



Impact of the $pdfs$ measurements on Higgs production via gluon fusion

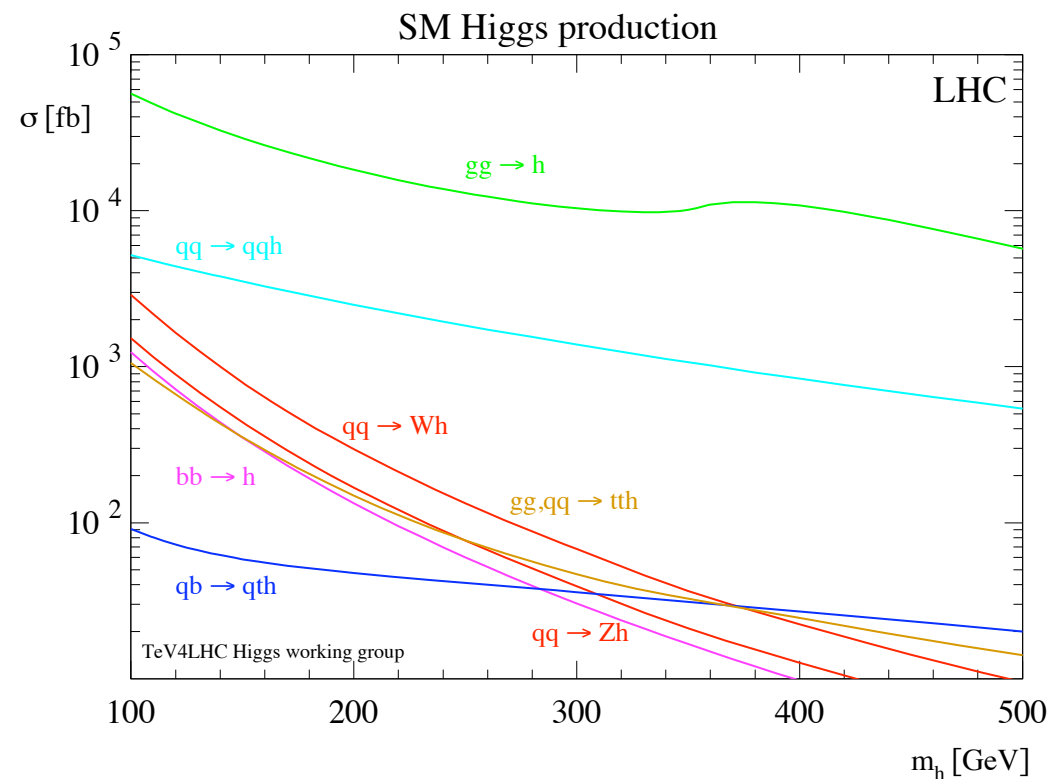
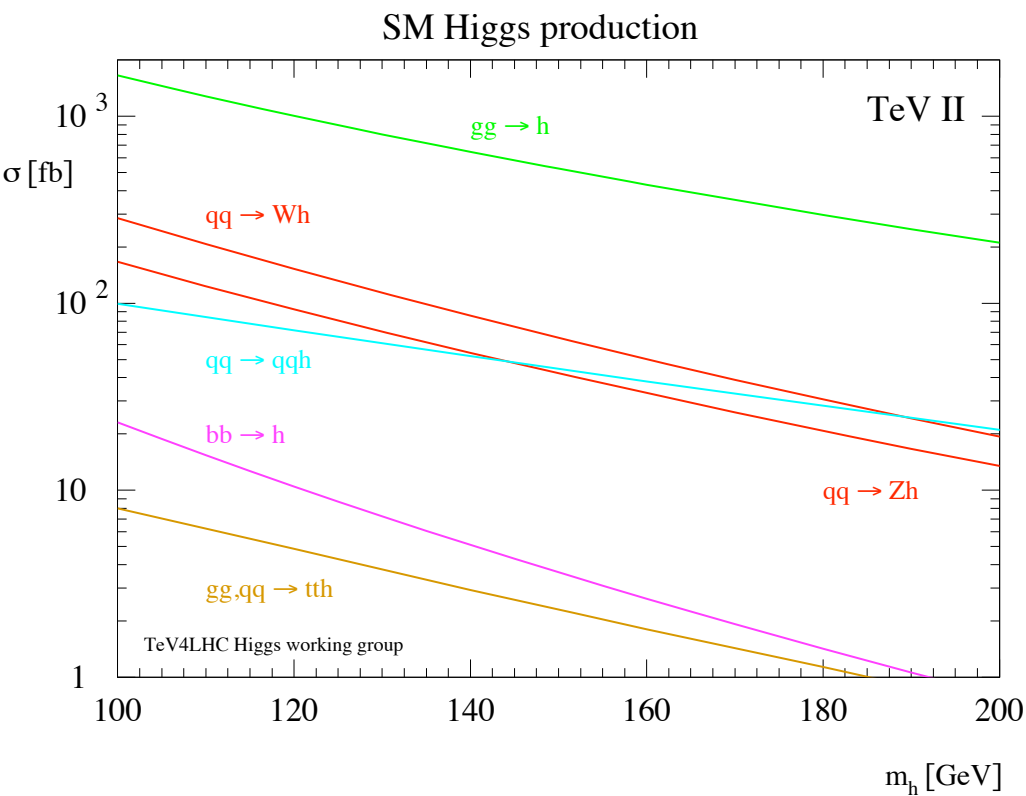
Alessandro Vicini

University of Milano, INFN Milano

Torino, November 23rd 2009

- The gluon fusion Higgs cross section
(what we exactly use to prepare the plots)
- Experimental uncertainties due to: the *pdfs*
 α_s
 m_t
- Theoretical uncertainties: scale variations
different quark content at NLO

Production mechanisms



The gluon fusion channel is by far the largest production mechanism and requires our best predictions to discover and to study the Higgs boson

Although it starts at 1-loop,
it is enhanced by the large gluon density of the proton

Higgs production

$$\sigma(h_1 + h_2 \rightarrow H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 \boxed{f_{a,h_1}(x_1, \mu_F^2) f_{b,h_2}(x_2, \mu_F^2)} \times \\ \times \int_0^1 dz \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \boxed{\hat{\sigma}_{ab}(z)},$$

proton parton density functions

partonic cross sections

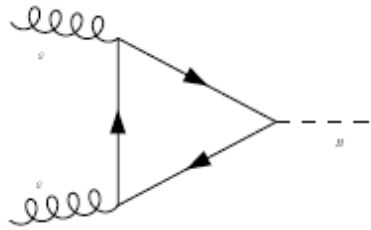
According to [factorization theorems](#), the hadronic cross section is written as the convolution of the proton *pdfs* with the partonic cross sections of the relevant parton-parton subprocesses

The partonic cross sections are computed at a given order in QCD

A consistent evaluation of the hadronic cross section requires that:

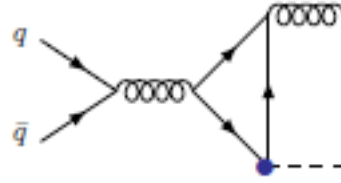
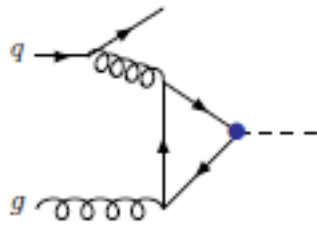
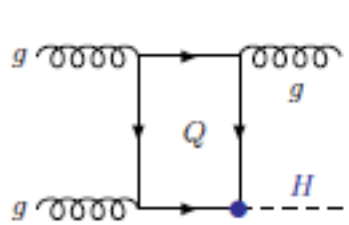
- the *pdfs* are extracted with the same accuracy;
- the strong coupling is evolved with the same accuracy.

Higgs production at NLO-QCD



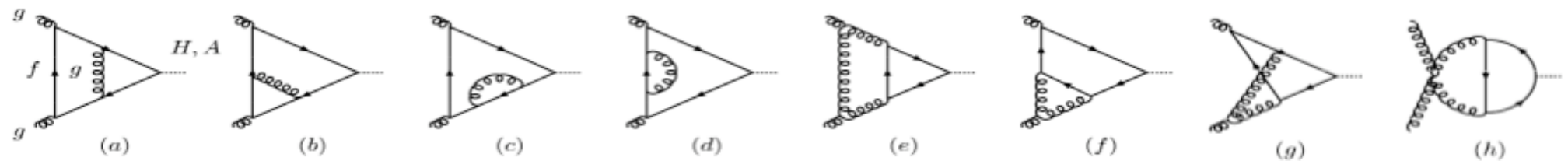
Georgi, Glashow, Machacek, Nanopoulos (1978)

LO



Dawson (1991),
Djouadi, Spira, Zerwas (1991)
Djouadi, Graudenz, Spira, Zerwas (1995)
Harlander, Kant (2005)
Aglietti, Bonciani, Degrandi, Vicini (2007)
Anastasiou, Beerli, Bucherer, Daleo, Kunszt (2007)

NLO-QCD real



NLO-QCD virtual

In the fermion loop run all possible quarks

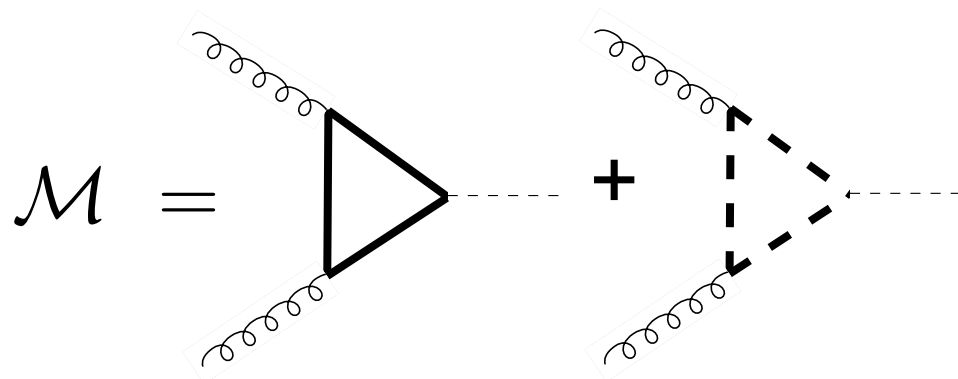
Gluon fusion: NLO-QCD corrections in the SM and beyond

Aglietti, Bonciani Degrassi, AV, JHEP 0701 (2007) 021

JHEP 0711 (2007) 095

program GGSCA

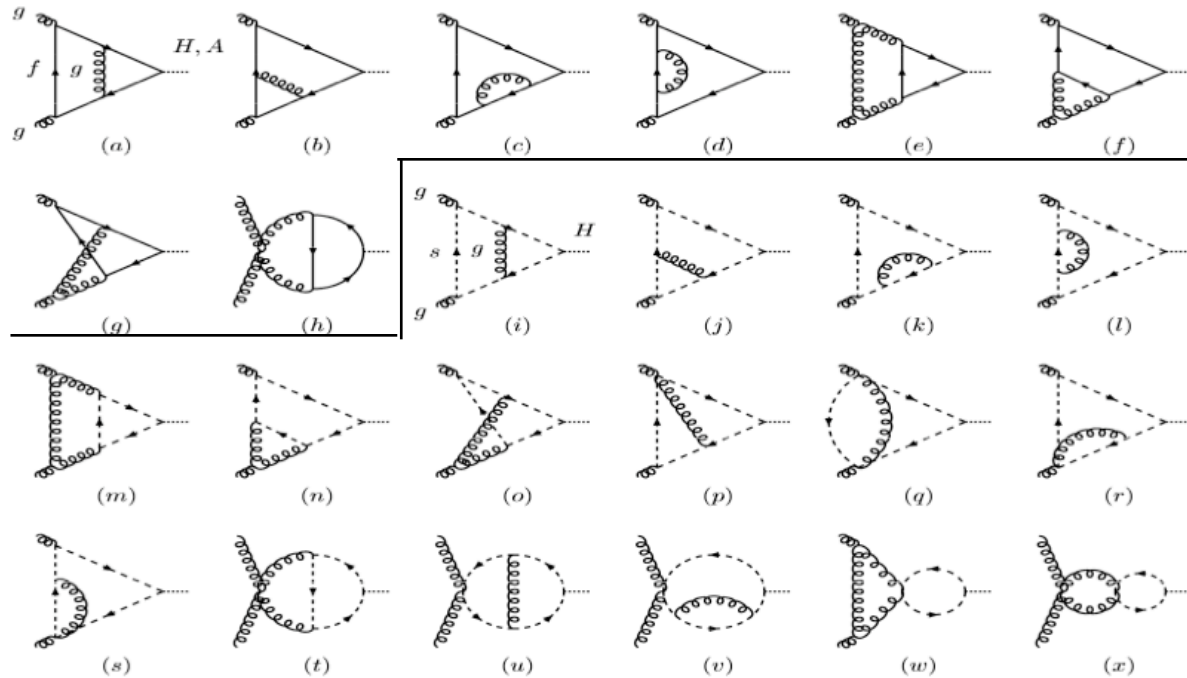
$$\sigma(h_1 + h_2 \rightarrow H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \\ \times \int_0^1 dz \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z)$$



$$\hat{\sigma}_{ab}(z) = \sigma^{(0)} z G_{ab}(z)$$

$$\sigma^{(0)} = \frac{G_\mu \alpha_S^2(\mu_R^2)}{128 \sqrt{2} \pi} \left| \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2} \right)^{1-2i} T(R_i) \mathcal{G}_i^{(1l)} \right|^2$$

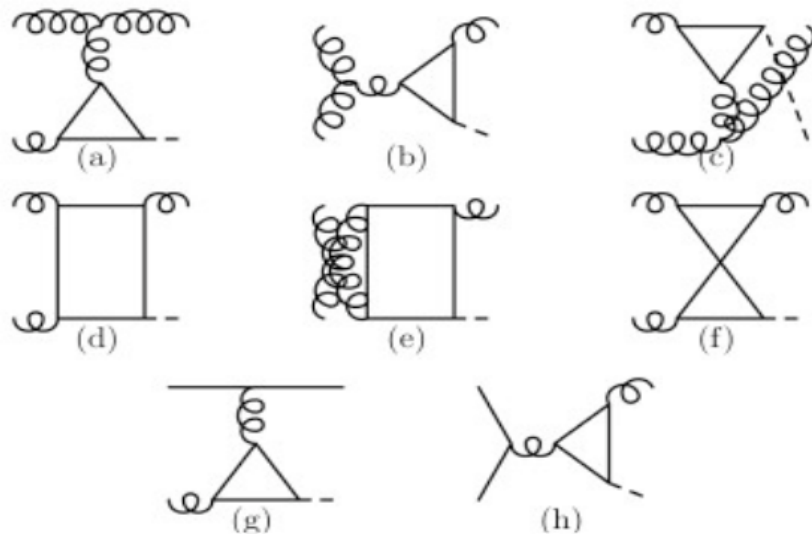
Gluon fusion: NLO-QCD corrections in the SM and beyond



$$\mathcal{G}_i^{(2l)} = \lambda_i \left(\frac{A^2}{m_0^2} \right)^{1-2i} T(R_i) \left(C(R_i) \mathcal{G}_i^{(2l, C_R)}(x_i) + C_A \mathcal{G}_i^{(2l, C_A)}(x_i) \right) \\ \times \left(\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) \mathcal{G}_j^{(1l)} \right)^{-1} + h.c.$$

Gluon fusion: NLO-QCD corrections in the SM and beyond

Baur, Glover Ellis, Hinchliff, Soldate, van der Bij Bonciani Degrassi, AV



also all the function A_{ij}
expressed in terms of HPLs

$$\mathcal{R}_{gg} = \frac{1}{z(1-z)} \int_0^1 \frac{dv}{v(1-v)} \left\{ \frac{8z^4 |\mathcal{A}_{gg}(\hat{s}, \hat{t}, \hat{u})|^2}{\left| \sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) \mathcal{G}_j^{(1l)} \right|^2} - (1-z+z^2)^2 \right\}$$

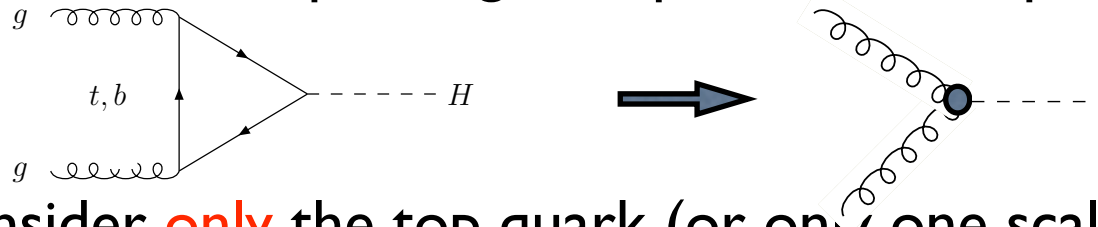
$$\mathcal{R}_{q\bar{q}} = \frac{128}{27} \frac{z(1-z) |\mathcal{A}_{q\bar{q}}(\hat{s}, \hat{t}, \hat{u})|^2}{\left| \sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) \mathcal{G}_j^{(1l)} \right|^2}$$

$$\mathcal{R}_{qg} = C_F \int_0^1 \frac{dv}{(1-v)} \left\{ \frac{1+(1-z)^2 v^2}{[1-(1-z)v]^2} \frac{2z |\mathcal{A}_{qg}(\hat{s}, \hat{t}, \hat{u})|^2}{\left| \sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) \mathcal{G}_j^{(1l)} \right|^2} - \frac{1+(1-z)^2}{2z} \right\} + \frac{1}{2} C_F z$$

QCD K-factor and the large- m_t limit

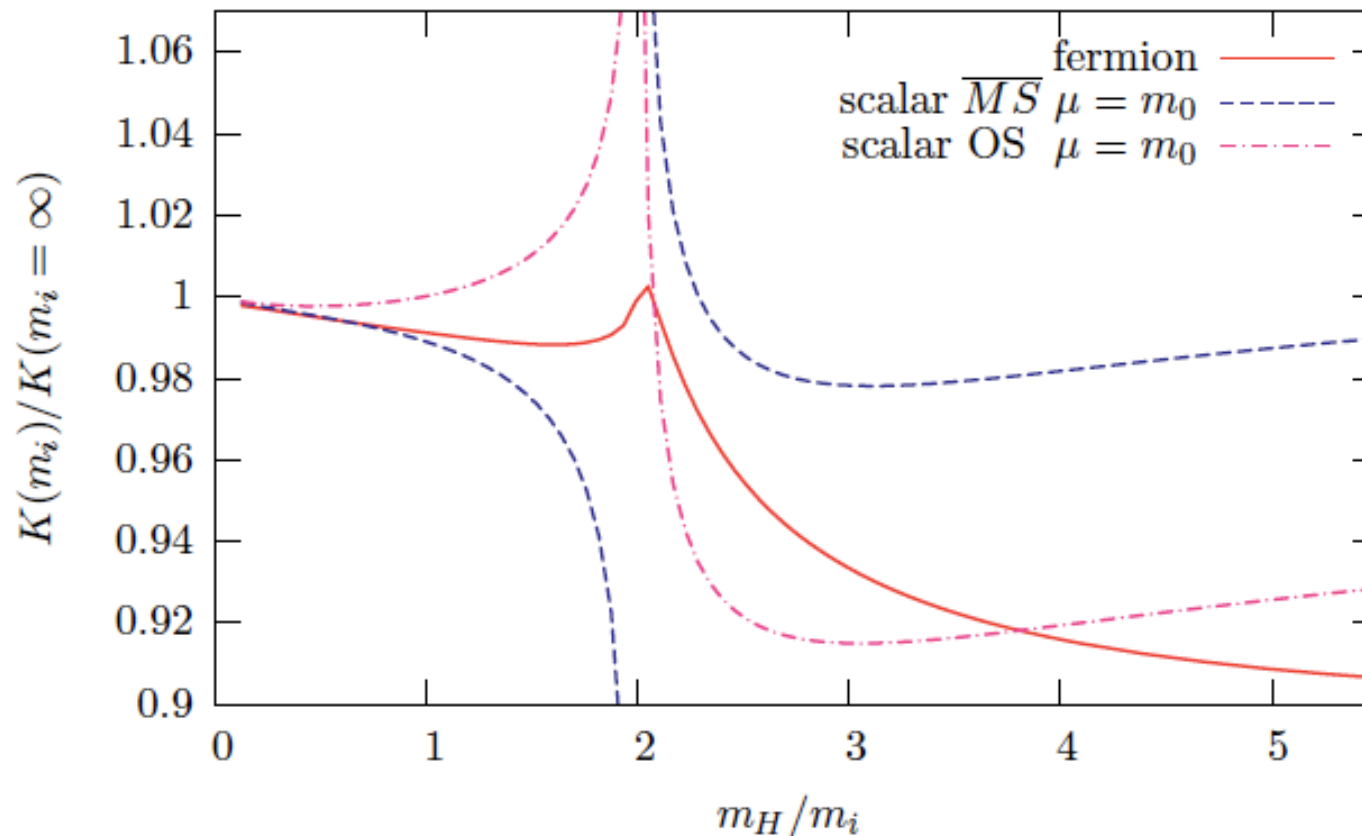
Bonciani Degrassi, AV

large- m_t limit: the top triangle loop shrinks to a pointlike interaction vertex



if we consider **only** the top quark (or **only** one scalar) in the loop, then the K-factor in the large- m_t limit is already a good approximation of the exact K-factor

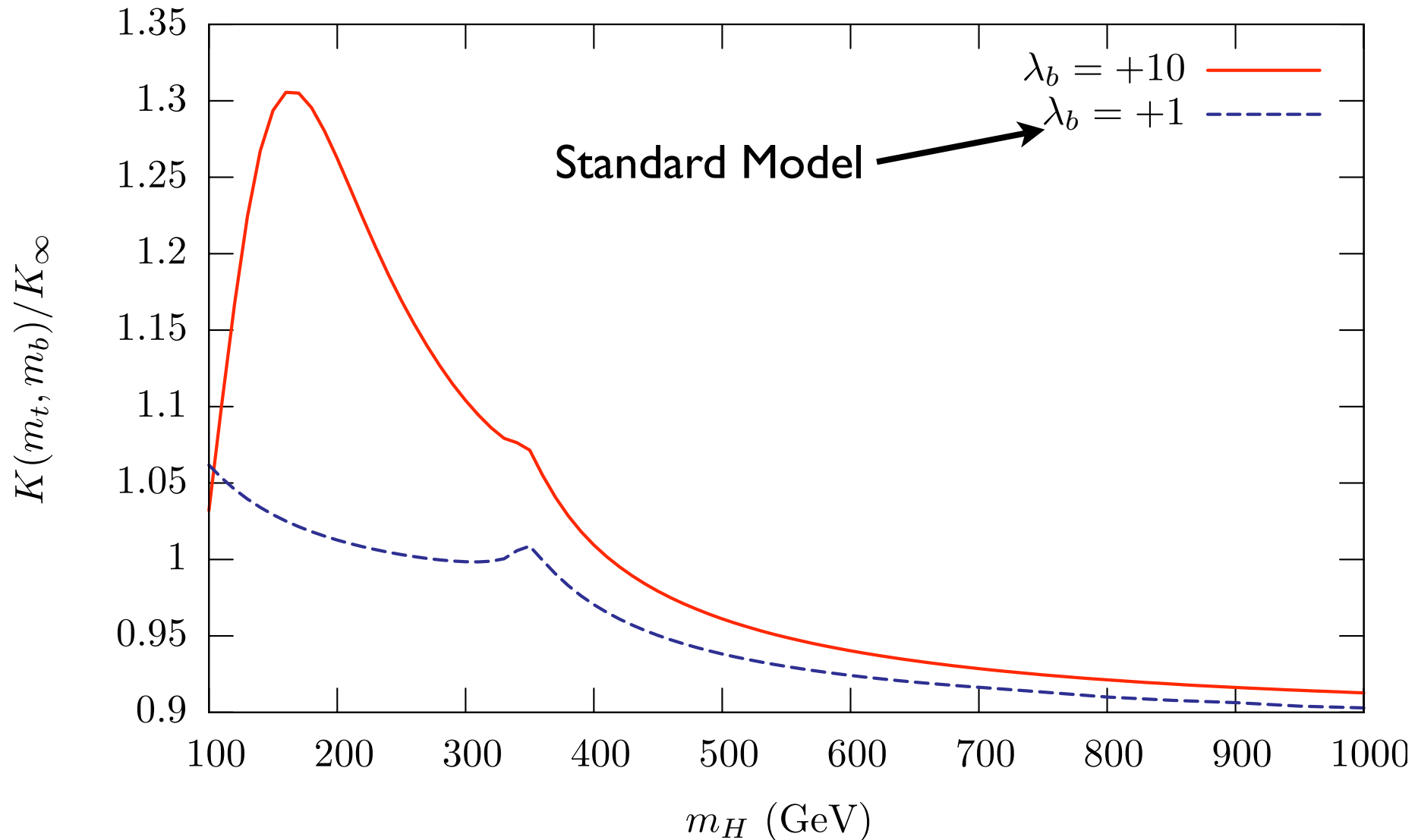
but we have to include at least also the bottom!



Exact QCD K-factor

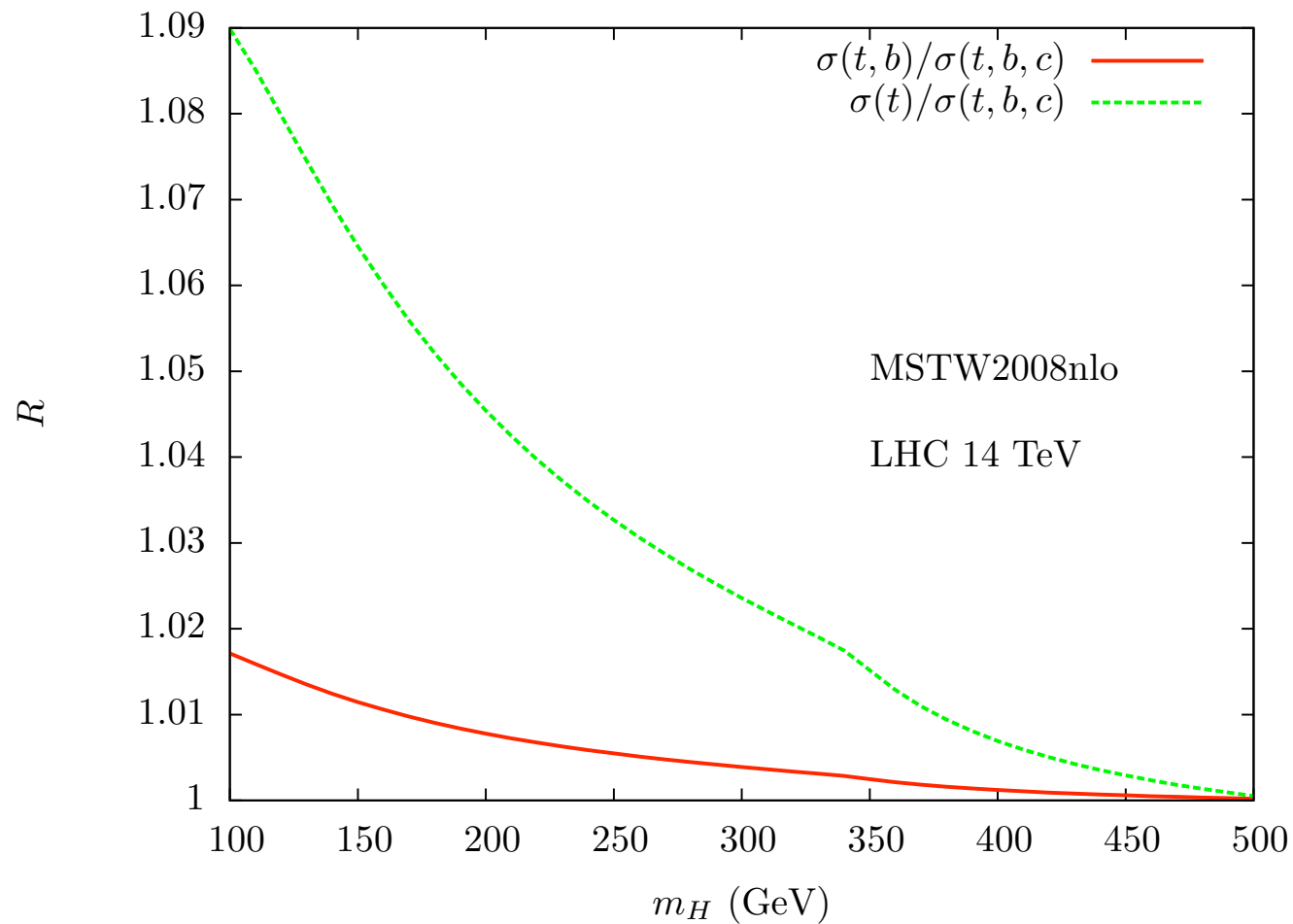
Bonciani Degrassi, AV

→ if we include QCD corrections to the light particles in the loop
the K-factor in the exact calculation may differ from the approximated one



also at NLO non trivial interferences between
the top and bottom (and charm, strange,...) amplitudes

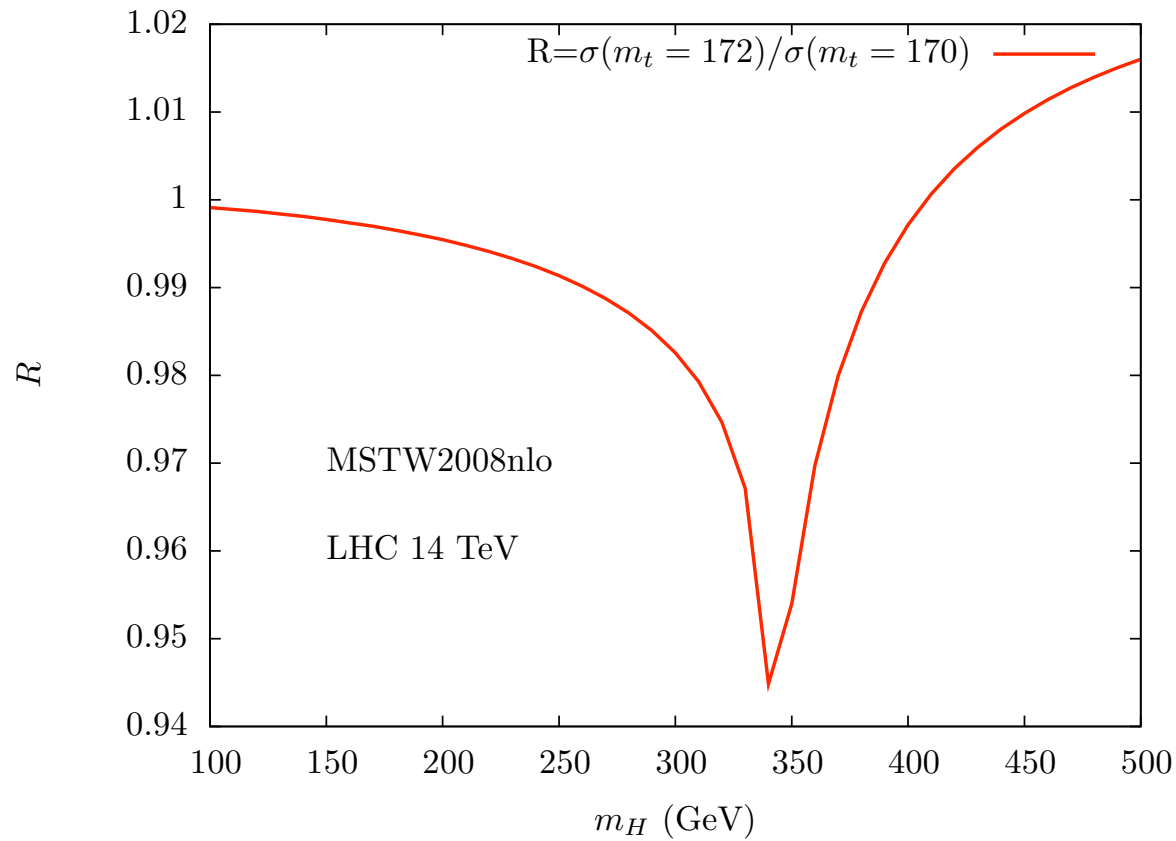
Effects of the inclusion of the amplitude with the charm quark



Alike the bottom, also the charm quark has a negative effect on the cross section because of a negative interference effect between top and charm amplitudes

All these ingredients are available since 1995 in HIGLU
(very good numerical agreement of our code with HIGLU)!

m_t dependence



For light Higgs, dependence on the precise value of the top mass below 0.5%

Around and above the $t\bar{t}$ threshold, non trivial dependence

Higgs production beyond NLO-QCD

- NNLO-QCD results in the $m_t \rightarrow \infty$ limit (+15%)
Anastasiou, Melnikov (2002), Harlander, Kilgore (2002)
- finite m_t effects at NNLO-QCD ($\sim 0.5\%$)
Marzani, Ball, Del Duca, Forte, Vicini (2008) Harlander, Ozeren (2009)
- soft-gluon resummation at NNLL-QCD (+6%)
Catani, De Florian, Grazzini, Nason (2003)
- inclusion of leading NNNLO-QCD contributions (+5%)
Moch, Vogt (2005)
- full NLO-EW corrections (+4-7%)
Aglietti, Bonciani, Degrandi, Vicini (2004, 2005) Actis, Passarino, Sturm, Uccirati (2007, 2008), Keung, Petriello (2009)

Further **increase**, beyond NLO-QCD, of the total cross section:

+25-30% of the Born

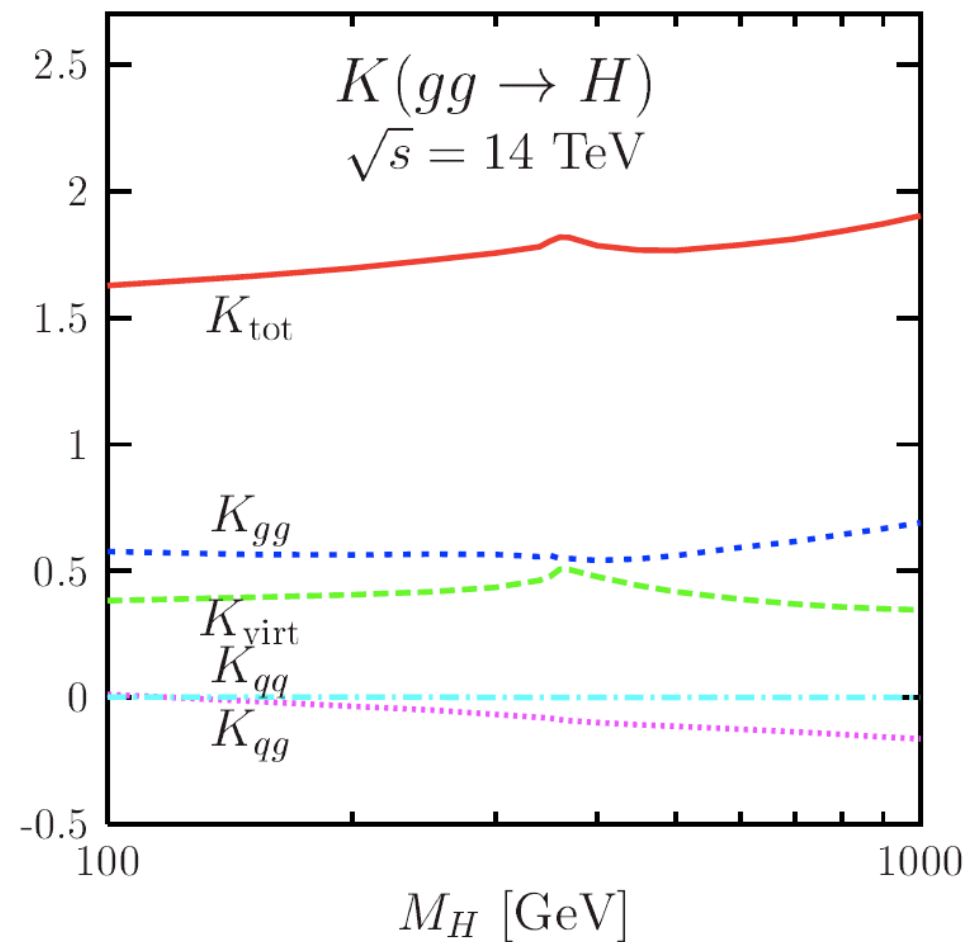
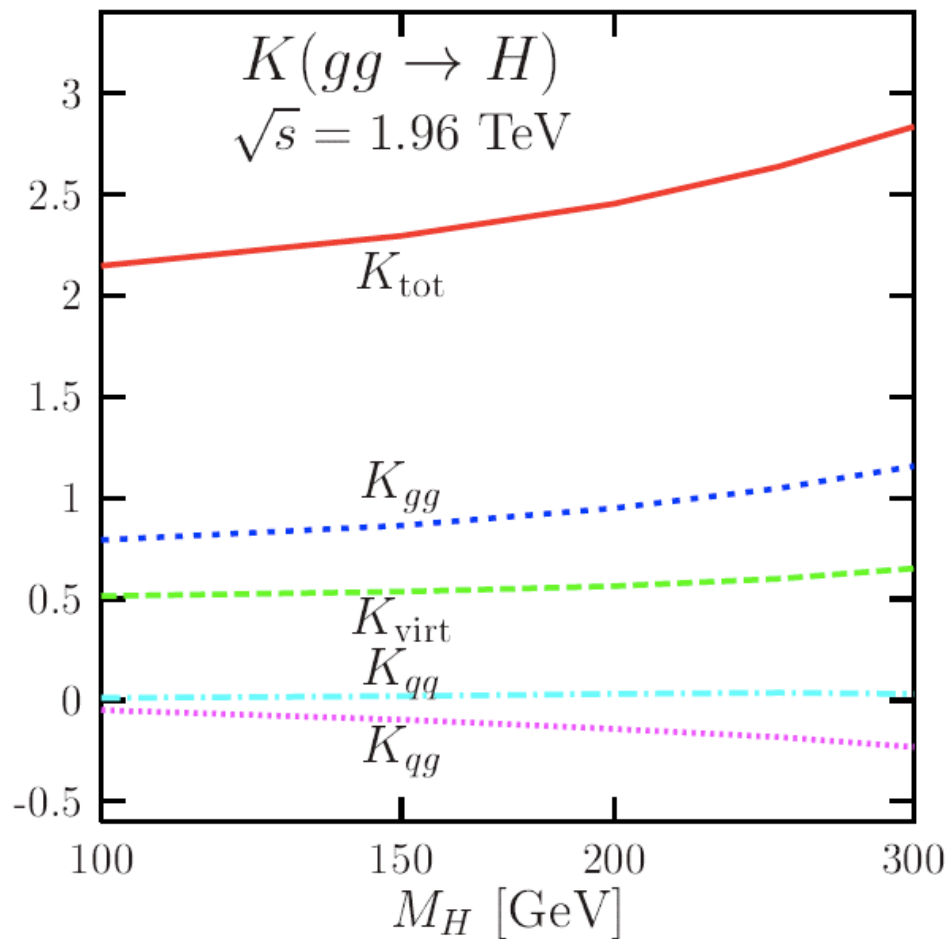
Stability against renormalization/factorization scale variation

Good accuracy of the partonic cross section

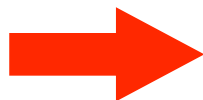
→ In this talk I will mostly discuss NLO-QCD results where *pdfs* can be compared

extraction from the data of *pdfs* with NNLO-QCD accuracy would require the use of full set (unavailable) of NNLO-QCD calculations

Higgs production at NLO-QCD

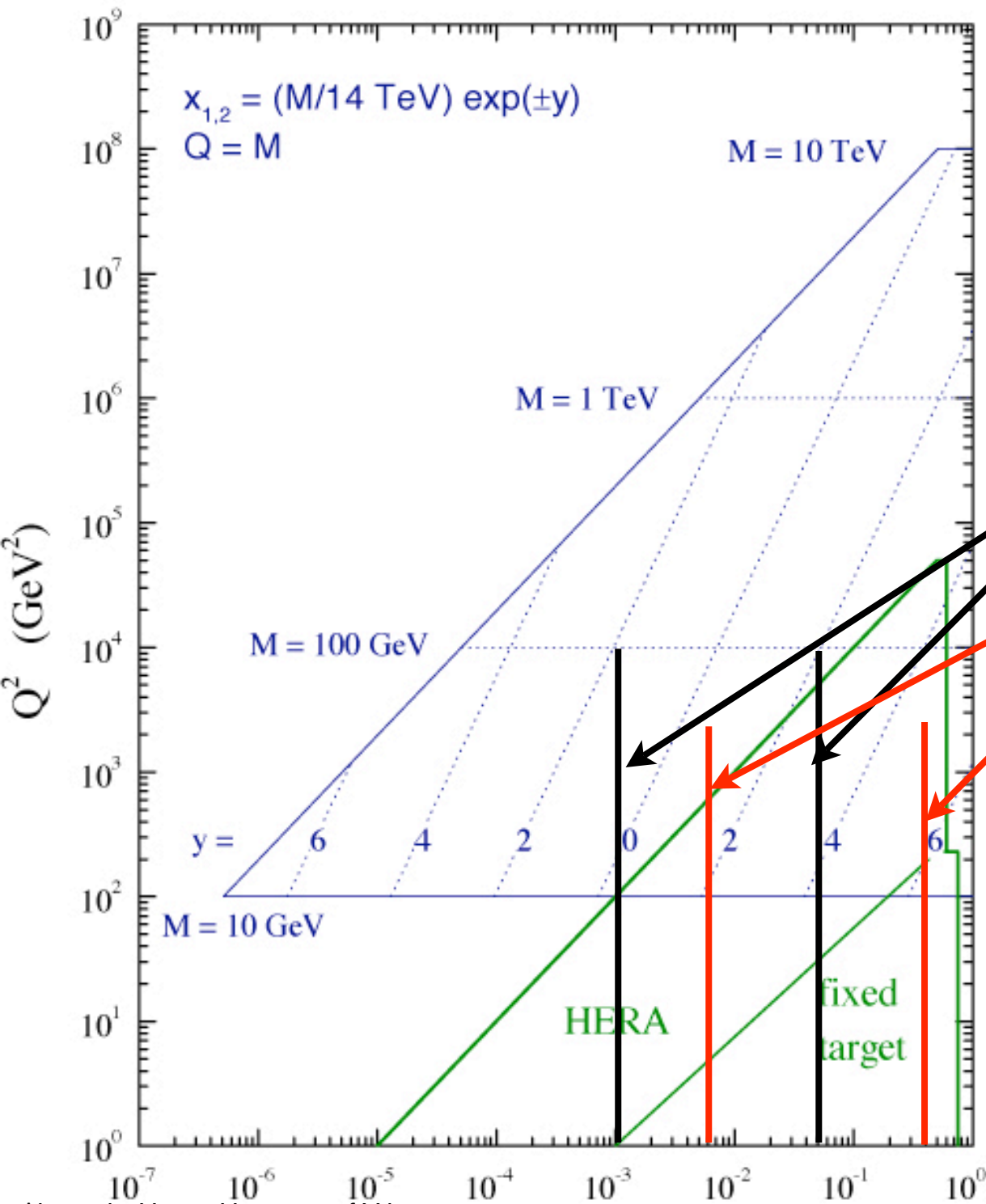


The gluon subprocesses
(both real and virtual)
give the largest contribution
to the total cross-section



Need to reduce the uncertainty
on the gluon pdf

parton kinematics



values of x relevant
to produce a final state with
invariant mass of 100 GeV
in the central rapidity interval

$$|y_H| \leq 2$$

$$[0.00096, 0.052]$$

$$[0.0069, 0.376]$$

The small- x region plays a **minor** role in the evaluation of the **inclusive** cross section (dominated by lowest-order threshold kinematics)

Cross section dominated by the lowest order partonic threshold kinematics
Large contribution due to soft gluon emission at the threshold
One small- x value in one pdf requires a large- x value in the other, but the steep fall of the gluon density suppresses these contributions

 The uncertainties associated to the small- x region do not affect the total cross section

Differences of the gluon density in the large- x region, due e.g. to different treatment of jet data, may appear at intermediate x because normalization/ momentum sum rules constraints

Higgs production at NLO-QCD: *pdfs* uncertainties

The uncertainty due to the experimental errors of the data, from which the *pdfs* are extracted, is parametrized in different ways:

- Montecarlo replicas
- Hessian method

The corresponding definitions to compute the standard deviation associated to an observable \mathcal{F} is

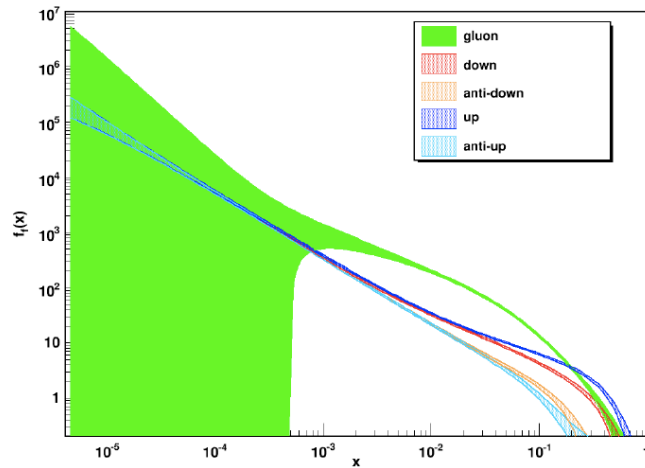
$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{\text{set}} - 1} \sum_{k=1}^{N_{\text{set}}} \left(\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}] \rangle \right)^2 \right)^{1/2}$$
$$\sigma_{\mathcal{F}}^{\text{hepdata}} = \frac{1}{2C_{90}} \left(\sum_{k=1}^{N_{\text{set}}/2} \left(\mathcal{F}[\{q^{(2k-1)}\}] - \mathcal{F}[\{q^{(2k)}\}] \right)^2 \right)^{1/2}$$

The factor C_{90} is not necessary any more for MSTW2008, where different sets are provided, which parametrize the uncertainties at 68% and at 90% C.L.

In our exercise \mathcal{F} is: the inclusive Higgs production cross section at NLO-QCD
the parton densities

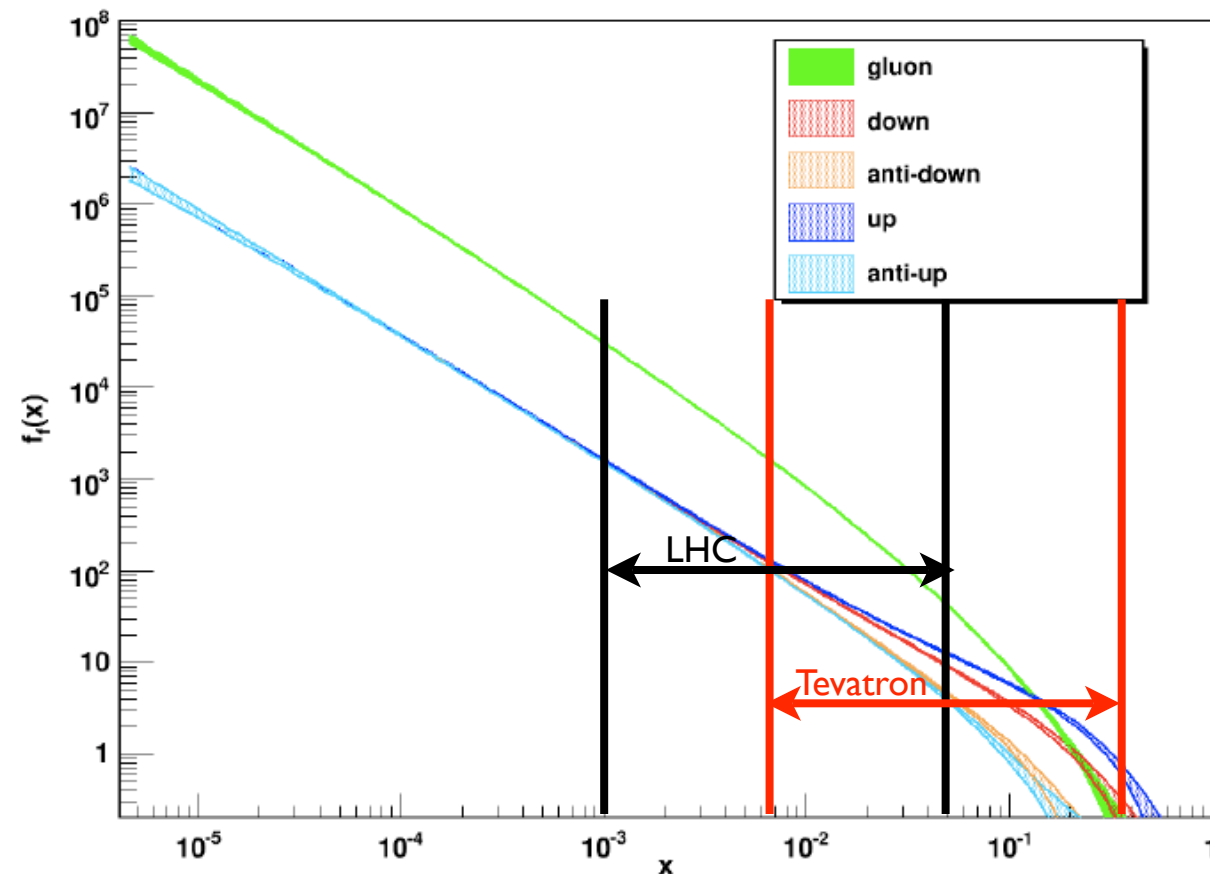
Effects of Altarelli-Parisi evolution

Parton PDFs at $Q = 1.4 \text{ GeV}$ - MSTW 2008 nlo

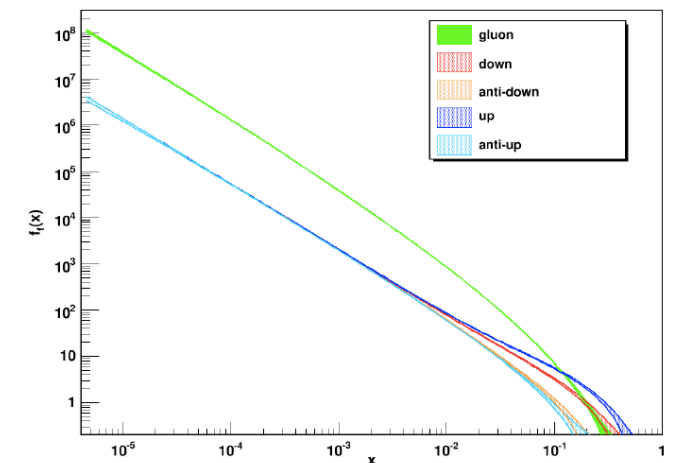


the x relevant for the gluon fusion
are well measured at HERA

Parton PDFs at $Q = 130 \text{ GeV}$ - MSTW 2008 nlo



Parton PDFs at $Q = 1000 \text{ GeV}$ - MSTW 2008 nlo



Comparison of *pdfs*

CTEQ6.6

DIS data, DY data, jets at colliders (not included: Tevatron run II, NuTeV, CHORUS data)
the data are fitted with a functional form fixed a priori
propagation of experimental errors to the *pdfs* described using the Hessian method

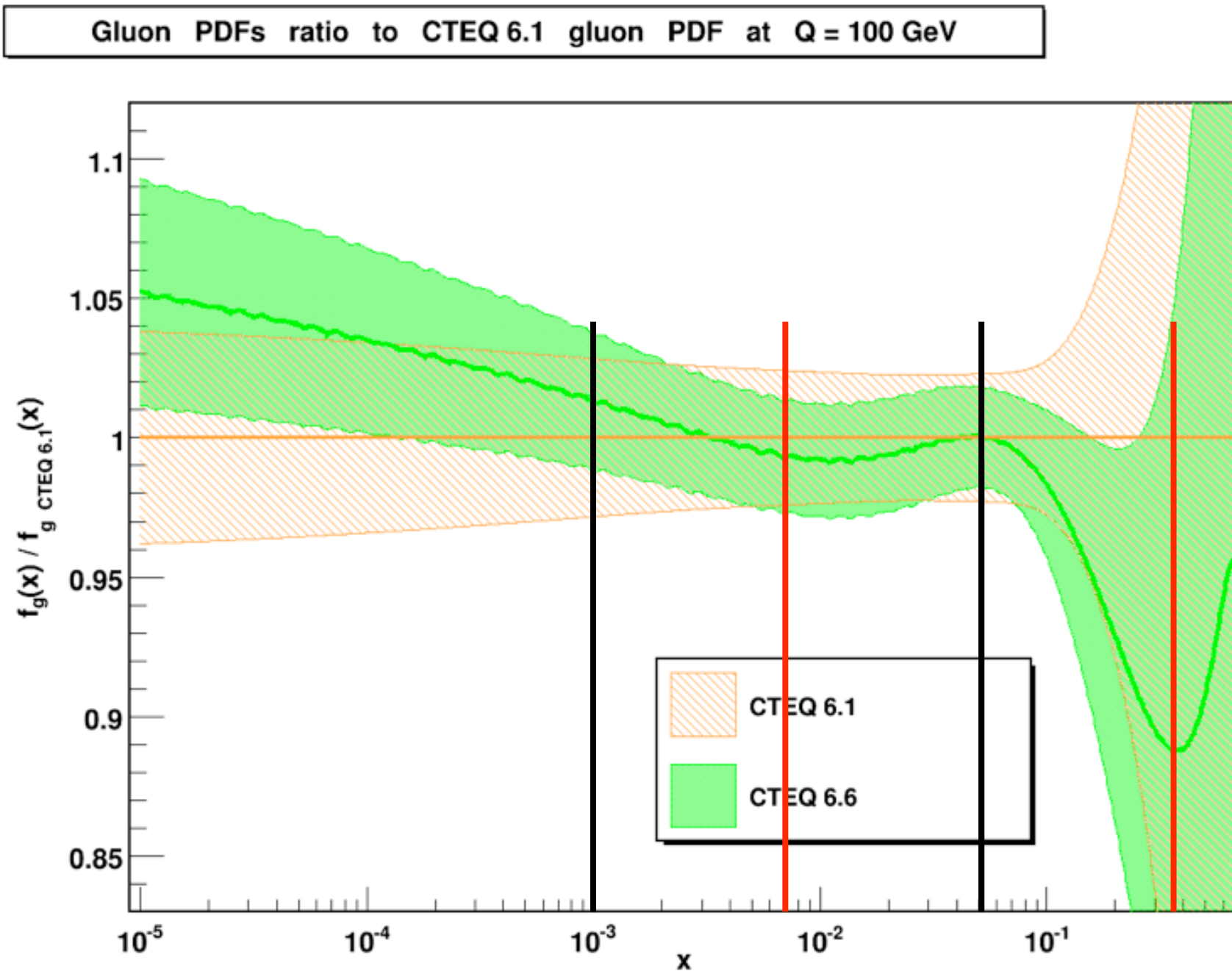
MSTW2008

DIS data, DY data, jets at colliders
the data are fitted with a functional form fixed a priori
propagation of experimental errors to the *pdfs* described using the Hessian method

NNPDF1.2

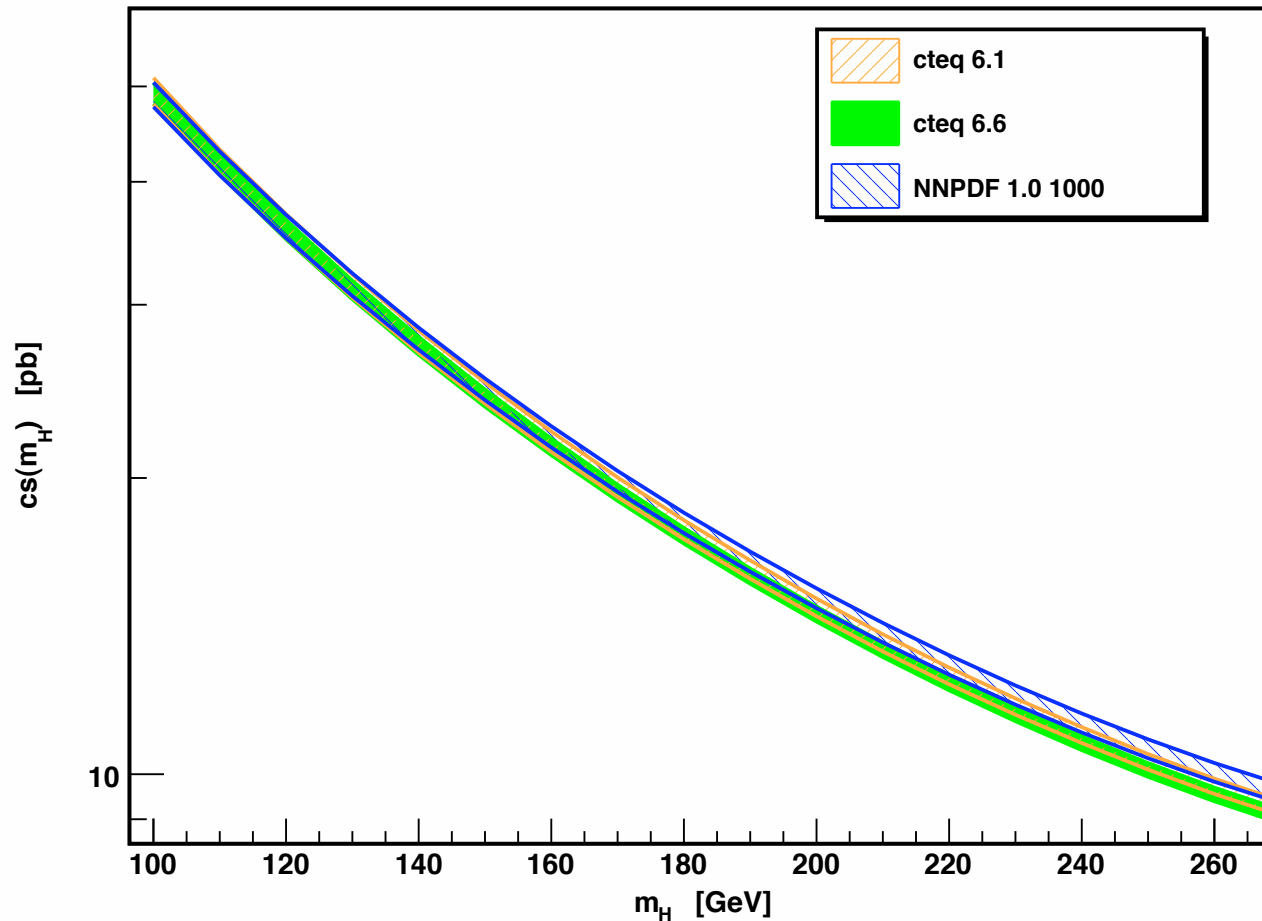
only DIS scattering data (NC and CC)
MC replicas of the data are generated within the experimental error
a Neural Network learns from the replicas a possible parametrization of the *pdfs*
the final set of *pdfs* provides a sampling of the *pdfs* parameter space
and allows to measure, in a statistically meaningful way, the spread of any observable
not yet full treatment of heavy quarks

CTEQ gluon densities

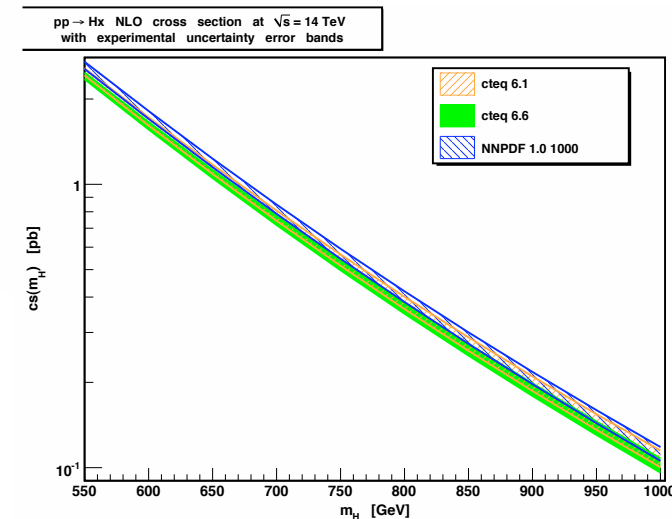
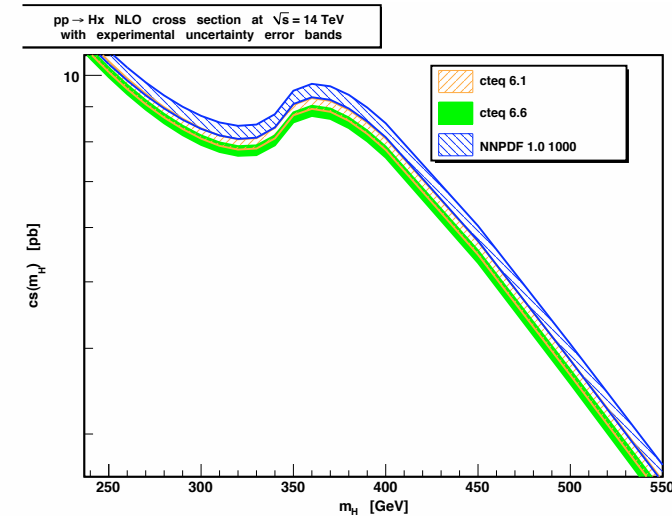


Higgs production at NLO-QCD: changes using CTEQ pdfs

pp → Hx NLO cross section at $\sqrt{s} = 14$ TeV
with experimental uncertainty error bands

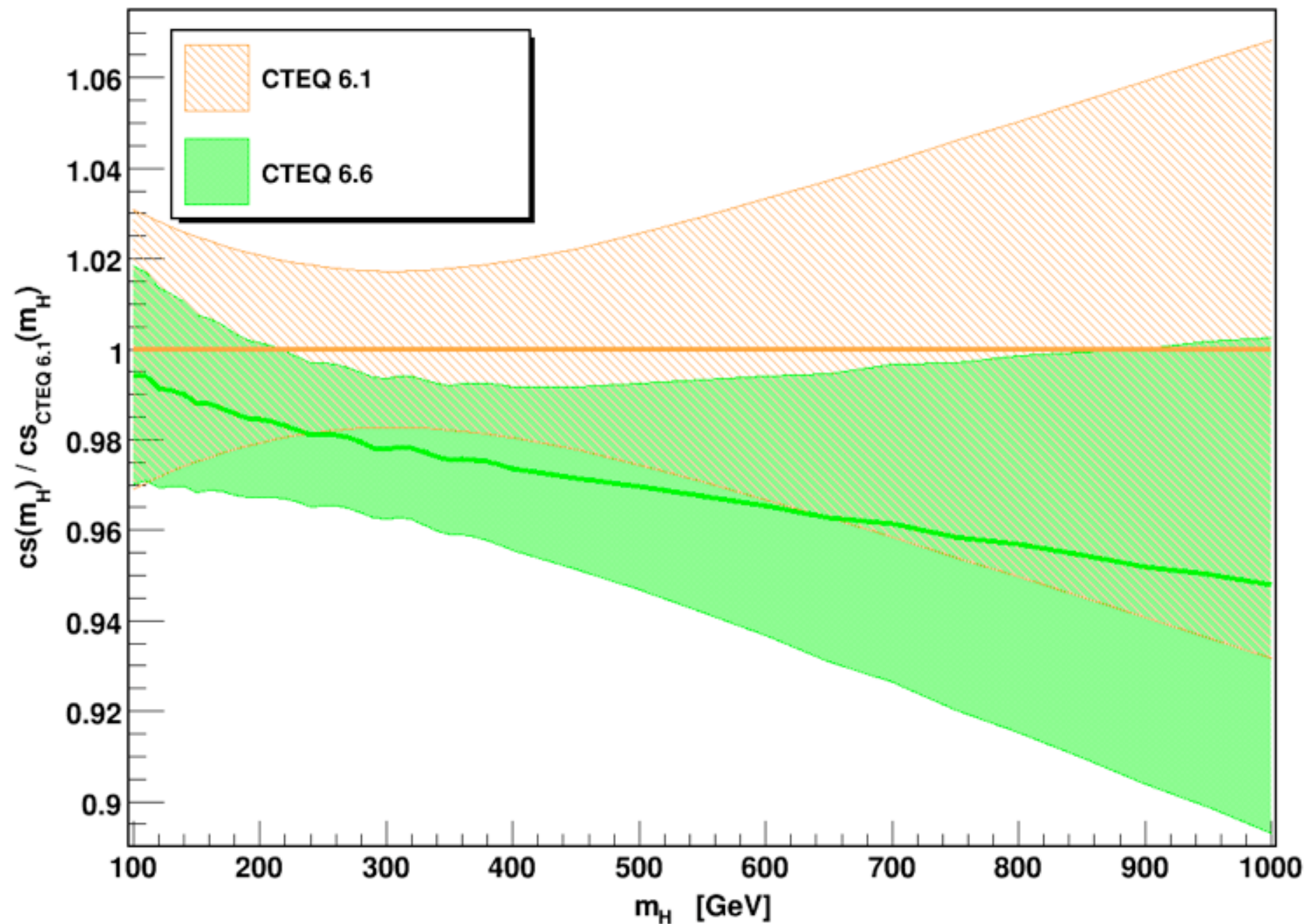


larger changes for large M_H



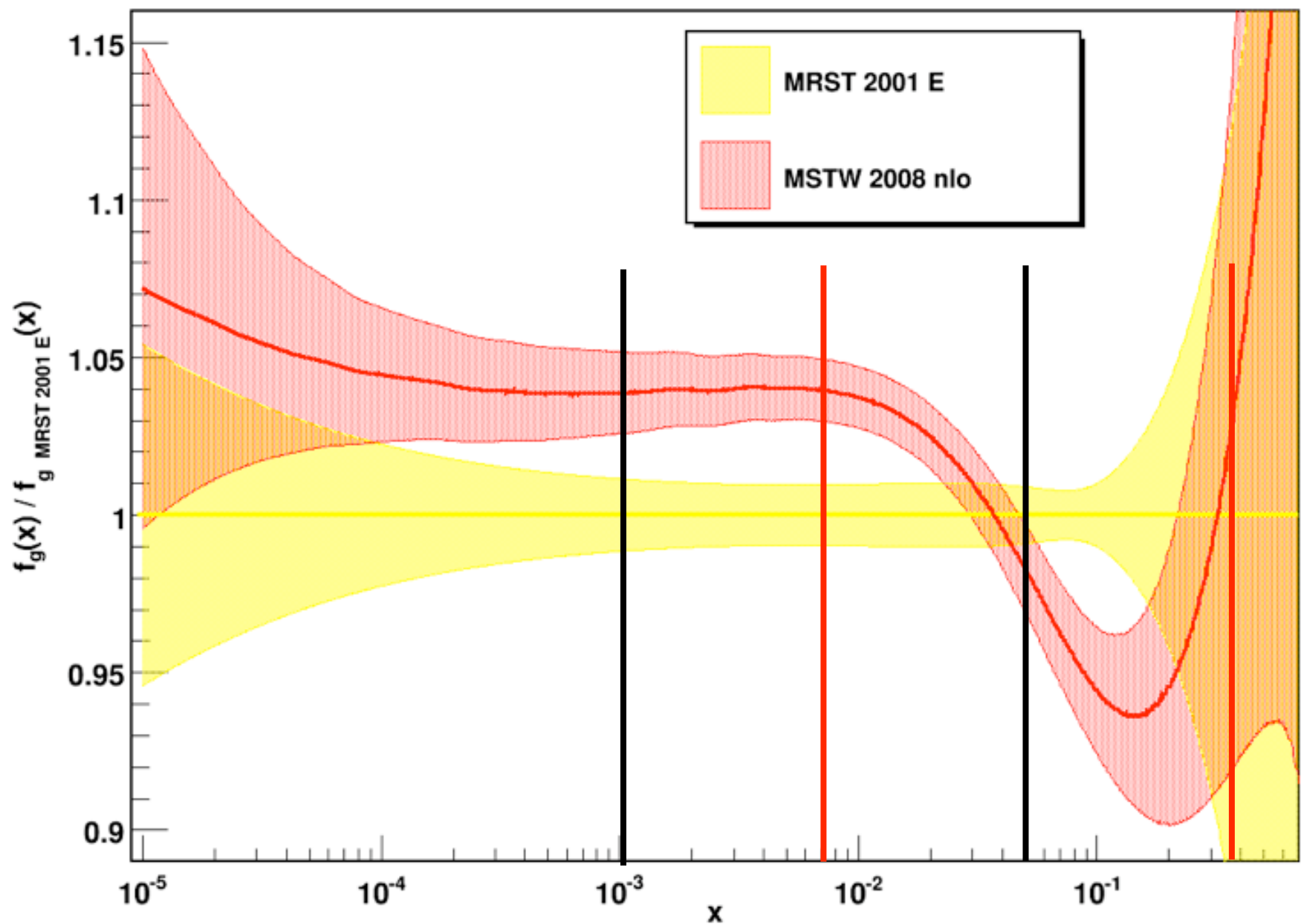
CTEQ cross sections

$pp \rightarrow Hx$ NLO cross section ratio to CTEQ 6.1 cross section
at $\sqrt{s} = 14$ TeV with experimental uncertainty error bands



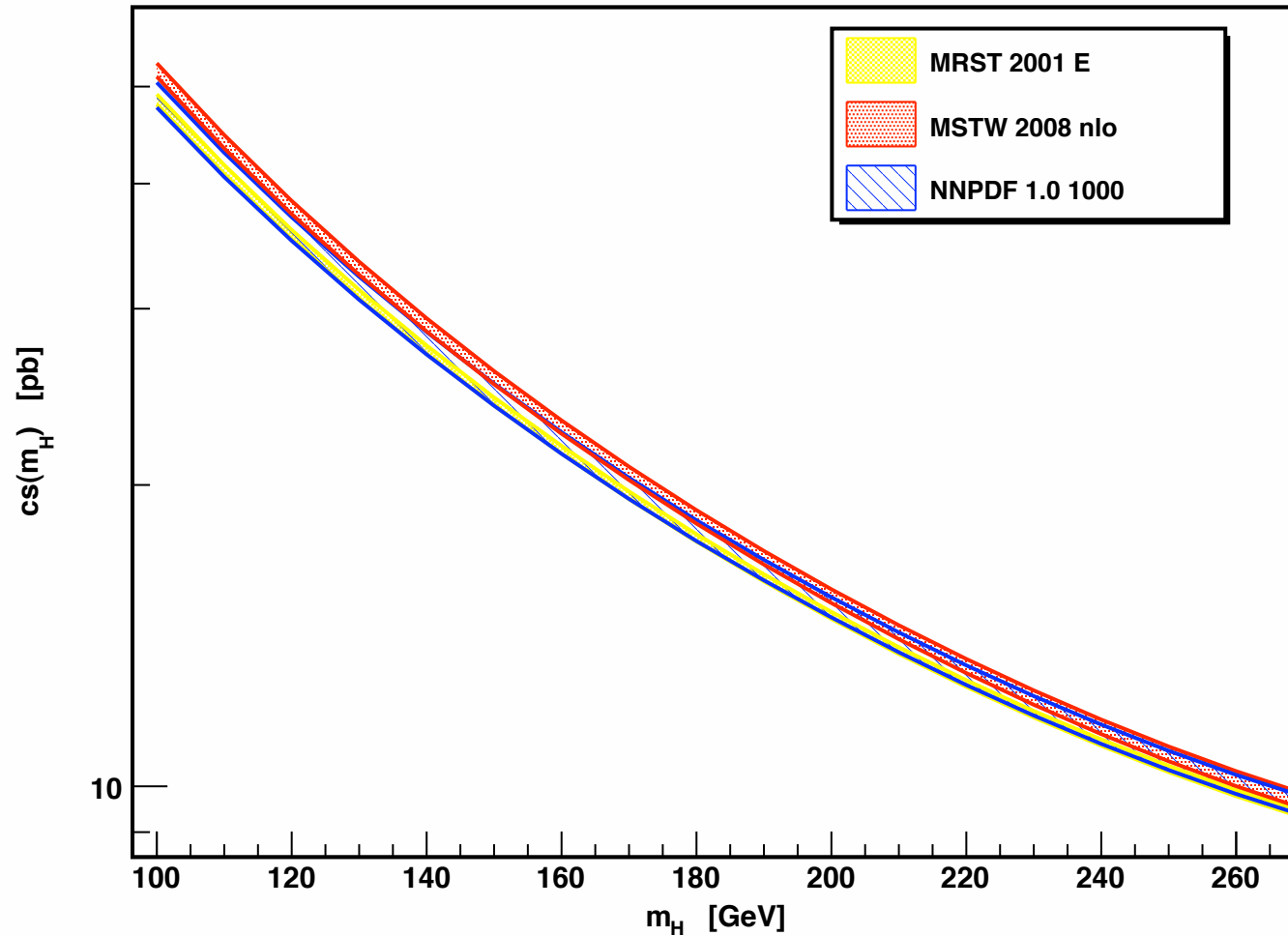
MSTW gluon densities

Gluon PDFs ratio to MRST 2001 E gluon PDF at $Q = 100 \text{ GeV}$



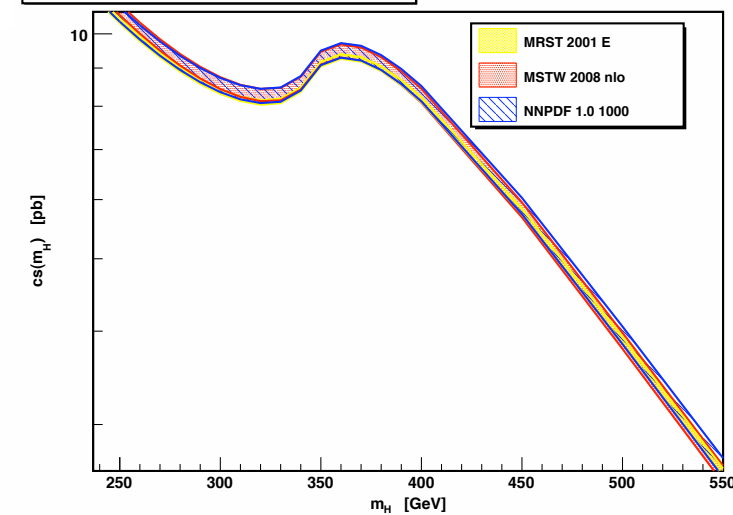
Higgs production at NLO-QCD: changes using M(R)ST(W) sets

$pp \rightarrow Hx$ NLO cross section at $\sqrt{s} = 14$ TeV
with experimental uncertainty error bands

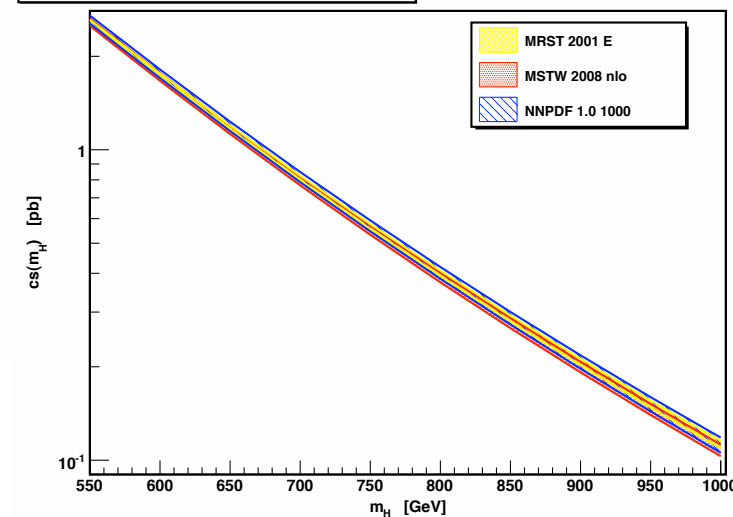


larger changes for small M_H

$pp \rightarrow Hx$ NLO cross section at $\sqrt{s} = 14$ TeV
with experimental uncertainty error bands

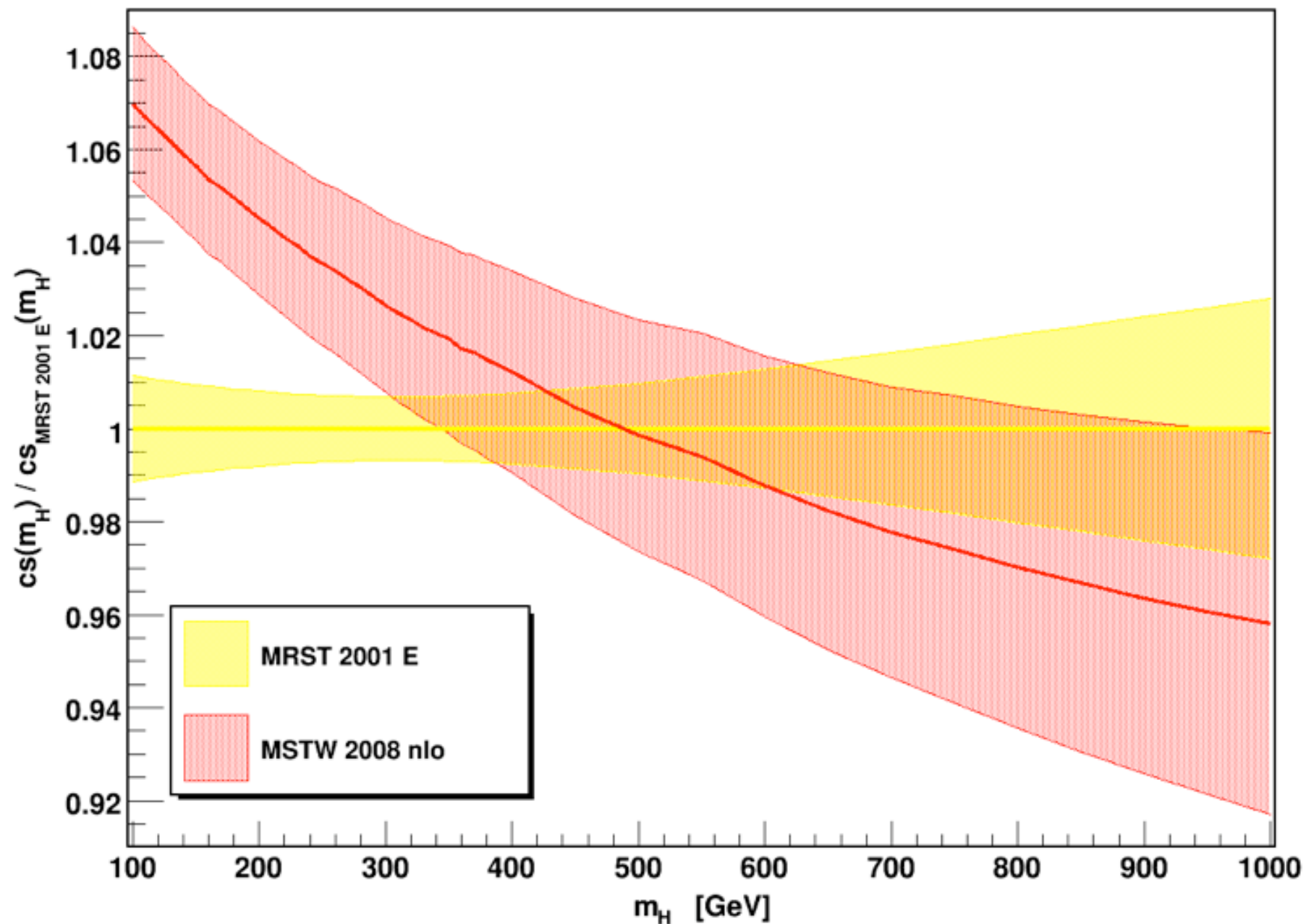


$pp \rightarrow Hx$ NLO cross section at $\sqrt{s} = 14$ TeV
with experimental uncertainty error bands



MSTW cross sections

$pp \rightarrow Hx$ NLO cross section ratio to MRST 2001 E cross section
at $\sqrt{s} = 14$ TeV with experimental uncertainty error bands



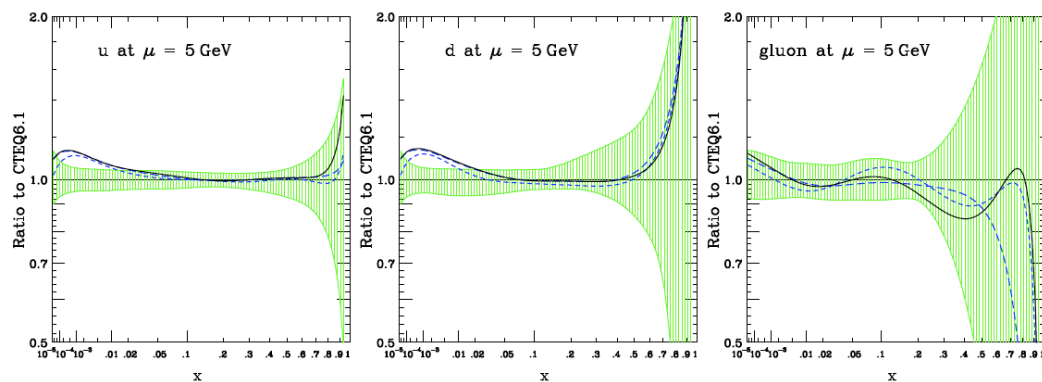
Comparison of pdfs: historical changes

CTEQ

Heavy quark mass effect were absent in 6.1 and have been introduced in 6.5/6.6

New treatment of the strange quark density

Important changes for up, down and gluon



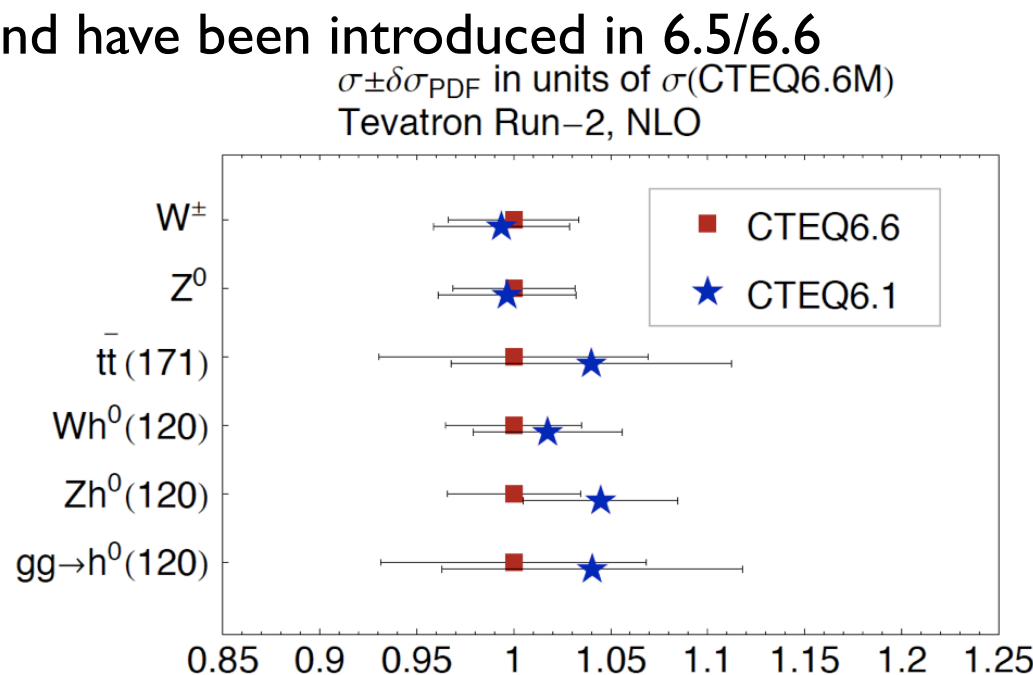
MSTW

New data: neutrino str.fun., CCFR and NuTeV, Z rapidity, Run II inclusive jet prod., HERA

GM-VFNS to treat HQ effects (first introduced in 2006),

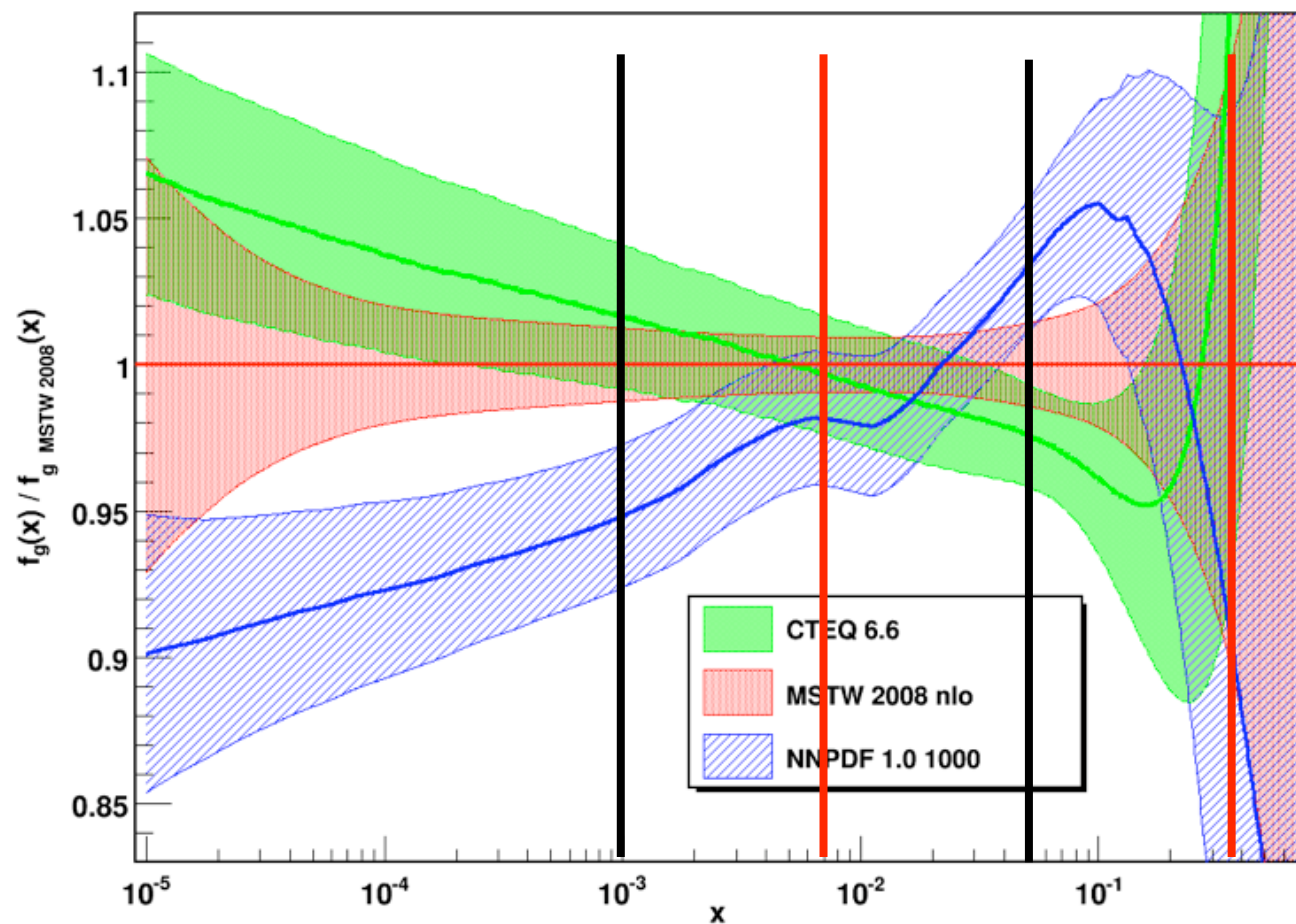
new richer parametrization

change in the α_s value (reduced from 2006 to 2008) yields larger gluon at small- x

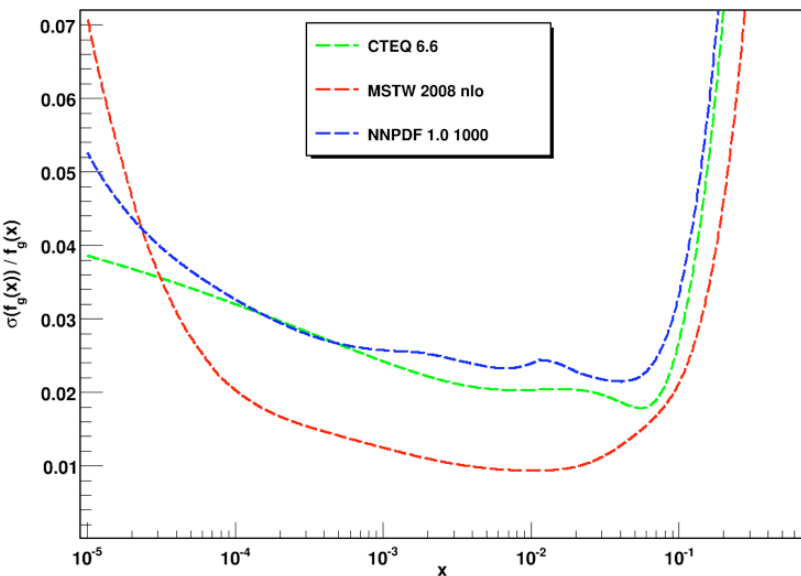


Comparison of pdfs: latest released sets

Gluon PDFs ratio to MSTW 2008 nlo gluon PDF at $Q = 100$ GeV



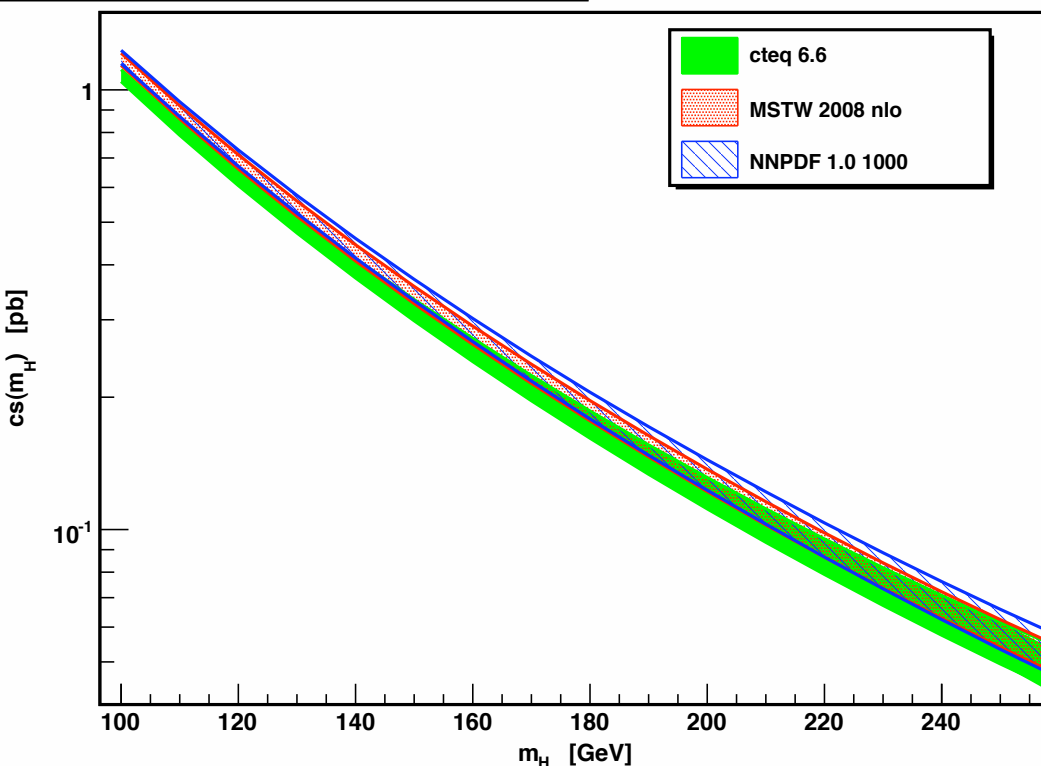
Relative uncertainties on gluon PDFs at $Q = 100$ GeV



Size of uncertainties similar for CTEQ and NNPDF
Smaller by a factor 2 for MSTW

In the region constrained by HERA data
NNPDF agrees with CTEQ and MSTW

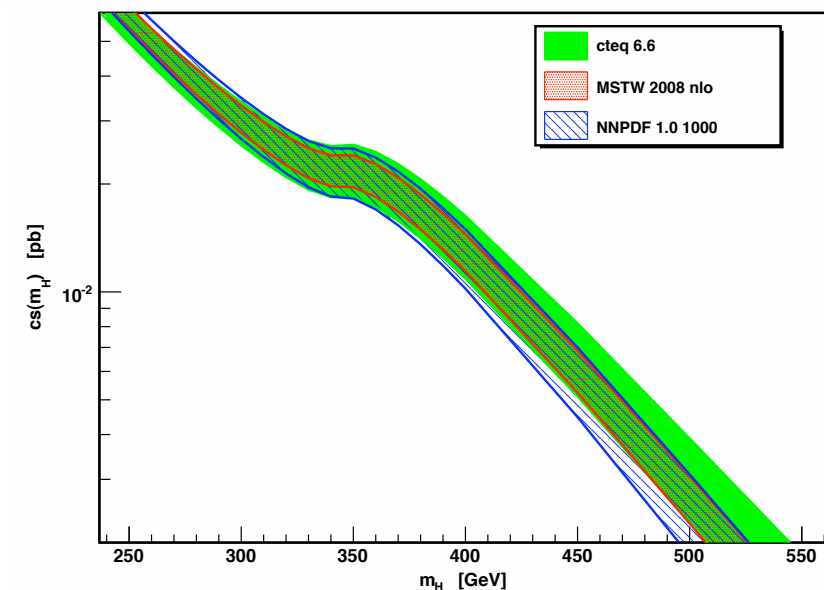
$p\bar{p} \rightarrow Hx$ NLO cross section at $\sqrt{s} = 1.96$ TeV
with experimental uncertainty error bands



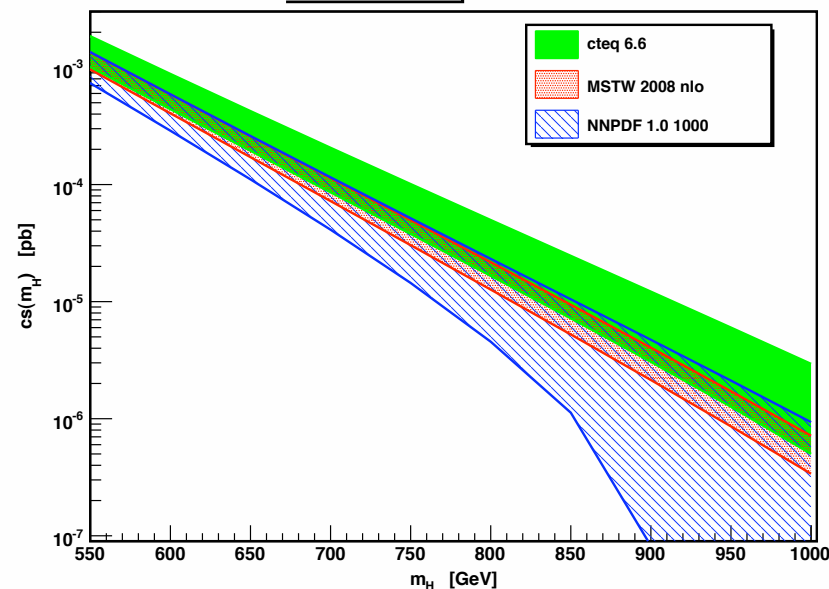
MSTW2008 and CTEQ6.6
do not overlap for $M_H < 150$ GeV

NNPDF1.0 agrees
with CTEQ6.6 for $M_H > 200$ GeV
with MSTW over entire range

$$\alpha_s(m_Z) = \begin{array}{ll} \text{CTEQ6.6} & 0.118 \\ \text{NNPDF1.0} & 0.119 \\ \text{MSTW2008} & 0.120 \end{array}$$



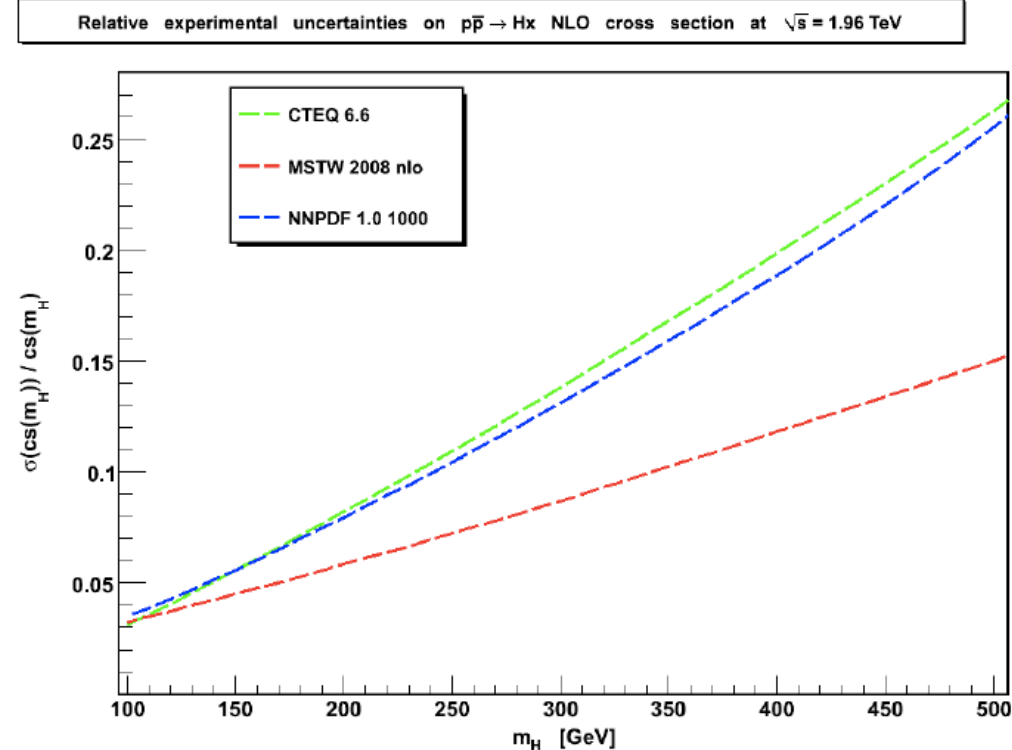
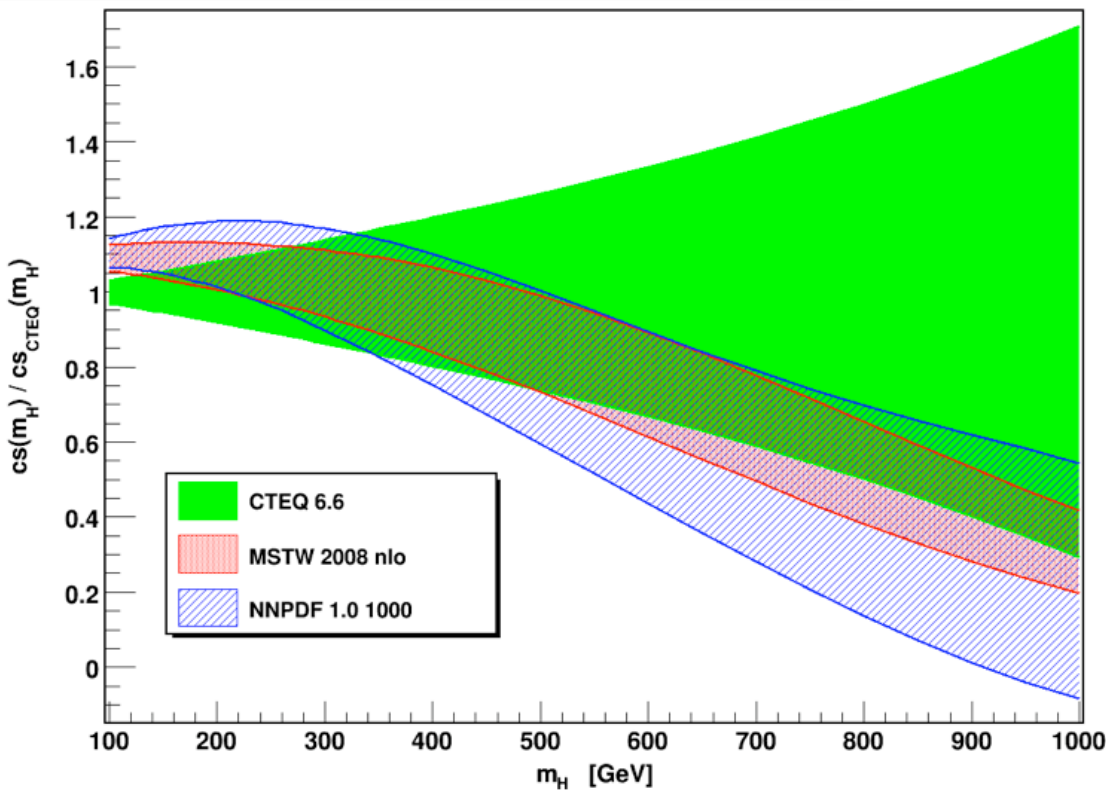
at $\sqrt{s} = 1.96$ TeV
y error bands



Higgs total cross section at NLO-QCD:

Tevatron

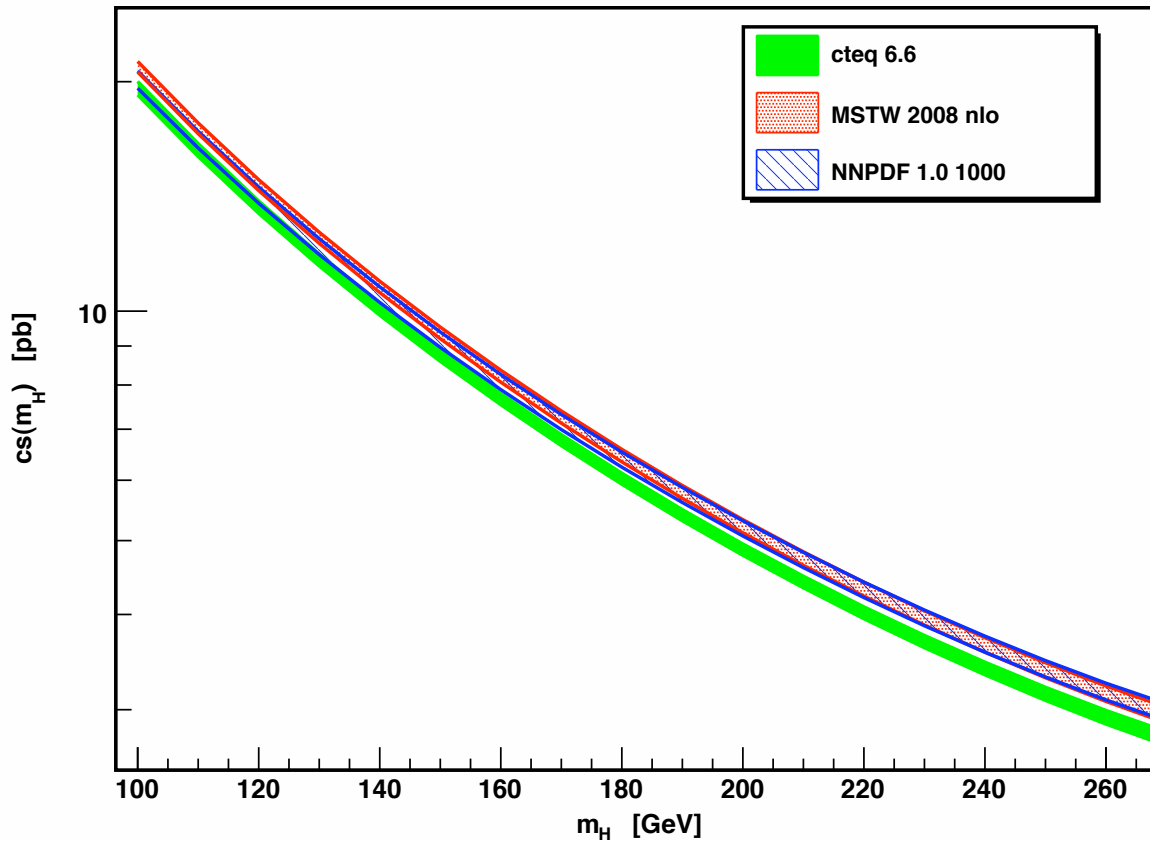
$p\bar{p} \rightarrow Hx$ NLO cross section relative to CTEQ 6.6 cross section
at $\sqrt{s} = 1.96$ TeV with experimental uncertainty error bands



Similar uncertainties quoted by the 3 collaborations

For light M_H , significant (up to 3σ) discrepancy (up to 9%) in the central values

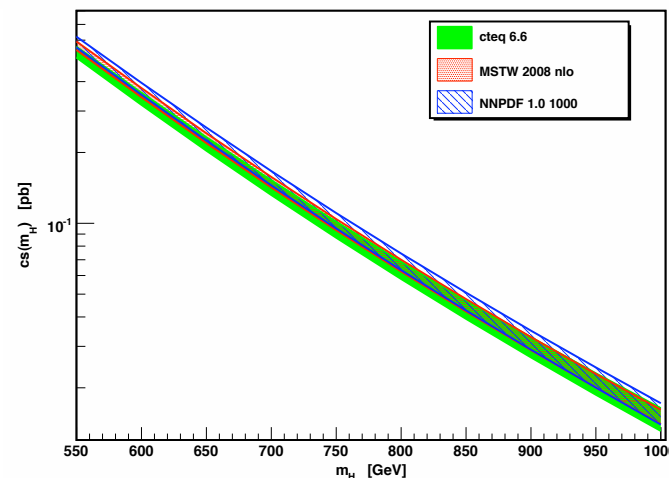
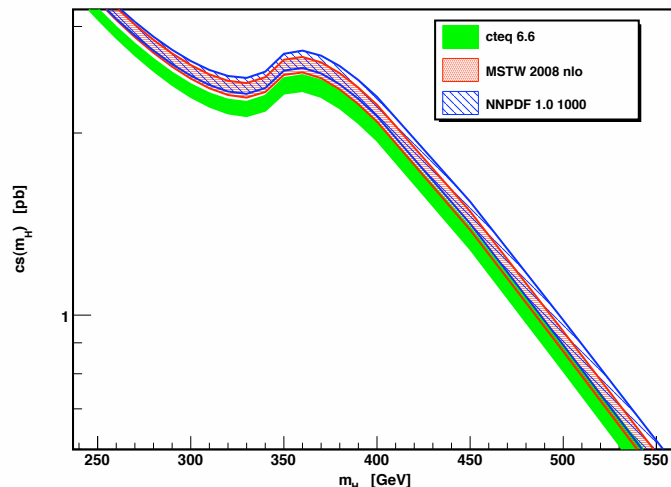
pp → Hx NLO cross section at $\sqrt{s} = 8$ TeV
with experimental uncertainty error bands



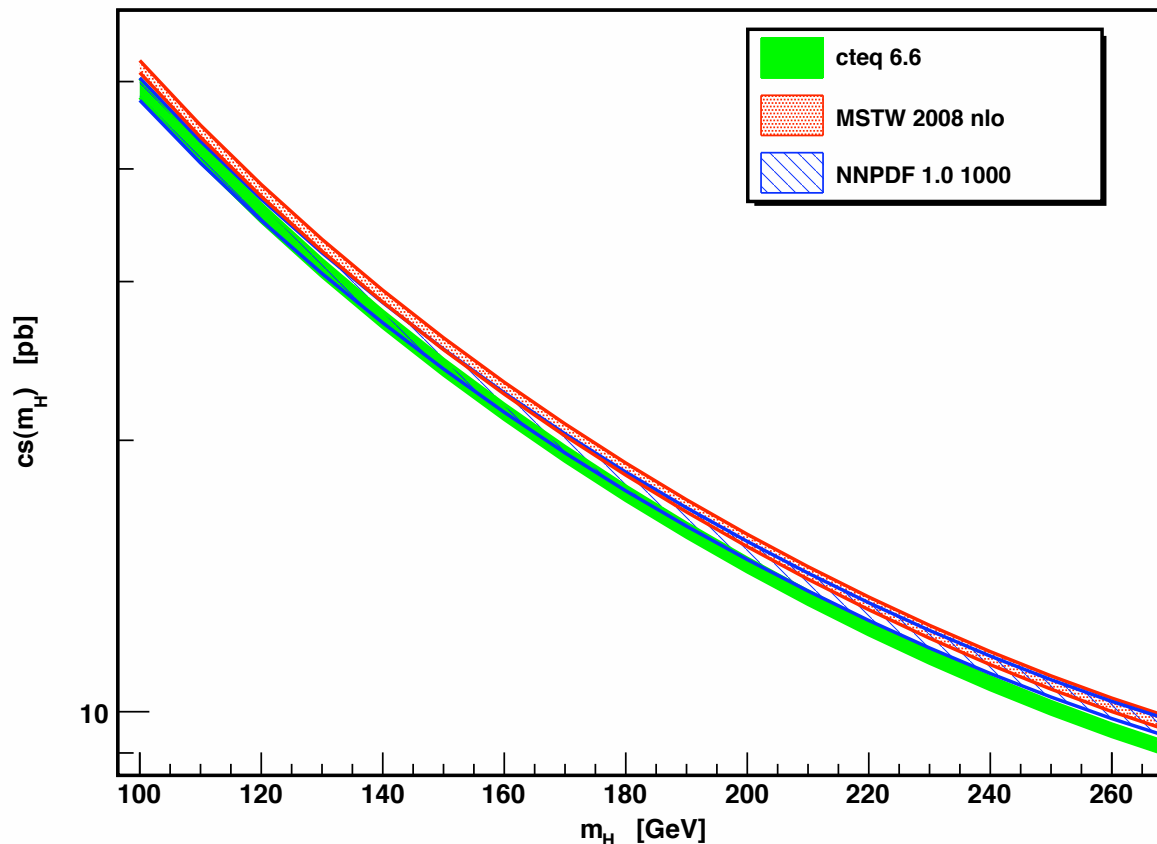
MSTW2008 and CTEQ6.6
do not overlap for $M_H < 500$ GeV

NNPDF1.0 overlaps
with CTEQ6.6 for $M_H < 120$ GeV
with MSTW for $M_H > 200$ GeV

$$\alpha_s(m_Z) = \begin{array}{ll} \text{CTEQ6.6} & 0.118 \\ \text{NNPDF1.0} & 0.119 \\ \text{MSTW2008} & 0.120 \end{array}$$



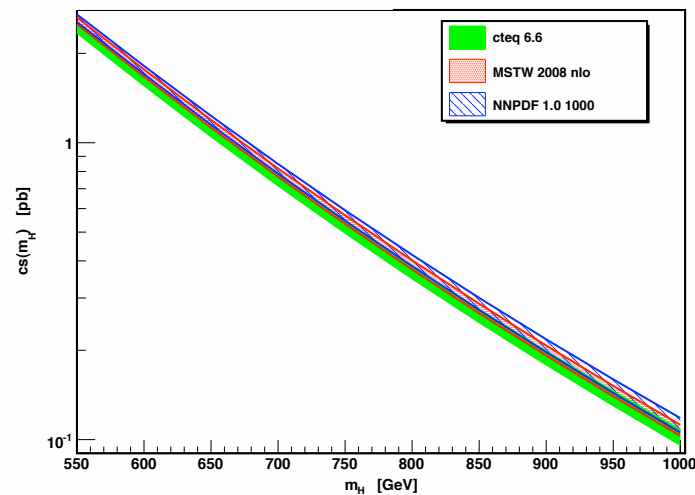
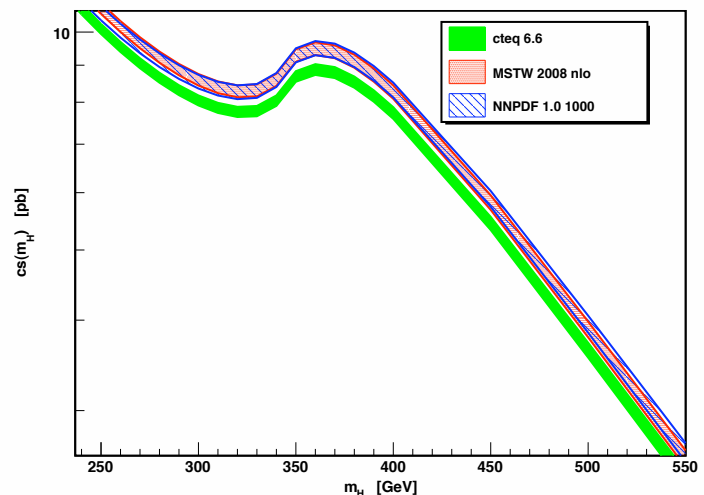
pp → Hx NLO cross section at $\sqrt{s} = 14$ TeV
with experimental uncertainty error bands



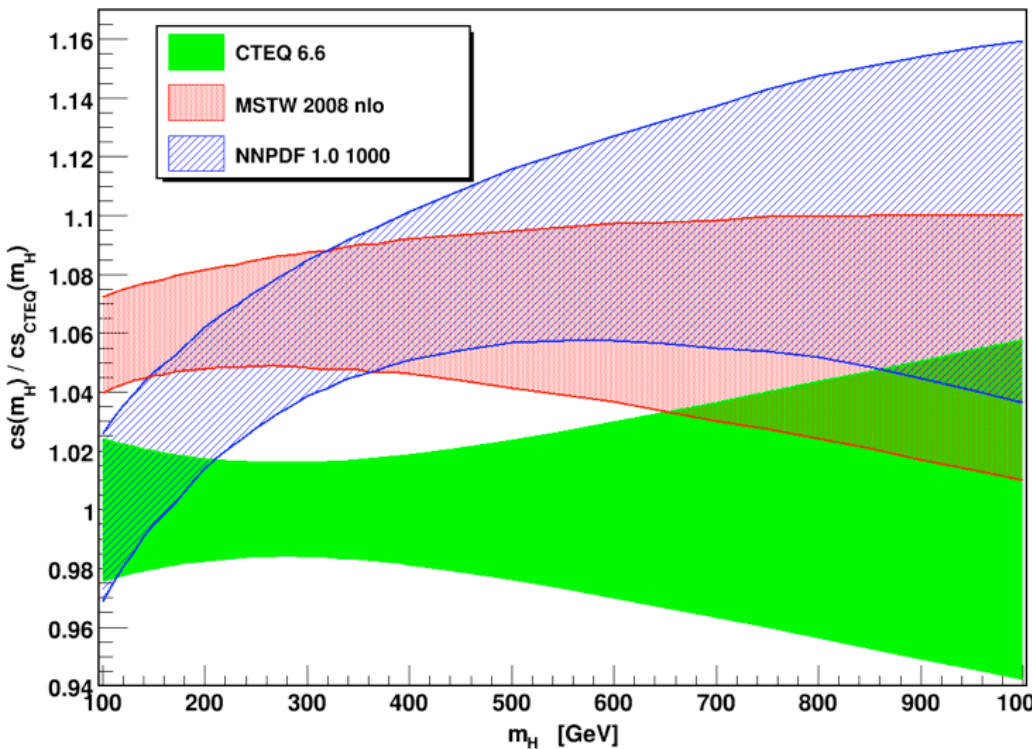
MSTW2008 and CTEQ6.6
do not overlap for $M_H < 500$ GeV

NNPDF1.0 agrees
with CTEQ6.6 for $M_H < 180$ GeV
with MSTW for $M_H > 200$ GeV

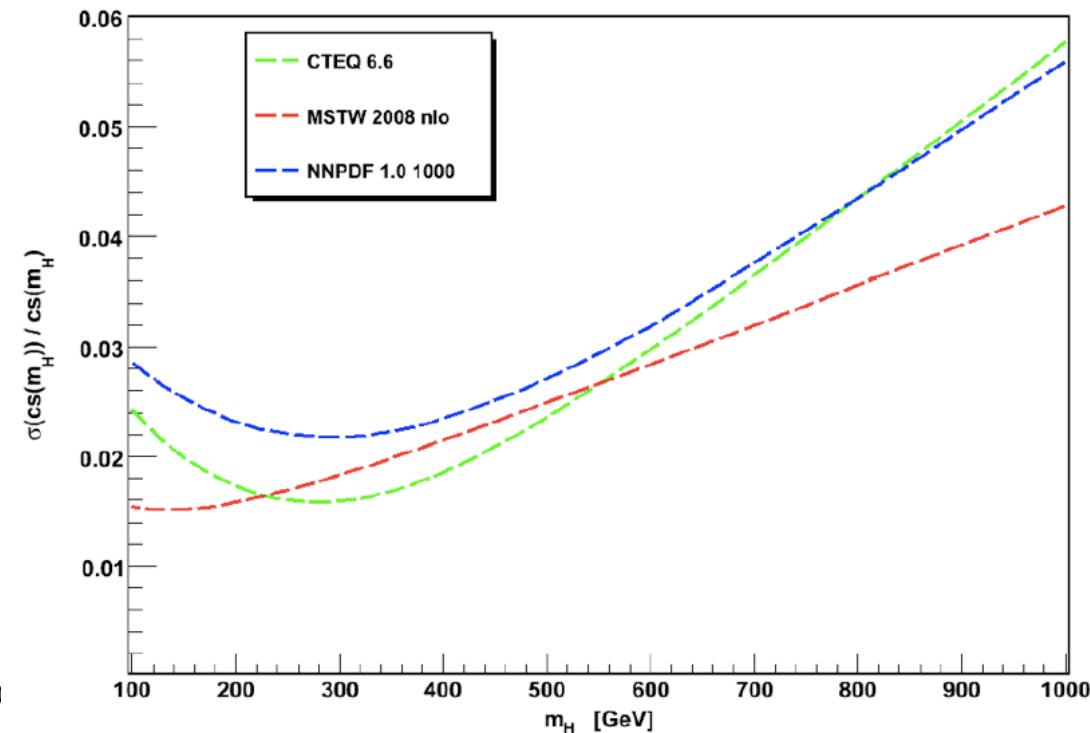
$$\alpha_s(m_Z) = \begin{array}{ll} \text{CTEQ6.6} & 0.118 \\ \text{NNPDF1.0} & 0.119 \\ \text{MSTW2008} & 0.120 \end{array}$$



pp → Hx NLO cross section relative to CTEQ 6.6 cross section
at $\sqrt{s} = 14$ TeV with experimental uncertainty error bands



Relative experimental uncertainties on pp → Hx NLO cross section at $\sqrt{s} = 14$ TeV



Similar uncertainties (worst agreement w.r.t. Tevatron)
quoted by the 3 collaborations

Significant (up to 3σ) discrepancy (6%) in the central values

The minimum of uncertainty for a range of Higgs masses corresponding
to a range of x where HERA data play a major role on the gluon determination

Possible origin of the discrepancy

Differences in the gluon density can account for part of the discrepancy but

the 3 collaborations use different values for $\alpha_s(m_Z)$
and the cross section has a strong dependence on this coupling

$$\sigma_{tot} = \alpha_s^2 \sigma_0 + \alpha_s^3 \sigma_1 + \dots \quad \sigma_0 \sim \sigma_1$$

$$\alpha_s(m_Z) = 0.1176(20)$$

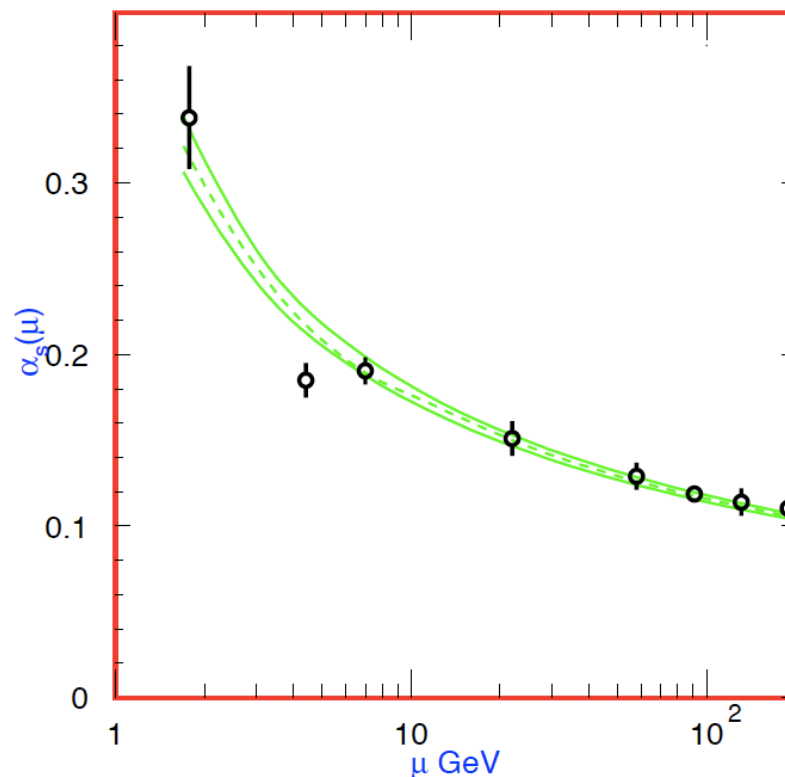
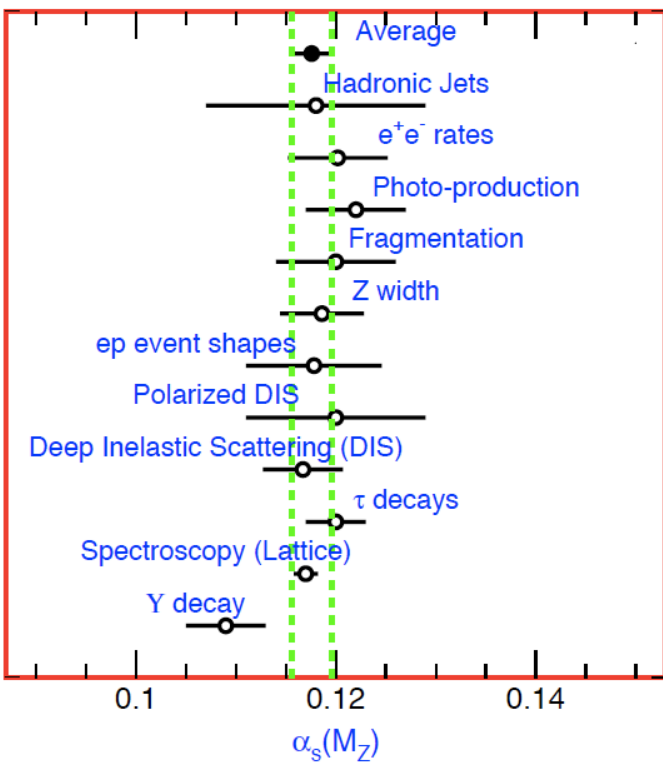
Naively, a change of α_s by 1σ implies a change of the cross section of 3-5 %

$$\frac{2}{117} \sim 0.017 \quad \text{for constant } pdfs \quad 0.034 < \delta\sigma_{tot} < 0.051$$

but

parton densities and strong coupling should be consistent with each other

it is necessary to study the role of the uncertainties on α_s in the global fit of the *pdfs*

α_s 

$$\alpha_s(m_Z) = 0.1176(20)$$

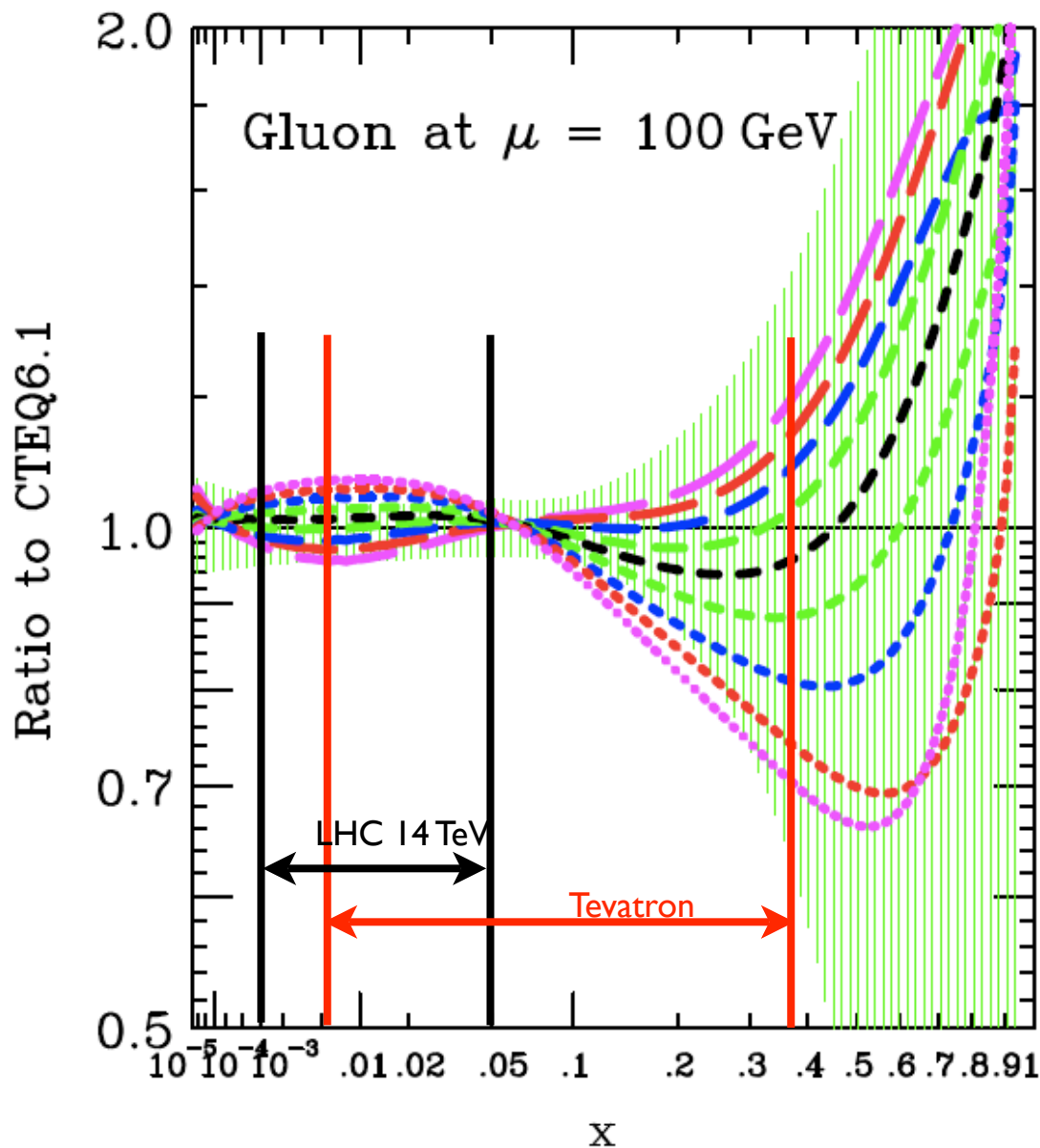
The strong coupling can be determined from observables that do not depend on the *pdfs* and then it can be used, with its error, to evaluate the impact of this parameter on the cross section

but

the strong coupling is very precisely measured in processes that involve the *pdfs*

it is necessary to evaluate **to which extent α_s and the *pdfs* are correlated** (e.g. gluon density is directly proportional to α_s)

α_s and gluon density in CTEQ6.6



gluon density obtained from a global fit with different values of $\alpha_s(M_Z)$

$$0.110 \leq \alpha_s(m_Z) \leq 0.126$$

compared with the uncertainty band of the best fit with central α_s

the vertical lines mark the x range to produce an object of 100 GeV with $|y| < 2$

α_s dependence, MSTW

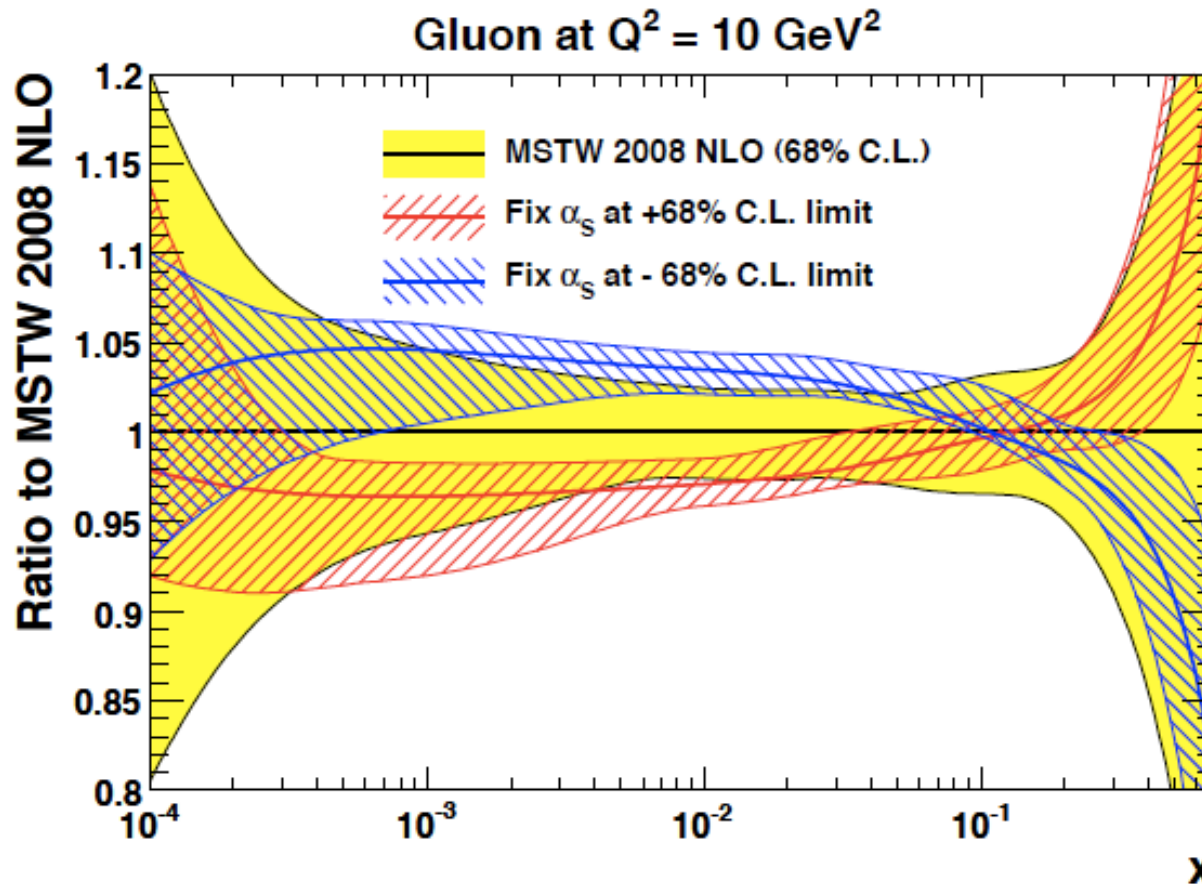
For MSTW, the uncertainty on the *pdfs* and on α_s are correlated

$$\alpha_s \in [\alpha_s^0 - 1\sigma, \alpha_s^0 + 1\sigma] \quad \alpha_s^0 \equiv \alpha_s(m_Z) = 0.1202_{-0.0015}^{+0.0012}$$

For each of the 5 values:

$$\alpha_s^0 - 1\sigma, \quad \alpha_s^0 - 0.5\sigma, \quad \alpha_s^0, \quad \alpha_s^0 + 0.5\sigma, \quad \alpha_s^0 + 1\sigma$$

there are 40 pdf sets



Some *pdfs* spreads are much smaller than the central-value spread

α_S dependence, MSTW

For each of the 5 values compute the *pdf* spread (not necessarily symmetric)

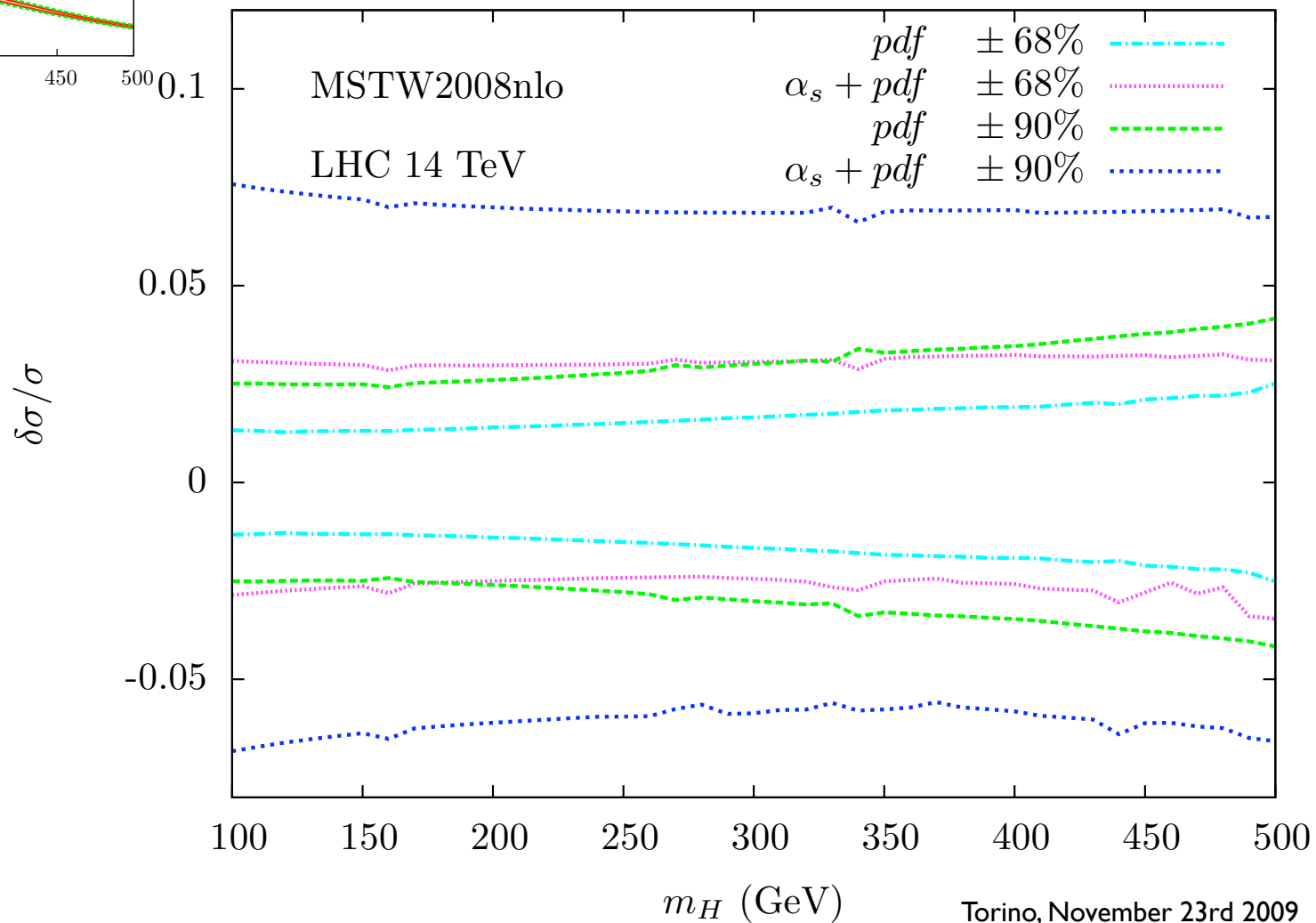
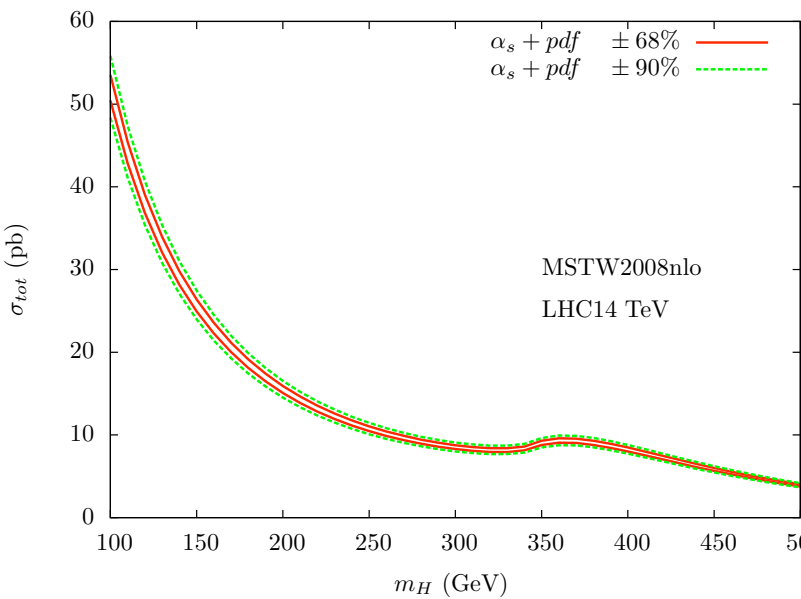
$$(\Delta F_{\text{PDF}}^{\alpha_S})_+ = \sqrt{\sum_{k=1}^n \left\{ \max \left[F^{\alpha_S}(S_k^+) - F^{\alpha_S}(S_0), F^{\alpha_S}(S_k^-) - F^{\alpha_S}(S_0), 0 \right] \right\}^2},$$
$$(\Delta F_{\text{PDF}}^{\alpha_S})_- = \sqrt{\sum_{k=1}^n \left\{ \max \left[F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^+), F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^-), 0 \right] \right\}^2},$$

The (pdf+alpha_s) spread is obtained as follows

$$(\Delta F_{\text{PDF}+\alpha_S})_+ = \max_{\alpha_S} (\{F^{\alpha_S}(S_0) + (\Delta F_{\text{PDF}}^{\alpha_S})_+\}) - F^{\alpha_S^0}(S_0),$$
$$(\Delta F_{\text{PDF}+\alpha_S})_- = F^{\alpha_S^0}(S_0) - \min_{\alpha_S} (\{F^{\alpha_S}(S_0) - (\Delta F_{\text{PDF}}^{\alpha_S})_-\}),$$

Some *pdfs* spreads are much smaller than the central-value spread
This recipe is quite conservative

α_s dependence, MSTW



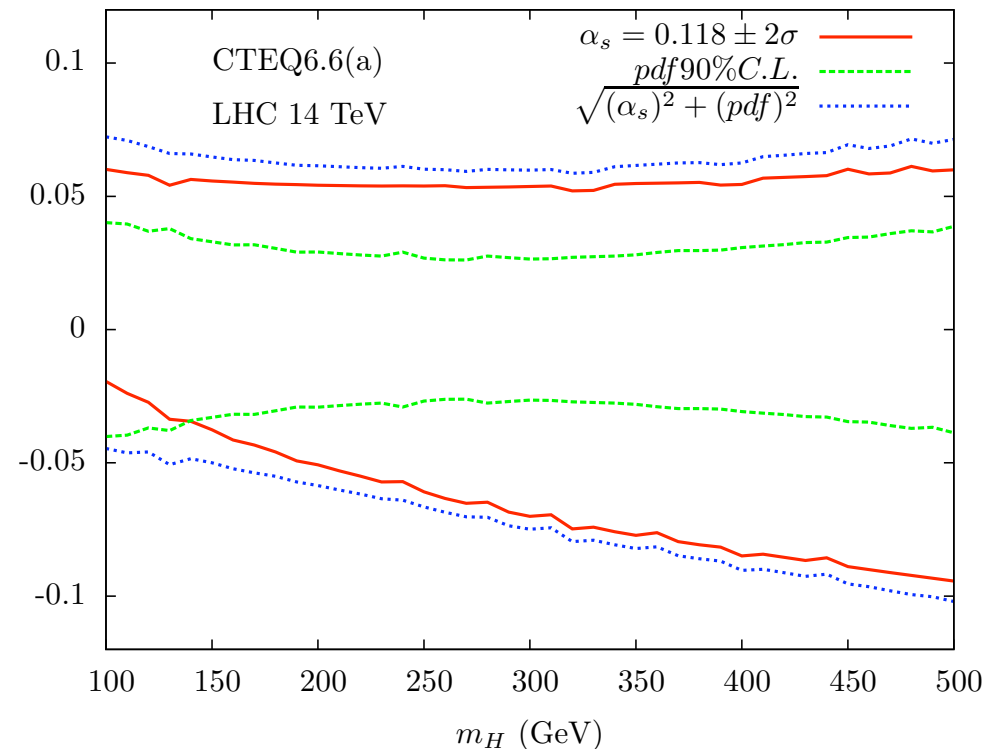
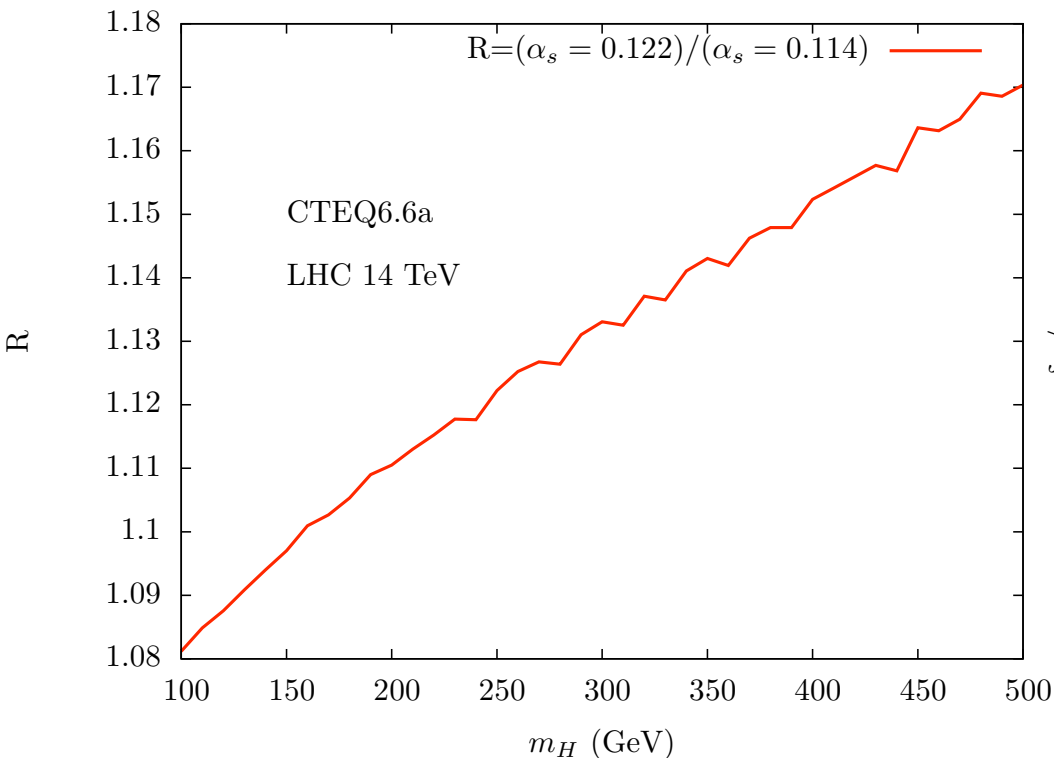
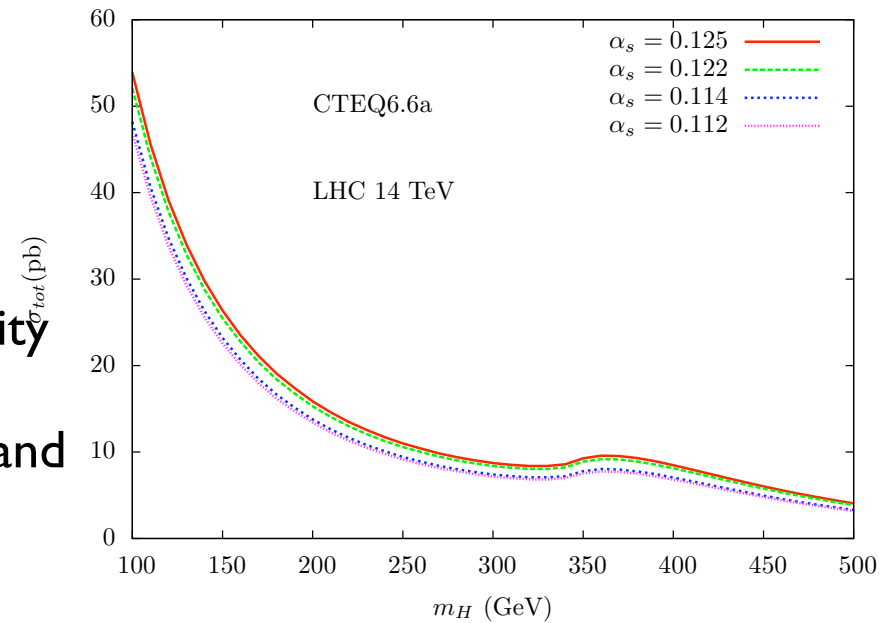
α_s dependence, CTEQ

Series of global fits compatible with existing data using as reference value for α_s values within the uncertainty band of $\alpha_s(m_Z)$

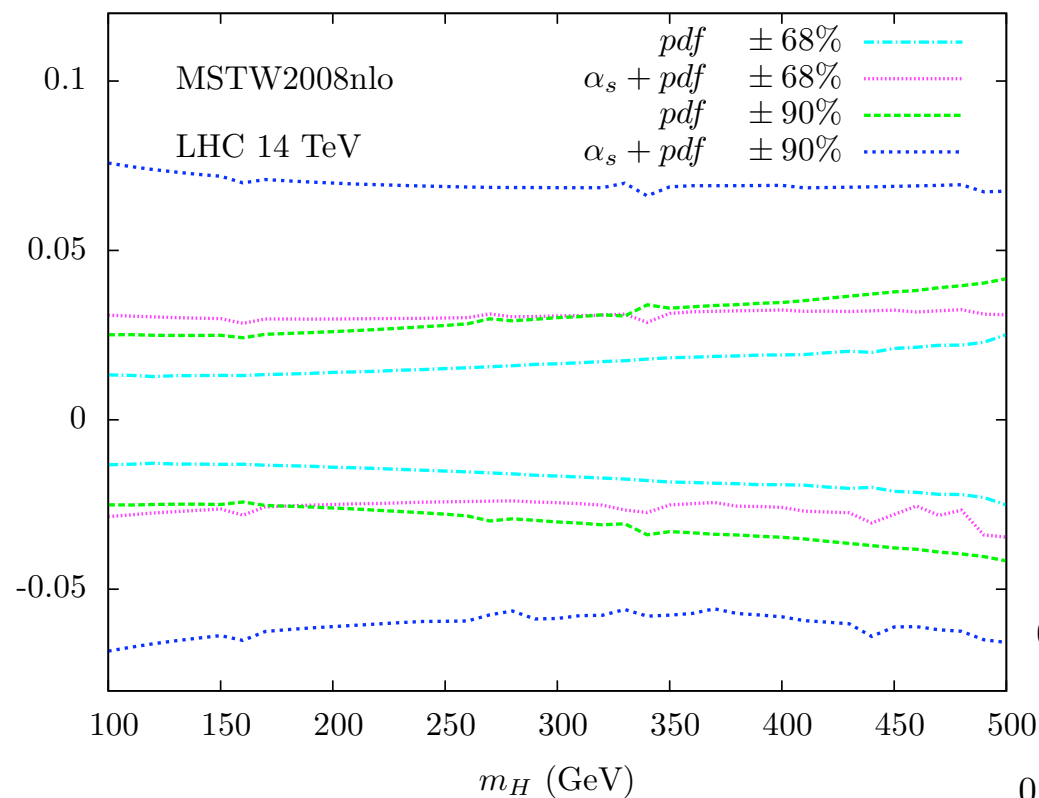
Every choice of $\alpha_s(m_Z)$ yields a different gluon density

CTEQ assumes that the uncertainties due to the pdf and those due to $\alpha_s(m_Z)$ are not correlated and can be combined in quadrature

The central values are instead strongly correlated



α_s dependence, comparison CTEQ6.6 vs MSTW2008



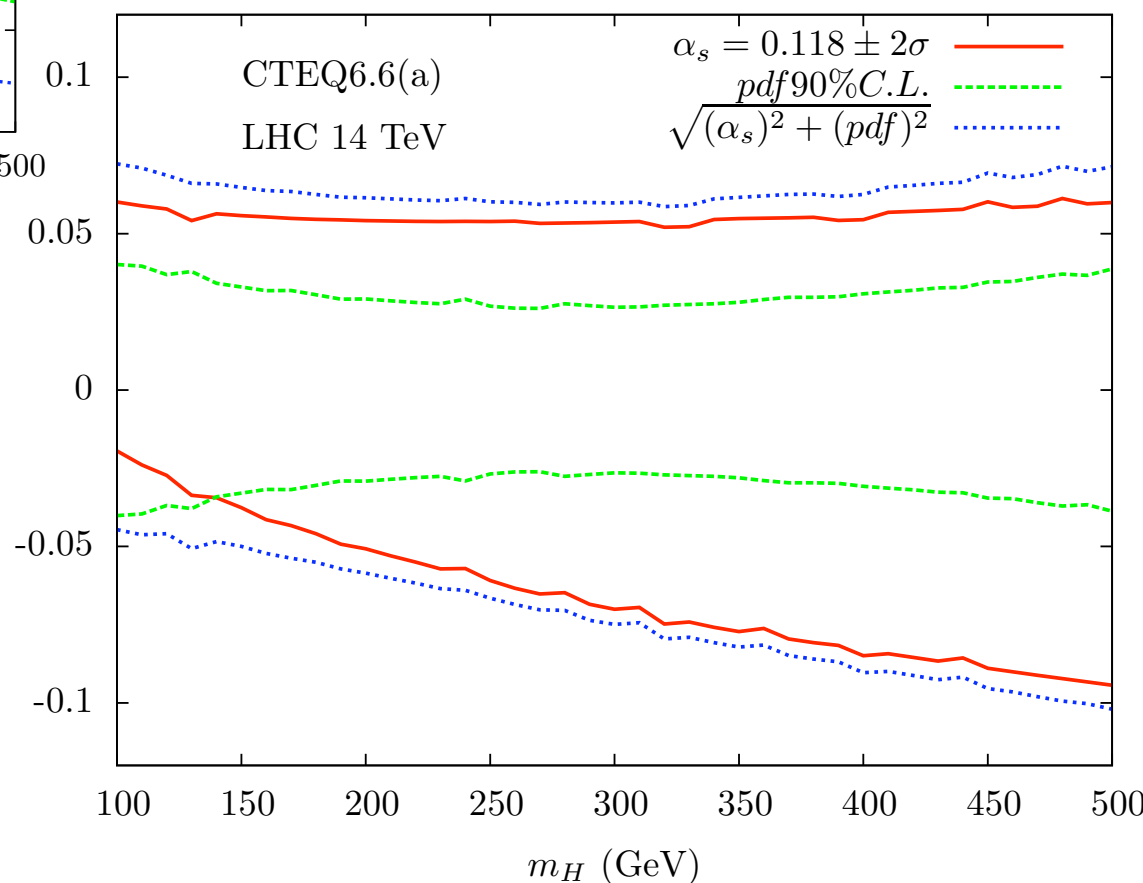
The assumptions are different

the overall size of the combined effect is similar

the estimate at small m_H is different

These larger uncertainty bands reduce the significance of the discrepancy of the two sets

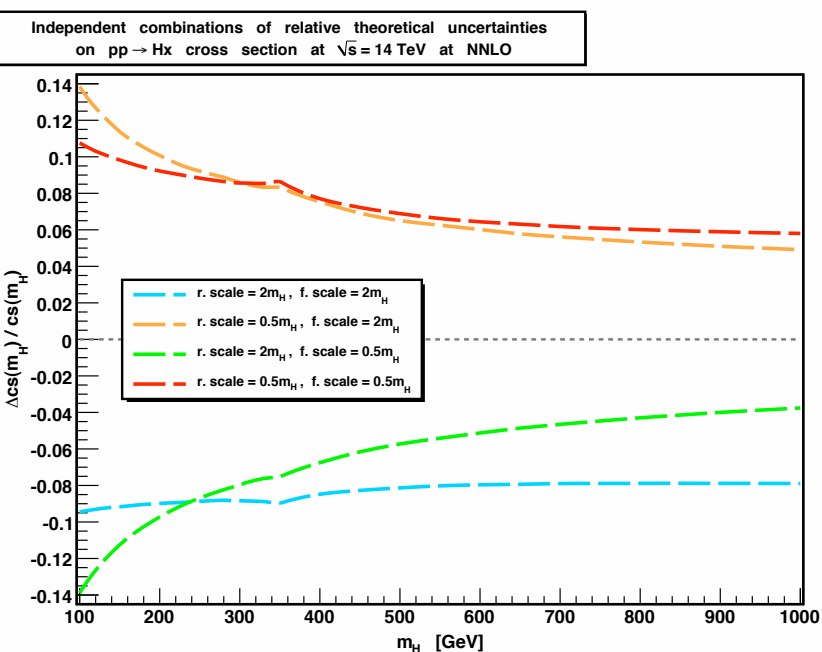
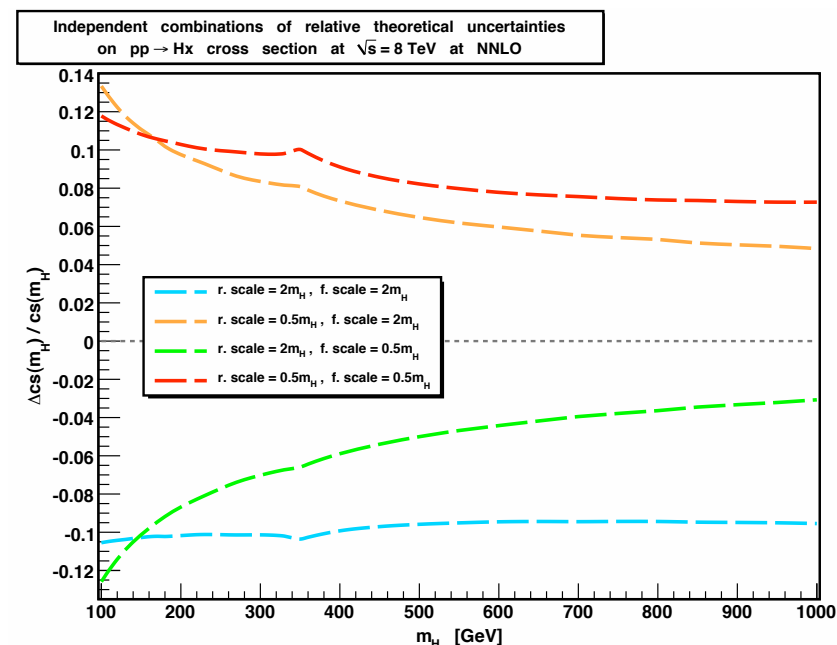
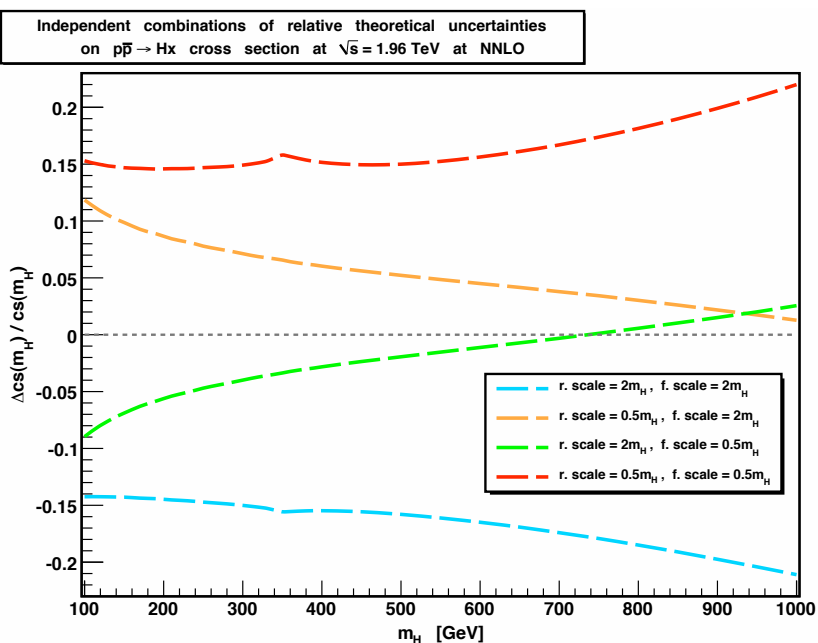
$\delta\sigma/\sigma$



Scale dependence (NNLO-QCD):

Independent variation of factorization and renormalization scales.

NNLO-QCD contributions in the infinite top mass limit added to the exact NLO-QCD

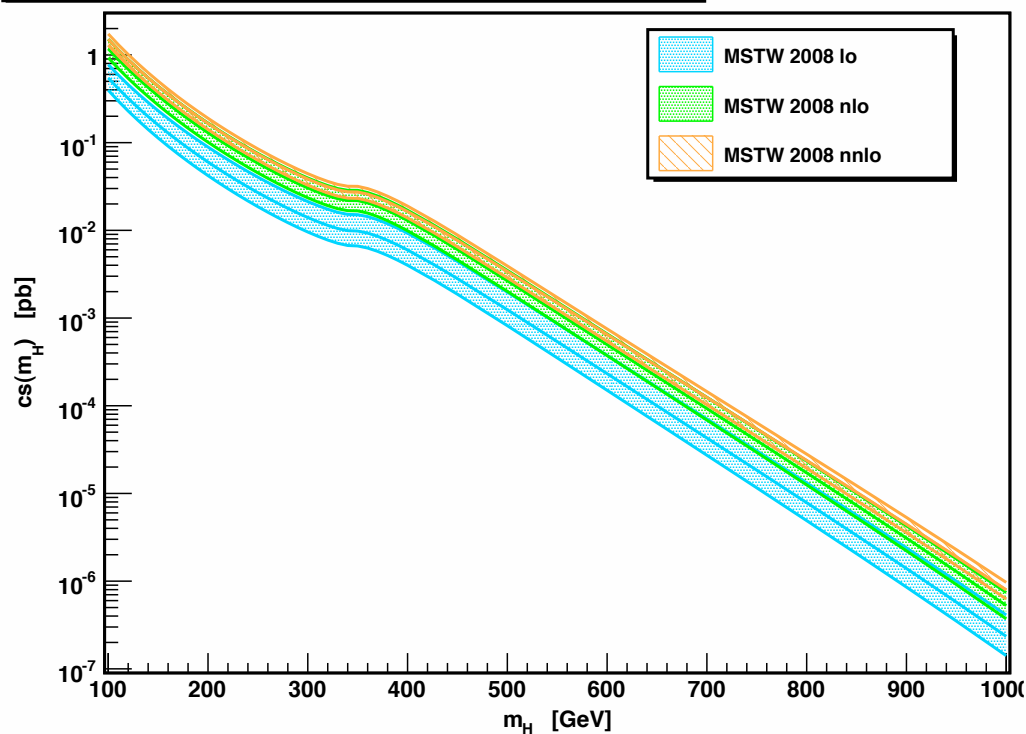


The lines show the 4 combinations
in units MH:

$$(\mu_{ren}, \mu_{fac}) = \\ (0.5, 0.5), (0.5, 2), (2, 0.5), (2, 2)$$

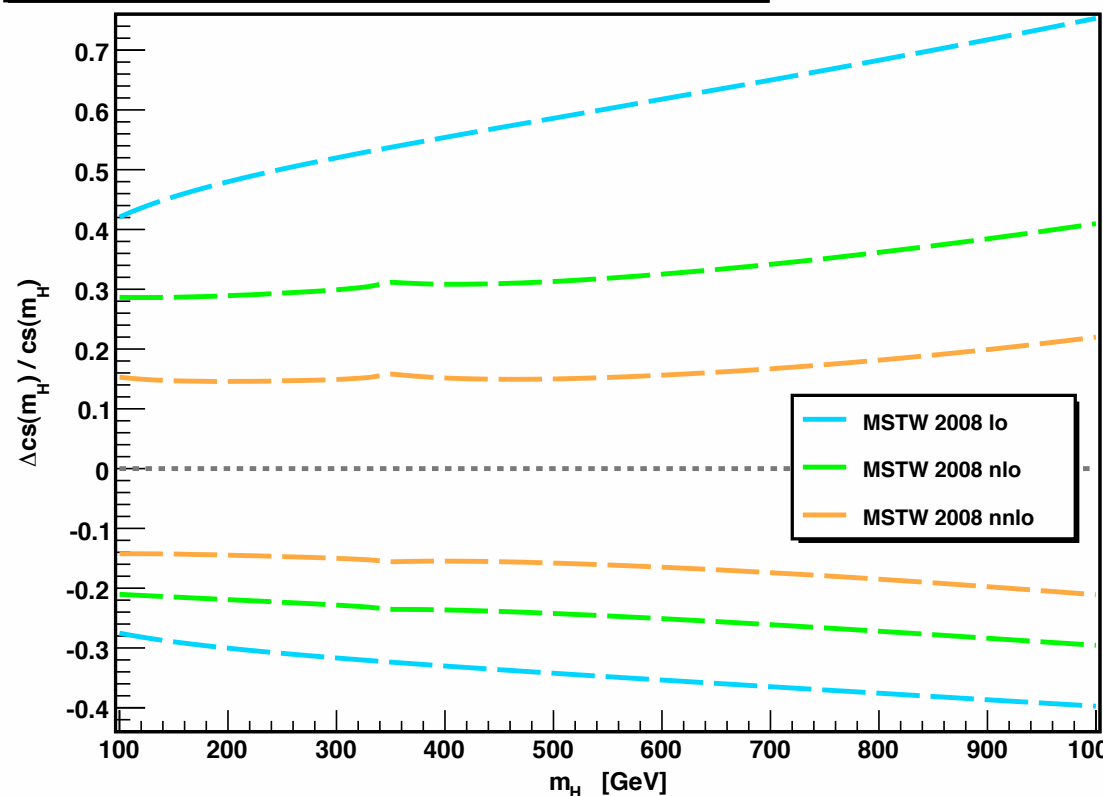
Scale dependence: Tevatron

$p\bar{p} \rightarrow Hx$ cross section at $\sqrt{s} = 1.96$ TeV at different orders
with theoretical uncertainty error bands



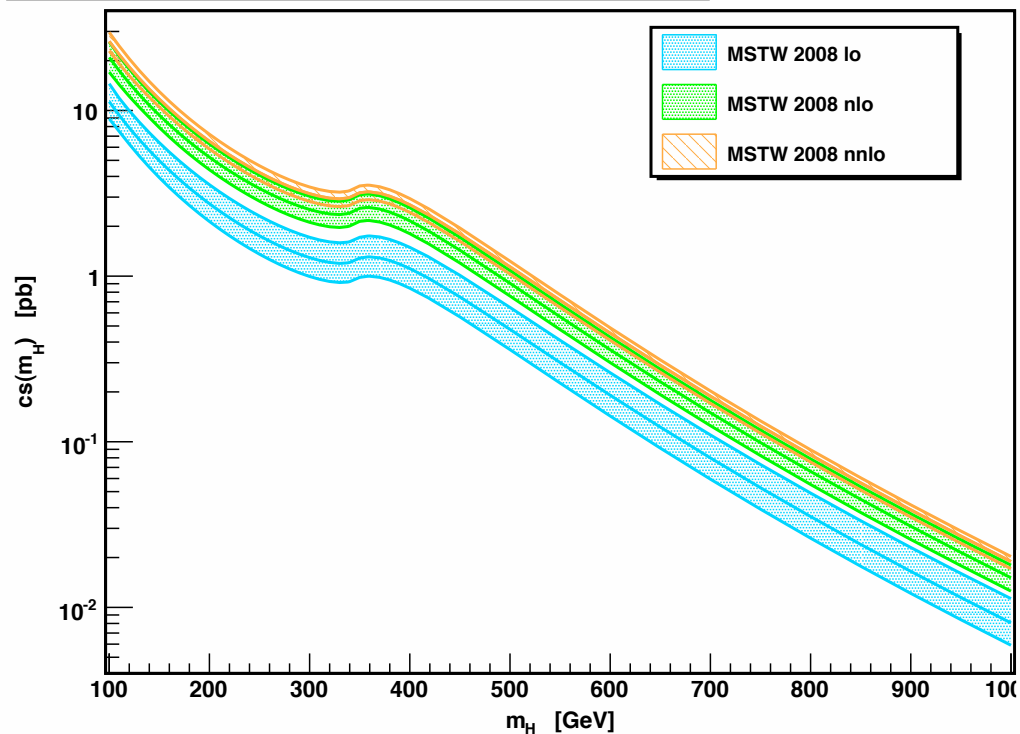
The bands are large
and have partial overlap

Relative theoretical uncertainties on $p\bar{p} \rightarrow Hx$ cross section
at $\sqrt{s} = 1.96$ TeV at different orders

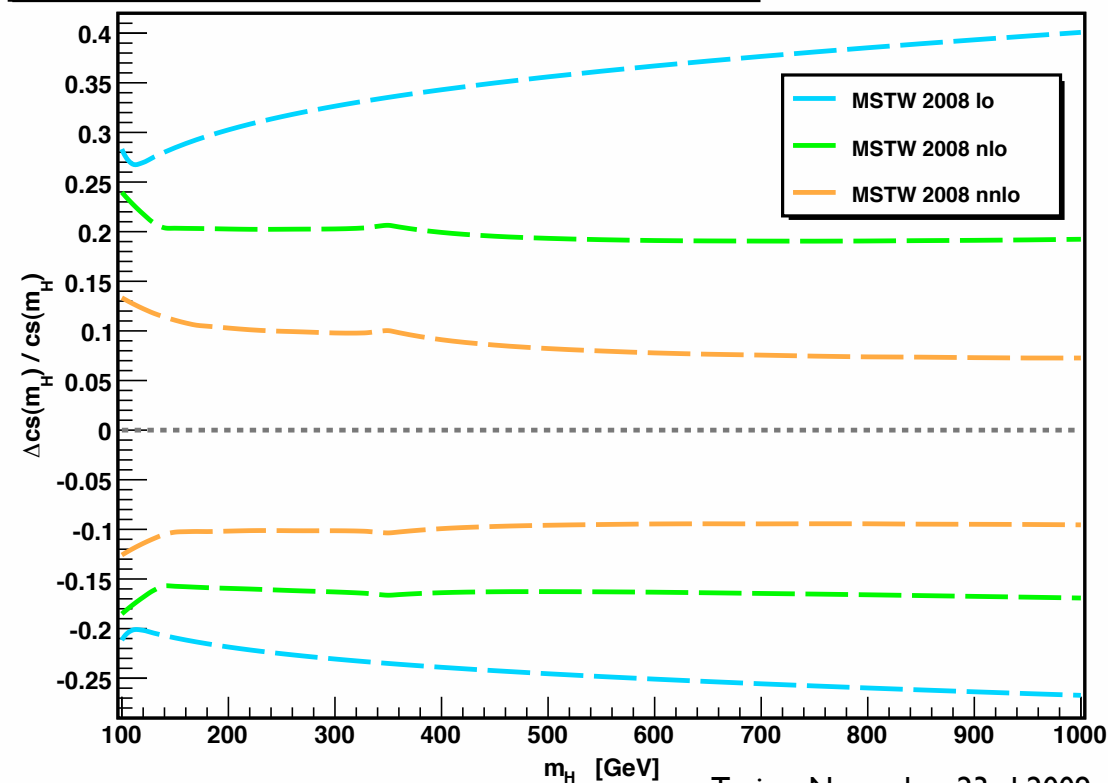


Scale dependence: LHC 8 TeV

$pp \rightarrow Hx$ cross section at $\sqrt{s} = 8$ TeV at different orders
with theoretical uncertainty error bands

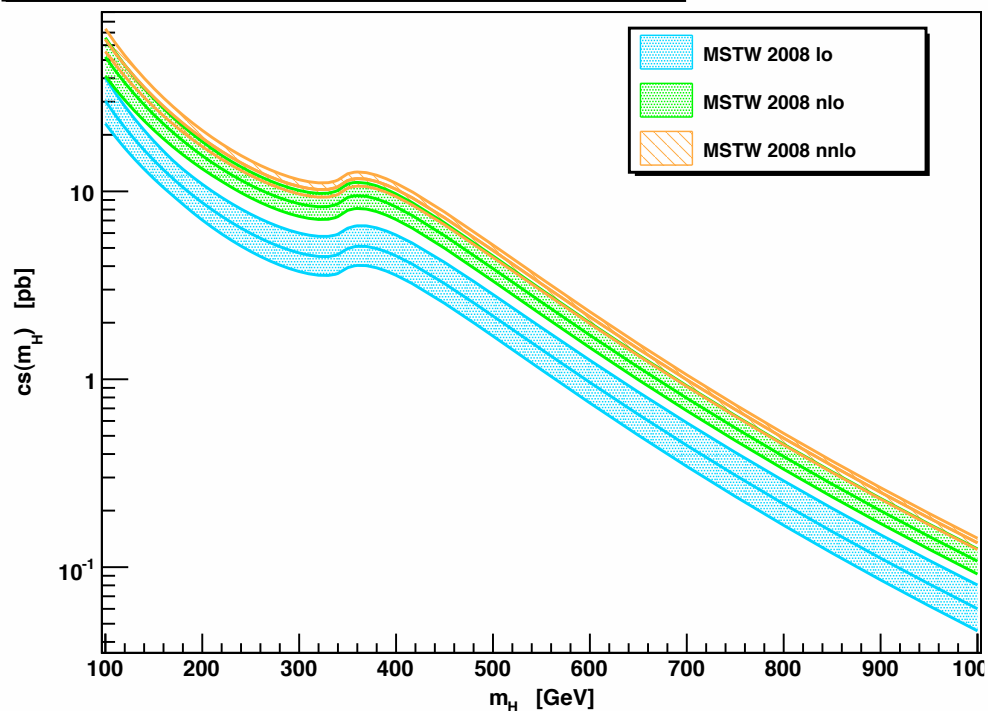


Relative theoretical uncertainties on $pp \rightarrow Hx$ cross section
at $\sqrt{s} = 8$ TeV at different orders



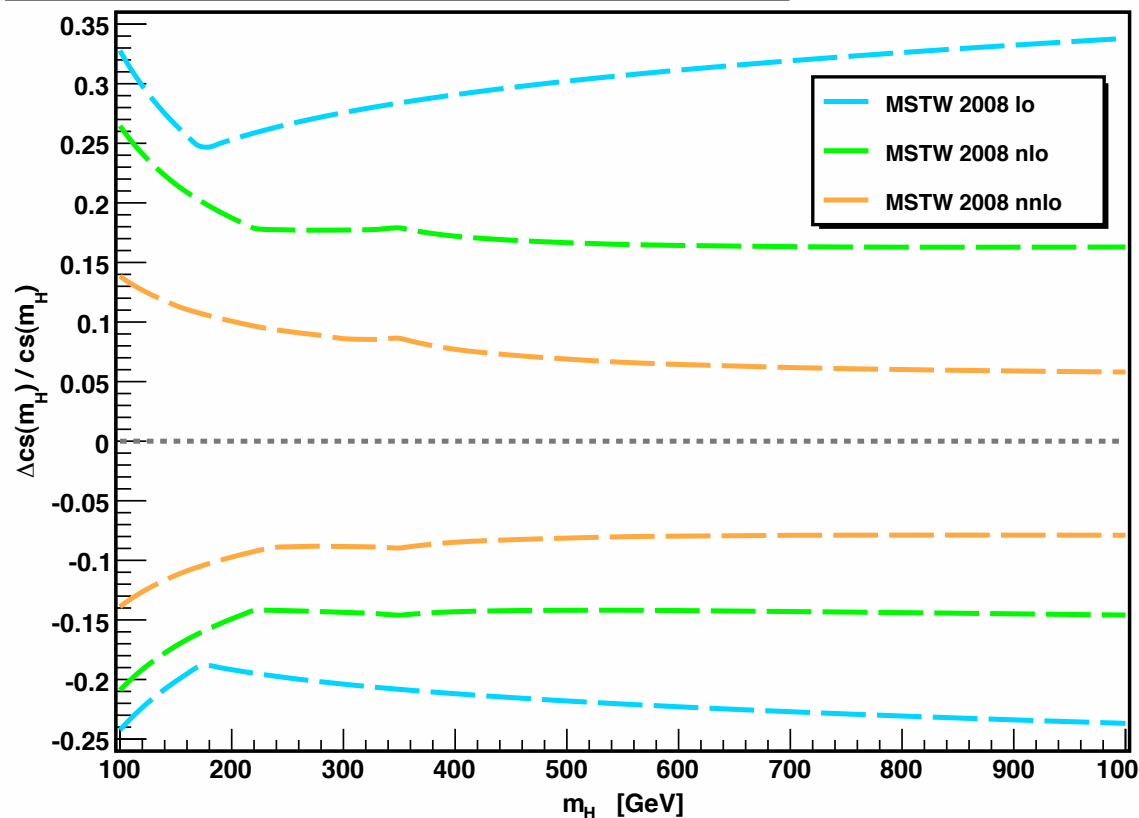
Scale dependence: LHC 14 TeV

$pp \rightarrow Hx$ cross section at $\sqrt{s} = 14$ TeV at different orders with theoretical uncertainty error bands



The bands are thinner than
at the Tevatron
but have smaller overlap

Relative theoretical uncertainties on $pp \rightarrow Hx$ cross section at $\sqrt{s} = 14$ TeV at different orders



Conclusions

- The experimental uncertainty on the *pdfs* affects the gluon fusion cross section at the few per cent level (the relevant x are well measured at HERA)
- Estimate of the size of the uncertainty similar for CTEQ, MSTW and NNPDF
- Not negligible differences of the central values between CTEQ6.6, MSTW2008 at NLO-QCD
- The role of α_s can partially reduce the significance of the discrepancy, enlarging the error bands
- Is the difference of the σ , at NLO-QCD, part of the *pdf* uncertainty?
- The size of the uncertainty bands due to variation of the scales are at the level of 6-15% at NNLO-QCD; non trivial shape of the bands
- The accurate prediction of the gluon fusion total cross section requires, at least, the use of exact NLO-QCD predictions with all quark flavors