QCD and Hadronic Interactions with Initial-State-Radiation at B-Factories

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On behalf of the *BABAR* Collaboration

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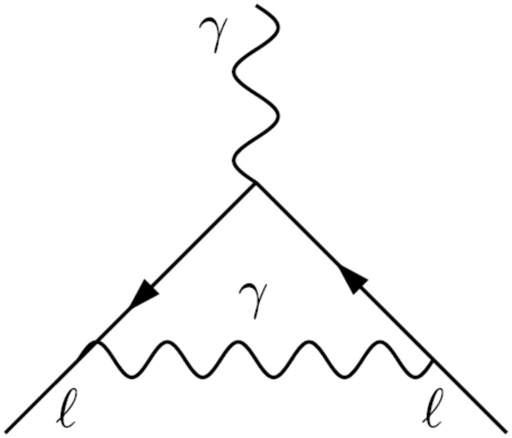
La Thuile, Italie, 2010

The “Anomalous” Magnetic Moment of the Lepton

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad\qquad a = (g-2)/2$$

(1928) Pointlike Dirac particles : $g = 2$, $a = 0$.

$g \neq 2$ due to higher order contributions :



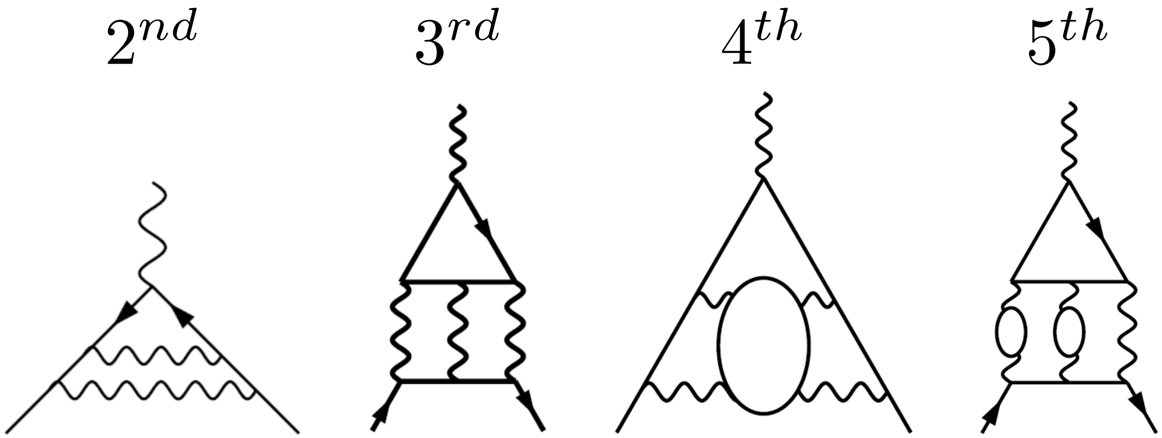
- (1947) Nafe et al. measure $a_e = (2.6 \pm 0.5) \times 10^{-3}$
- (1948) Schwinger (1st order) $a^{(1)} = \alpha/2\pi \approx 1.2 \times 10^{-3}$

Lepton universality at this 1st order

Our belief in QED and in the gauge-theory-based SM originates from this 1st success.

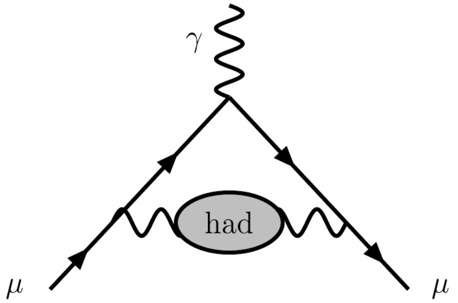
Higher Orders

One graph given as example out of many ..

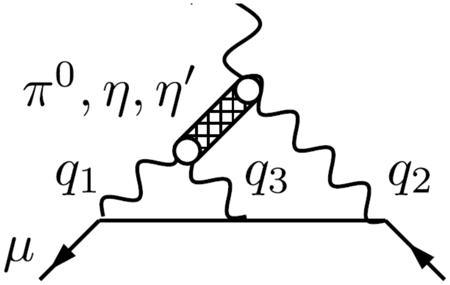


$$a = a^{\text{QED}} + a^{\text{had}} + a^{\text{weak}}$$

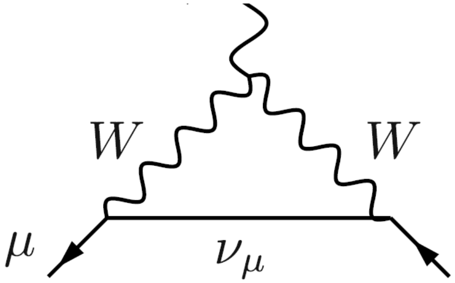
Hadronic Vacuum Polarisation (VP)



Hadronic light-by-light Scattering



Weak Interactions



a_e, α and a_μ

- Heavy-to-Light and Light-to-Heavy mass ratios take part differently (e/μ) in the loops (QED, QCD, weak)

$$a_e = \frac{\alpha}{2\pi} - 0.3\left(\frac{\alpha}{\pi}\right)^2 + 1.2\left(\frac{\alpha}{\pi}\right)^3 - 1.9\left(\frac{\alpha}{\pi}\right)^4 + 0.0(4.6)\left(\frac{\alpha}{\pi}\right)^5 + 1.72(2)10^{-12}(\mathrm{QCD} + \mathrm{weak})$$

$$a_\mu = \frac{\alpha}{2\pi} + 0.8\left(\frac{\alpha}{\pi}\right)^2 + 24.\left(\frac{\alpha}{\pi}\right)^3 - 131.\left(\frac{\alpha}{\pi}\right)^4 + 663.\left(\frac{\alpha}{\pi}\right)^5 + 7.07(7)10^{-8}(\mathrm{QCD} + \mathrm{weak})$$

Numbers truncated !

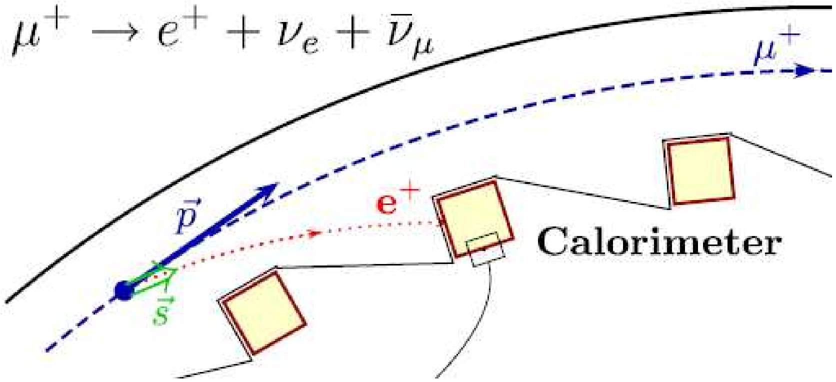
- a_e measured in a one-electron quantum cyclotron

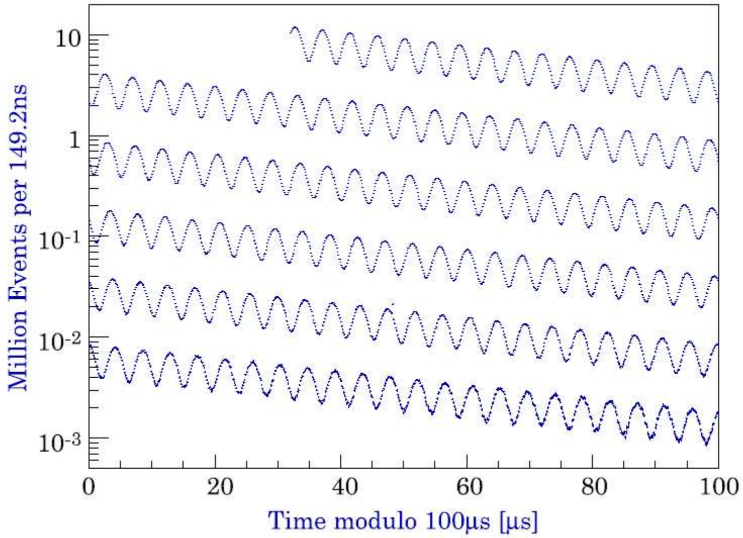
$$a_e = 1159652180.73(\pm 0.28)10^{-12}, \quad \text{(0.24 ppb)}$$

- $\Rightarrow$ α known to 0.37 ppb
- In total the QED uncertainty on a_μ^{QED} is tiny : 1.7 ppb

Odom PRL 97 (2006) 030801, Gabrielse PRL 97 (2006) 030802

a_μ *Measurement E821 @ BNL*





- $\pi^- \rightarrow \mu^- \nu$ violates P, μ^- longitudinally polarized.
- μ^- stored in a cyclotron, constant $\vec{B}$.
  - μ^- rotating with freq ω_c ; μ^- spin precessing with freq ω_s
  - freq. difference $\omega_a = \omega_s - \omega_c = a_\mu e B / m_\mu$
- $\mu \rightarrow e\nu\overline{\nu}$ violates P, e direction (energy in lab) remembers μ^- polarization.
- Fraction of e above $E_{\mathrm{threshold}}$ is modulated with freq. ω_a

$a_\mu(\mathrm{expt}) = (11659208.0 \pm 5.4(\mathrm{stat}) \pm 3.3(\mathrm{syst})) \times 10^{-10}$, (0.54 ppm)

E821 @ BNL, $\mu^+ - \mu^-$ charge average Bennett Phys.Rev.D73 :072003,2006

Theoretical prediction for a_μ – May 2009

SM-to-experiment comparison [in units 10^{-10}]

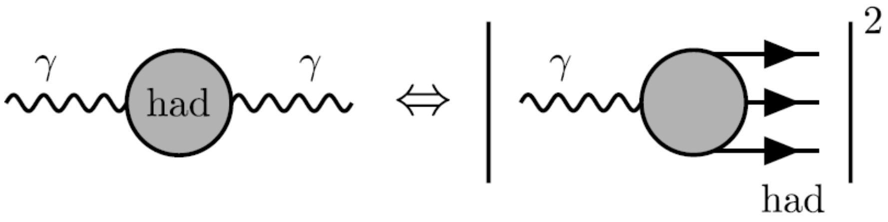
QED	116 584 71.81	± 0.02
Leading hadronic vacuum polarization (VP)	690.30	± 5.26
Sub-leading hadronic vacuum polarization	-10.03	± 0.11
Hadronic light-by-light	11.60	± 3.90
Weak (incl. 2-loops)	15.32	± 0.18
Theory	11659179.00	± 6.46
Experiment	11659208.00	± 6.30
Exp − theory	29.00	± 9.03

Assuming Gaussian statistics, a 3.2 σ discrepancy.

Jegerlehner, Nyffeler / Phys Rept 477 (2009) 1110

uses e^+e^- input only for VP

Theoretical prediction : The Hadronic VP (1)



- Quark loops not computable from first principles – QCD.
- Vacuum polarization : energy dependent running charge :

$$e^2 \rightarrow e^2/[1+(\Pi'(k^2)-\Pi'(0))]$$

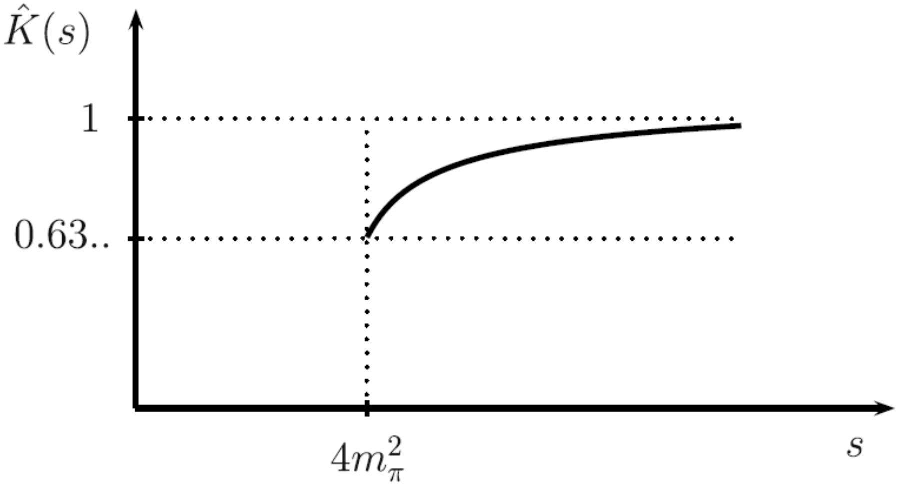
- Dispersion relation from analyticity

$$\Pi'(k^2)-\Pi'(0)=\frac{k^2}{\pi}\int_0^\infty \frac{Im\Pi'(s)}{s(s-k^2-i\epsilon)}\mathrm{d}s$$

- Optical theorem (unitarity)

$$Im\Pi'(s)=\alpha(s)R_{\mathrm{had}}(s)/3, \quad \text{with } R_{\mathrm{had}}(s)=\sigma_{\mathrm{had}}\frac{3s}{4\pi\alpha(s)}=\frac{\sigma_{e^+e^-\rightarrow hadrons}}{\sigma_{e^+e^-\rightarrow\mu^+\mu^-}}$$

Theoretical prediction : The Hadronic VP (2)

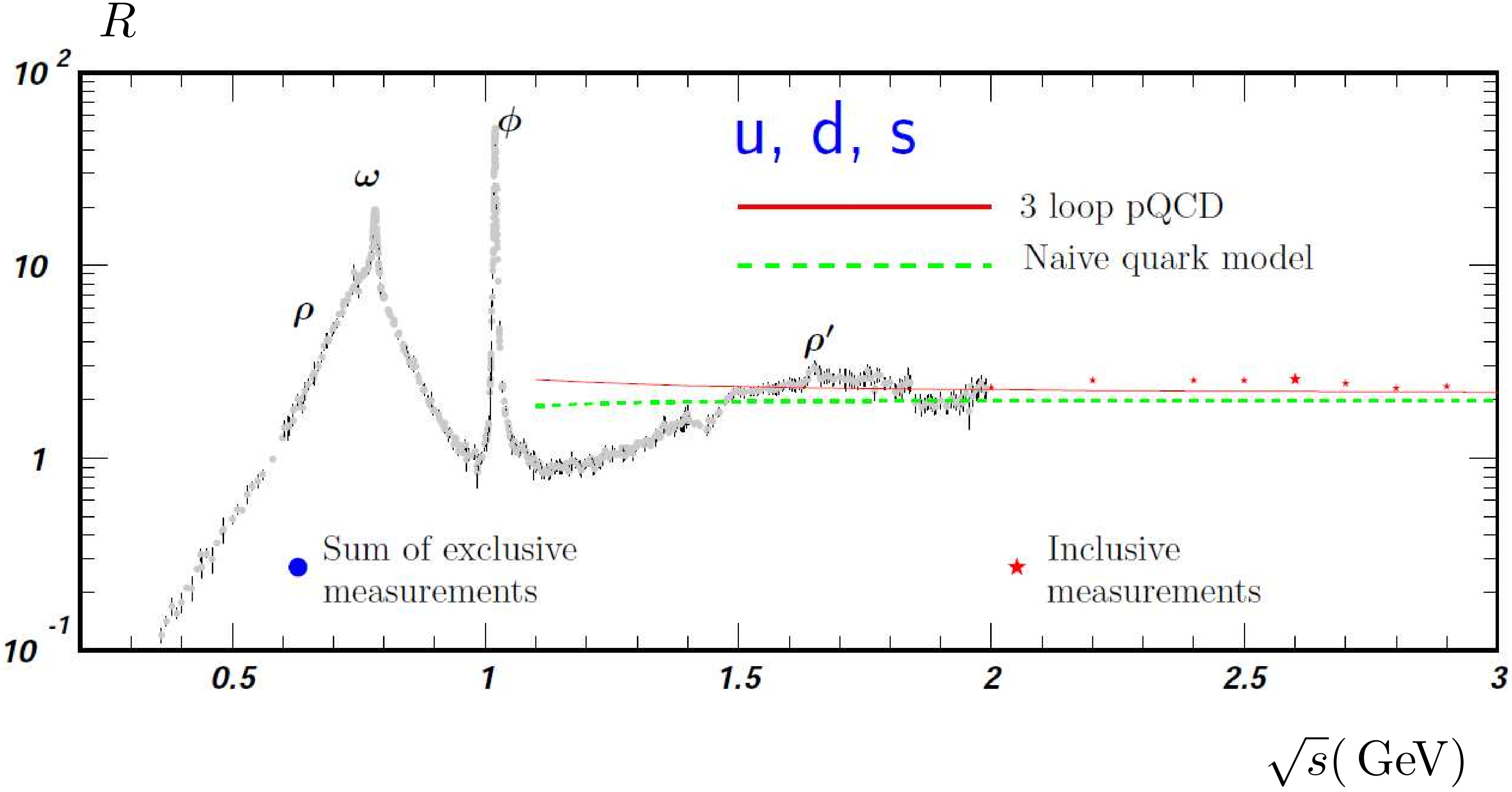


Wrapping it up, the “dispersion integral” :

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R_{\text{had}}(s)\hat{K}(s)}{s^2}\mathrm{d}s$$

- Technically, $\int_{4m_\pi^2}^{E_{cut}^2}$ is obtained from the data, $\int_{E_{cut}^2}^{\infty}$ from pQCD.
- The estimation of the contribution with the largest uncertainty to a_μ(theory) boils down to a precise measurement of $R_{\text{had}}(s)$
- Most precision on $R_{\text{had}}(s)$ needed at low $\sqrt{s}$

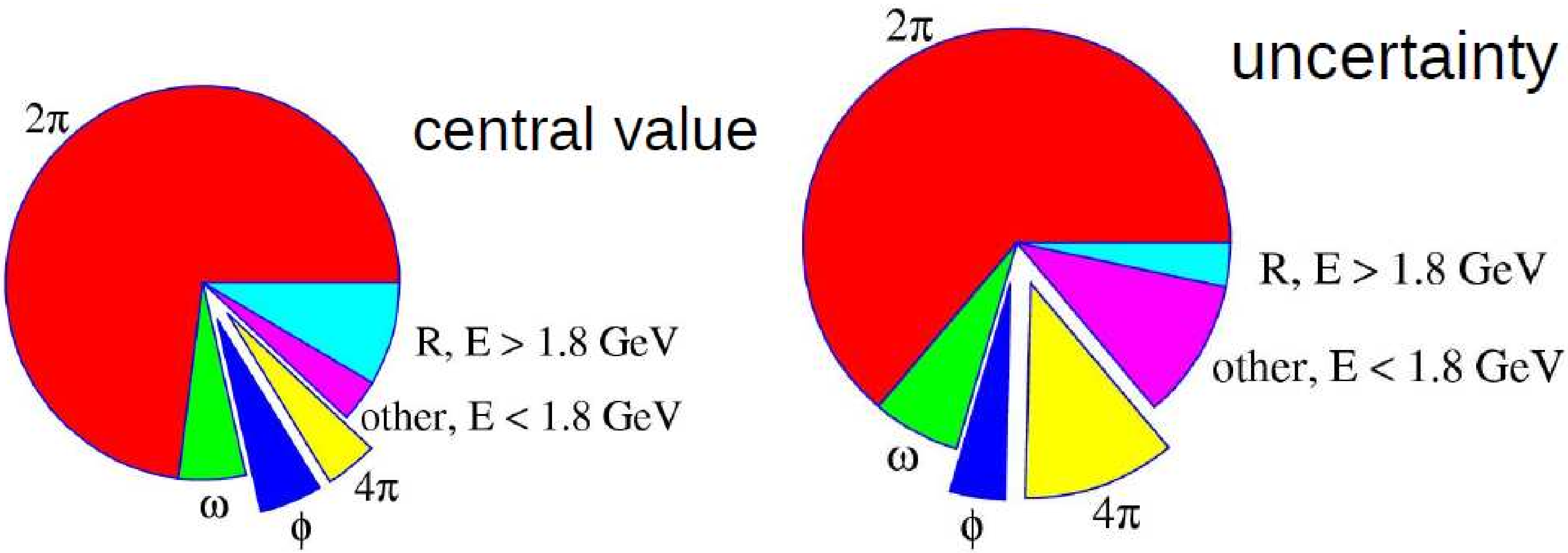
$R_{\text{had}}(s)$: *Direct Measurements* $e^+e^- \to$ *Hadrons*



PDG, Phys. Lett. B667,1 (2008)

$e^+e^- \to$ *Hadrons : channel break-down*

Contributions to the "dispersion integral".



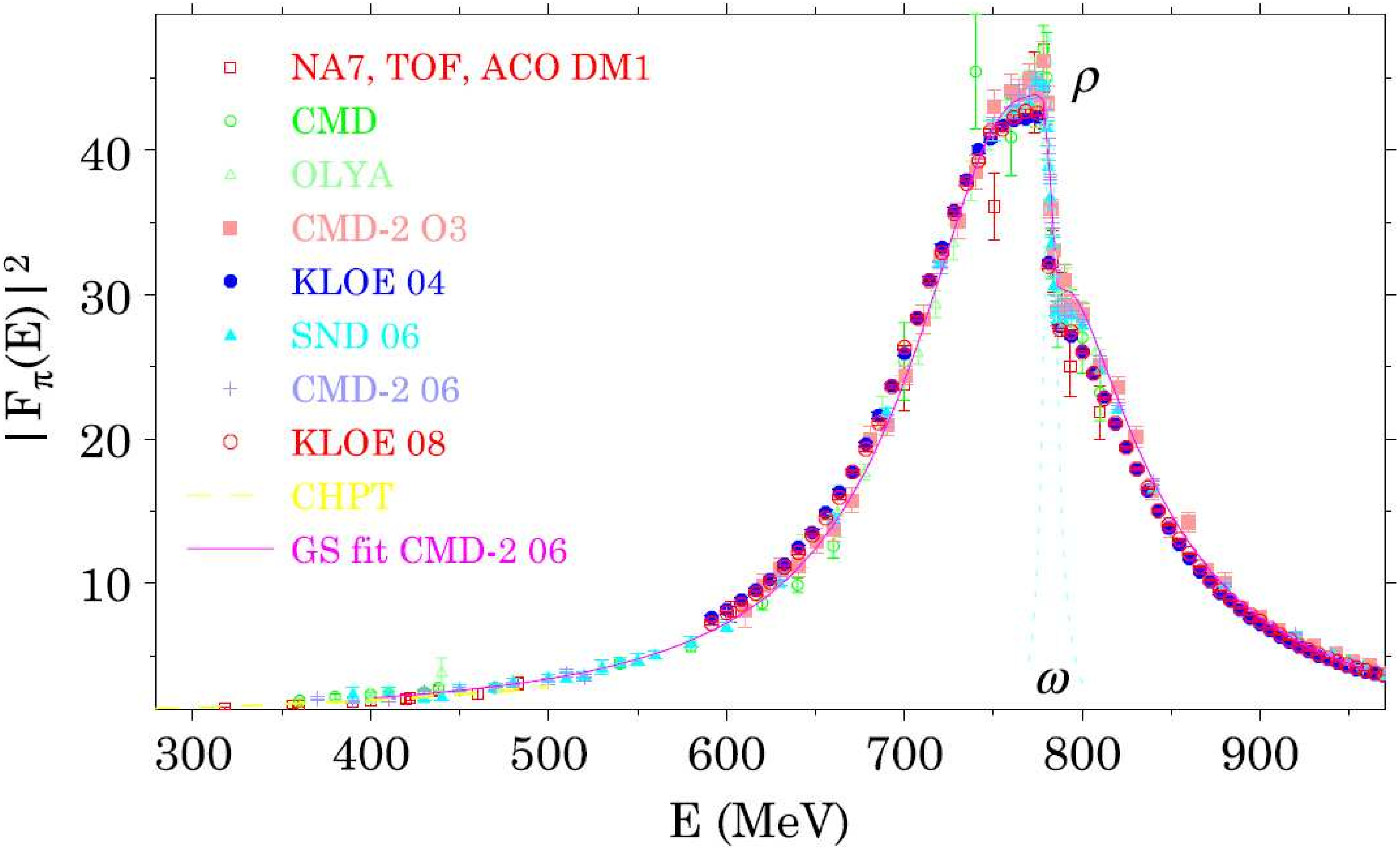
$e^+e^- \to \pi^+\pi^-$ dominates (73 %)

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\mathrm{GeV}/c^2] = (504.6 \pm 3.1(\mathrm{exp}) \pm 0.9(\,rad)) \times 10^{-10}$$

$$a_\mu^{\mathrm{had}} = (690.9 \pm 5.3) \times 10^{-10}$$

Davier, Nucl. Phys. Proc. Suppl. 169, 288 (2007)

$e^+e^- \to \pi^+\pi^-$: *Direct Measurements*



- (KLOE 08 supersedes KLOE04)
- The 3.2 σ discrepancy mentioned above is based on this input

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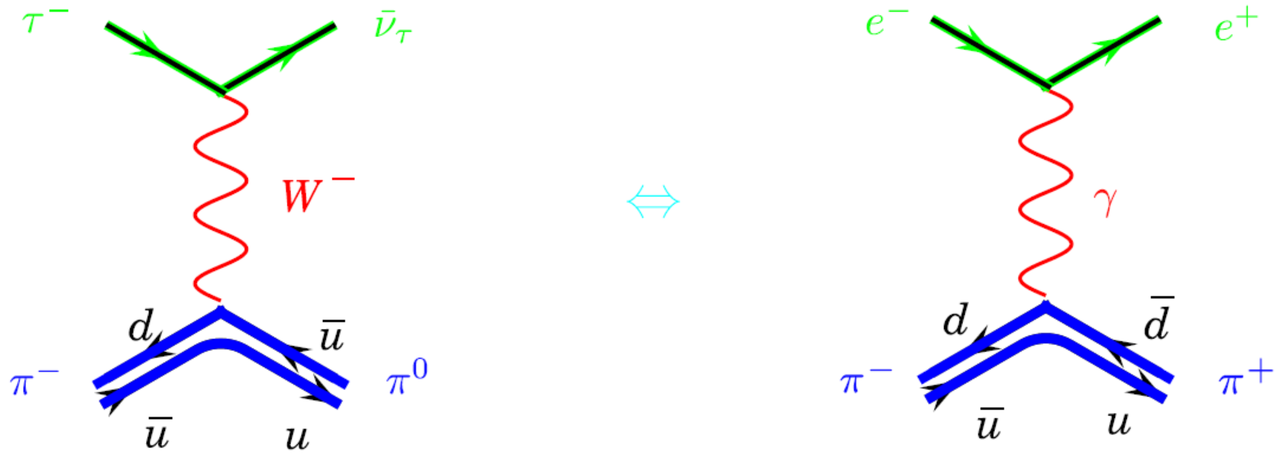
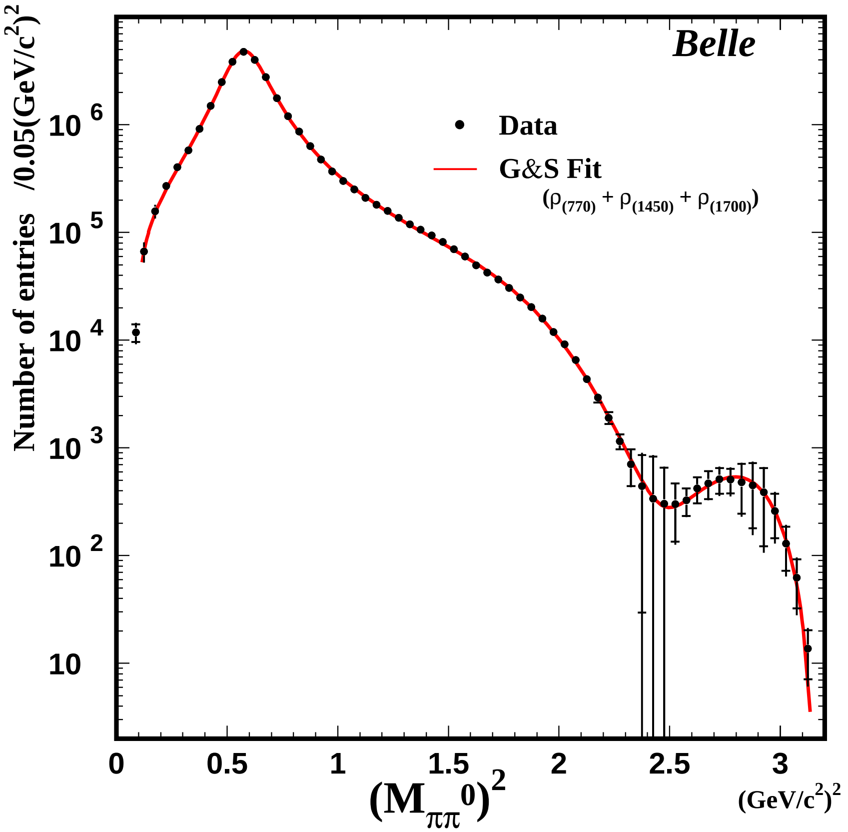
τ *Decay Spectral Functions*

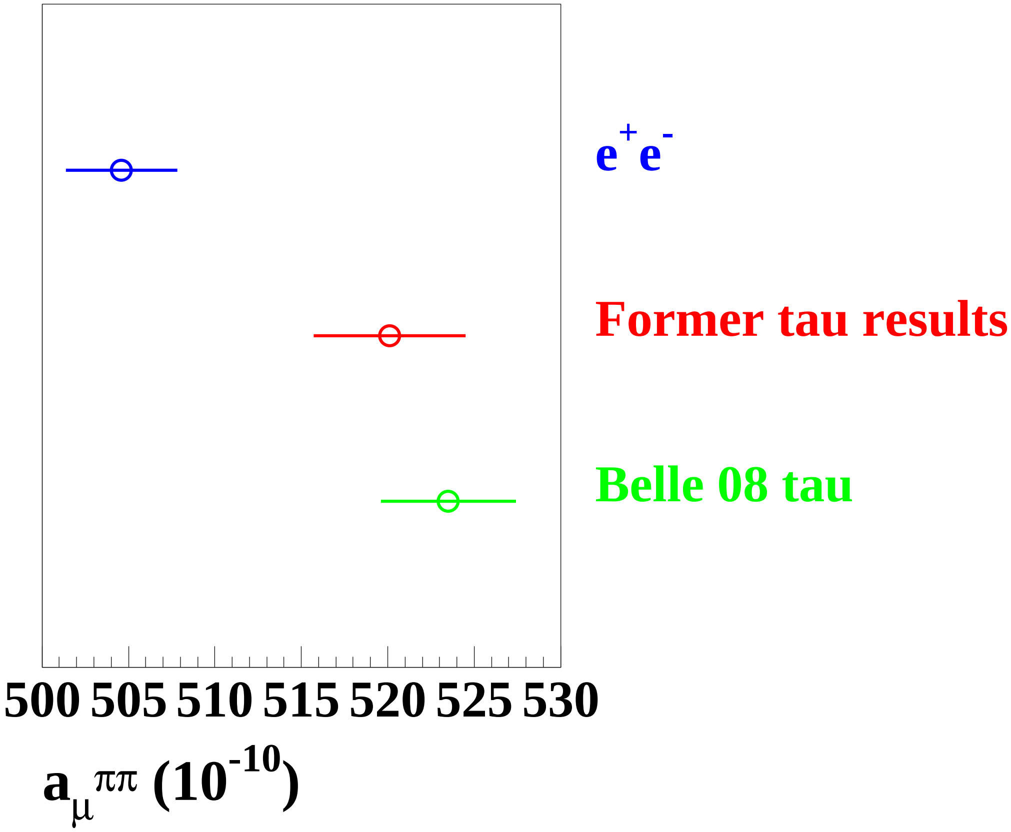
$I = 1$ part of $e^+e^- \to \mathrm{had}$ from $\tau \to \nu_\tau + \mathrm{had}$ by isospin rotation



- example $\pi^0\pi^- \leftrightarrow \pi^+\pi^-$
- $\sigma(e^+e^- \to \pi^+\pi^-) = \dfrac{4\pi\alpha^2}{s}\nu_0(s)$, $\nu(s)$ "spectral function"
- $\dfrac{1}{\Gamma}\dfrac{\mathrm{d}\Gamma}{\mathrm{d}s} = F(s)\dfrac{\mathcal{B}(\tau \to e\nu_\tau\overline{\nu}_e)}{\mathcal{B}(\tau \to \pi^-\pi^0\nu_\tau)} \times \nu_-(s)$, where $F(s)$ is a known function of s
- CVC : $\nu_0(s) = \nu_-(s)$ isospin breaking (IB) corrections $\cdots$
- ALEPH (1997), OPAL (1999), CLEO (2000)
- $\mathcal{B}(\tau \to \pi^-\pi^0\nu_\tau)$ ALEPH's most precise $(25.471 \pm 0.097 \pm 0.085)\%$

Belle's High Statistics Results on $\tau \longrightarrow \pi^-\pi^0\nu_\tau$





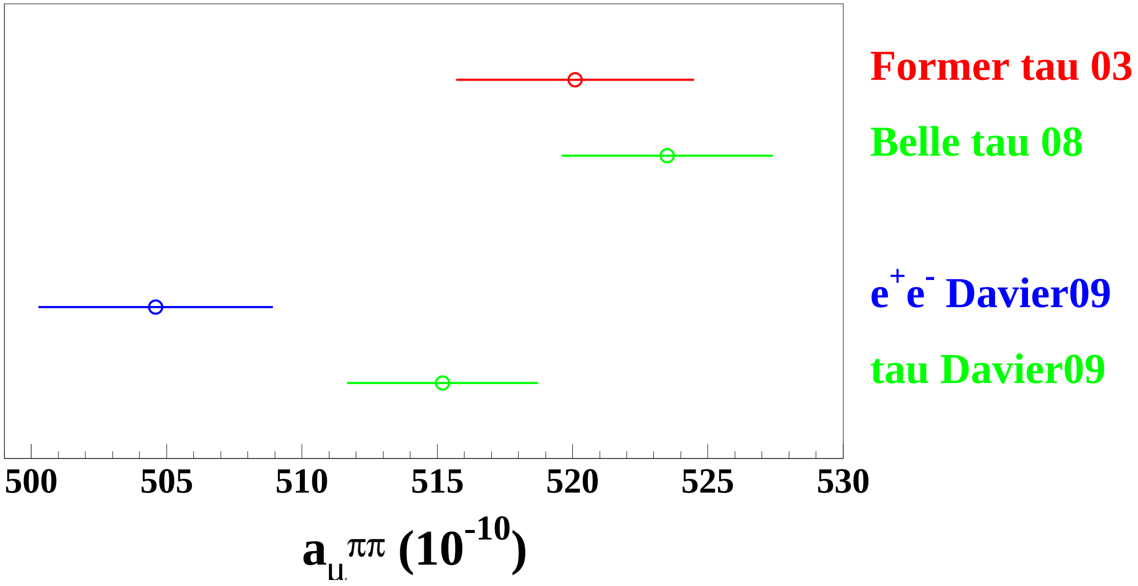
	$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\mathrm{GeV}/c^2]$ (units 10^{-10})
ALEPH, CLEO, OPAL	520.1 ± 2.4 (exp) ± 2.7 (Br.) ± 2.5 (IB)
Belle	523.5 ± 1.5 (exp) ± 2.6 (Br.) ± 2.5 (IB)

72 fb^{-1} of e^+e^- data taken at ≈ 10.6 GeV

Fujikawa Phys.Rev.D78 :072006,2008.

Combination from Davier Eur. Phys. J. C 27, 497 (2003).

τ *Spectral Functions : Isospin Breaking (IB) Corrections*



Several contributions to the IB corrections, among which :

- Short- and long-distance radiative corrections
- $\pi^0 - \pi^\pm$ and $\rho^0 - \rho^+$ mass differences
- FSR correction included

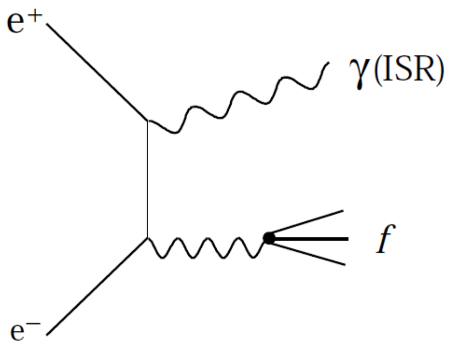
IB corrections are still being worked on :

$-13.8 \quad \pm 2.4 + 4.2(\mathrm{FSR})$ Eur. Phys. J. C 27, 497 (2003).

$-16.07 \quad \pm 1.85,$ 0906.5443v3 [hep-ph] Accepted by Eur. Phys. J. C

$\Delta = \quad -6.9$ Units 10^{-10}

Initial State Radiation (ISR)



- Optimal use of the available luminosity
- Covers whole energy range with same detector condition and analysis.
    - And good efficiency down to threshold
- If observe the whole final state (γ + hadrons)
  $\Rightarrow$ over-constrained kinematical fit $\Rightarrow$ powerful background noise rejection.

$$\frac{\mathrm{d}\sigma_{[e^+e^-\to f\gamma]}}{\mathrm{d}s'}(s') = \frac{2m}{s} W(s,x)\sigma_{[e^+e^-\to f]}(s') \ , \qquad x = \frac{E_\gamma}{\sqrt{s}} = 1 - \frac{s'}{s}$$

- $W(s,x)$ "radiator function", density of probability to radiate a photon with energy $E_\gamma = x\sqrt{s}$: a known function

Binner, Physics Letters B 459 (1999) 279

ISR : “Old” BaBar Results

Vigourous campaign that is still in progress

$K^+K^-\eta$, $K^+K^-\pi^0$, $K^0K^\pm\pi^\mp$	Phys.Rev.D77 :092002,2008.
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$	Phys.Rev.D76 :092005,2007.
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$	Phys.Rev.D76 :012008,2007.
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$	Phys.Rev.D73 :052003,2006.
$\overline{p}p$	Phys.Rev.D73 :012005,2006.
$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K^+K^-K^+K^-$	Phys.Rev.D71 :052001,2005.
$\pi^+\pi^-\pi^0$	Phys.Rev.D70 :072004,2004.

- First observations

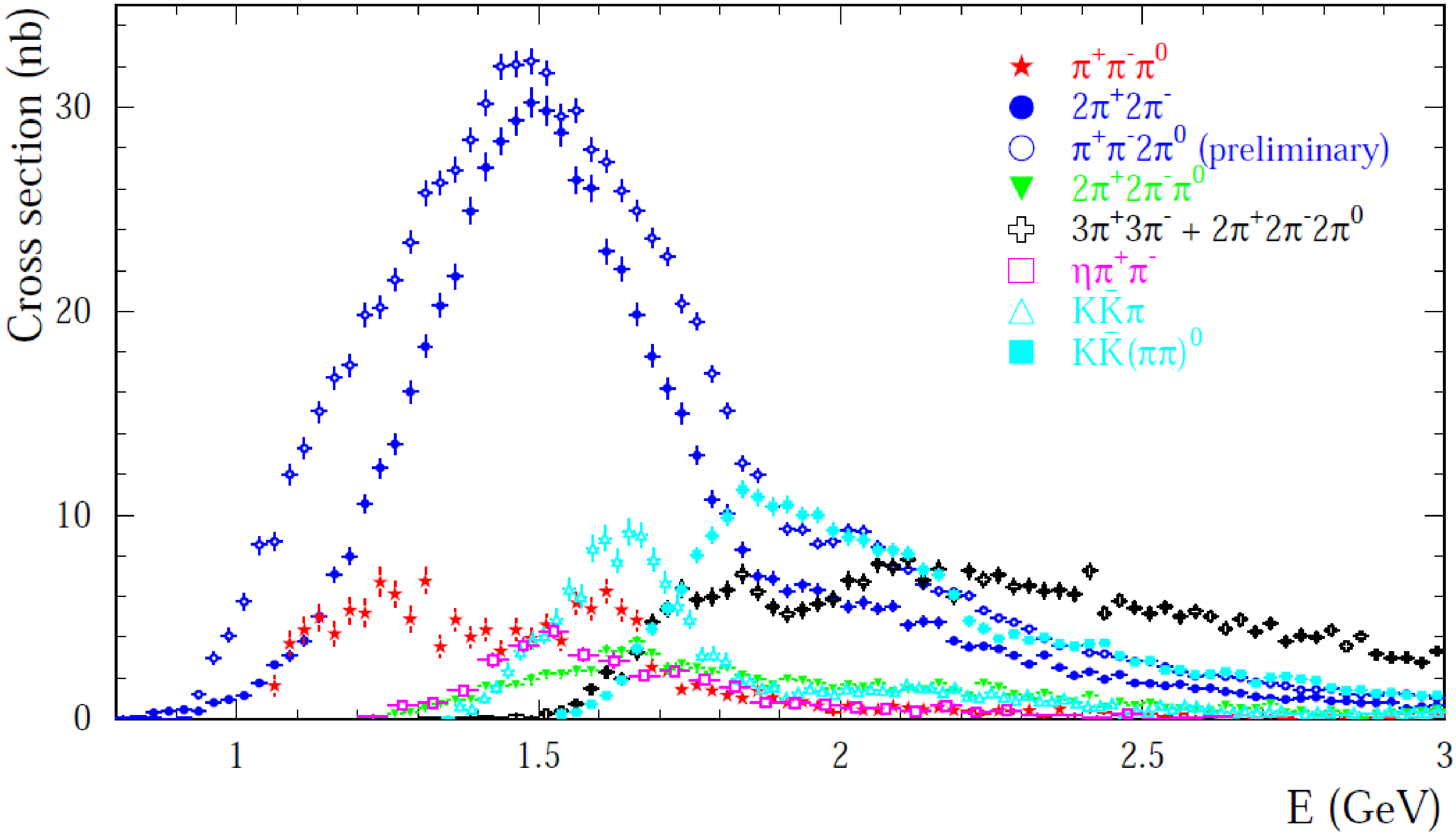
232 fb^{-1}, 89 fb^{-1} @ 10.6 GeV

- ISR γ tagging ⇒ efficient background rejection
- Only charmless mesons in this slide
- Unprecedented accuracy :

$a_\mu(< 1.8\,\text{GeV}/c^2)$		without *BABAR*	with *BABAR*
	$\pi^+\ \pi^-\ \pi^0$	2.45 ± 0.26	3.25 ± 0.09
	$2(\pi^+\ \pi^-)$	14.20 ± 0.90	13.09 ± 0.44
	$3(\pi^+\ \pi^-)$	0.10 ± 0.10	0.11 ± 0.02
	$2(\pi^+\ \pi^-\ \pi^0)$	1.42 ± 0.30	0.89 ± 0.09

Davier, Nucl. Phys. Proc. Suppl. 169, 288 (2007)

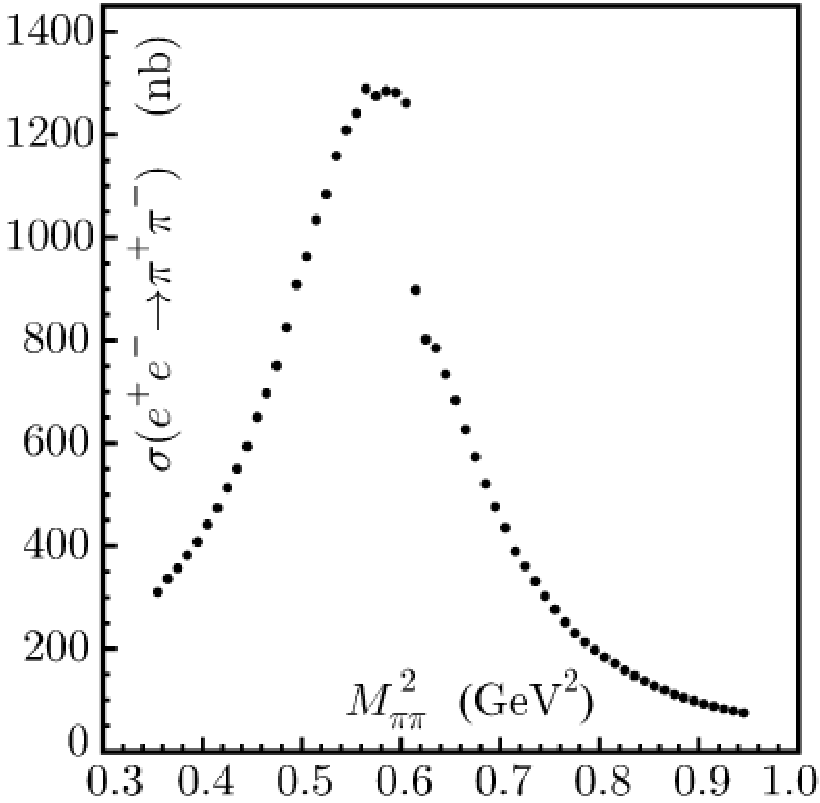
ISR : "Old" BaBar results



Druzhinin, (LP07) arXiv :0710.3455 [hep-ex]

ISR : KLOE $\pi^+\pi^-$

- 240 pb^{-1}
- On the ϕ
- No ISR photon tagging
- $\vec{p}_{\text{had}}$ compatible with a γ in beam pipe
- LO : No additional photon reco'ed
- $W(s,x)$ from NLO PHOKHARA generator (precision 0.5 %)
- Luminosity (0.3 %) from Bhabha scat. (BABAYAGA generator (0.1 %))



$$a_\mu^{\pi^+\pi^-}[592 - 975\,\text{MeV}] = (387.2 \pm 0.5(\text{stat}) \pm 2.4(\text{expt}) \pm 2.3(\text{th})) \times 10^{-10}$$

- Compatible with CMD-2 & SND on (630 - 958 MeV) with similar precision : $\Delta = (-4.6 \pm 4.2) \times 10^{-10}$

Ambrosino, Phys.Lett.B670 :285,2009

ISR : BaBar $\pi^+\pi^-$

Only attempt in BaBar, to my knowledge, to master systematics at the 10^{-3} level

- ISR γ in EMC (thus : at large angle)
- 1 (for efficiency) or 2 (for physics) tracks of good quality
- Particle identification (PID) of the charged particles
- separate $\pi\pi, KK, \mu\mu$ event samples
- kinematic fit (using only direction of ISR γ) including 1 additional γ : NLO !
- obtain all efficiencies (trigger, filter, tracking, PId, fit) from same data
- measure ratio of $\pi\pi$ to $\mu\mu$ cross sections to cancel : ee luminosity, additional ISR, vacuum polarization, ISR γ efficiency

Correct for FSR in $\mu\mu$ and ISR + additional FSR, both calc. in QED, and checked in data

$$R_{\mathrm{exp}}(s') = \frac{\sigma_{[\pi\pi\gamma(\gamma)]}(s')}{\sigma_{[\mu\mu\gamma(\gamma)]}(s')} = \frac{\sigma^0_{[\pi\pi(\gamma)]}(s')}{(1+\delta^{\mu\mu}_{\mathrm{FSR}})\sigma^0_{[\mu\mu(\gamma)]}(s')} = \frac{R(s')}{(1+\delta^{\mu\mu}_{\mathrm{FSR}})(1+\delta^{\mu\mu}_{add,FSR})}$$

A Comment : γ-Tag or not γ-Tag ?

A key issue is the difficulty (impossibility ?) to control the ISR γ efficiency to the desired precision.

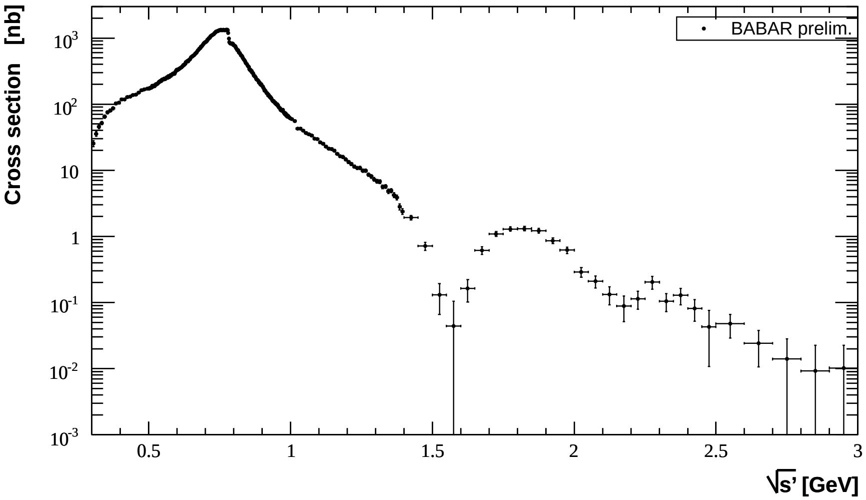
Two ways out :

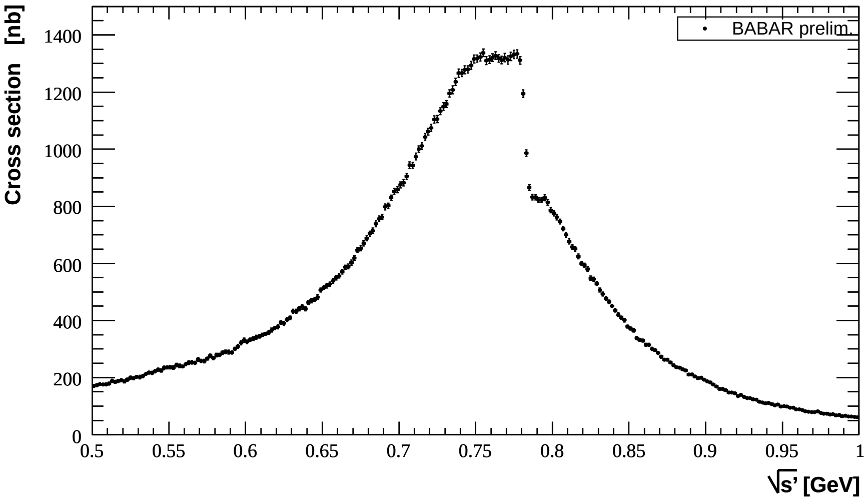
– KLOE : no γ tag, at the cost of a significant background. (mitigated by requesting that the non-observed γ would be in the beam pipe)

– BaBar : γ tag, and use the $\pi\pi/\mu\mu$ ratio
$\Rightarrow$ the ISR γ efficiency cancels in the ratio (to first order)
$\Rightarrow$ γ tag costs a loss of 9/10 in statistics

Note that KLOE has a tagged analysis in progress.

ISR : BaBar $\pi^+\pi^-$





Bare (VP removed), unfolded $\sigma_{e^+e^-\to\pi^+\pi^-}$ $232\,\mathrm{fb}^{-1}$ @ $\sqrt{s} \approx 10.6\,\mathrm{GeV}$

$$\boxed{a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\mathrm{GeV}/c^2] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}}$$

- Similar precision as combination of previous e^+e^- results.
- 2.0 σ larger than previous e^+e^- average PRL 103, 231801 (2009)
- Longer paper, in preparation, to be submitted to PRD

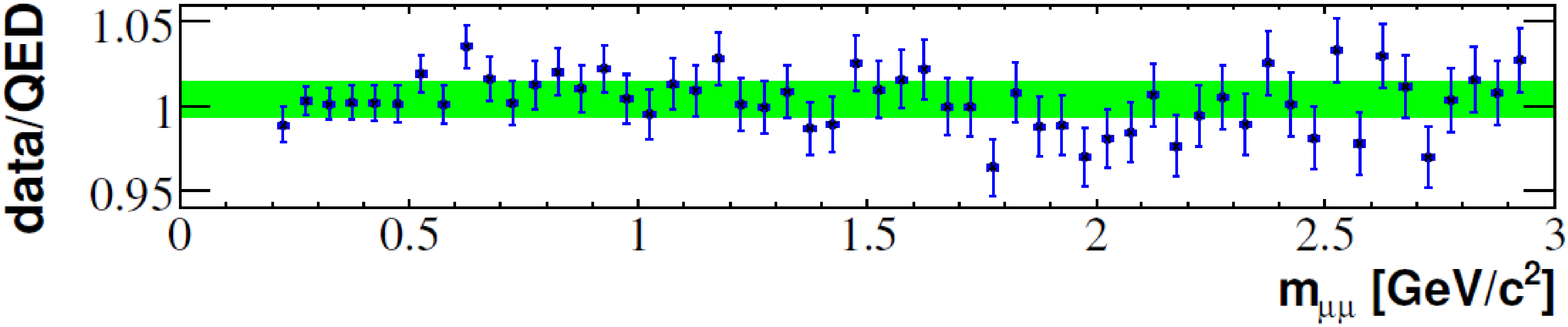
Systematics

Relative systematic uncertainties (in 10^{-3}) on the $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section by $\sqrt{s'}$ intervals (in GeV) up to $1.2\,\mathrm{GeV}$.

Source of Uncertainty	CM Energy Interval (GeV)				
	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2
trigger/ filter	5.3	2.7	1.9	1.0	0.5
tracking	3.8	2.1	2.1	1.1	1.7
π-ID	10.1	2.5	6.2	2.4	4.2
background	3.5	4.3	5.2	1.0	3.0
acceptance	1.6	1.6	1.0	1.0	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9
correlated $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3
unfolding	1.0	2.7	2.7	1.0	1.3
ISR luminosity ($\mu\mu$)	3.4	3.4	3.4	3.4	3.4
total uncertainty	13.8	8.1	10.2	5.0	6.5

PRL 103, 231801 (2009)

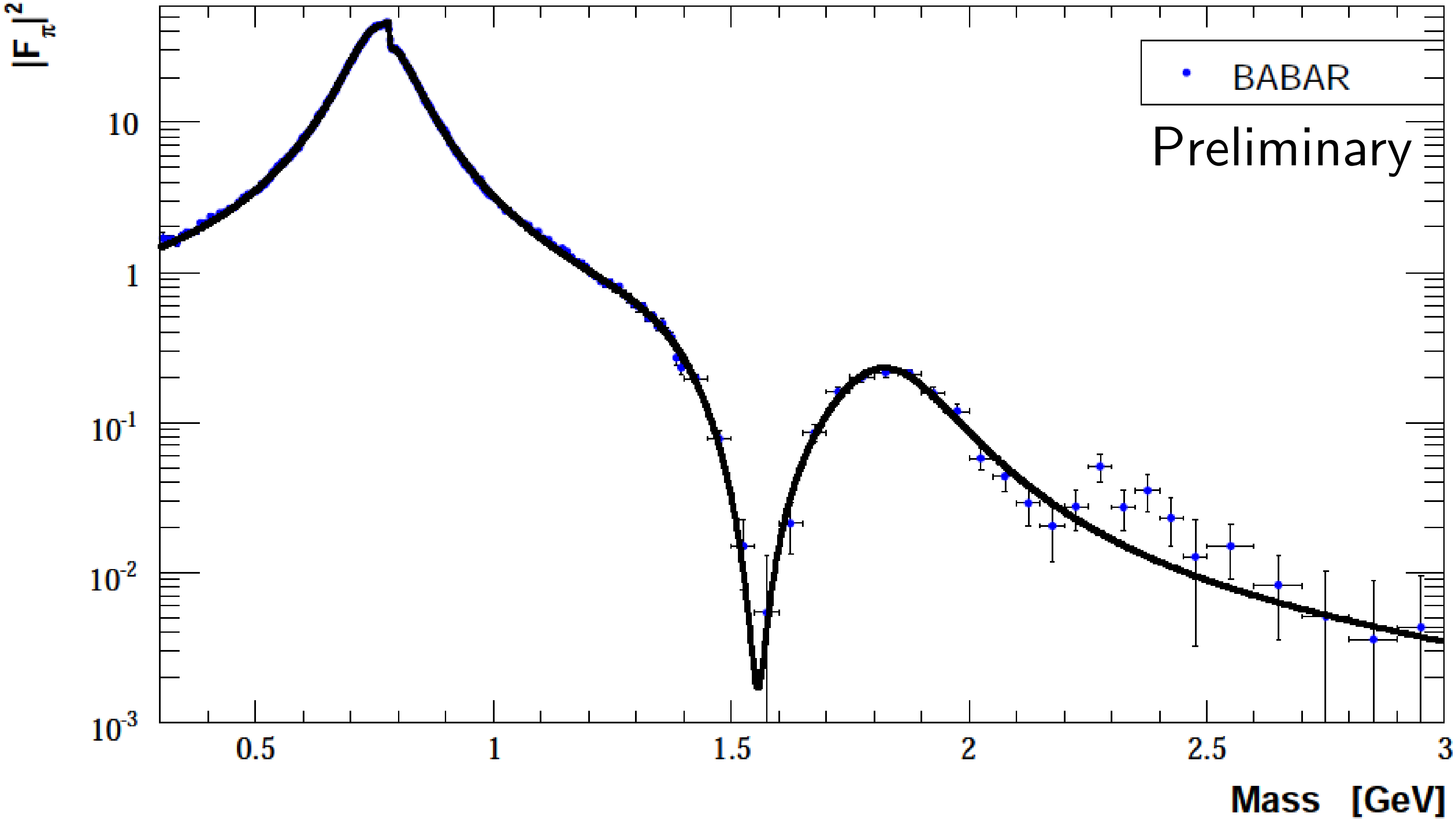
BaBar : Sanity check : Comparison of the $\mu\mu$ spectrum with QED



- Here the radiator function is needed.
- MC simulation corrected for all known MC/data differences.
    - e.g. : ISR γ efficiency measured in data, from $\mu\mu$-only reco'ed evts.
    - MC corrected for known NLO deficiencies by comparing to PHOKHARA

Good agreement within $0.4 \pm 1.1\%$; dominated by $\mathcal{L}_{e^+e^-}$ $(\pm 0.9\%)$

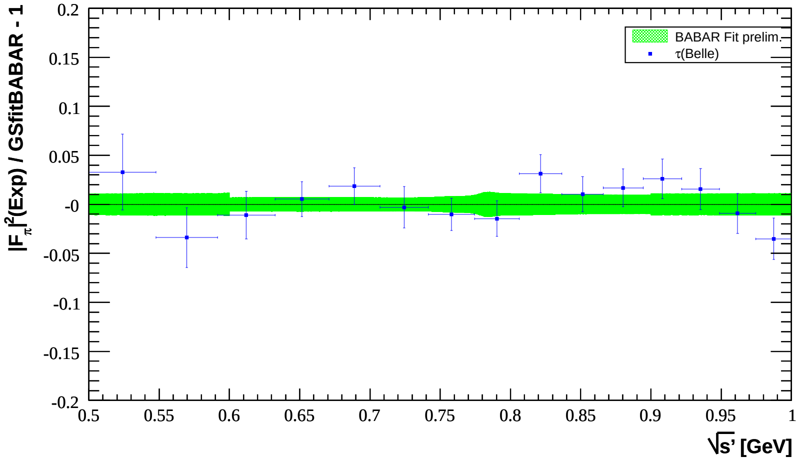
$VDM\ Fit\ of\ |F_\pi(s)|^2$



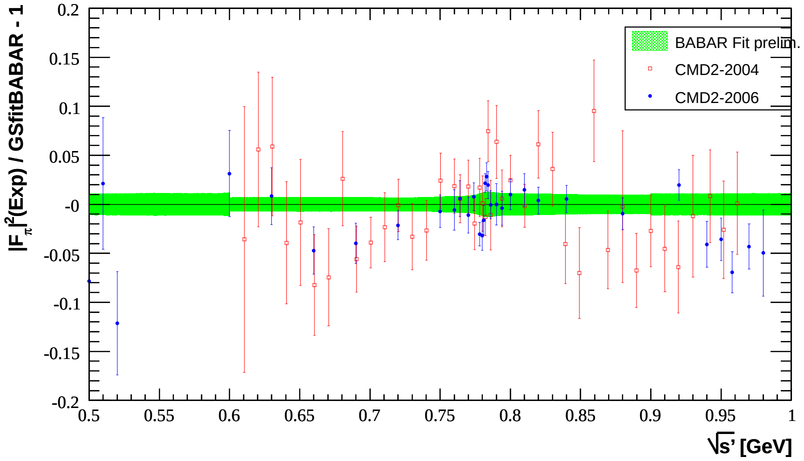
- $|\text{form factor}|^2$ fitted with a vector dominance model, $\rho, \rho', \rho'', \omega$.
- ρ's described by the Gounaris-Sakurai model $\chi^2/n_{df} = 334/323$

BaBar $\pi^+\pi^-$ *: comparison with previous results*

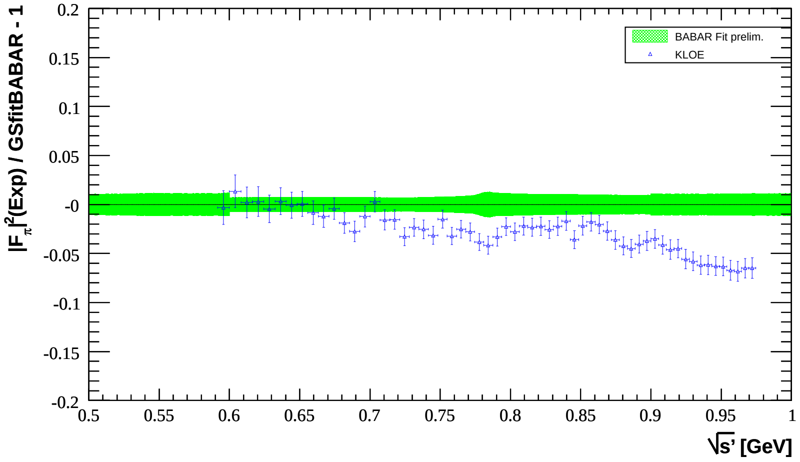
Belle τ



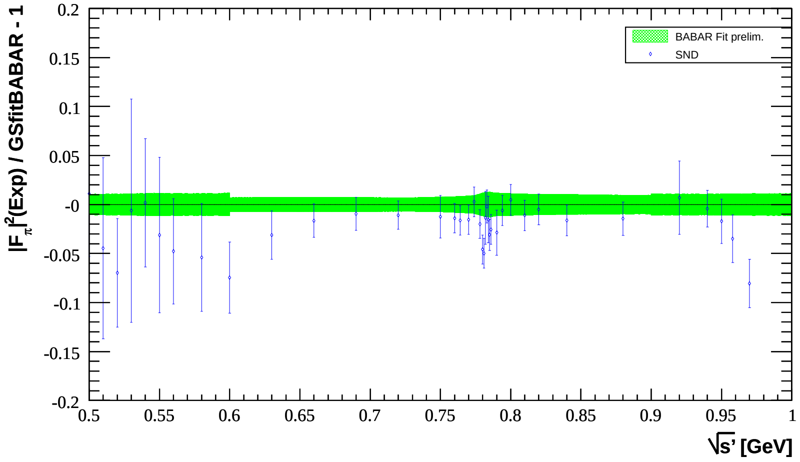
CMD2 e^+e^-



KLOE e^+e^- ISR

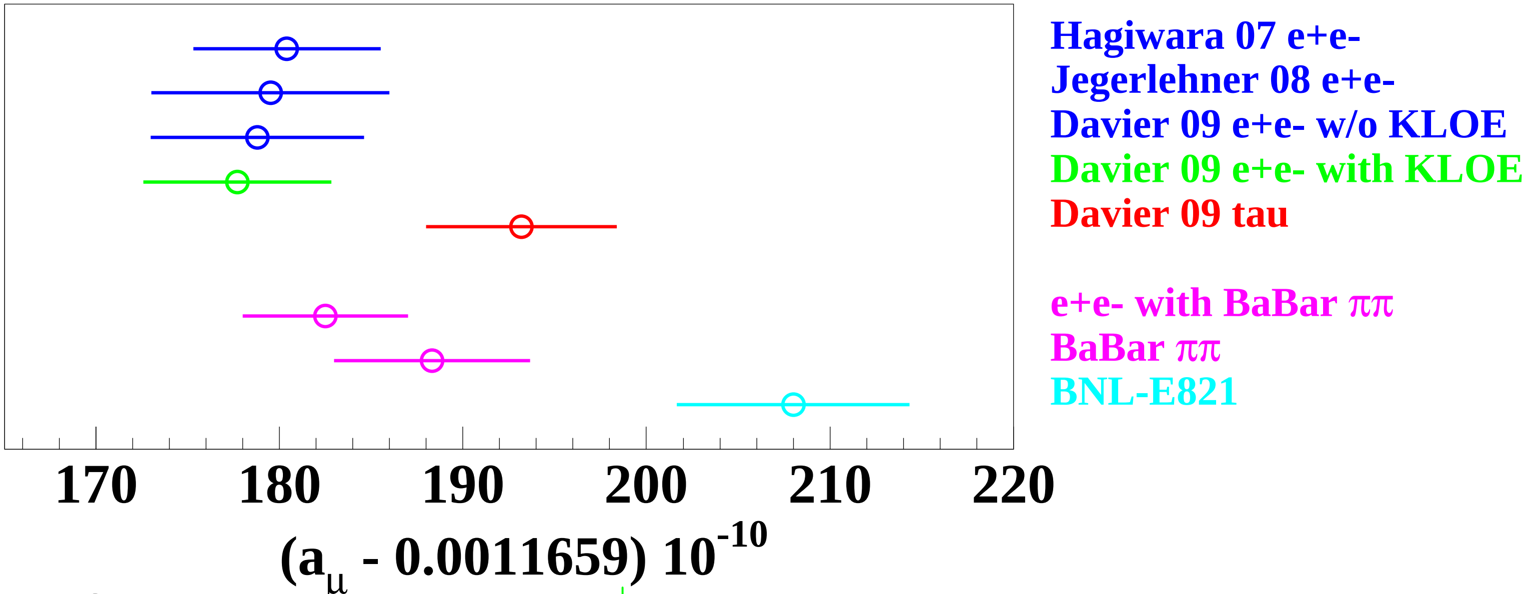


SND e^+e^-



The green band is the representation of the VDM fit to the BaBar ISR data

a_μ : *Where do we stand ?*



- Good agreement between e^+e^--based predictions
- BNL a_μ measurement 3.4 σ higher than e^+e^--based
- τ-based "only" 1.8 σ from BNL.

My combinations, assuming Gaussian statistics

- BaBar compatible with the τ-based
- BaBar 2.4 σ from BNL.
- $(e^+e^- +$ BaBar$)$ 3.3 σ from BNL.

The $(e^+e^-$- BNL) shift has decreased, so has the uncertainty .. $> 3\sigma$ still

What could happen during the present decade

- $a_\mu(\mathrm{expt})$: 0.54 → 0.14 ppm   Carey, FERMILAB-PROPOSAL-0989 (2009)
  - was statistics limited (0.46 ppm) ; move μ storage ring BNL → FNAL.
- Vacuum polarization (hadronic) : $a_\mu^{\pi^+\pi^-}$ is the bottle-neck
  - e^+e^-, “direct”
    - upgrades, CMD2 → CMD3 and SND
    - BES III : $R : 2.0 - 4.6\,\mathrm{GeV}$,   Zhemchugov @ TAU08
  - e^+e^-, ISR : Statistics is not an issue.
    - Belle checks BaBar’s ISR result ?
    - KLOE checks it’s ISR result with $\pi\pi/\mu\mu$ ratio
  - τ : Statistics is not an issue.
    - Theorists converge to a narrow range calculation of Δ IB
    - BES III : τ spectral fns, $\mathcal{B}(\tau \to \pi^-\pi^0\nu)$ Zhemchugov @ TAU08
    - BaBar checks Belle’s τ result ?
- Light-by-Light : Theory + $\gamma\ \gamma$ program at DAΦNE-2   Prades @ TAU08
- QED : The 5^{th} order is being evaluated   Kinoshita

Je vous remercie de votre attention

Back-up slides

Jargon

ISR	Initial State Radiation
FSR	Final State Radiation
VP	Vacuum Polarization
IB	Isospin Breaking
CVC	Conserved vector current
P	Parity
pQCD	perturbative QCD
VDM	Vector dominance model

Form Factor and $R_{\text{had}}(s)$

$$R_{\text{had}}(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} \left|F_\pi^{(0)}(s)\right|^2$$