Overview about extractions of Fragmentation Functions

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Structure of the Nucleon: $N = p, n$

theory: $q - l$ interactions $\Rightarrow$ SM, SUSY...

exp: always hadrons $h =$ bound states of q:

$$p = (uud), n = (ddu), \pi^{\pm} = (u\bar{d}), K^{\pm} = (s\bar{u})...$$

exp. $\Leftrightarrow$ theory: 2 basic quantities needed $q \Leftrightarrow h$:

- the parton distributions (PDFs): $N \Rightarrow q$: $q(x)$
- fragmentation functions (FFs): $q \Rightarrow h$: $D_q^h(z)$

at low energies – wave functions

at high energies – almost free quarks – PDF and FF

our info about $q - l$ & $q - q$ ints. depends on our knowledge of

PDF and FF

PDF & FF = equally important $\Rightarrow$ EXP.

PDF – quite well known, but FF – rather new objects!

subject of the talk:

How to determine the FF?

How to determine the kaon FF?

$$D_q^h(z) = ?$$

FF are universal: $\Rightarrow$ **FF are the same in all processes**

$\Rightarrow$ **determined in a suitable process – can be used in any other process, at any other energy scale.**

Which are the processes for FF?

- e^+e^- **annihilation:**

$$e^+ + e^- \to h + X, \quad h = \pi^\pm, K^\pm, p/\bar{p}...$$

$$d\sigma^h = \sum_q e_q^2 \otimes D^h_{q+\bar{q}}$$

$\Rightarrow$ **the "cleanest" process, but determines only**

$$D^h_{u+\bar{u}}, \quad D^h_{d+\bar{d}}, \quad D^h_{s+\bar{s}}, \quad D^h_g$$

q **and** $\bar{q}$ **are not distinguished**

- semi-inclusive DIS:

$$l + N \to l' + h + X, \qquad h = \pi^{\pm}, K^{\pm}, p/\bar{p}...$$

$$d\sigma_N^h = \sum_q e_q^2 \left[q \otimes D_q^h + \bar{q} \otimes D_{\bar{q}}^h \right]$$

- hadron production in pp collisions:

$$p + p \to h + X, \quad h = \pi^{\pm},\ K^{\pm},\ p/\bar{p}..$$

$$d\sigma_{pp}^h = \sum_{a,b,c} q_a \otimes q_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$

D_q^h & $D_{\bar{q}}^h$ determined separately but PDF involved

- e^+e^-, **SIDIS** and pp are used to determine D_q^h

too many unknowns: always relats. among the FF assumed

$\Rightarrow$ **TH uncertainties introduced**

What are the assumptions on FF?

fav. FFs $=$ constituent quarks fragment into h

unfav. FFs $=$ other quarks fragment into h

different assumptions used by different groups!

$$\underline{\underline{K^+ = (u\bar{s})}}$$

I) all fav. FFs and all unfav. FFs are equal $\Rightarrow$ 2 FFs (BKK)

$$\begin{aligned} D_u^{K^+} &= D_{\bar{s}}^{K^+} \Leftarrow fav. \\ D_{\bar{u}}^{K^+} &= D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+} \Leftarrow unfav. \end{aligned}$$

II) fav. FFs are not equal, all unfav. FFs equal $\Rightarrow$ 3 FFs (DSS)

$$D_u^{K^+}, \qquad D_{\bar{s}}^{K^+} \quad \Leftarrow \quad m_s >> m_{u,d}$$

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+}$$

III) fav. FFs and unfav. FFs are power suppressed (Kre):

$$D_u^{K^+}, \qquad D_{\bar{s}}^{K^+} \quad \Leftarrow \quad m_s >> m_{u,d}$$

$$D_u^{K^+} = (1-z) D_{\bar{s}}^{K^+},$$

$$D_d^{K^+} = D_{\bar{d}}^{K^+} = (1-z)^2 D_{\bar{s}}^{K^+}$$

IV) fav. FFs are not equal and unfav. FFs are not equal

$\Rightarrow$ 5 FFs (AKK)

$$D_u^{K^+}, \qquad D_{\bar{s}}^{K^+} \Leftarrow fav.$$

$$D_{\bar{u}}^{K^+}, \qquad D_s^{K^+}$$

$$D_d^{K^+} = D_{\bar{d}}^{K^+}$$

$$in\ addition: \quad D_{c,\bar{c}}^{K^+}, \quad D_{b,\bar{b}}^{K^+}, \quad D_g^{K^+}$$

$$\underline{\underline{\pi^+ = (u\bar{d})}}$$

I) all fav. FFs and all unfav. FFs are equal $\Rightarrow$ 2 FFs (HKNS):

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} : \quad SU(2) \Leftarrow fav.$$

$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} \Leftarrow unfav.$$

II) all fav. FFs are equal and 2 unfav. FFs $\Rightarrow$ 3 FFs (AKK)

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} : \quad SU(2) \Leftarrow fav.$$

$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} : \quad SU(2) \Leftarrow unfav.$$

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+}$$

III) **the fav. & unfav. FFs are proportional (DSS)**

$$3FFs + \mathcal{N} + \mathcal{N}' : \qquad \begin{aligned} D_u^{\pi^+} &= \mathcal{N} D_{\bar{d}}^{\pi^+} : \quad \mathcal{N} = 1.10 \;\Leftarrow\; fav. \\ D_{\bar{u}}^{\pi^+} &= D_d^{\pi^+} : \quad SU(2) \;\Leftarrow\; unfav. \\ D_s^{\pi^+} &= \mathcal{N}' D_{\bar{u}}^{\pi^+} \Leftarrow \mathcal{N}' = 0.82 \end{aligned}$$

IV) **unfav. are power suppressed (Kre):**

$$\begin{aligned} D_u^{\pi^+} &= D_{\bar{d}}^{\pi^+} : \;\; SU(2) \;\Leftarrow\; fav. \\ D_s^{\pi^+} &= D_{\bar{s}}^{\pi^+} = D_d^{\pi^+} = (1-z) D_u^{\pi^+} \;\Leftarrow\; unfav. \end{aligned}$$

$$in\ addition : \quad D_{c,\bar{c}}^{\pi^+}, \quad D_{b,\bar{b}}^{\pi^+}, \quad D_g^{\pi^+}$$

and always

C-inv for h^-: $D_{\bar{q}}^{h^-} = D_q^{h^+}$

SU(2)-inv for neutral hadrons:

$$D_q^{\pi^0} = \frac{D_q^{\pi^+} + D_q^{\pi^-}}{2}$$

$$D_u^{K_s^0} = \frac{D_d^{K^+} + D_d^{K^-}}{2}, \qquad D_d^{K_s^0} = \frac{D_u^{K^+} + D_u^{K^-}}{2}$$

Different parametrizations for FFs exist

1) B. Kniehl et al, KKP (2000) $\Rightarrow e^+e^-$

2) S. Kretzer (2000) $\Rightarrow e^+e^-$

3) EMC (1989) $\Rightarrow$ SIDIS

4) Kretzer et al, (2001) $\Rightarrow e^+e^-$ & SIDIS

5) M. Hirai et al, HKNS (2007) $\Rightarrow e^+e^-$

6) S. Albino et al, AKK (2008) $\Rightarrow e^+e^-$ & pp

7) De Florian et al, DSS (2007) $\Rightarrow e^+e^-$, SIDIS & pp

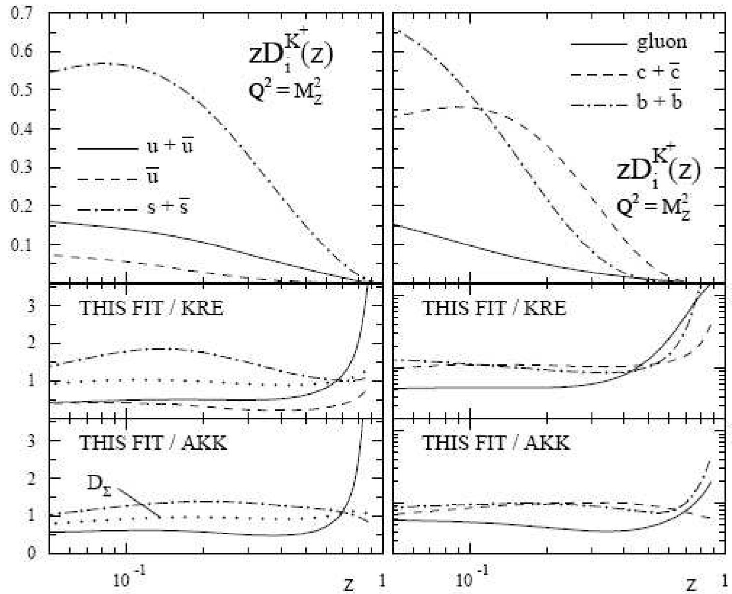
2 general features about them:

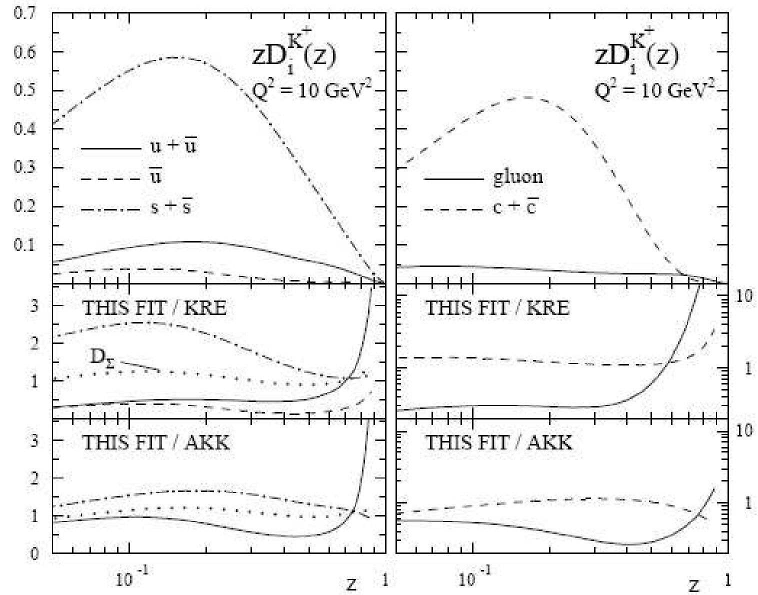
1) different assumptions used

2) they don't agree among themselves

3) they describe the data

The Kaon paramts: $Q^2 = GeV^2$ & M_Z^2 [*De Florian etc. (DSS)*]





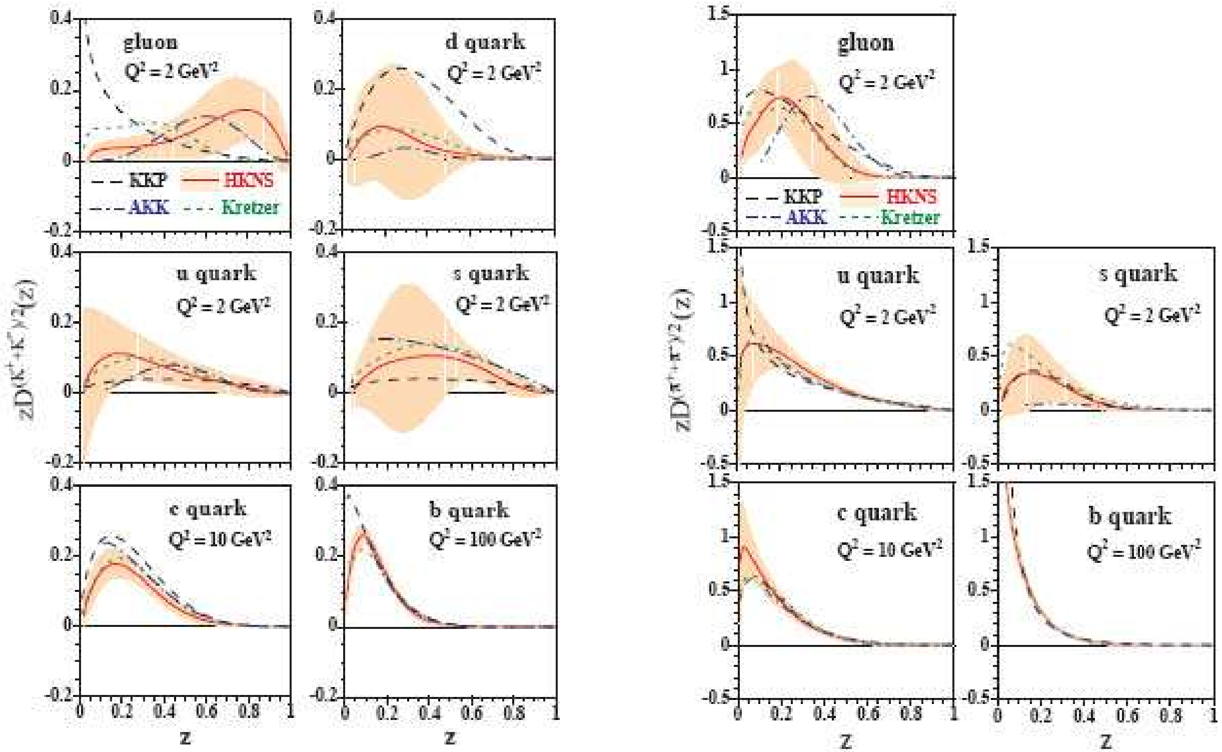
DSS: $\underbrace{e^+e^-}_{\simeq m_Z} + \underbrace{SIDIS + pp}_{Q << m_Z}$; **AKK, KRE:** e^+e^-

- diff. processes, • diff. Q^2, • diff. assumptions

if fact. theorem: the difference is only in the assumptions!

- $D_\Sigma = D_u + D_d + D_s;\quad \sigma_{e^+e^-}^{h^\pm}(M_Z^2) \sim \hat{e}_q^2(M_Z^2)\, D_\Sigma$

The parametrs. from e^+e^-, NLO [*Hirai etc. (HKNS)*]

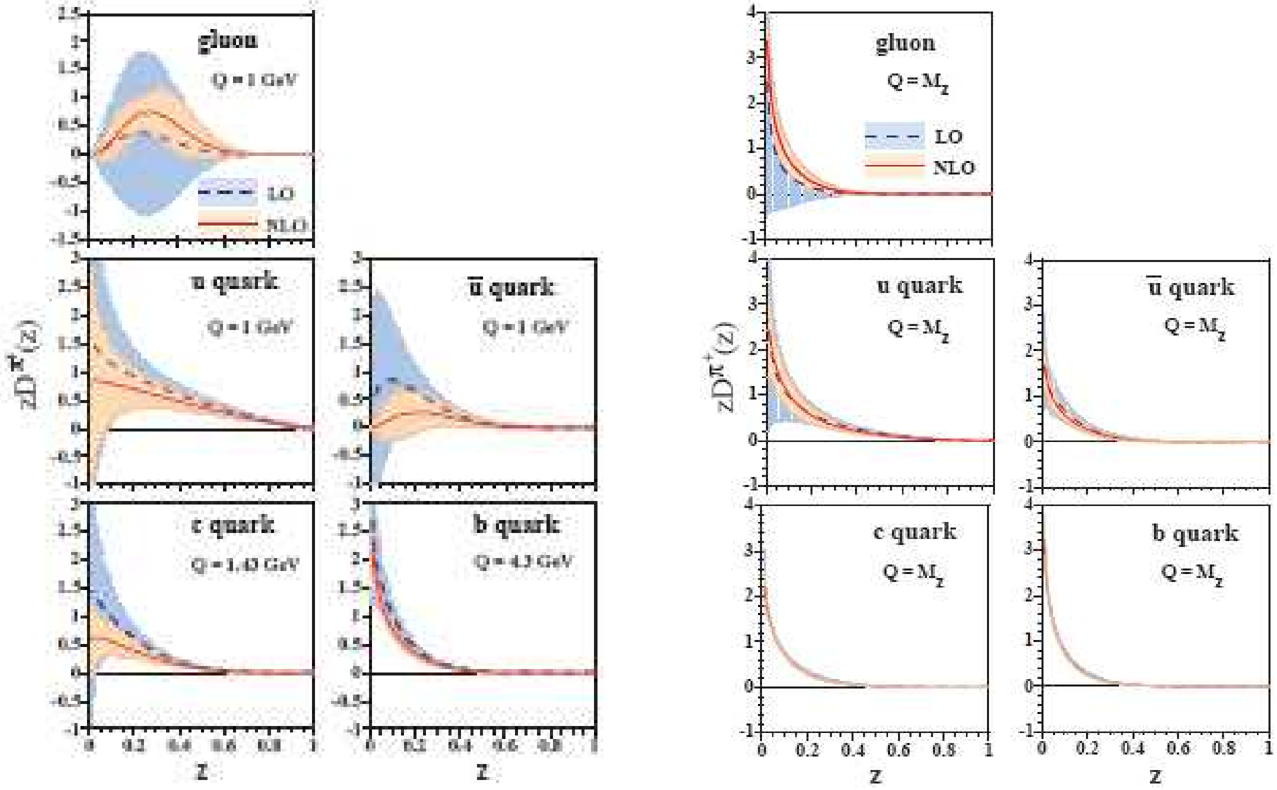


KKP: $D_u^{K^\pm} = D_s^{K^\pm}$; $\sqrt{s} = 29, 91.2$ **GeV;**

$D_u^{K^\pm}$, $D_s^{K^\pm} \rightarrow$ **AKK :** $\sqrt{s} = 29, 91.2$ **GeV; HKNS :** $\sqrt{s} = 29, 58, 91.2$;

- **diff. assumps & diff.** Q^2 **– all params. within uncertainties!**

The uncertainties at $Q^2 = 1\ GeV^2$ and M_Z^2 [*Hirai etc. (HKNS)*]



- D_g = the biggest uncertainties - Q^2-evol. increase uncertns!

Why uncertainties?

- different data used
- different assumptions
- D_g – with biggest uncertainties – affects FFs through evolution
- low Q^2 with bigger uncertainties (due to evolution $\Rightarrow D_g$)

The goal

To obtain info about FFs that is:

1) model independent
   - with no assumptions about FFs
   - with no assumptions about PDFs
2) correct in any QCD order
3) with less uncertainties in Q^2-evolution

Why we need this?

1) test the existing parametrizations
2) test the assumptions
3) test Q^2-evolution

We work with non-singlets

Why NS's?

- no new FF in σ in LO, NLO...
- no new FF in Q^2-evol.
- no uncerts. from D_g in Q^2-evolution

$$\underline{Recall:} \quad DIS: \quad g_1^p - g_1^n = \frac{1}{6}\Delta q_3 \otimes \left(1 + \frac{\alpha_s}{2\pi}\delta C_q + \ldots\right)$$

$$\underbrace{\Delta q_3}_{\text{NS}} = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})$$

- uniquely determines Δq_3 in any QCD order

We ask:

- How do determine the NS of FF in e^+e^-, SIDIS & pp?

We present the formula in NLO:

LO: simple products $q(x).D(z)$

NLO: convolutions $q \otimes D = \int dx'\, dz'\, q(\frac{x}{x'})\, D(\frac{z}{z'})$

The difference cross sections

We suggest: the difference cross section between h and $\bar{h}$:

$$\sigma^h,\, \sigma^{\bar{h}} \Rightarrow \sigma^{h-\bar{h}} = \sigma^h - \sigma^{\bar{h}}$$

We show: they determine **the NS:** [*E.Ch. & E. Leader*]

$$D^h_{u-\bar{u}}, \quad D^h_{d-\bar{d}}, \quad D^h_{s-\bar{s}}$$

without any assumptions, only C-inv!

- notation: $[D^h_{q-\bar{q}} = D^{h-\bar{h}}_q \equiv D^h_q - D^h_{\bar{q}}]$

The processes

$$e + N \to e + h + X: \qquad \sigma_N^{h^+ - h^-} = \sigma_N^{h^+} - \sigma_N^{h^-}$$

$$p + p \to h + X: \qquad \sigma_{pp}^{h^+ - h^-} = \sigma_{pp}^{h^+} - \sigma_{pp}^{h^-}$$

$$\mathrm{C-inv}: \qquad D_g^{h^+ - h^-} = 0, \qquad D_q^{h^+ - h^-} = -D_{\bar{q}}^{h^+ - h^-}$$

The difference cross sections in SIDIS: $\sigma_N^{h-\bar{h}}$

The general formula for $e+N \to e+h+X$ is:

$$\sigma_N^h \;\propto\; \sum_{q,\bar{q}} e_q^2 \Big\{ q \otimes \hat{\sigma}_{qq}(\gamma q \to qX) \otimes D_q^h$$
$$+\, q \otimes \hat{\sigma}_{qg}(\gamma q \to gX) \otimes D_g^h$$
$$+\, g \otimes \hat{\sigma}_{gq}(\gamma g \to q\bar{q}X) \otimes (D_q^h + D_{\bar{q}}^h) \Big\}$$

C-inv. implies: $D_g^{h-\bar{h}} = 0$, $D_q^{h-\bar{h}} = -D_{\bar{q}}^{h-\bar{h}}$

in all QCD orders all gluons cancel: – no g, no D_g

$$\sigma_N^{h-\bar{h}} \propto \left[4u_V \otimes D_u^{h-\bar{h}} + d_V \otimes D_d^{h-\bar{h}} + (s-\bar{s}) \otimes D_s^{h-\bar{h}} \right] \otimes \hat{\sigma}(\gamma q \to qX)$$

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}_{qq}^{(1)} + \ldots$$

• $eN \to e + h + X$: in $\sigma_N^{h-\bar{h}}$ each term = NS of both PDFs and FFs – no g & D_g^h in any order

• $pp \to h + X$: $\sigma_{pp}^{h-\bar{h}}$ = NS only on the FFs – no D_g^h in any order

The difference cross sections, NLO in QCD

$$\sigma_p^{h^+-h^-} \simeq [4u_V \otimes D_u^{h^+-h^-} + d_V \otimes D_d^{h^+-h^-} + s_V \otimes D_s^{h^+-h^-}] \otimes (1+\alpha_s C_{qq}),$$

$$\sigma_d^{h^+-h^-} \simeq [(u_V + d_V) \otimes (4D_u + D_d)^{h^+-h^-} + s_V \otimes D_s^{h^+-h^-}] \otimes (1+\alpha_s C_{qq})$$

$$\sigma_{pp}^{h^+-h^-} \simeq [L_u \otimes u_V \otimes D_u + L_d \otimes d_V \otimes D_d + L_s \otimes s_V \otimes D_s]^{h^+-h^-}$$

$$s_V \equiv (s - \bar{s})$$

- 3 indep. meas. for 3 FFs with no assumptions!

common for the difference cross sections

only the combinations $q_V \, D_q^{h^+ - h^-}$ enter:

$\Downarrow$

- $(D_u, \, D_d)^{h^+ - h^-}$ are enhanced by u_V & d_V
- $D_s^{h^+ - h^-}$ is suppressed by $s - \bar{s}$

ν-exps: $|s - \bar{s}| \leq .025$ [*C. Bourelly, J. Soffer, F. Buccella, 2007*]

- $\sigma_p^{h^+-h^-} \simeq [4u_V \otimes D_u^{h^+-h^-} + d_V \otimes D_d^{h^+-h^-}] \otimes (1 + \alpha_s C_{qq}),$
- $\sigma_d^{h^+-h^-} \simeq [(u_V + d_V) \otimes (4D_u + D_d)^{h^+-h^-}] \otimes (1 + \alpha_s C_{qq})$
- $\sigma_{pp}^{h^+-h^-} \simeq [L_u \otimes u_V \otimes D_u + L_d \otimes d_V \otimes D_d]^{h^+-h^-}$

- only $D_u^{h^+-h^-}$ and $D_d^{h^+-h^-}$ enter
- $(D_u, D_d)^{h^+-h^-}$ are enhanced by u_V & d_V
- D_g does not enter
- these hold in any QCD order – due to sym. properties!

the info on $D_q^{h-\bar{h}}$ depends on $h = \pi^\pm, K^\pm$... and the process

L_q : known functs. of $\tilde{q} = q + \bar{q}$ & $\hat{\sigma}_{ab}^{cX}(a + b \longrightarrow c + X)$:

$$L_u(x,t,u) = \tilde{u}(x)\, d\hat{\Sigma} + (\tilde{d} + \tilde{s})(x)\, d\hat{\sigma}_{qq'}^{qX} + g(x)\, d\hat{\sigma}_{qg}^{(q-\bar{q})X}$$

$$L_d(x,t,u) = \tilde{d}(x)\, d\hat{\Sigma} + (\tilde{u} + \tilde{s})(x)\, d\hat{\sigma}_{qq'}^{qX} + g(x)\, d\hat{\sigma}_{qg}^{(q-\bar{q})X}$$

$$L_s(x,t,u) = \tilde{s}(x)\, d\hat{\Sigma} + (\tilde{u} + \tilde{d})(x)\, d\hat{\sigma}_{qq'}^{qX} + g(x)\, d\hat{\sigma}_{qg}^{(q-\bar{q})X}$$

where

$$d\hat{\Sigma}(s,t,u) \equiv \left[d\hat{\sigma}_{qq}^{qX}(s,t,u) + \frac{1}{2} d\hat{\sigma}_{q\bar{q}}^{(q-\bar{q})X}(s,t,u) \right], \qquad \tilde{q} = q + \bar{q}.$$

$$\hat{\sigma}_{ab}^{cX} = \alpha_s\, \sigma^{(0)} + \alpha_s^2\, \sigma^{(1)} + \ldots$$

- $\sigma_{pp}^{h-\bar{h}}$ is much simpler than σ_{pp}^{h}

$\hat{\sigma}_{ab}^{cX}$: LO: 4 (in $\sigma_{pp}^{h-\bar{h}}$) $\Leftrightarrow$ 8 (in σ_{pp}^{h})

NLO: 6 (in $\sigma_{pp}^{h-\bar{h}}$) $\Leftrightarrow$ 21 (in σ_{pp}^{h})

The difference cross sections for $h = K^{\pm}$

1) $D_u^{K^+-K^-}$ & $D_d^{K^+-K^-}$

$\Rightarrow$ determined without assumptions

2) $\underline{\underline{D_d^{K^+-K^-} = 0}}$?

- $\sigma_p^{K^+-K^-} \simeq 4u_V \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+-K^-}$,
- $\sigma_d^{K^+-K^-} \simeq 4(u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+-K^-}$
- $\sigma_{pp}^{K^+-K^-} \simeq L_u \otimes u_V \otimes D_u^{K^+-K^-}$

- only if $D_d^{K^+-K^-} = 0$ all $\sigma^{K^+-K^-}$ are fitted with one FF!

$\Rightarrow$ we test only this assumption!

- $D_d^{K^+-K^-} = 0$ in all analysis

The difference cross sections for $h = \pi^{\pm}$

1) $D_u^{\pi^+ - \pi^-}$ & $D_d^{\pi^+ - \pi^-}$

$\Rightarrow$ determined without assumptions

2) $D_u^{\pi^+ - \pi^-} = -D_d^{\pi^+ - \pi^-}$?

$$\bullet\, \sigma_p^{\pi^+ - \pi^-} \simeq [4u_V - d_V] \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) \otimes D_u^{\pi^+ - \pi^-}$$

$$\bullet\, \sigma_d^{\pi^+ - \pi^-} \simeq [u_V + d_V] \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) \otimes D_u^{\pi^+ - \pi^-}$$

$$\bullet\, \sigma_{pp}^{\pi^+ - \pi^-} \simeq [L_u \otimes u_V - L_d \otimes d_V] \otimes D_u^{\pi^+ - \pi^-}$$

- only the best determined u_V and d_V enter
- only if $D_u^{\pi^+ - \pi^-} = -D_d^{\pi^+ - \pi^-}$ all $\sigma^{\pi^+ - \pi^-}$ are fitted with 1 FF!

$\Rightarrow$ we test only this assumption! *[De Florian et al: $\simeq$ 10%]*

Processes with kaons

up to now: $\forall\, h^{\pm}: \quad \sigma^{h^+} - \sigma^{h^-} \Rightarrow D_u^{h^+-h^-},\ D_d^{h^+-h^-}$

now: only kaons: both $K^{\pm}$ & K_s^0

Why $K^{\pm}$ and K^0? – no new FFs appear:

- SU(2) relates $K^{\pm}$ and K_s^0:

$$D_u^{K^+ + K^-} = D_d^{K^0+\bar{K}^0}, \qquad D_d^{K^+ + K^-} = D_u^{K^0+\bar{K}^0}$$

$$D_s^{K^+ + K^-} = D_s^{K^0+\bar{K}^0}, \qquad D_g^{K^+ + K^-} = D_g^{K^0+\bar{K}^0}$$

$$D_c^{K^+ + K^-} = D_c^{K^0+\bar{K}^0}, \qquad D_b^{K^+ + K^-} = D_b^{K^0+\bar{K}^0}$$

used in all analysis!

Processes with $K^{\pm}$ and K_s^0

We consider: the diff. & sum cross sections of $K^{\pm}$ and K_s^0:

$$\sigma^{\mathcal{K}} = \sigma^{K^+} + \sigma^{K^-} - 2\,\sigma^{K_s^0} \Rightarrow (D_u - D_d)^{K^+ + K^-} = NS$$

$$\sigma^{K} = \sigma^{K^+} + \sigma^{K^-} + 2\,\sigma^{K_s^0} \Rightarrow (D_u + D_d)^{K^+ + K^-},\; D_s^{K^{\pm}}$$

[*E.Ch. & E.Leader*]

assumed: only C-inv. and SU(2)-inv!

The difference of $K^{\pm}$ and K_s^0

$$\sigma^{K^{\pm}}, \sigma^{K_s^0} \quad \Rightarrow \quad \sigma^{\mathcal{K}} = \sigma^{K^+} + \sigma^{K^-} - 2\sigma^{K_s^0}, \quad K_s^0 = (K^0 + \bar{K}^0)/\sqrt{2}$$

the processes $[K = K^{\pm}, K_s^0]$:

- $e^+e^- \to K + X$: $\quad \sigma^{\mathcal{K}}_{e^+e^-} = \sigma^{K^+}_{e^+e^-} + \sigma^{K^-}_{e^+e^-} - 2\sigma^{K_s^0}_{e^+e^-}$
- $eN \to e + K + X$: $\quad \sigma^{\mathcal{K}}_{N} = \sigma^{K^+}_{N} + \sigma^{K^-}_{N} - 2\sigma^{K_s^0}_{N}$
- $pp \to K + X$: $\quad \sigma^{\mathcal{K}}_{pp} = \sigma^{K^+}_{pp} + \sigma^{K^-}_{pp} - 2\sigma^{K_s^0}_{pp}$

- Note: e^+e^- can be used $\to$ the most precise data!

The cross sections for $\sigma^{\mathcal{K}}$:

$$\bullet\, d\sigma^{\mathcal{K}}_{e^+e^-} \simeq 6\sigma_0(\hat{e}_u^2 - \hat{e}_d^2)(1 + \alpha_s\, C_q \otimes)\, \mathrm{D}_{\mathrm{u-d}}^{\mathrm{K^+ + K^-}}$$

$$\bullet\, d\sigma^{\mathcal{K}}_{p} \simeq [(4u + d) \otimes (1 + \alpha_s\, C_{qq} \otimes) + \alpha_s\, g \otimes C_{gq} \otimes]\, \mathrm{D}_{\mathrm{u-d}}^{\mathrm{K^+ + K^-}}$$

$$\bullet\, d\sigma^{\mathcal{K}}_{d} \simeq [(u + d) \otimes (1 + \alpha_s\, C_{qq} \otimes) + \alpha_s\, g \otimes C_{gq} \otimes]\, \mathrm{D}_{\mathrm{u-d}}^{\mathrm{K^+ + K^-}}$$

$$\bullet\, d\sigma^{\mathcal{K}}_{pp} \simeq \sum_{ab} f_a \otimes f_b \otimes \hat{\Sigma}_{ab} \otimes \mathrm{D}_{\mathrm{u-d}}^{\mathrm{K^+ + K^-}}$$

$$\hat{\Sigma}_{ab} = \hat{\sigma}_{ab}^{uX} + \hat{\sigma}_{ab}^{\bar{u}X} - \hat{\sigma}_{ab}^{dX} - \hat{\sigma}_{ab}^{\bar{d}X}, \qquad D_{u-d}^{K^+ + K^-} \equiv (D_u - D_d)^{K^+ + K^-}$$

common for $\sigma^{\mathcal{K}}$

- each process in LO, NLO... $\longrightarrow$ the same FF $= \mathbf{D_{u-d}^{K^+ + K^-}}$

$\Rightarrow$ test of factorization!

- $\mathbf{D_{u-d}^{K^+ + K^-}} =$ NS $\Rightarrow$ no new FFs appear in Q^2 evolution

- $\sigma^{\mathcal{K}}_{e^+e^-}$, $\sigma^{\mathcal{K}}_{N}$ & $\sigma^{\mathcal{K}}_{pp}$ measure $D_{u-d}^{K^+ + K^-}$ at very different Q^2:

$$e^+e^- \longrightarrow K + X \qquad high \quad Q^2, \quad \sim \quad Z^0 - exchange$$

$$eN \longrightarrow e + K + X \qquad low \quad Q^2, \quad \sim \quad \gamma - exchange$$

$\Rightarrow$ test of Q^2 evolution! – $D_{u-d}^{K^+ + K^-}$ doesn't mix

• **both** $D_u^{K^+ + K^-}$ **&** $D_d^{K^+ + K^-}$ **well determined in** $e^+e^- \longrightarrow K^\pm + X$

but different assumps. $\Rightarrow$ **test of the assumptions!**

$\mathrm{D}_{\mathbf{u-d}}^{\mathrm{K^++K^-}}$ from $e^+e^- \to K + X$

Why e^+e^- is most attractive?

$$d\sigma^{\mathcal{K}}_{e^+e^-} \simeq 6\sigma_0(\hat{e}_u^2 - \hat{e}_d^2)(1 + \alpha_s\, C_q \otimes)\, \mathrm{D}_{\mathbf{u-d}}^{\mathrm{K^++K^-}}$$

1) the most precise and abundant data

2) no uncertainties from PDFs

3) the most accurate th. calculations − NLO, NNLO

4) use all available data: $z > 0.001 \Rightarrow$ no divergs. at small z

$\Rightarrow$ global fit analysis: $z > 0.05$ − AKK, HNKNS, $z > 0.1$ - DSS

$\Rightarrow$ most data and the most accurate data are at small z

5) the price: $|\overrightarrow{p}^h| \neq E^h$ at small z & $\sqrt{s} - m_K$ corrs.

The sum of $K^{\pm}$ and K_s^0

$$\sigma^{K^{\pm}},\ \sigma^{K_s^0} \quad \Rightarrow \quad \sigma^{K} = \sigma^{K^+} + \sigma^{K^-} + 2\sigma^{K_s^0}$$

the processes:

- $e^+e^- \to K + X$: $\quad \sigma^{K}_{e^+e^-} = \sigma^{K^+}_{e^+e^-} + \sigma^{K^-}_{e^+e^-} + 2\sigma^{K_s^0}_{e^+e^-}$

- $eN \to e + K + X$: $\quad \sigma^{K}_{N} = \sigma^{K^+}_{N} + \sigma^{K^-}_{N} + 2\sigma^{K_s^0}_{N}$

$$LO: \quad \sigma^{K}_{e^+e^-} \simeq (\hat{e}_u^2 + \hat{e}_d^2)\, D_{u+d} + 2\hat{e}_d^2\; D_s$$

$$\sigma^{K}_{p} \simeq (4\tilde{u} + \tilde{d})\, D_{u+d} + 2\,\tilde{s}\, D_s$$

$$\sigma^{K}_{n} \simeq (4\tilde{d} + \tilde{u})\, D_{u+d} + 2\,\tilde{s}\, D_s$$

- only $D_{u+d}^{K^++K^-}$ and $D_s^{K^++K^-}$ enter, $\quad [\tilde{q} = q + \bar{q}]$

- $\sigma_p^K,\ \sigma_n^K \Rightarrow (D_{u+d},\ D_s)^{K^++K^-}$

- $\sigma_{p,n}^{\mathcal{K}} \Rightarrow (D_{u-d})^{K^++K^-}$

– $(D_{u\pm d},\ D_s)^{K^\pm} \Leftarrow$ only in SIDIS, no need of e^+e^- at high Q^2

– less uncertainties from Q^2-evolution – **but LO only!**

Test of LO:

$$\frac{(\sigma_p - \sigma_n)^K(x, z)}{(\tilde{u} - \tilde{d})(x)} = f(z) = D_{u+d}^{K^++K^-}(z)$$

NLO for $\sigma^K = \sigma^{K^+} + \sigma^{K^-} + 2\sigma^{K_s^0}$:

$$\sigma^K_{e^+e^-},\ \sigma^K_p,\ \sigma^K_n \Rightarrow (D_{u+d},\ D_s \text{ and } D_g)^{K^++K^-}$$

$$LO: (\sigma_p - \sigma_n)^K \simeq \underbrace{(\tilde{u} - \tilde{d})}_{NS} D_{u+d}$$

$$NLO: (\sigma_p - \sigma_n)^K \simeq (\tilde{u} - \tilde{d}) \otimes \{D_{u+d}(1 + \alpha_s C_{qq}) + \alpha_s CqgD_g\}$$

- advantage: D_{u+d} and D_g – no uncertainties of s and g.

$$\sigma^K_{e^+e^-} \simeq (\hat{e}_u^2 + e_d^2) D_{u+d} + \hat{e}_d^2 D_s + \alpha_s D_g$$

$$\sigma^{\mathcal{K}}_{e^+e^-},\ \sigma^{\mathcal{K}}_N,\ \sigma^{\mathcal{K}}_{pp} \Rightarrow D^{K^++K^-}_{u-d}$$

- advantage: D_s not suppressed by s

Summary

We define: meas. quantities that uniquely determine NS of FFs in any QCD order and without assumps. on FFs & PDFs:

1) $\sigma_p^{h^+-h^-}$, $\sigma_d^{h^+-h^-}$ & $\sigma_{pp}^{h^+-h^-} \simeq (D_u \,\&\, D_d)^{h^+-h^-}$ = NS, $\forall h^\pm$

2) $\sigma_p^{\pi^+-\pi^-}$, $\sigma_d^{\pi^+-\pi^-}$ & $\sigma_{pp}^{\pi^+-\pi^-} \simeq$ test for $D_u^{\pi^+-\pi^-} = -D_d^{\pi^+-\pi^-}$

3) $\sigma_p^{K^+-K^-}$, $\sigma_d^{K^+-K^-}$ & $\sigma_{pp}^{K^+-K^-} \simeq$ test for $D_d^{K^+-K^-} = 0$

4) if $K^\pm \,\&\, K_s^0 \Rightarrow \sigma^{\mathcal{K}} = \sigma^{K^+} + \sigma^{K^-} - 2\sigma^{K_s^0}$

$\sigma_{e^+e^-}^{\mathcal{K}}$, $\sigma_p^{\mathcal{K}}$, $\sigma_d^{\mathcal{K}}$ & $\sigma_{pp}^{\mathcal{K}} \simeq (D_u - D_d)^{K^+ + K^-}$ = NS

- possible test for the commonly used assumptions
- possible test for factorization and Q^2 evolution

5) Cross section differences determine only NS,

but without assumps. and uncerts. in Q^2-evol.

6) our mssg: $\sigma^{K^+-K^-}$, $\sigma^{\pi^+-\pi^-}$ and $\sigma^{\mathcal{K}}$ can be used also as model independent constraints in global fit analysis

7) if $K^\pm$ and $K^0 \Rightarrow \sigma^{K^+} + \sigma^{K^-} \pm 2\sigma^{K_s^0}$:

LO: SIDIS $\Rightarrow (D_{u\pm d} \text{ and } D_s)^{K^+ + K^-}$

NLO: SIDIS & $e^+e^- \Rightarrow (D_{u\pm d},\ D_s \text{ and } D_g)^{K^+ + K^-}$ without the uncertainties of s and g

8) control of the acceptances needed to form these differences – we look forward to SIDIS at COMPASS and JLab and pp at RHIC.