drago. P. Ratcliffe, Phys. Rep. 359, 1 (2002)
- E. Leader, Spin in Particle Physics, Cambridge UP (2001)
- B.W. Filippone, X.-D. Ji, Adv. Nucl. Phys. 26, 1 (2001)
- G. Bunce, N. Saito, J. Soffer, W. Vogelsang, Ann. Rev. Nucl. Part. Sci. 50, 525 (2000)
- B. Lampe, E. Reya, Phys. Rep. 332, 1 (2000)
- Z.-t. Liang, C. Boros, Int. J. Mod. Phys. A 15, 927 (2000)
- J. Felix, Mod. Phys. Lett. A 14, 827 (1999)
- S.B. Nurushev, Int. J. Mod. Phys. A 12, 3433 (1997)
- U. Stiegler, Phys. Rep. 277, 1 (1996)
- M. Anselmino, A. Efremov, E. Leader, Phys. Rep. 261, 1 (1995)
- S.M. Troshin, N.E. Tyurin, Spin Phenomena in Particle Interactions, WSP (1994)
- L.G. Pondrom, Phys. Rep. 122, 57 (1985)
- C. Bourrely, J. Soffer, E. Leader, Phys. Rep. 59, 95 (1980)