Neutrino Physics and LFV:
a theoretical overview

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Les Rencontres de Physique
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La Thuile
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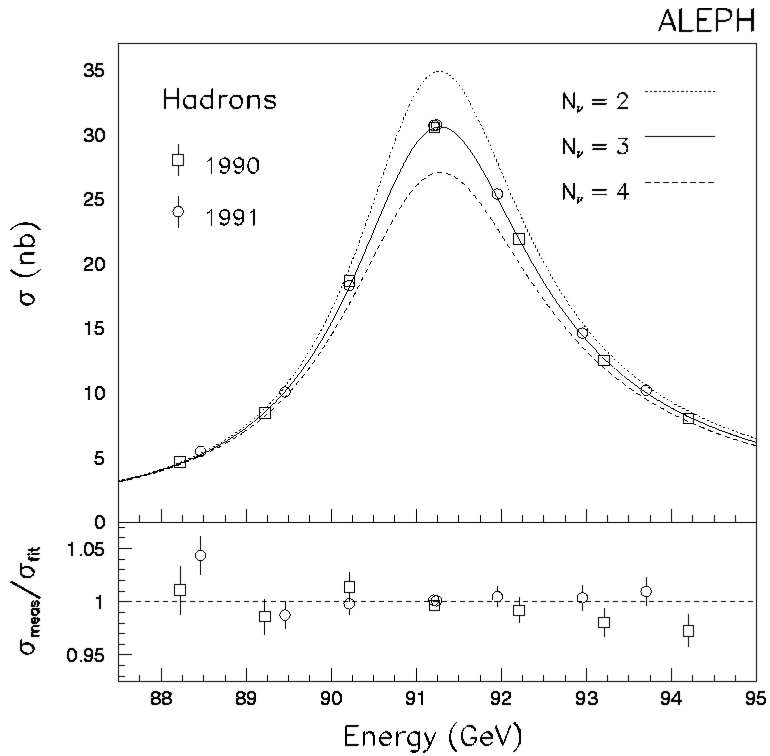
Facts in neutrino physics

1- There are three light active neutrinos

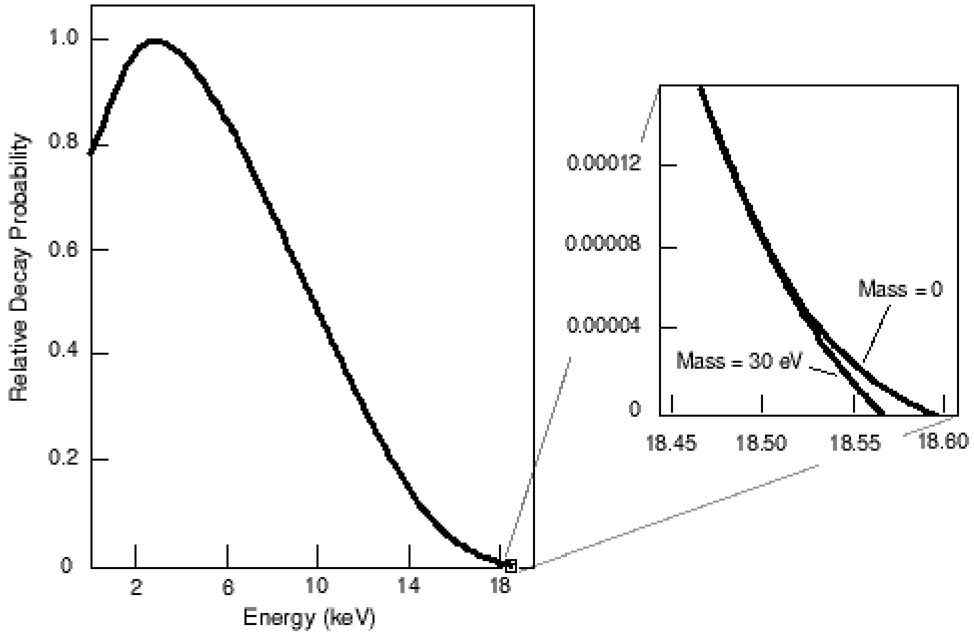
(three neutrinos which electroweak charge lighter than 45 GeV)

From measurements of the invisible decay width of the Z^0 at LEP

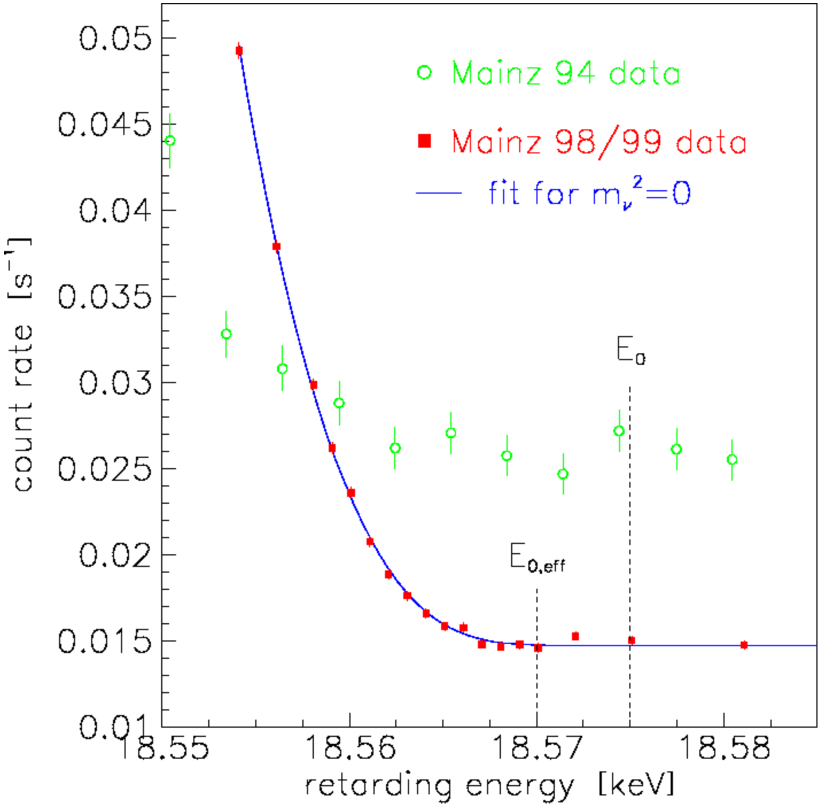


2- The electron neutrino is very light ($m_{\nu_e} < 2.2$ eV)

From measurements of the endpoint of the tritium beta decay electron spectrum: $^3He \rightarrow H\ e^- \bar{\nu}_e$

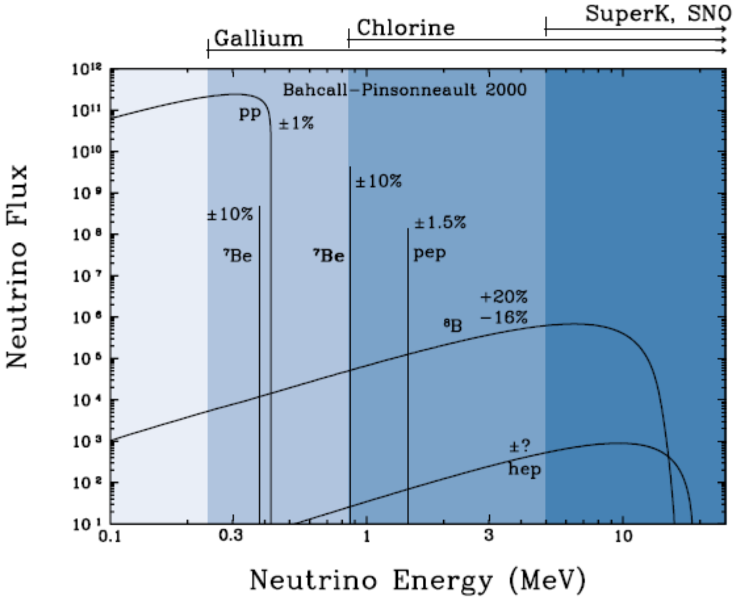


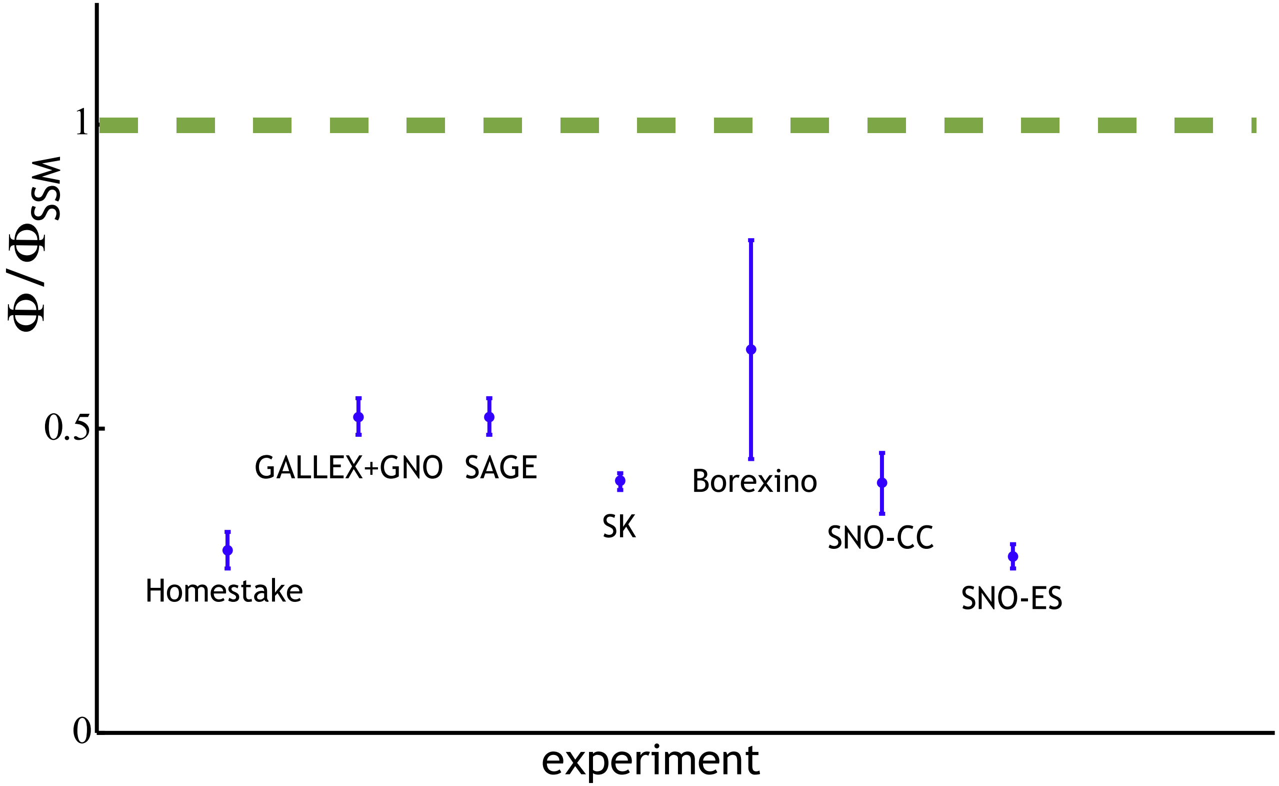
The Beta Decay Spectrum for Molecular Tritium



Future sensitivity: 0.2 eV (KATRIN)

3- Electron neutrinos produced in the Sun disappear on their way to the Earth...





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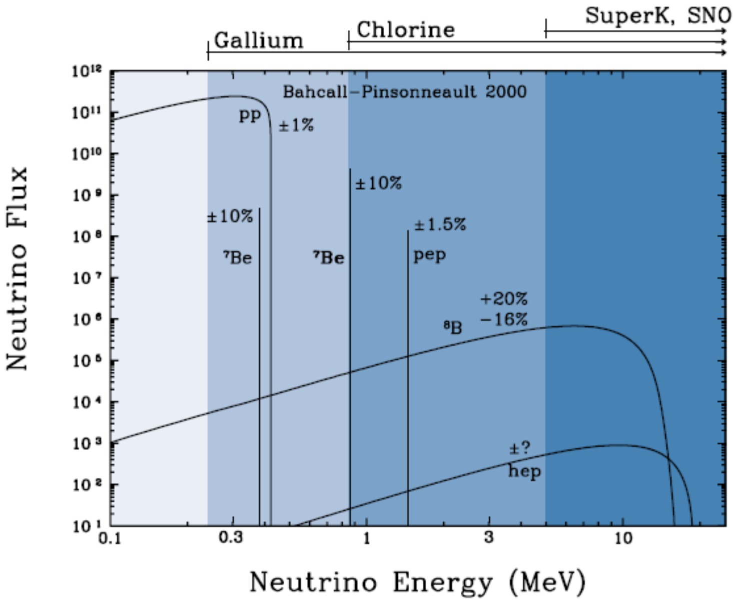
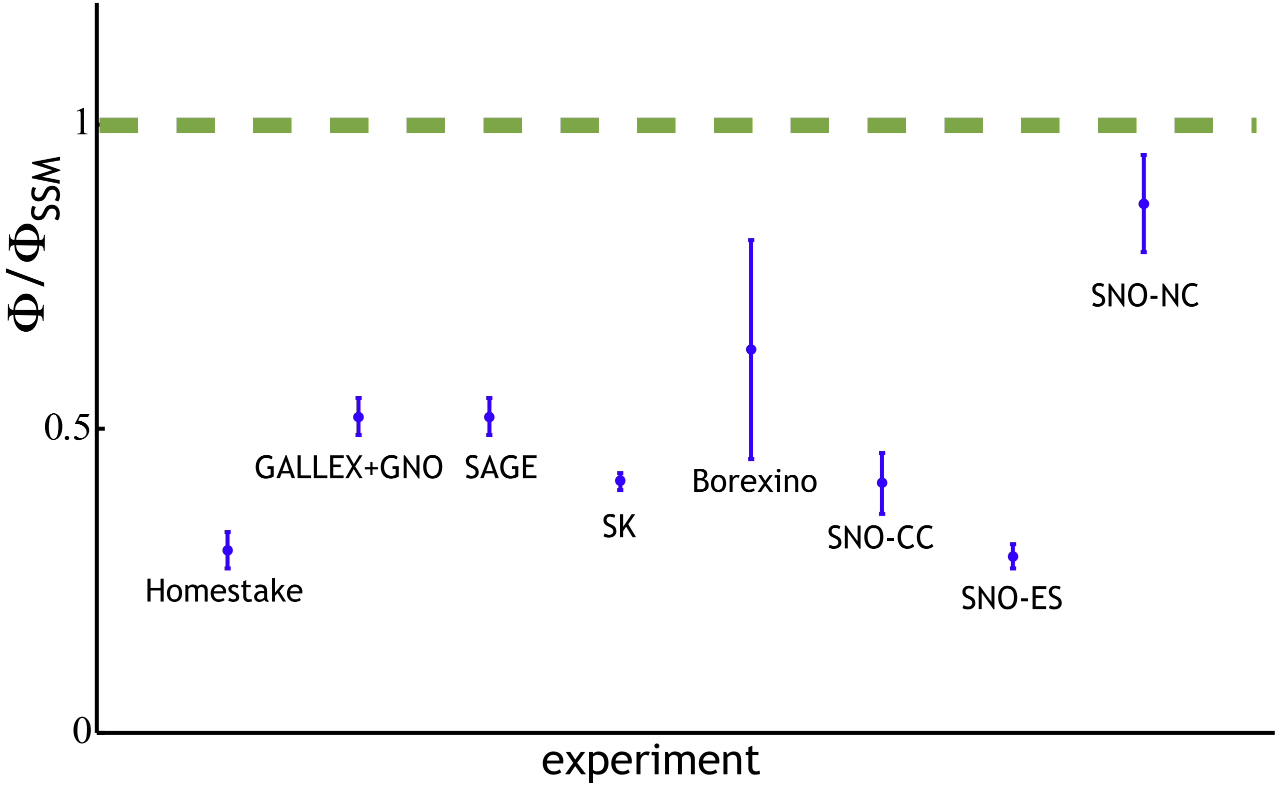
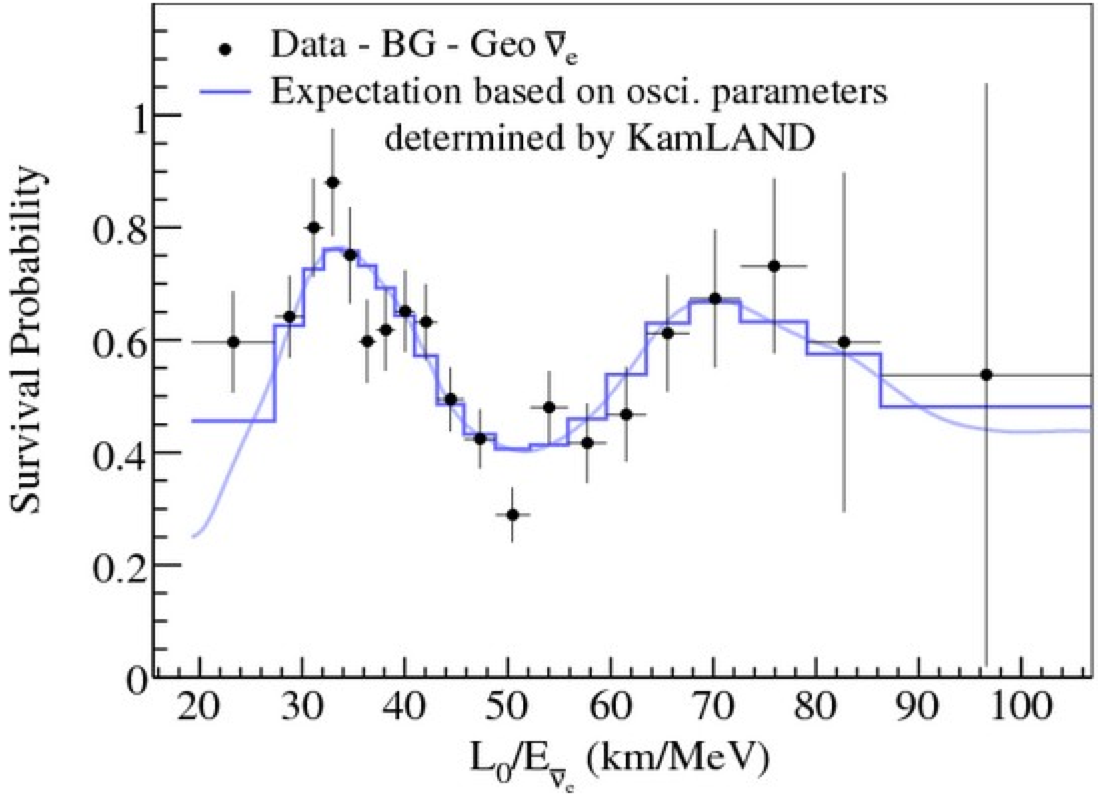




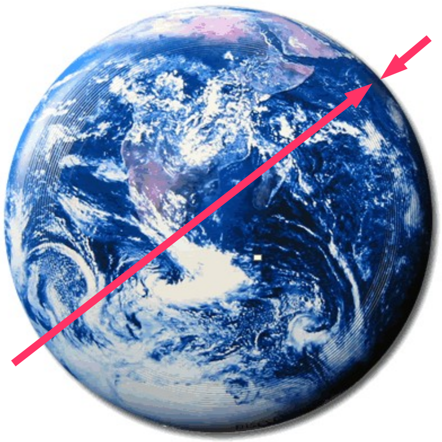
... but the total neutrino flux is conserved

4- Electron antineutrinos produced in nuclear power plants disappear after travelling a few kilometers, and then reappear!



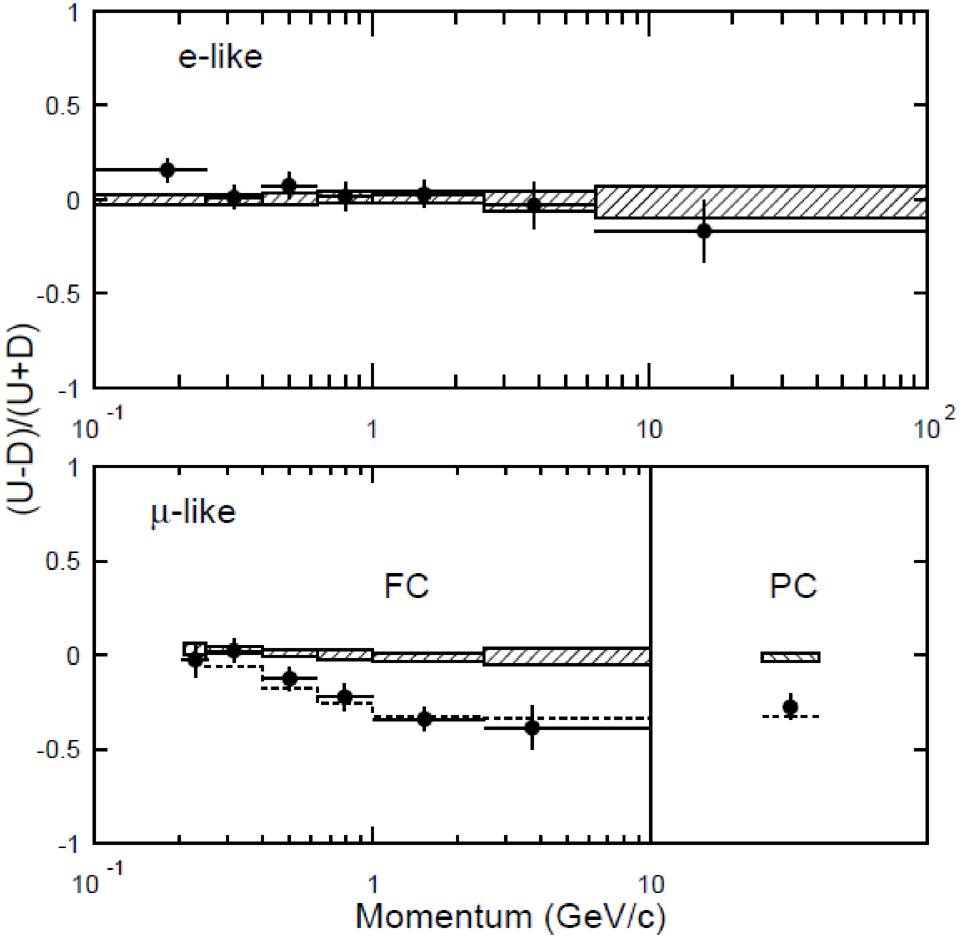
But no disappearance has been observed in shorter baselines (~1km, CHOOZ)

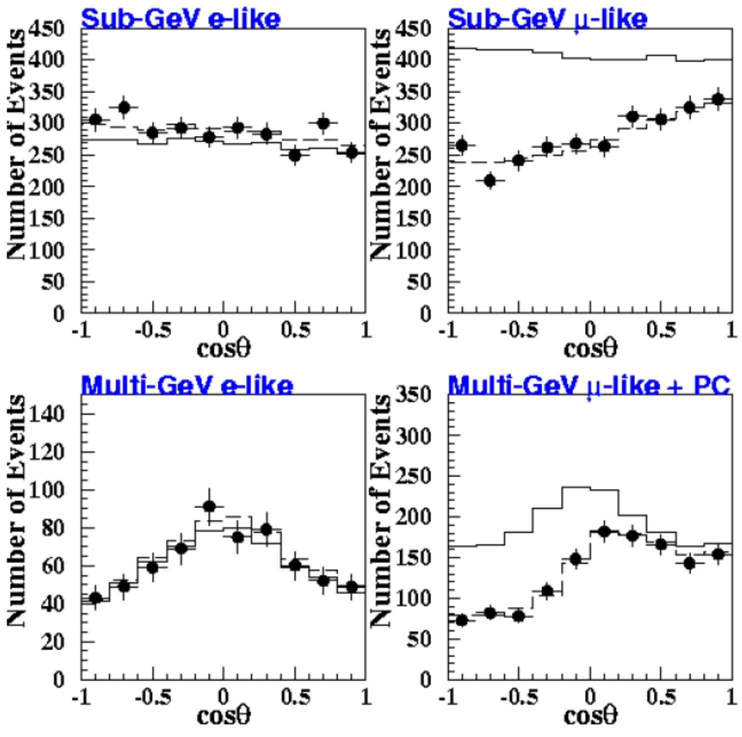
5- Muon neutrinos coming from the other side of the Earth disappear



Downward moving
neutrinos

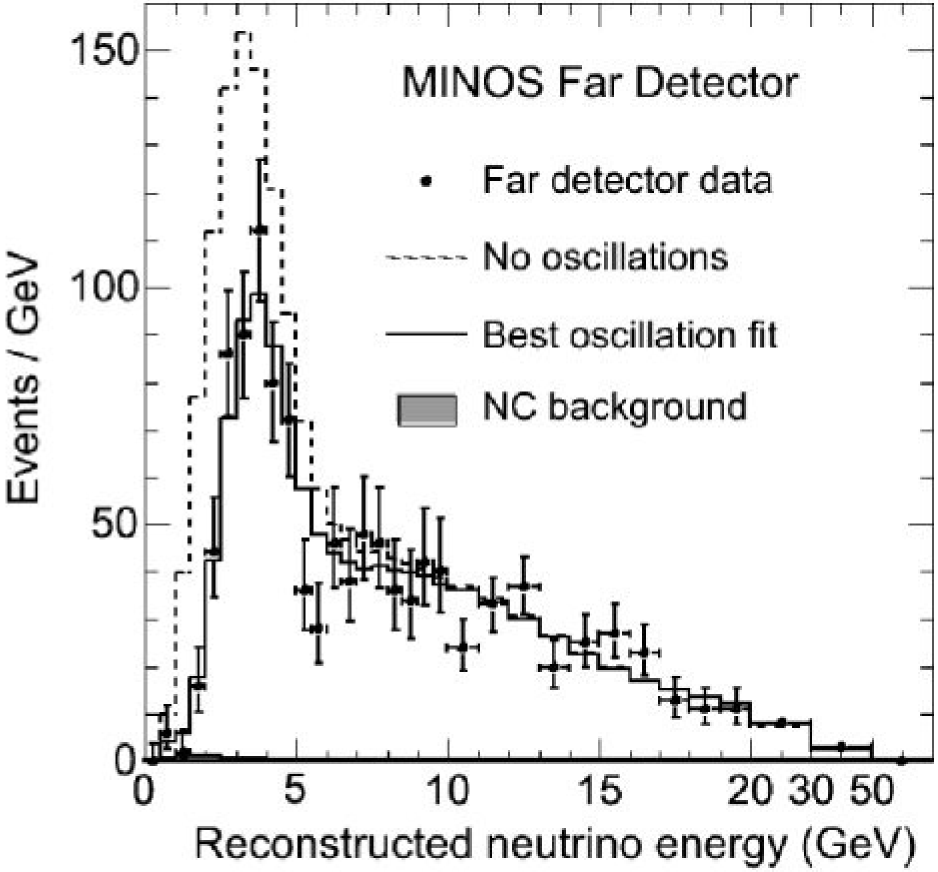
Upward moving
neutrinos





Super-Kamiokande

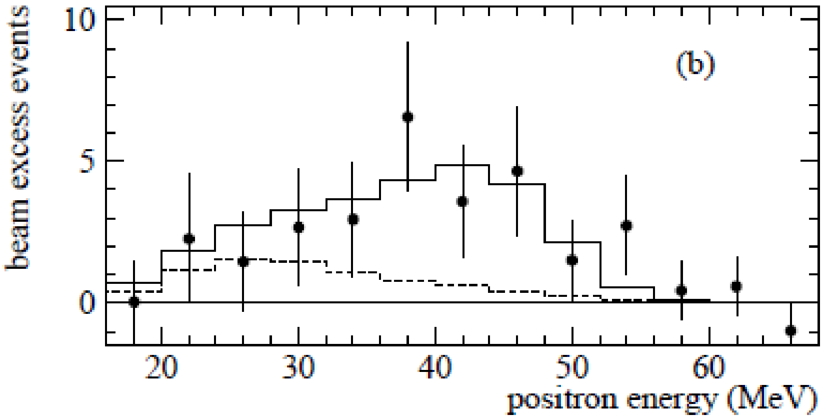
6- Muon neutrinos produced in pion decay disappear on their way to a far detector (K2K/MINOS)

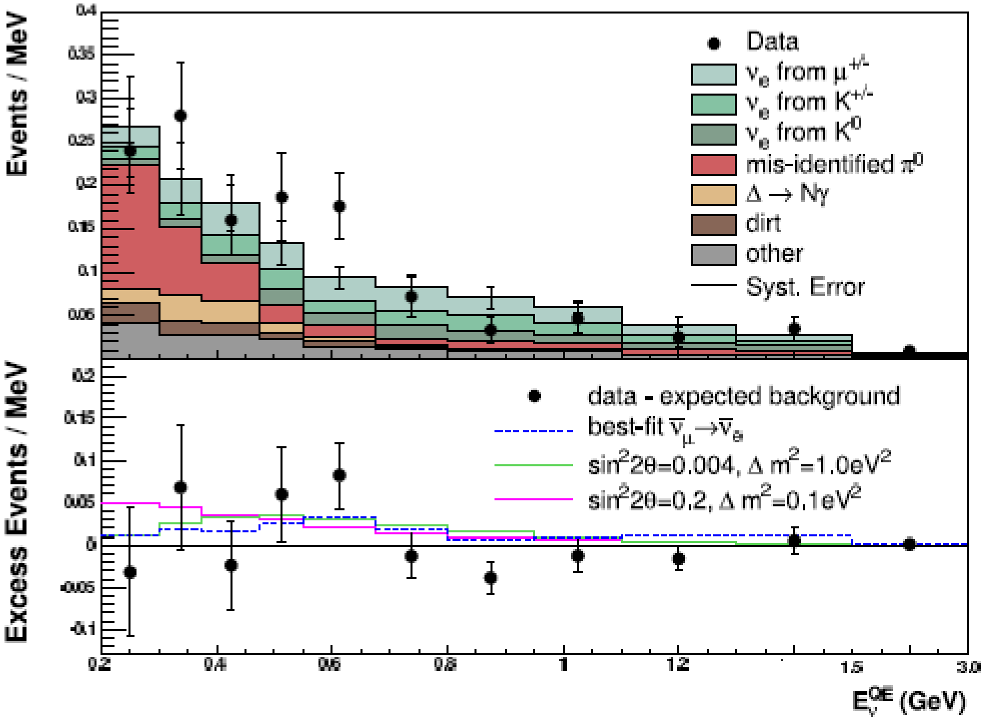


7- electron antineutrinos appear in a muon antineutrino beam from the decay of pions.

Observed only by LSND...

... and not confirmed by MiniBooNE





These facts have dramatic implications for Particle Physics

In the Standard Model $-\mathcal{L}^{\text{Yuk}}_{\text{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \text{h.c.}$

$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau$$

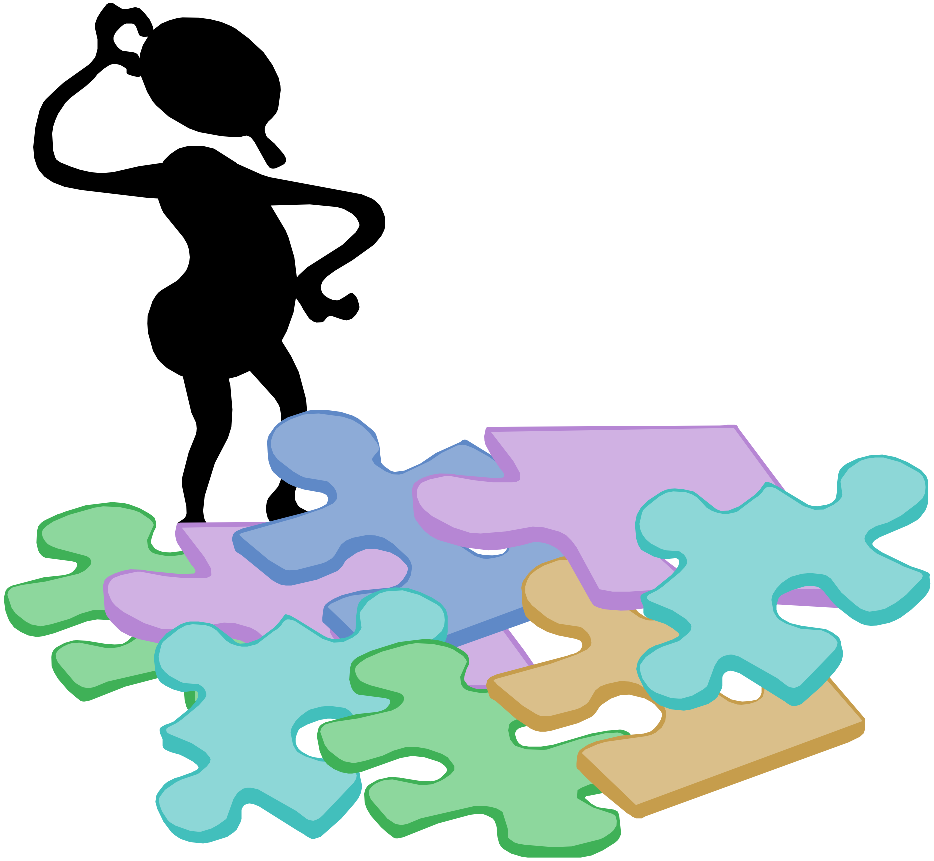
Charged lepton masses

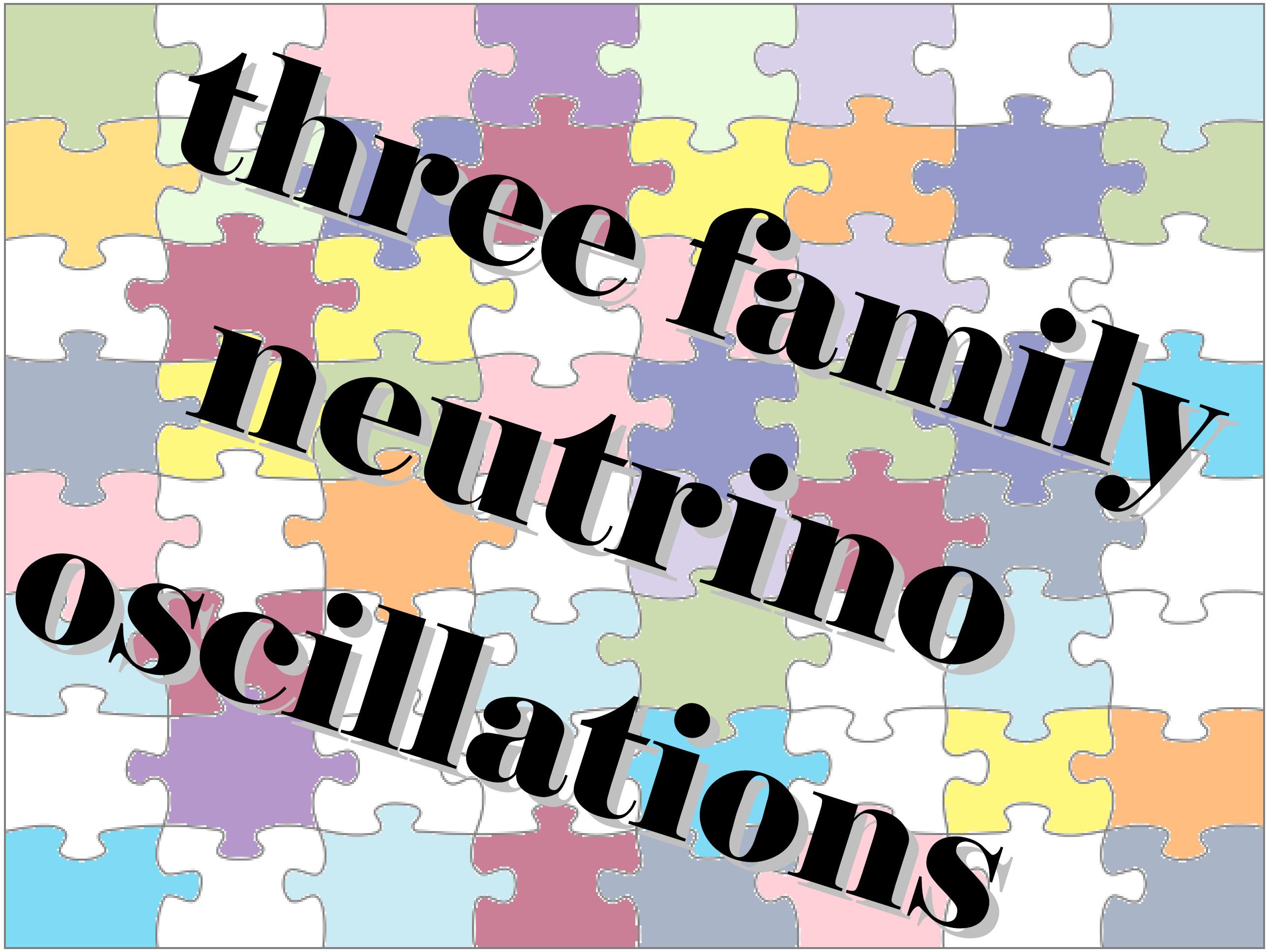
Family lepton numbers and total lepton number are strictly conserved.

The disappearance of electron/muon neutrinos and antineutrinos constitute *evidences* that lepton flavour is not conserved.

Physics beyond the Standard Model

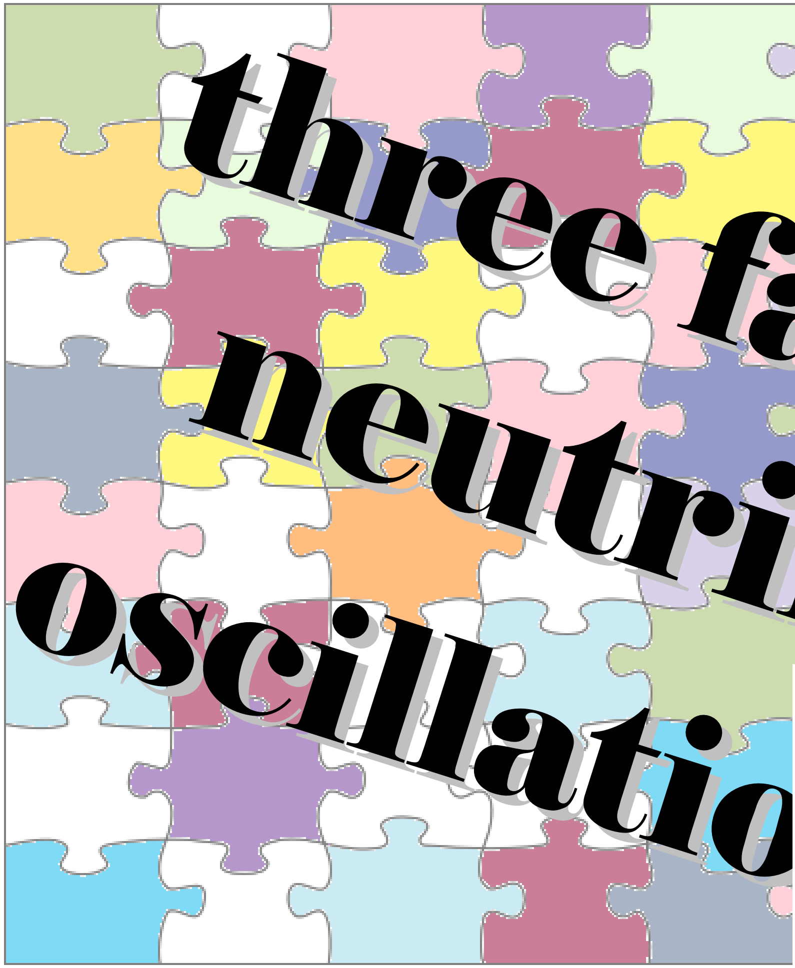
Which physics beyond the Standard Model?





three family neutrino oscillations

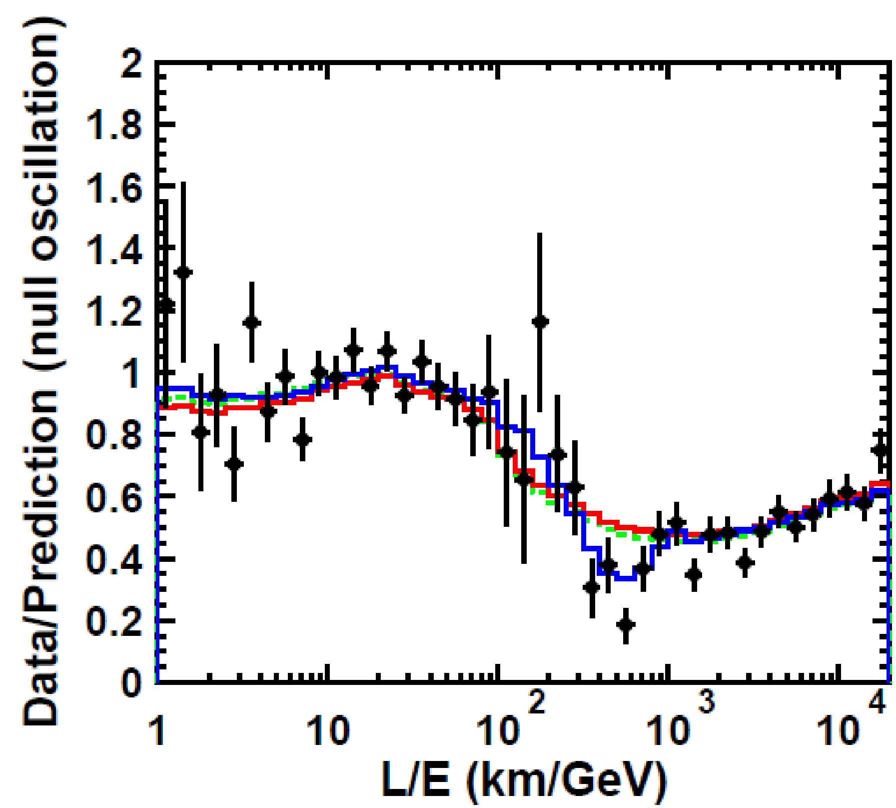
LSND

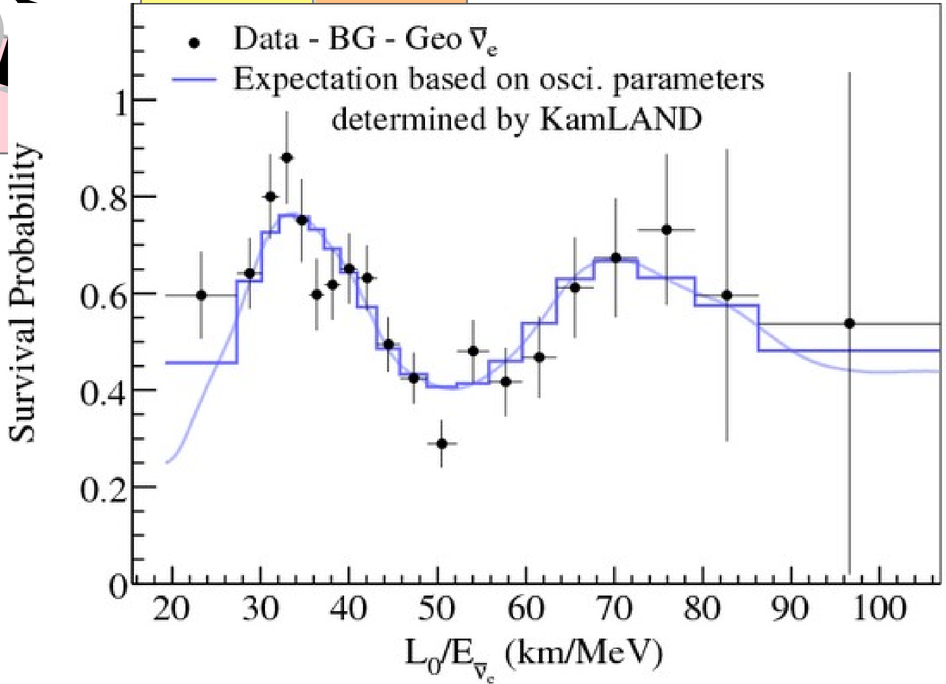


three family neutrino oscillatio

~~Neutrino decay~~

~~Quantum decoherence~~





~~Neutrino decay~~

~~Quantum decoherence~~

~~Resonant spin-flip flavour conversion in the Sun~~

Status of neutrino oscillations

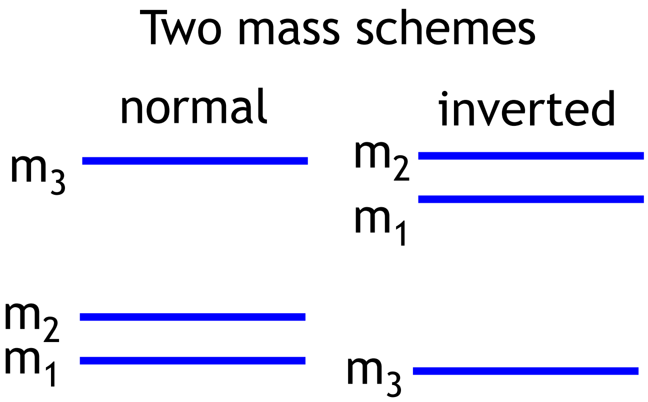
All the experimental results (except LSND) can be simultaneously explained by flavour oscillations of three active neutrinos.

If neutrinos are massive,

$$|\nu_\alpha\rangle = (U_{\rm lep})_{\alpha i}|\nu_i\rangle$$

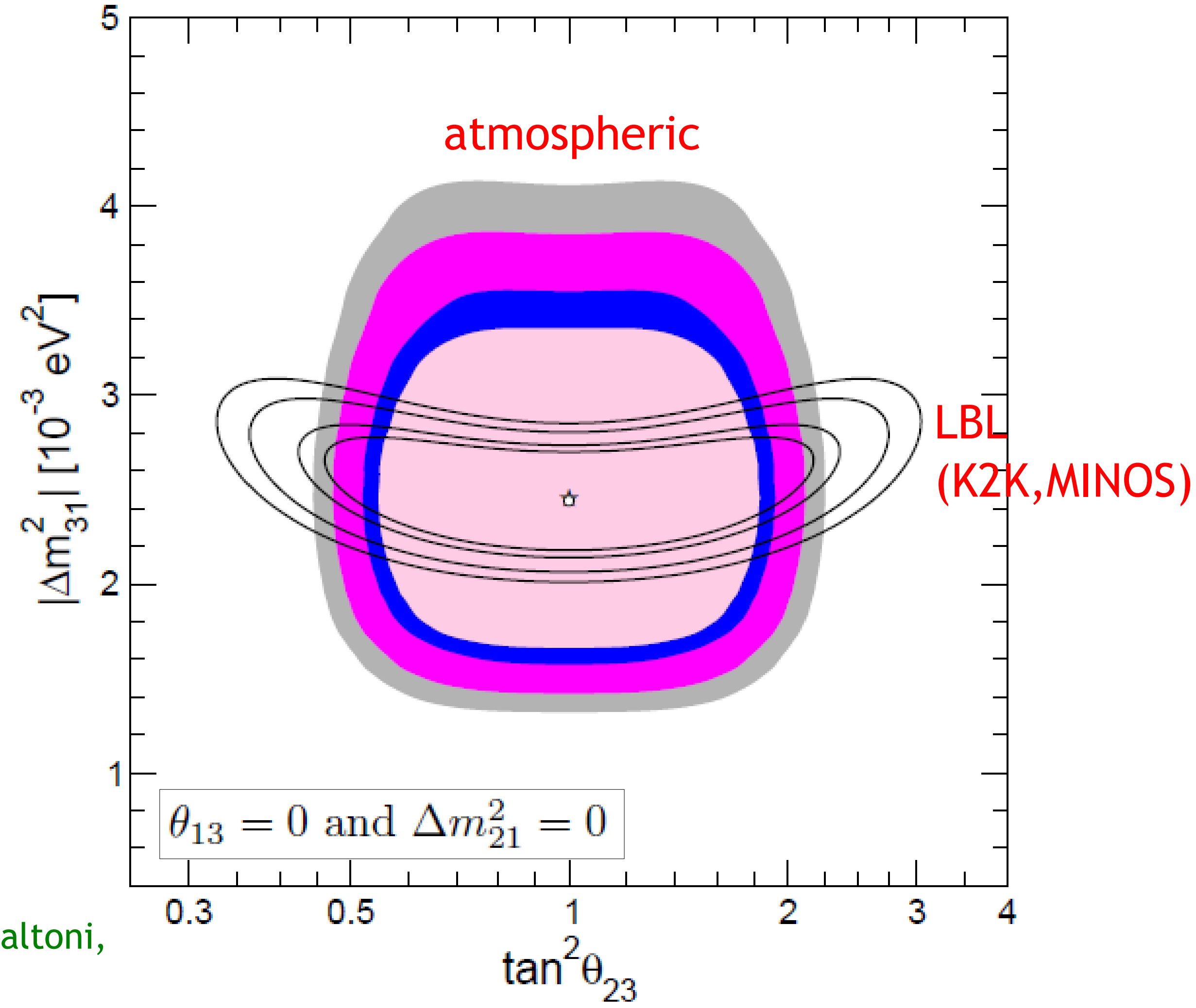
Flavour eigenstate
α=e, μ, τ

mass eigenstate
i=1, 2, 3



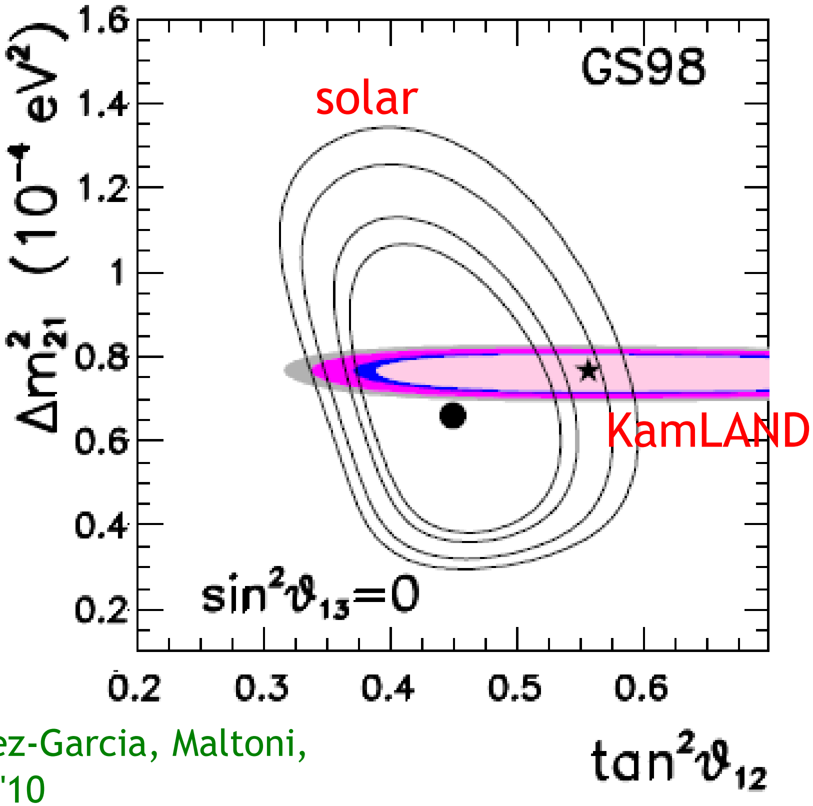
$$U_{\rm lep} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} (e^{-i\phi/2}, e^{-i\phi'/2}, 1)$$

Atmospheric data + LBL



Gonzalez-Garcia, Maltoni, Salvado'10

Solar data + KamLAND



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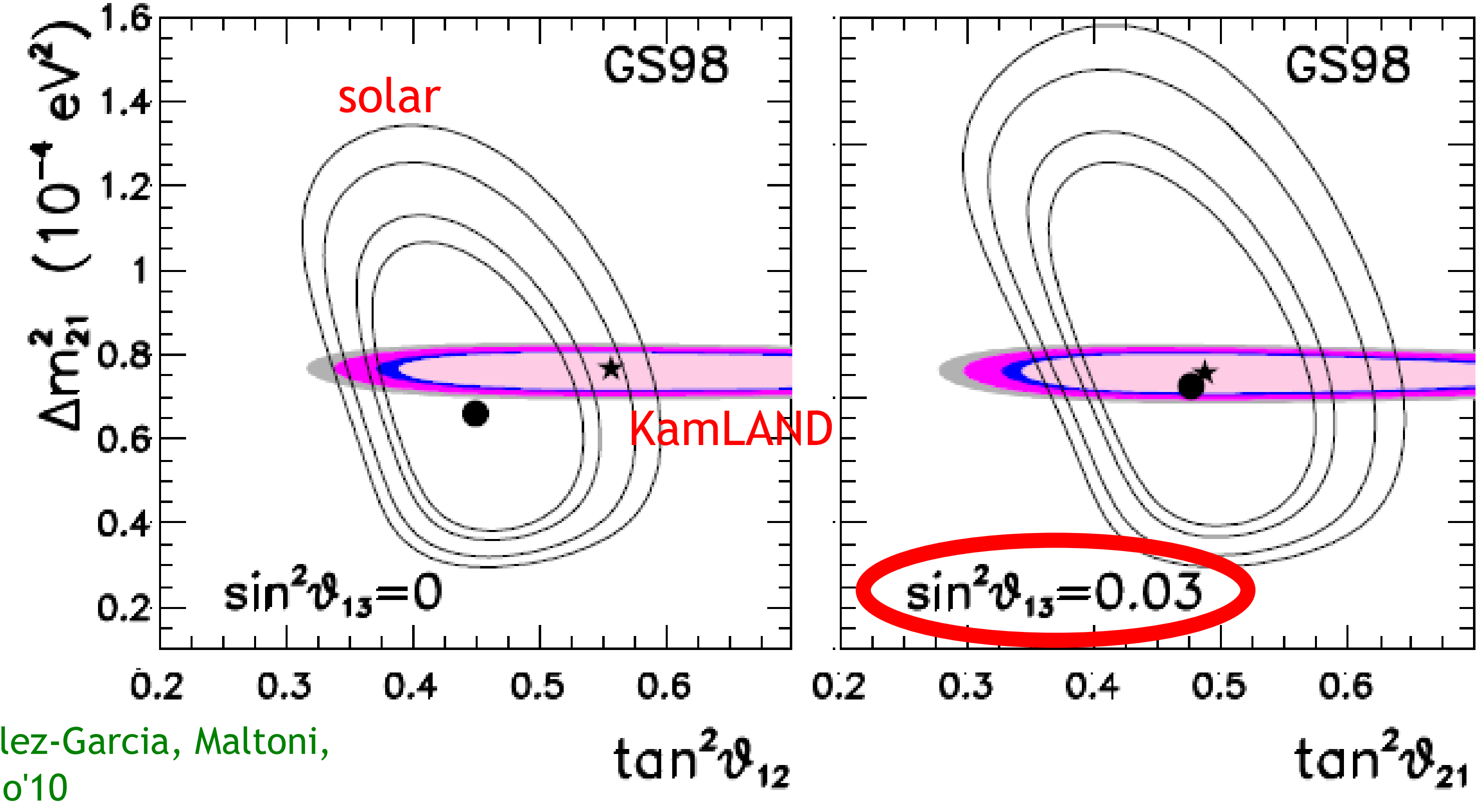

Gonzalez-Garcia, Maltoni,
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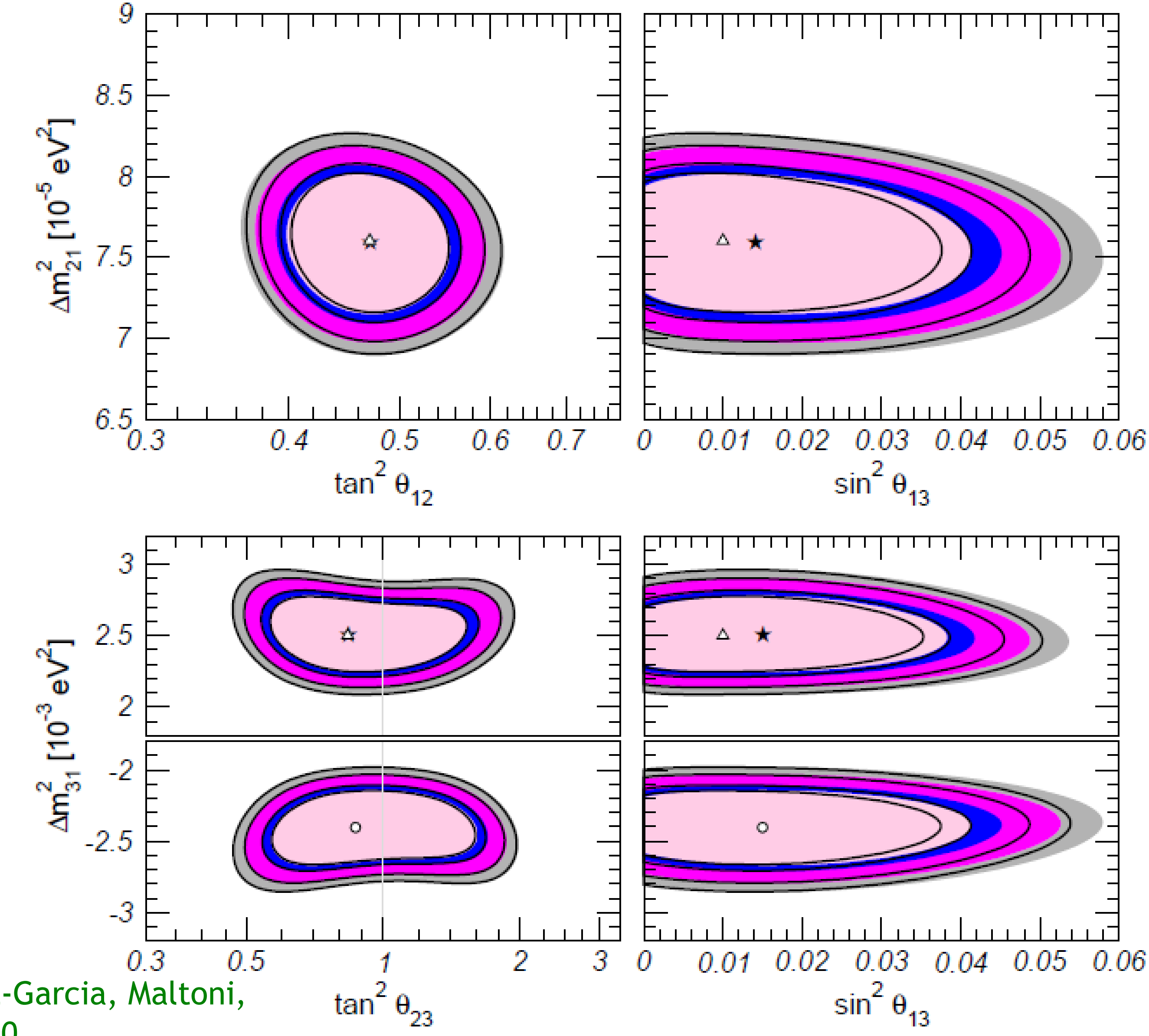
Global 3ν oscillation analysis



Gonzalez-Garcia, Maltoni, Salvado'10

Global 3ν oscillation analysis

$$\Delta m^2_{21} = 7.59 \pm 0.20 \; \left({}^{+0.61}_{-0.69}\right) \times 10^{-5} \text{ eV}^2$$

$$\Delta m^2_{31} = \begin{cases} -2.40 \pm 0.11 \; \left({}^{+0.37}_{-0.39}\right) \times 10^{-3} \text{ eV}^2 \text{ (inverted)} \\ +2.51 \pm 0.12 \; \left({}^{+0.39}_{-0.36}\right) \times 10^{-3} \text{ eV}^2 \text{ (normal)} \end{cases}$$

$$\theta_{12} = 34.4 \pm 1.0 \; \left({}^{+3.2}_{-2.9}\right)$$

$$\theta_{23} = 42.3\,^{+5.3}_{-2.8} \; \left({}^{+11.4}_{-7.1}\right)$$

$$\theta_{13} = 6.8\,^{+2.6}_{-3.6} \; (\leq 13.2)$$

$$\left[\, \sin^2\theta_{13} = 0.014\,^{+0.013}_{-0.011} \; (\leq 0.052) \,\right]$$

$$\delta_{\text{CP}} \in [0,\, 360]$$

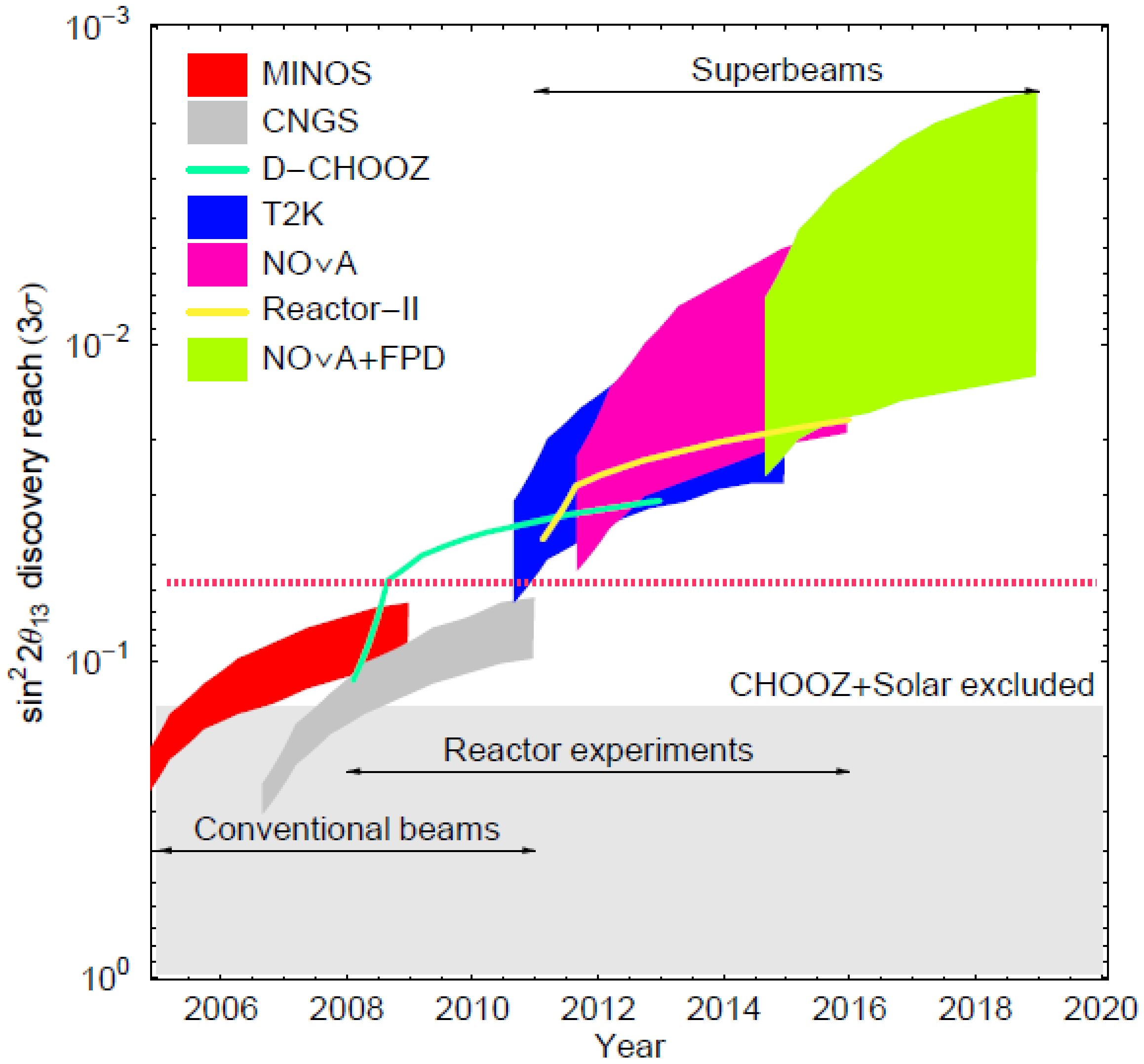
Open questions in neutrino oscillations:

θ_{13}

sign(Δm^2_{31})/neutrino mass scheme

CP violation

Absolute neutrino mass scale



Albrow et al.'05

If the hint from the global fit is correct, we will have soon evidences for non-vanishing θ_{13} from reactor experiments and superbeams.

Even with this limited information, we can already notice some features:

- Neutrino masses are tiny, $m_\nu < \mathcal{O}(1 \text{ eV})$
- Two large mixing angles ($\theta_{\text{atm}} \simeq \pi/4$, $\theta_{\text{sol}} \simeq \pi/6$)

  One small mixing angle ($\theta_{13} \simeq 0$)

$$U_{\text{lep}} \simeq \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

- The two heaviest neutrinos present a mild mass hierarchy

$$\Delta m^2_{\text{atm}} = m_3^2 - m_1^2 \quad \longrightarrow \quad m_3 = \sqrt{\Delta m^2_{\text{atm}} + m_1^2}$$

$$\Delta m^2_{\text{sol}} = m_2^2 - m_1^2 \quad \longrightarrow \quad m_2 = \sqrt{\Delta m^2_{\text{sol}} + m_1^2}$$

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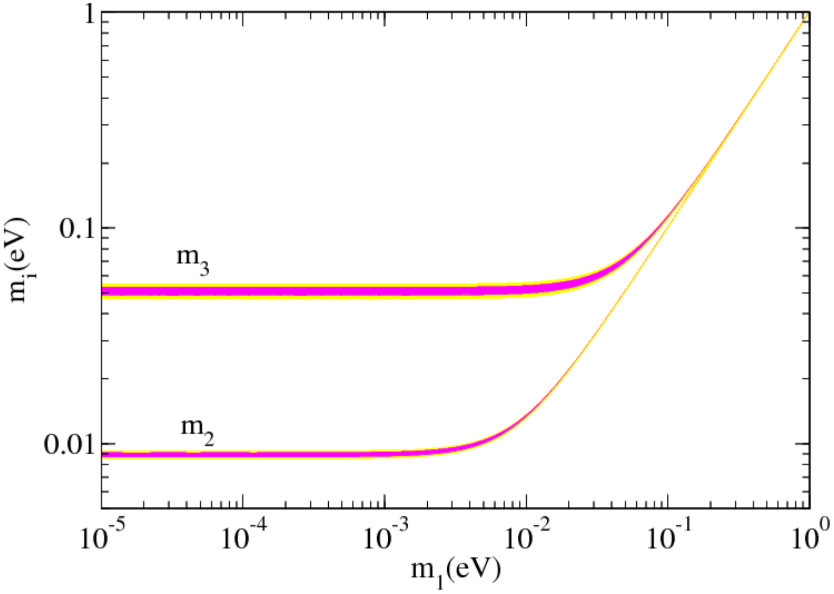
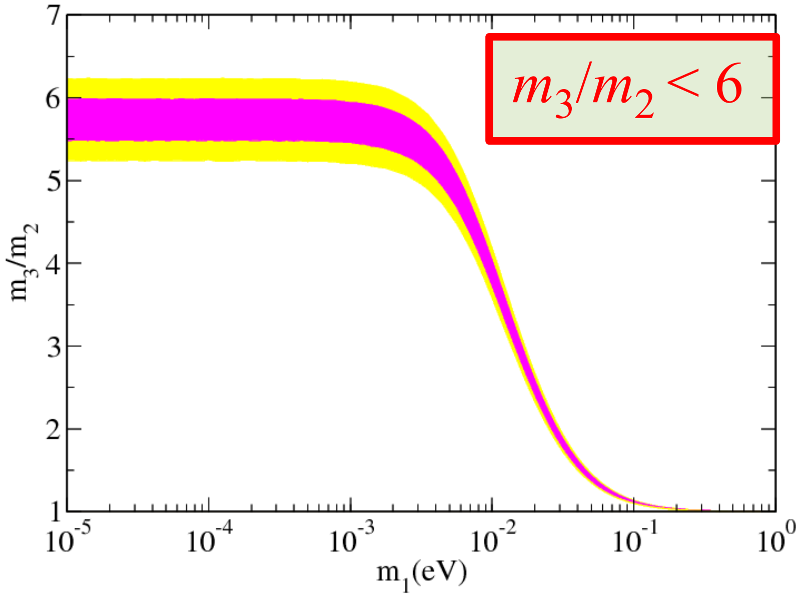




$m_3/m_2 < 6$

Compare with the quark sector

$$\left.\begin{aligned} m_u &= 1.5 \text{ to } 3.3\,\mathrm{MeV}\\ m_c &= 1.27^{+0.07}_{-0.11}\,\mathrm{GeV}\\ m_t &= 171.3 \pm 1.1 \pm 1.2\,\mathrm{GeV}\end{aligned}\right\} \quad \begin{aligned} m_t/m_c &\simeq 140\\ m_c/m_u &\simeq 550\end{aligned}$$

vs. $m_3/m_2 < 6$ in ν sector

$$\left.\begin{aligned} m_d &= 3.5 \text{ to } 6.0\,\mathrm{MeV}\\ m_s &= 105^{+25}_{-35}\,\mathrm{MeV}\\ m_b &= 4.20^{+0.17}_{-0.07}\,\mathrm{GeV}\end{aligned}\right\} \quad \begin{aligned} m_b/m_s &\simeq 44\\ m_s/m_d &\simeq 19\end{aligned}$$

$$|U_{\mathrm{CKM}}| \simeq \begin{pmatrix} 0.97 & 0.23 & 0.004\\ 0.23 & 0.973 & 0.04\\ 0.008 & 0.04 & 1 \end{pmatrix} \qquad |U_{\mathrm{lep}}| \simeq \begin{pmatrix} 0.82 & 0.56 & 0\\ 0.41 & 0.56 & 0.71\\ 0.41 & 0.56 & 0.71 \end{pmatrix}$$

Compare also with the charged lepton sector

$$\left.\begin{aligned} m_e &= 0.51\,\mathrm{MeV}\\ m_\mu &= 106\,\mathrm{MeV}\\ m_\tau &= 1.78\,\mathrm{GeV}\end{aligned}\right\} \quad \begin{aligned} m_\tau/m_\mu &\simeq 17\\ m_\mu/m_e &\simeq 208\end{aligned}$$

vs. $m_3/m_2 < 6$ in ν sector

A model of neutrino masses should address the following questions:

- Why tiny masses?
- Why mild mass hierarchy?
- Why large mixing angles?

And preferable, the model has to be testable.

Dozens (hundreds?) of models

Dirac neutrinos

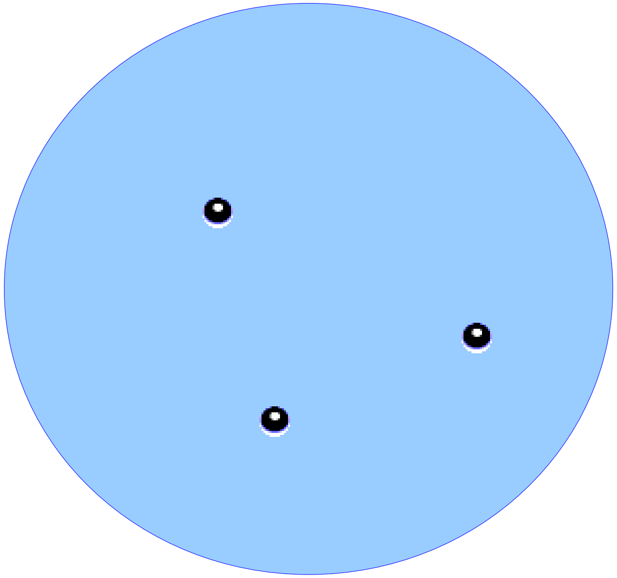
$$-\mathcal{L} = \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

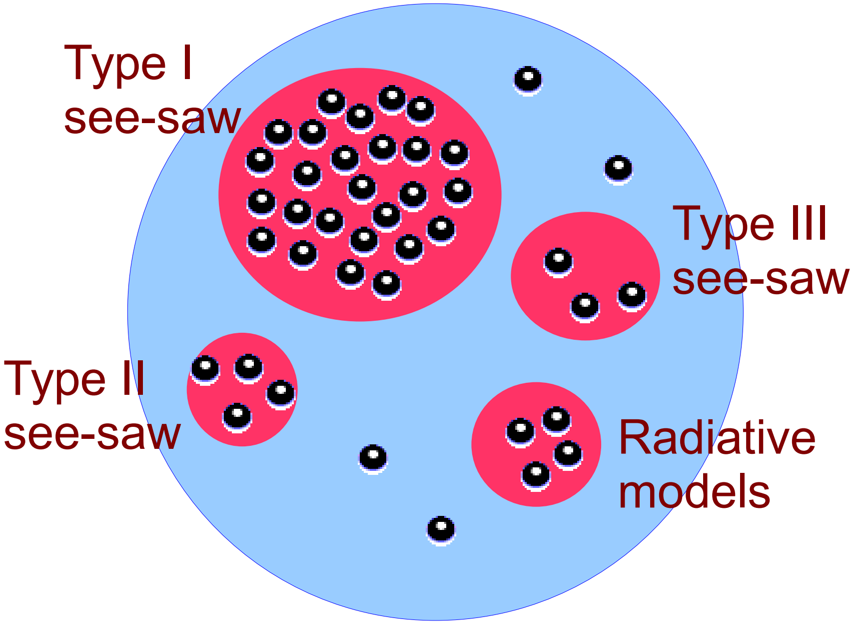
$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_{\text{lep}}$$

Majorana neutrinos

$$-\mathcal{L} = \frac{1}{2} \bar{\nu}_L^c m_M \nu_L + \text{h.c.}$$

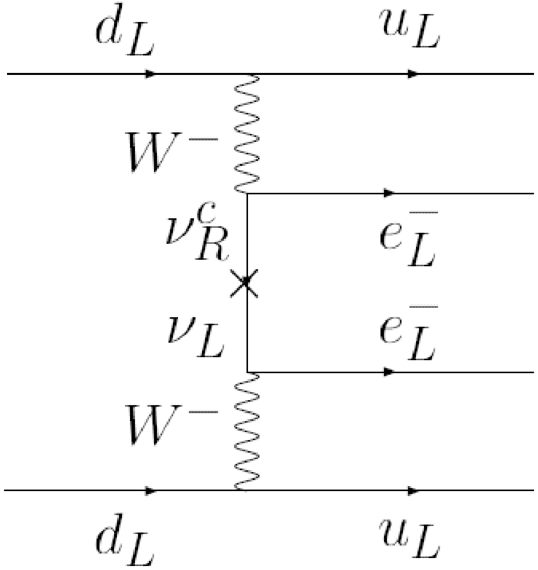
$U(3)_{e_R} \times U(3)_L \longrightarrow$ nothing





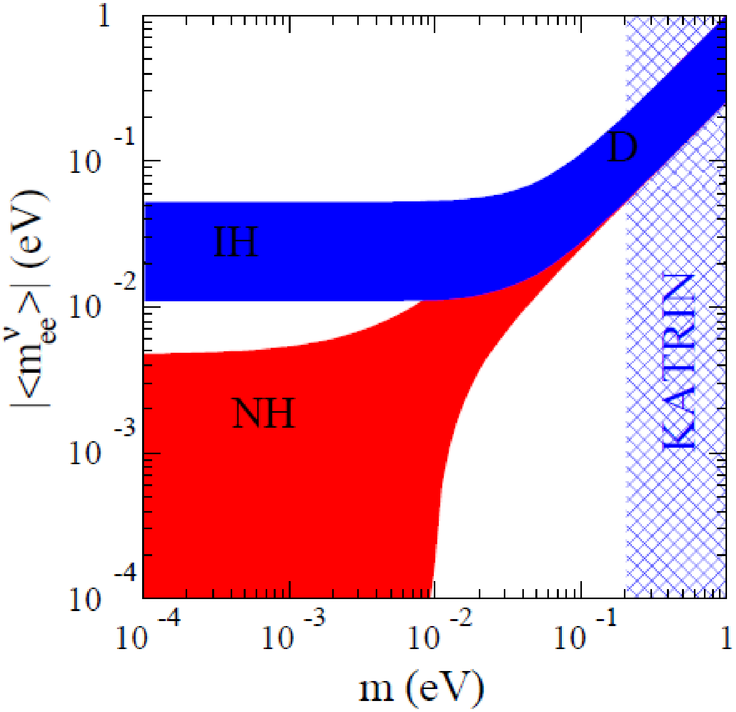
Dirac or Majorana?

The smoking gun for Majorana neutrinos: neutrinoless double beta decay



If neutrinos are Majorana particles, the nuclear process $(A,Z) \rightarrow (A,Z+2)+e^-+e^-$ is allowed

Not observed so far. Lifetime $>10^{24}$–10^{25} years



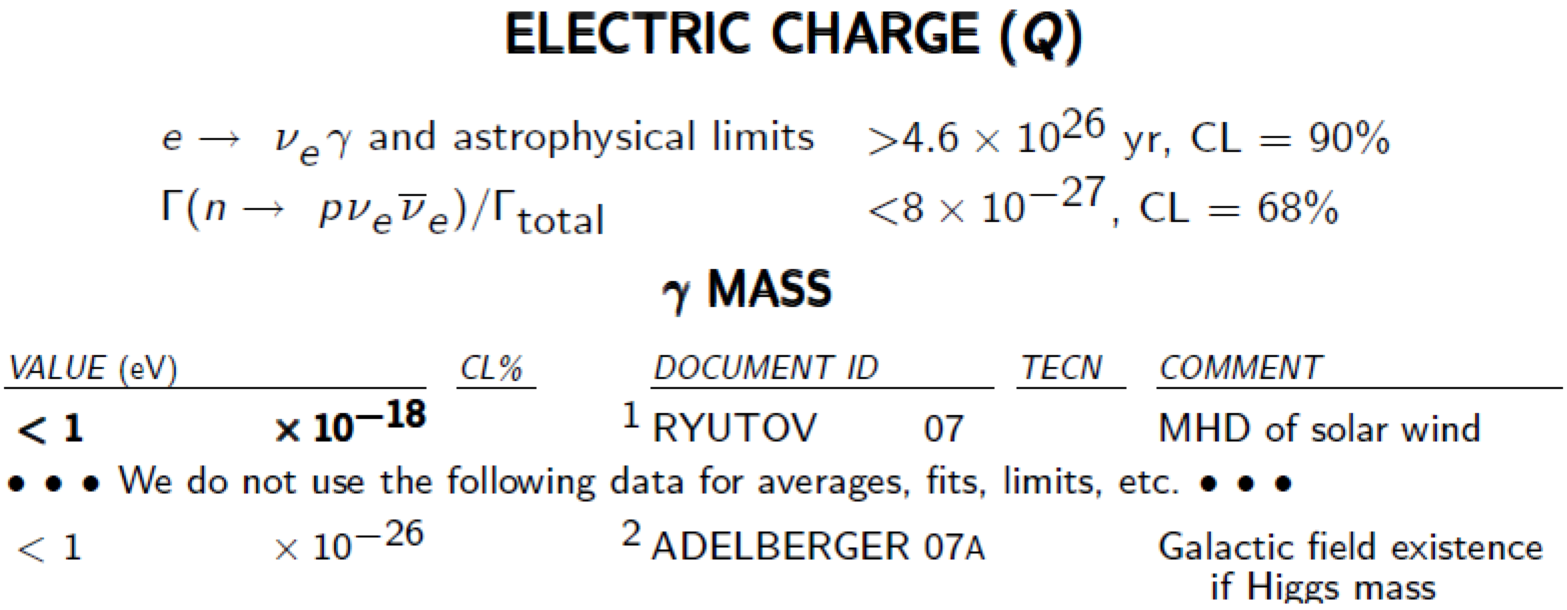
The rate of 0ν2β depends crucially on the spectrum. If neutrinos are degenerate or inverse hierarchical, 0ν2β could be observed in the next generation of experiments (CUORE, GERDA...)

Bahcall, Murayama
Peña-Garay

Is there a smoking gun for Dirac neutrinos?

A strong hint: neutrino charge

If neutrinos are Majorana particles, a neutrino charge would imply that electric charge is not a conserved quantity. However, present experiments are very well consistent with electric charge conservation:



PDG'09

ELECTRIC CHARGE (Q)

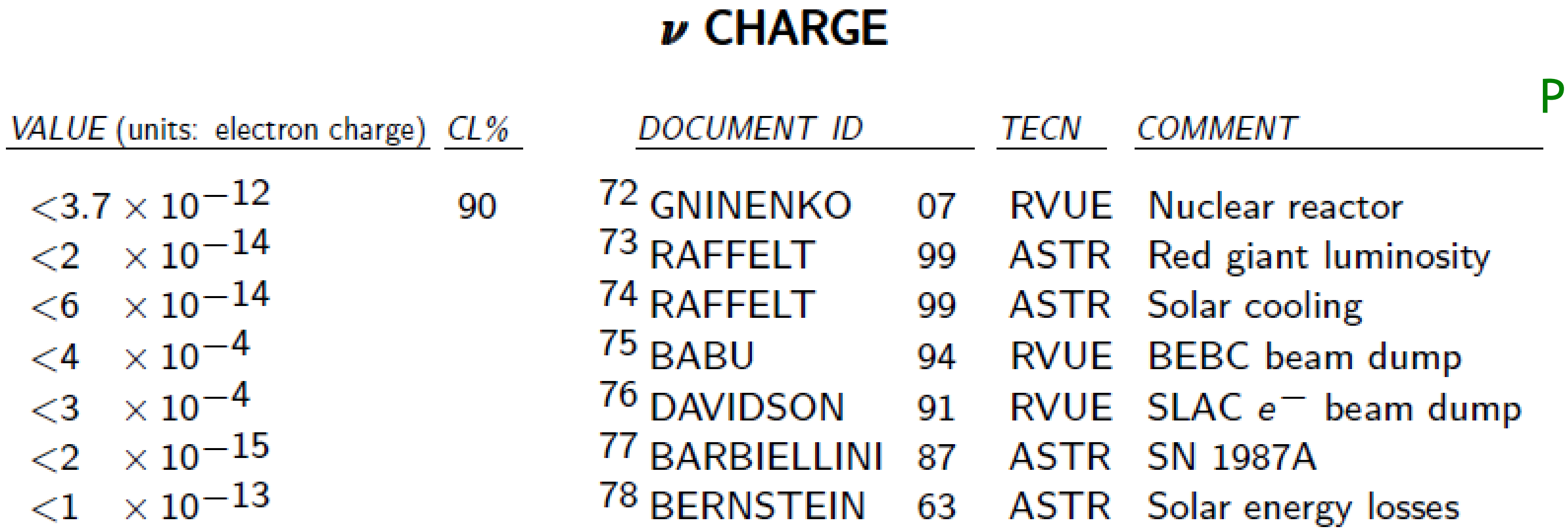
$e \to \nu_e \gamma$ and astrophysical limits	$>4.6 \times 10^{26}$ yr, CL = 90%
$\Gamma(n \to p \nu_e \overline{\nu}_e)/\Gamma_{\text{total}}$	$<8 \times 10^{-27}$, CL = 68%

γ MASS

VALUE (eV)	*CL%*	*DOCUMENT ID*	*TECN*	*COMMENT*
$< 1 \times 10^{-18}$		[1] RYUTOV 07		MHD of solar wind
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$< 1 \times 10^{-26}$		[2] ADELBERGER 07A		Galactic field existence if Higgs mass

Therefore, the observation of a neutrino charge would point to Dirac neutrino masses (or accept electric charge violation).

Experimental bounds on the neutrino charge

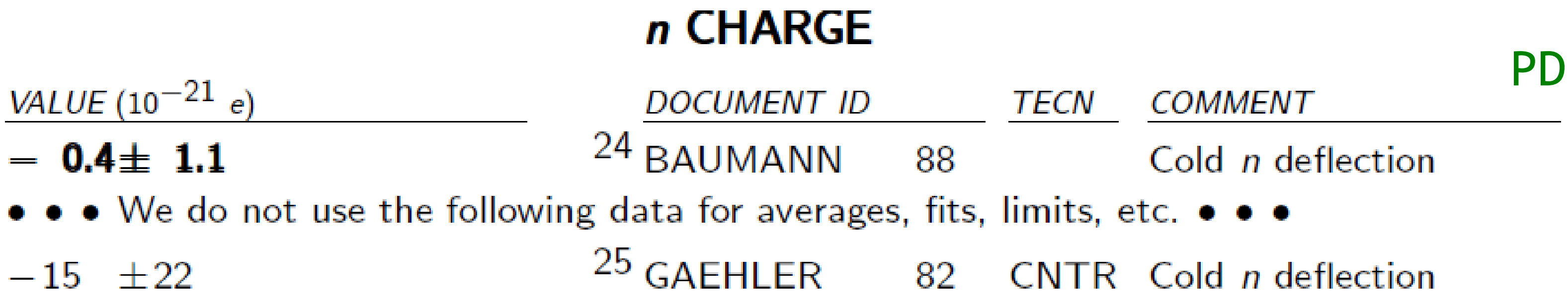


ν CHARGE

PDG'09

VALUE (units: electron charge)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
$<3.7 \times 10^{-12}$	90	[72] GNINENKO	07	RVUE	Nuclear reactor
$<2 \times 10^{-14}$		[73] RAFFELT	99	ASTR	Red giant luminosity
$<6 \times 10^{-14}$		[74] RAFFELT	99	ASTR	Solar cooling
$<4 \times 10^{-4}$		[75] BABU	94	RVUE	BEBC beam dump
$<3 \times 10^{-4}$		[76] DAVIDSON	91	RVUE	SLAC e^- beam dump
$<2 \times 10^{-15}$		[77] BARBIELLINI	87	ASTR	SN 1987A
$<1 \times 10^{-13}$		[78] BERNSTEIN	63	ASTR	Solar energy losses

In a model with B-L preserved (as in the SM extended with three massive Dirac neutrinos), the cancellation of gauge anomalies implies that the neutron charge is equal to minus the neutrino charge



n CHARGE

PDG'09

VALUE (10^{-21} *e*)	*DOCUMENT ID*		*TECN*	*COMMENT*
= 0.4± 1.1	[24] BAUMANN	88		Cold *n* deflection
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−15 ±22	[25] GAEHLER	82	CNTR	Cold *n* deflection

Present experiments are consistent both with Dirac neutrino masses and Majorana neutrino masses

Dirac masses?

The rest of the known fermions are Dirac particles. Why not neutrinos too?

The leptonic Lagrangian reads:

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + {\rm h.c.}$$

(Note that lepton number conservation has to be imposed *by hand*)

- Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $h_\nu \sim 10^{-12}$

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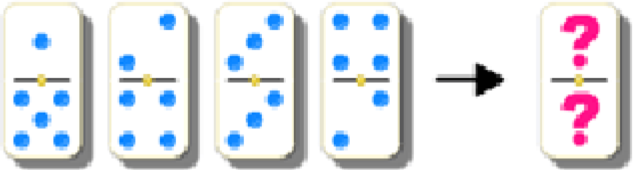
- Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $h_\nu \sim 10^{-12}$
- Why mild hierarchy? All the Dirac Yukawa matrices we know (h_u, h_d, h_e) have very hierarchical eigenvalues.



h_u very hier.

h_d very hier.

h_e very hier.

h_ν ???

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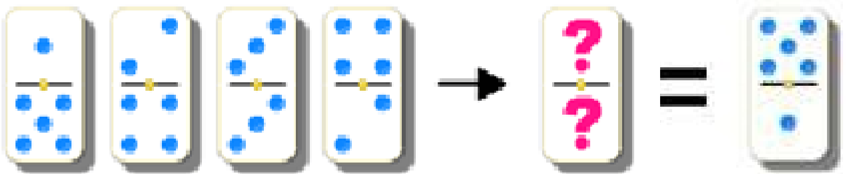

$\longrightarrow m_3/m_2 \sim 20\text{–}150$

but experimentally $m_3/m_2 < 6$

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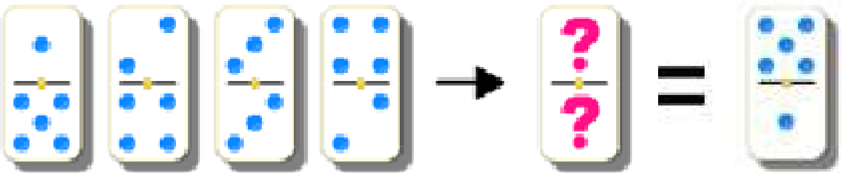

h_u very hier. h_d very hier. h_e very hier. h_ν ??? h_ν very hier. $\longrightarrow m_3/m_2 \sim 20\text{–}150$

but experimentally $m_3/m_2 < 6$

- Why large angles? Possible: the matrices that diagonalize the Yukawa couplings could have large mixing angles. We only know that $U_{CKM}=U_u^\dagger U_d$ has small angles. No guidance from other sectors.

Dirac neutrino masses have ugly features, but *are not excluded*

Majorana masses?

Option preferred by most theorists, even though no Majorana fermion has been positively discovered.

The leptonic Lagrangian reads:

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + {\rm h.c.}$$

Two remarkable facts:

- No new particle at low energies,
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New opportunities to explain the striking differences between neutrino parameters and quark parameters

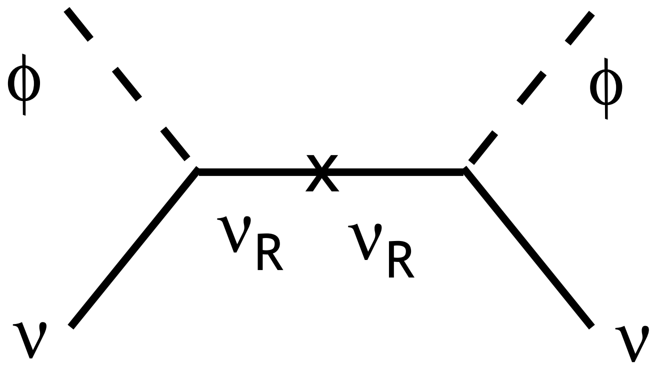
Origin of Majorana neutrino masses

Many proposals!

The most popular one (perhaps the simplest and most elegant) consists on introducing new heavy degrees of freedom:

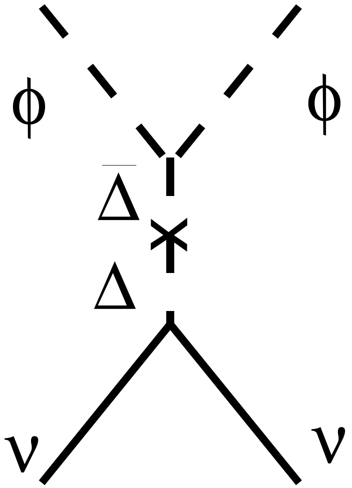
See-saw mechanism

Type I



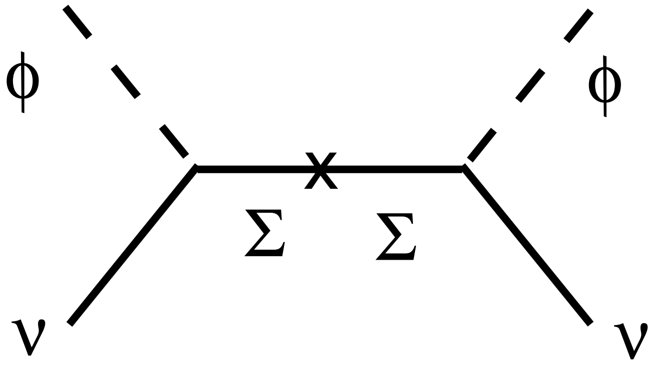
(fermion singlets)

Type II



(scalar triplets)

Type III



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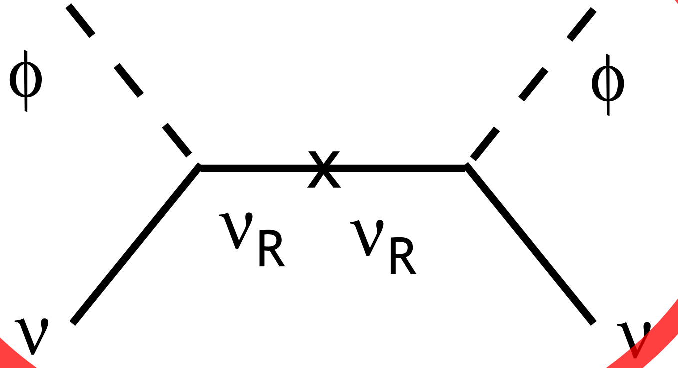

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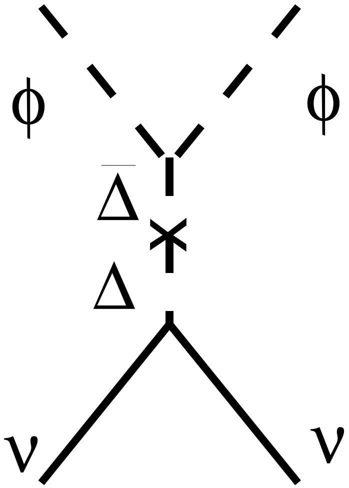

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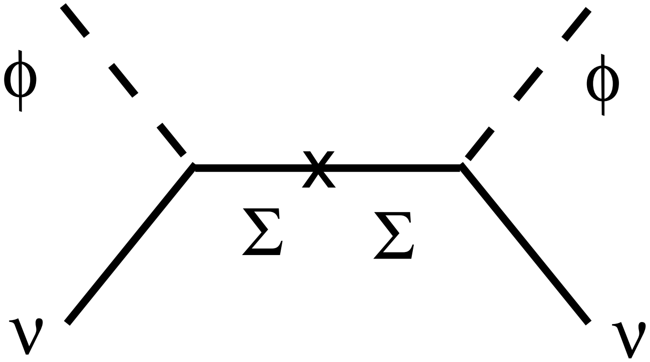

(fermion triplets)

Type I see-saw mechanism: Introduce heavy right-handed neutrinos (at least two).

That's it!!

The most general Lagrangian compatible with the Standard Model gauge symmetry is:

$$-\mathcal{L}_{\text{lep}} = (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} - \frac{1}{2}M_{ij}\bar{\nu}_{Ri}\nu^c_{Rj} + \text{h.c.}$$

$$\Big\downarrow \quad M \gg \langle\phi^0\rangle$$

$$-\mathcal{L}_{\text{lep}} = \frac{1}{2}(L_i\tilde{\phi})[h_\nu^T M^{-1} h_\nu]_{ij}(L_j\tilde{\phi}) + \text{h.c.}$$

$$\boxed{\mathcal{M}_\nu = h_\nu^T M^{-1} h_\nu \langle\phi^0\rangle^2}$$

- Naturally small due to the suppression by the large right-handed neutrino masses
- The Dirac Yukawa coupling enters in a complicated way. Could this be the origin of the difference between quark parameters and neutrino parameters?

Pros:

- Natural, simple and elegant.
- The particle content is left-right symmetric.
- Nicely compatible with GUTs.
- Could account for the observed matter-antimatter asymmetry in our Universe (leptogenesis) $\Longrightarrow \mathrm{M} \gtrsim 10^9\ \mathrm{GeV}$

“Most standard extension of the Standard Model”

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  - 500 GeV Linear Collider needs 100 MW
  - 10^{12} GeV Linear Collider needs 10^{11} MW

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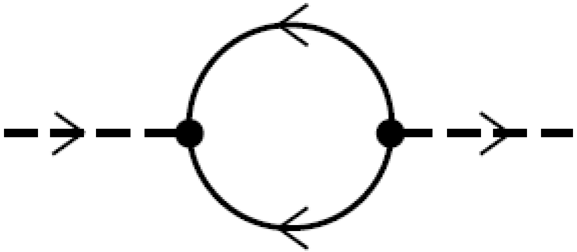
- The best motivated see-saw scenario (with $M \gg M_Z$) suffers a serious fine-tuning problem

An explicit hierarchy problem

The see-saw Lagrangian is:

$$-\mathcal{L}_{\text{lep}} = (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} - \frac{1}{2}M_{ij}\bar{\nu}_{Ri}\nu^c_{Rj} + \text{h.c.}$$

The Higgs doublet interacts with heavy degrees of freedom



$$\delta m_\phi^2 \sim \frac{1}{16\pi^2}h_\nu^2 M^2$$

Quadratic divergence!

$$m_\nu \sim \frac{h_\nu^2\langle\phi^0\rangle^2}{M}$$

$$\delta m_\phi^2 \sim \frac{1}{16\pi^2}\frac{m_\nu M^3}{\langle\phi^0\rangle^2}$$

$$\delta m_\phi^2 \lesssim m_\phi^2 \Rightarrow M \lesssim 10^7 \text{ GeV}$$

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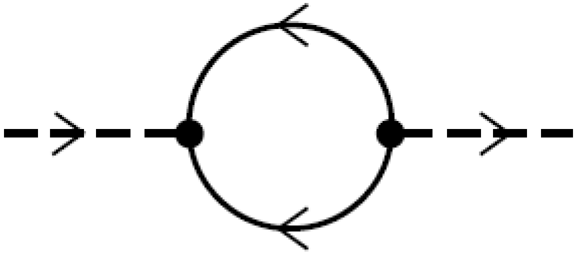

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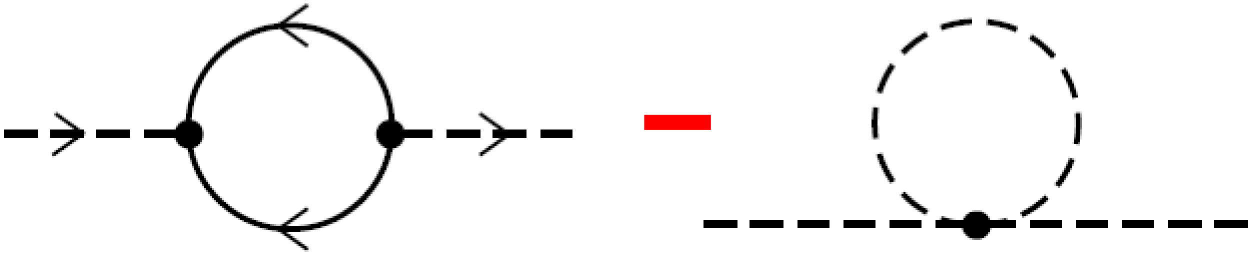
In the SUSY version of the see-saw



$$\delta m_H^2 \sim \frac{1}{16\pi^2}Y^2\log\frac{M^2}{\Lambda^2}$$

SUSY is the natural framework to implement the (high-scale) see-saw mechanism

New opportunities to test the see-saw mechanism!

What's next in leptonic physics?

- Leptonic Lagrangian (until ~1998)

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \mathrm{h.c.}$$

- Charged leptons massive
- neutrinos massless
- lepton flavour conserved
- total L number conserved

- Leptonic Lagrangian (until ?)

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + \mathrm{h.c.}$$

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \mathrm{h.c.}$$

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- Challenge: find evidences of the next term in the effective Lagrangian

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \mathrm{h.c.}$$

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \mathrm{h.c.}$$

LFV in the charged lepton sector

Some dimension 6 operators are:

$$\left.\begin{aligned} \mathcal{O}_B^{ij} &= \bar{e}_{Ri}\sigma_{\mu\nu}L_j\phi B_{\mu\nu} \\ \mathcal{O}_W^{ij} &= \bar{e}_{Ri}\sigma_{\mu\nu}\tau_I L_j\phi W^I_{\mu\nu} \end{aligned}\right\} \begin{array}{l} \mu\to e\gamma \\ \tau\to\mu\gamma \\ Z\to\mu e \end{array}$$

$$\left.\begin{aligned} \mathcal{O}_{LL}^{ijkl} &= (\bar{L}_i\gamma^\mu L_j)(\bar{L}_k\gamma_\mu L_l) \\ \mathcal{O}_{RR}^{ijkl} &= (\bar{e}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu e_l) \\ \mathcal{O}_{LR}^{ijkl} &= (\bar{L}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu L_l) \end{aligned}\right\} \begin{array}{l} \mu\to eee \\ \tau\to\mu\mu\mu \end{array}$$

(+ dim. 6 operators involving quarks and leptons
+ dim. 6 operators that violate total lepton number)

Neutrino masses violate flavour ⇒ they induce all these operators

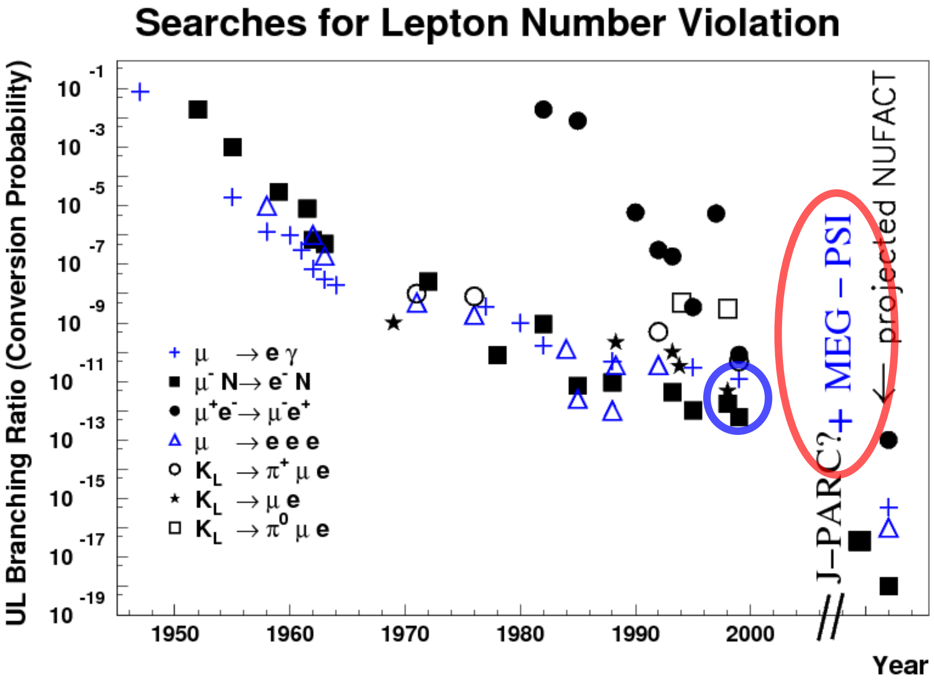
If the only source of LFV are neutrino masses, the dim-6 operators are very suppressed, giving

$$\mathrm{BR}(\mu \to e\gamma) \sim \frac{3\alpha}{32\pi}\left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \sin^2\theta$$

The predictions for the rare lepton decays are

$\mathrm{BR}(\mu\to e\gamma) \simeq 10^{-57}$, $\mathrm{BR}(\tau\to\mu\gamma) \simeq 10^{-54}$, $\mathrm{BR}(\tau\to e\gamma) \simeq 10^{-57}$,

Well consistent with experiments searching for rare charged lepton decays.



Äystö et al.

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However, there could be new sources of LFV apart from neutrino masses

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \mathrm{h.c.}$$

Scale of lepton number violation $\overset{?}{\neq}$ Scale of lepton flavour violation

Bounds on new physics from μ→eγ

Lowest dimension operator which induces μ→eγ

$$-\mathcal{L} = m_\mu \bar{\mu}(f^{\mu e}_{M1} + \gamma_5 f^{\mu e}_{E1})\sigma^{\mu\nu} e F_{\mu\nu} + \mathrm{h.c.}$$

The rate for the rare muon decay is:

$$\mathrm{BR}(\mu \to e\gamma) = \frac{96\pi^3\alpha}{G_F^2}(|f^{\mu e}_{E1}|^2 + |f^{\mu e}_{M1}|^2)$$

The present experimental bound BR(μ→eγ)<1.2×10^{-11} gives:

$$|f^{\mu e}_{E1}|, |f^{\mu e}_{M1}| \lesssim 10^{-12}\mathrm{GeV}^{-2}$$

Naively,

$$f^{\mu e} \sim \frac{1}{\Lambda^2} \longrightarrow \Lambda \gtrsim 300\mathrm{TeV}$$

In most models the contact interaction arises as a result of quantum effects (new particles interacting with the muon and the electron circulating in loops).

$$f^{\mu e} \sim \frac{\theta_{\mu e}^2 \alpha}{\Lambda^2}$$

Then, the present bound on BR($\mu \rightarrow e\gamma$) requires

$$\Lambda \gtrsim 20\mathrm{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$

$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\mathrm{GeV}$$

A large mass scale for the new particles and/or small coupling between the electron or muon with the new particles.

Rare tau decays

Complementary probe of lepton flavour violation.

Until very recently, not as interesting as $\mu\to e\gamma$ for constraining models.

PDG 2004

$$\mathrm{BR}(\tau \to e\gamma) \leq 2.7 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 1.1 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$

The experimental bound $\mathrm{BR}(\tau\to\mu\gamma)<1.1\times10^{-6}$ yields:

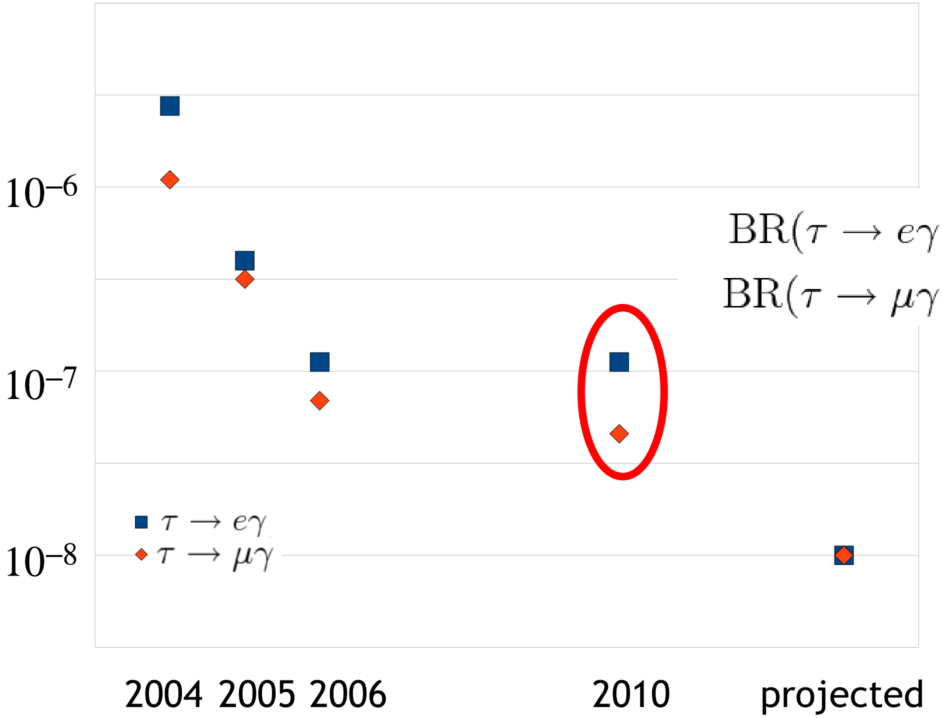
$$\Lambda \gtrsim 800\mathrm{GeV} \quad \text{if} \quad \theta_{\tau\mu} \sim \frac{1}{\sqrt{2}}$$

$$\theta_{\tau\mu} \lesssim 0.3 \quad \text{if} \quad \Lambda \sim 300\mathrm{GeV}$$

(compare to 20TeV!)

Impressive experimental progress in the last years!!

BELLE



$$\mathrm{BR}(\tau \to e\gamma) \leq 1.1 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$
$$\mathrm{BR}(\tau \to \mu\gamma) \leq 4.5 \times 10^{-8} \qquad \mathrm{CL} = 90\%$$

The present experimental bounds on the rare tau decays yield:

From τ→eγ $\Lambda \gtrsim 1300\mathrm{GeV}$ if $\theta_{\tau e} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau e} \lesssim 0.2$ if $\Lambda \sim 300\mathrm{GeV}$

From τ→μγ $\Lambda \gtrsim 1700\mathrm{GeV}$ if $\theta_{\tau\mu} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau\mu} \lesssim 0.1$ if $\Lambda \sim 300\mathrm{GeV}$

fairly stringent constraints

Implications for Physics BSM

DRAMATIC! Many extensions of the Standard Model postulate new particles at the electroweak scale (hierarchy problem, “WIMP miracle”, cosmic ray anomalies...)

Recall: the present bound on BR($\mu \to e\gamma$) requires

$$\Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$

$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$

Very stringent constraints on models. Or on the positive side, detection might be around the corner.

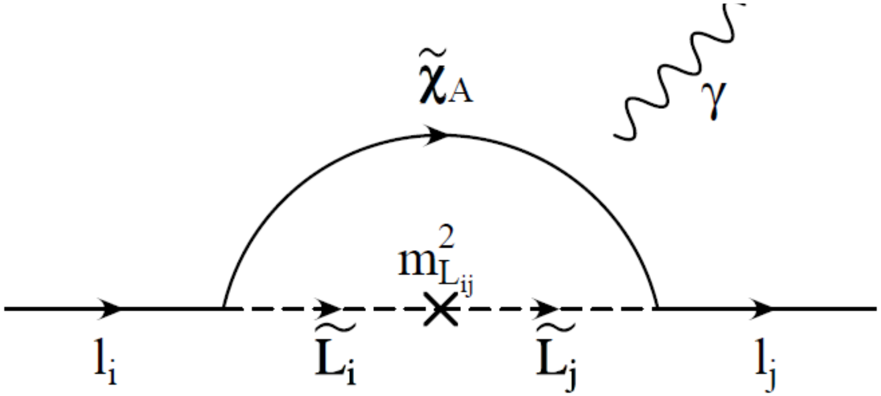
This is the case for:

- Supersymmetric models
- Extra dimensional models
- Little Higgs models
- ...

LFV in SUSY scenarios

A SUSY model contains in general new sources of flavour violation in the soft-SUSY breaking Lagrangian

$$-\mathcal{L}_{\rm soft}^{\rm lep} = (\mathbf{m}_L^2)_{ij}\widetilde{L}_i^*\widetilde{L}_j + (\mathbf{m}_e^2)_{ij}\widetilde{e}_{Ri}^*\widetilde{e}_{Rj} + \left(\mathbf{A}_{eij}\widetilde{e}_{Ri}^*\widetilde{L}_j H_d + \text{h.c.}\right)$$



Back of the envelope calculation of BR($l_i \rightarrow l_j \gamma$):

$$\mathrm{BR}(\ell_j \to \ell_i\gamma) \simeq \frac{\alpha^3}{G_F^2}\frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8}\tan^2\beta\,\mathrm{BR}(\ell_j \to \ell_i\nu_j\bar{\nu}_i)$$

Suppressed by the soft mass scale (1 TeV)

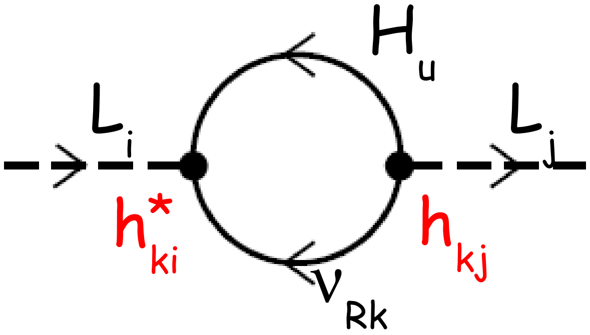
$$\begin{aligned}(\mathbf{m}_L^2)_{12}/m_S^2 &< 3\times10^{-4}\\(\mathbf{m}_L^2)_{13}/m_S^2 &< 0.09\\(\mathbf{m}_L^2)_{23}/m_S^2 &< 0.09\end{aligned}$$

(for m_s=400GeV and tanβ=10)

LFV in the Type I see-saw model

SUSY is the natural framework to implement the (high-scale) see-saw mechanism

Quantum corrections induced by heavy particles generate flavour violating terms in the slepton sector: Borzumati, Masiero



$$\left(\mathbf{m}_L^2\right)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(h_\nu^\dagger h_\nu)_{ij}\log\left(\frac{M_X}{M_{\rm maj}}\right)$$

$$\left(\mathbf{m}_e^2\right)_{ij} \simeq 0$$

$$(\mathbf{A}_e)_{ij} \simeq \frac{-3}{8\pi^2}A_0 h_e (h_\nu^\dagger h_\nu)_{ij}\log\left(\frac{M_X}{M_{\rm maj}}\right)$$

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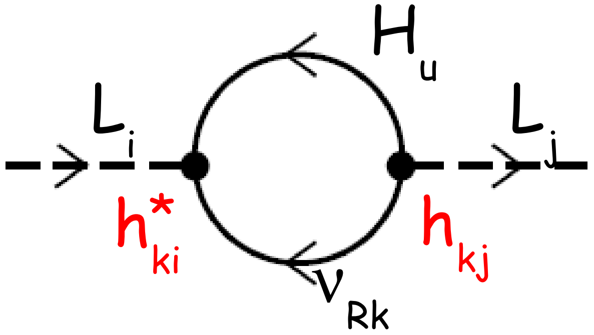

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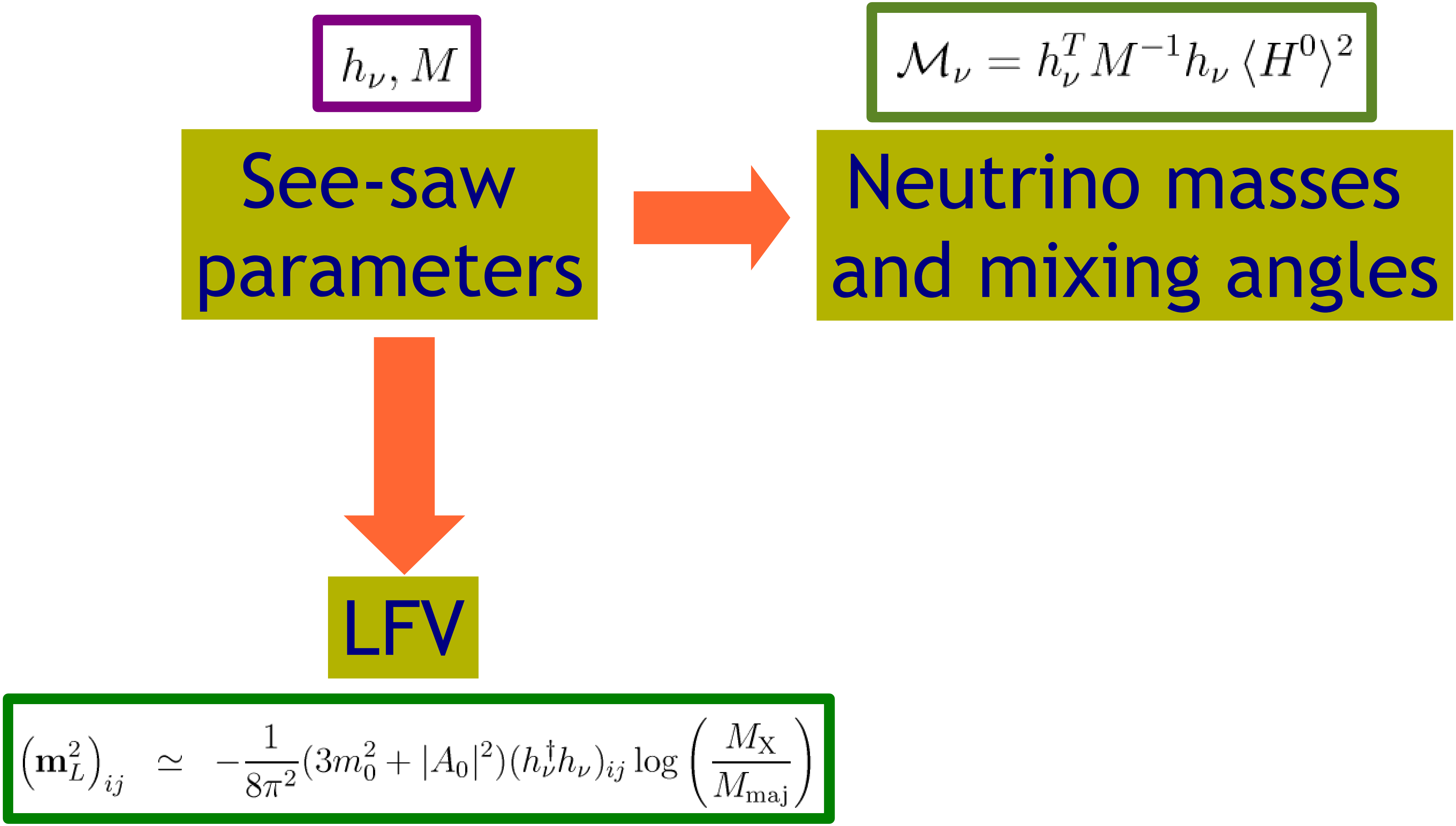
Logarithmic dependence with M. There is a possibly large low energy imprint of the heavy RH neutrinos



$$h_\nu, M$$

See-saw parameters

$$\mathcal{M}_\nu = h_\nu^T M^{-1} h_\nu \langle H^0 \rangle^2$$

Neutrino masses and mixing angles

LFV

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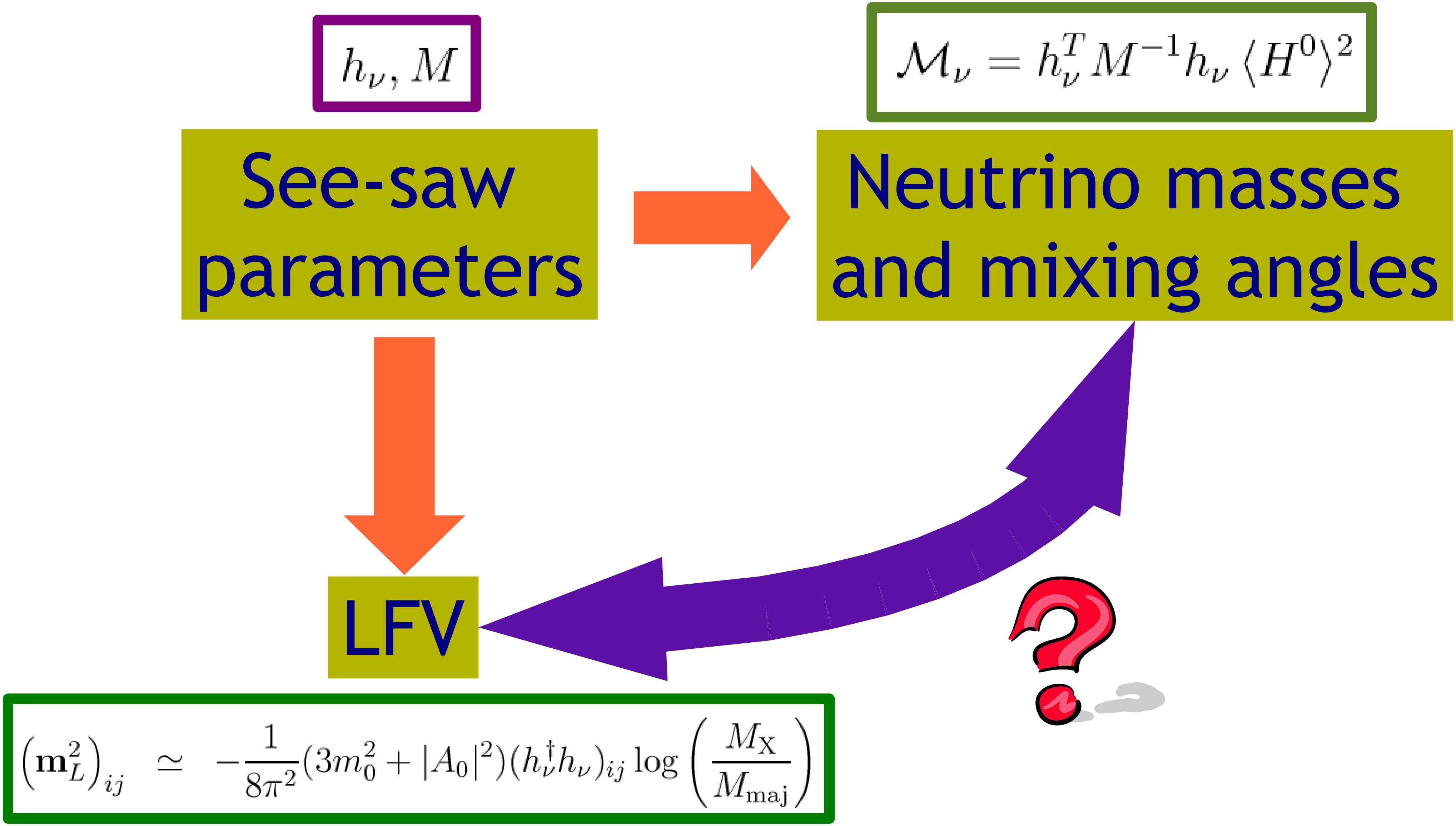

h_ν, M

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$\mathcal{M}_\nu = h_\nu^T M^{-1} h_\nu \langle H^0 \rangle^2$

Neutrino masses and mixing angles

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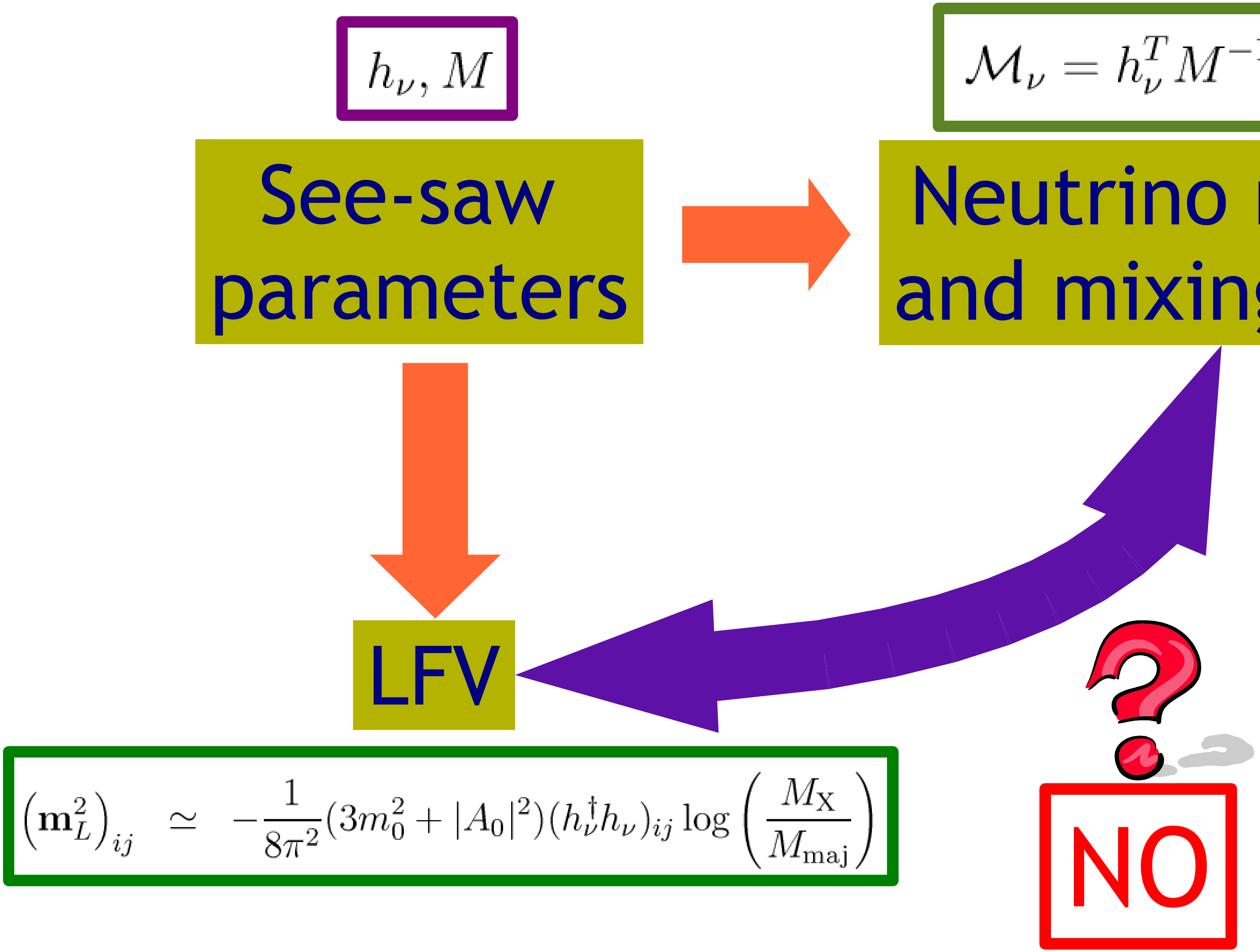

h_ν, M

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Neutrino masses and mixing angles

LFV

$$\left(\mathbf{m}_L^2\right)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(h_\nu^\dagger h_\nu)_{ij} \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)$$

NO

The see-saw Lagrangian has 12+6 new parameters.
Neutrino observations at most can fix 6+3 parameters.
Still, there are 6+3 free parameters.

There are, compatible with the observed neutrino parameters, an infinite set of Yukawa couplings!

Casas, AI

Right-handed neutrino masses

$$h_\nu = \frac{1}{\langle H^0 \rangle} \sqrt{D_M}\, R\, \underbrace{\sqrt{D_m}\, U^\dagger_{\rm lep}}$$

"Fixed" by experiments

Complex orthogonal matrix

Changing R and the right-handed neutrino masses, any $Y^\dagger Y$ can be obtained.

In fact, there is a one-to-one correspondence between

$$\{h_\nu, M\} \leftrightarrow \left\{\mathcal{M}, h_\nu^\dagger h_\nu\right\}$$

Davidson, AI

High-energy parameters of the see-saw Lagrangian

Low energy observables: neutrino mass matrix, BR($l_i \rightarrow l_j \gamma$), EDMs

From a *model independent* perspective, the type-I see-saw can accommodate anything at low energies!! **No predictions**

Is this a dead-end? Is it impossible to test the SUSY see-saw?

Remarkably, under some well motivated assumptions, it is possible to derive predictions for the LFV processes, in the form of lower bounds.

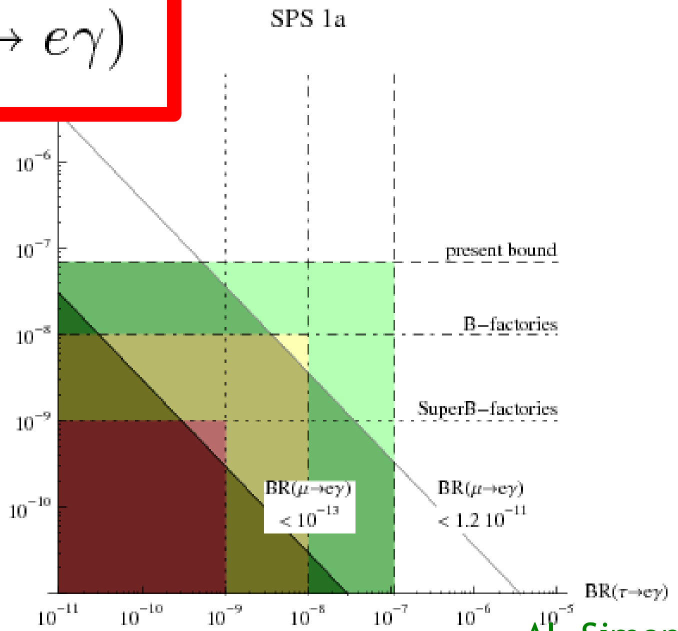
- Absence of tunings
- Hierarchical neutrino Yukawa couplings

1- Assume that the processes τ→μγ and τ→eγ are observed. This means that all flavour numbers are violated in the slepton sector.

The process μ→eγ is necessarily generated at higher orders:

$$\boxed{\mathrm{BR}(\mu \to e\gamma) \gtrsim C \times \mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma)}$$

If both τ→μγ and τ→eγ are observed in the near future, the lower bound on BR(μ→eγ) violates present experiments. The SUSY see-saw would be disfavoured.



AI, Simonetto

2- A careful calculation of the rate for $\mu\rightarrow e\gamma$ shows that:

$$\boxed{\mathrm{BR}(\mu \rightarrow e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{M_1}{5 \times 10^{12}\mathrm{GeV}}\right)^2 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2}$$

AI, Simonetto

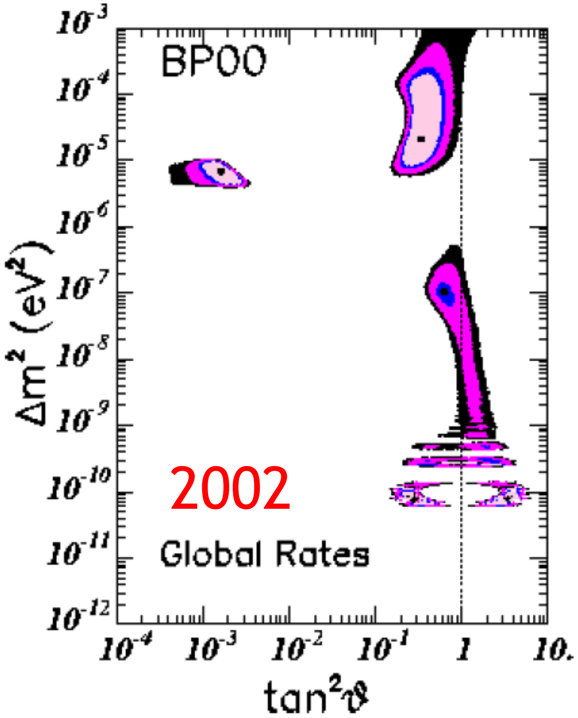
What is the mass of the lightest right-handed neutrino?

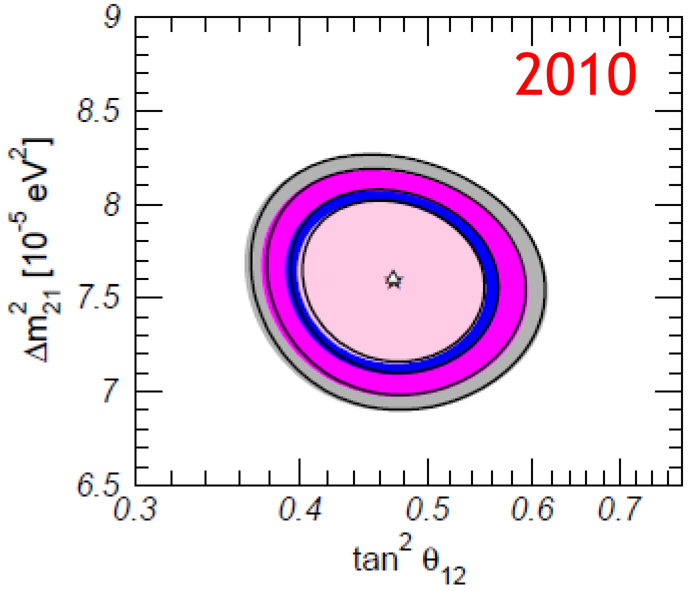
Hint from baryogenesis. The leptogenesis mechanism requires the lightest right handed neutrino to be heavier than 10^9 GeV (more realistically, 5×10^{10} GeV)

SUSY leptogenesis implies $\mathrm{BR}(\mu\rightarrow e\gamma)\gtrsim10^{-18}$. In the more realistic case $\mathrm{BR}(\mu\rightarrow e\gamma)\gtrsim10^{-16}$ (equivalent to a rate $R(\mu\mathrm{Ti} \rightarrow \mathrm{e\ Ti})\gtrsim10^{-18}$)

Conclusions

- Many experimental results have shown that lepton flavour is violated in the neutrino sector. These results can be nicely explained in the framework of three family neutrino oscillations.
- Huge progress over the last ten years in the determination of the neutrino masses and mixing angles.





- Unfortunately, not enough to unravel the origin of neutrino masses. Next challenge: discover LFV in the charged lepton sector.