Kaon Fragmentation Function from NJL-Jet

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CSSM

Collaborators:
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Outlook

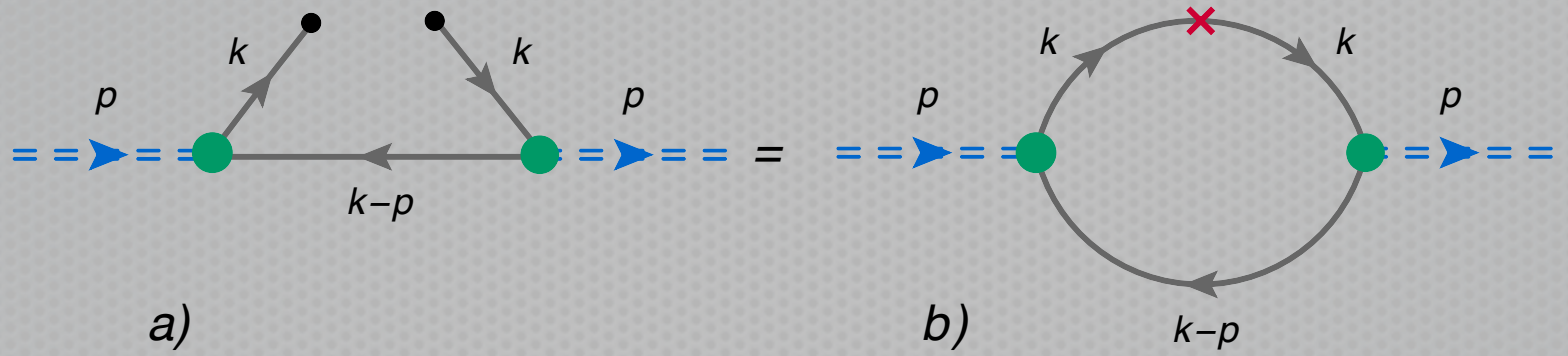
- Motivation
- Strange NJL-jet: Distribution & Fragmentation Functions.
- Monte-Carlo simulations:
  - Vector mesons, Nucleon--Anti-Nucleon.
  - Inclusion of $p_\perp$.

Motivation

- Providing guidance on a bases of a sophisticated model for applications to problems where phenomenology is difficult.
- Unfavored fragmentation functions from model.
- Transverse-momenta dependent fragmentations in a model where structure functions (both unpolarized and polarized) were calculated.

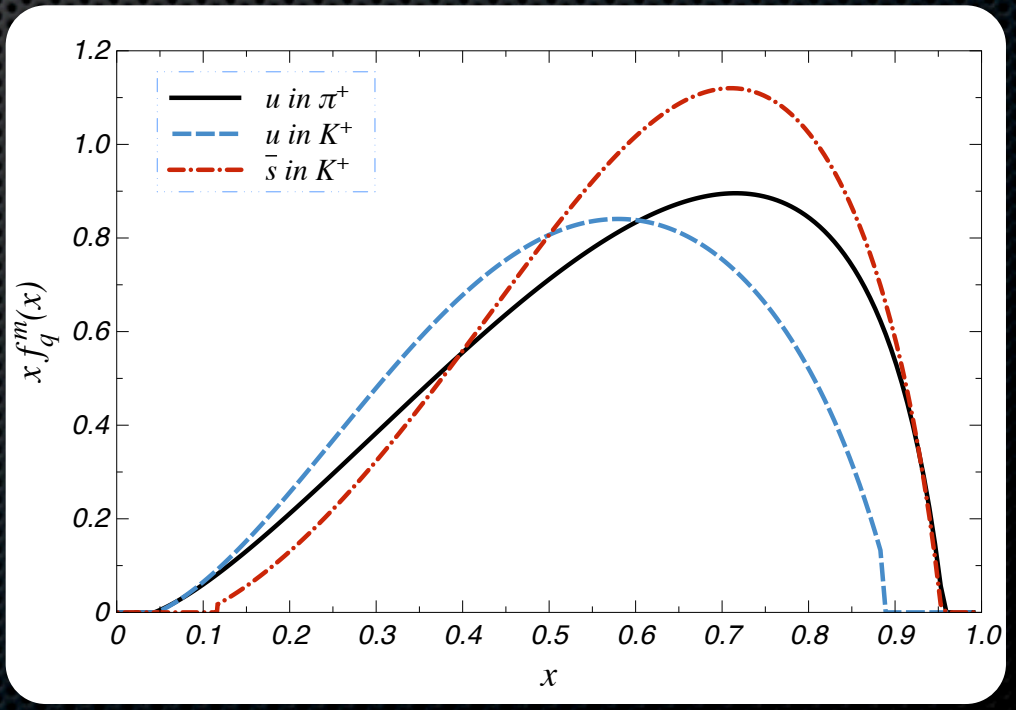
K Distribution Function in NJL

Bentz et al.: Nucl.Phys.A651:143-173,1999.



- Regularization:

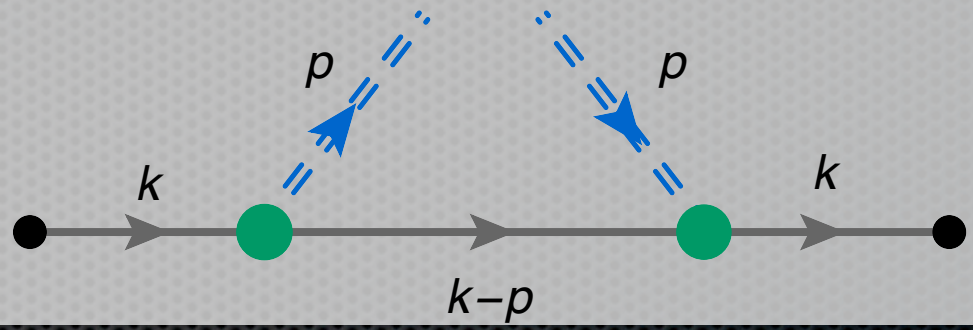
  Lepage-Brodsky (LB)
  Invariant Mass Cutoff



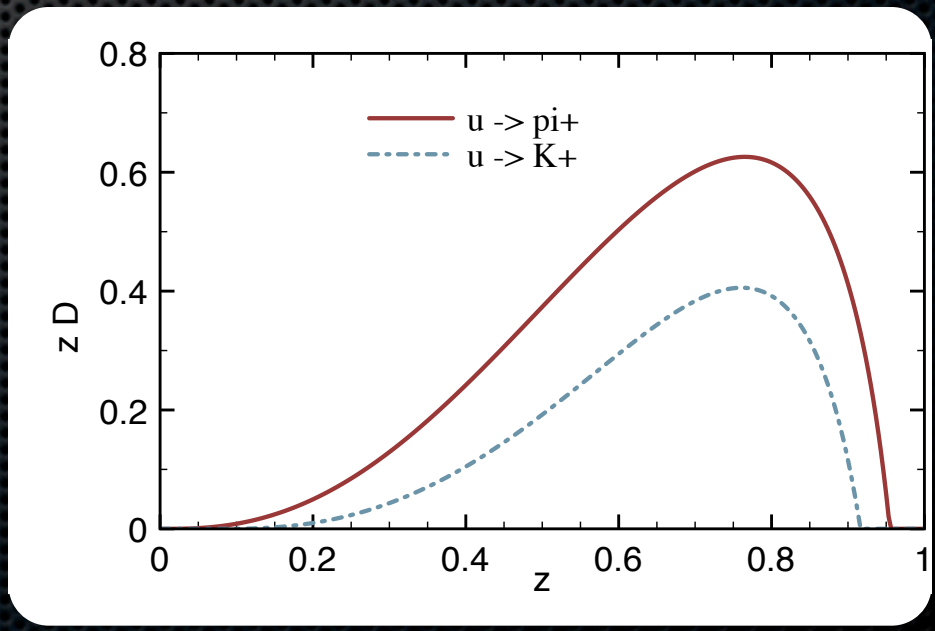
Splitting Functions

- One-quark truncation of the wavefunction:

$$d_q^m(z) : q \to Qm$$
$$m = q\bar{Q}$$



$$u \to d\pi^+$$
$$u \to sk^+$$

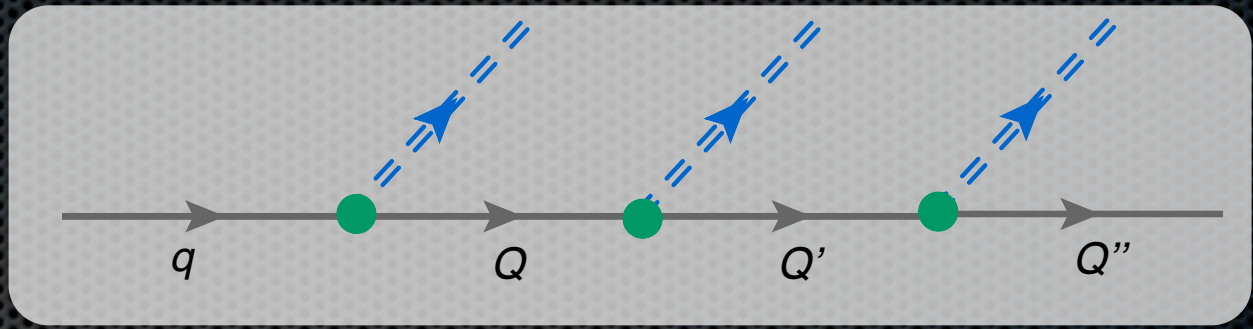


NJL-jet Model for Fragmentation Function

Field, Feynman.Nucl.Phys.B136:1,1978.

- Chain Decay:

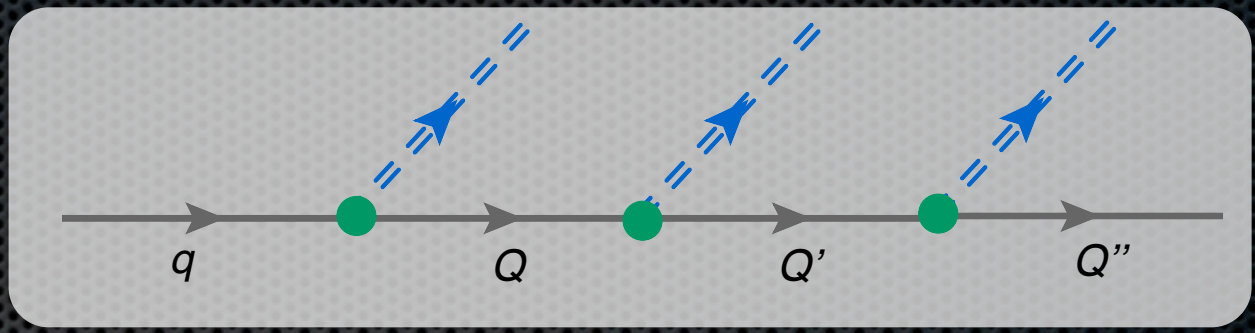
  No re-absorption



$$D_q^m(z) = \hat{d}_q^m(z) + \int_z^1 \frac{dy}{y} \hat{d}_q^Q(\frac{z}{y}) \cdot D_Q^m(y)$$

$$\hat{d}_q^m(z) = \hat{d}_q^{Q'}(1-z)|_{m=\bar{Q}'q}$$

Probabilistic Interpretation of Integral Equation



The probability of finding mesons m with mom. fraction z in a jet of quark q

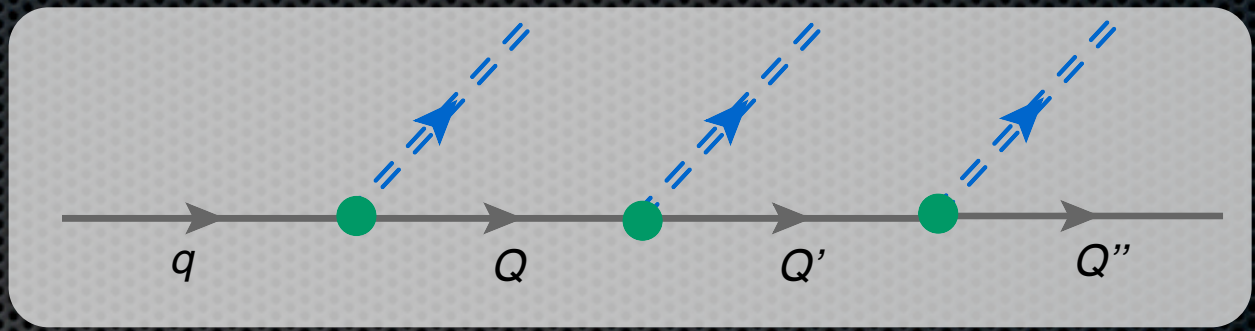
$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m(\frac{z}{y})\frac{dz}{y}$$

Probability of emittng the meson at link 1

Probability of Momentum fraction y is transferred to jet at step 1

The probability scales with mom. fraction

Probabilistic Interpretation of Integral Equation



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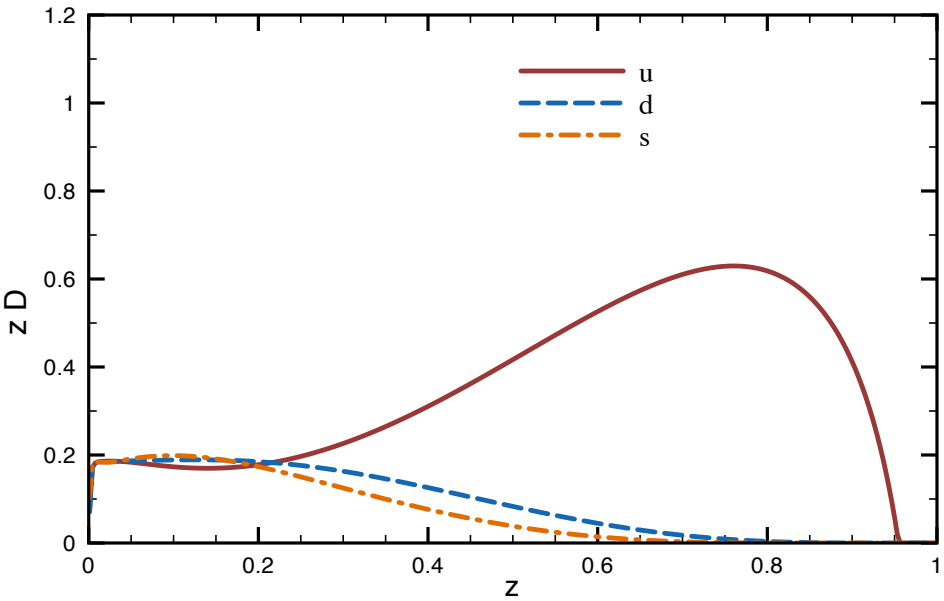
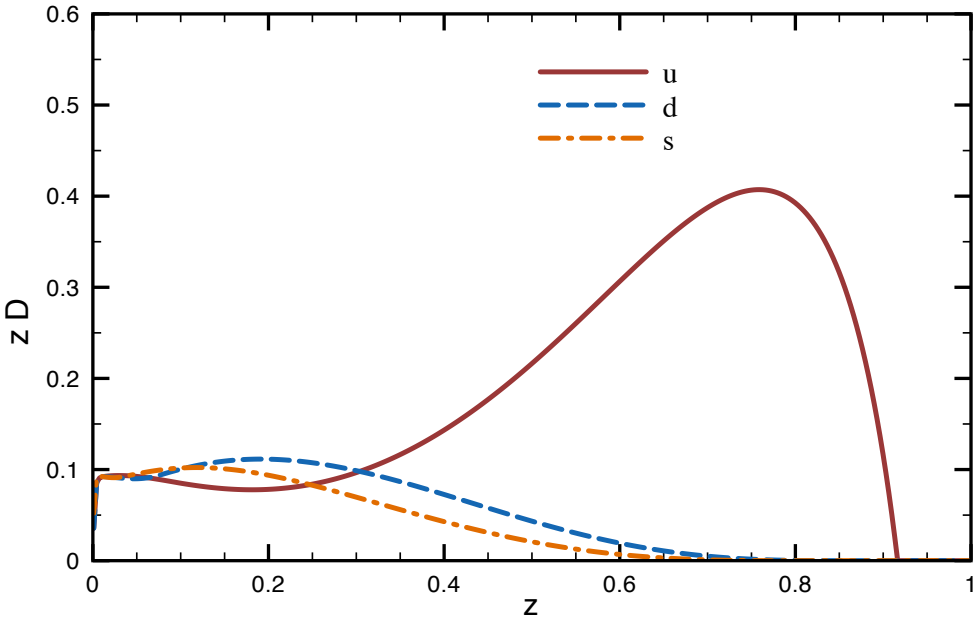
Solutions at the Model Scale

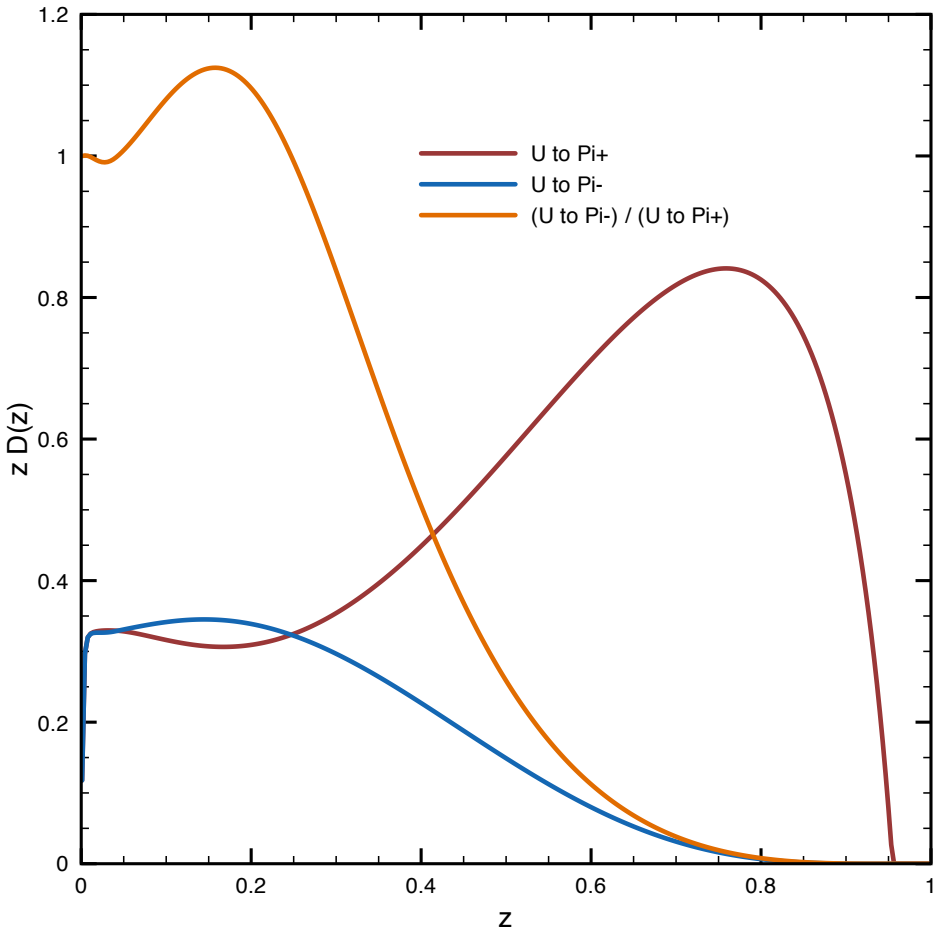


π^+



K^+

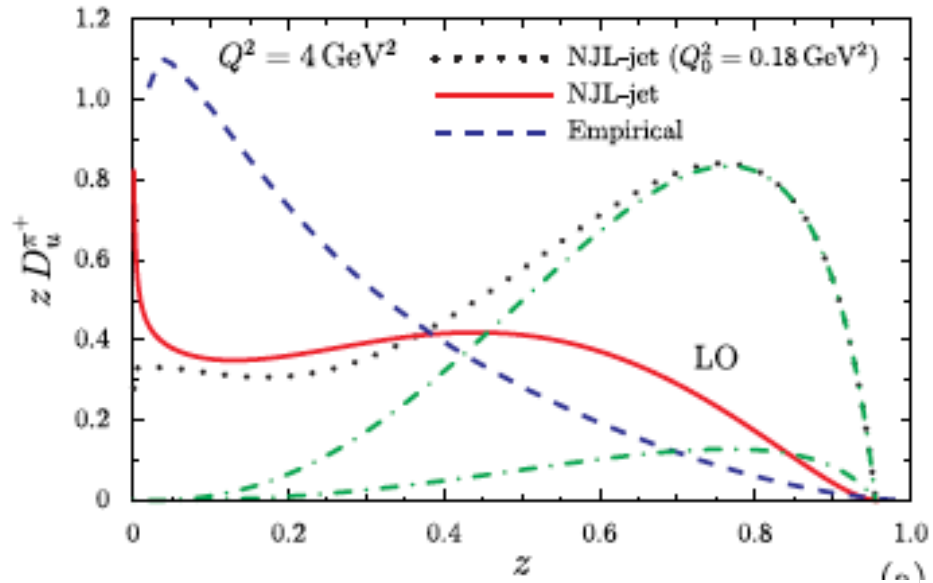
The Ratio of Unfavored to Favored

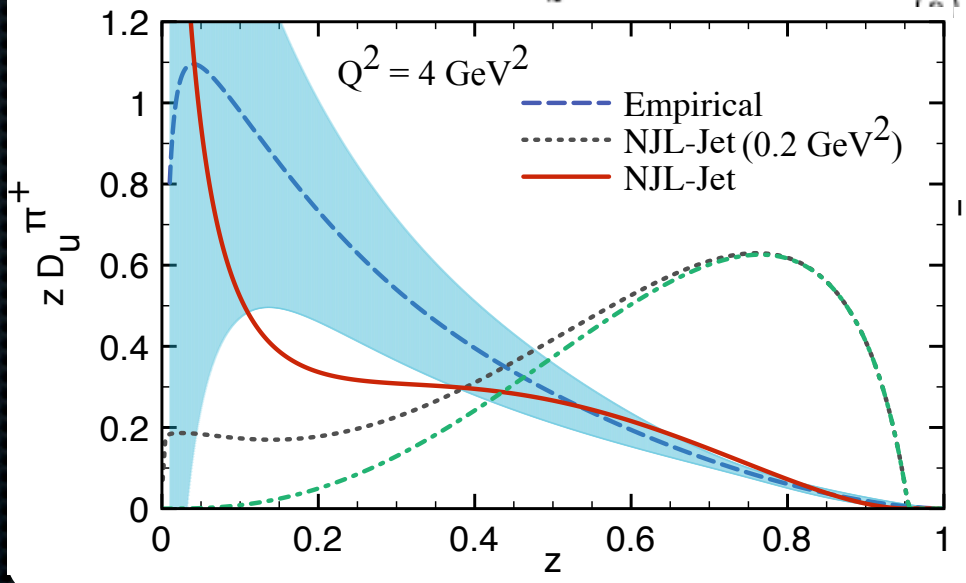


Strangeness Effect in Pion

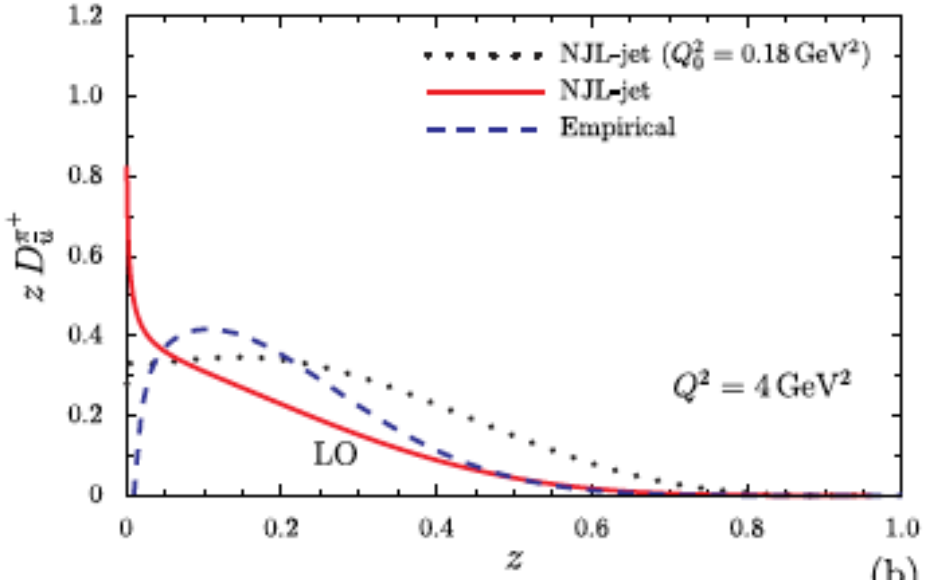
Ito et al. Phys.Rev.D80:074008,2009

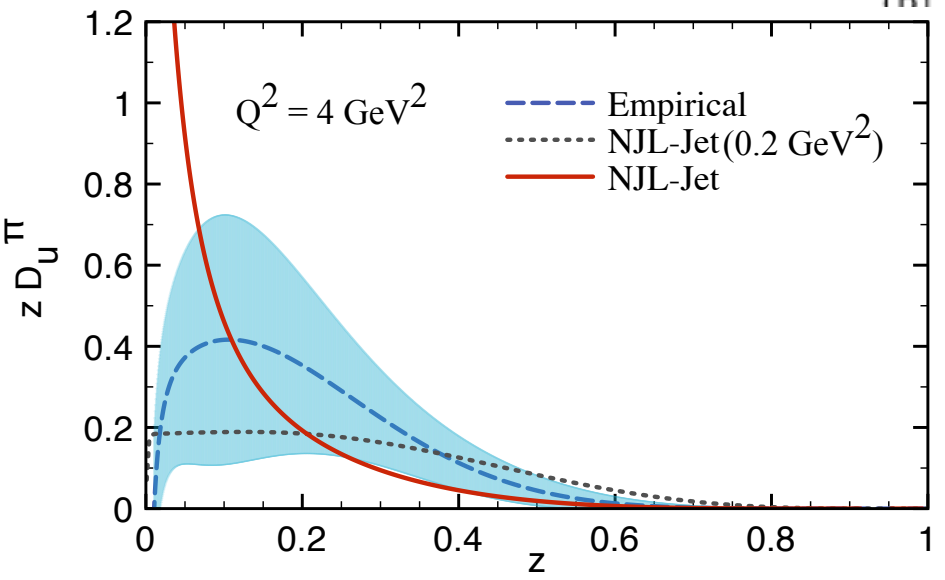
Favored





Unfavored

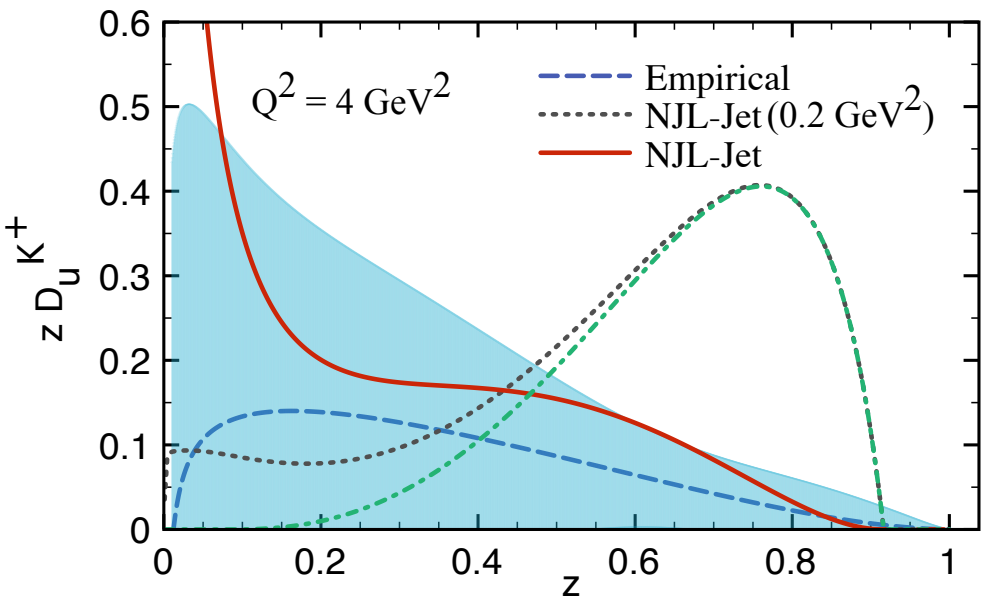


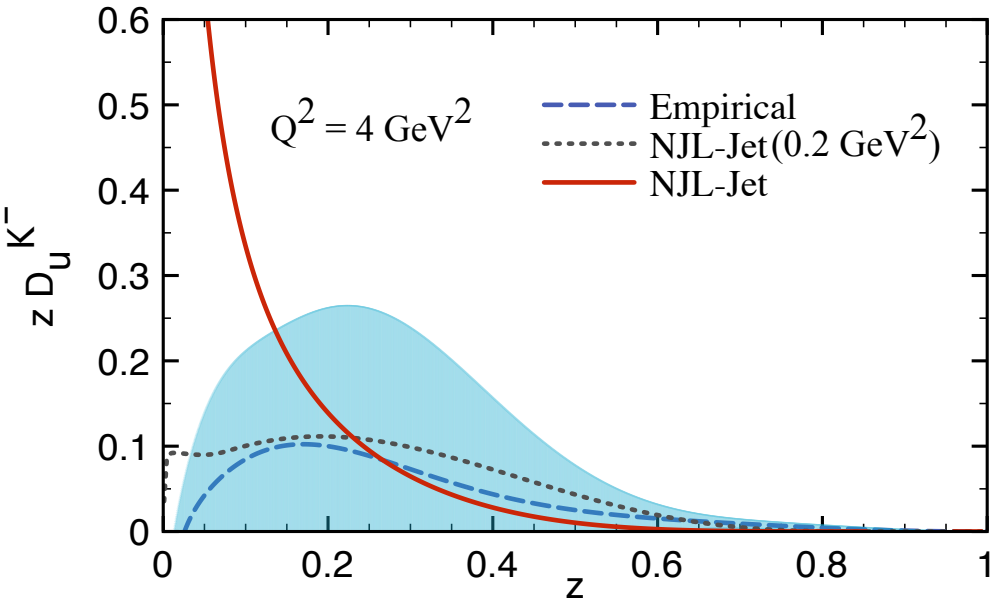


no s quark

with s quark

Results for Kaon





Monte-Carlo (MC) Simulations

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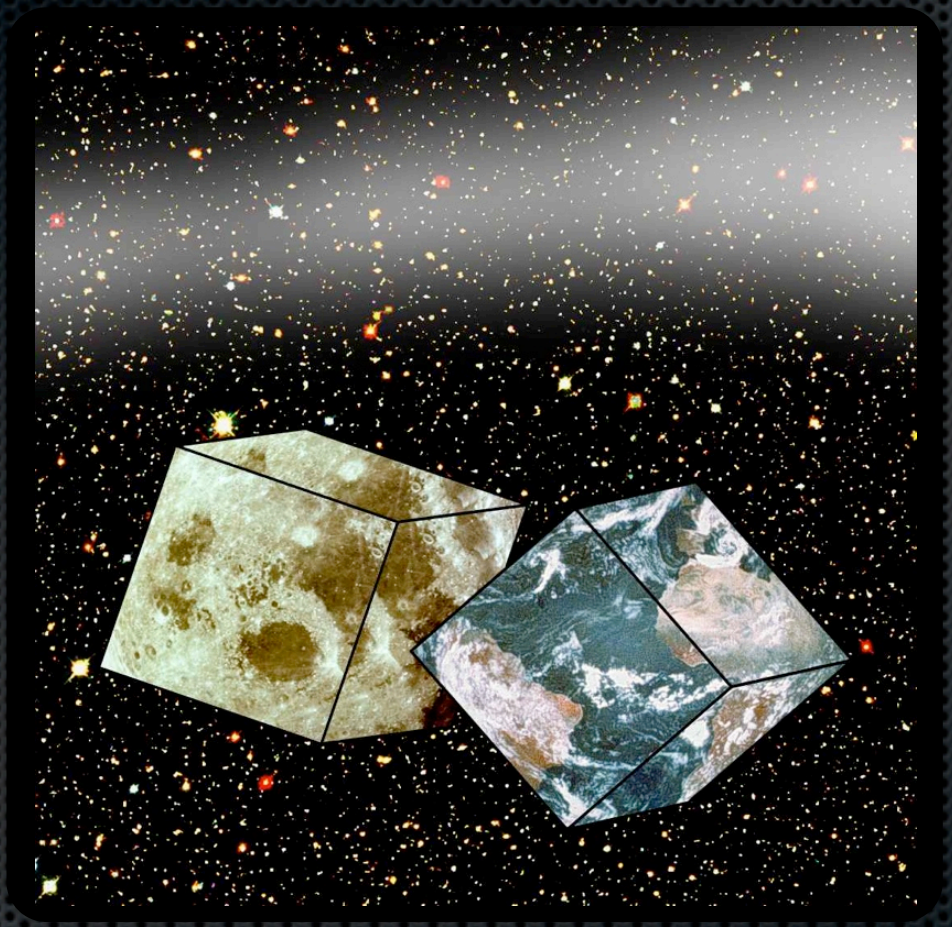

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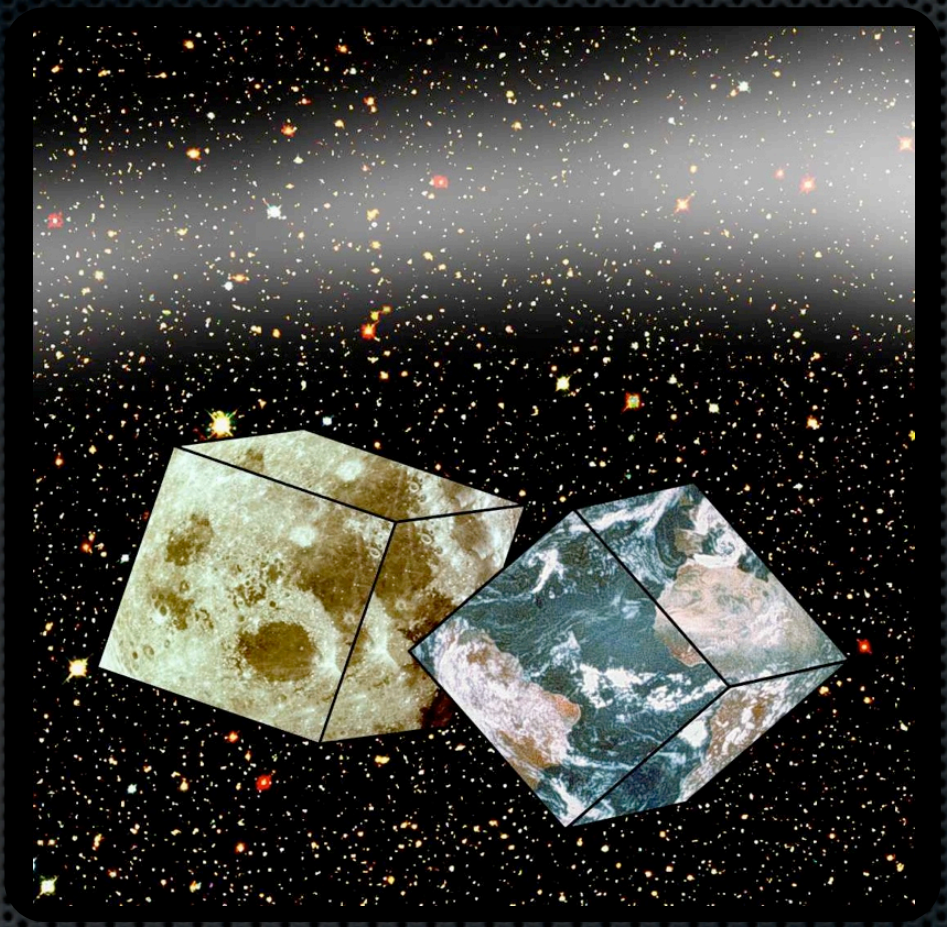
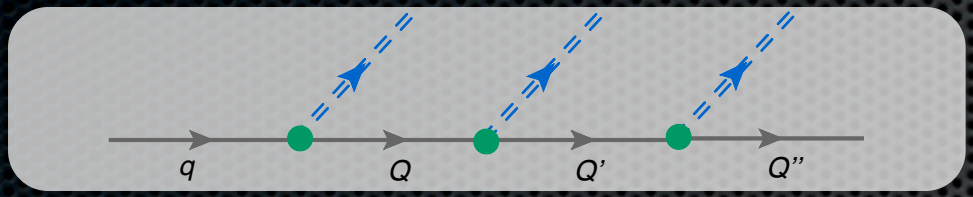


- Simulate decay chains to extract probabilities.
- Allows for inclusion of $p_\perp$ and experimental cut-offs.
- Numerically trivially parallelizeable (MPI, GPGPU).

Fragmentations from MC

Starting with Pions

- Assume Cascade process:



- Sample the emitted hadron according to splitting weight.
- Randomly sample z from input splittings.
- Evolve to sufficiently large number of decay links.
- Repeat for decay chains with the same initial quark.

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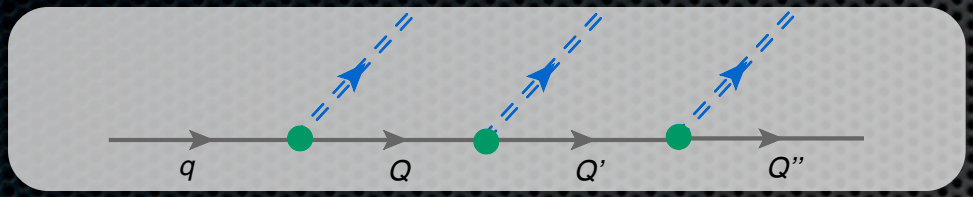

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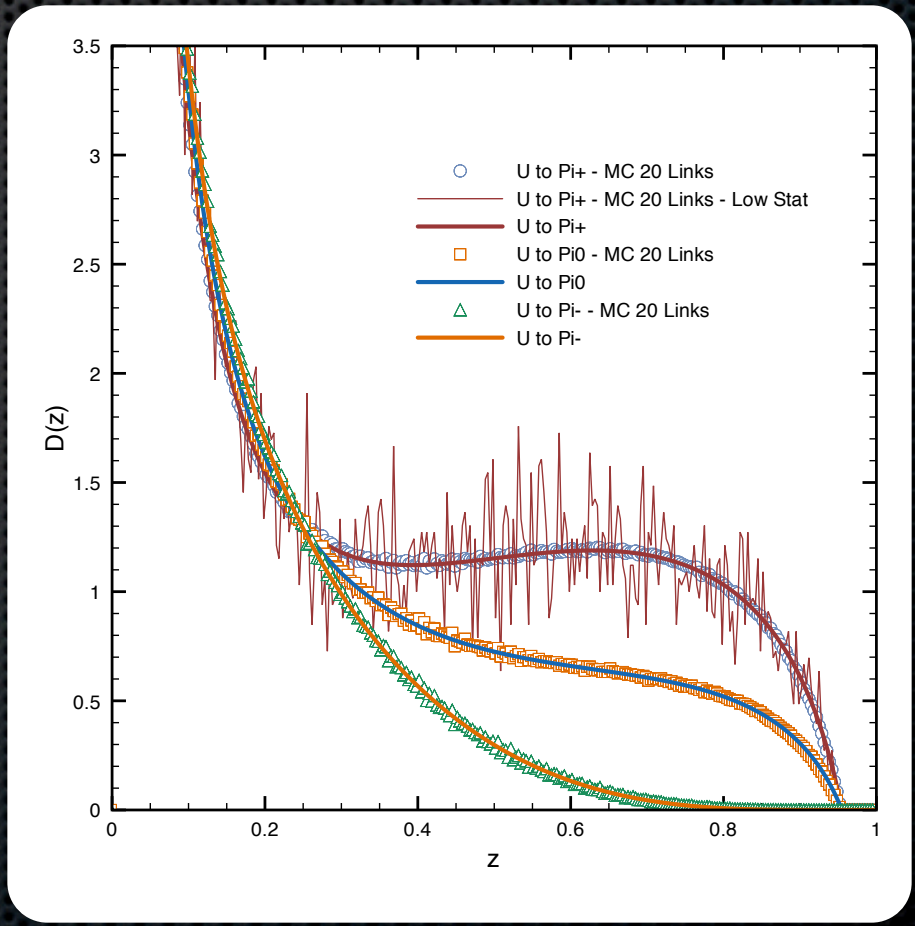
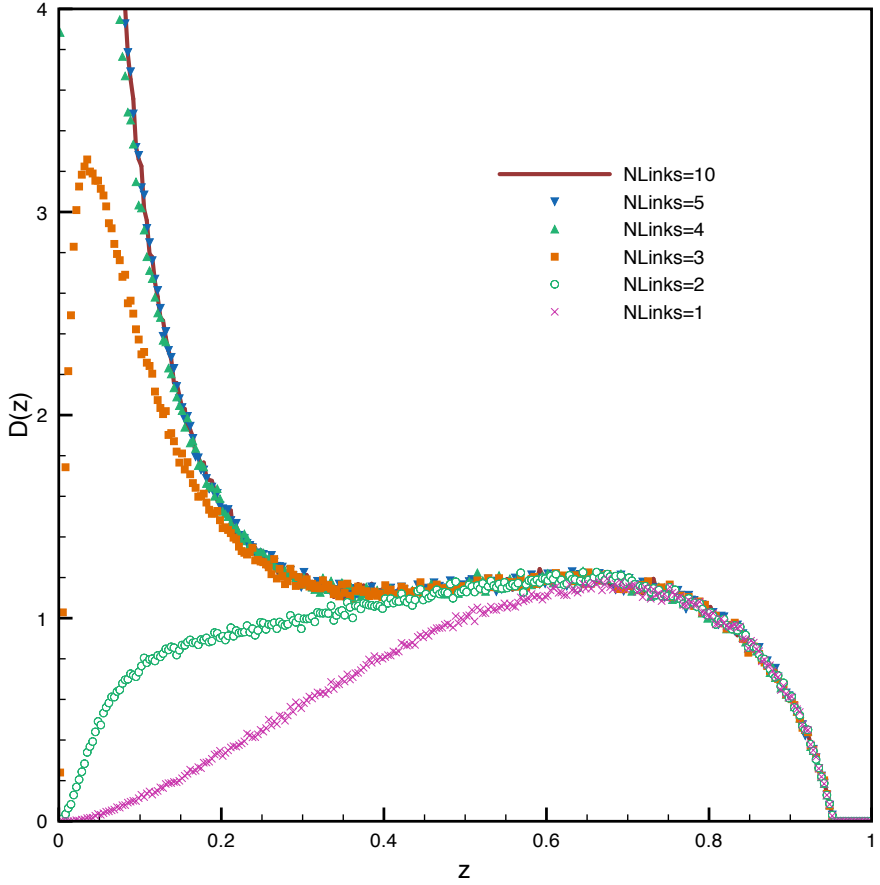
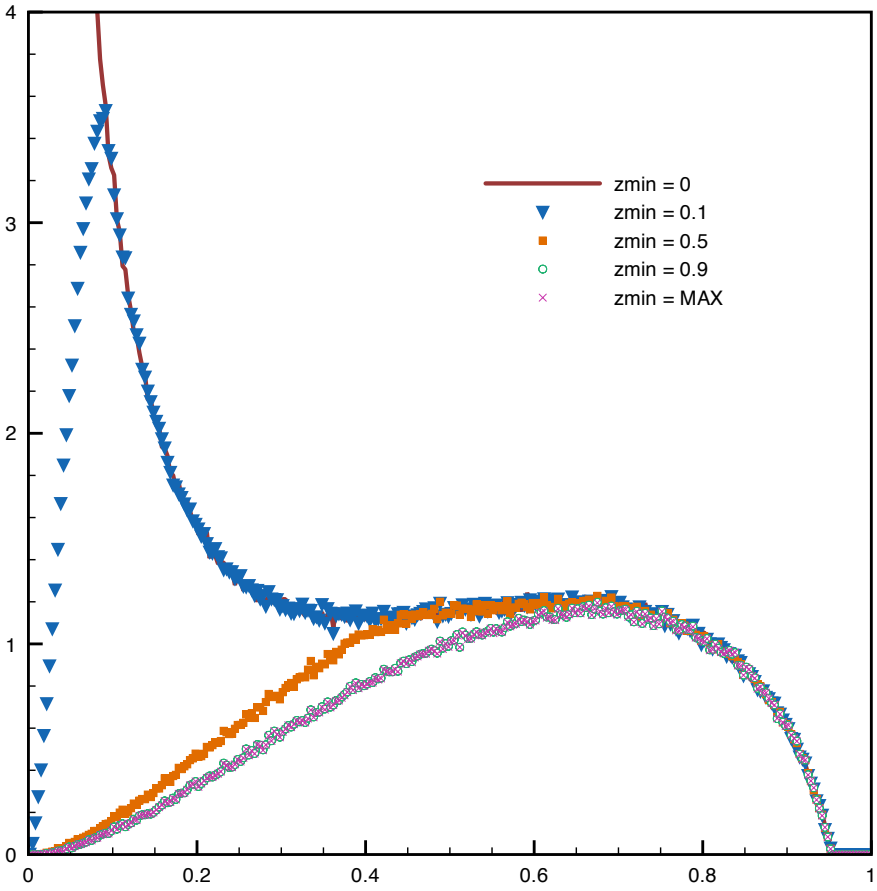


Dependence on Chain cut-off

- Variation with # of links



- Variation with z_{min}

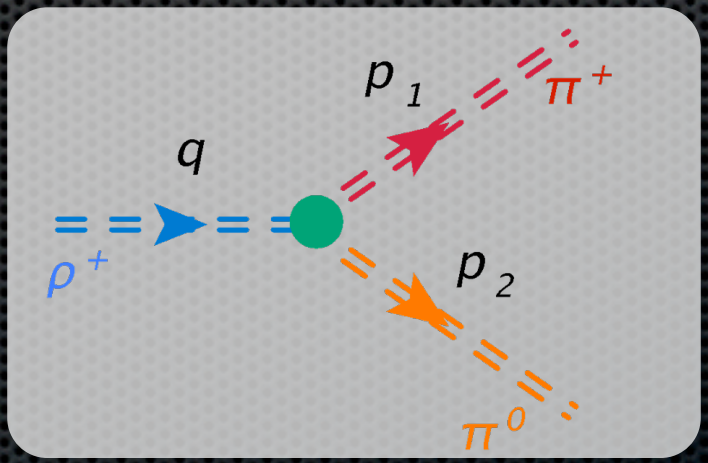


More Channels: Vector Mesons

- Calculate quark splittings $d_q^m(z)$ in vector channel:

$$m = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}$$

- Add the decay of the resonances:

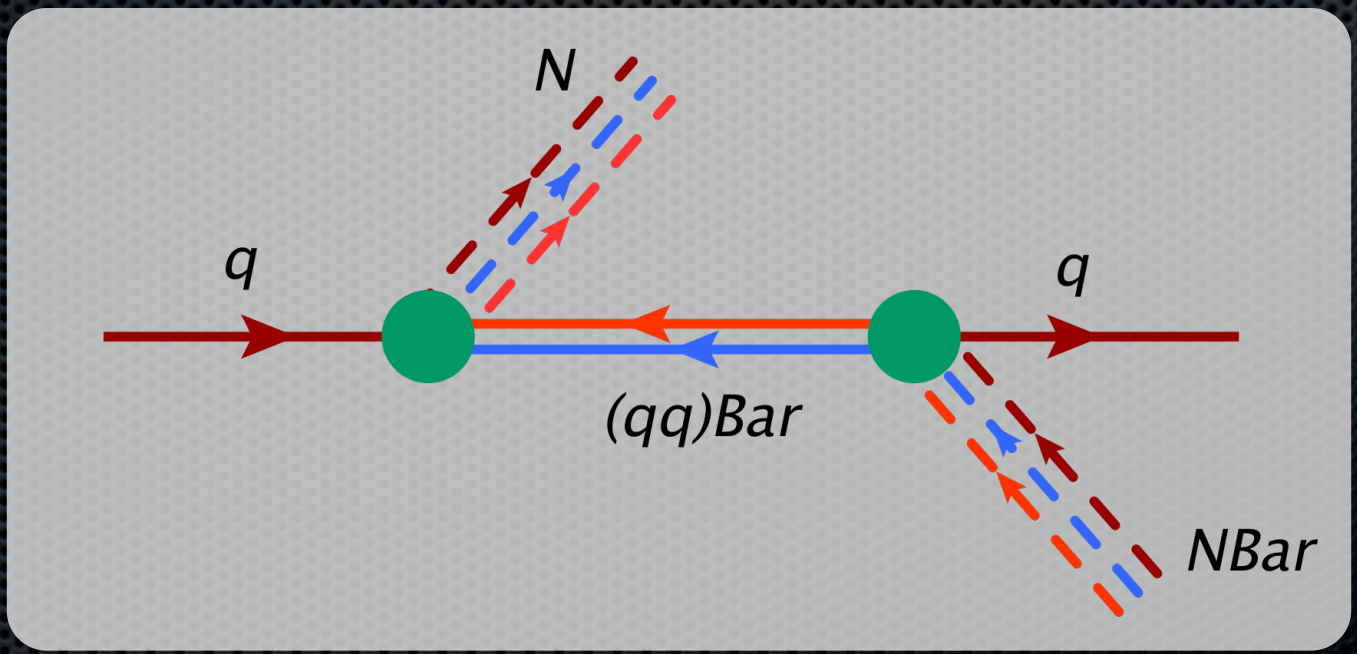


- Note:

Only $\rho \to \pi\pi$ at the moment.

More Channels: Nucleon Anti-Nucleon

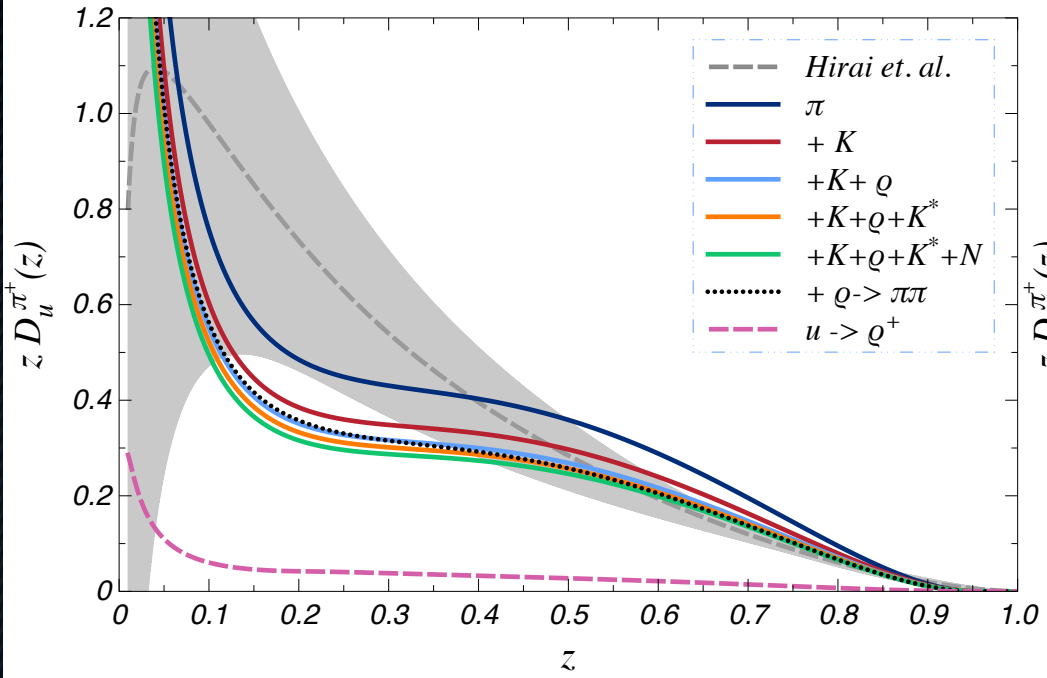
- Invoke quark-diquark model for nucleon.
- Calculate splittings $d_q^N(z)$ and $d_{\overline{qq}}^{\overline{N}}(z)$ (quark to nucleon and anti-diquark to anti-nucleon):



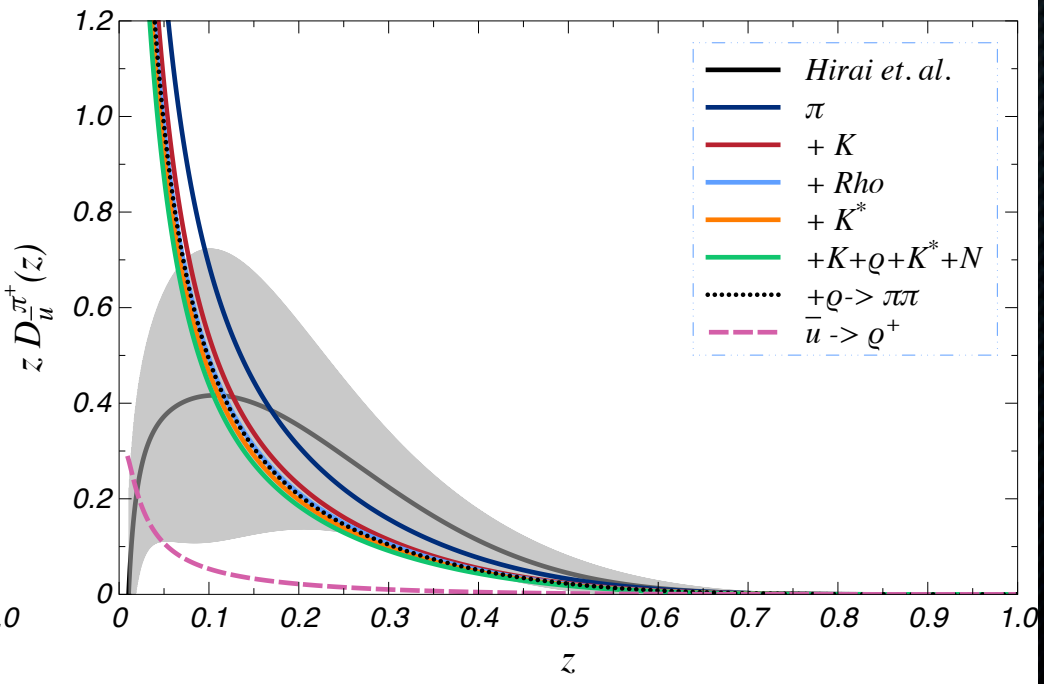
- We considered only scalar (anti-)diquarks (for now).

Results: Pion Fragmentations

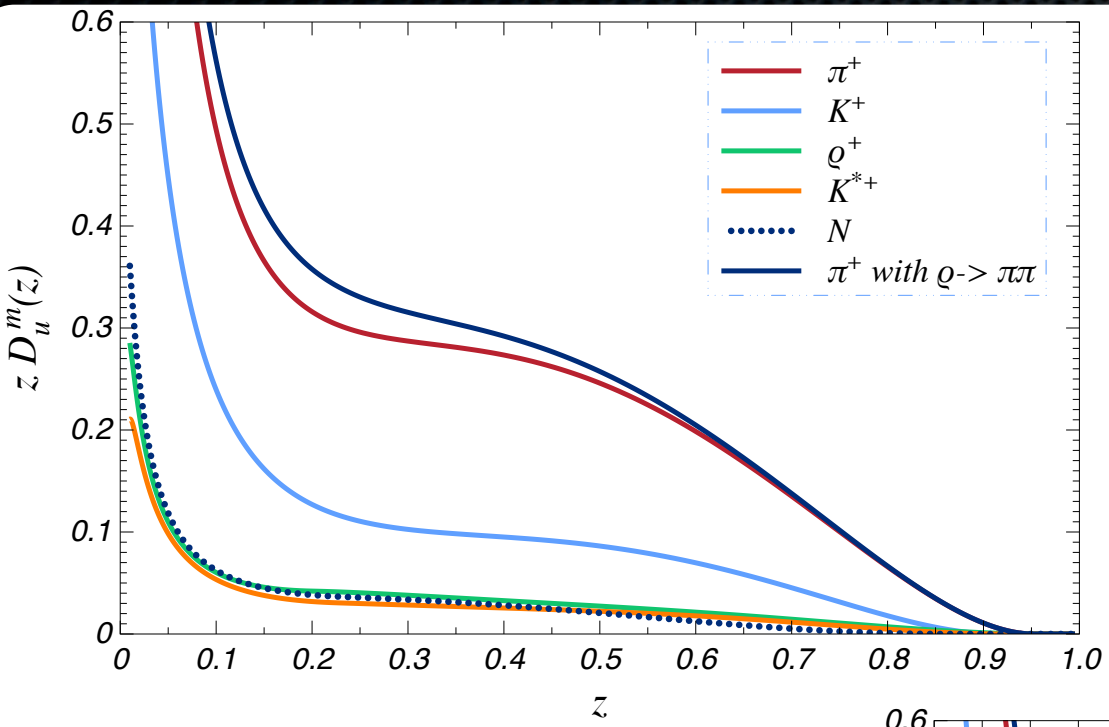
Favored



Unfavored



Results: Fragmentations to All Hadrons



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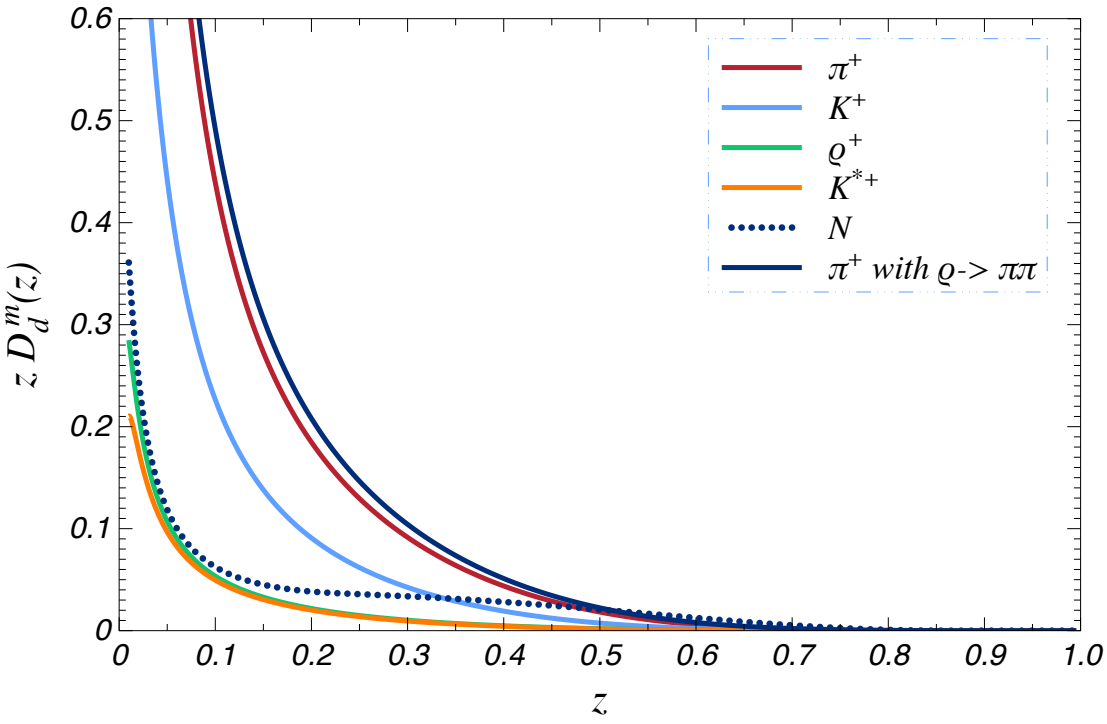
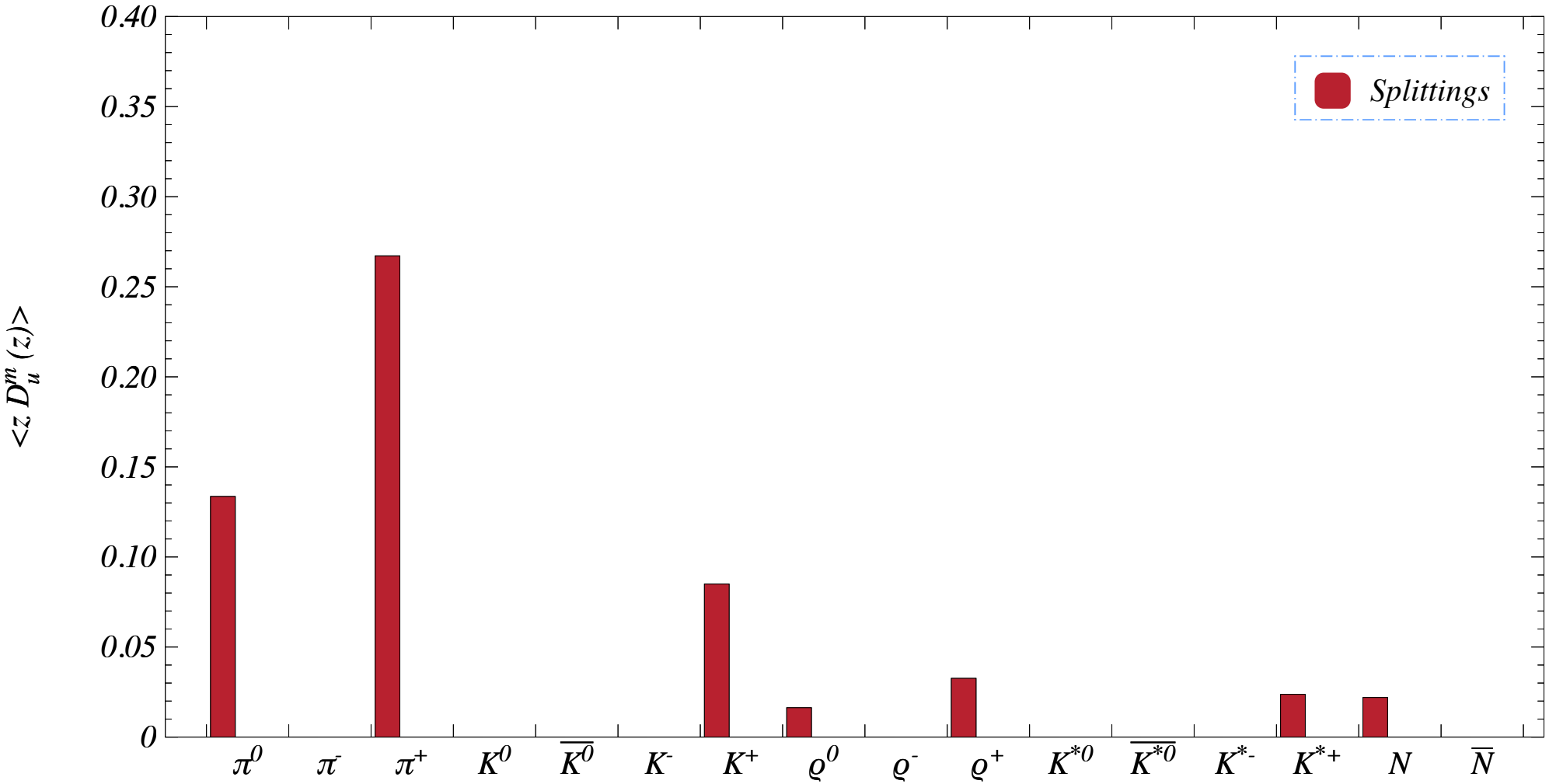


Unfavored

Results: Momentum Fractions



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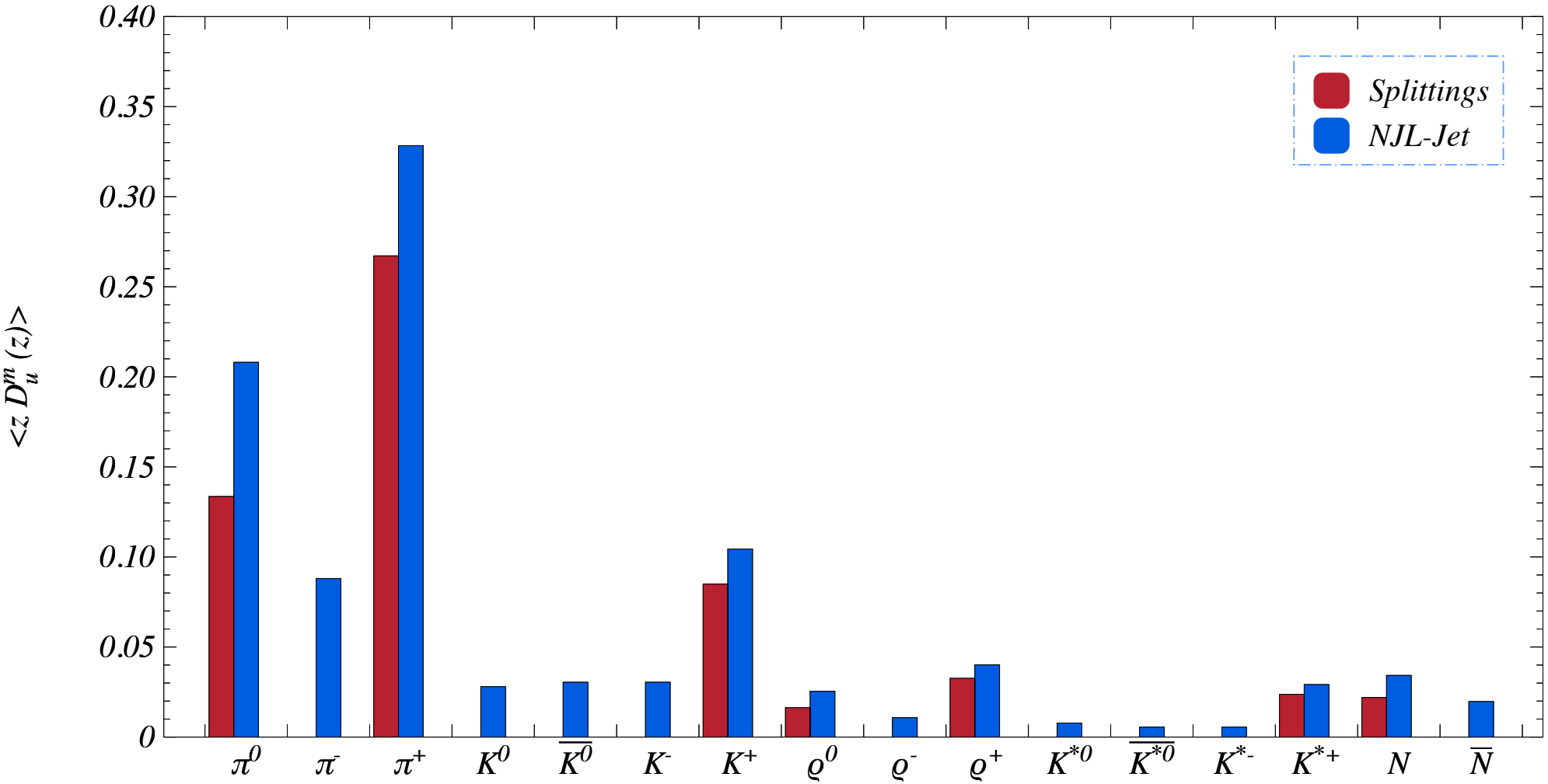

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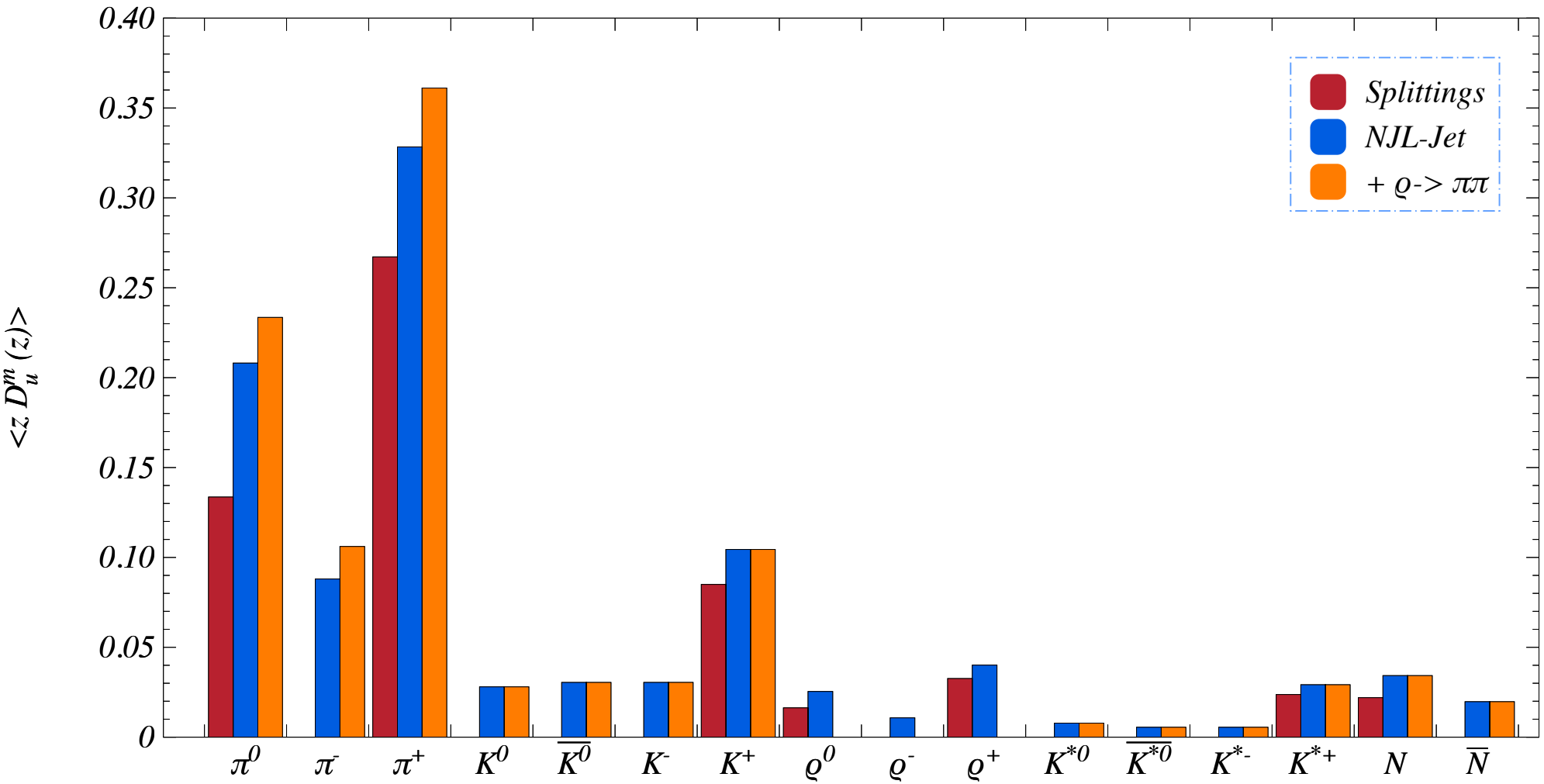

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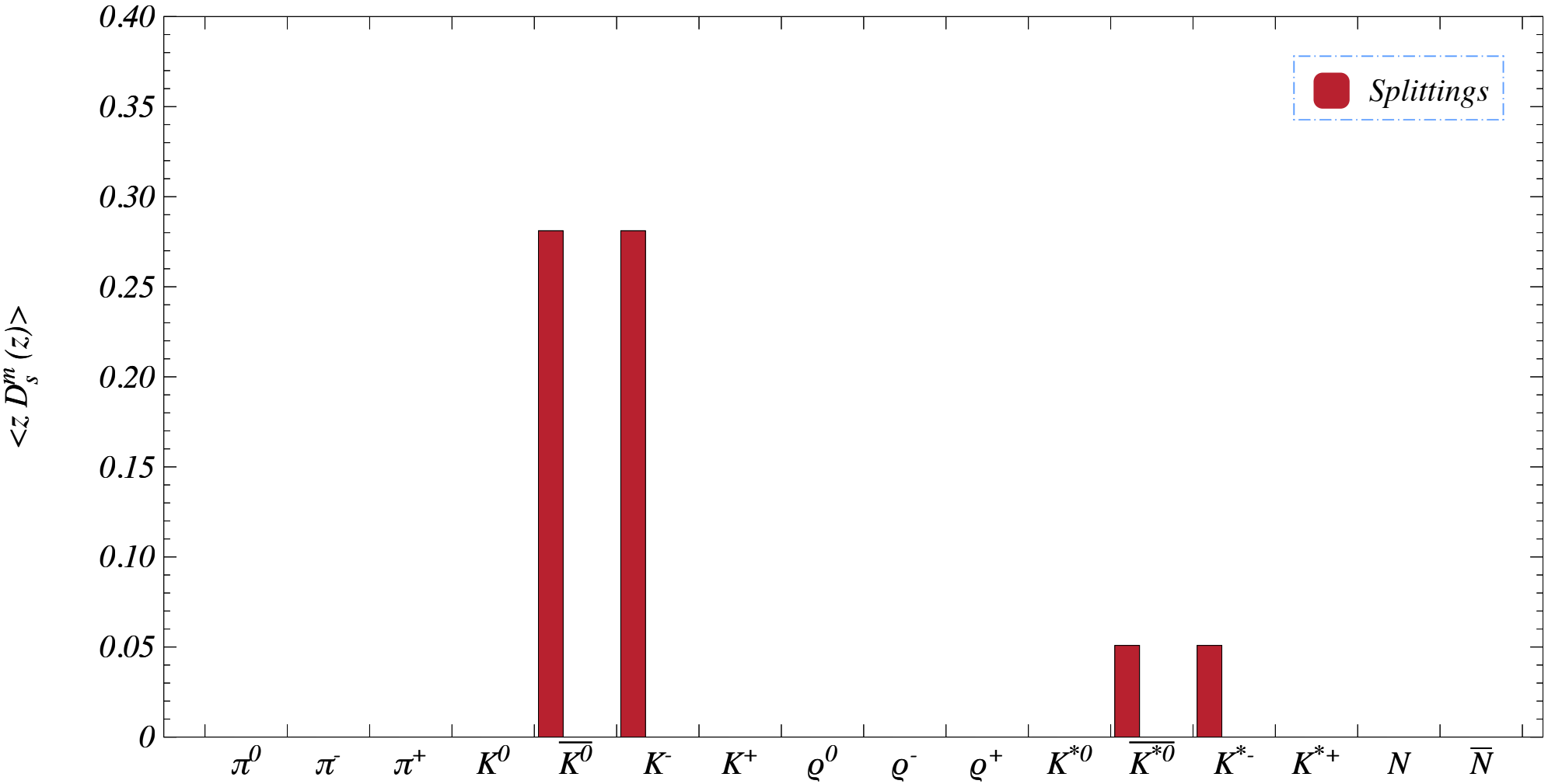

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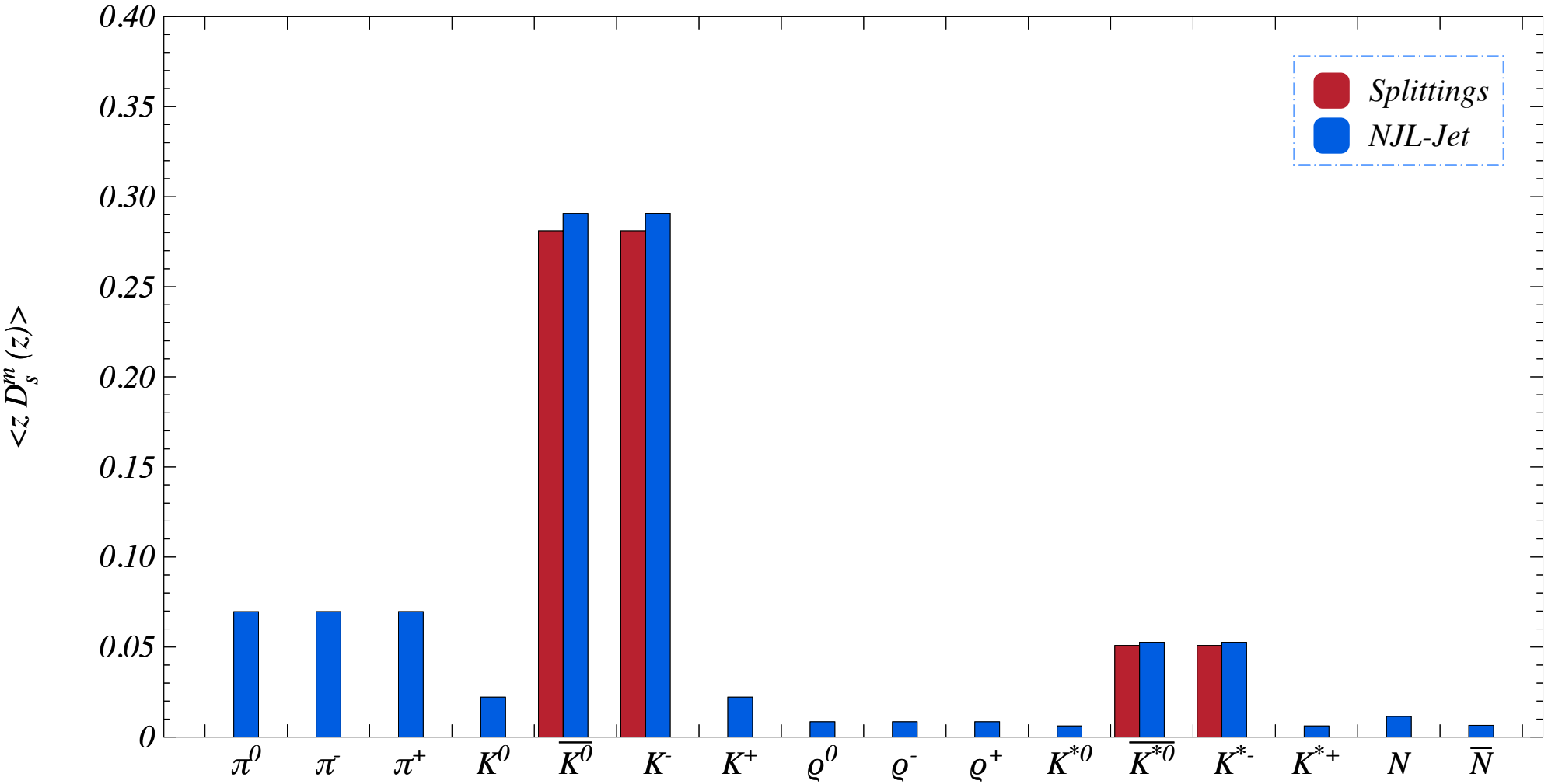

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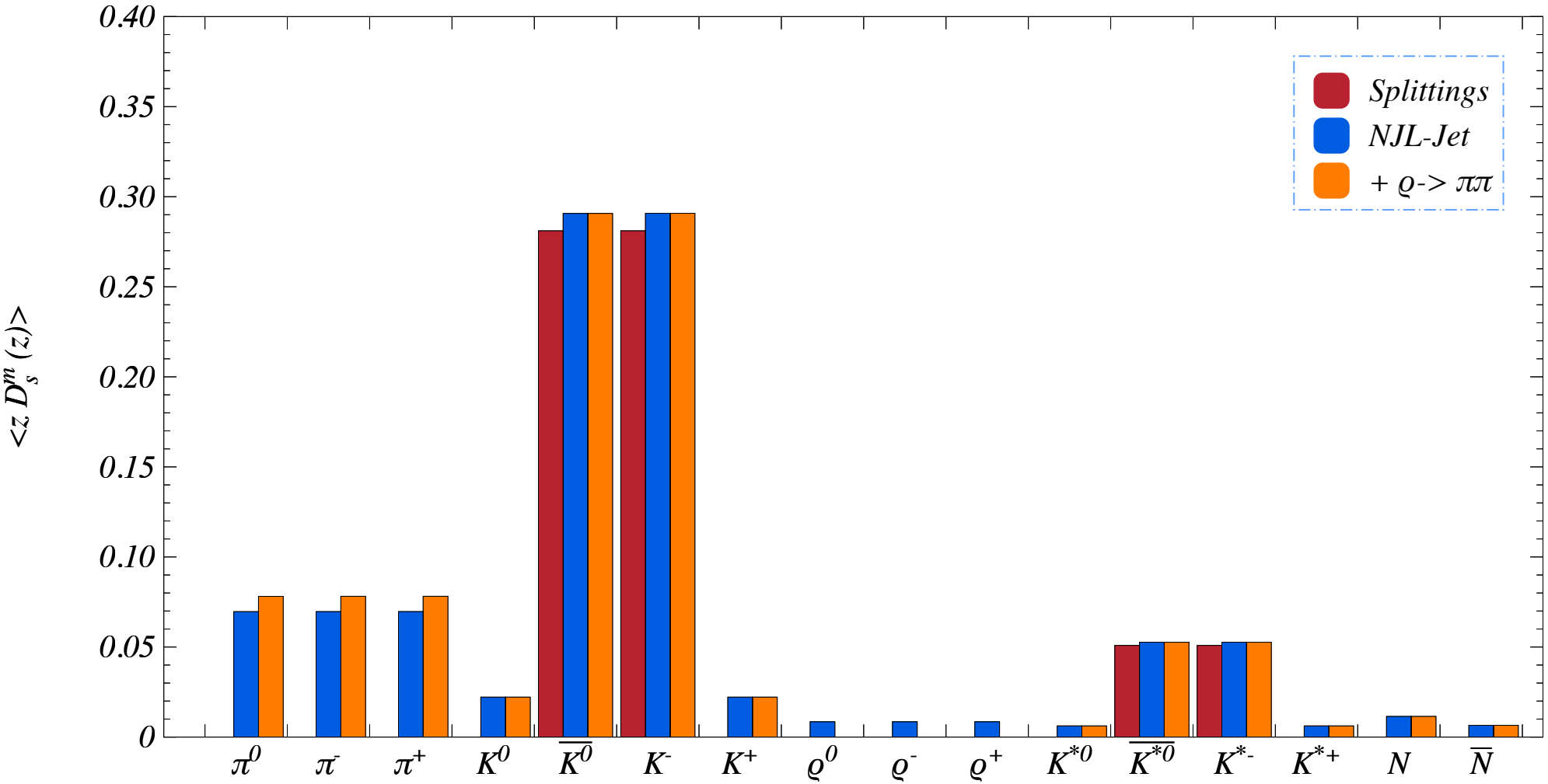
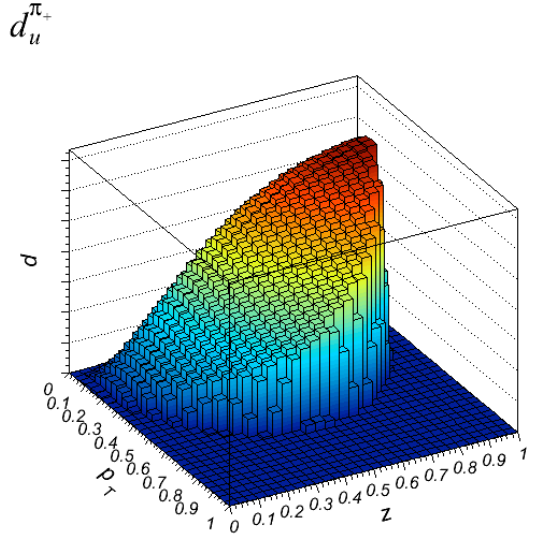


Including Transverse Momenta

- $p_\perp$ -dependent splittings: $d_q^m(z, p_\perp)$

$$\frac{C_I}{2} g_{mqq}^2 z \frac{p_\perp^2 + ((z-1)M_1 + M_2)^2}{(p_\perp^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_m^2)^2}.$$



- Conserve transverse momenta at each link.

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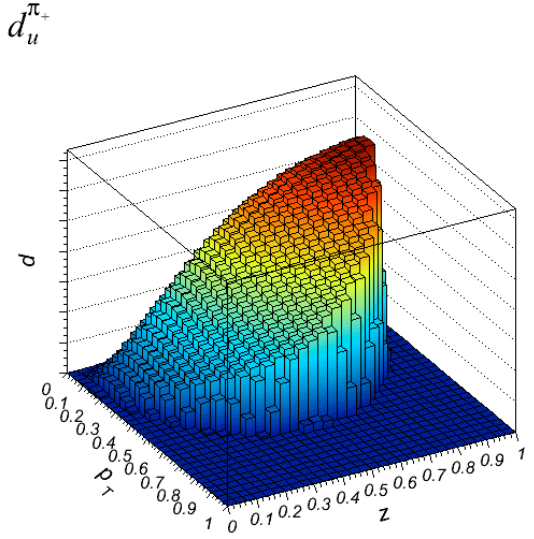
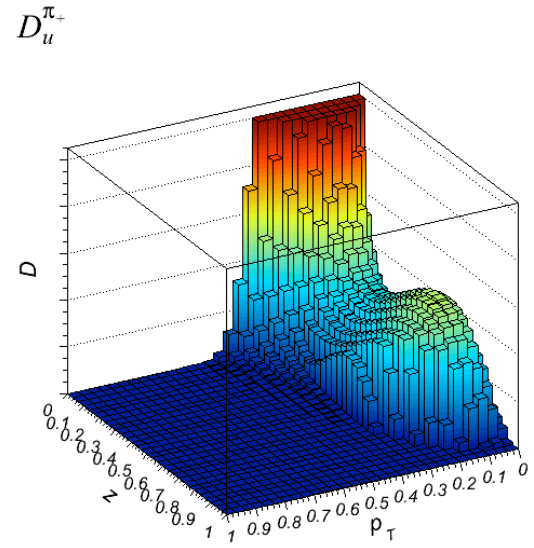
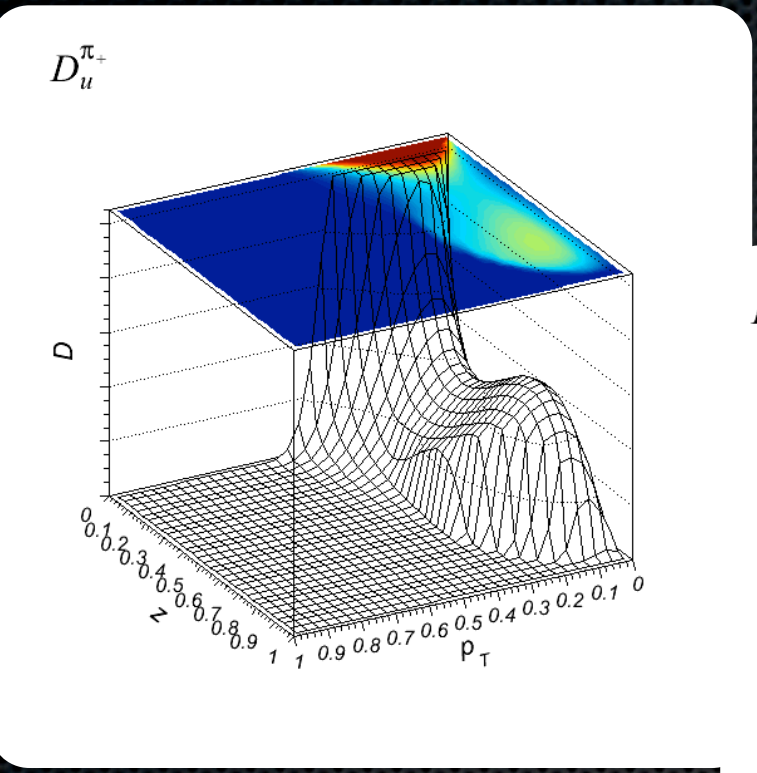
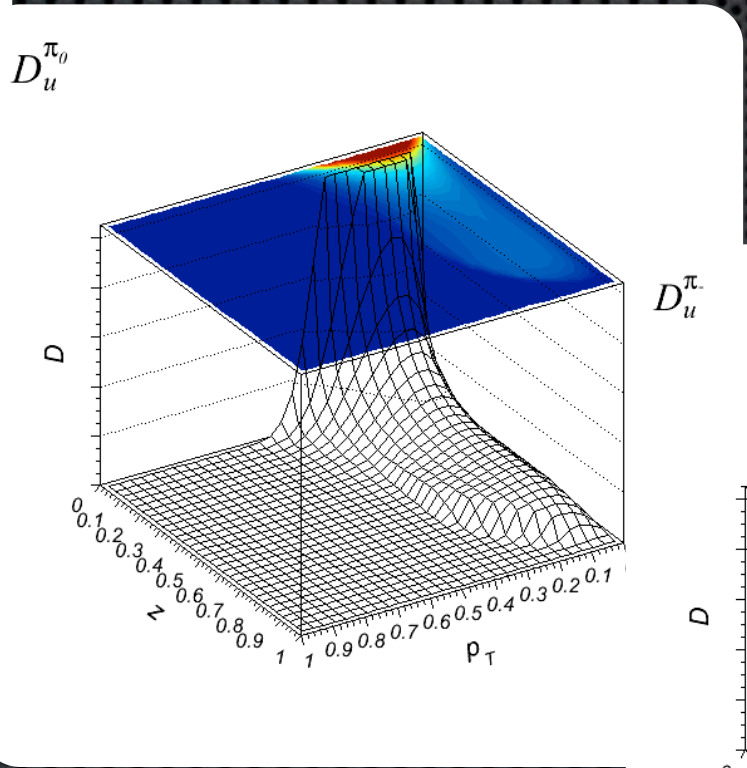
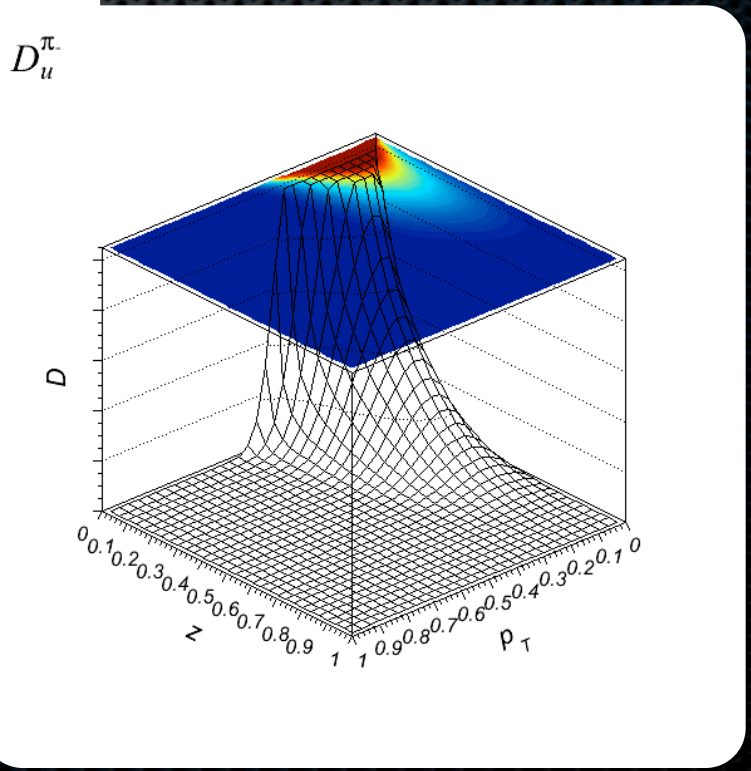




More Results with $p_\perp$







Summary

- NJL-jet model: coupled channel cascade description of fragmentation function, both favored and unfavored, that includes pseudoscalar, vector mesons and nucleon anti-nucleon channels.
- Shows convergence as we include more channels.
- Reasonable agreement with empirical parametrizations of experimental data.
- Generalization to finite energy is possible through MC.
- $p_\perp$ - dependence together with structure function, SIDIS cross-sections within the same framework.

Outlook

- Improvements:
  - Inclusion of possible relevant production and decay channels.
  - In MC simulation: inclusion of quark remnant hadronization, etc.
- Perspectives:
  - Collins Fragmentation Functions.
  - Di-hadron Fragmentations.
  - In-Medium Effects.

Setting the Model Scale

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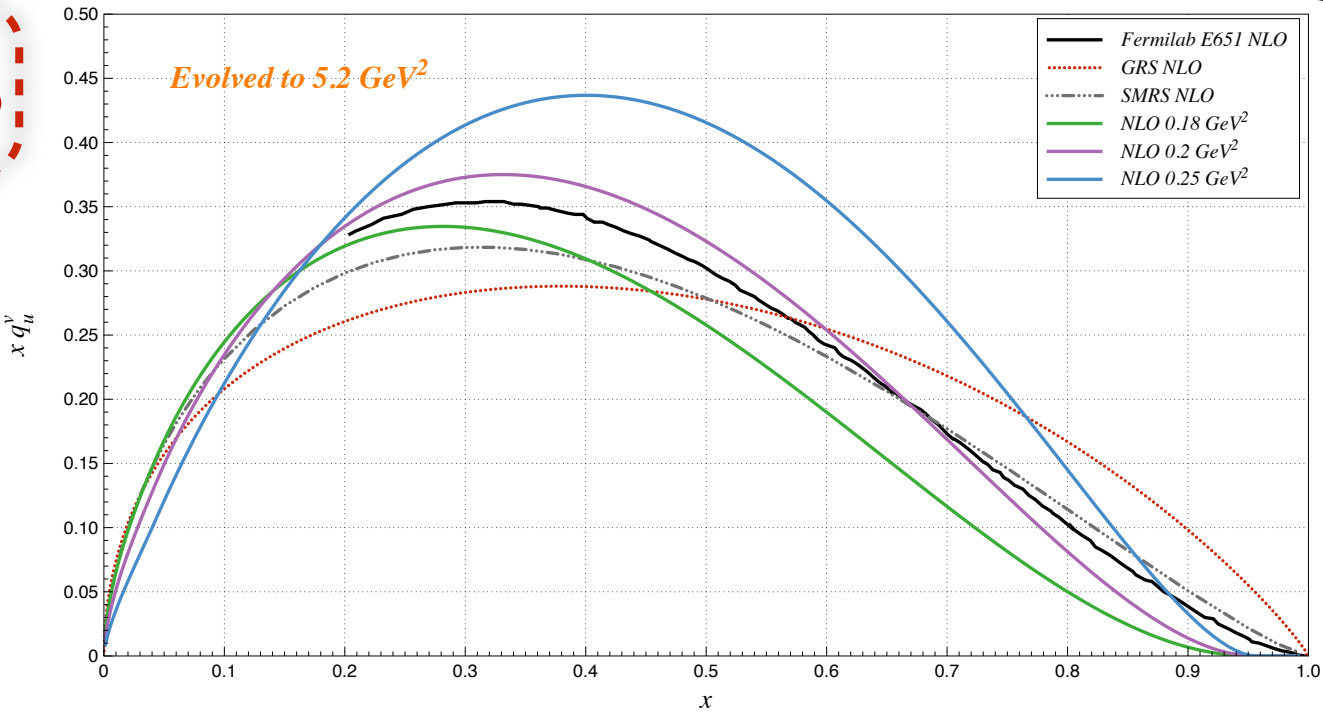

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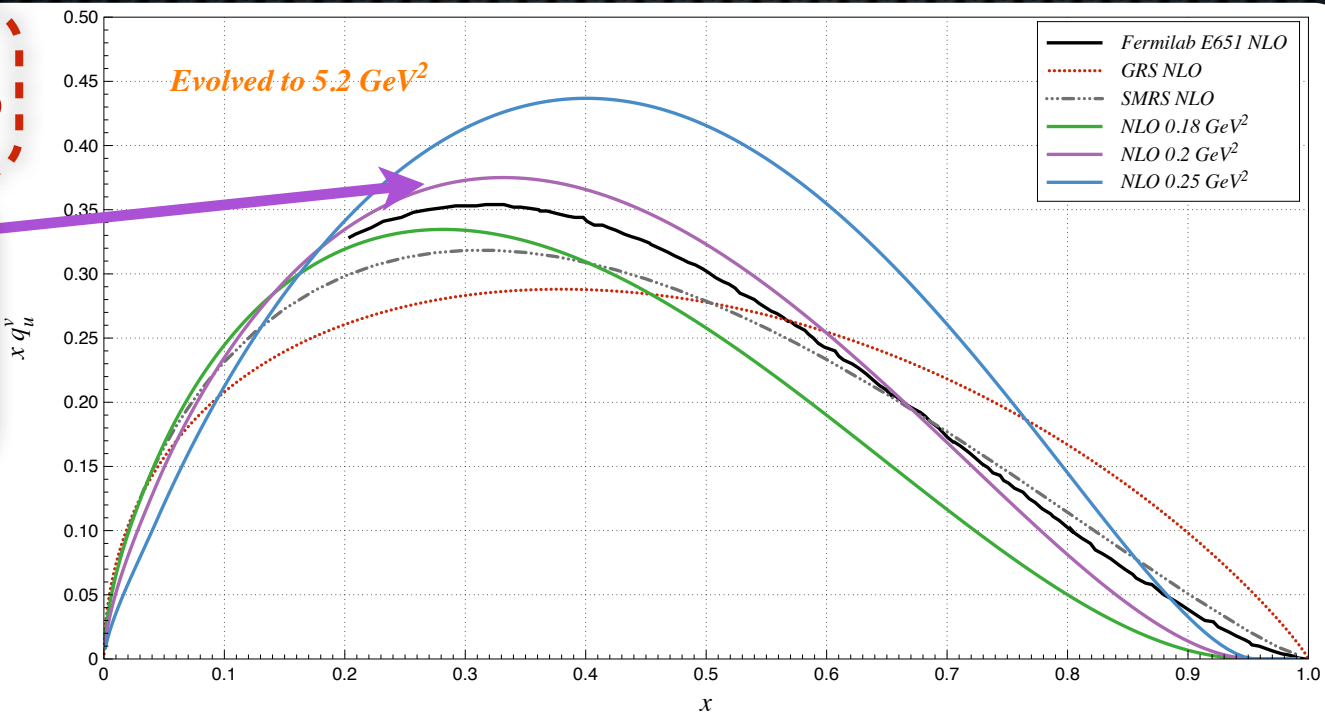
$$\alpha_s^{NLO}(Q_0^2) = 1.67$$



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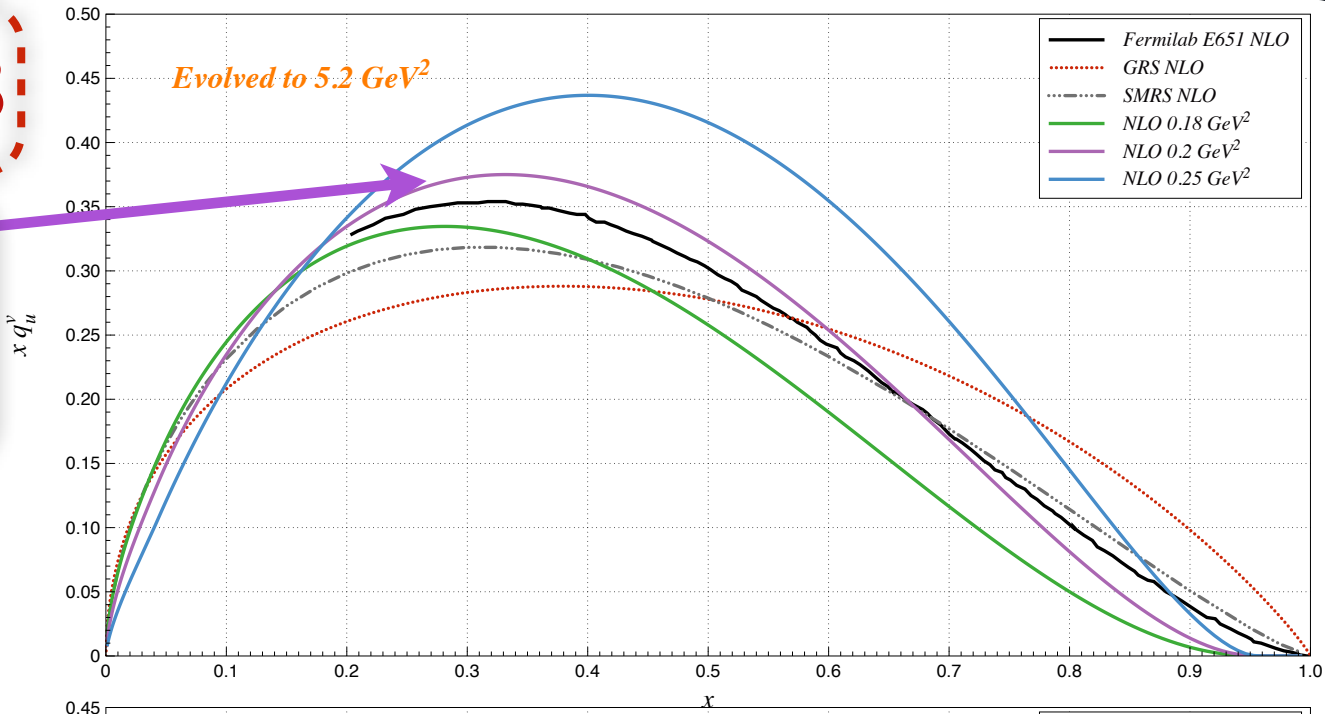

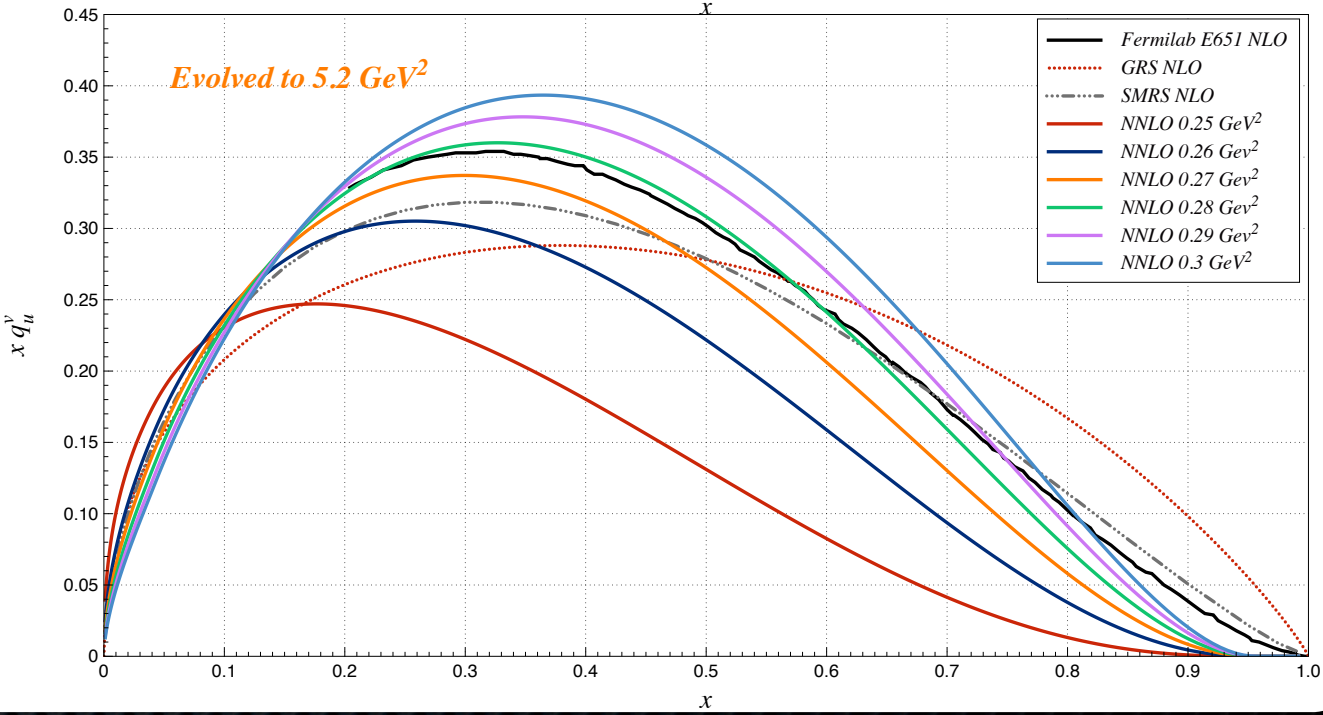

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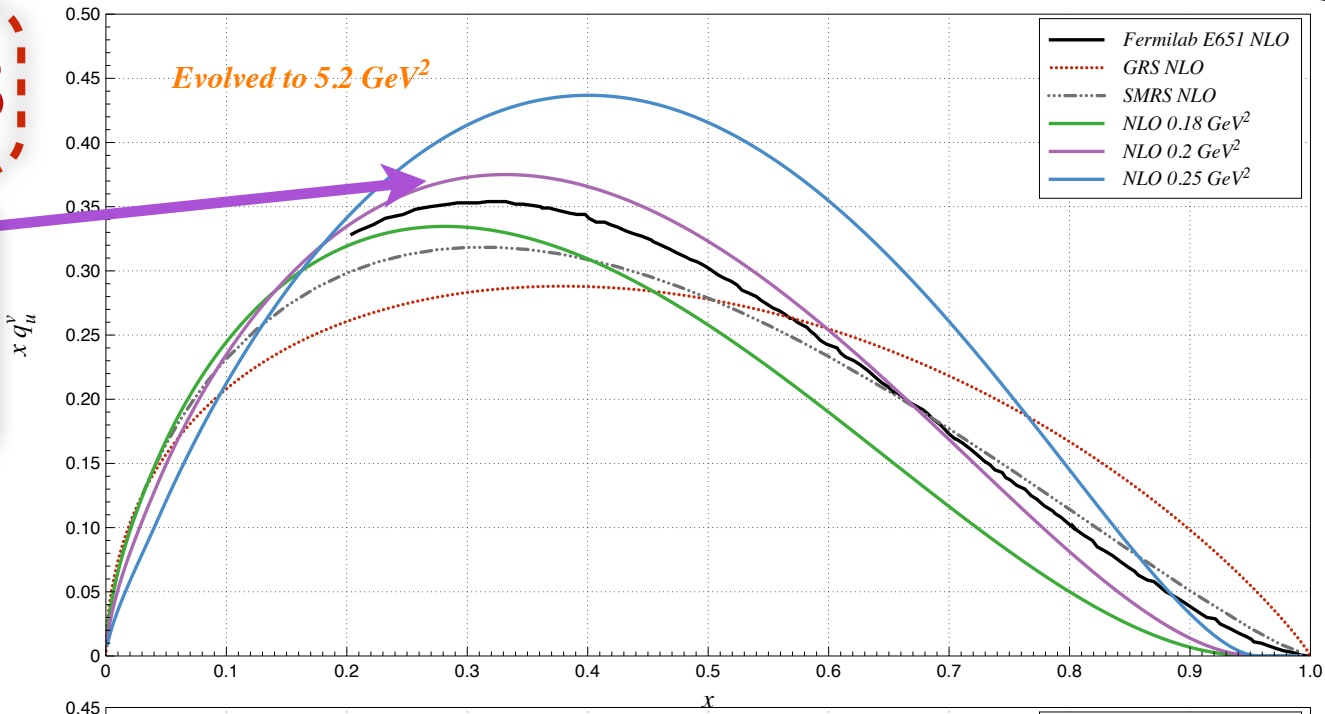

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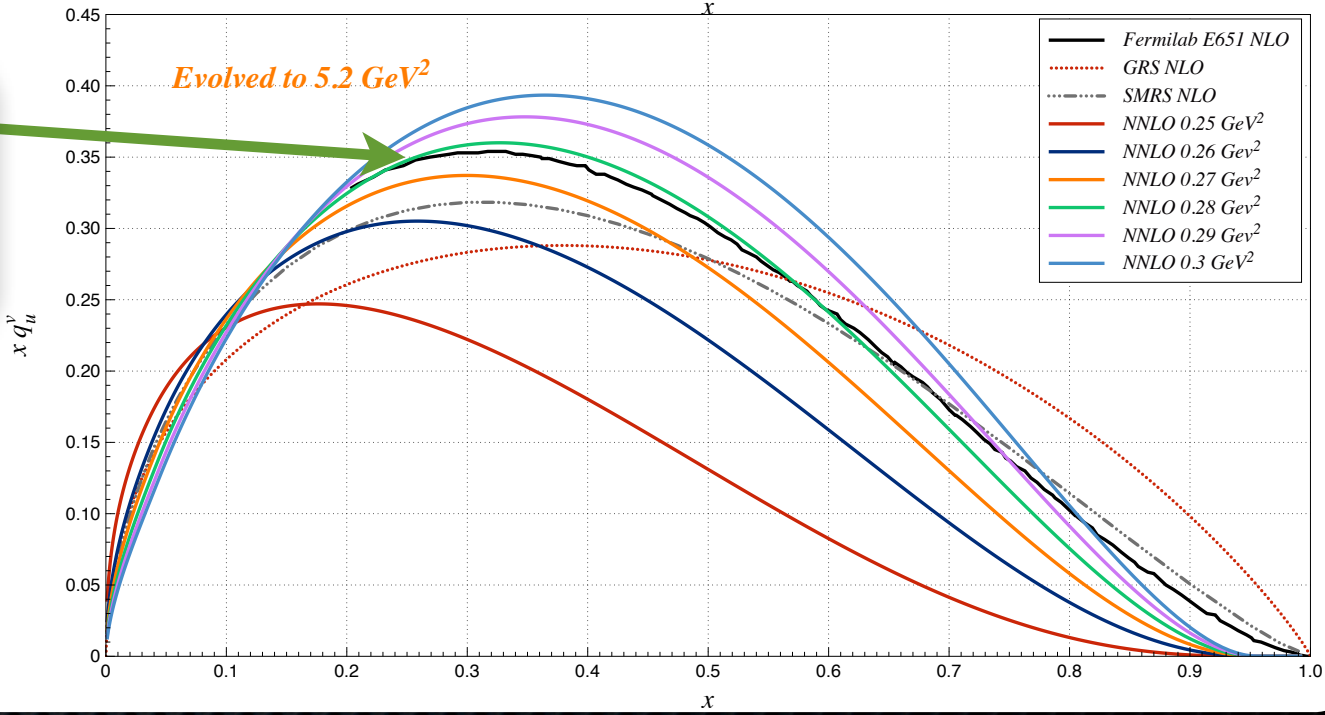

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$$Q^2_{0\,NNLO} = 0.28\ GeV^2$$

$$\alpha_s^{NNLO}(Q_0^2) = 1.59$$

$$\alpha_s(Q_0^2)/2\pi \approx 0.25$$