Introduction to Fracture Functions

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Fracture Functions
Introduction & motivations

Semi-inclusive distributions (initial state)

Applications (Diffraction, Polarization,...)

Scaling in diffraction (recent results: H1)

some more recent applications

Initial states in hadronic processes:

Multiple hadrons within FF formalism

New Jet-like Fracture Functions

Semi Inclusive Deep Inelastic Scattering
the
Current + Target
Fragmentation issue

The idea of Fracture Functions originates from the need to extend the description of the semi-inclusive hadronic processes in deep inelastic scattering to include the initial state target fragmentation region. It could seem a natural task, in fact, the one of a complete description of the final state entirely in terms of the collinear and infrared logarithmic structure of QCD in its

perturbative phase. The formulation of the initial state dynamics to include the rich complexity of the QCD-improved parton model with his quark and gluon degrees of freedom was not considered. This fact appared even more

needed at the time when the new HERA lepton-proton collider was beginning to operate in DESY.

The dynamics of the target fragmentation naturally extends the perturbative region of applicability of the QCD theory. It involves the description of quantitively important processes which are softer than the hard current fragmentation. It, therefore, deals with physics scales which are smaller and at the limits of the perturbative region and, also for this reason, it constitutes a complementary dynamics with respect to the current fragmentation. Both target and current fragmentation have to be taken into account in order to reproduce the entire final state without imposing unnatural cuts to separate them.

motivations

Let us at this point recall the idea and the motivations for the fracture functions with the same words we used as taken from Ref.[5]: "When one or two hadrons are present in the initial state, collinear singularities cannot be avoided. Asymptotic freedom, however, is still of much importance. Together with general factorization theorems for collinear singularities[8], it allows to justify the so-called QCD-improved parton model whereby experimental cross-sections can be computed by convoluting some uncalculable, but process independent, quantities with process-dependent, but calculable, elementary cross-sections. The best known case of this type is undoubtfully that of structure functions, which can be measured in deep inelastic lepton-hadron collisions in some kinematical regime and then used to compute either the same process or a completely new hard reaction at a different scale. Besides this utilitaristic value, structure functions have also provided, for many years, an invaluable

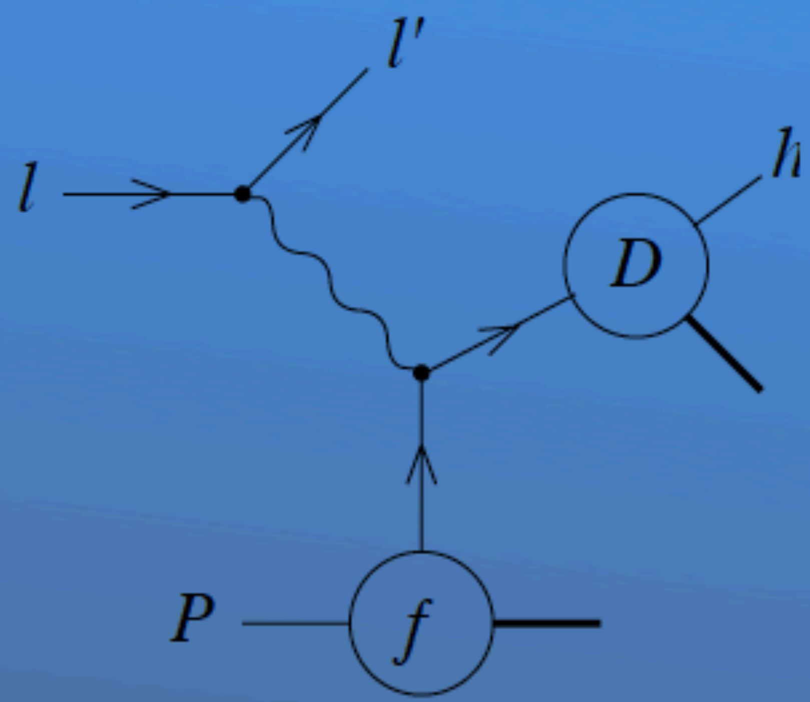
sourc … nce

and s … po-

larization state. Another much studied set of uncalculable, universal functions is that of so-called fragmentation functions, providing the probability that a given hadron is produced (inclusively) in a jet initiated by a given parton. A typical use of factorization resides here in the possibility of computing multihadron final states in jet physics, by convoluting the above fragmentation functions with the calculable perturbative jet evolution[9]. With the advent

Structure & Fragmentation

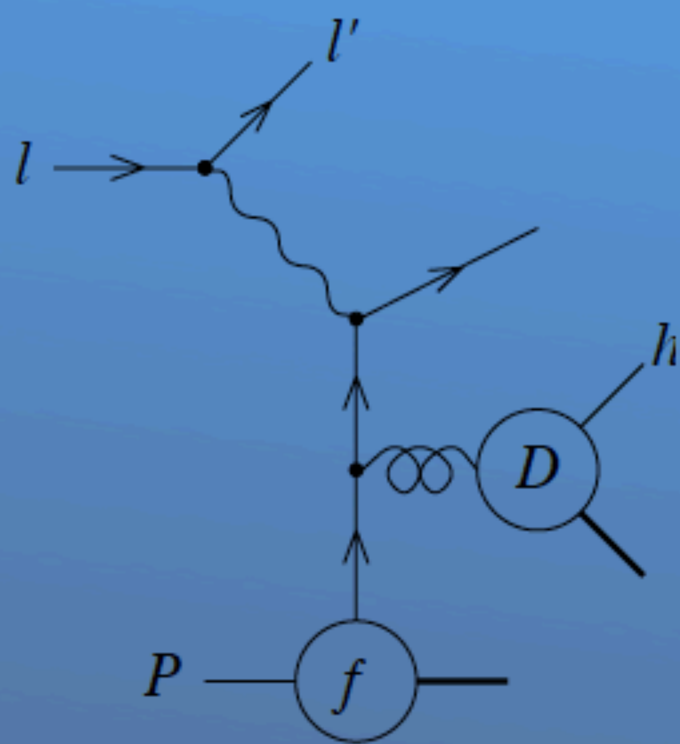
Semi Inclusive Deep Inelastic Scattering
Current Fragmentation

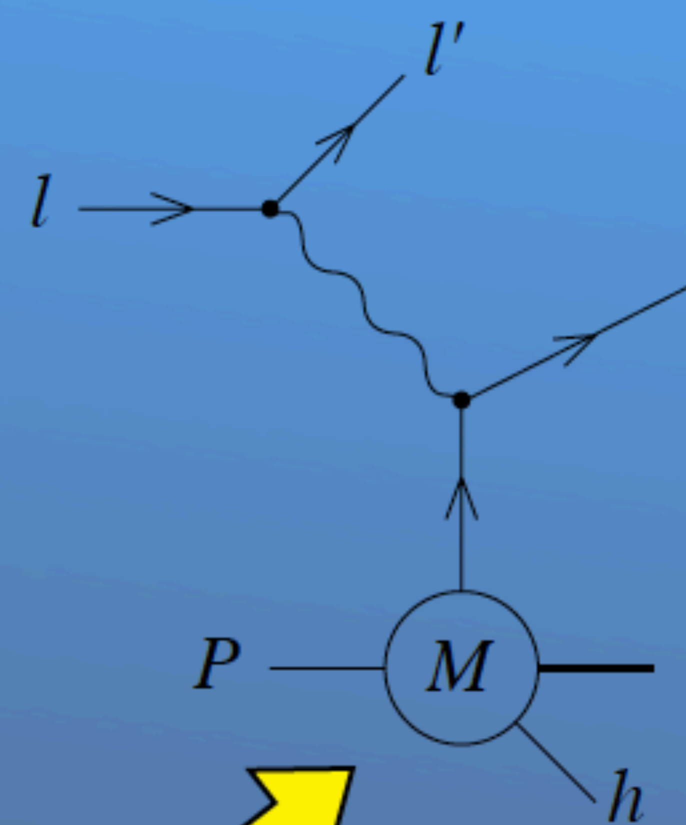


$$\sigma_C = \int \frac{dx'}{x'} \frac{dz'}{z'} F_P^i(x', Q^2)\, \hat{\sigma}_{ij}(x/x', z_h/z', Q^2)\, D_h^j(z', Q^2)$$

Hadrons may also come from elsewhere !

Semi Inclusive Deep Inelastic Scattering
Target Fragmentation





Fracture Function

$$\sigma_T = \int \frac{dx'}{x'} M^i_{Ph'}(x', z_h, Q^2)\, \hat{\sigma}_i(x/x', Q^2)$$

L.Trentadue and G.Veneziano, Phys.Lett. **B323** (1994) 201

Fracture Functions = Fragmentation & structure functions with the calculable perturbative jet evolution[9]. With the advent of the new powerful electron-proton collider HERA at DESY, more phase space is becoming available together with a richer variety of channels. One may thus wonder if the only QCD-inspired use of the machine should be the refined measurements of structure and fragmentation functions together with tests of their predictable evolution and factorization properties. There seems to be some widespread consensus that this should not be the case and that, on the contrary, the study of hadron structure can be extended at HERA in new directions. Actually, already at hadronic colliders, there have been studies[7] of quantities such as the Pomeron structure function, diffractive hard scattering and the like, with stimulating outcomes. The aim of this paper is to not diffraction only !

give a proper framework in which to talk about these extensions of "bread and butter" QCD physics. We shall argue that, within perturbatlve QCD, it is possible to introduce new uncalculable, but measurable and universal functions, that we call "fracture" functions, which tell us about the structure function of a given target hadron once it has fragmented (hence its name) into another given final state hadron. Fracture functions (besides exhibiting a mild, calculable Q^2 dependence) depend upon two hadronic and one partonic label and on two momentum fractions, a Bjorken x and a Feynman z variable $M = M^j_{p,h}(x, z, Q^2)$. One can also say that M measures the parton distribution of the object exchanged between the target and the final hadron, without making a (possibly doubtful) model about what that object actually is, a single particle, a Regge trajectory, a multiparticle continuum, or else. As for ordinary structure functions, the importance of measuring such an object will be twofold: i) it will teach us about the structure of hadronic systems other than the usual targets, and ii) it can be used as input for computing other hard semi-inclusive processes at other machines, such as some future hadronic colliders. By a judicious choice of the final hadron and of its momentum, one will be able, for instance, to enrich the gluonic component of the partonic flux and thus to enhance signal to background ratios for interesting gluon-induced processes in hadron-hadron collisions".

Properties:

- Do not depend on the arbitrary choosen scale Q_0^2 i.e.

$$\frac{\partial}{\partial Q_0^2} M_{p,h}^j(x,z,Q^2) = 0$$

- Both $D_l^h(x,Q^2)$ and $F_p^i(x,Q^2)$ satisfy the usual Altarelli Parisi evolution equations and $\sum_h \int_0^1 dz\, z\, D_l^h(x,Q^2) = 1$ and $\sum_i \int_0^1 dx\, x\, F_p^i(x,Q^2) = 1$ with

$$\sum_i \int_0^1 du\, u\, P_i^j(u) = 0 \qquad (8)$$

$M_{p,h}^j(x,z,Q^2)$ satisfies the momentum sum rule:

$$\sum_h \int_0^1 dz\, z\, M_{p,h}^j(x,z,Q^2) = (1-x)\, F_p^j(x,Q^2)$$

accounting for the s-channel unitarity constraint.

proof within the jet-calculus formalism

The combination of the Fracture Function with the target-Fragmentation evolution gives the evolution equation:

$$\frac{\partial}{\partial \log Q^2} M_{i,h/p}\left(\xi,\zeta,Q^2\right) = \frac{\alpha_s(Q^2)}{2\pi}\int_{\xi/(1-\zeta)}^{1}\frac{du}{u}\,P_j^i(u)\,M_{j,h/N}\left(\frac{\xi}{u},\zeta,Q^2\right)$$
$$+\frac{\alpha_s(Q^2)}{2\pi}\int_{\xi}^{\xi/(\xi+\zeta)}\frac{du}{\xi(1-u)}\,\hat{P}_j^{i,l}(u)\,f_{j/p}\left(\frac{\xi}{u},Q^2\right)\,D_{h/l}\left(\frac{\zeta u}{\xi(1-u)},Q^2\right)$$

Homogeneus (as the usual Gribov Lipatov Altarelli Parisi type) term + Inhomogeneus term

1) Fracture Functions satisfy unitarity

$$\sum_h \int_0^1 dz\, z\, M^j_{p,h}(x,z;\mu^2) = (1-x) \cdot F^j_p(x,\mu^2).$$

2) Fracture Functions factorize

*M.Grazzini, L.Trentadue and G.Veneziano, Nucl.Phys.B*519(1998)394
*J.Collins, Phys.Rev.D*57(1998)3051

3) Extended $M(x,z,t,Q^2)$ - Fracture Functions satisfy Gribov-Lipatov-Altarelli-Parisi type evolution equations

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}^j_{A,A'}(x,z,t,Q^2) = \frac{\alpha_S(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) \mathcal{M}^i_{A,A'}(x/u,z,t,Q^2)$$

$$\sigma_{current} \simeq \int F_p^i \, \hat{\sigma}_i^j \, D_j^h .$$

$$\sigma_{target} \simeq \int M_{p,h}^i \, \hat{\sigma}_i .$$

$$\sigma_{target} \simeq \int M_{p,h}^i \, \hat{\sigma}_i + \int F_p^i \, D_k^h \, \hat{\sigma}_i .$$

$$\frac{\partial M_{p,h}^j(x,z,Q^2)}{\partial ln \, Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) \, M_{p,h}^i(\frac{x}{u}, z, Q^2)$$

$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{u \, du}{x(1-u)} \hat{P}_i^{j,l}(u) \, D_l^h(\frac{zu}{x(1-u)}, Q^2) \, F_p^i(\frac{x}{u}, Q^2)$$

$$M_{p,h}^j(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^{1-z} \frac{dw}{w} \, E_i^j(\frac{x}{w}, Q^2, Q_0^2) \, M_{p.h}^i(w.z.Q_0^2)$$

$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \int_{x+z}^1 \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{x}{w}} \frac{du}{u(1-u)} E_k^j(\frac{x}{wu}, Q^2, k^2) \, \hat{P}_i^{kl}(u)$$

$$\cdot D_l^k(\frac{z}{w(1-u)}, k^2) \, F_p^i(w, k^2).$$

Two separate contributions can be isolated in the target cross-section[5]:

$$\sigma_{target} \simeq \int M^i_{p,h}\, \hat{\sigma}_i + \int F^i_p\, D^h_k\, \hat{\sigma}_i \,. \tag{3}$$

Correspondingly one can associate to the cross-section two terms i.e. $\sigma_{target} = M^{NP} + M^P$. The first is a Non-Perturbative contribution and the second a Perturbative one. They can be defined at a given scale Q_0^2 by requiring that $M^P|_{Q^2=Q_0^2} = 0$. It is possible to obtain an evolution equation to determine the fracture function $M^i_{p,h}(x,z,Q^2)$ at any other scale Q^2. The evolution equation has the form:

$$\frac{\partial M^j_{p,h}(x,z,Q^2)}{\partial ln\, Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u)\, M^i_{p,h}(\frac{x}{u}, z, Q^2)$$
$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{u\, du}{x(1-u)} \hat{P}^{j,l}_i(u)\, D^h_l(\frac{zu}{x(1-u)}, Q^2)\, F^i_p(\frac{x}{u}, Q^2) \tag{4}$$

being $P^j_i(u)$ and $\hat{P}^{j,l}_i(u)$ the regularized and real Altarelli-Parisi vertices respectively[9]. $D^h_l(z,Q^2)$ represents the fragmentation function of the parton l into hadron h and $F^i_p(x,Q^2)$ is the ordinary deep inelastic proton structure function. The evolution equation can be solved and the solution reads:

$$M^j_{p,h}(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^{1-z} \frac{dw}{w} E^j_i(\frac{x}{w},Q^2,Q_0^2)\, M^i_{p.h}(w.z.Q_0^2)$$
$$+\frac{\alpha_s(Q^2)}{2\pi} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \int_{x+z}^{1} \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{z}{w}} \frac{du}{u(1-u)} E^j_k(\frac{x}{wu},Q^2,k^2)\, \hat{P}^{kl}_i(u) \tag{5}$$
$$\cdot\, D^k_l(\frac{z}{w(1-u)},k^2)\, F^i_p(w,k^2).$$

The first term describes the hadron distribution at a given arbitrary scale Q_0^2 evolving it to a scale Q^2 by means of the perturbative evolution function $E^j_i(\frac{x}{w},Q^2,Q_0^2)$ which satisfies the equation[9]:

$$Q^2 \frac{\partial}{\partial\, Q^2} E^j_i(x,Q^2,Q_0^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} P^j_k(u)\, E^k_i(\frac{x}{u},Q^2). \tag{6}$$

The second term describes the perturbative evolution from Q_0^2 to Q^2 of the active exchanged parton i. The perturbatively generated partonic shower accompaining the evolution of the parton i contains an inclusive distribution for an additional parton l which finally fragments into the hadron h. Fracture functions do satisfy several properties[5]:

- Do not depend on the arbitrary choosen scale Q_0^2 i.e.

$$\frac{\partial}{\partial Q_0^2} M^j_{p,h}(x,z,Q^2) \;=\; 0 \tag{7}$$

- Both $D_l^h(x,Q^2)$ and $F_p^i(x,Q^2)$ satisfy the usual Altarelli Parisi evolution equations and $\sum_h \int_0^1 \; dz \; z \; D_l^h(x,Q^2) = 1$ and $\sum_i \int_0^1 \; dx \; x \; F_p^i(x,Q^2) = 1$ with

$$\sum_i \int_0^1 du \; u \; P_i^j(u) \;=\; 0 \tag{8}$$

$M^j_{p,h}(x,z,Q^2)$ satisfies the momentum sum rule:

$$\sum_h \int_0^1 dz \; z \; M^j_{p,h}(x,z,Q^2) \;=\; (1-x)\; F_p^j(x,Q^2) \tag{9}$$

accounting for the s-channel unitarity constraint.

- In terms of moments. by defining

$$\int_0^1 dz \; z^m \int_0^{1-z} dx \; x^n \; M^i_{p,h}(x,z,Q^2) \;=\; M^{i,ph}_{m,n}(Q^2) \tag{10}$$

Diffraction

Applications:

Diffraction:

$$e^-(k) + A(P) \to e^-(k') + A(P') + X$$

- According to the Ingelman-Schlein model,

[G.Ingelman and P.Schlein, Phys.Lett. **B152** (1985) 256.]

the diffractive structure function $F_2^{diff}(x, Q^2)$ is

$$F_2^{diff}(x, Q^2) = \sum_a \int d\xi \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} \hat{\sigma}(\frac{x}{\xi}, Q^2, \mu)$$

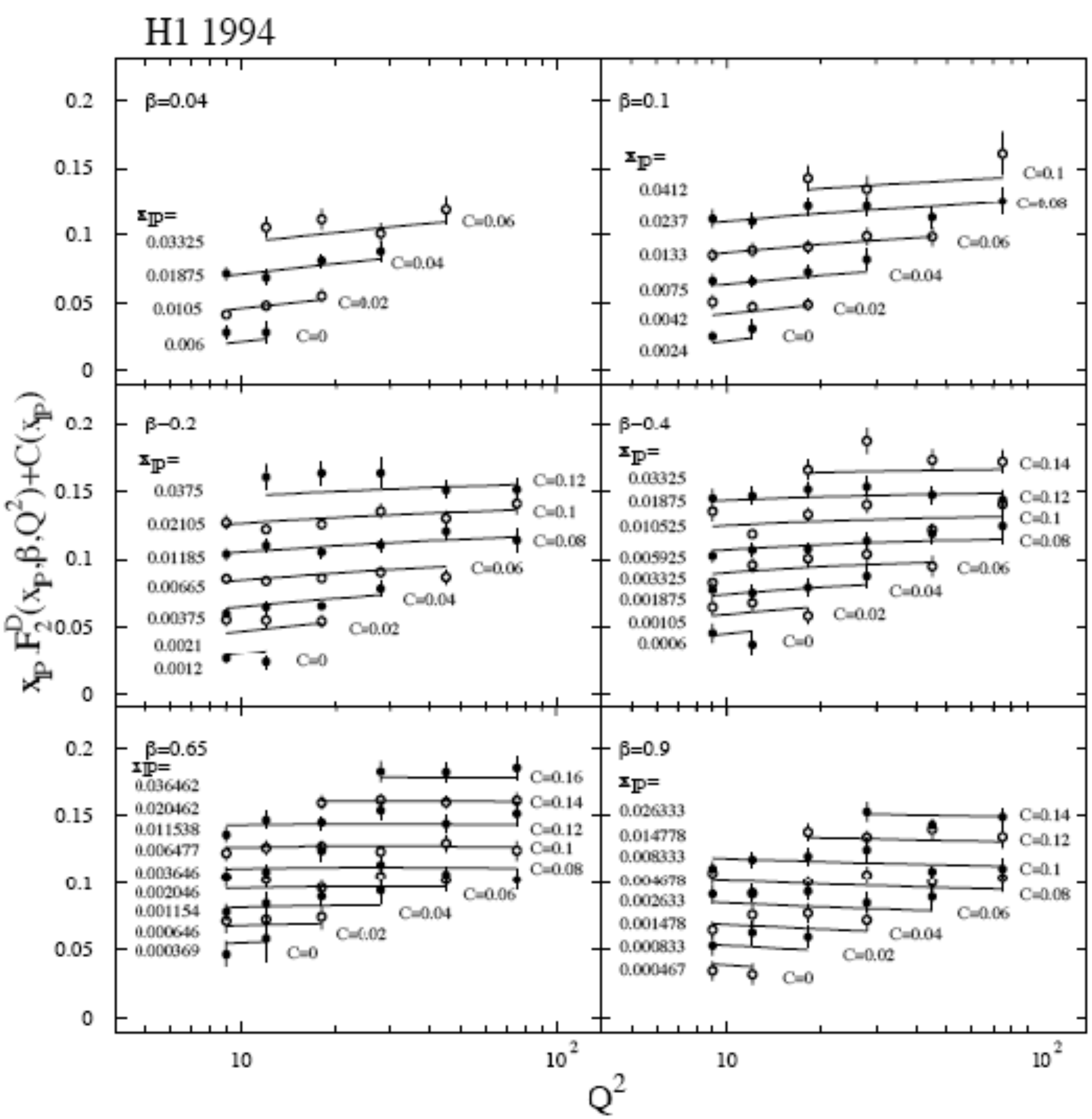
But

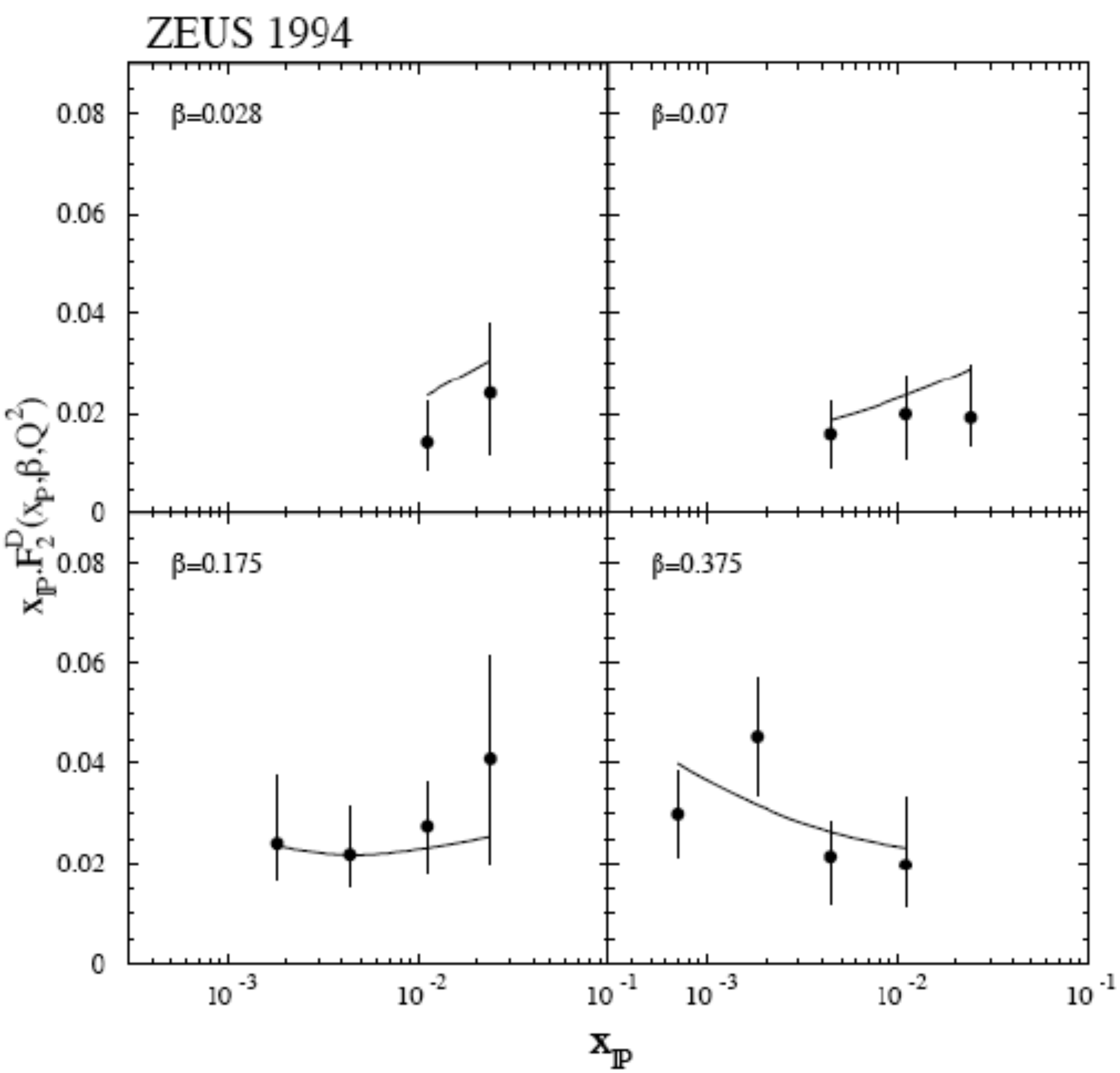
$$\int_0^\infty d|t \, \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} = M_{AA}(\xi, \mu, z = 1 - x_P)$$

- Analyzing $F_2^{D(3)}(\beta, \mu, x_P)$, H1 collaboration at HERA observed a possible Q^2 dependence of the diffractive distributions

$\Downarrow$

A logarithmic dependence on Q^2 is implicitely contained in fracture functions!


H1 1994
β=0.04
β=0.1
β=0.2
β=0.4
β=0.65
β=0.9
$x_{I\!P}F_2^D(x_{I\!P},\beta,Q^2)+C(x_{I\!P})$
Q^2



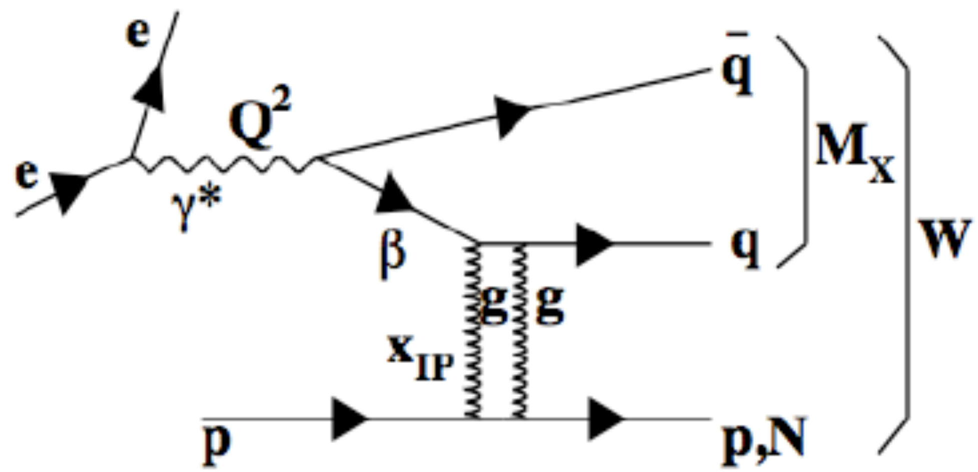
ZEUS 1994
β=0.028
β=0.07
β=0.175
β=0.375
0.08
0.06
0.04
0.02
0
$x_{IP} \cdot F_2^D(x_{IP}, \beta, Q^2)$
10^{-3}
10^{-2}
10^{-1}
x_{IP}

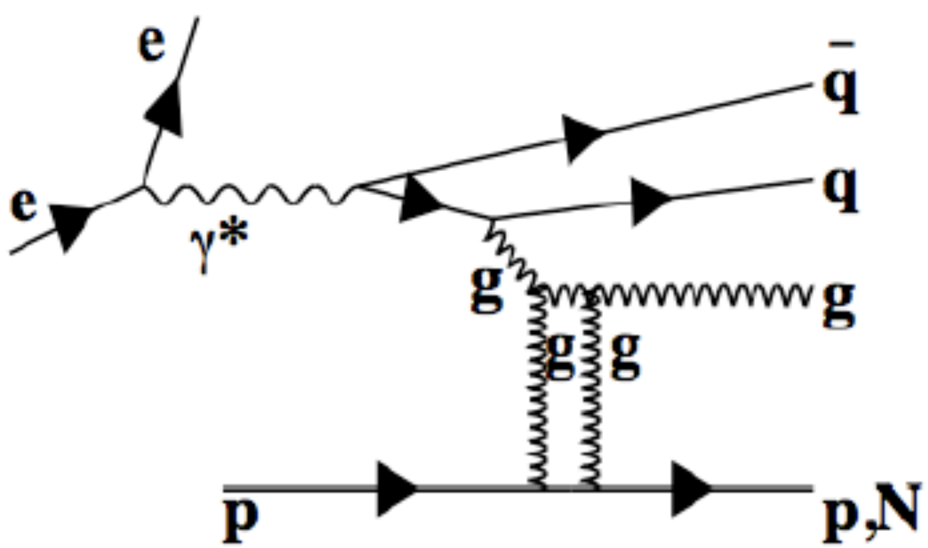
DESY 05-011
January 2005

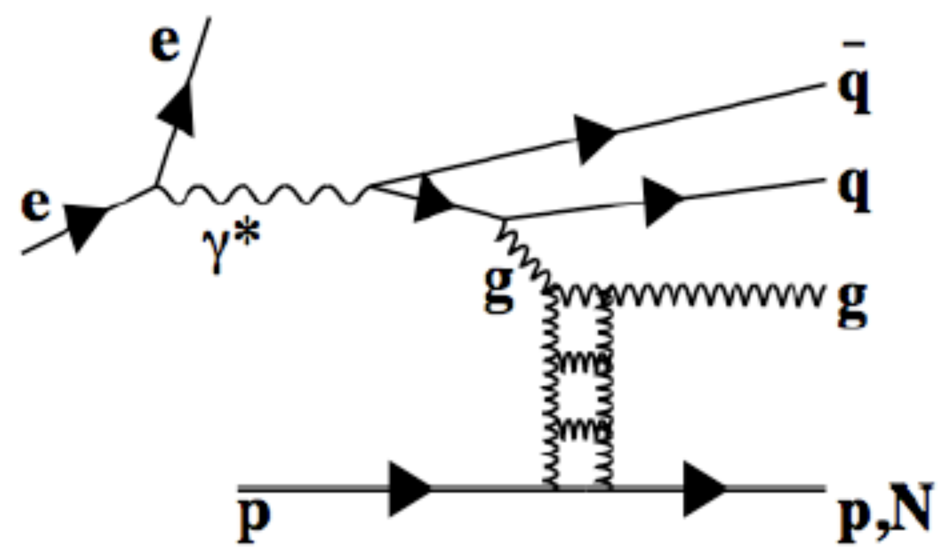
Study of deep inelastic inclusive and diffractive scattering with the ZEUS forward plug calorimeter

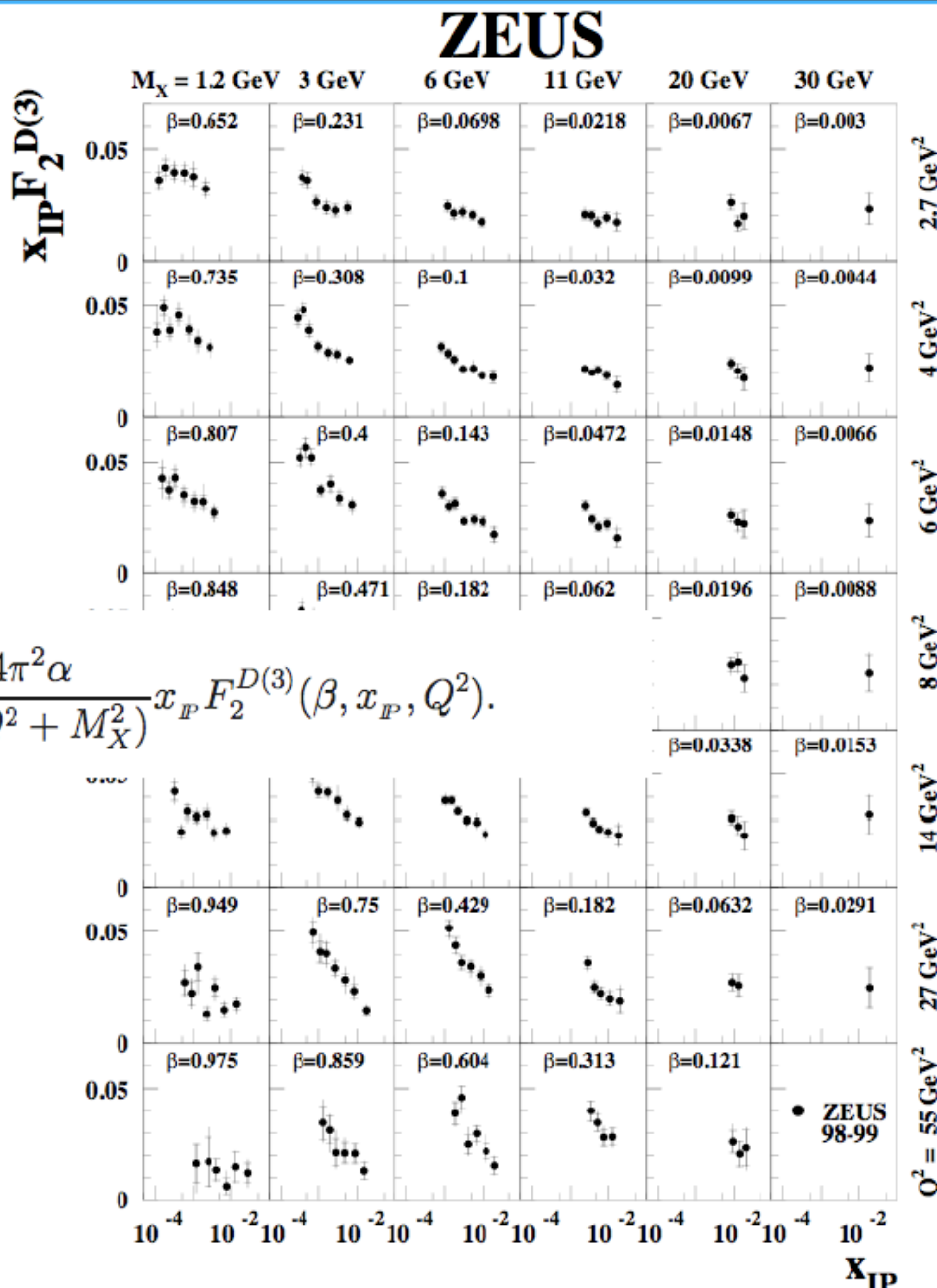
ZEUS Collaboration

tio is flat with Q^2, indicating a leading-twist behaviour of the diffractive cross section. The data are also presented in terms of the diffractive structure function, $F_2^{D(3)}(\beta, x_{I\!P}, Q^2)$, of the proton. For fixed β, the Q^2 dependence of $x_{I\!P} F_2^{D(3)}$ changes with $x_{I\!P}$ in violation of Regge factorisation. For fixed $x_{I\!P}$, $x_{I\!P} F_2^{D(3)}$ rises as $\beta \to 0$, the rise accelerating with increasing Q^2. These positive scaling violations suggest substantial contributions of perturbative effects in the diffractive DIS cross section.


e
e
Q2
γ*
β
q̄
q
M_X
W
g g
x_IP
p
p,N


e
e
γ*
q̄
q
g
g
g g
p
p,N


e
e
γ*
q̄
q
g
g
p
p,N



$$\frac{1}{2M_X}\frac{d\sigma^{\text{diff}}_{\gamma^* p\to XN}(M_X,W,Q^2)}{dM_X}=\frac{4\pi^2\alpha}{Q^2(Q^2+M_X^2)}x_{I\!P}F_2^{D(3)}(\beta,x_{I\!P},Q^2).$$

More recent data
H1 2006

H1- 2006

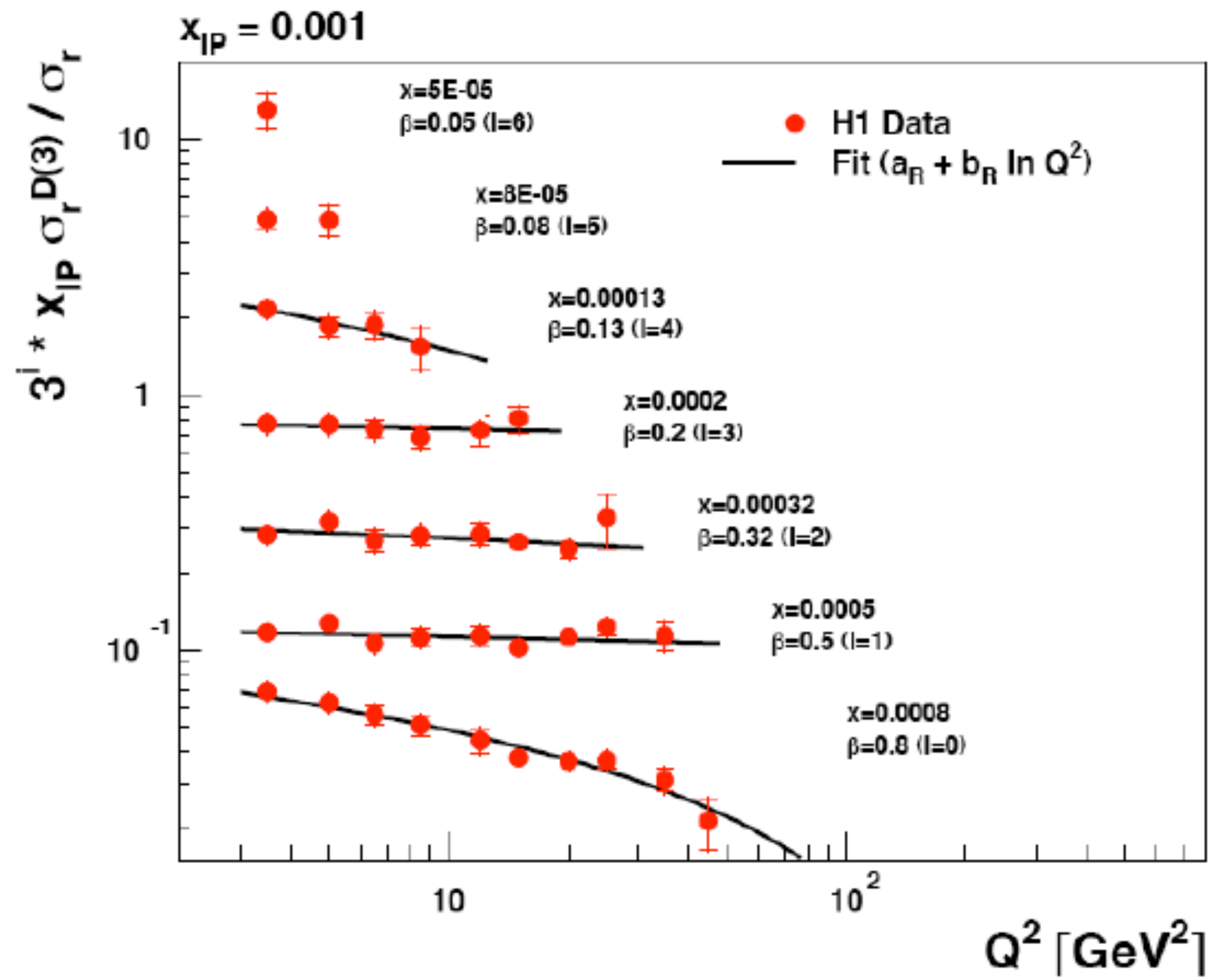


Fig. 14. As Fig.13 with $x_{IP} = 0.001$. From Ref.[38]

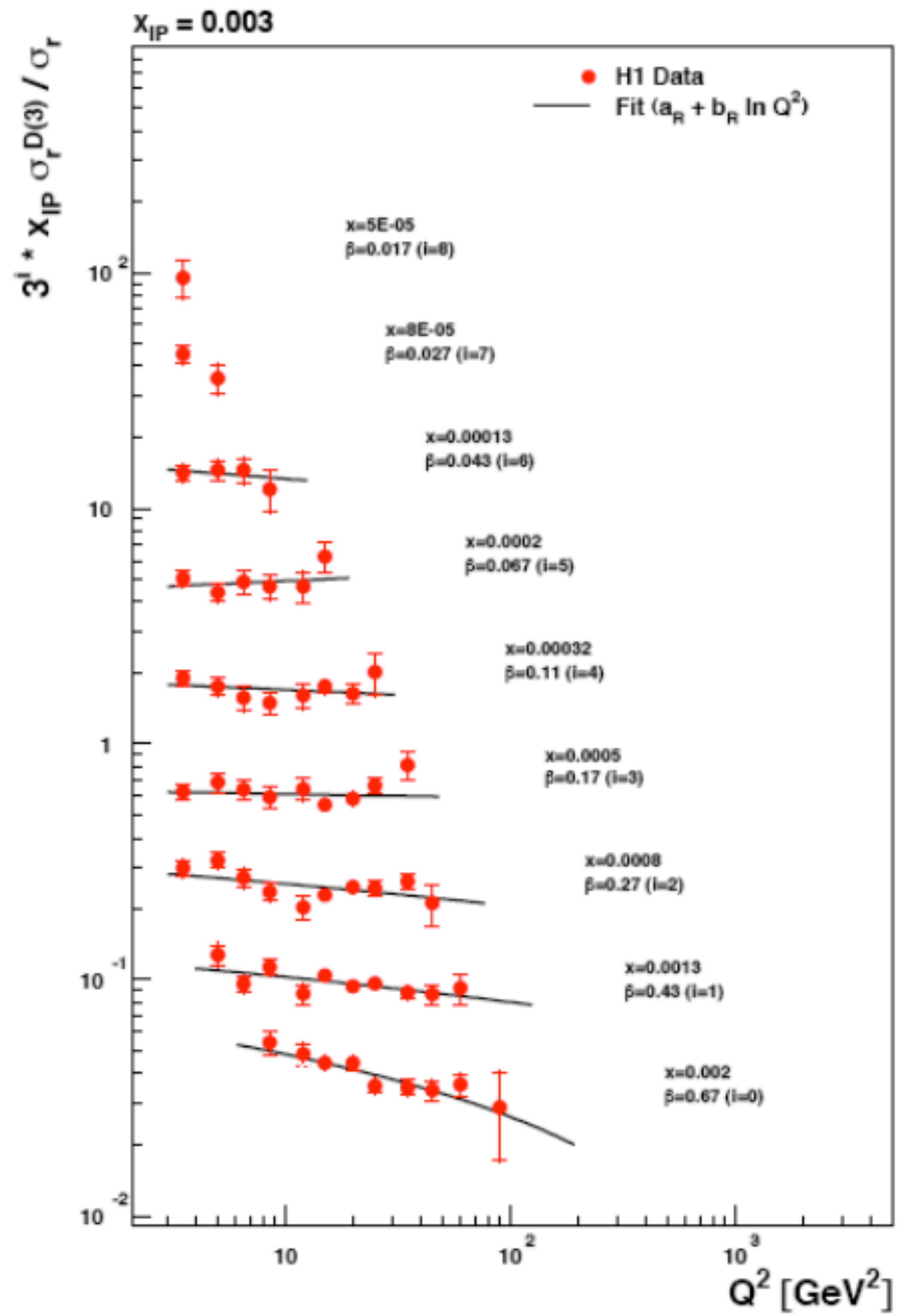
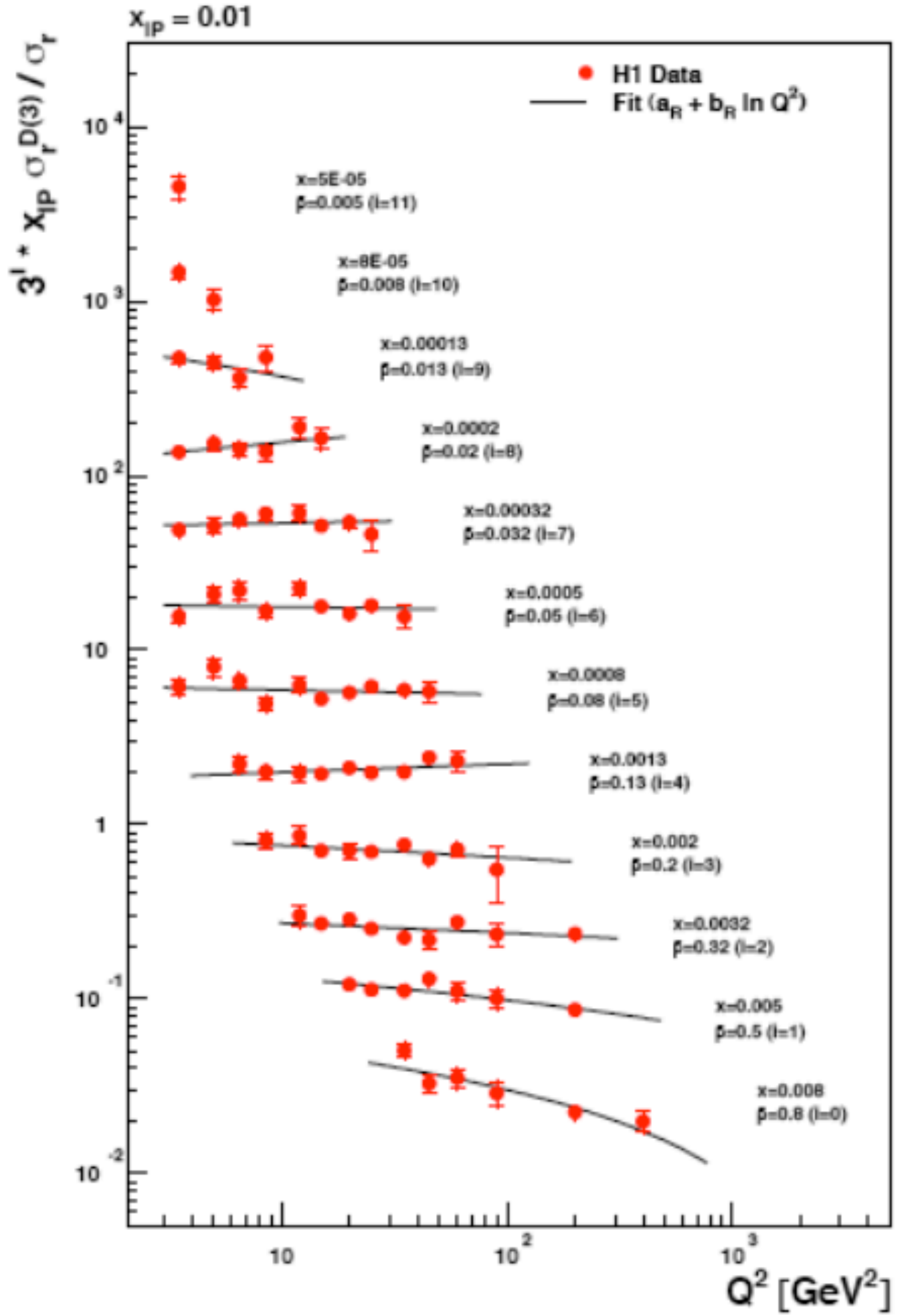


Fig. 15. As Fig.13 with $x_{I\!P} = 0.003$. From Ref.[38]



x_IP = 0.01
H1 Data
Fit (a_R + b_R ln Q²)
3^i * x_IP σ_r^D(3) / σ_r
x=5E-05
β=0.005 (i=11)
x=8E-05
β=0.008 (i=10)
x=0.00013
β=0.013 (i=9)
x=0.0002
β=0.02 (i=8)
x=0.00032
β=0.032 (i=7)
x=0.0005
β=0.05 (i=6)
x=0.0008
β=0.08 (i=5)
x=0.0013
β=0.13 (i=4)
x=0.002
β=0.2 (i=3)
x=0.0032
β=0.32 (i=2)
x=0.005
β=0.5 (i=1)
x=0.008
β=0.8 (i=0)
Q² [GeV²]

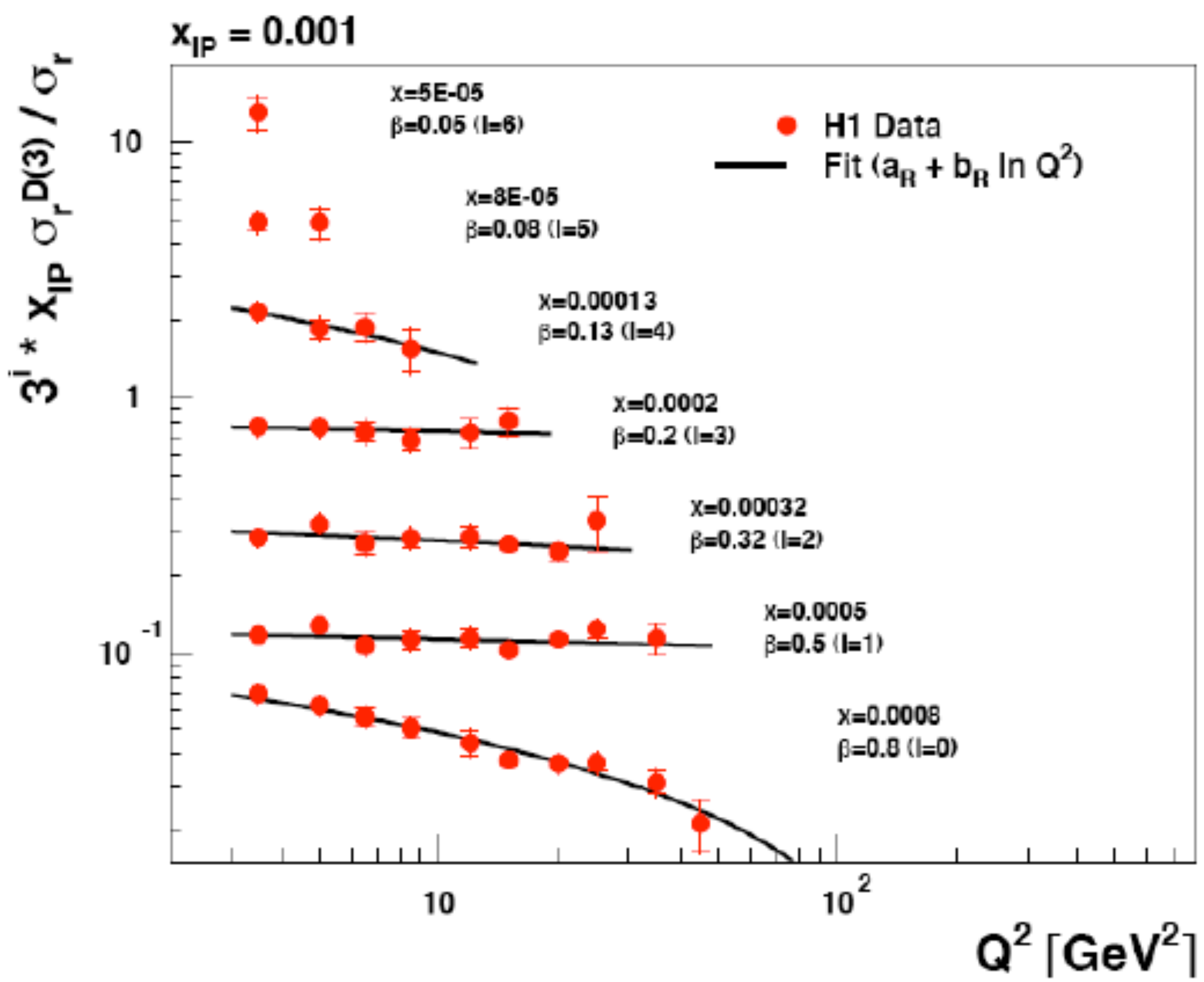


Fig. 14. As Fig.13 with $x_{I\!P} = 0.001$. From Ref.[38]

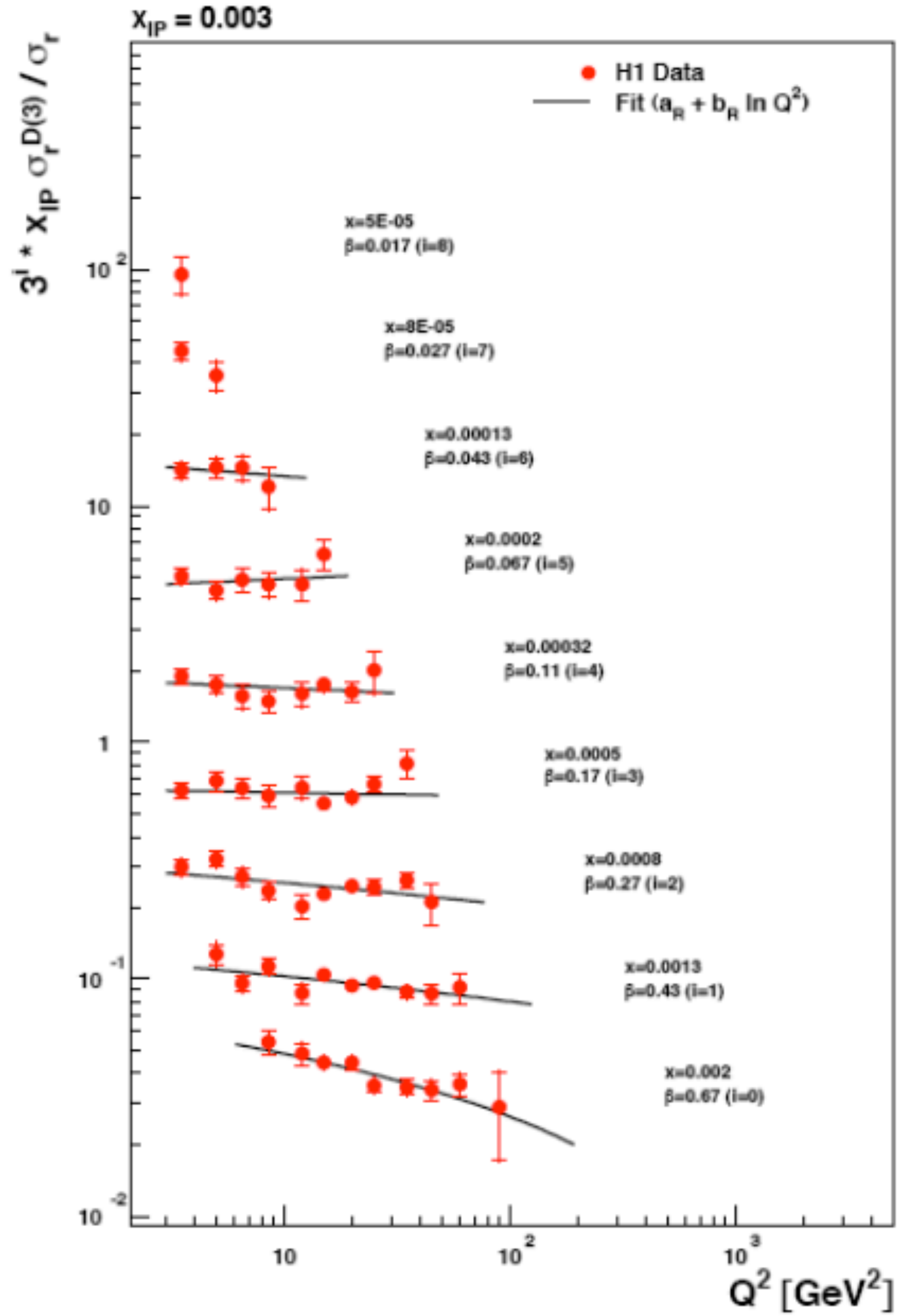
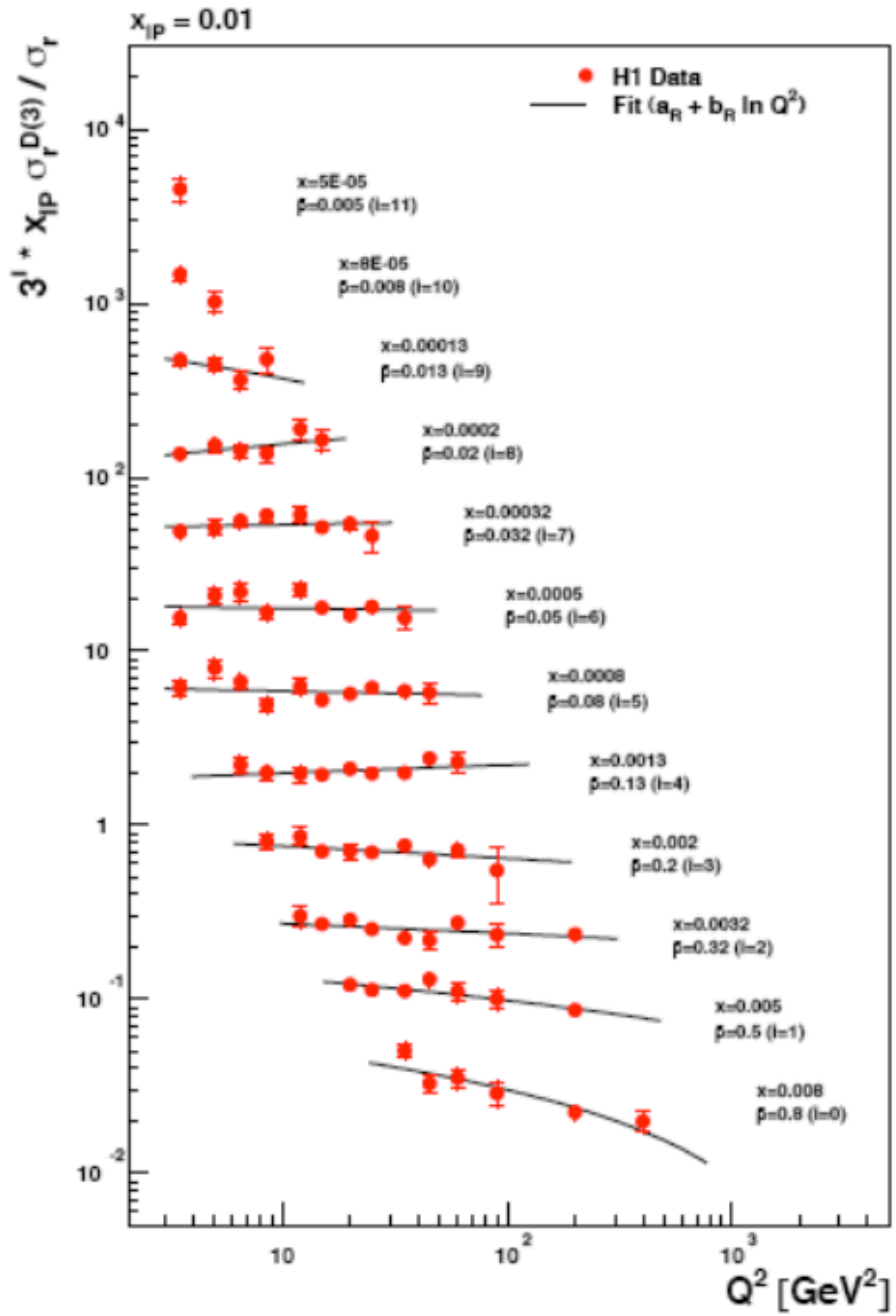


Fig. 15. As Fig.13 with $x_{I\!P} = 0.003$. From Ref.[38]



x_{IP} = 0.01
$3^i * x_{IP}\ \sigma_r^{D(3)} / \sigma_r$
H1 Data
Fit ($a_R + b_R \ln Q^2$)
x=5E-05
β=0.005 (i=11)
x=8E-05
β=0.008 (i=10)
x=0.00013
β=0.013 (i=9)
x=0.0002
β=0.02 (i=8)
x=0.00032
β=0.032 (i=7)
x=0.0005
β=0.05 (i=6)
x=0.0008
β=0.08 (i=5)
x=0.0013
β=0.13 (i=4)
x=0.002
β=0.2 (i=3)
x=0.0032
β=0.32 (i=2)
x=0.005
β=0.5 (i=1)
x=0.008
β=0.8 (i=0)
Q^2 [GeV^2]

Next-to-Leading Fracture Functions

Next-to-leading Fracture Functions

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Next to leading order evolution of SIDIS processes in the forward region*

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We compute the order α_s^2 quark initiated corrections to semi-inclusive deep inelastic scattering extending the approach developed recently for the gluon contributions. With these corrections we complete the order α_s^2 QCD description of these processes, verifying explicitly the factorization of collinear singularities. We also obtain the corresponding NLO evolution kernels, relevant for the scale dependence of fracture functions. We compare the non-homogeneous evolution effects driven by these kernels with those obtained at leading order accuracy and discuss their phenomenological implications.

PACS numbers: 12.38.Bx, 13.85.Ni
Keywords: Semi-Inclusive DIS; perturbative QCD; Fracture functions

$$\frac{\partial M^r_{i,h/P}(\xi,\zeta,M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_{\frac{\xi}{1-\zeta}}^{1}\frac{du}{u}\left[P^{(0)}_{i\leftarrow j}(u)+\frac{\alpha_s(M^2)}{2\pi}P^{(1)}_{i\leftarrow j}(u)\right]M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M^2\right)$$

$$+\frac{\alpha_s(M^2)}{2\pi}\frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\left[\hat{P}^{(0)}_{ki\leftarrow j}(u,v)+\frac{\alpha_s(M^2)}{2\pi}P^{(1)}_{ki\leftarrow j}(u,v)\right]f^r_{j/P}\left(\frac{\xi}{u},M^2\right)D^r_{h/k}\left(\frac{\zeta}{\xi\, v},M^2\right),$$

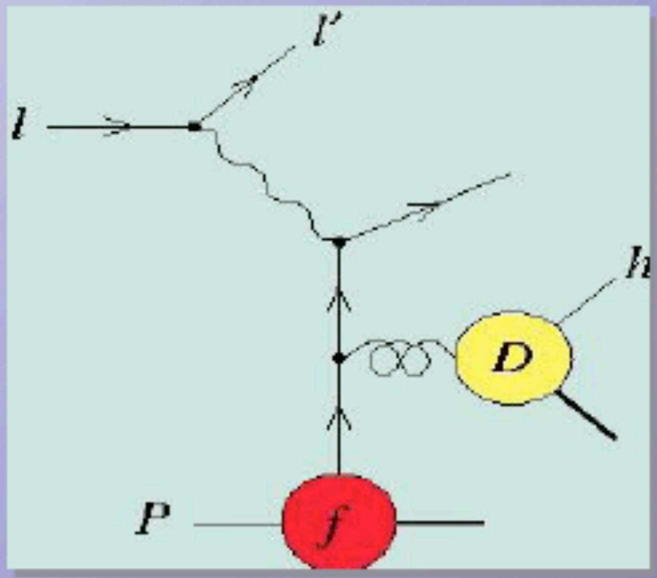
Transverse Momenta

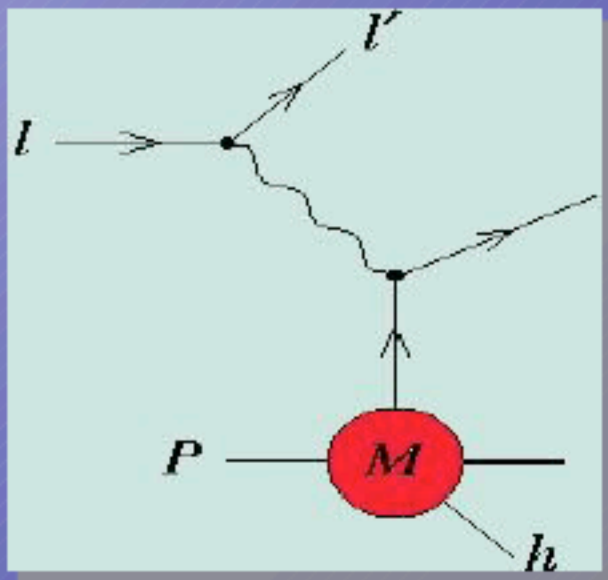
F. Ceccopieri, L.T.,
Phys.Lett.B (2006)

- QCD predicts the scale dependence of M :

$$\frac{\partial}{\partial \log Q^2} M_{i,h/p}(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{x/(1-z)}^{1} \frac{du}{u} P_j^i(u) M_{j,h/N}\left(\frac{x}{u}, z, Q^2\right) +$$

$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_{x}^{x/(x+z)} \frac{du}{x(1-u)} \hat{P}_j^{i,l}(u) F_{j/p}\left(\frac{x}{u}, Q^2\right) D_{h/l}\left(\frac{zu}{x(1-u)}, Q^2\right)$$

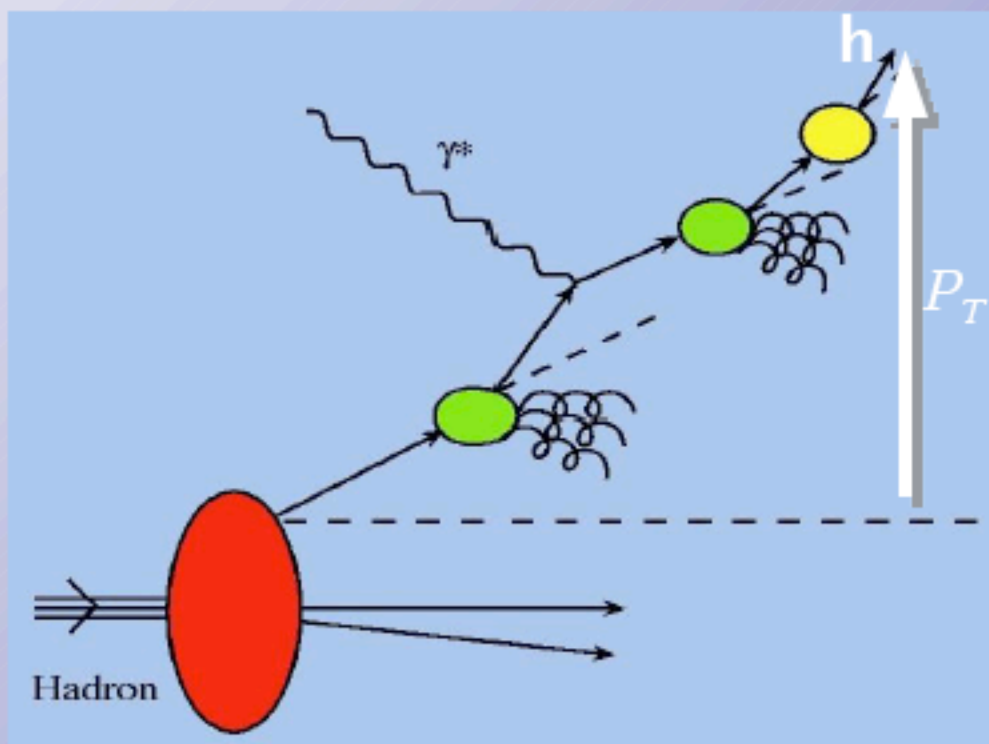




↳ Leading twist SIDIS cross section is thus:

$$\frac{d^3\sigma_p^h}{dx_B\, dQ^2\, dz_h} \propto \sum_{i=q,\bar{q}} e_i^2 \left[\underbrace{F_{i/p}(x_B, Q^2) D_{h/i}(z_h, Q^2)}_{\sigma_{current}} + \underbrace{(1-x_B) M_{i,h/p}(x_B, (1-x_B) z_h, Q^2)}_{\sigma_{target}} \right]$$

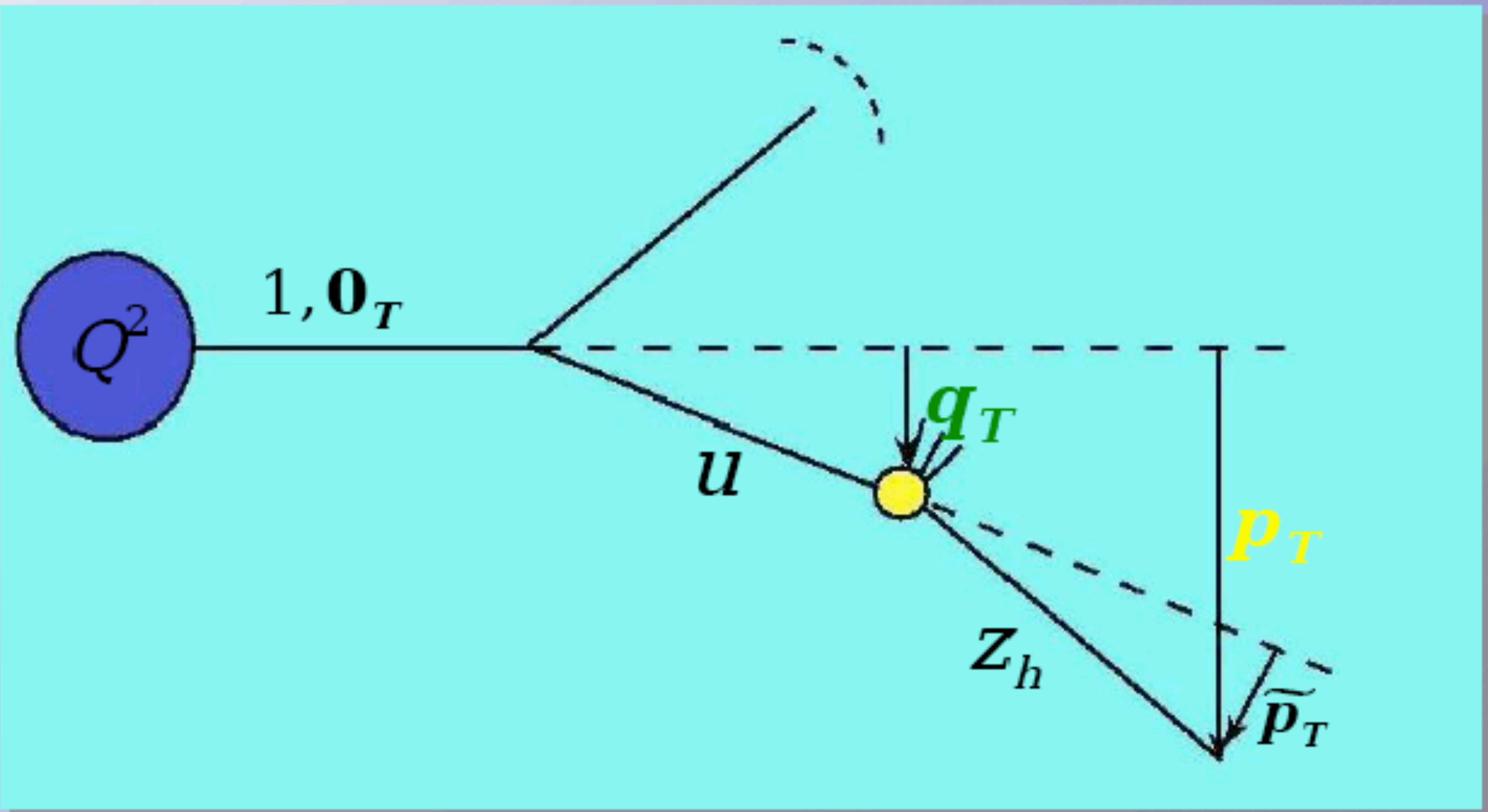
- Sources of transverse momentum in $l+P\rightarrow l+h+X$:



- Intrinsic distribution k_T [8] ✓
- Radiative q_T ✗
- Intrinsic fragmentation p_T [8] ✓

Detected hadron transverse momentum : $P_T \approx p_T + z_h k_T$

- Time-like TMD DGLAP evolution equation



Branching kinematics:

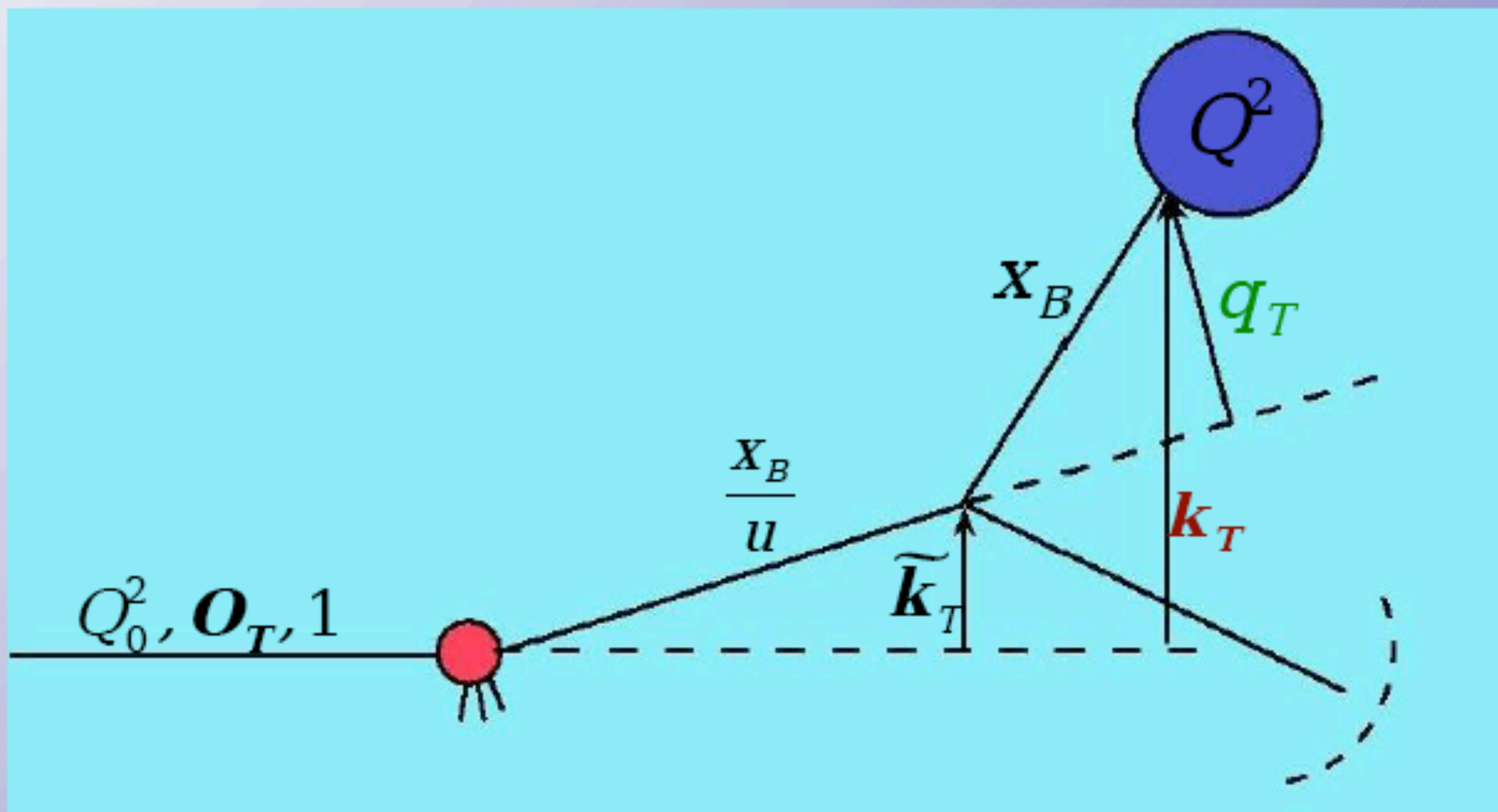
$$\tilde{p}_T = P_T - \frac{z_h}{u} q_T$$

$$u(1-u)Q^2 = P_T^2$$

$$Q^2 \frac{\partial D_a^b(Q^2, z_h, p_T)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{du}{u} \int \frac{d^2 q_T}{\pi} \delta[u(1-u)Q^2 - q_T^2] P_a^c(w) D_c^b\left(Q^2, \frac{z_h}{u}, p_T - \frac{z_h}{u} q_T\right)$$

- Space-like TMD DGLAP evolution equation



Branching kinematics:

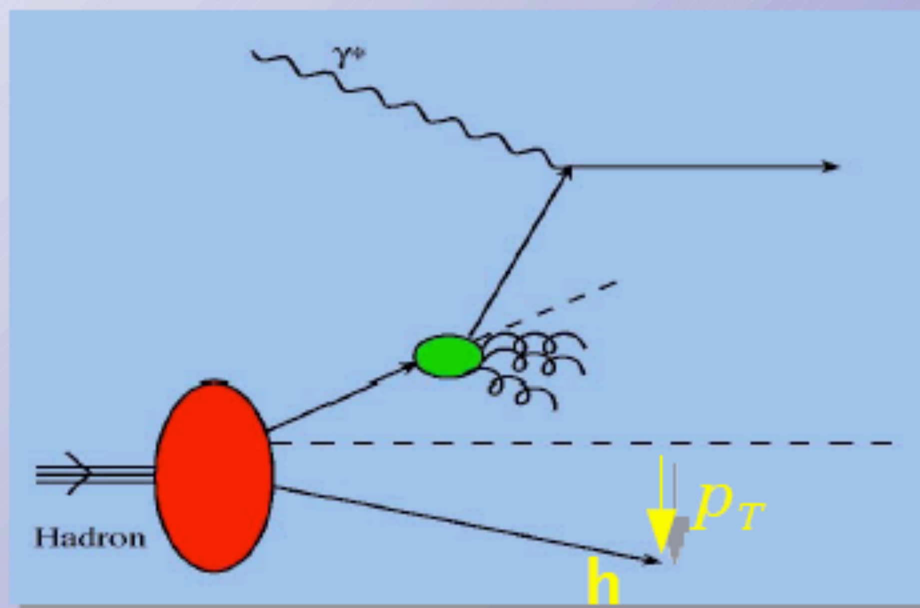
$$\tilde{\boldsymbol{k}}_T = \frac{K_T - q_T}{u}$$

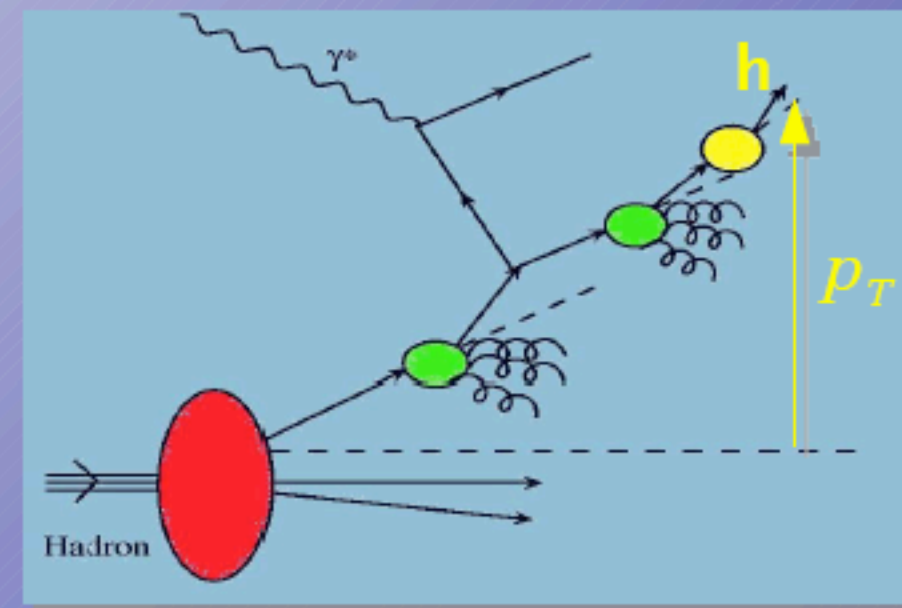
$$(1-u)\, Q^2 = \boldsymbol{q}_T^2$$

$$Q^2 \frac{\partial f_a^b(Q^2, x_B, \boldsymbol{k}_T)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \int \frac{d^2 \boldsymbol{q}_T}{\pi} \delta[(1-u)\, Q^2 - \boldsymbol{q}_T^2]\, P_a^c(u)\, f_c^b\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k}_T - \boldsymbol{q}_T}{u}\right)$$

- Fracture functions TMD evolution equation





$$Q^2 \frac{\partial M^j_{p,h}(Q^2, x_B, \mathbf{k_T}, z_h, \mathbf{p_T})}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u^3} \int \frac{d^2 \mathbf{q_T}}{\pi} \delta[(1-u)Q^2 - \mathbf{q_T^2}] P_i^j(u) M^i_{p,h}\left(Q^2, \frac{x_B}{u}, \frac{\mathbf{k_T} - \mathbf{q_T}}{u}, z_h, \mathbf{p_T}\right) +$$

$$+ \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u} \int \frac{d^2 \mathbf{q_T}}{\pi} \delta[(1-u)Q^2 - \mathbf{q_T^2}] \frac{u}{x_B(1-u)} \hat{P}_i^{j,1}(u) \cdot$$

$$\cdot F_P^i\left(Q^2, \frac{x_B}{u}, \frac{\mathbf{k_T} - \mathbf{q_T}}{u}\right) D_1^h\left(Q^2, \frac{z_h u}{x_B(1-u)}, \mathbf{p_T} - \frac{z_h u}{x_B(1-u)} \mathbf{q_T}\right)$$

Factorization: ☐

Increasing phase space

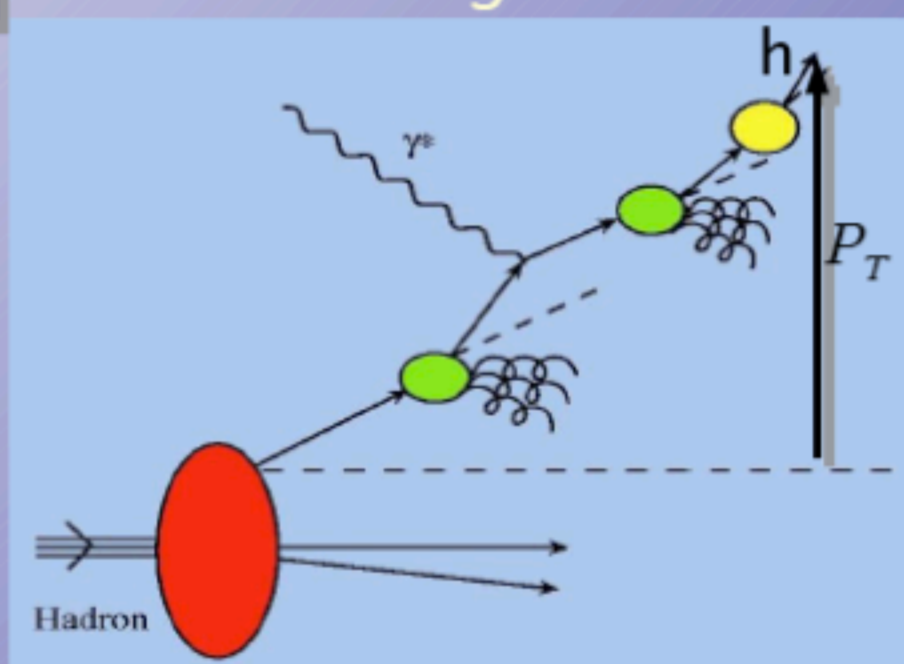
$$d\sigma \propto f(x) \circ D(z)$$ ✓

$$d\sigma \propto f(x) \circ D(z) + M(x,z)$$ ✓

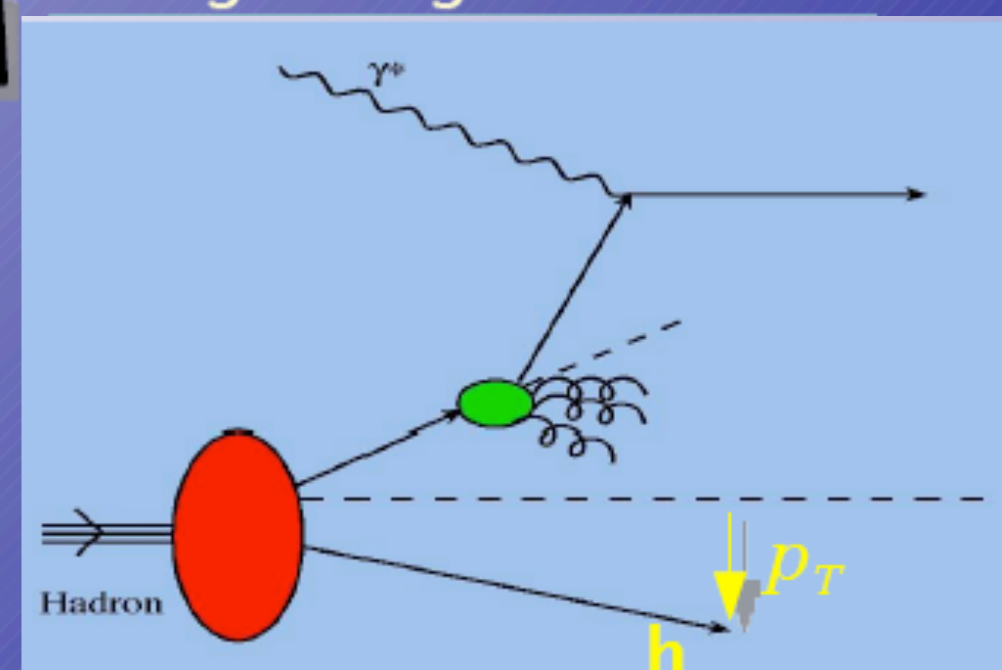
$$\frac{d\sigma}{dP_T} \propto f(x, k_T) \circ D(z, p_T)$$ ✓ [9]

$$\frac{d\sigma}{dP_T} \propto f(x, k_T) \circ D(z, p_T) + M(x, k_T, z, p_T)$$ ✗

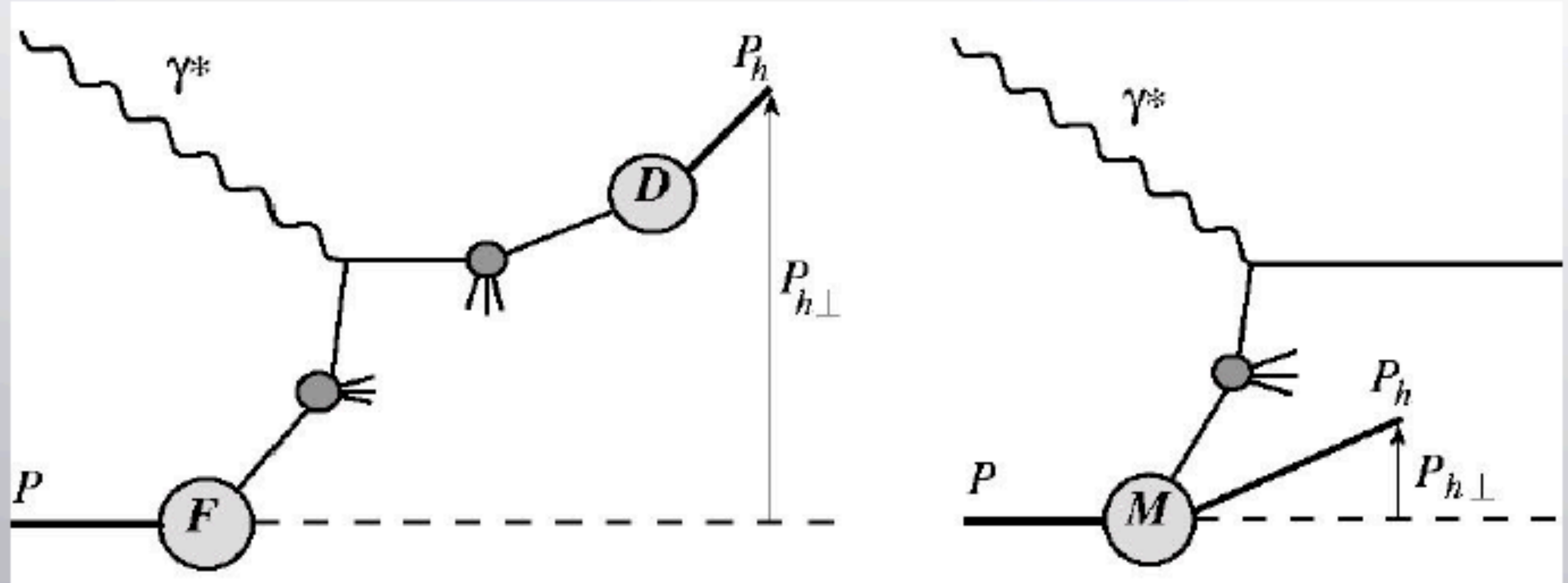
Current fragmentation



Target fragmentation



[9] X. Ji, J. Ma, F. Yuan *Phys. Rev.* **D71**, 034005 (2005)



$$Q^2 \frac{\partial \mathcal{M}^i_{P,h}(x, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \Bigg\{ \int_{\frac{x}{1-z}}^1 \frac{du}{u^3} P^i_j(u) \int \frac{d^2 \boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 -$$

$$\cdot \mathcal{M}^j_{P,h}\Big(Q^2, \frac{x}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}, z, \boldsymbol{p}_\perp\Big) + \int_x^{\frac{x}{x+z}} \frac{du}{x(1-u)u^2} \hat{P}^{i,l}_j(u) \frac{d^2 \boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 -$$

$$\cdot \mathcal{F}^j_P\Big(\frac{x}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}, Q^2\Big) \mathcal{D}^h_l\Big(\frac{zu}{x(1-u)}, \boldsymbol{p}_\perp - \frac{zu}{x(1-u)} \boldsymbol{q}_\perp, Q^2\Big) \Bigg\} .$$

$$\int d^2 \boldsymbol{k}_\perp \int d^2 \boldsymbol{p}_\perp \mathcal{M}^i_{P,h}(x, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2) = \mathcal{M}^i_{P,h}(x, z, Q^2) ,$$

Fracture Functions

and

Polarization

Application of FF to polarized processes

The application of fracture functions to describe polarized processes has been studied by De Florian, Garcia Canal, Sampayo and Sassot[50, 51]. The aim is to extend to the target fragmentation region the description of polarized processe. They discuss the factorization of the collinear singularities related to the polarized processes, particularly those which are absorbed in the re-definition of the spin dependent analogue of fracture functions[1]. In Ref.[50]

50. D. de Florian, C. A. Garcia Canal, R. Sassot: Nucl. Phys. B **470**, 195 (1996)
51. D. de Florian, O. A. Sampayo, R. Sassot: Phys. Rev. D **66**, 010001 (2002)

The potential relevance of the fracture functions to describe spin dependent distributions has been advocated by Teryaev[42]. They may be applied at fixed target energies and may also include interference and final state interaction, providing a source for azimuthal asymmetries at HERMES and polarization at NOMAD. Accordingly the work of Ref.[44] can be rephrased in terms of fracture functions (see also [43]).

42. O. V. Teryaev: Acta Phys. Polon. B **33**, 3749 (2002)
43. O. V. Teryaev: Phys. Part. Nucl. **35**, 524 (2004)
44. S. J. Brodsky, D. S. Hwang, I. Schmidt: Phys. Lett. B **530**, 99 (2002)
45. A. Kotzinian: Phys. Lett. B **552**, 172 (2003)

Fracture Functions : spin dependent

/0211027 v2 11 Nov 2002

Spin-dependent, interference and T−odd fragmentation and fracture functions

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Joint Institute for Nuclear Research, Dubna, 141980 Russia

Fracture functions, originally suggested to describe the production of diffractive and leading hadrons in semi-inclusive DIS, may be also applied at fixed target energies. They may also include interference and final state interaction, providing a source for azimuthal asymmetries at HERMES and (especially) Λ polarization at NOMAD. The recent papers by Brodsky, Hwang and Schmidt, and by Gluck and Reya, may be understood in terms of fracture functions.

The Fractured Boer-Mulders Effect in the Production of Polarized Baryons

Dennis Sivers

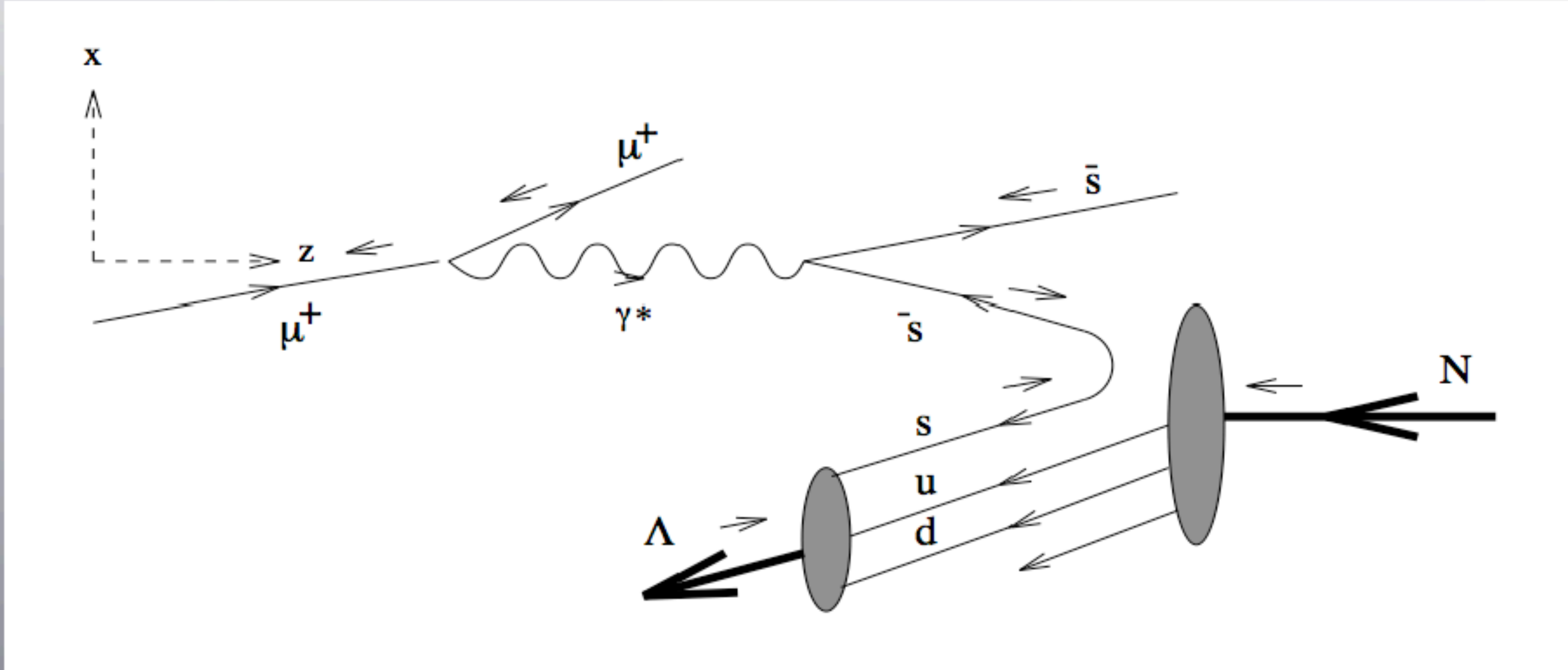
Abstract

The fractured Boer-Mulders functions, $\Delta^N M^q_{B\uparrow/\{q,q\}\uparrow:p}(x, p_{TN}; z, \vec{p}_T \cdot \vec{k}_T; Q^2)$, describe an intriguing class of polarization effects for the production of baryons in the target fragmentation region of deep-inelastic processes. These functions characterize transverse momentum asymmetries related to the spin orientation for different flavors of axial-vector diquarks, $\{q_i, q_j\}\uparrow$, in an unpolarized ensemble of protons just as the familiar Boer-Mulders functions characterize transverse momentum asymmetries connected to the spin orientation of quarks in unpolarized targets. The asymmetries in p_{TN} of the fractured Boer-Mulders effect originating in the proton distribution function can be separated kinematically, both in SIDIS and in the Drell-Yan process, from the asymmetries in k_{TN} of the polarizing fracture functions, $\Delta^N M^q_{B\uparrow/(q,q):p}(x, {p_T}^2; z, k_{TN}; Q^2)$, generated during the soft color rearrangement of the fragmentation process. The experimental requirements for this separation are presented in this article and it is shown that the fractured Boer-Mulders effect should change sign between Drell-Yan and SIDIS while the polarizing fracture functions remain the same. Simple isospin arguments indicate the two polarization mechanisms should give significantly different results for the production of polarized Λ's and Σ's.

PPI 0912
Draft 10/26/09

Lambda production

a while ago...



CERN–TH/95–135
hep-ph/9506280

THE PROTON SPIN PUZZLE AND
Λ POLARIZATION IN DEEP–INELASTIC SCATTERING

John Ellis[a,1], Dmitri Kharzeev[a,b,2] and Aram Kotzinian[c,d,3]

CERN-TH/96-261
hep-ph/9610362

arXiv:hep-ph/9610362v2 6 Jan 1997

Next to Leading Order QCD Corrections to Polarized Λ Production in DIS

D. de Florian[1]

Theoretical Physics Division, CERN, CH 1211 Geneva 23, Switzerland
e-mail: Daniel.de.Florian@cern.ch

R. Sassot

Departamento de Física, Universidad de Buenos Aires
Ciudad Universitaria, Pab.1 (1428) Bs.As., Argentina

Abstract

We calculate next to leading order QCD corrections to semi-inclusive polarized deep inelastic scattering and e^+e^- annihilation cross sections for processes where the polarization of the identified final-state hadron can also be determined. Using dimensional regularization and the HVBM prescription for the γ_5 matrix, we compute corrections for different spin-dependent observables, both in the $\overline{MS}$ and $\overline{MS}_p$ factorization schemes, and analyse their structure. In addition to the well known corrections to polarized parton distributions, we also present those for final-state polarized fracture functions and polarized fragmentation functions, in a consistent factorization scheme.

Tuesday, October 19, 2010

3. Semi-Inclusive Deep Inelastic Scattering

With the definition for polarized fragmentation functions given in the previous section, eq. (14), we are now able to compute the NLO corrections for semi-inclusive DIS in the case in which the final-state hadron is polarized. NLO contributions for processes with unpolarized final-state hadrons (with either polarized or unpolarized initial states) have been computed in refs. [10] and [8], respectively, so we refer the reader to these for most of the definitions and conventions.

Using the usual kinematical DIS variables for the interaction between a lepton of momentum l and helicity λ_l and a nucleon A of momentum P and helicity λ_A

$$x = \frac{Q^2}{2\,P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad Q^2 = -q^2 \text{ and } S_H = (P+l)^2, \tag{17}$$

the differential cross section for the production of a hadron h with energy $E_h = z\,E_A(1-x)$ and helicity λ_h (with n partons in the final state) can be written as

$$\begin{aligned}
\frac{d\sigma^{\lambda_l\lambda_A\lambda_h}}{dx\,dy\,dz} &= \int\frac{du}{u}\sum_n\sum_{j=q,\bar{q},g}\int dPS^{(n)}\,\frac{\alpha^2}{S_H x}\,\frac{1}{e^2(2\pi)^{2d}} \\
&\times \left[Y_M(-g^{\mu\nu}) + Y_L\frac{4x^2}{Q^2}P_\mu P_\nu + \lambda_l Y_P\,\frac{x}{Q^2}\,i\epsilon^{\mu\nu qP}\right] \\
&\times \sum_{\lambda_1,\lambda_2=\pm1} H_{\mu\nu}(\lambda_1,\lambda_2)\left\{M_{j,h/A}\left(\frac{x}{u},\frac{E_h}{E_A},\frac{\lambda_1}{\lambda_A},\frac{\lambda_h}{\lambda_A}\right)(1-x)\right. \\
&\left. + \; f_{j/A}\left(\frac{x}{u},\frac{\lambda_1}{\lambda_A}\right)\sum_{i_\alpha=q,\bar{q},g} D_{h/i_\alpha}\left(\frac{z}{\rho},\frac{\lambda_h}{\lambda_2}\right)\frac{1}{\rho}\right\}
\end{aligned} \tag{18}$$

Lambda production

a while ago...experiment

受入
84-12-11
高工研図書室

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/84-148
November 5th, 1984

STUDIES OF QUARK AND DIQUARK FRAGMENTATION INTO IDENTIFIED HADRONS IN DEEP INELASTIC MUON-PROTON SCATTERING

The European Muon Collaboration

Aachen[1], CERN[2], DESY (Hamburg)[3], Freiburg[4], Hamburg (University)[5], Kiel[6], LAL (Orsay)[7], Lancaster[8], LAPP (Annecy)[9], Liverpool[10], Marseille[11], Mons[12], MPI (München)[13], Oxford[14], RAL (Chilton)[15], Sheffield[16], Torino[17], Uppsala[18] and Wuppertal[19].

M. Arneodo[17], A. Arvidson[18], J.J. Aubert[11], B. Badelek[18o)], J. Beaufays[12], K.H. Becks[19], C. Bee[8], C. Benchouk[11], G. Berghoff[1], I. Bird[8], D. Blum[7], E. Böhm[6], X. de Bouard[9], F.W. Brasse[3], H. Braun[19], C. Broll[9], S. Brown[10], H. Brück[19], H. Calen[18], D. Callebaut[12], J. Carr[15a)], J.S. Chima[15], J. Ciborowski[5o)], R. Clifft[15], J.H. Cobb[8b)], G. Coignet[9], F. Combley[16], J. Coughlan[8], G.R Court[10], G. D'Agostini[11], S. Dahlgren[18], J.K. Davies[14c)], F. Dengler[13], I. Derado[13], U. Dosselli[2d)], T. Dreyer[4], J. Drees[19], J.J. Dumont[1], M. Düren[1], V. Eckardt[13], A. Edwards[19], M. Edwards[15], T. Ernst[4], G. Eszes[9e)], J. Favier[9], M.I. Ferrero[17f)], J. Figiel[5g)], W. Flauger[3], J. Foster[16], E. Gabathuler[2], J. Gajewski[5o)], R. Gamet[10], J. Gayler[3], N. Geddes[14], P. Giubellino[17], C. Gössling[3], P. Grafström[18], F. Grard[12], L. Gustafsson[18], J. Haas[2], E. Hagberg[18], F.J. Hasert[1], P. Hayman[10], P. Heusse[7], C. Hoppe[5], M. Jaffré[7], A. Jacholkowska[7], F. Janata[5], G. Jancso[9e)], A.S. Johnson[14)], E.M. Kabuss[4], G. Kellner[2], V. Korbel[3], J. Krüger[19], S. Kullander[18], U. Landgraf[4], D. Lanske[1], J. Loken[14], K. Long[14], M. Maire[9], A. Menz[13], W. Mohr[4], F. Montanet[9h)], H.E. Montgomery[2m)], R.P. Mount[14i)],

Tuesday, October 19, 2010

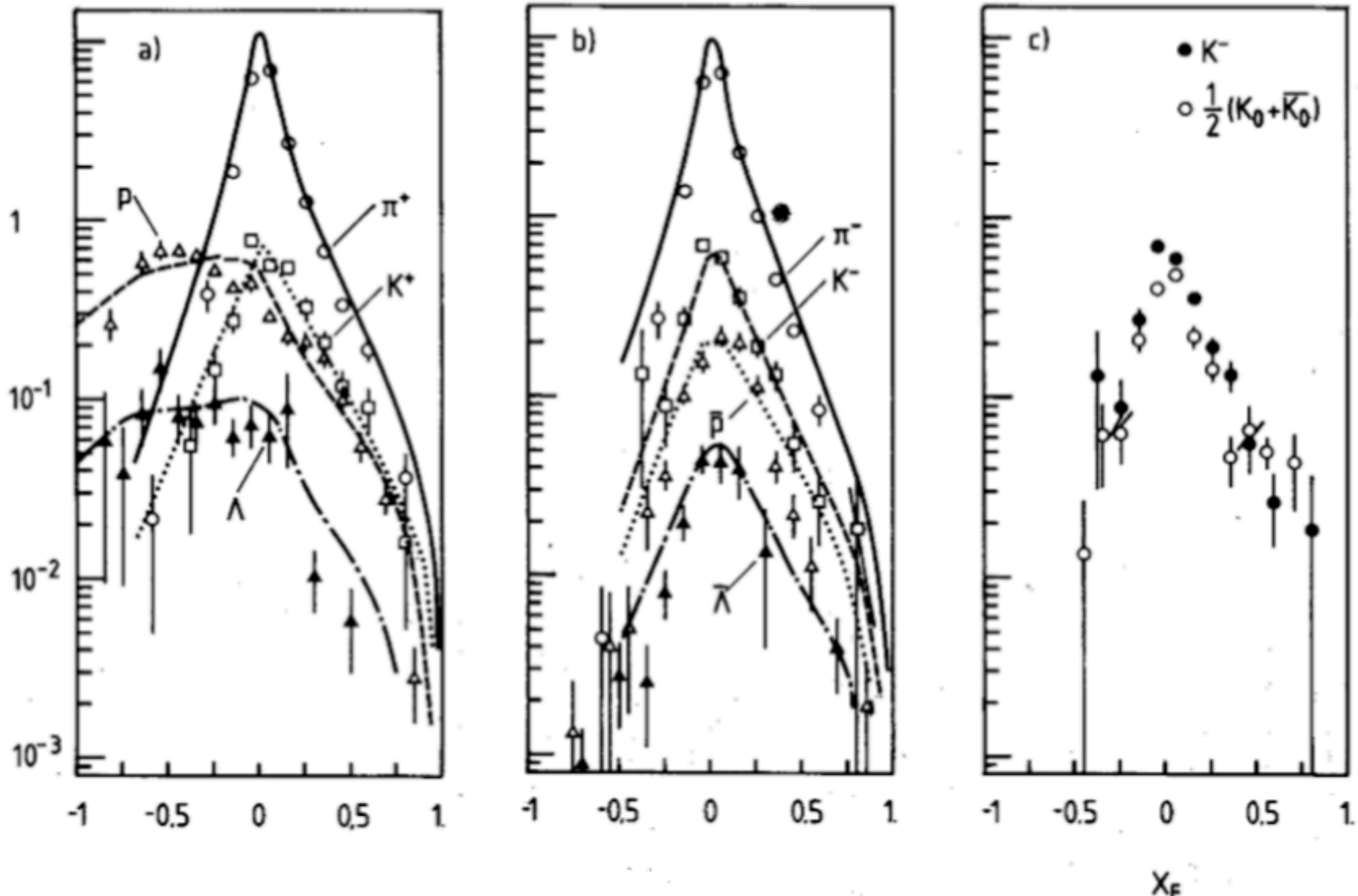


Fig. 2 Feynman x distributons normalized to the number of scattered muons (N_μ) for positive and negative hadrons

a) π^+, K^+, p and Λ

b) π^-, K^-, $\bar{p}$ and $\bar{\Lambda}$

c) K^- and $(K^o+\bar{K}^o)/2$

The curves represent the predictions of the Lund model

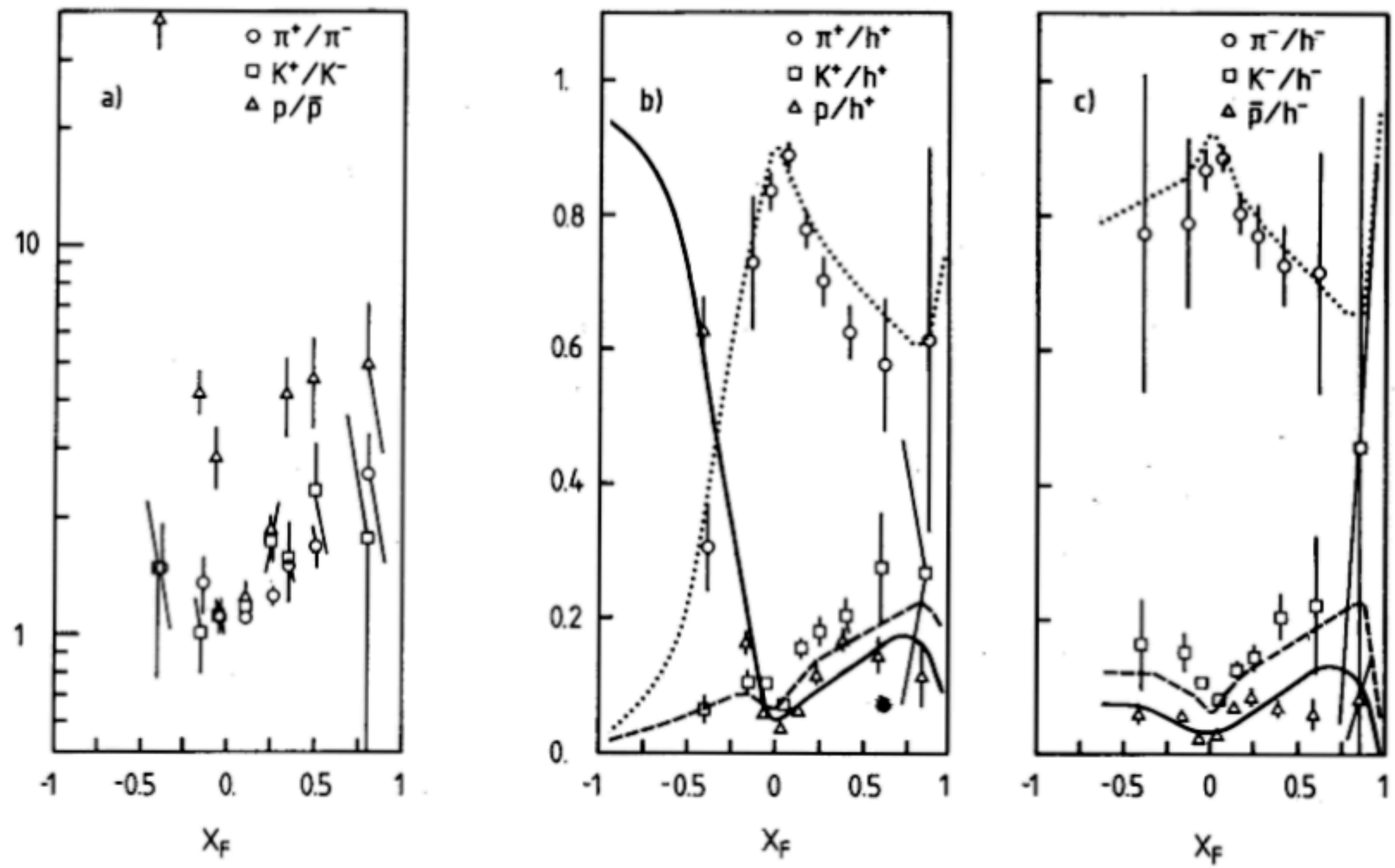


Fig. 3

Fig. 3 Ratio of pions, kaons and protons as a function of Feynman x

a) π^+, K^+, p normalized to π^-, K^-, $\bar{p}$

b) π^+, K^+, p normalized to all positive hadrons

c) π^-, K^-, $\bar{p}$ normalized to all negatve hadrons

The curves represent the predictions of the Lund model

... to conclude

FractureFunctions are a well established QCD based approach to semi inclusive processes
(a comprehensive Current+Target fragmentation)

QCD evolution known   Diffraction (well tested at HERA)

It appears they might be a useful framework to investigate Polarization

In all applications NLO and NNLO improvements possible and partly done

Lambda production is a potential application

FF are a potentially useful tool also for multiple hadron final states

Further studies will assess any potential application of the FF idea

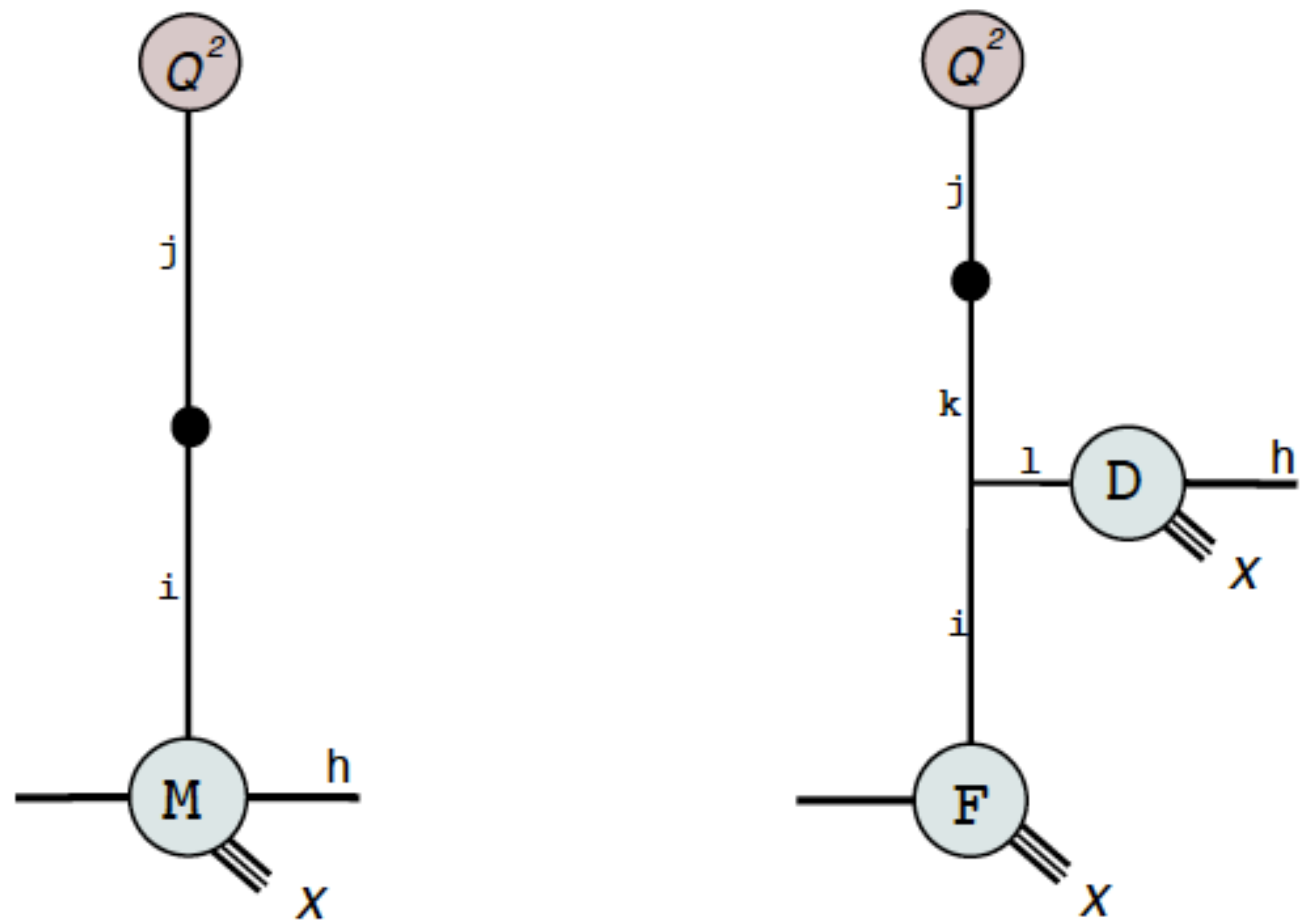
Extension to multiple hadron distributions

$$M^{j}_{h/P}(x,z,Y) = M^{j}_{A,h/P}(x,z,Y) + M^{j}_{B,h/P}(x,z,Y)\,,$$

$$M^{j}_{A,h/P}(x,z,Y) = \int_{x}^{1-z}\frac{dw}{w}E^{j}_{i}\left(\frac{x}{w},Y-y_0\right)M^{i}_{A,h/P}(w,z,y_0)\,,$$

$$M^{j}_{B,h/P}(x,z,Y) = \int_{y_0}^{Y}dy\int_{x+z}^{1}\frac{dw}{w^2}\int_{\frac{x}{w}}^{1-\frac{z}{w}}\frac{du}{u(1-u)}\,.$$

$$\cdot E^{j}_{k}\left(\frac{x}{wu},Y-y\right)\hat{P}^{kl}_{i}(u)D^{h}_{l}\left(\frac{z}{w(1-u)},y\right)F^{i}_{P}(w,y)\,.$$



(A) (B)

$$\frac{\partial}{\partial Y} M^j_{A,h/P}(x,z,Y) = \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^j_{A,h/P}(x/u,z,Y)\,,$$

$$\frac{\partial}{\partial Y} M^j_{B,h/P}(x,z,Y) = \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^j_{B,h/P}(x/u,z,Y) + \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl}_i(u) D^h_l\Big(\frac{zu}{x(1-u)}, Y\Big) F^i_P(x/u,Y)\,.$$

$$Y = \frac{1}{2\pi\beta_0} \ln\Big[\frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)}\Big], \quad dY = \frac{\alpha_s(Q^2)}{2\pi}\frac{dQ^2}{Q^2}\,.$$

$$Q^2 \frac{\partial}{\partial Q^2} M^j_{h/P}(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^i_{h/P}(x/u,z,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl}_i(u) D^h_l\Big(\frac{zu}{x(1-u)}, Q^2\Big) F^i_P(x/u,Q^2)\,.$$

Di-hadron Fracture Functions

$$\sigma_T = \int \frac{du}{u} M^j_{h_1,h_2/P}(u, z_1, z_2, Q^2)\,\hat{\sigma}_j(x/u, Q^2)\,.$$

$$M^j_{h_1,h_2/P}(x, z_1, z_2, Y) = \sum_{X=A,B,C,D} M^j_{X,h_1,h_2/P}(x, z_1, z_2, Y)\,.$$