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MSSM Higgs Physics

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1. Why Higgs?
2. The SM Higgs boson
3. The MSSM Higgs sector
4. SUSY Higgses at the LHC

1. Why Higgs?

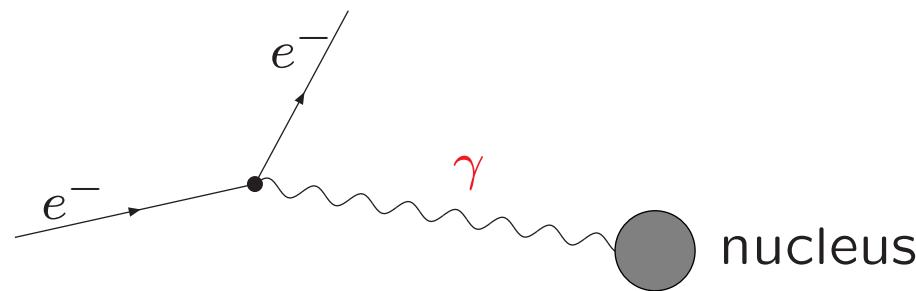
Standard Model (SM) of the electroweak and strong interaction

SM: Quantum field theory \Rightarrow interaction: exchange of field quanta

Construction principle of the SM: **gauge invariance**

Example: Quantum electro-dynamics (QED)

field quanta: photon A_μ

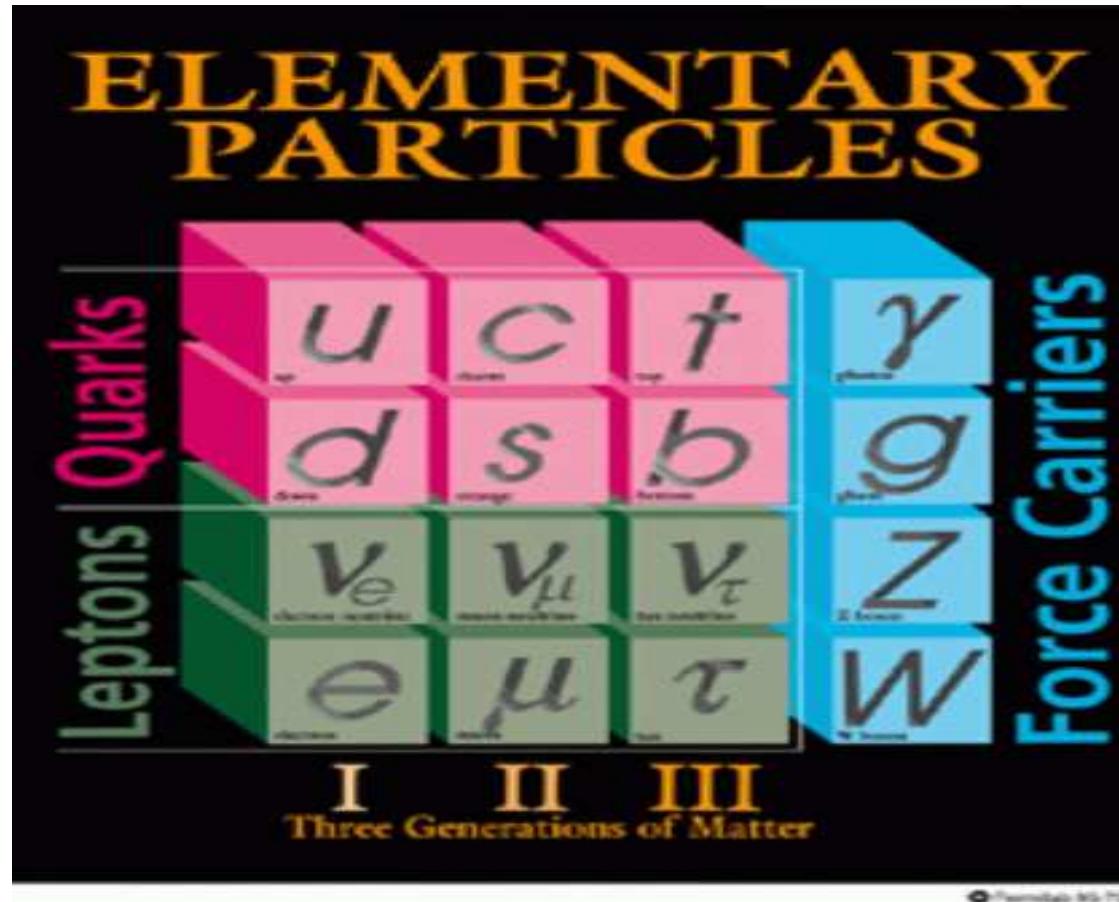


\mathcal{L}_{QED} invariant under **gauge transformation**:

$$\Psi \rightarrow e^{ie\lambda(x)}\Psi, A_\mu \rightarrow A_\mu + \partial_\mu\lambda(x)$$

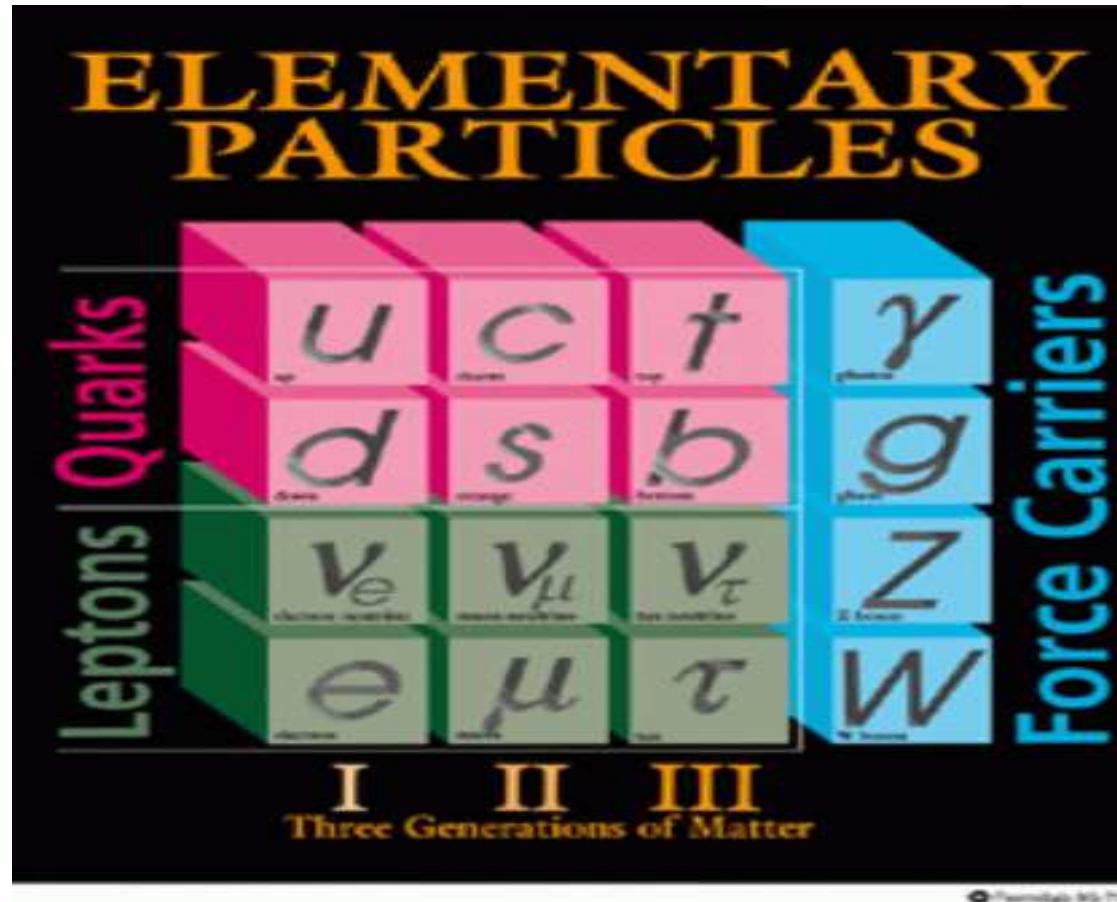
mass term for photon: $m^2 A^\mu A_\mu$ not gauge invariant
 $\Rightarrow A_\mu$ is massless gauge field

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

⇒ but theory predicts massless gauge bosons . . .

Problem:

Gauge fields Z, W^+, W^- are **massive**

explicite mass terms in the Lagrangian \Leftrightarrow breaking of gauge invariance

Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

Higgs sector in the Standard Model:

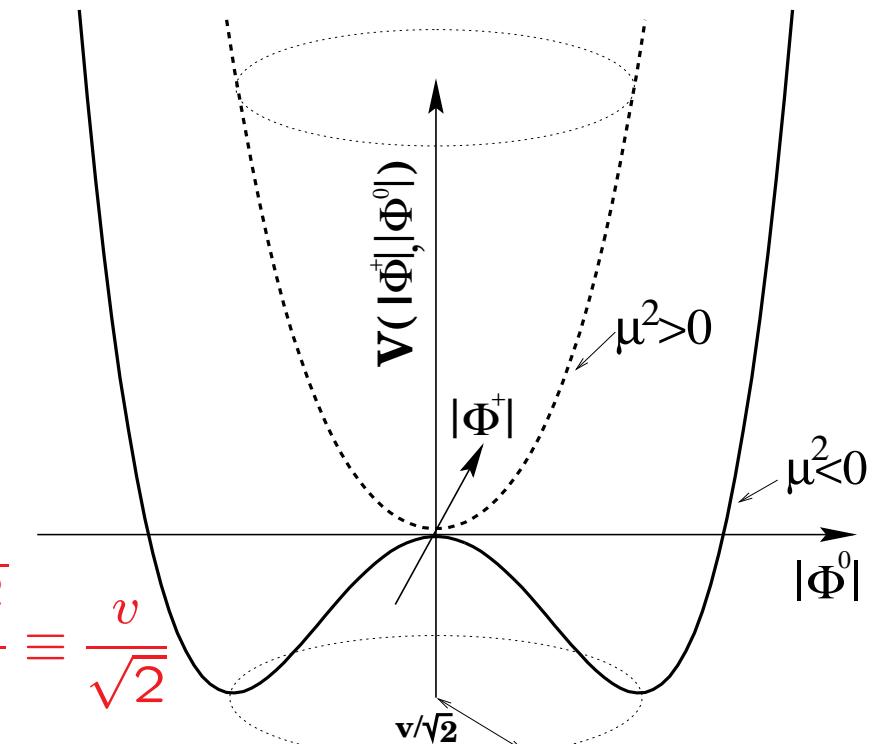
$$\text{Scalar SU(2) doublet: } \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Higgs potential:

$$V(\phi) = \mu^2 |\Phi^\dagger \Phi| + \lambda |\Phi^\dagger \Phi|^2, \quad \lambda > 0$$

$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle \Phi_0 \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$



$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad (\text{unitary gauge})$$

H : elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R - g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

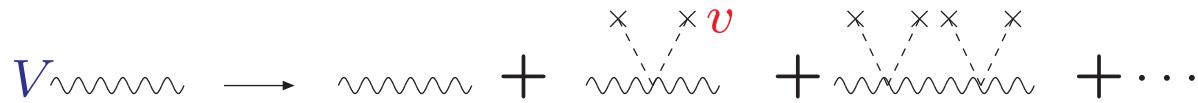
with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{I} \vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^\dagger \qquad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

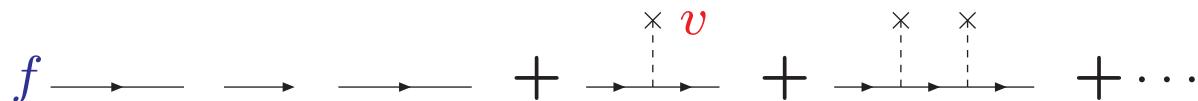
⇒ mass terms for gauge bosons and fermions

1.) $VV\Phi\Phi$ coupling:



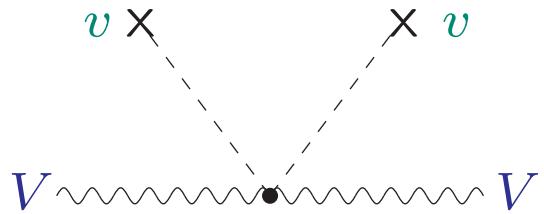
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} + \sum_j \frac{1}{q^2} \left[\left(\frac{gv}{\sqrt{2}} \right)^2 \frac{1}{q^2} \right]^j = \frac{1}{q^2 - M^2} : M^2 = g^2 \frac{v^2}{2}$$

2.) fermion mass terms: Yukawa couplings:

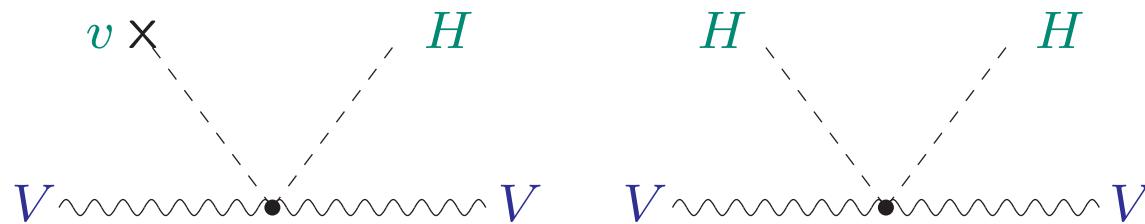


$$\frac{1}{q} \rightarrow \frac{1}{q} + \sum_j \frac{1}{q} \left[\frac{g_f v}{\sqrt{2}} \frac{1}{q} \right]^j = \frac{1}{q - m_f} : m_f = g_f \frac{v}{\sqrt{2}}$$

1.) $VV\Phi\Phi$ coupling:



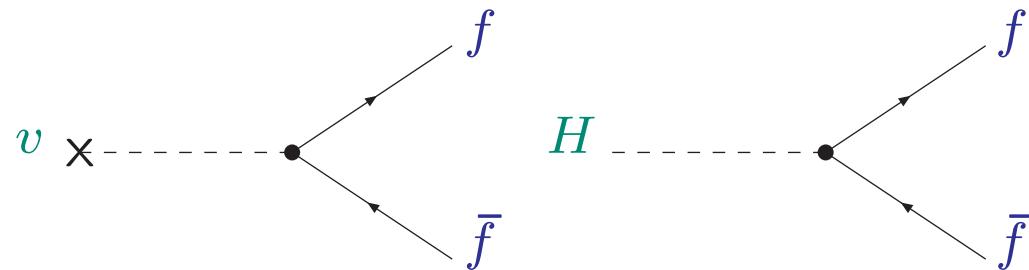
⇒ VV mass terms: $g_2^2 v^2 / 2 \equiv M_W^2$, $(g_1^2 + g_2^2)v^2 / 2 \equiv M_Z^2$



⇒ triple/quartic couplings to gauge bosons

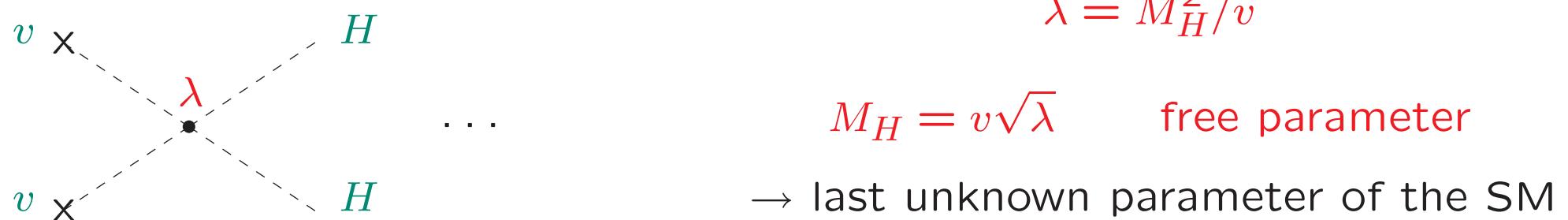
⇒ coupling \propto masses

2.) fermion mass terms: Yukawa couplings



$$m_f = v g_f \Rightarrow \text{coupling} \propto \text{masses}$$

3.) mass of the Higgs boson: self coupling



⇒ establish Higgs mechanism ≡ find the Higgs ⊕ measure its couplings

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \rightarrow W_L W_L$

$$\mathcal{M}_V = \text{Diagram showing two incoming } W \text{ bosons scattering into } \gamma, Z + \text{Diagram showing two incoming } W \text{ bosons scattering into } \gamma, Z + \text{Diagram showing two incoming } W \text{ bosons scattering into } W = -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1) \text{ for } E \rightarrow \infty$$

⇒ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$$\mathcal{M}_S = \text{Diagram showing two incoming } W \text{ bosons scattering into } H + \text{Diagram showing two incoming } W \text{ bosons scattering into } H = g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \text{ for } E \rightarrow \infty$$

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4} (g_{WWH}^2 - g^2 M_W^2) + \dots$$

⇒ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g M_W$$

2. The SM Higgs boson

1.) Decay to fermions:

coupling:

$$g_{f\bar{f}H} = [\sqrt{2} G_\mu]^{1/2} m_f$$

decay width:

$$\Gamma(H \rightarrow f\bar{f}) = N_c \frac{G_\mu M_H}{4\sqrt{2}\pi} m_f^2(M_H^2) \left(1 - 4 \frac{m_f^2}{M_H^2}\right)^{3/2}$$

with N_c = number of colors

Bulk of QCD corrections for decays to quarks are mapped into

$$m_q^2(\text{pole}) \rightarrow m_q^2(M_H^2)$$

Dominant decay process: $H \rightarrow b\bar{b}$

2.) Decay to heavy gauge bosons ($V = W, Z$):

coupling:

$$g_{V V H} = 2 \left[\sqrt{2} G_\mu \right]^{1/2} M_V^2$$

on-shell decay width ($M_H > 2M_V$):

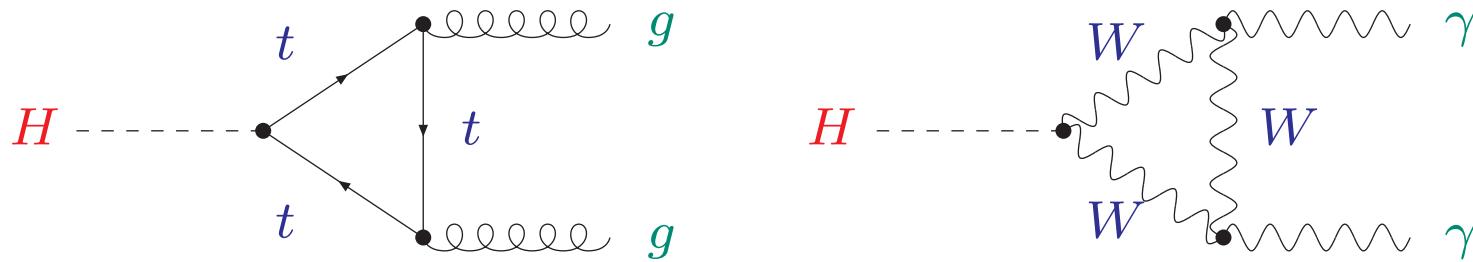
$$\Gamma(H \rightarrow VV) = \delta_V \frac{G_\mu M_H^3}{16 \sqrt{2} \pi} \left(1 - 4 \frac{M_V^2}{M_H^2} + 12 \frac{M_V^4}{M_H^4} \right) \left(1 - 4 \frac{M_V^2}{M_H^2} \right)^{1/2}$$

with $\delta_{W,Z} = 2, 1$

off-shell decay width ($M_H < 2M_V$):

$$\Gamma(H \rightarrow VV^*) = \delta'_V \frac{3G_\mu^2 M_H}{16 \pi^3} M_V^4 \times \text{Integral}$$

3.) Decay to massless gauge bosons (gg , $\gamma\gamma$):



$$\Gamma(H \rightarrow gg) = \frac{G_\mu \alpha_s^2(M_H^2) M_H^3}{36 \sqrt{2} \pi^3} \left[1 + C \frac{\alpha_s(\mu)}{\pi} \right]$$

via the top quark loop with

$$C = \frac{215}{12} - \frac{23}{6} \log \left(\frac{\mu^2}{M_H^2} \right) + \mathcal{O}(\alpha_s)$$

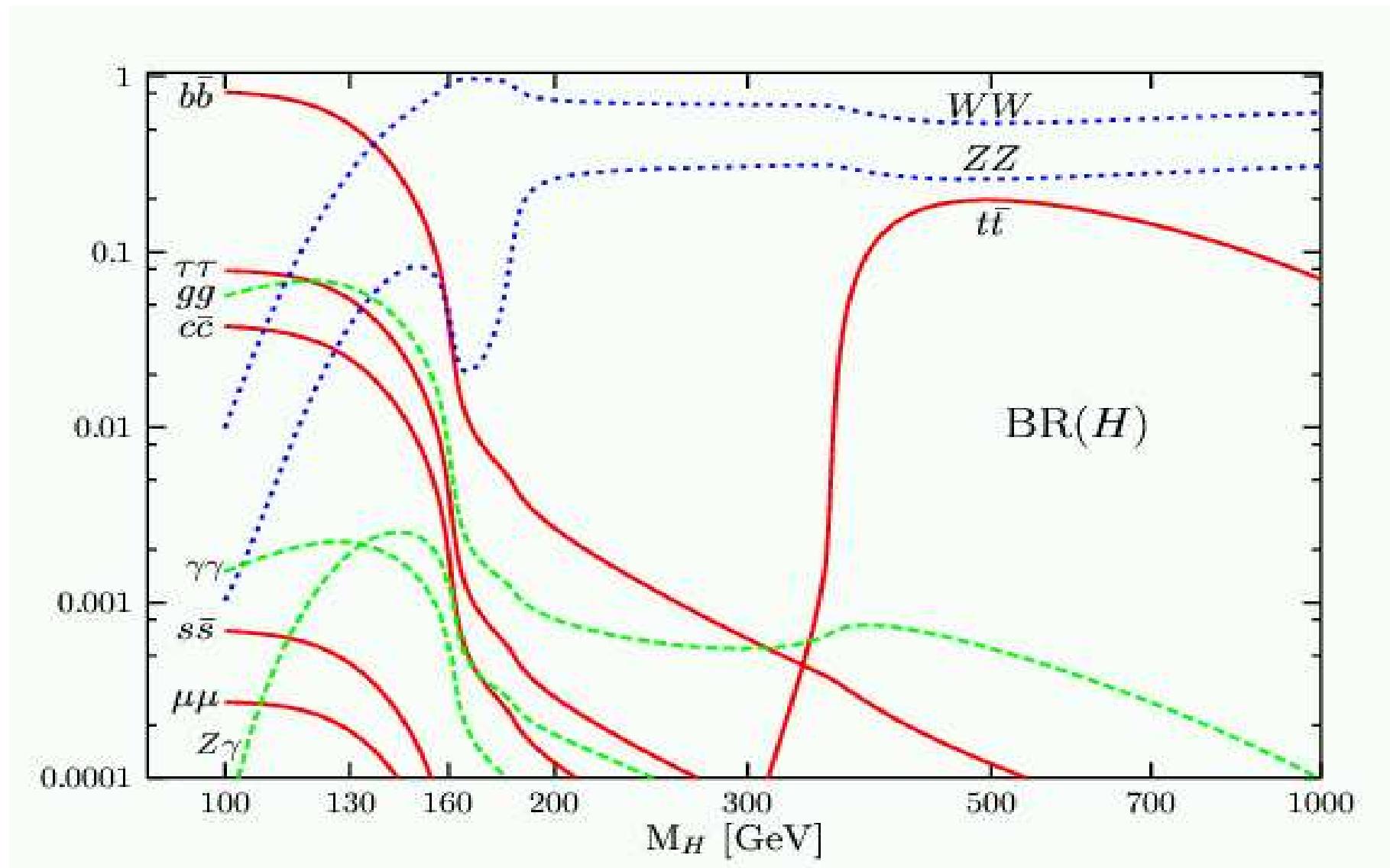
\Rightarrow huge QCD corrections

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_\mu \alpha^2 M_H^3}{128 \sqrt{2} \pi^3} \left| \frac{4}{3} e_t^2 - 7 \right|^2$$

via the top quark and W boson loop

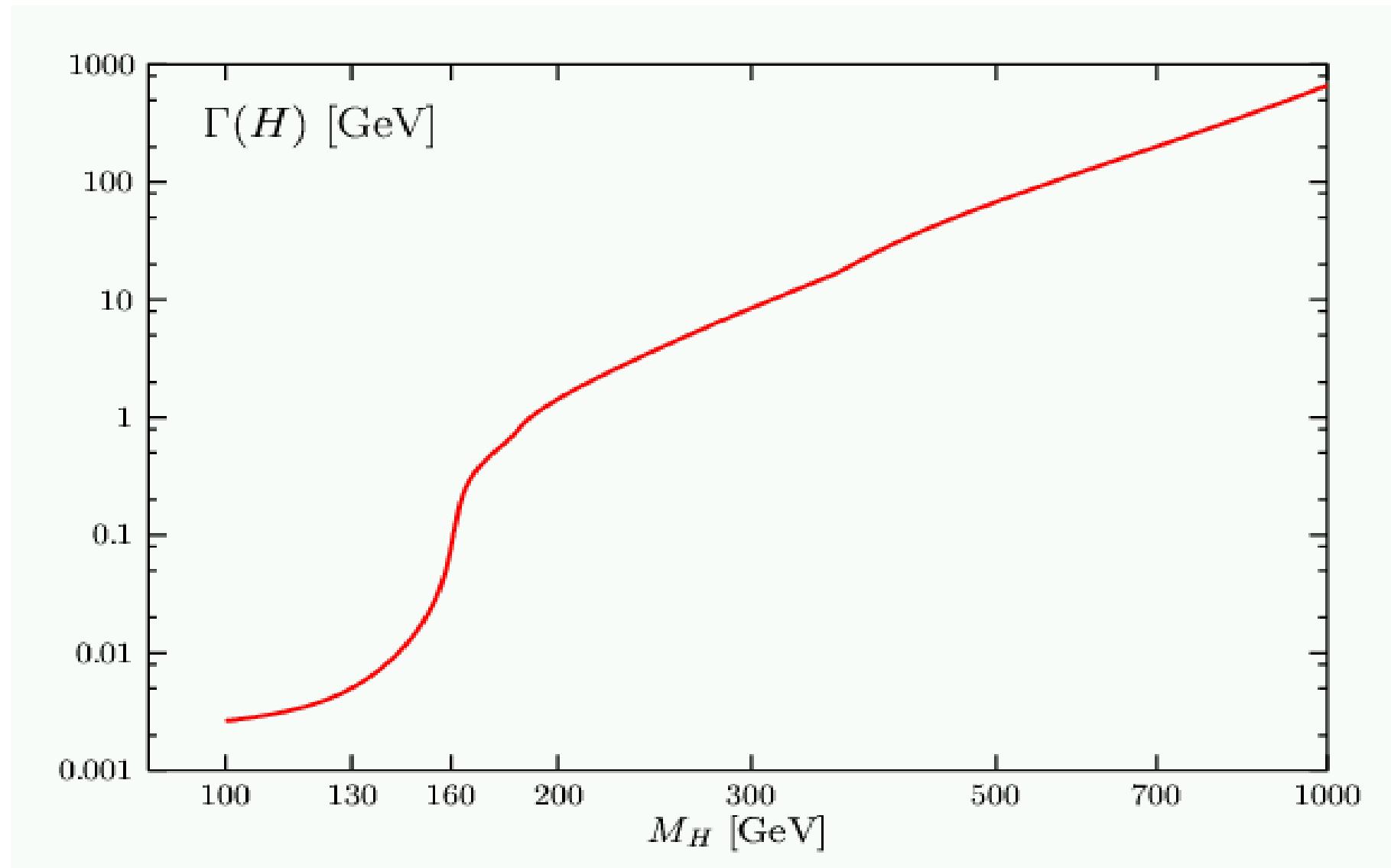
Overview of the branching ratios:

[taken from [hep-ph/0503172](#)]



The total SM Higgs boson width:

[taken from [hep-ph/0503172](#)]



Global fit to all SM data:

[LEPEWWG '09]

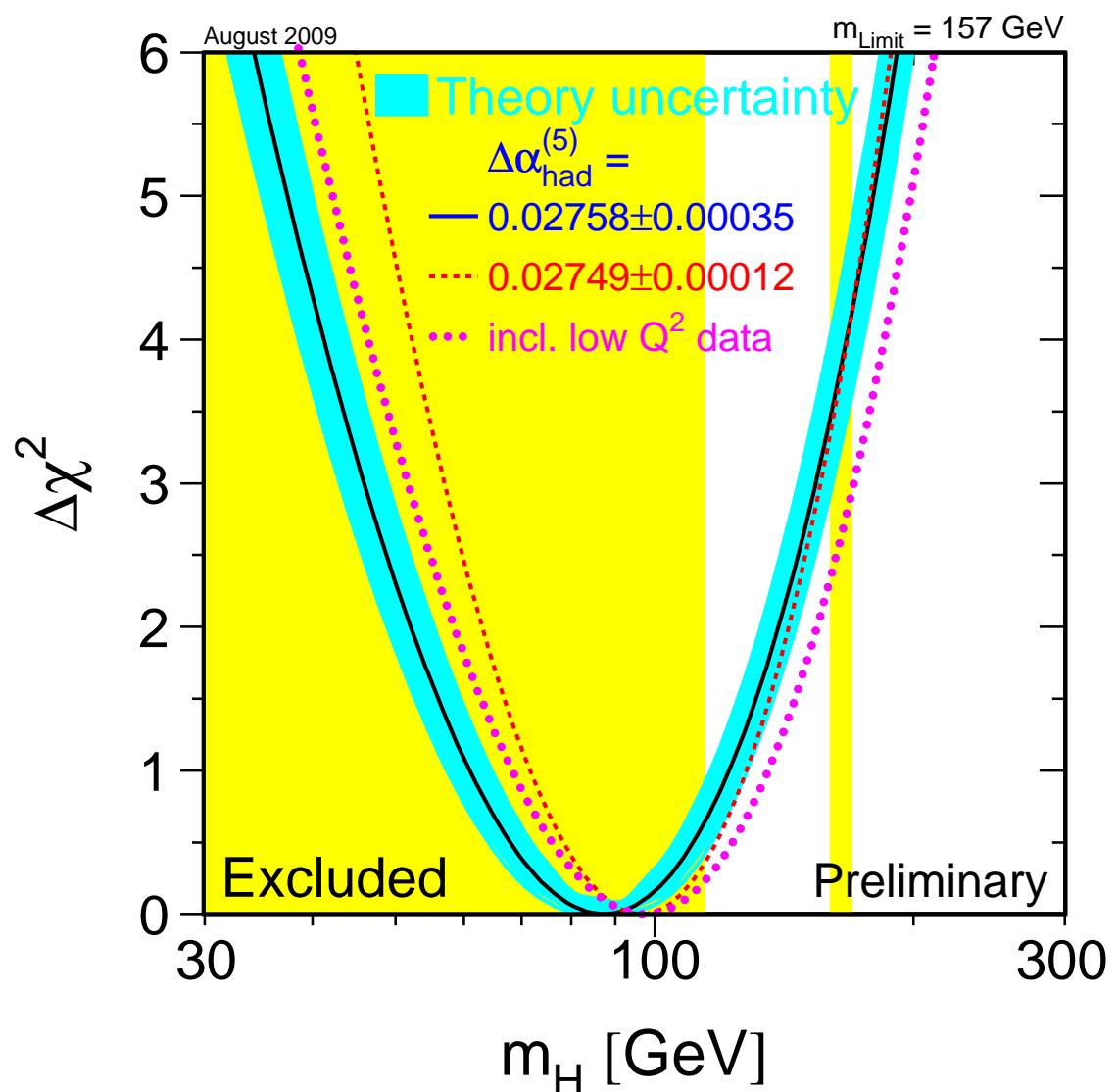
$$\Rightarrow M_H = 87^{+35}_{-26} \text{ GeV}$$

$M_H < 157$ GeV, 95% C.L.

Assumption for the fit:

SM incl. Higgs boson

\Rightarrow no confirmation of
Higgs mechanism



\Rightarrow Higgs boson seems to be light, $M_H \lesssim 160$ GeV

Global fit to all SM data incl. direct searches:

[*GFitter* '09]

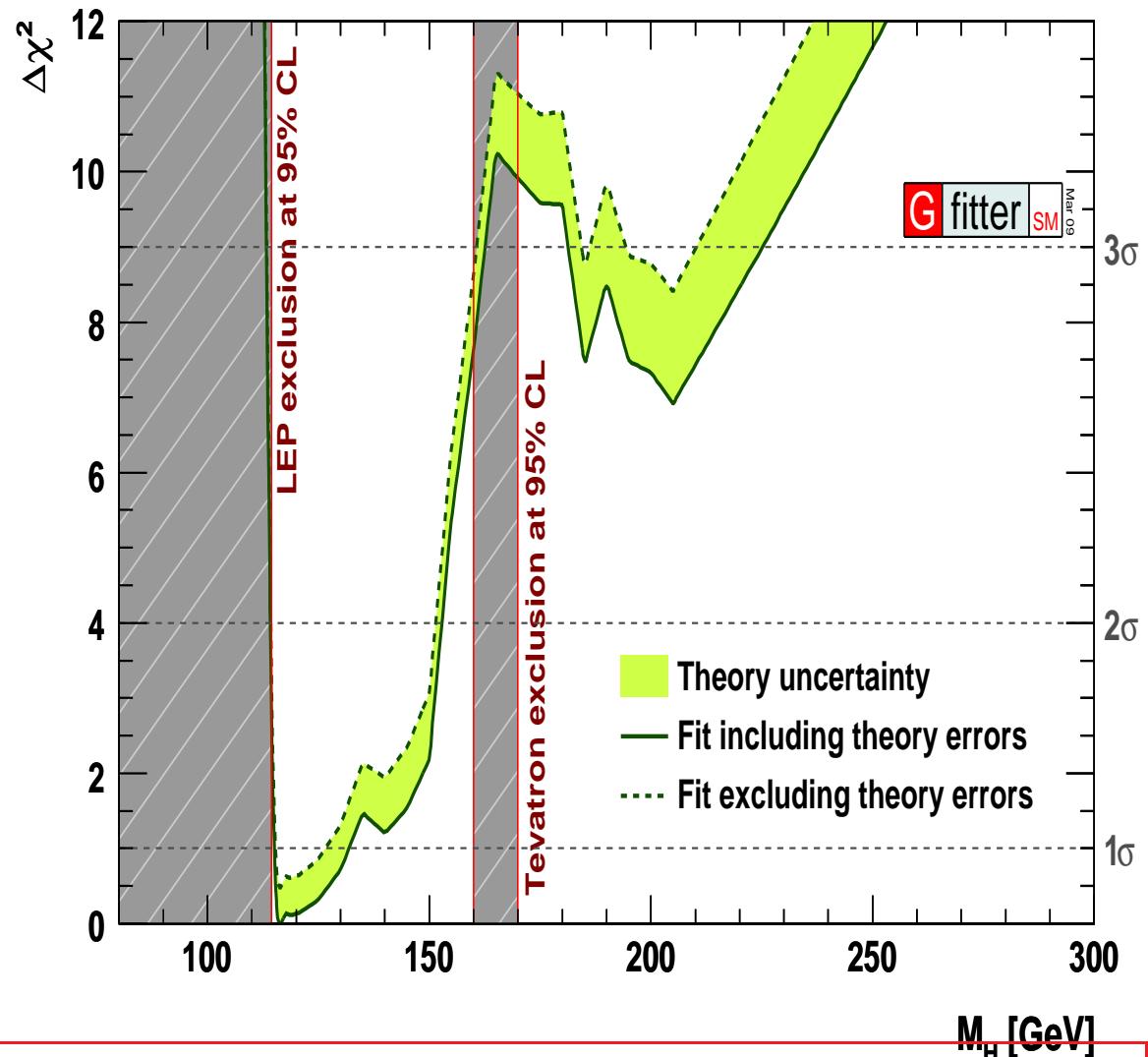
$$\Rightarrow M_H = 116.4^{+18.3}_{-1.4} \text{ GeV}$$

$$M_H < 152 \text{ GeV, 95% C.L.}$$

Assumption for the fit:

SM incl. Higgs boson

\Rightarrow no confirmation of
Higgs mechanism



\Rightarrow Higgs boson seems to be light, $M_H \lesssim 150 \text{ GeV}$

3. The MSSM Higgs sector

Supersymmetry (SUSY) : Symmetry between

Bosons \leftrightarrow Fermions

$$Q \text{ |Fermion} \rangle \rightarrow \text{|Boson} \rangle$$

$$Q \text{ |Boson} \rangle \rightarrow \text{|Fermion} \rangle$$

Simplified examples:

$$Q \text{ |top, } t \rangle \rightarrow \text{|scalar top, } \tilde{t} \rangle$$

$$Q \text{ |gluon, } g \rangle \rightarrow \text{|gluino, } \tilde{g} \rangle$$

\Rightarrow each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}}$ \Rightarrow SUSY is broken . . .

. . . via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $M_{\text{SUSY}} = \mathcal{O}(1 \text{ TeV})$

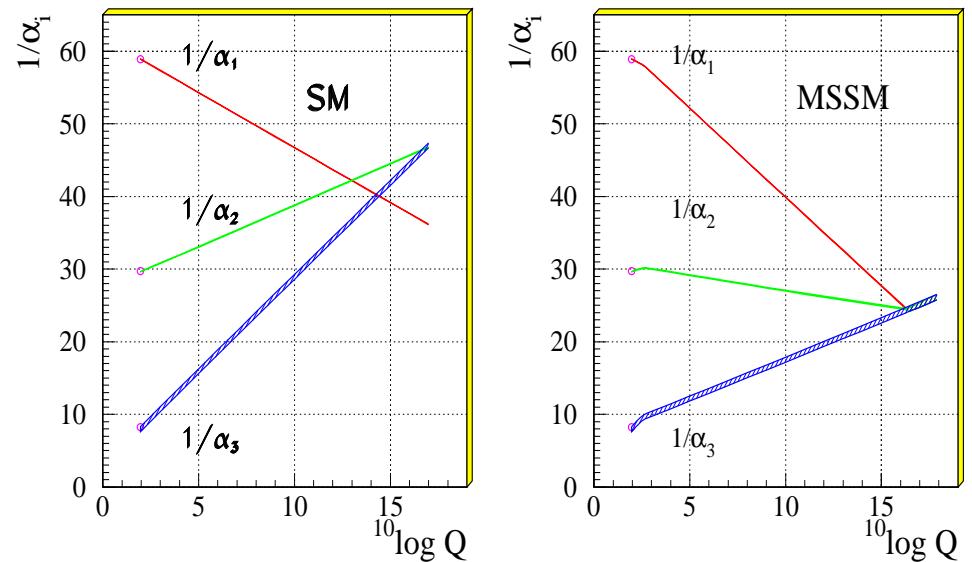
Five reasons as a SUSY motivation

The SM is in a pretty good shape.

Why MSSM? (Is it worth to double the particle spectrum?)

- 1.) Stability of the Higgs mass
against higher-order corr.
- 2.) Unification of gauge couplings:
Not possible in the SM, but in
the **MSSM** (although it was **not**
designed for it.)
- 3.) Spontaneous symmetry breaking
via Higgs mechanism is
automatic in **SUSY GUTs**
- 4.) SUSY provides CDM candidate
- 5.) ...

Unification of the Coupling Constants
in the SM and the minimal MSSM



[Amaldi, de Boer, Fürstenau '92]

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$[u, d, c, s, t, b]_{L,R}$	$[e, \mu, \tau]_{L,R}$	$[\nu_{e,\mu,\tau}]_L$	Spin $\frac{1}{2}$
$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R}$	$[\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R}$	$[\tilde{\nu}_{e,\mu,\tau}]_L$	Spin 0
g	$\underbrace{W^\pm, H^\pm}_{\text{}}$	$\underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{}}$	Spin 1 / Spin 0
\tilde{g}	$\tilde{\chi}_{1,2}^\pm$	$\tilde{\chi}_{1,2,3,4}^0$	Spin $\frac{1}{2}$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

\tilde{t}/\tilde{b} sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^*\tan\beta$):

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}$$

$$\mathcal{M}_{\tilde{b}}^2 = \begin{pmatrix} M_{\tilde{b}_L}^2 + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M_{\tilde{b}_R}^2 + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m_{\tilde{b}_1}^2 & 0 \\ 0 & m_{\tilde{b}_2}^2 \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ - \tilde{t}/\tilde{b} couplings

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

\Rightarrow relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

The Higgs sector in SUSY

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L \Phi d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \Phi_c u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi_c = i\sigma_2 \Phi^\dagger, \quad \Phi \rightarrow \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \rightarrow \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L \Phi^\dagger$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i , not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow H_d (\equiv H_1)$ and $H_u (\equiv H_2)$ needed to give masses
to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies,
quadratic divergences

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: h^0, H^0, A^0, H^\pm

Goldstone bosons: G^0, G^\pm

Input parameters: (to be determined experimentally)

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$$

Enlarged Higgs sector: Two Higgs doublets with \mathcal{CP} violation

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} e^{i\xi}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states: h^0, H^0, A^0, H^\pm

2 \mathcal{CP} -violating phases: $\xi, \arg(m_{12}) \Rightarrow$ can be set/rotated to zero

Input parameters: (to be determined experimentally)

$$\tan \beta = \frac{v_2}{v_1}, \quad M_{H^\pm}^2$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \quad \tan(2\alpha) = \tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Three Goldstone bosons (as in SM): G^0, G^\pm

→ longitudinal components of W^\pm, Z

⇒ Five physical states: h^0, H^0, A^0, H^\pm

h, H : neutral, \mathcal{CP} -even, A^0 : neutral, \mathcal{CP} -odd, H^\pm : charged

Gauge-boson masses:

$$M_W^2 = \frac{1}{2} g'^2 (v_1^2 + v_2^2), \quad M_Z^2 = \frac{1}{2} (g^2 + g'^2) (v_1^2 + v_2^2), \quad M_\gamma = 0$$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_1, v_2, m_1, m_2, m_{12}$$

relation for $M_W^2, M_Z^2 \Rightarrow 1$ condition

minimization of V w.r.t. neutral Higgs fields $H_1^1, H_2^2 \Rightarrow 2$ conditions

\Rightarrow only two free parameters remain in V , conventionally chosen as

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$$

$\Rightarrow m_h, m_H, \text{mixing angle } \alpha, m_{H^\pm}$: no free parameters, can be predicted

In lowest order:

$$m_{H^\pm}^2 = M_A^2 + M_W^2$$

Predictions for m_h , m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} =$$
$$\begin{pmatrix} M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(M_A^2 + M_Z^2) \sin \beta \cos \beta \\ -(M_A^2 + M_Z^2) \sin \beta \cos \beta & M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix}$$

$\Downarrow \leftarrow$ Diagonalization, α

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h , m_H :

$$m_{H,h}^2 = \frac{1}{2} \left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

\Rightarrow Light Higgs boson h required in SUSY

Measurement of m_h , Higgs couplings

\Rightarrow test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{hVV} = \sin(\beta - \alpha) g_{HVV}^{\text{SM}}, \quad V = W^\pm, Z$$

$$g_{HVV} = \cos(\beta - \alpha) g_{HVV}^{\text{SM}}$$

$$g_{hAZ} = \cos(\beta - \alpha) \frac{g'}{2 \cos \theta_W}$$

$$g_{hb\bar{b}}, g_{h\tau^+\tau^-} = -\frac{\sin \alpha}{\cos \beta} g_{Hb\bar{b}, H\tau^+\tau^-}^{\text{SM}}$$

$$g_{ht\bar{t}} = \frac{\cos \alpha}{\sin \beta} g_{Ht\bar{t}}^{\text{SM}}$$

$$g_{Ab\bar{b}}, g_{A\tau^+\tau^-} = \gamma_5 \tan \beta g_{Hb\bar{b}}^{\text{SM}}$$

$\Rightarrow g_{hVV} \leq g_{HVV}^{\text{SM}}$, $g_{hVV}, g_{HVV}, g_{hAZ}$ cannot all be small

$g_{hb\bar{b}}, g_{h\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

The decoupling limit:

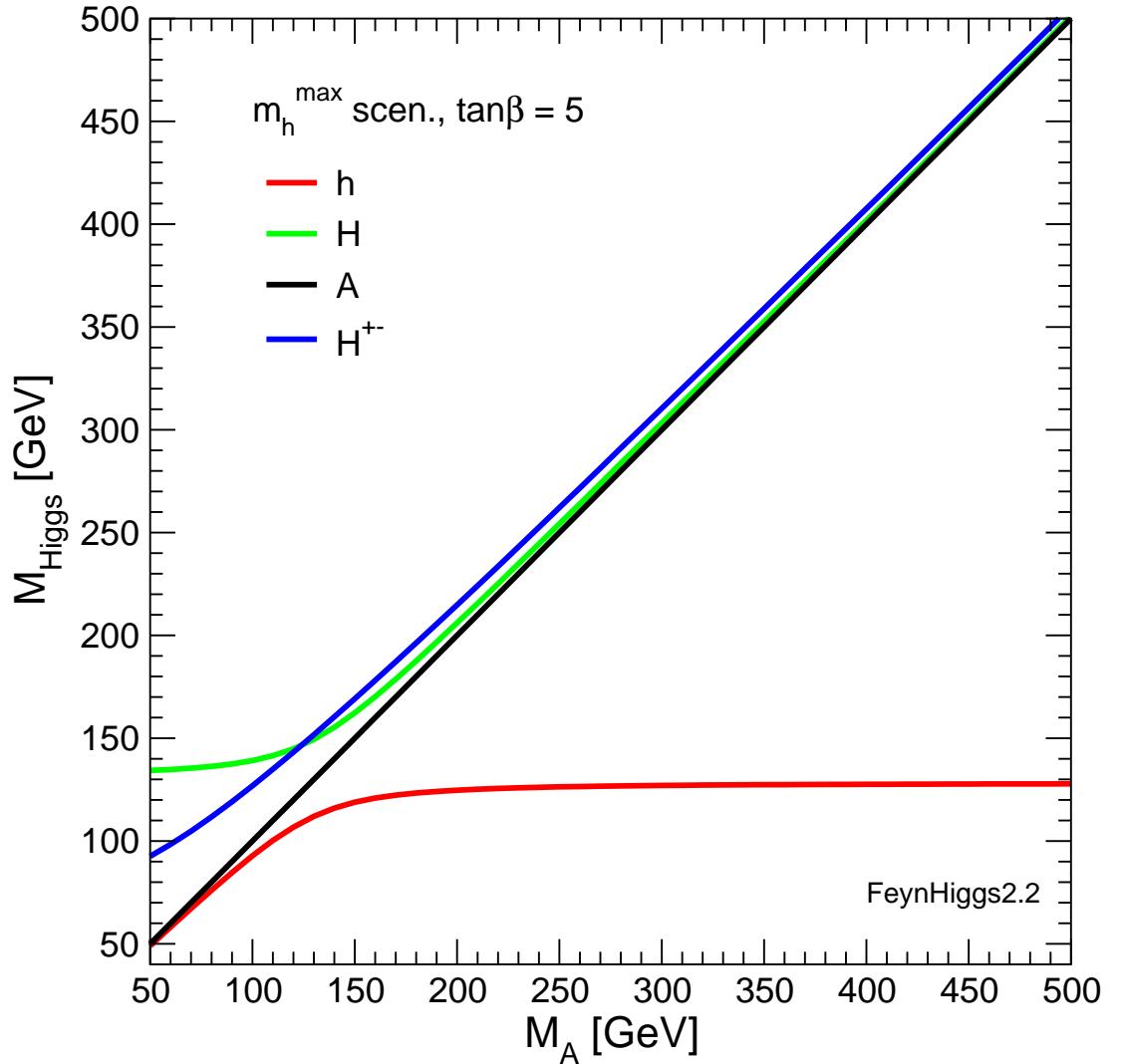
For $M_A \gtrsim 150$ GeV:

The lightest MSSM Higgs
is SM-like

The heavy MSSM Higgses:

$M_A \approx M_H \approx M_{H^\pm}$

of course there are exceptions . . .



The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h :

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{e m_t}{2 M_W s_W}, \frac{e m_t^2}{M_W s_W}, \dots$

⇒ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete one-loop and ‘almost complete’ two-loop result and leading three-loop corrections exist.

Upper bound on M_h in the MSSM:

“Unconstrained MSSM”:

M_A , $\tan \beta$, 5 parameters in \tilde{t} – \tilde{b} sector, μ , $m_{\tilde{g}}$, M_2

$$M_h \lesssim 135 \text{ GeV}$$

for $m_t = 173.1 \pm 1.3 \text{ GeV}$

(including theoretical uncertainties from unknown higher orders)
⇒ observable at the LHC

Obtained with:

FeynHiggs

[S.H., W. Hollik, G. Weiglein '98 – '02]

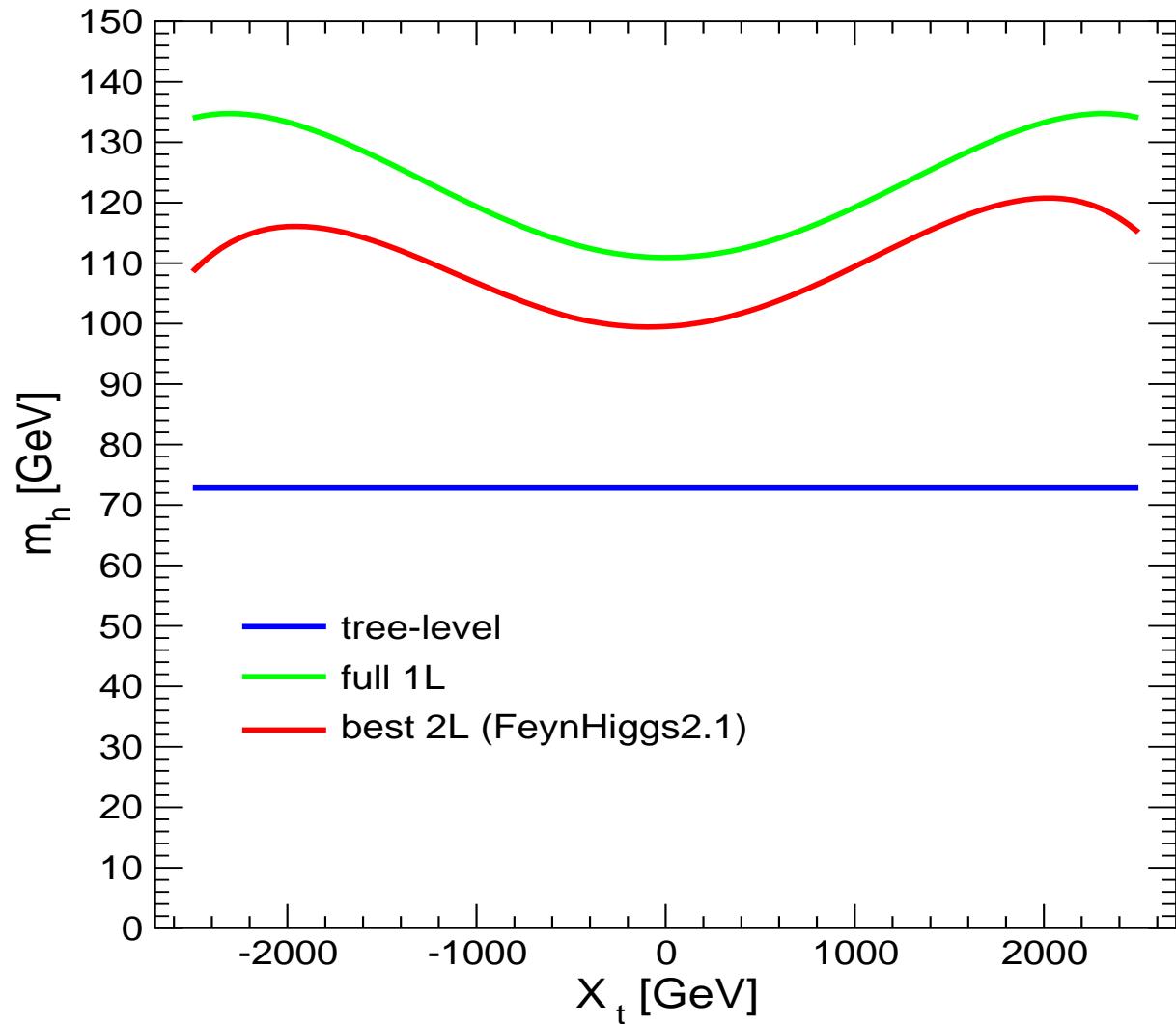
[T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '03 – '09]

www.feynhiggs.de

→ all Higgs masses, couplings, BRs (easy to link, easy to use :-)

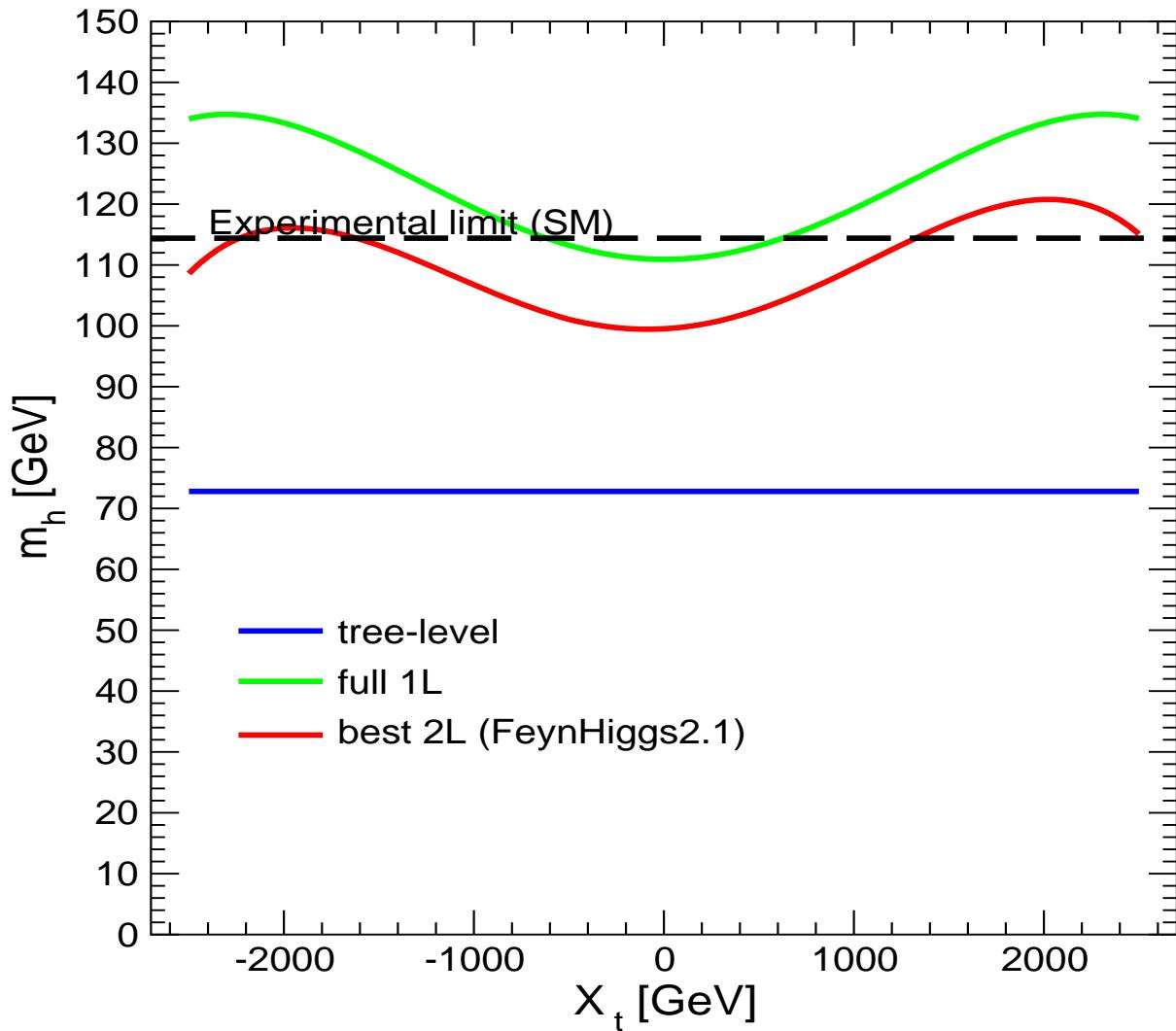
Effects of the two-loop corrections to the lightest Higgs mass:

Example for one set of MSSM parameters



Effects of the two-loop corrections to the lightest Higgs mass:

Example for one set of MSSM parameters



Comparison with
experimental limits
⇒ strong impact on
bound on SUSY parameters

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

- From unknown higher-order corrections:

$$\Rightarrow \Delta M_h \approx 3 \text{ GeV}$$

- From uncertainties in input parameters

$$m_t, \dots, M_A, \tan \beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \dots$$

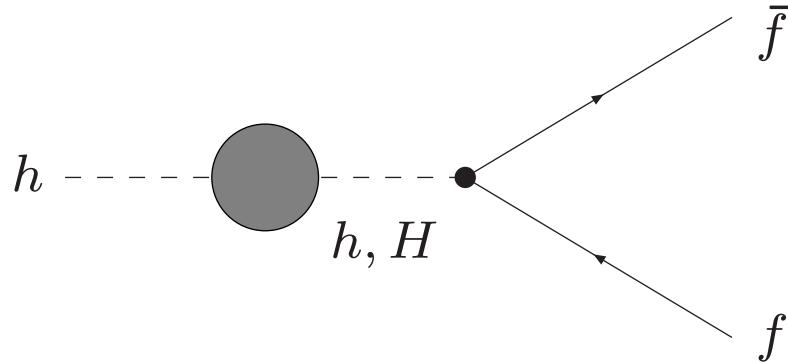
$$\Delta m_t \approx 1 \text{ GeV} \Rightarrow \Delta M_h \approx 1 \text{ GeV}$$

Higgs couplings, production cross sections

⇒ also affected by large SUSY loop corrections

... see below

$h f \bar{f}$ coupling:



$$A(h \rightarrow f \bar{f}) = \sqrt{Z_h} \left(\Gamma_h - \frac{\hat{\Sigma}_{hH}(M_h^2)}{M_h^2 - m_H^2 + \hat{\Sigma}_{HH}(M_h^2)} \Gamma_H \right)$$

⇒ Effective $h f \bar{f}$ coupling can vanish for large $\hat{\Sigma}_{hH}$

Gluino vertex corrections to $h \rightarrow q\bar{q}$:

⇒ ratio $\Gamma(h \rightarrow \tau^+ \tau^-)/\Gamma(h \rightarrow b\bar{b})$ can significantly differ from SM value for large $\tan \beta$

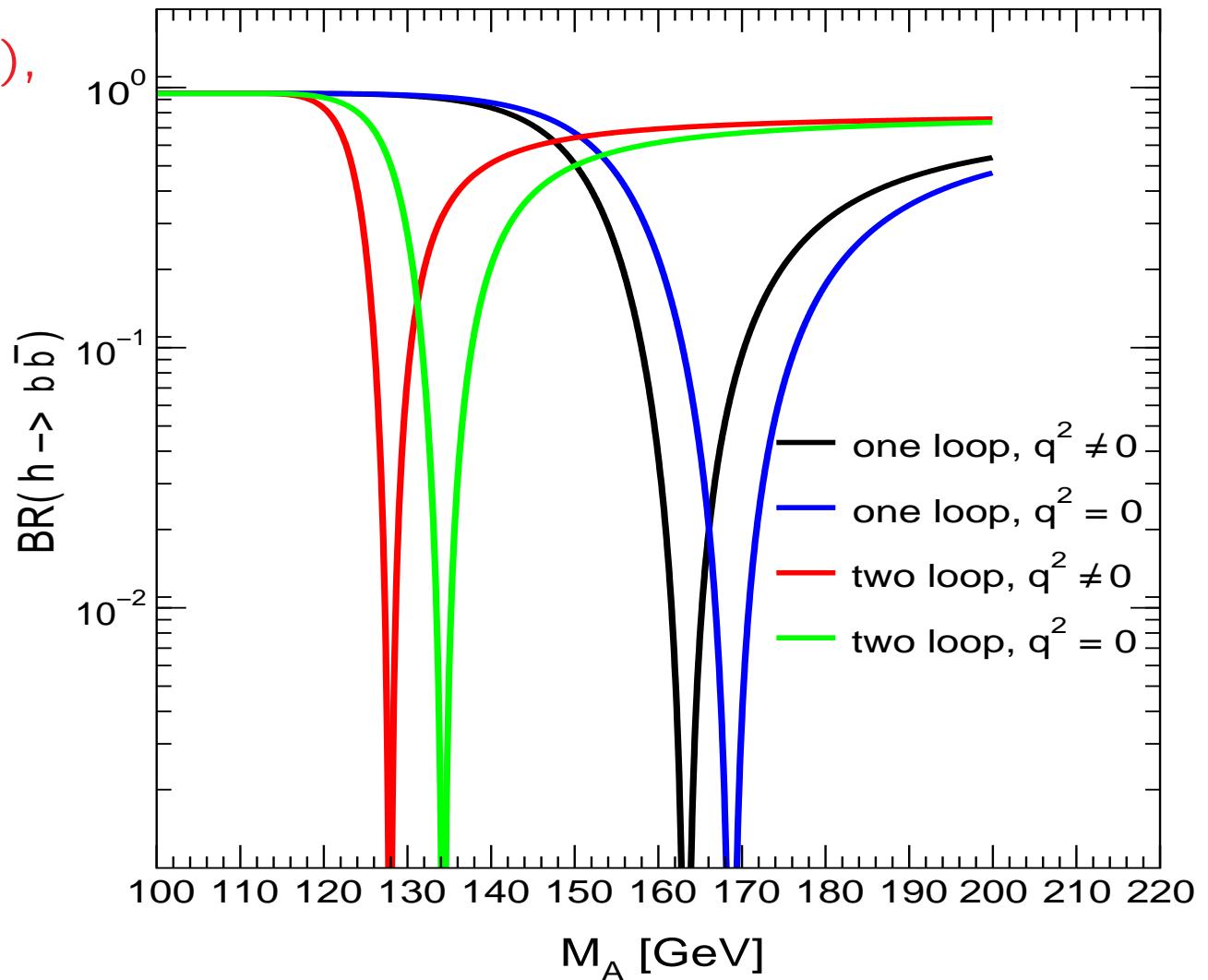
Effective $h f \bar{f}$ coupling can go to zero for large $\hat{\Sigma}_{hH}$

⇒ “Pathological regions”

[W. Loinaz, J. Wells '98] [M. Carena, S. Mrenna, C. Wagner '99]

⇒ Suppression of $\text{BR}(h \rightarrow b\bar{b})$,
 $\text{BR}(h \rightarrow \tau\tau)$, ...

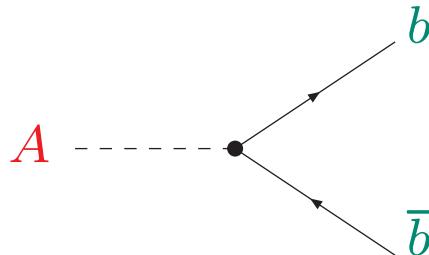
[S.H., W. Hollik, G. Weiglein '00]



The heavy MSSM Higgs bosons

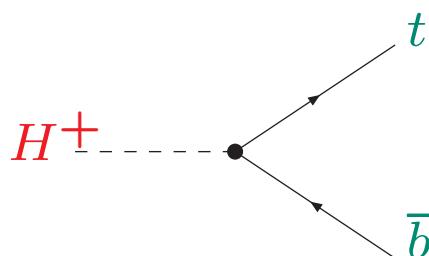
Differences compared to the SM Higgs:

Additional enhancement factors compared to the SM case:



$$y_b \rightarrow y_b \frac{\tan \beta}{1 + \Delta_b}$$

At large $\tan \beta$: either $H \approx A$ or $h \approx A$



$$y_b \frac{\tan \beta}{1 + \Delta_b}$$

$$\begin{aligned} \Delta_b &= \frac{2\alpha_s}{3\pi} m_{\tilde{g}} \mu \tan \beta \times I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) \\ &+ \frac{\alpha_t}{4\pi} A_t \mu \tan \beta \times I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu) \end{aligned}$$

⇒ other parameters enter ⇒ strong μ dependence

Effects of complex phases:

Propagator/Mass matrix at tree-level without \mathcal{CPV} :

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
(→ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ ($i, j = h, H$) : renormalized Higgs self-energies

\mathcal{CP} -even fields can mix

⇒ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2$ ($i = 1, 2$): $\mathcal{M}^2 = M^2 - iM\Gamma$

The Higgs sector of the cMSSM at the loop-level:

Complex parameters enter via loop corrections:

- μ : Higgsino mass parameter
- $A_{t,b,\tau}$: trilinear couplings $\Rightarrow X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot \beta, \tan \beta\}$ complex
- $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)
- M_3 : gluino mass parameter

\Rightarrow can induce \mathcal{CP} -violating effects

Result:

$$(A, H, h) \rightarrow (h_3, h_2, h_1)$$

with

$$M_{h_3} > M_{h_2} > M_{h_1}$$

\Rightarrow strong changes in Higgs couplings to SM gauge bosons and fermions

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_A^2 & 0 & 0 \\ 0 & q^2 - m_H^2 & 0 \\ 0 & 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
 (→ Feynman-diagrammatic approach):

$$M_{hHA}^2(q^2) = \begin{pmatrix} q^2 - m_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

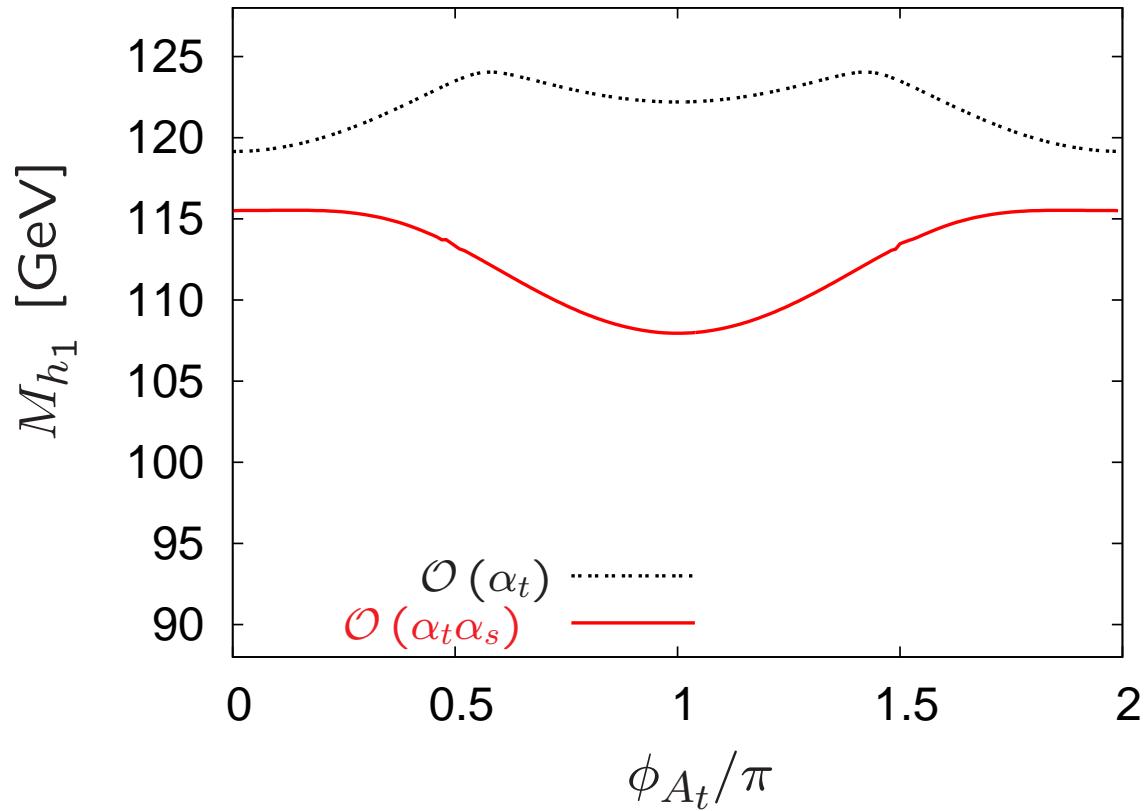
$\hat{\Sigma}_{ij}(q^2)$ ($i, j = h, H, A$) : renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \Rightarrow \mathcal{CP}\text{V}$, \mathcal{CP} -even and \mathcal{CP} -odd fields can mix

⇒ complex roots of $\det(M_{hHA}^2(q^2))$: $\mathcal{M}_{h_i}^2$ ($i = 1, 2, 3$): $\mathcal{M}^2 = M^2 - iM\Gamma$

M_{h_1} as a function of ϕ_{A_t} :

[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]



$M_{\text{SUSY}} = 1000 \text{ GeV}$

$|A_t| = 2000 \text{ GeV}$

$\tan \beta = 10$

$M_{H^\pm} = 150 \text{ GeV}$

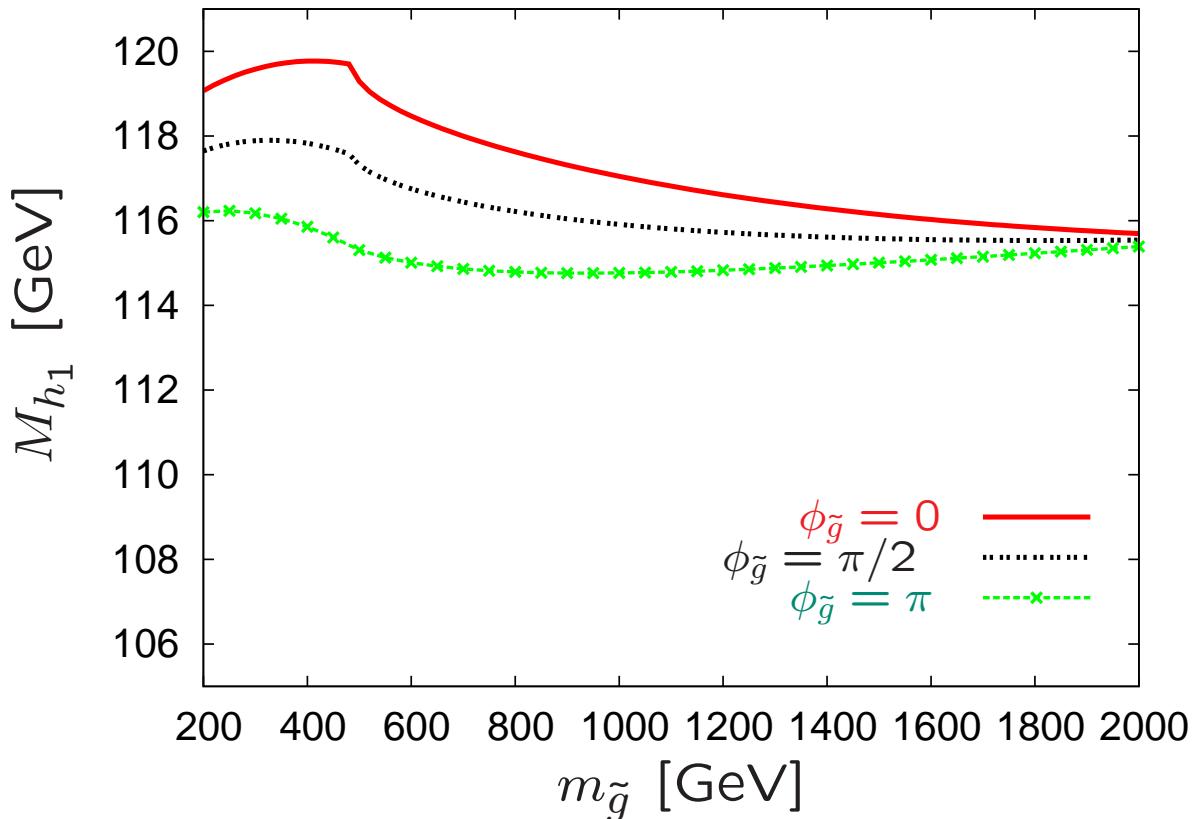
OS renormalization

⇒ modified dependence

on ϕ_{A_t} at the 2-loop level

M_{h_1} as a function of $\phi_{\tilde{g}}$:

[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]



$M_{\text{SUSY}} = 500$ GeV

$A_t = 1000$ GeV

$\tan \beta = 10$

$M_{H^\pm} = 500$ GeV

OS renormalization

⇒ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

⇒ large effects around
threshold

⇒ phase dependence
has to be taken
into account

4. SUSY Higgses at the LHC



Discovering the Higgs boson

What has to be done?

1. Find the new particle

Discovering the Higgs boson

What has to be done?

1. Find the new particle
2. measure its mass (\Rightarrow ok?)

Discovering the Higgs boson

What has to be done?

1. Find the new particle
2. measure its mass (\Rightarrow ok?)
3. measure coupling to gauge bosons
4. measure couplings to fermions

Discovering the Higgs boson

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4. measure couplings to fermions
5. measure self-couplings

Discovering the Higgs boson

What has to be done?

1. Find the new particle
2. measure its mass (\Rightarrow ok?)
3. measure coupling to gauge bosons
4. measure couplings to fermions
5. measure self-couplings
6. measure spin, . . .

Discovering the Higgs boson

What has to be done?

1. Find the new particle T
2. measure its mass (\Rightarrow ok?) T
3. measure coupling to gauge bosons
4. measure couplings to fermions
5. measure self-couplings
6. measure spin, . . .

T = Tevatron,

Discovering the Higgs boson

What has to be done?

- | | |
|--|-------|
| 1. Find the new particle | T L |
| 2. measure its mass (\Rightarrow ok?) | T L |
| 3. measure coupling to gauge bosons | L |
| 4. measure couplings to fermions | L |
| 5. measure self-couplings | |
| 6. measure spin, . . . | |

T = Tevatron, L = LHC,

Discovering the Higgs boson

What has to be done?

1. Find the new particle	T	L	I
2. measure its mass (\Rightarrow ok?)	T	L	I
3. measure coupling to gauge bosons	L	I	
4. measure couplings to fermions	L	I	
5. measure self-couplings		I	
6. measure spin, . . .	L	I	

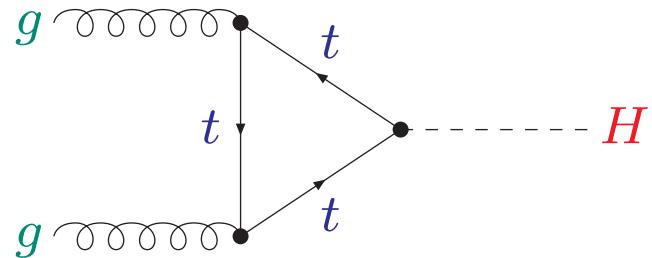
T = Tevatron, L = LHC, I = ILC

We need the **ILC** to **find the Higgs**
and to **establish the Higgs mechanism!**

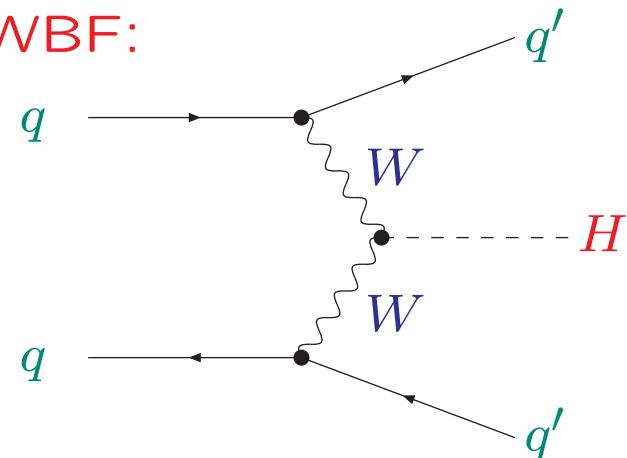
But the **LHC** can do a crucial part already!

Important SM production channel at the LHC:

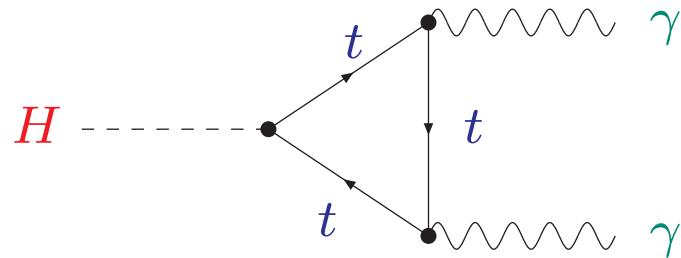
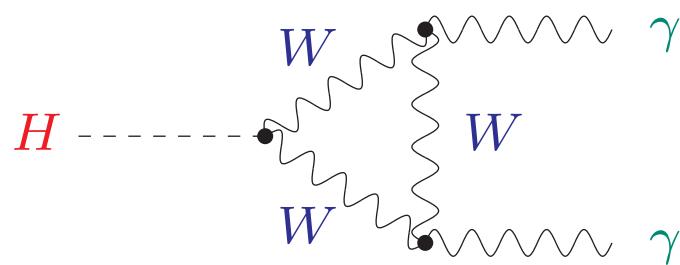
Gluon-Fusion:



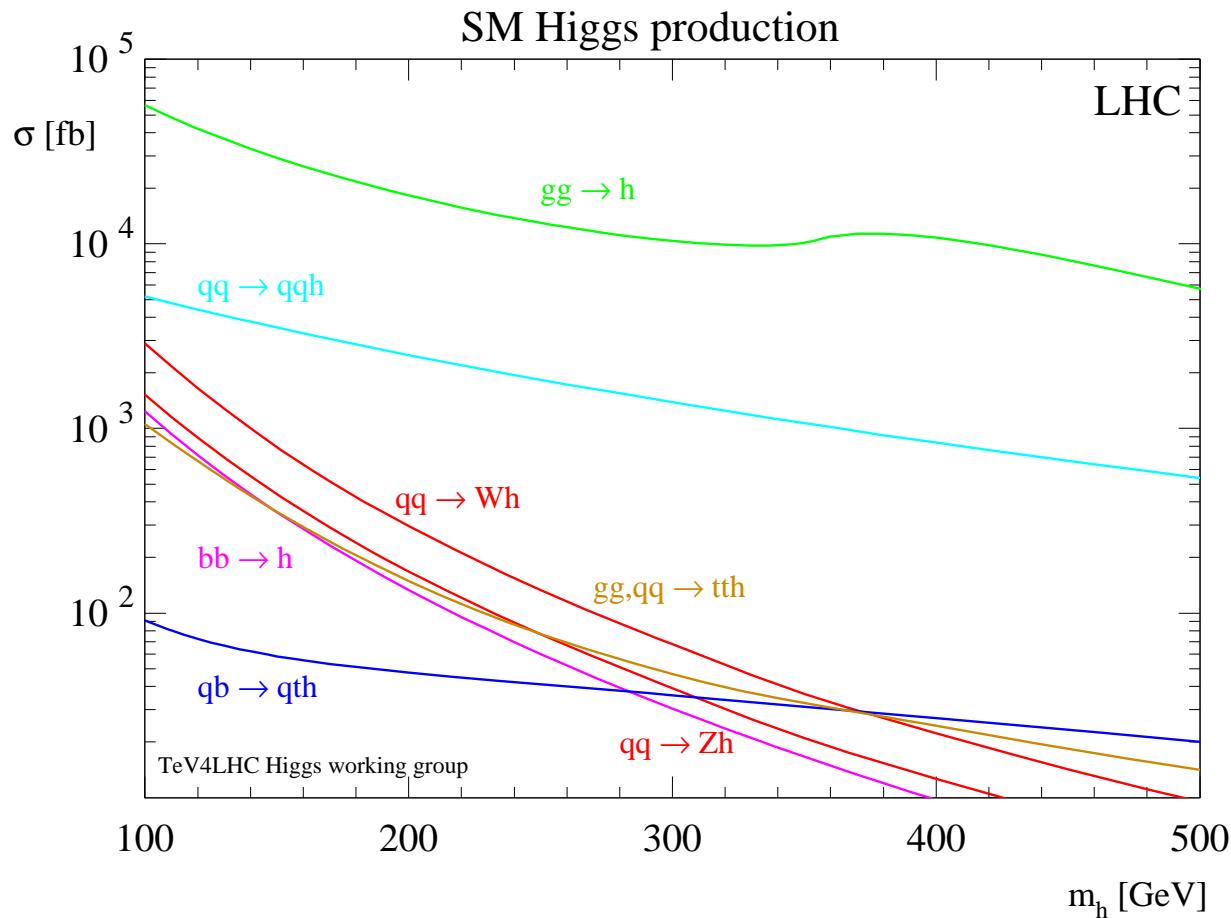
WBF:



Important decay for Higgs mass measurement:



Overview of SM Higgs production at the LHC:



gluon fusion: $gg \rightarrow H$

weak boson fusion (WBF):

$q\bar{q} \rightarrow q'\bar{q}'H$

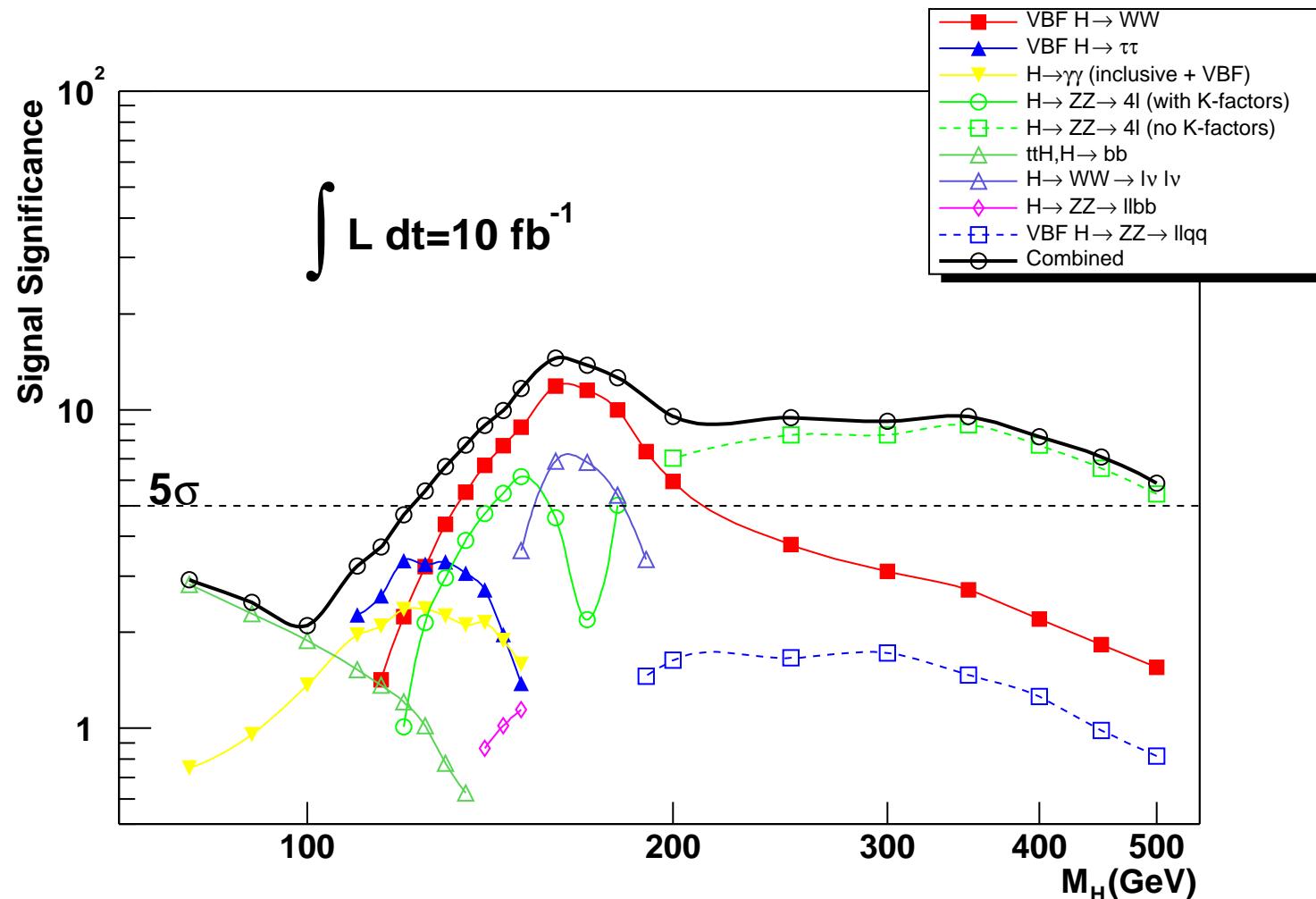
top quark associated production: $gg, q\bar{q} \rightarrow t\bar{t}H$

weak boson associated production: $q\bar{q}' \rightarrow WH, ZH$

SM Higgs search at the LHC: \Rightarrow full parameter space accessible!?

SM Higgs search at the LHC: \Rightarrow full parameter space accessible

[ATLAS '05]

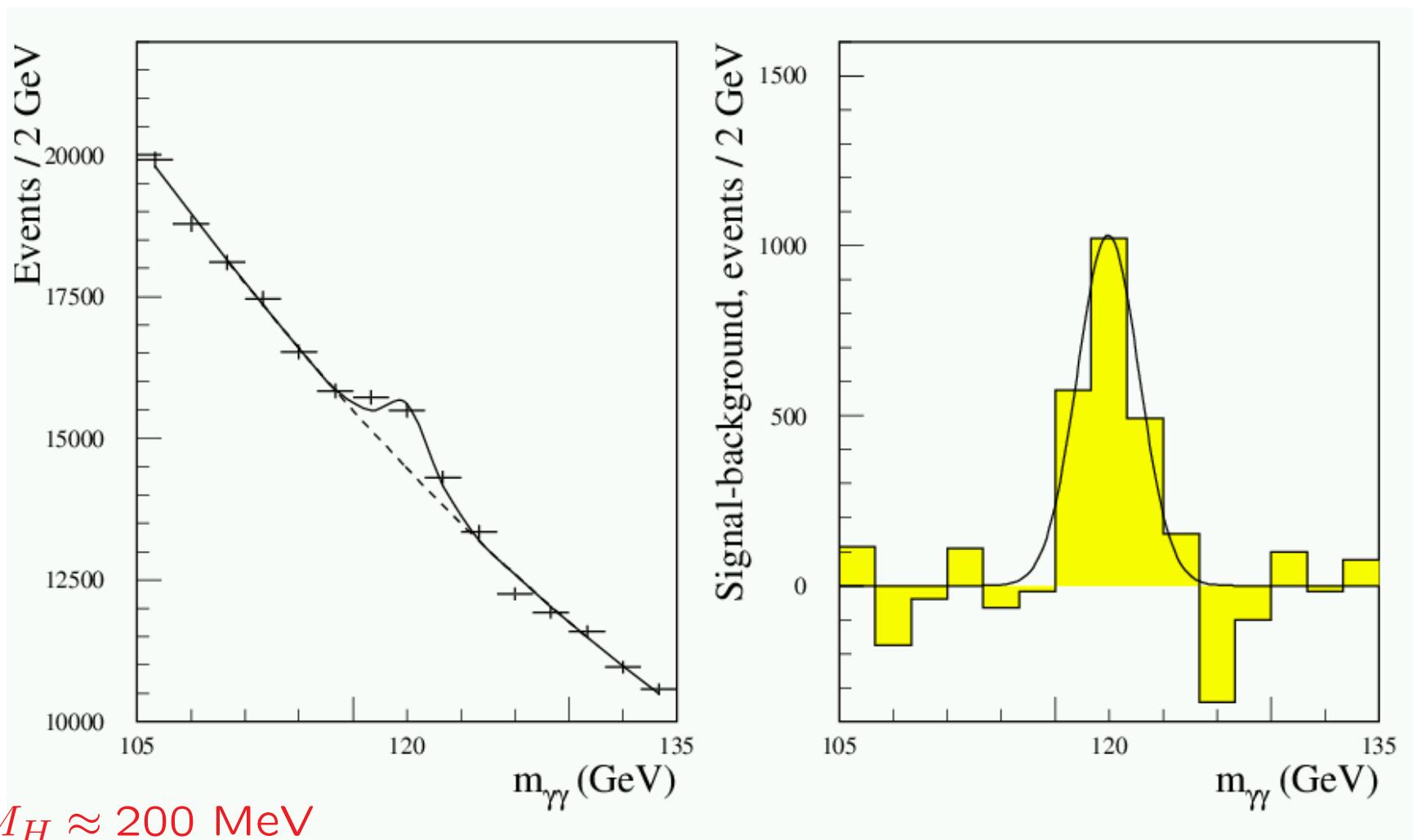


\Rightarrow most problematic case also at the LHC: $M_H = 115 \dots 120 \text{ GeV}$

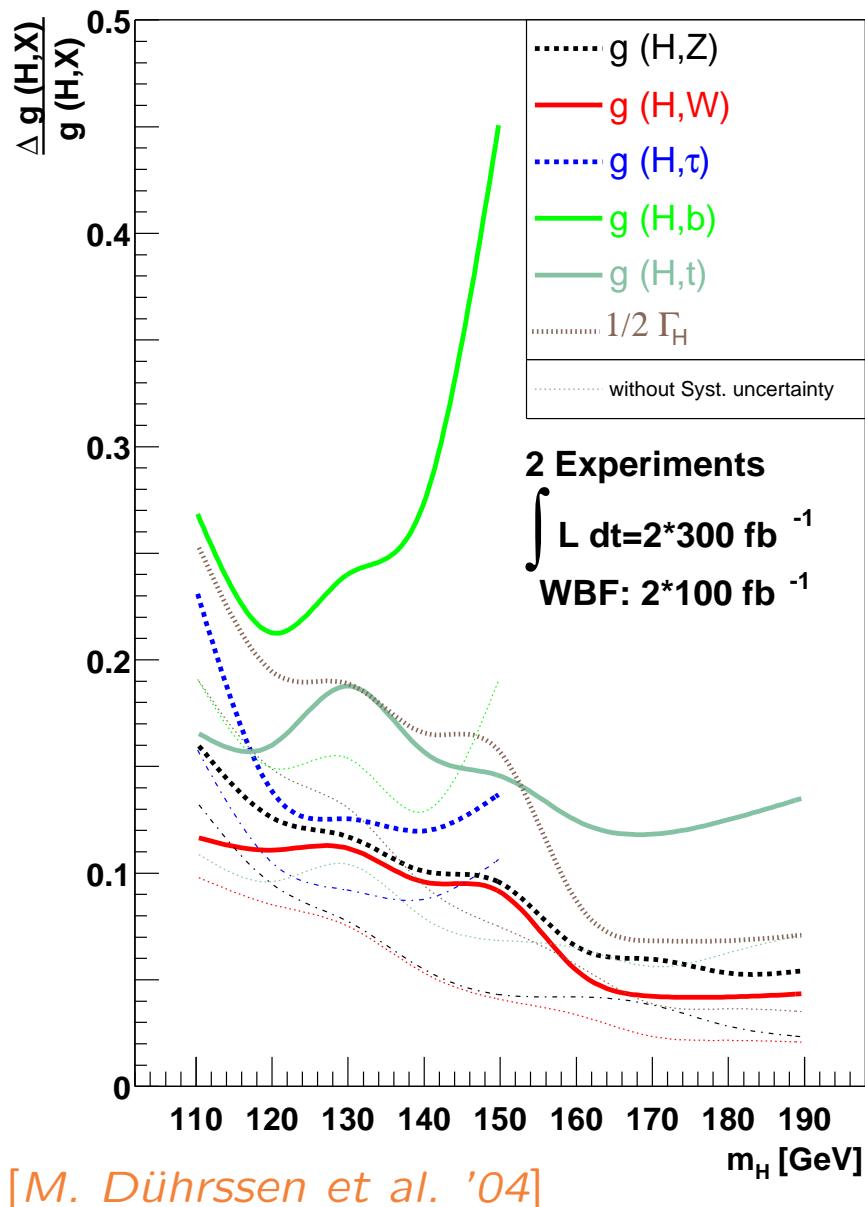
Step 2: Measurement of the mass

Best channel for mass measurement in the SM: $H \rightarrow \gamma\gamma$

[ATLAS '99]



Higgs couplings at the LHC:



- mass: $\delta M_h \approx 200$ MeV
- couplings: $(2 * 300 + 2 * 100)$ fb^{-1} : typical accuracies of 20-30% for $m_H \leq 150$ GeV 10% accuracies for HVV couplings above WW threshold

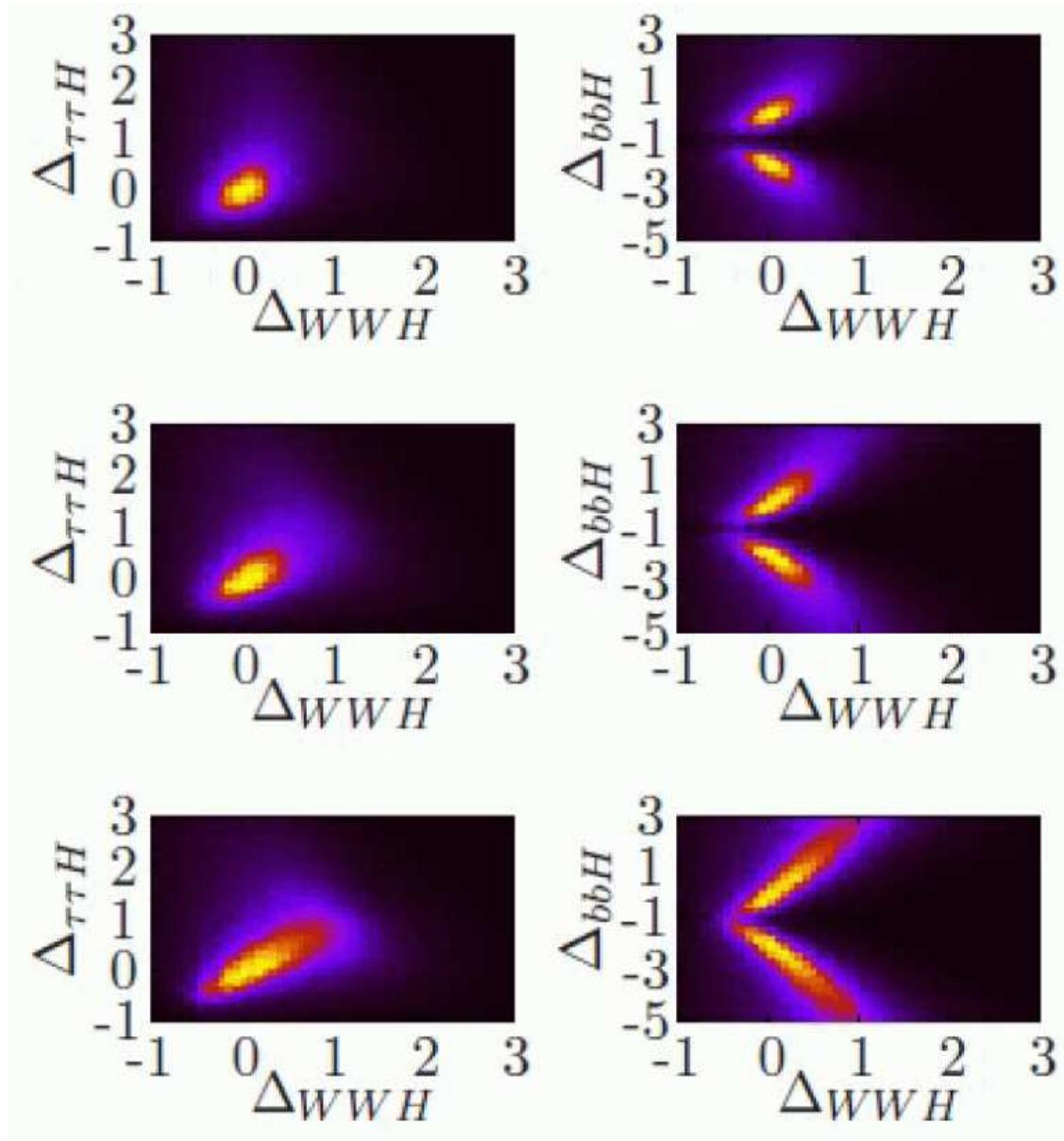
Assumption:

- $g_{HVV}^2 \leq g_{HVV,\text{SM}}^2 \times 1.05$
- SM rates for the Higgs

Problems:

- old $t\bar{t}H, H \rightarrow b\bar{b}$ studies used
- valid in weakly interacting models
- rates much lower than in SM ??
- physics can/will hide in 5% margin
- self-couplings out of reach

Impact of $H \rightarrow b\bar{b}$ analyses:



old: $t\bar{t}H, H \rightarrow b\bar{b}$
no longer viable :-(

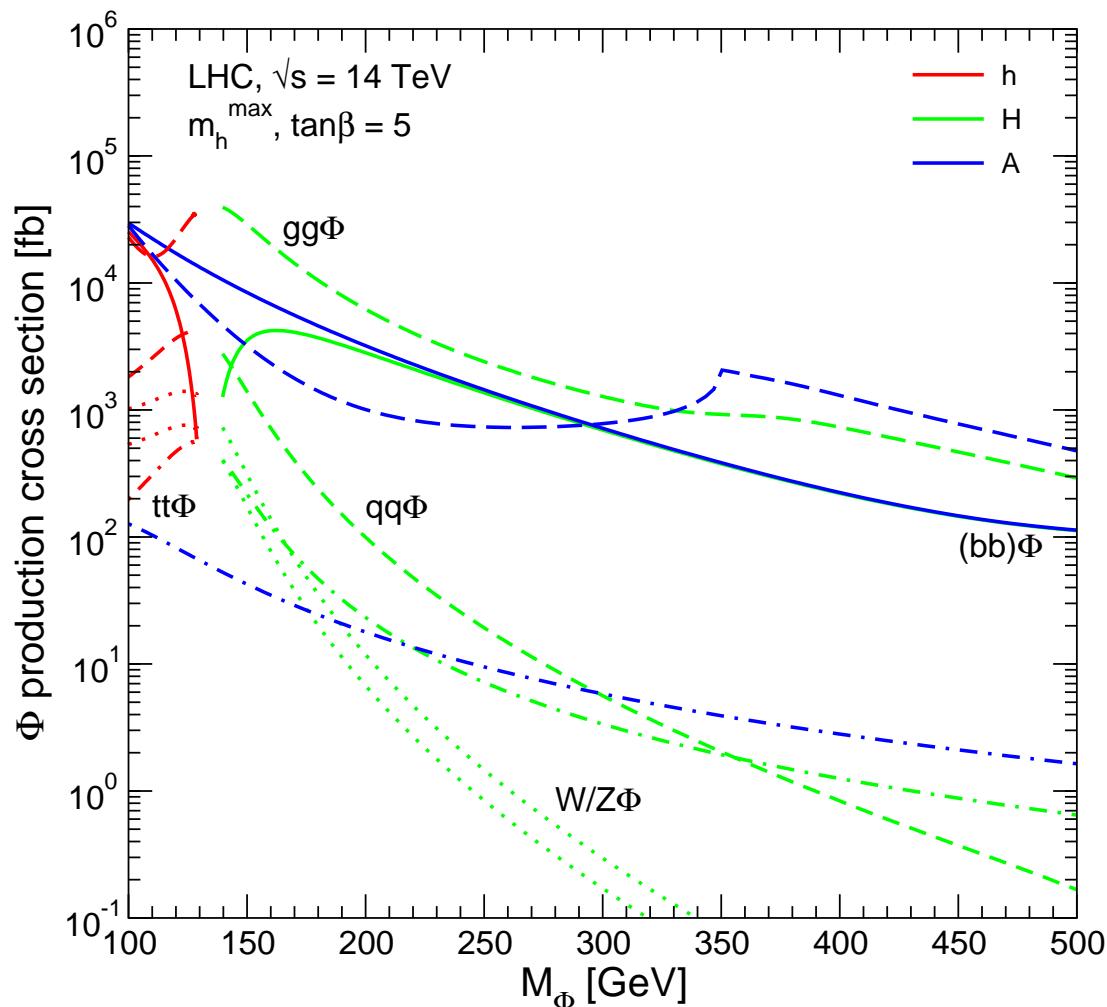
new idea:
 $WH, H \rightarrow b\bar{b}$ in boosted system
[Butterworth et al. '08]

recently (partially) confirmed
by ATLAS

Impact: [SFitter '09]
 $H b\bar{b}$ crucial!

Situation is a bit more complicated for SUSY Higgses ($\phi = h, H, A$)

[*Tev4LHC Higgs working group report '06*]



gluon fusion: $gg \rightarrow \phi$

weak boson fusion (WBF):

$q\bar{q} \rightarrow q'\bar{q}'\phi$

top quark associated
production: $gg, q\bar{q} \rightarrow t\bar{t}\phi$

weak boson associated
production: $q\bar{q}' \rightarrow W\phi, Z\phi$

NEW: $b\bar{b}\phi$

Search for the lightest MSSM Higgs at the LHC:

⇒ full parameter accessible But there might be problems . . .

Possible problem in SUSY:

$$gg \rightarrow h \rightarrow \gamma\gamma$$

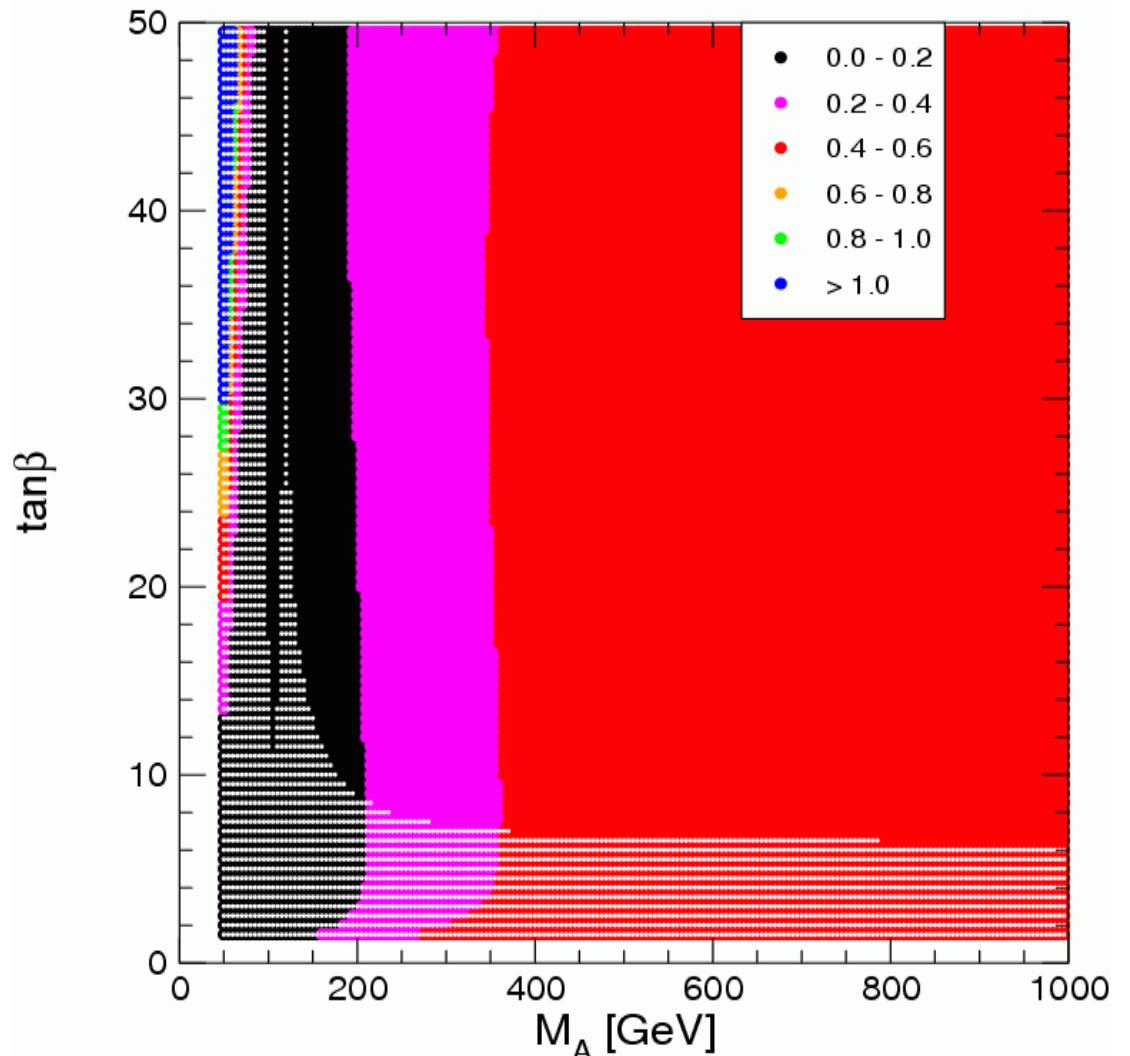
can be **strongly suppressed**

→ “gluophobic Higgs scenario”

[*M. Carena, S.H., C. Wagner,
G. Weiglein '02*]

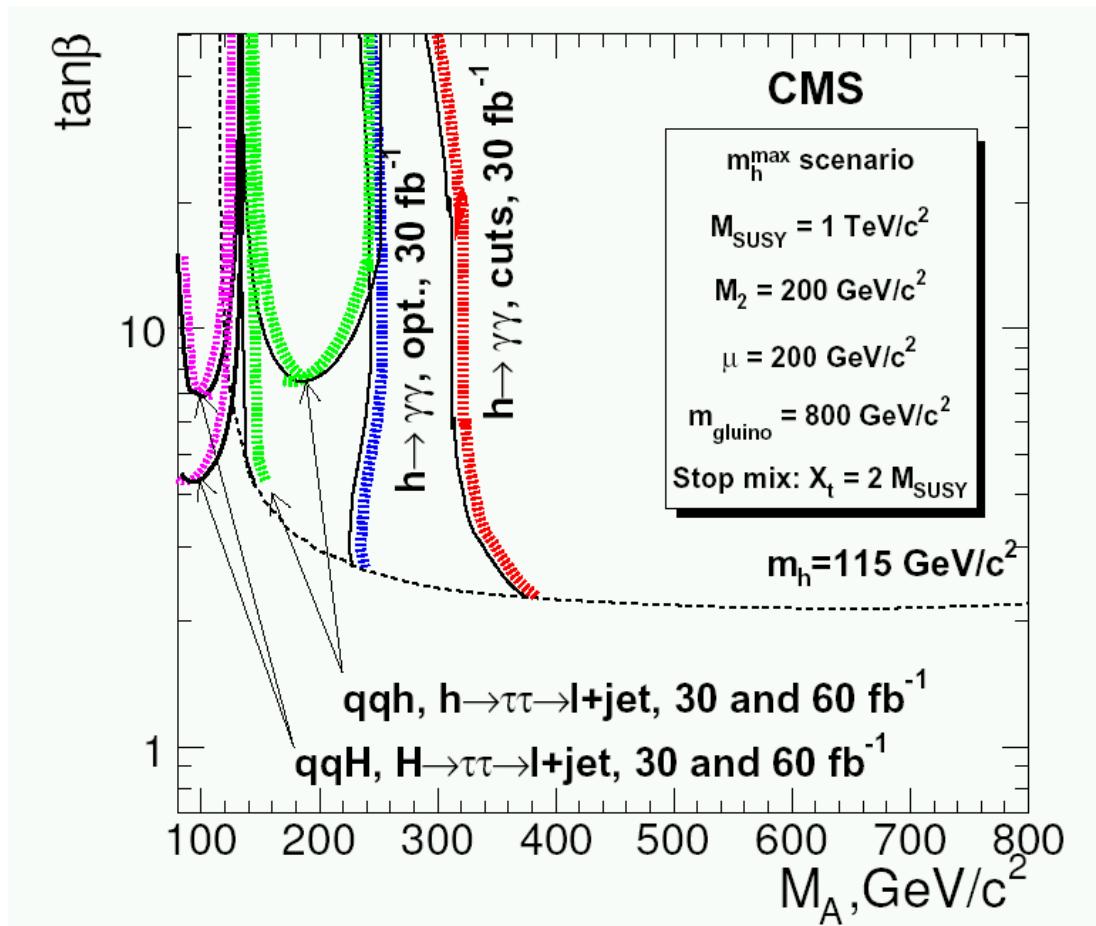
⇒ Strong suppression of
 $gg \rightarrow h \rightarrow \gamma\gamma$ possible
over the whole parameter space

(not realized in
mSUGRA/CMSSM, GMSB,
AMSB, . . .)



M_h measurement in the “nice” m_h^{\max} scenario:

[CMS '06]



Measurement possible only for
 $M_A \gtrsim 250 \text{ GeV}$
 $\Rightarrow \delta M_h \approx 200 \text{ MeV}$

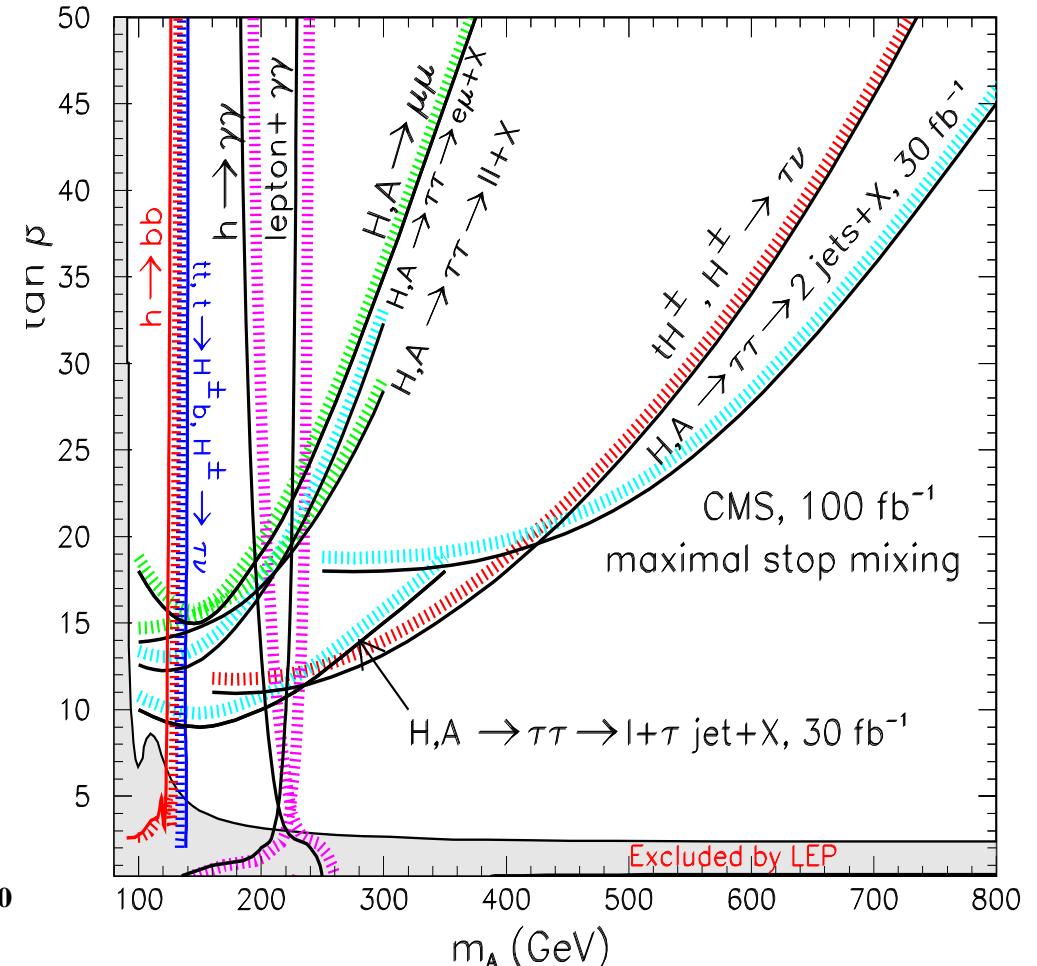
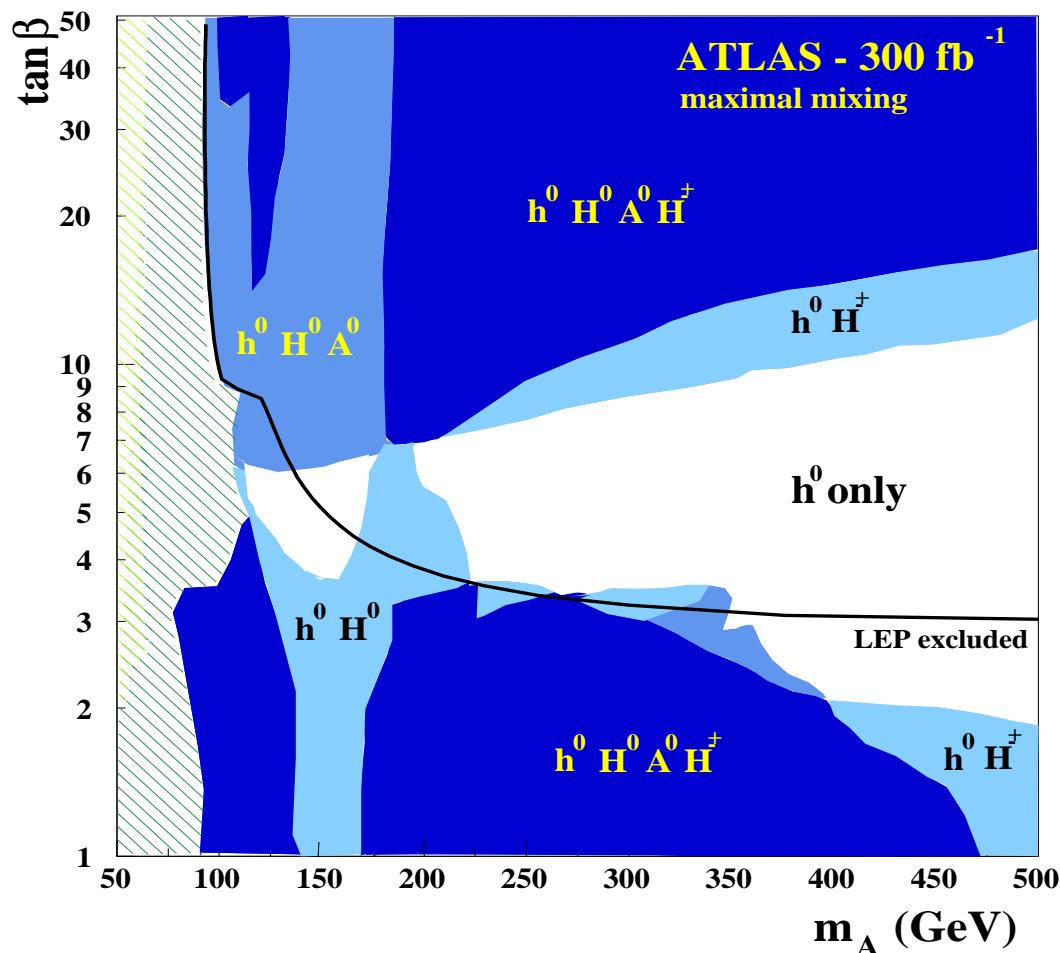
other channels:
 $h \rightarrow ZZ^* \rightarrow 4\mu$ ($M_h \gtrsim 130 \text{ GeV}$)

otherwise: $\delta M_h \gtrsim 1 - 2 \text{ GeV}$

The heavy MSSM Higgs bosons

MSSM Higgs discovery contours in M_A - $\tan\beta$ plane

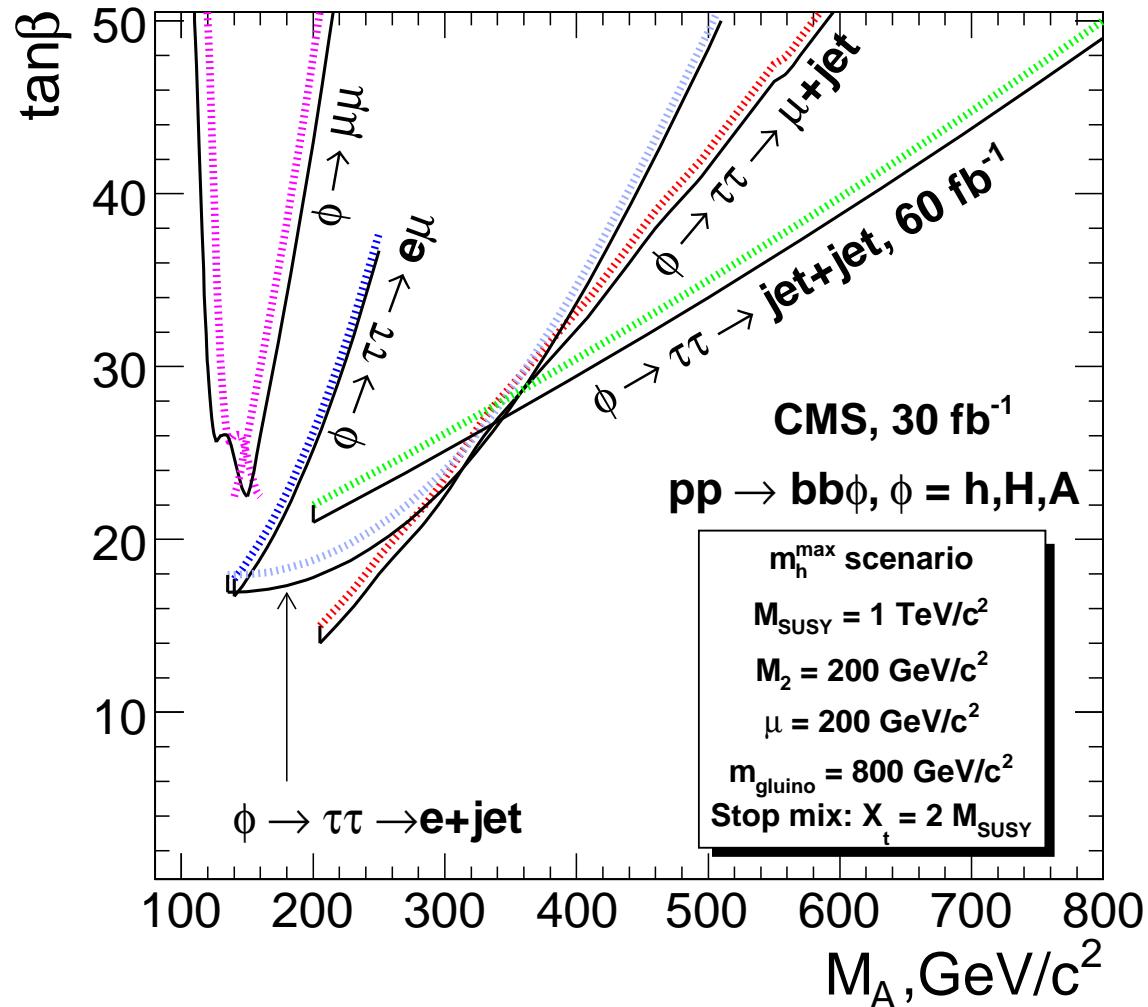
(m_h^{\max} benchmark scenario): [ATLAS '99] [CMS '03]



areas where only h is observable \Rightarrow “LHC wedge”

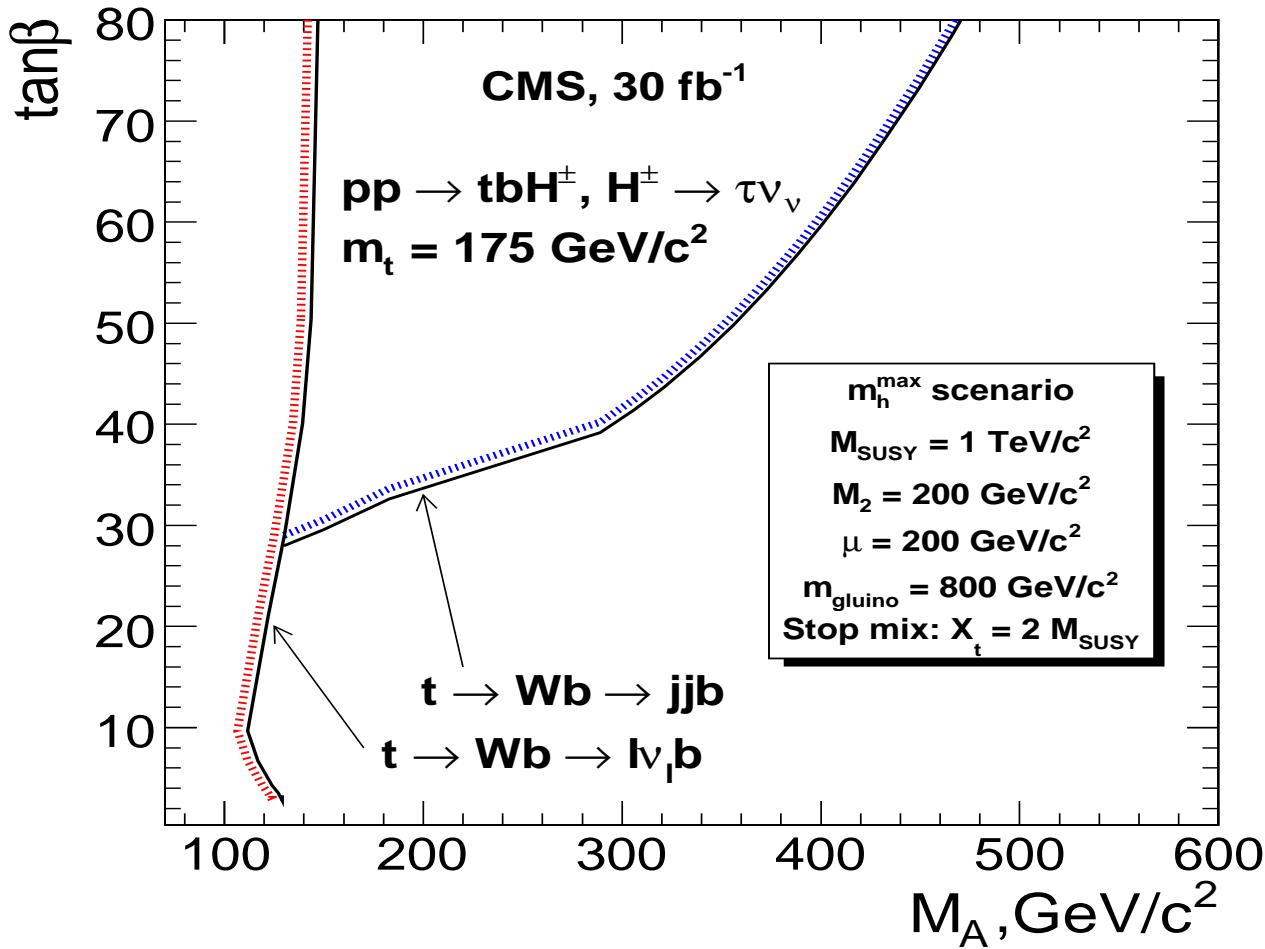
Latest results for neutral heavy Higgs bosons:

MSSM Higgs discovery contours in M_A – $\tan\beta$ plane ($\phi = H, A$)
(m_h^{\max} benchmark scenario): [CMS PTDR '06]



Charged Higgs boson searches:

MSSM Higgs discovery contours in M_A - $\tan\beta$ plane
(m_h^{\max} benchmark scenario): [CMS PTDR '06]



light charged Higgs:

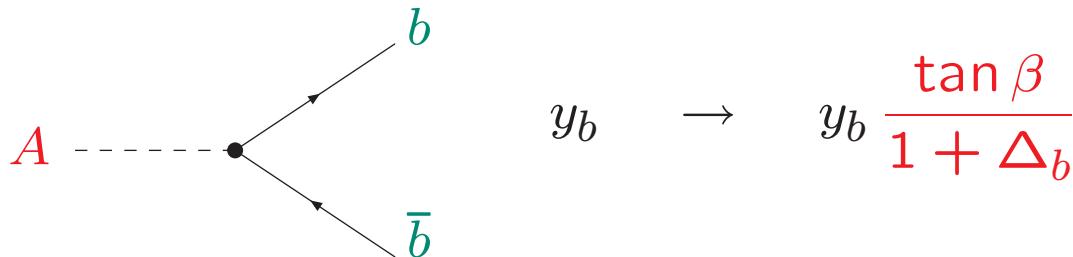
$$M_{H^\pm} < m_t$$

heavy charged Higgs:

$$M_{H^\pm} > m_t$$

Differences compared to the SM Higgs:

Additional enhancement factors compared to the SM case:



At large $\tan \beta$: either $H \approx A$ or $h \approx A$



$$\begin{aligned} \Delta_b &= \frac{2\alpha_s}{3\pi} m_{\tilde{g}} \mu \tan \beta \times I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) \\ &+ \frac{\alpha_t}{4\pi} A_t \mu \tan \beta \times I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu) \end{aligned}$$

⇒ other parameters enter ⇒ strong μ dependence

Most powerful search modes for heavy MSSM Higgs bosons:

$$\boxed{\begin{aligned} b\bar{b} &\rightarrow H/A \rightarrow \tau^+\tau^- + X \\ g\bar{b} &\rightarrow tH^\pm + X, \quad H^\pm \rightarrow \tau\nu_\tau \\ p\bar{p} &\rightarrow t\bar{t} \rightarrow H^\pm + X, \quad H^\pm \rightarrow \tau\nu_\tau \end{aligned}}$$

Enhancement factors compared to the SM case:

$$H/A : \frac{\tan^2 \beta}{(1 + \Delta_b)^2} \times \frac{\text{BR}(H \rightarrow \tau^+\tau^-) + \text{BR}(A \rightarrow \tau^+\tau^-)}{\text{BR}(H \rightarrow \tau^+\tau^-)_{\text{SM}}}$$

$$H^\pm : \frac{\tan^2 \beta}{(1 + \Delta_b)^2} \times \text{BR}(H^\pm \rightarrow \tau\nu_\tau)$$

$\Rightarrow \Delta_b$ effects so far neglected by ATLAS/CMS

also relevant for $\text{BR}(H/A \rightarrow \tau^+\tau^-)$, $\text{BR}(H^\pm \rightarrow \tau\nu_\tau)$

also relevant: correct evaluation of $\Gamma(H/A/H^\pm \rightarrow \text{SUSY})$

\Rightarrow additional effects on $\text{BR}(H/A \rightarrow \tau^+\tau^-)$, $\text{BR}(H^\pm \rightarrow \tau\nu_\tau)$

Suggestion for new benchmark scenarios:

[M. Carena, S.H., C. Wagner, G. Weiglein '05]

→ investigate benchmark scenarios:

- Vary only M_A and $\tan \beta$ (large!)
- Keep all other SUSY parameters fixed

- Vary in addition μ : $\mu = \pm 1000, \pm 500, \pm 200$ GeV
(if perturbativity allows)

1. m_h^{\max} scenario:

→ obtain conservative $\tan \beta$ exclusion bounds ($X_t = 2 M_{\text{SUSY}}$)

A_t large \Rightarrow large $\mathcal{O}(\alpha_t)$ contribution to Δ_b

2. no-mixing scenario

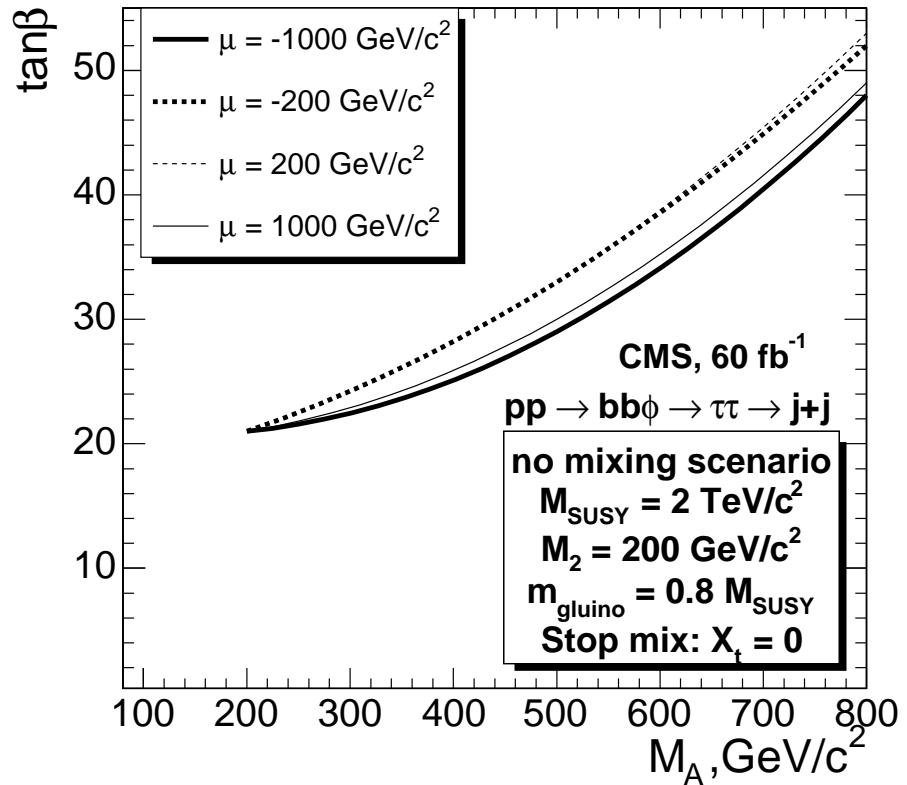
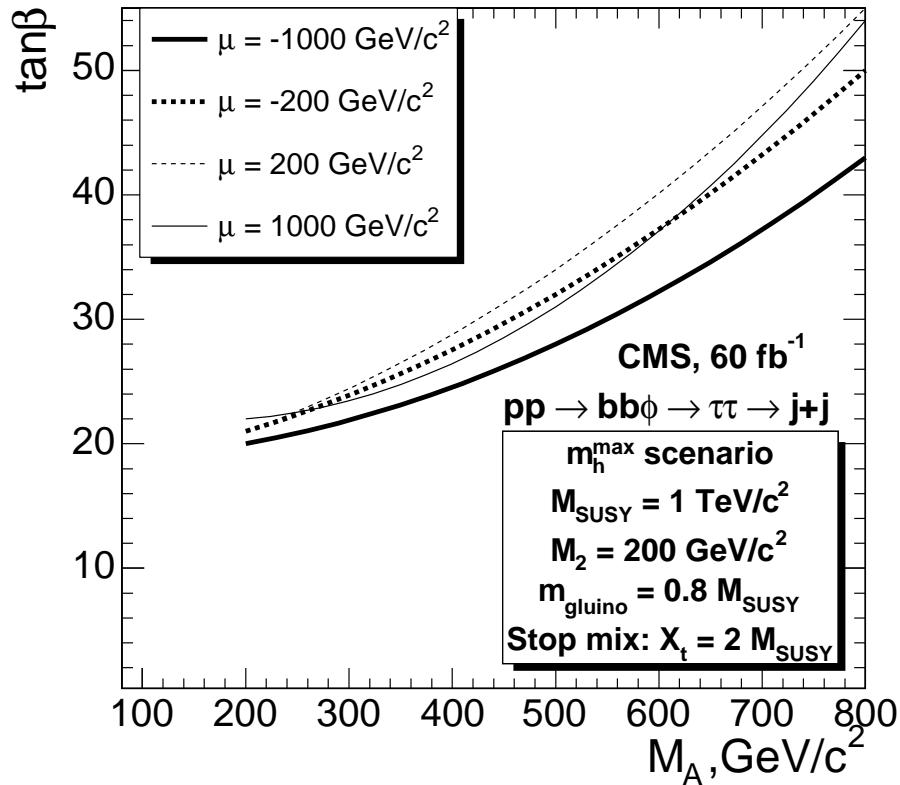
→ no mixing in the scalar top sector ($X_t = 0$)

A_t small \Rightarrow small $\mathcal{O}(\alpha_t)$ contribution to Δ_b

\Rightarrow large difference to m_h^{\max} scenario

Dependence of LHC wedge from $b\bar{b} \rightarrow H/A \rightarrow \tau^+\tau^- \rightarrow 2 \text{jets}$ on μ :

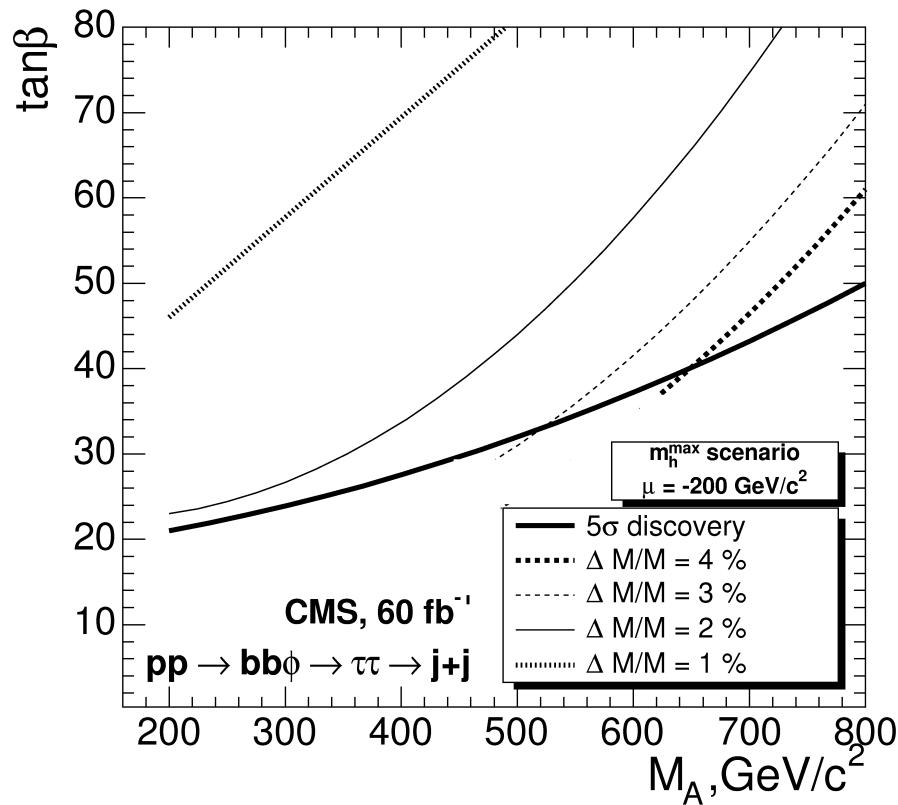
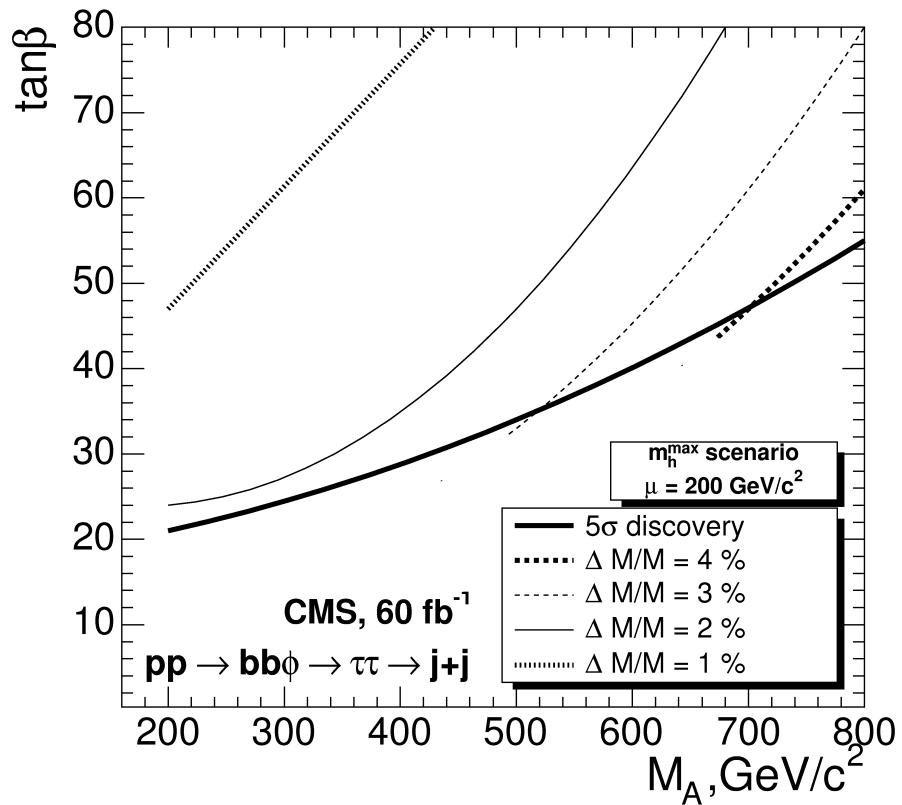
[S.H., A. Nikitenko, G. Weiglein et al. '06]



- ⇒ now based on full CMS simulation
- ⇒ non-negligible variation with the sign and absolute value of μ
(→ numerical compensations in production and decay)

Precision of $\delta M/M$ from $b\bar{b} \rightarrow H/A \rightarrow \tau^+\tau^- \rightarrow 2 \text{jets}$:

[S.H., A. Nikitenko, G. Weiglein et al. '06]

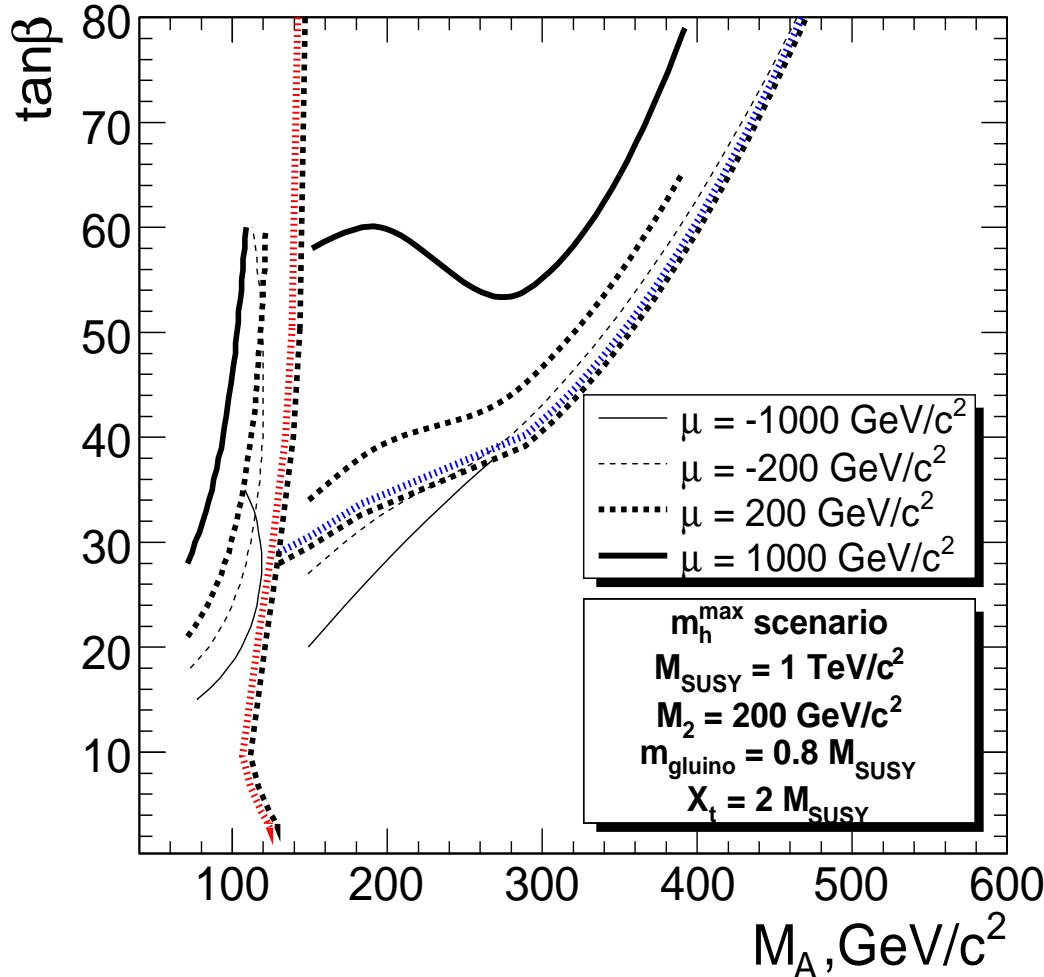


⇒ now based on full CMS simulation

⇒ high precision measurement of heavy Higgs boson masses possible

Charged Higgs: comparison with CMS PTDR (m_h^{\max} scenario):

[M. Hashemi, S.H., R. Kinnunen, A. Nikitenko, G. Weiglein '07]



→ note: M_A – $\tan\beta$ plane

light charged Higgs:

always worse than PTDR
better M_{H^\pm} calculation!
inclusion of Δ_b effects

heavy charged Higgs:

PTDR in “the middle”
new results partially
substantially worse

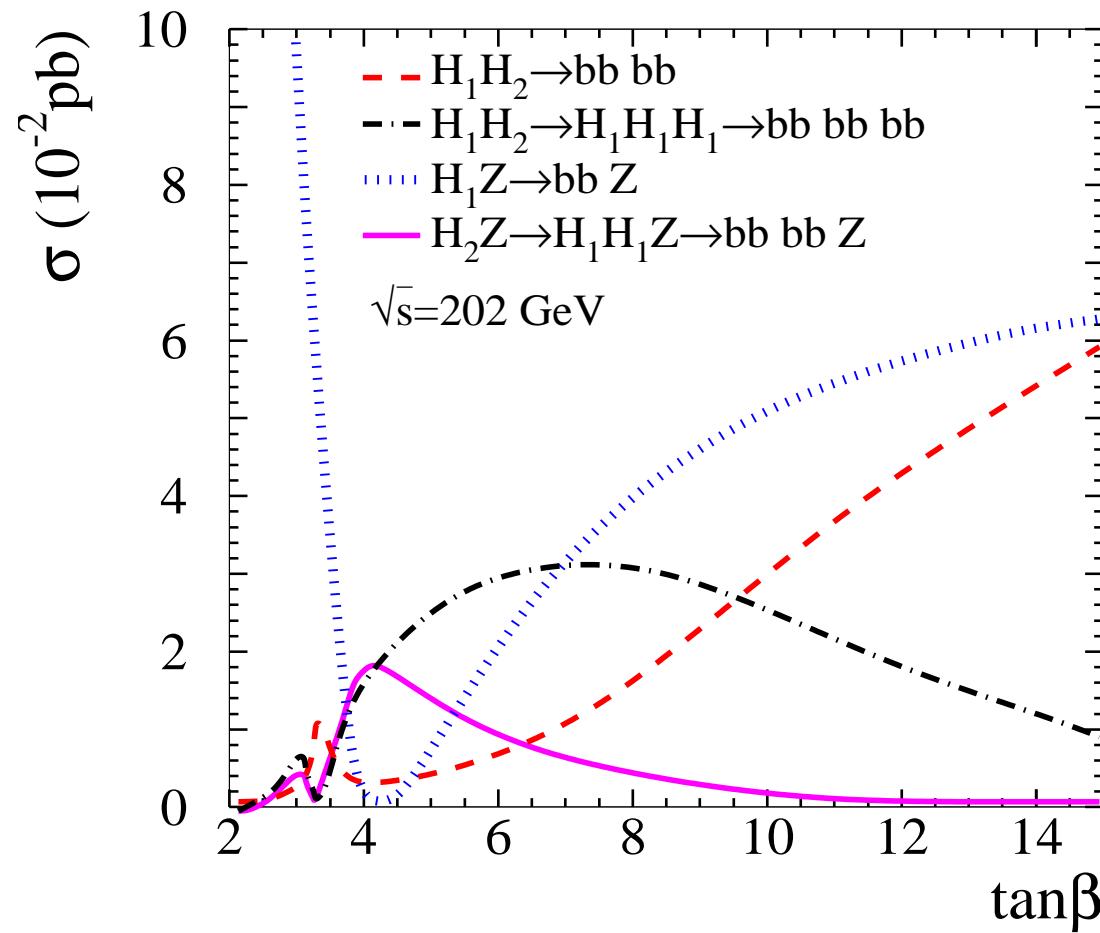
\mathcal{CPV} effects on Higgs boson searches:

CPX : benchmark scenario in the cMSSM

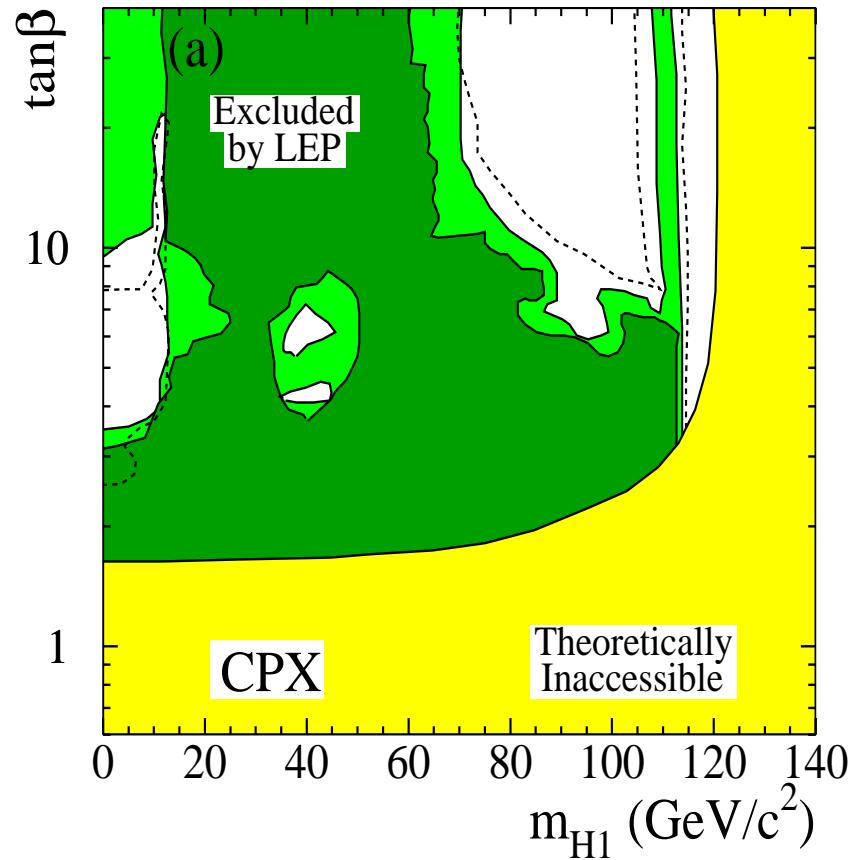
[*M. Carena, J. Ellis, A. Pilaftsis, C. Wagner '00*]

LEP Higgs production cross sections:

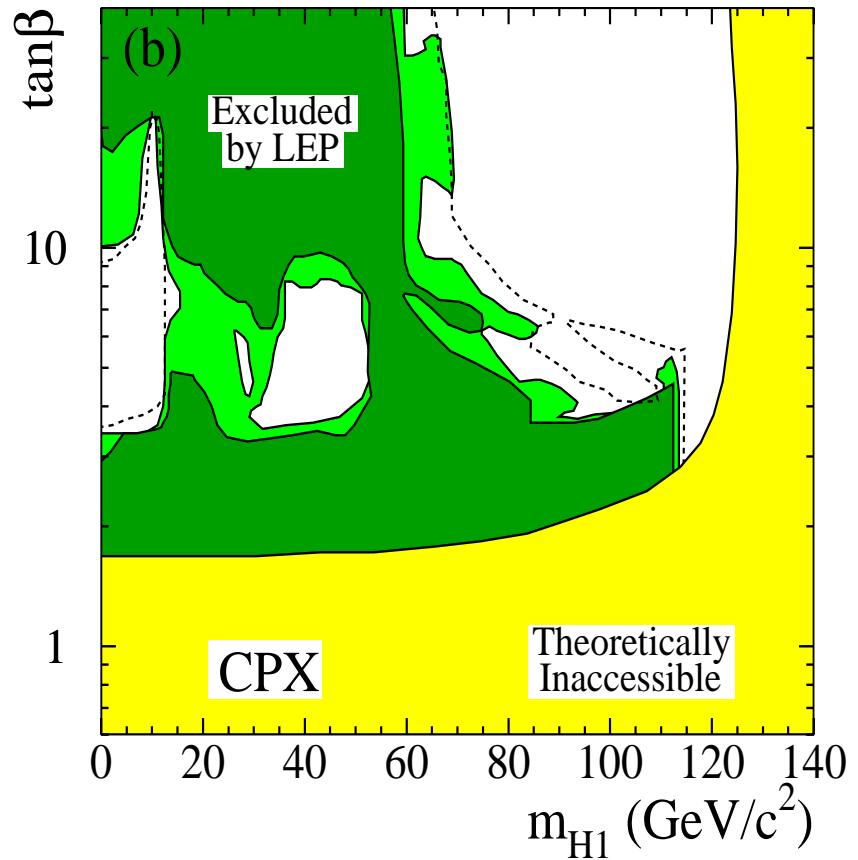
[*LEPHiggsWG '06*]



$m_t = 169.3 \text{ GeV}$



$m_t = 174.3 \text{ GeV}$



The LEP analysis showed an unexcluded hole in the m_{h_1} – $\tan\beta$ plane at $m_{h_1} \approx 45 \text{ GeV}$, $\tan\beta \approx 8$

Reevaluating the CPX hole(s) at LEP

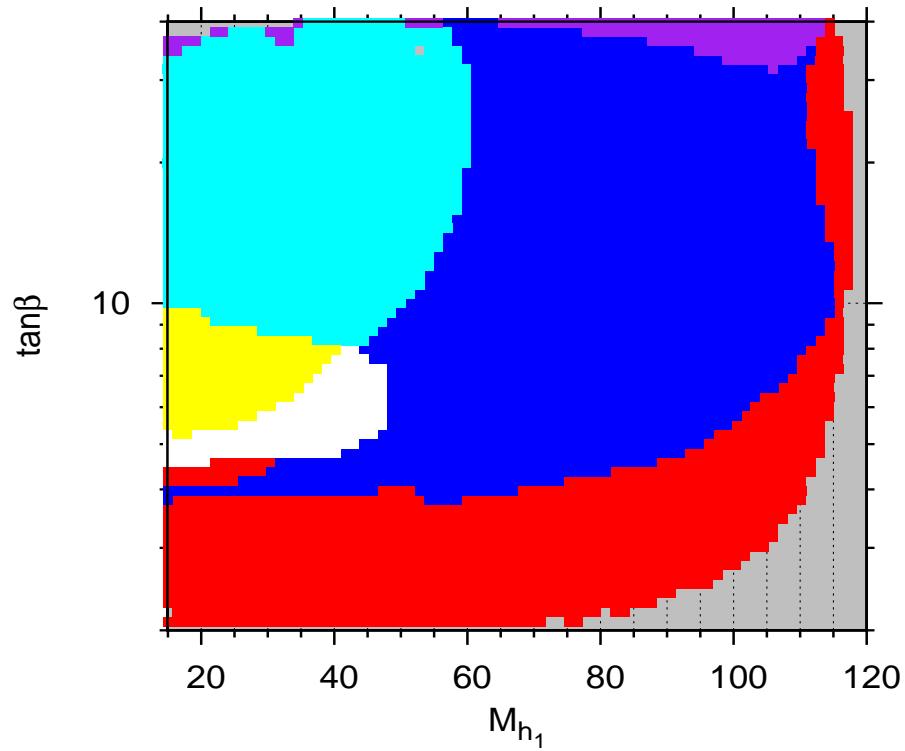
The LEP analysis showed an **unexcluded hole** in the m_{h_1} – $\tan\beta$ plane at
 $m_{h_1} \approx 45$ GeV, $\tan\beta \approx 8$

New theoretical developments:

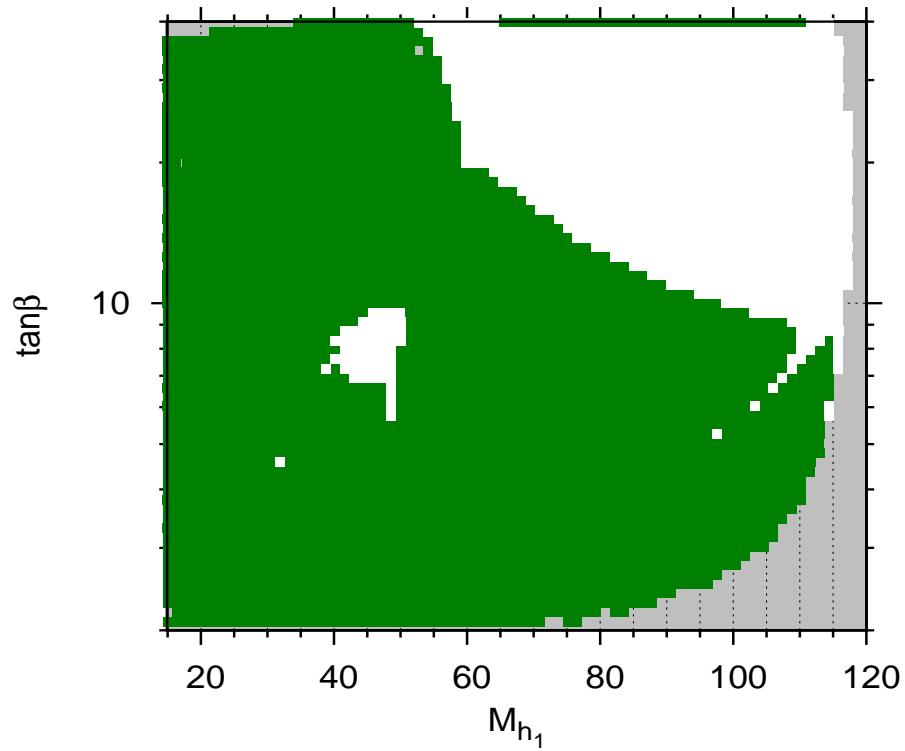
- phase dependent $\mathcal{O}(\alpha_t \alpha_s)$ corrections to Higgs self-energies
[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]
- phase dependent one-loop correction to $\Gamma(h_a \rightarrow h_b h_c)$
[G. Weiglein, K. Williams '08]

⇒ effects on CPX hole(s)?

Reevaluating the CPX hole(s) at LEP using HiggsBounds



- : $h_1 Z \rightarrow b\bar{b}Z$
- : $h_2 Z \rightarrow b\bar{b}Z$
- : $h_2 Z \rightarrow h_1 h_1 Z \rightarrow b\bar{b}b\bar{b}Z$
- : $h_2 h_1 \rightarrow b\bar{b}b\bar{b}$
- : $h_2 h_1 \rightarrow h_1 h_1 h_1 \rightarrow b\bar{b}b\bar{b}b\bar{b}$
- : theoretical inaccessible



- : excluded
- : not excluded
- : theoretical inaccessible
- ⇒ hole(s) confirmed

Outlook

- The quest for electroweak symmetry breaking continues!
- Low-energy Supersymmetry continues to be our best bet for physics beyond the Standard Model
- Data rules:
We need experimental information from Tevatron, LHC, ILC,
 ν experiments, dark matter searches, low-energy experiments, . . .
to verify / falsify our ideas about electroweak symmetry breaking,
the Higgs, extensions of the SM, . . .
- The experiments in the next years will bring a decisive test of our ideas about the Higgs and electroweak symmetry breaking

⇒ Very exciting prospects for the coming years

Expect the unexpected!

Back-up

The models: 1.) CMSSM (or mSUGRA):

⇒ Scenario characterized by

$$m_0, m_{1/2}, A_0, \tan\beta, \text{sign } \mu$$

m_0 : universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0 : universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

$\text{sign}(\mu)$: sign of supersymmetric Higgs parameter

} at the GUT scale

⇒ particle spectra from renormalization group running to weak scale

The models: 2.) NUHM1: (Non-universal Higgs mass model)

Assumption: no unification of scalar fermion and scalar Higgs parameter at the GUT scale

⇒ effectively M_A or μ as free parameters at the EW scale

⇒ besides the CMSSM parameters

M_A or μ

Further extension: NUHM2:

Assumption: no unification of the Higgs parameters at the GUT scale

⇒ effectively M_A and μ as free parameters at the EW scale

⇒ besides the CMSSM parameters

M_A and μ

Prediction of M_h in the CMSSM

[Buchmüller, Cavanaugh, De Roeck, Ellis, Flächer, S.H., Isidori, Olive, Ronga, Weiglein '09]

General idea:

Take the most simple MSSM version: CMSSM/NUHM1

→ just three/four GUT scale parameters + $\tan \beta$

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Take the most simple MSSM version: CMSSM/NUHM1

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- combine all electroweak precision data as in the SM
- combine with B physics observables
- combine with CDM and $(g - 2)_\mu$
- include SM parameters with their errors: m_t, \dots
- scan over the full CMSSM/NUHM1 parameter space

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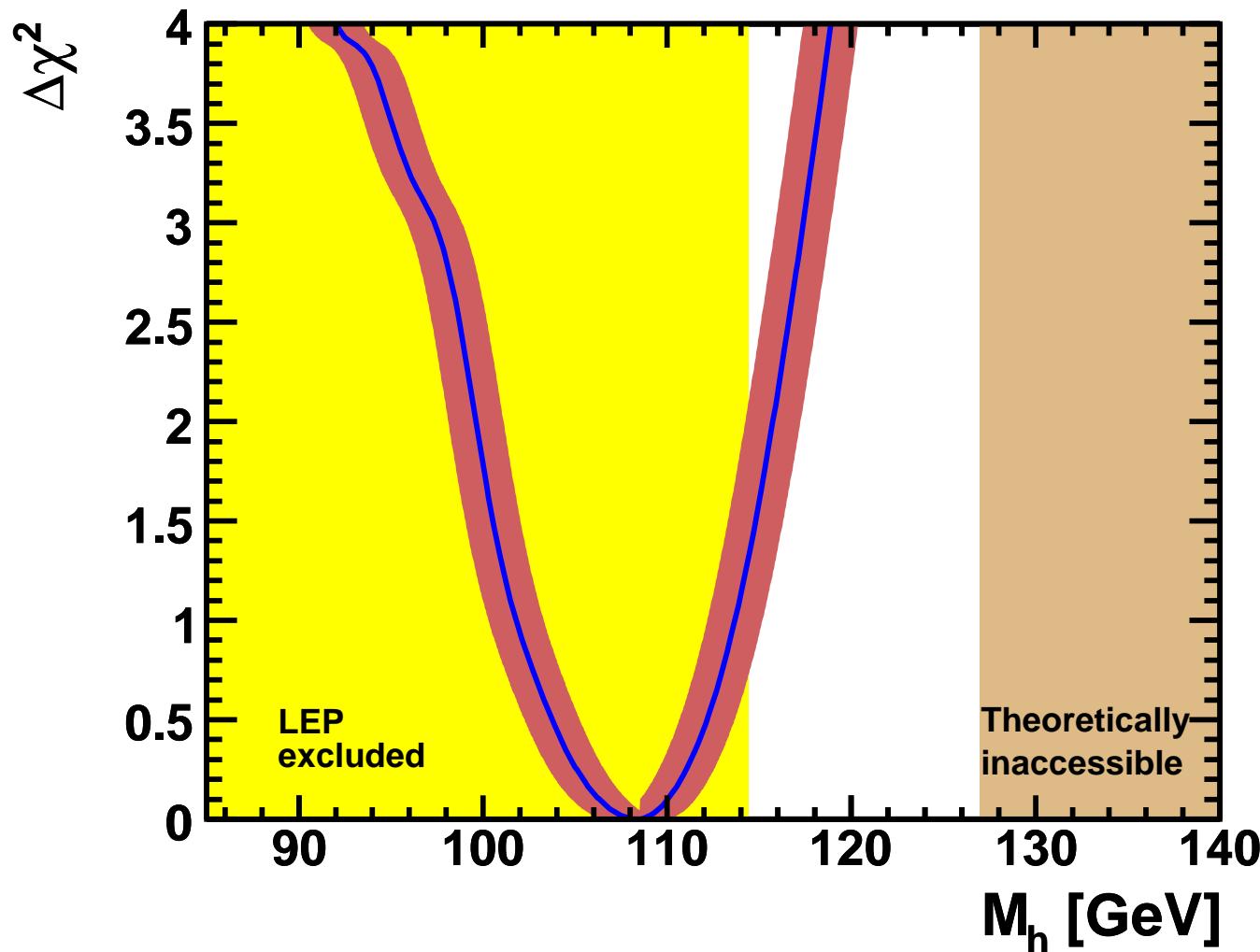
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- scan over the full CMSSM/NUHM1 parameter space

⇒ preferred M_h values

CMSSM: red band plot:

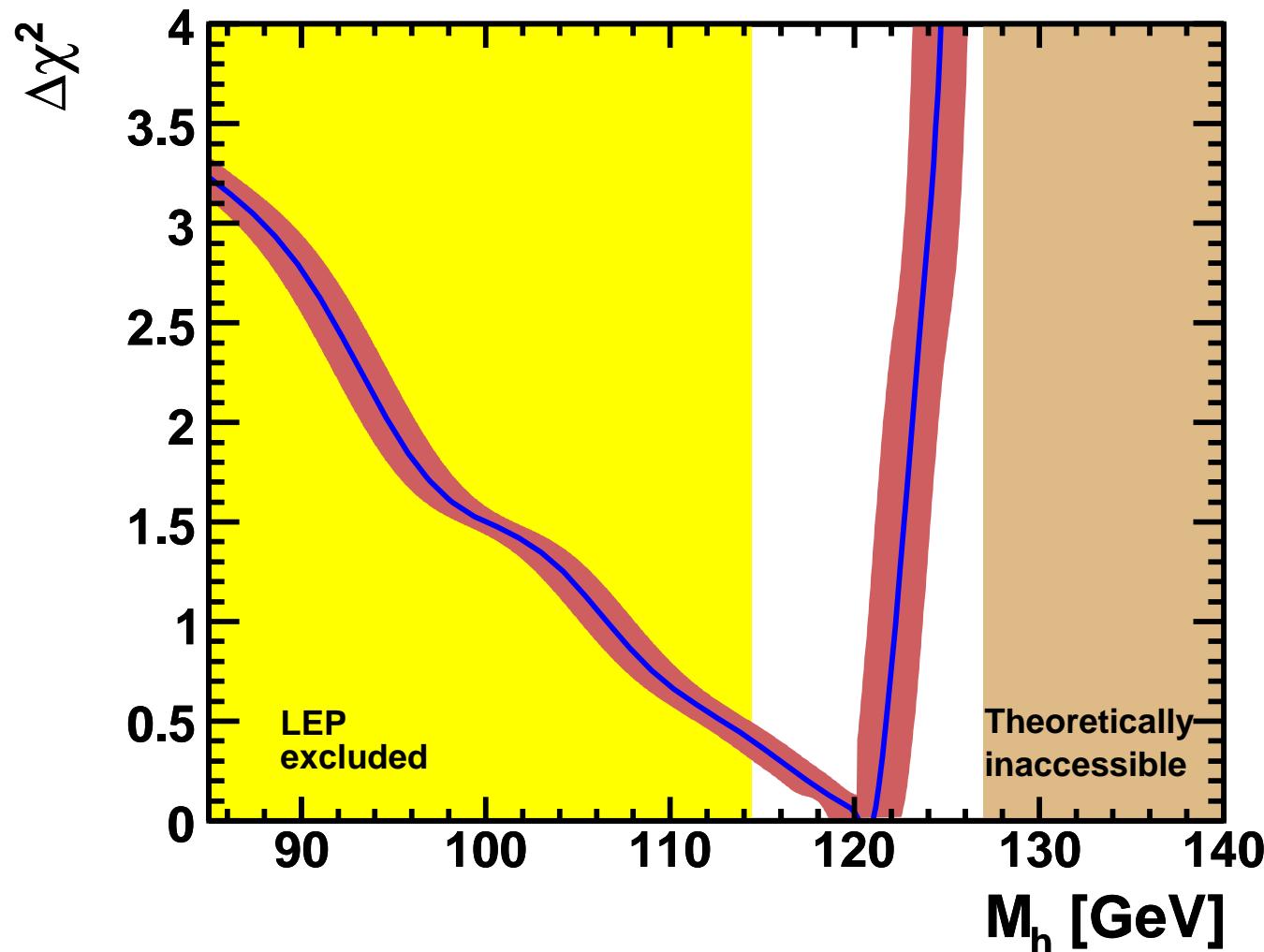
[MasterCode '09]



$$M_h = 108 \pm 6 \text{ (exp)} \pm 1.5 \text{ (theo)} \text{ GeV}$$

NUHM1: red band plot:

[MasterCode '09]



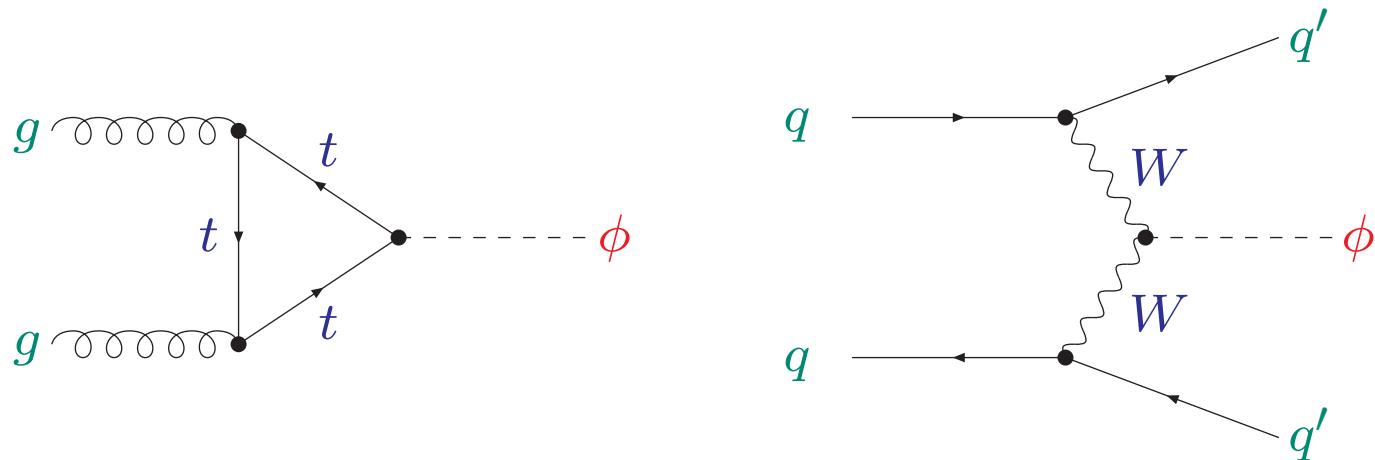
$$M_h = 121^{+1}_{-14} \text{ (exp)} \pm 1.5 \text{ (theo)} \text{ GeV}$$

\Rightarrow naturally above LEP limit

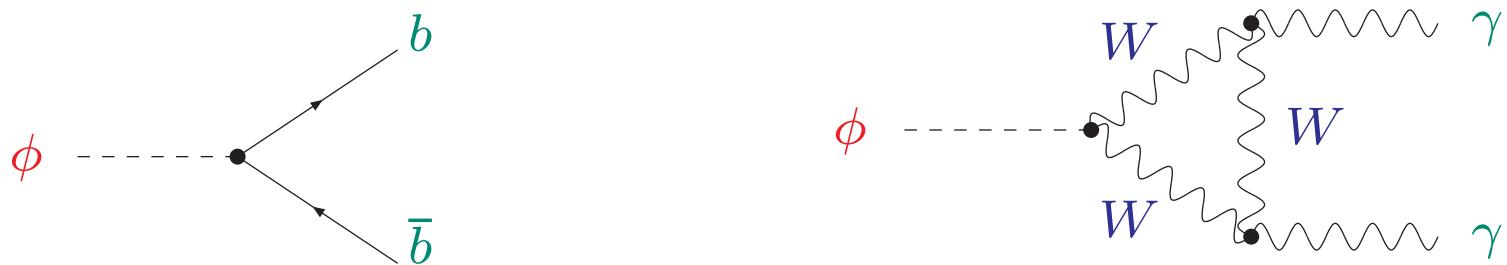
Higgs boson mixings with \mathcal{CPV} :

Examples for external (on-shell) Higgs bosons ($\phi = h_1, h_2, h_3$):

Higgs production:



Higgs decays:



⇒ important to ensure on-shell properties of external Higgs boson

The Z matrix:

Amplitude with external Higgs h_i :

$$A(h_i) = \sqrt{Z_i} \left(\Gamma_{h_i} + Z_{ij} \Gamma_{h_j} + Z_{ik} \Gamma_{h_k} \right)$$

$\sqrt{Z_i}$: ensures that the residuum of the external Higgs boson is set to 1

Z_{ij} : describes the transition from $i \rightarrow j$

$$Z_i = [1 + (\hat{\Sigma}_{ii}^{\text{eff}})'(\mathcal{M}_i^2)]^{-1}$$

$$\begin{aligned} \hat{\Sigma}_{ii}^{\text{eff}}(p^2) &= \hat{\Sigma}_{ii}(p^2) \\ &\quad - i \frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{ki}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)} \end{aligned}$$

$$Z_{ij} = \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)} \Big|_{p^2=\mathcal{M}_i^2}$$

$$\hat{\Gamma}(p^2) = iM_{hHA}^2(p^2) \quad \Delta(p^2) = (-\Gamma(p^2))^{-1}$$

m_i : tree-level masses M_i : higher-order corrected masses

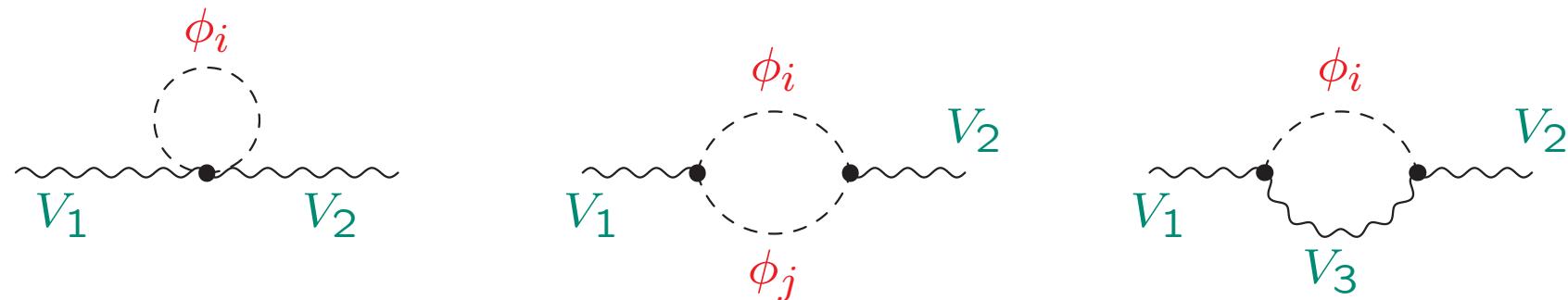
Written more compact with the **Z matrix** : $\mathbf{Z}_{ij} = \sqrt{Z_i} Z_{ij}$

Internal Higgs bosons

Examples for Higgs bosons entering loop corrections:

Vector boson self-energies:

e.g. in μ decay, precision observables, . . .
 $(V_{1,2,3} = Z, W^\pm)$



$\phi_{i,j} = h, H, A$ (tree-level states): \Rightarrow ok

But what if $\phi_{i,j} = h_1, h_2, h_3$?

\Rightarrow How to include higher-order corrections to the Higgs bosons properly?

\Rightarrow How to define “effective couplings” ?

Two possibilities:

1.) “ p^2 on-shell”: \mathbf{U}

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2 \text{ on-shell}} = \mathbf{U} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \quad p^2 \text{ on - shell : } \begin{aligned} \hat{\Sigma}_{ii}(p^2) &\rightarrow \hat{\Sigma}_{ii}(m_i^2) \\ \hat{\Sigma}_{ij}(p^2) &\rightarrow \hat{\Sigma}_{ij}((m_i^2 + m_j^2)/2) \end{aligned}$$

$$\mathbf{U} \operatorname{Re}(\mathbf{M}_{hHA}(p^2 \text{ on - shell})) \mathbf{U}^\dagger = \begin{pmatrix} M_{h_1,p^2 \text{ os}}^2 & 0 & 0 \\ 0 & M_{h_2,p^2 \text{ os}}^2 & 0 \\ 0 & 0 & M_{h_3,p^2 \text{ os}}^2 \end{pmatrix}$$

2.) “ $p^2 = 0$ ”: \mathbf{R} ($\mathcal{CP}C$ case, 2×2 mixing $\Rightarrow \alpha_{\text{eff}}$)

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{R} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \quad \mathbf{R} \mathbf{M}_{hHA}(0) \mathbf{R}^\dagger = \begin{pmatrix} M_{h_1,p^2=0}^2 & 0 & 0 \\ 0 & M_{h_2,p^2=0}^2 & 0 \\ 0 & 0 & M_{h_3,p^2=0}^2 \end{pmatrix}$$

Limit $p^2 \rightarrow 0$:

$$\mathbf{Z} \rightarrow \mathbf{R} : \quad \mathbf{R} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{R} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \quad \mathbf{R} \mathbf{M}_{hHA}(0) \mathbf{R}^\dagger = \begin{pmatrix} M_{h_1,p^2=0}^2 & 0 & 0 \\ 0 & M_{h_2,p^2=0}^2 & 0 \\ 0 & 0 & M_{h_3,p^2=0}^2 \end{pmatrix}$$

- \mathbf{R} in the 2×2 case is exactly α_{eff}
- \mathbf{R} corresponds to the effective potential approach

What is better?

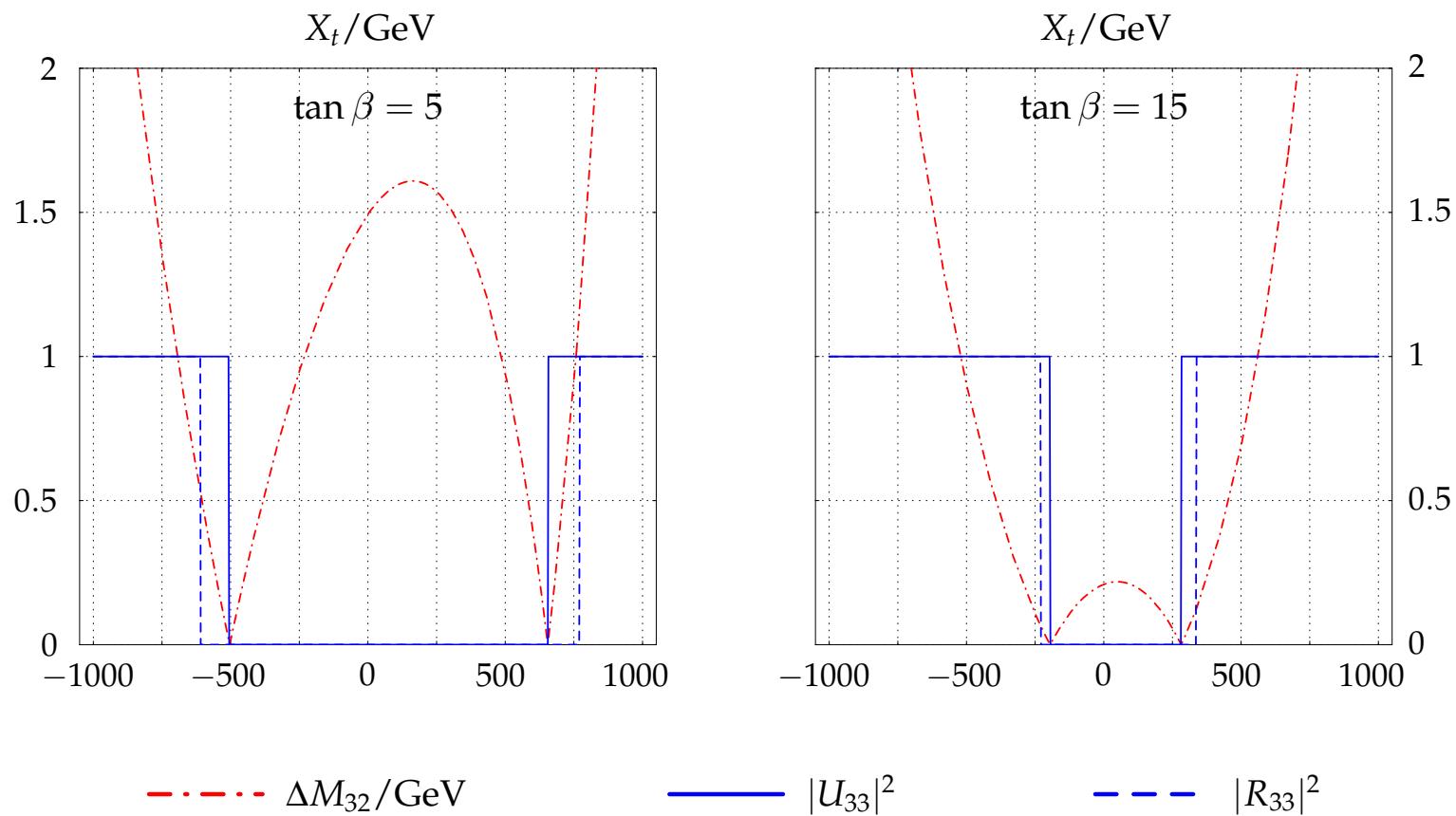
- 1.) “ p^2 on-shell”: \mathbf{U}
- 2.) “ $p^2 = 0$ ”: \mathbf{R}

Two possible tests:

1. Compare full decay width, evaluated with \mathbf{Z} ,
with approximations, evaluated with \mathbf{U} or \mathbf{R}
→ see later in “Numerical examples”
2. \mathbf{U}_{33}^2 and \mathbf{R}_{33}^2 correspond to the \mathcal{CP} -odd part of h_3
In the rMSSM: $\mathbf{U}_{33}^2, \mathbf{R}_{33}^2 = 0$ or 1 (depending on mass ordering)
Switch-over from 0 to 1 should happen for $\Delta M_{32} := M_{h_3} - M_{h_2} = 0$
→ compare switch-over with ΔM_{32}

→ Compare switch-over with ΔM_{32} :

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500 \text{ GeV}$, $\mu = 1000 \text{ GeV}$, $M_{H^\pm} = 150 \text{ GeV}$



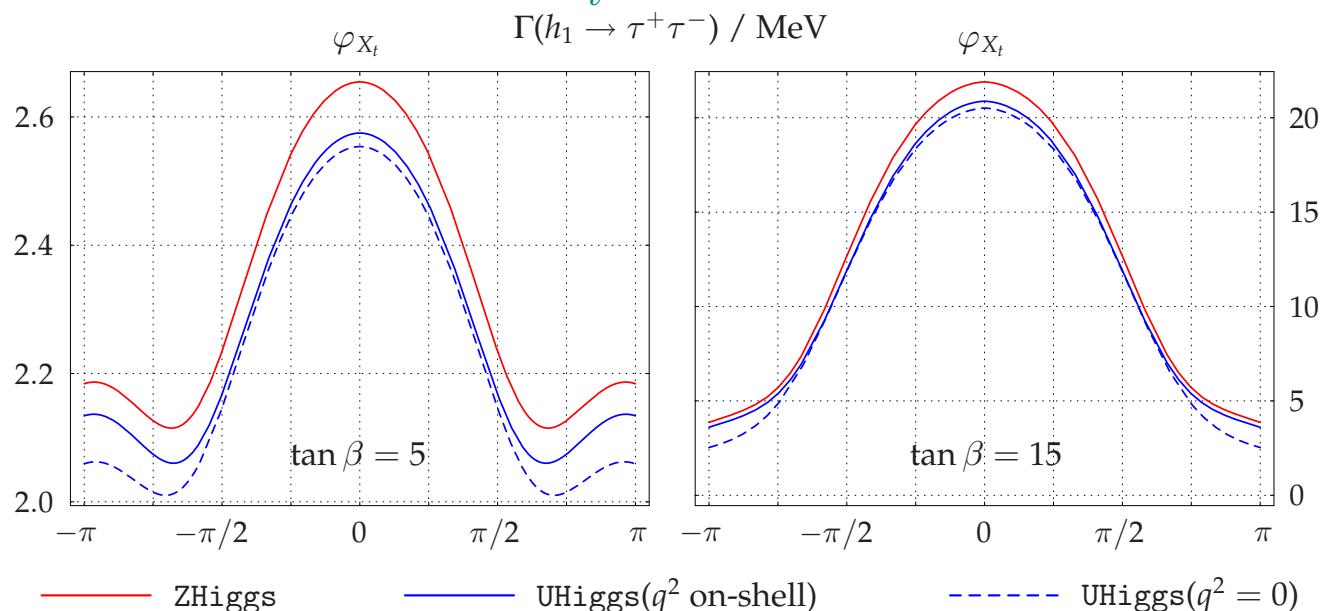
⇒ **U** gives the better results
⇒ use **U** for effective couplings

Numerical example for external Higgs bosons:

[T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '07]

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500 \text{ GeV}$, $A_t = 1000 \text{ GeV}$, $\mu = 1000 \text{ GeV}$, $M_{H^\pm} = 150 \text{ GeV}$

$\Gamma(h_1 \rightarrow \tau^+ \tau^-)$ as a function of ϕ_{X_t}



red solid: **Z** , blue solid: **U** , blue dashed: **R**

⇒ **U** gives results closer to full result than **R**

⇒ deviations at the 5-10% level

Summary: treatment of “higher-order” corrected Higgs bosons:

1. external/on-shell Higgs bosons

amplitude with on-shell Higgs boson i :

$$A_{h_i xy} \sim \sqrt{Z_i} (Z_{ih} C_{hxy} + Z_{iH} C_{Hxy} + Z_{iA} C_{Axy})$$

Z_i , Z_{ij} : finite wave function renormalizations

Written more compact with the **Z matrix**:

$$\mathbf{Z}_{ij} = \sqrt{Z_i} Z_{ij}$$

resulting in

$$A_{h_i xy} \sim \mathbf{Z}_{ih} C_{hxy} + \mathbf{Z}_{iH} C_{Hxy} + \mathbf{Z}_{iA} C_{Axy}$$

2. Higgs bosons in loop corrections

rotate tree-level couplings with **U** or **R**:

$$\begin{aligned} C_{h_i xy} &= \mathbf{U}_{ih} C_{hxy} + \mathbf{U}_{iH} C_{Hxy} + \mathbf{U}_{iA} C_{Axy} \\ C_{h_i xy} &= \mathbf{R}_{ih} C_{hxy} + \mathbf{R}_{iH} C_{Hxy} + \mathbf{R}_{iA} C_{Axy} \end{aligned}$$