## Aurore Courtoy -INFN Pavia

 withAlessandro Bacchetta<br>Marco Radici

## Flavor decomposition of Dihadron Fragmentation Function and

 its relevance for Transversity
## Transverse Spin \& TMDs

## From DIS to Semi-Inclusive DIS

- 3 leading-twist PDFs:

- Transversity not accessible through inclusive DIS
- chiral-odd
- we go to Semi Inclusive DIS
- one more variable $\mathbf{k}_{\perp}$
- Lorentz expansion of all the possible functions
- birth of TMDs



## Ways to Transversity

## SIDIS on pi



## TMD factorization

- Convolution
- Soft factors
- Evolution
- Complex universality
$d \sigma \propto \sum_{q}\left[h_{1}^{q} \otimes H_{1}^{\perp q}\right]\left(x, z, P_{h \perp}^{2}\right)$
chiral-odd partner


## Ways to Transversity

## SIDIS on p



## TMD factorization

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$d \sigma \propto \sum_{q}\left[h_{1}^{q} \otimes H_{1}^{\perp q}\right] \underset{\sim}{\left(x, z, P_{h \perp}^{2}\right)}$


## Ways to Transversity

## SIDIS on p



## Ways to Transversity

## SIDIS on p



## Transverse Spin from Fragmentation Functions

Distribution of hadrons inside the jet
$\longrightarrow$ Direction of the transverse polarization of the fragmenting quarks
Also unpolarized
(6 TMD FF $D_{1}^{q \rightarrow h}\left(z, \kappa_{T}^{2}\right)$


- DiFF

$$
D_{1}^{q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, R_{T}^{2}\right)
$$



## Transverse Spin from Fragmentation Functions

## Interference Fragmentation Functions



$$
H_{1, q \rightarrow h_{1} h_{2}}^{\varangle}\left(z_{1}, z_{2}, R_{T}^{2}\right)
$$

relates transverse polarization of the fragmenting quark to angular distribution of the hadron pairs in the transverse plane
$\checkmark$ Naive T-odd ; chiral-odd
$\checkmark$ Does not vanish if integrated over transverse momentum $\mathbf{k}_{\perp}$
$\checkmark$ The two hadrons must be distinguishable

## Framework for DiFF

## SIDIS on $\mathrm{p} \uparrow$



Collinear factorization

- Universality
- No convolution
- Evolution understood

e+e- to pions

$\mathrm{pp} \uparrow$ to pions



## Є+e- : q9 correlator for DiFF



$$
d \sigma \propto \frac{\alpha^{2}}{Q^{6}} L_{\mu \nu} W_{4 h}^{\mu \nu}
$$

Boer, Jakob, Radici, PRD 67 (03) Bacchetta, Radici , PRD 67(03)

$$
W_{4 h}^{\mu \nu} \propto \sum_{a} \int d \mathbf{k}_{T} d \overline{\mathbf{k}}_{T} \delta^{2}() \operatorname{Tr}\left[\left.\left.\int d \bar{k}^{-} \bar{\Delta}\right|_{\bar{k}+\ldots} \gamma^{\mu} \int d k^{+} \Delta\right|_{k^{-}-\ldots} \gamma^{\nu}\right]
$$

## Є+e- : q9 correlator for DiFF

Boer, Jakob, Radici, PRD 67 (03)


Bacchetta, Radici , PRD 67(03)


$$
\begin{aligned}
& \mathcal{P}_{-} \Delta_{a}\left(z, \cos \theta, M_{h}^{2}, \phi_{R}\right) \gamma^{-} \\
& =\frac{2|\vec{R}|}{8 \pi M_{h}}\left(D_{1}^{a}\left(z, \zeta(\cos \theta), M_{h}^{2}\right)+i H_{1}^{\varangle a}\left(z, \zeta(\cos \theta), M_{h}^{2}\right) \frac{|\vec{R}|}{M_{h}} \sin \theta \gamma^{\mu} n_{\mu}\right) \mathcal{P}_{-}
\end{aligned}
$$



## Є+e- : q9 correlator for DiFF



Boer, Jakob, Radici, PRD 67 (03) Bacchetta, Radici , PRD 67(03)


$$
\mathcal{P}_{-} \Delta_{a}\left(z, \cos \theta, M_{h}^{2}, \phi_{R}\right) \gamma^{-} \quad \text { Integrated over kT }
$$

$$
\frac{2|\vec{R}|}{M_{h}} F_{1}\left(z, \zeta(\cos \theta), M_{h}^{2}\right)=\sum_{n} F_{1, n}\left(z, M_{h}^{2}\right) P_{n}(\cos \theta)
$$



## Physics of the DiFF

## Main approximation:

 truncation of the partial wave analysis up to 2nd order$\Rightarrow L=0,1$ relative partial waves
$\Rightarrow$ terms $\propto 1, \cos \vartheta, \sin \vartheta, \cos \vartheta \sin \vartheta$
s-wave $\rightarrow$ unpolarized
interference $b /$ w unpolarized pair (s-wave) and longitudinally pol. pair (p-wave)
$D_{1}^{q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, R_{T}^{2}\right) \quad \rightarrow$ sor p waves
$H_{1, q \rightarrow h_{1} h_{2}}^{\varangle}\left(z_{1}, z_{2}, R_{T}^{2}\right) \quad \rightarrow$ interf. s \& p waves

## The Asymmetiry in ete-



$$
A\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\frac{\left\langle\cos \left(\phi_{R}+\phi_{\bar{R}}\right)\right\rangle}{\langle 1\rangle}
$$

$$
A^{\cos \left(\phi_{R}+\phi_{\bar{R}}\right)}\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right) \propto \frac{\sum_{q} e_{q}^{2} H_{1, q}^{\varangle}\left(z, M_{h}^{2}\right) H_{1, q}^{\varangle}\left(\bar{z}, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1, q}\left(z, M_{h}^{2}\right) \bar{D}_{1, q}\left(\bar{z}, \bar{M}_{h}^{2}\right)}
$$


$\checkmark Q^{2} \sim 100 \mathrm{GeV}^{2}$
$\checkmark$ (z, M_h) correlation
$\checkmark 4$ plots
$\Rightarrow$ limited range in z
$\checkmark$ large errors
$\checkmark 8 \times 8$ (Mh1, Mh2)
$\checkmark 9 x 9(z 1, z 2)$

$\Rightarrow$ red curves:
spectator model result

## Data not published yet



Peaks at
i. $\quad M_{h} \sim m_{\rho}=770 \mathrm{MeV}$
ii. $\quad M_{h} \sim m_{\omega}=782 \mathrm{MeV}$
iii. broad peak at $\quad M_{h} \sim 500 \mathrm{MeV}$

Most prominent channels at $M_{h} \leq 1.8 \mathrm{GeV}^{2}$

1. Background

$$
q \rightarrow \pi^{+} \pi^{-} X_{1}
$$

2. $\rho$ production

$$
q \rightarrow \rho X_{2} \rightarrow \pi^{+} \pi^{-} X_{2}
$$

3. $\omega$ production

$$
q \rightarrow \omega X_{3} \rightarrow \pi^{+} \pi^{-} X_{3}
$$

$$
q \rightarrow \omega \pi^{0} X_{4}^{\prime} \rightarrow \pi^{+} \pi^{-} \pi^{0} X_{4}^{\prime}
$$

undetected $\pi 0$

## Monte Carlo from BELLE

## Unpolarized cross section

## Monte Carlo from BELLE

## Unpolarized cross section



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## Monte Carlo from BELLE

## Unpolarized cross section



## Monte Carlo from BELLE

## Unpolarized cross section


e.g. uds from $\omega$ channels

-4 zbins

- Flavor decomposition
- uds
- charm
$\rho$ channel
$\omega$ channels
non resonant contrib.

NB: in our analysis, we neglect resonant channels contribution to the charm

## Unpolarized Cross Section

## Constraints on the Functional Form from

1. the kinematics
2. the ss and pp interference-like
3. physics model-inspired

I want to fit:
with a functional form like:

$$
d \sigma \propto 2 \frac{6 \alpha^{2}}{Q^{2}} \frac{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}{2 M_{h}} f_{D_{1}}^{a}\left(z, M_{h}\right) \int_{0.2}^{1} \int_{0.28}^{2} f_{D_{1}}^{\bar{a}}\left(\bar{z}, \bar{M}_{h}\right)
$$

$$
f_{D_{1}}^{a}\left(z, M_{h}\right)=2 M_{h} z^{2} \sum_{a} \sqrt{e_{a}^{2} D_{1 a}^{s s+p p}\left(z, M_{h}^{2}\right)}
$$

Error on the MC: Jnumber of events Functional form inside de integration routine Propagation of errors ...

## Unpolarized Cross Section

## Constraints on the Functional Form from

1. the kinematics
2. the ss and pp interference-like
3. physics model-inspire^

## Constraints coming while determining the Functional Form


with a functional form

Error on the MC: Jnumber of events Functional form inside de integration routine Propagation of errors ...

## A.C., Baccielite anuc Radici

 in preparation
## uds from p


$D_{1, \rho}^{u d s}\left(z, M_{h}\right) \propto A \sqrt{M_{h}^{2}-4 m_{\pi}^{2}} \times\left(1-R^{3}\right)^{B} z^{C} \times \frac{\exp f\left(M_{h}, z\right)}{\left(M_{h}^{2}-m_{\rho}^{2}\right)^{2}+m_{\rho}^{2} \Gamma_{\rho}^{2}}$


$$
\frac{\exp f\left(M_{h}, z\right)}{\left.2-m_{\rho}^{2}\right)^{2}+m_{\rho}^{2} \Gamma_{\rho}^{2}}
$$



A.C., Baccirilitelna Radioi, in preparation

Global fit result for $\mathrm{z}=0.25$


## uds from w

## ( $z, M \_h$ ) fit of the $4 z$-bins

$D_{1, \omega}^{u d s}\left(z, M_{h}\right) \propto \sqrt{M_{h}^{2}-4 m_{\pi}^{2}}\left(A \frac{(1-z)^{B} z^{C}}{\left(M_{h}-m_{\omega}^{2}\right)^{2}+m_{\omega}^{2} \Gamma_{\omega}^{2}}\right.$ $\left.+D \sqrt{M_{h}^{2}-4 m_{\pi}^{2}}(1-z)^{E} z^{F} \times \frac{\exp f\left(z, M_{h}\right) \exp f^{\prime}(z)}{\left(M_{h}^{2}-G^{2}\right)^{2}+H^{2}}\right)$

$\chi^{2} /$ d.o.f $\sim 1.3$

A.C., Baccheliatanu Radici, in preparation


$$
D_{1, b k g d}^{u d s}\left(z, M_{h}\right) \propto \sqrt{M_{h}^{2}-4 m_{\pi}^{2}} A(1-z)^{B} z^{C} e^{f\left(z, M_{h}\right)} e^{f^{\prime}(z)}
$$




## 

 in preparation
( $z, M_{-}$h ) fit
of the 4 -bins




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1 want to fit:
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$$
\begin{aligned}
& d \sigma \propto \frac{6 \alpha^{2}}{Q^{2}} \frac{\pi^{2}}{16} \frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{2 M_{h}} f_{H_{1}^{\triangleleft}}^{a}\left(z, M_{h}\right) \int_{0.2}^{1} \int_{0.28}^{2} f_{H_{1}^{\top}}^{\bar{a}}\left(\bar{z}, \bar{M}_{h}\right) \\
& f_{H_{1}^{\triangleleft}}^{a}\left(z, M_{h}\right)=2 M_{h} z^{2} \frac{|\vec{R}|}{M_{h}} \sum_{a} e_{a}^{2} H_{1 a}^{s p}\left(z, \xi, M_{h}^{2}\right) \\
&=z^{2} \sqrt{M_{h}^{2}-4 m_{\pi}^{2}} \sum_{a} e_{a}^{2} H_{1 a}^{s p}\left(z, M_{h}^{2}\right)
\end{aligned}
$$

Error on $\sigma$ : error on the data \& error on the fit of unpol. $\sigma$ 1st step: no integration but bin value from experiment. Propagation of errors

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&=z^{2} \sqrt{M_{h}^{2}-4 m_{\pi}^{2}} \sum_{a} e_{a}^{2} H_{1 a}^{s p}\left(z, M_{h}^{2}\right)
\end{aligned}
$$

further assumption
$H_{1}^{\varangle u}\left(z, M_{h}\right)=f\left(z, M_{h}\right) D_{1}^{u}\left(z, M_{h}\right)$
Error on $\sigma$ : error on the data \& error on the fit of unpol. $\sigma$ 1st step: no integration but bin value from experiment.
Propagation of errors

## IFF from the Asymmetry

Fit of the sum over flavors of

$$
\sum_{a} e_{a}^{2} H_{1 a}^{\varangle s p}\left(z, M_{h}^{2}\right) \bar{H}_{1 \bar{a}}^{\varangle s p}\left(\langle\bar{z}\rangle,\left\langle\bar{M}_{h}^{2}\right\rangle\right)
$$

## Assumptions

- role of flavor decomposition from UNPOLARIZED FF

$$
\begin{aligned}
& D_{1}^{u}\left(z, M_{h}\right)=D_{1}^{\bar{u}}\left(z, M_{h}\right)=D_{1}^{d}\left(z, M_{h}\right)=D_{1}^{d}\left(z, M_{h}\right) \\
& D_{1}^{s}\left(z, M_{h}\right)=D_{1}^{\bar{s}}\left(z, M_{h}\right) \\
& D_{1}^{c}\left(z, M_{h}\right)=D_{1}^{\bar{c}}\left(z, M_{h}\right)
\end{aligned}
$$

Montecarlo uds-c

- Two (or more) scenarios

$$
\begin{aligned}
& \text { I. } . \quad D_{1}^{s}\left(z, M_{h}\right)=0 \\
& \text { II. } . D_{1}^{s}\left(z, M_{h}\right)=D_{1}^{u}\left(z, M_{h}\right)
\end{aligned}
$$

$\underline{H_{1}<}$

- flavor decomposition

$$
\begin{aligned}
& H_{1}^{\varangle u}\left(z, M_{h}\right)=H_{1}^{\varangle d}\left(z, M_{h}\right)=-H_{1}^{\varangle d}\left(z, M_{h}\right)=-H_{1}^{\varangle \bar{u}}\left(z, M_{h}\right) \\
& H_{1}^{\varangle s}\left(z, M_{h}\right)=H_{1}^{\varangle \bar{s}}\left(z, M_{h}\right)=0 \\
& H_{1}^{\varangle c}\left(z, M_{h}\right)=H_{1}^{\varangle \bar{c}}\left(z, M_{h}\right)=0
\end{aligned}
$$

## Flavor Decomposition: The Scenarios

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## Flavor Decomposition: The Scenarios



## Flavor Decomposition: The Scenarios



## Flavor Decomposition: The Scenarios






## Flavor Decomposition: The Scenarios



Flavor decomposition influences $f(z, M h)$

## Strange and Transversity...

## SIDIS on $\mathrm{p} \uparrow$



$$
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right) \propto f(y) \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x)} \times \frac{H_{1}^{\varangle, u}\left(z, M_{h}\right)}{D_{1}^{u}\left(z, M_{h}\right)}
$$

- then we are left with a one variable fit


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$$
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$$

Flavor decomposítion scenario 1. and 11.

- then we are left with a one variable fit


## Strange and Transversity...

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$$
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$$

Flavor decomposition of $f_{1}(x)$ Depends on the parametrization we use To be studied...

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## Strange and Transversity...

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$$
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$$

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## Back to the Transversity...

## HERMIMES



Figure 2: The top panels show $A_{U \perp}^{\sin \left(\phi_{R \perp}+\phi_{S}\right) \sin \theta}$ versus $M_{\pi \pi}, x$, and $z$. The bottom panels show the average values of the variables that were integrated over. For the dependence on $x$ and $z$, $M_{\pi \pi}$ was constrained to the range $0.5 \mathrm{GeV}<M_{\pi \pi}<1.0 \mathrm{GeV}$, where the signal is expected to be largest. The error bars show the statistical uncertainty. A scale uncertainty of $8.1 \%$ arises from the uncertainty in the target polarization. Other contributions to the systematic uncertainty are summed in quadrature and represented by the asymmetric error band.

## Di-hadron Fragmentation Functions

probability for (un-)polarized quarks to fragment into the hadron pair (h1 h2)

## In particular, IFF

relates transverse polarization of the fragmenting quark to angular distribution of the hadron pairs in the transverse plane

- Collinear factorization
- Universality
- No convolution
- Evolution understood


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## Flavor Decomposition

- Rôle on the param of $H_{1}^{\varangle}$
- Monte Carlo input
- Data for Kaons
- BaBar data


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## DiFF way to Transversity

- We have fitted the $D_{1}$ DiFF from the BELLE experiment
- We have almost extracted $H_{1}^{\varangle}$ from the BELLE data
- Next step: Go down to SIDIS and extract Transversity

