Recent developments in QCD

Vittorio Del Duca

INFN LNF

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QCD

- an unbroken Yang-Mills gauge field theory featuring asymptotic freedom and confinement
- in non-perturbative regime (low Q^2) many approaches: lattice, Regge theory, χ PT, large N_c, HQET
- in perturbative regime (high Q^2) QCD is a precision toolkit for exploring Higgs & BSM physics

QCD at LHC

Precise determination of

- strong coupling constant α_s
- parton distributions
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

Goal: to make theoretical predictions of signals and backgrounds as accurate as the LHC data

LHC: present & future

- calibrate the detectors, and re-discover the SM
  i.e. measure known cross sections: jets, $W, Z, t\bar{t}$

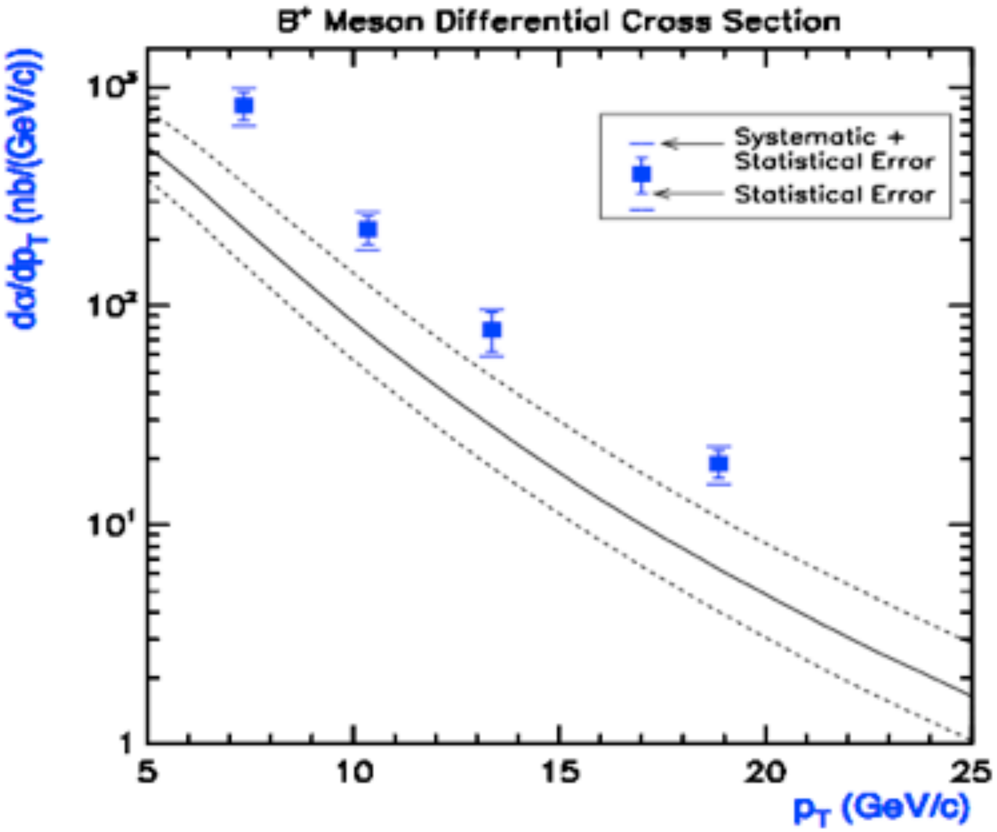
- understand the EWSB/find New-Physics signals
  (ranging from Z' to leptons, to gluinos in SUSY
  decay chains, to finding the Higgs boson)

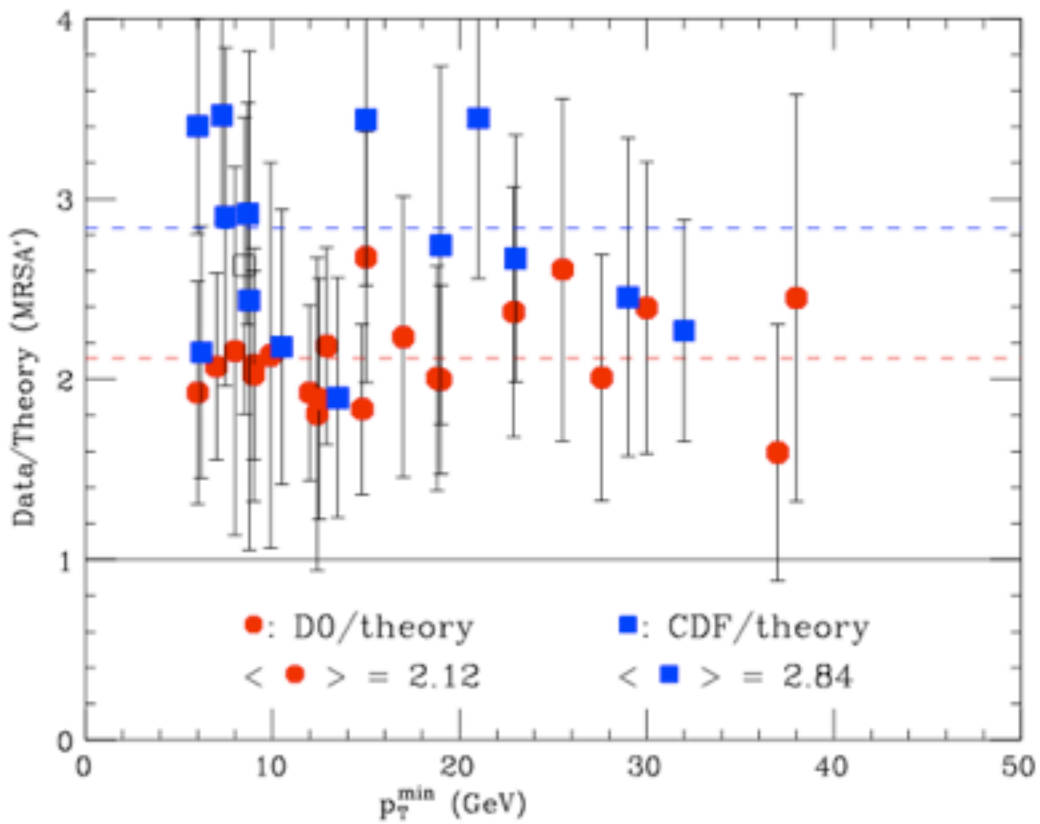
- constrain and model the New-Physics theories

in all the steps above (except probably Z' to leptons)
precise QCD predictions play a crucial role

Tales from the past

B production in the 90's

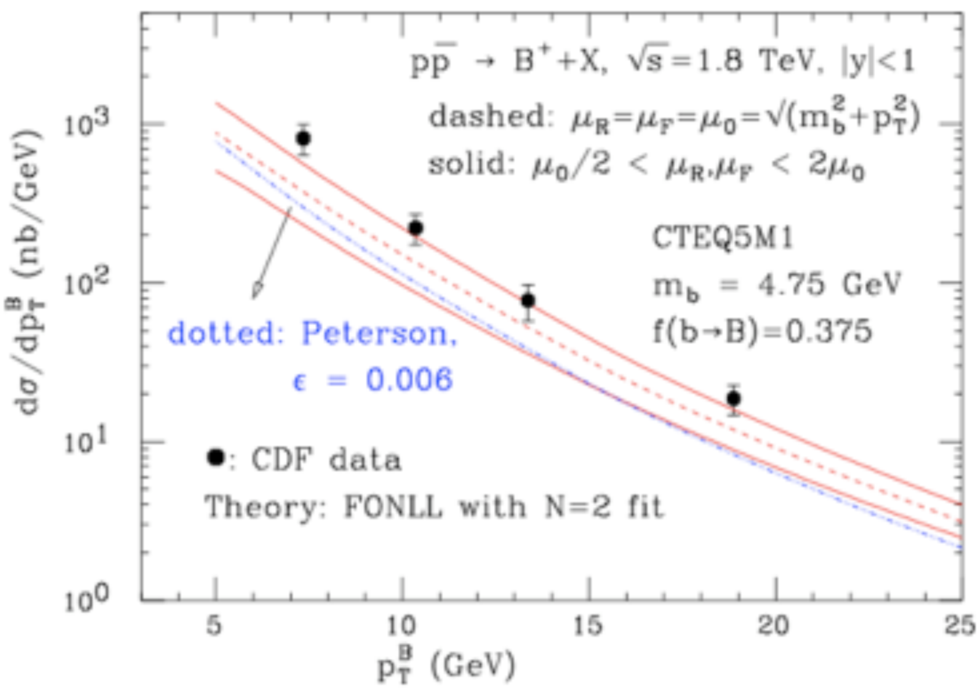


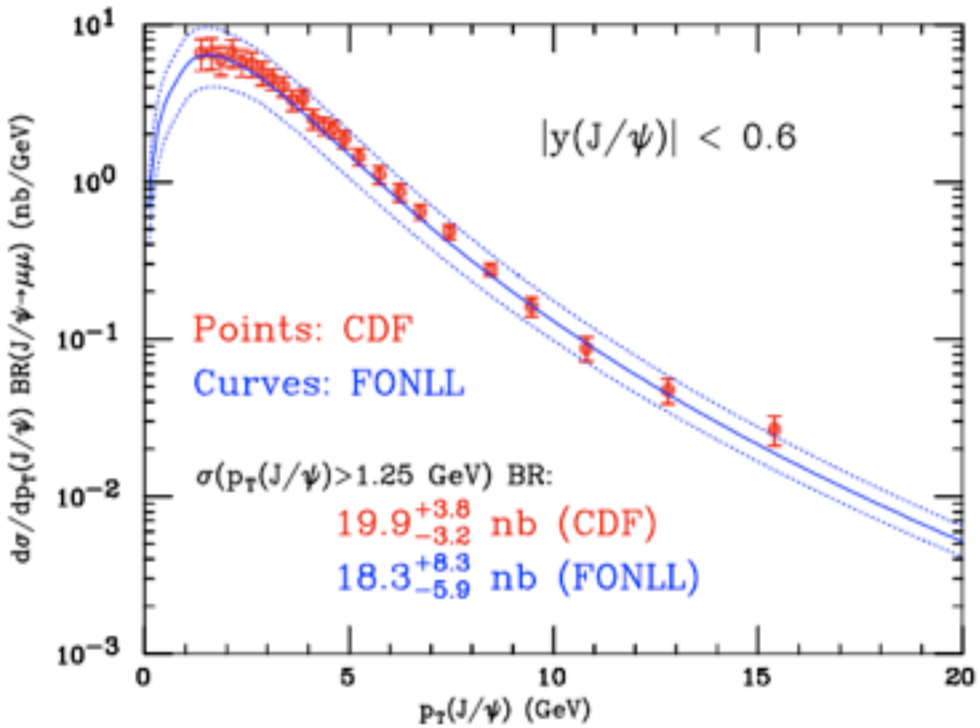


discrepancy between Tevatron data and NLO prediction

B cross section in $p\bar{p}$ collisions at 1.96 TeV

$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$





FONLL = NLO + NLL

total x-sect is $19.4 \pm 0.3(stat)^{+2.1}_{-1.9}(syst)$ nb

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

CDF hep-ex/0412071

use of updated fragmentation functions by Cacciari & Nason and resummation

→ good agreement with data ⟹ no New Physics

QCD at high Q^2

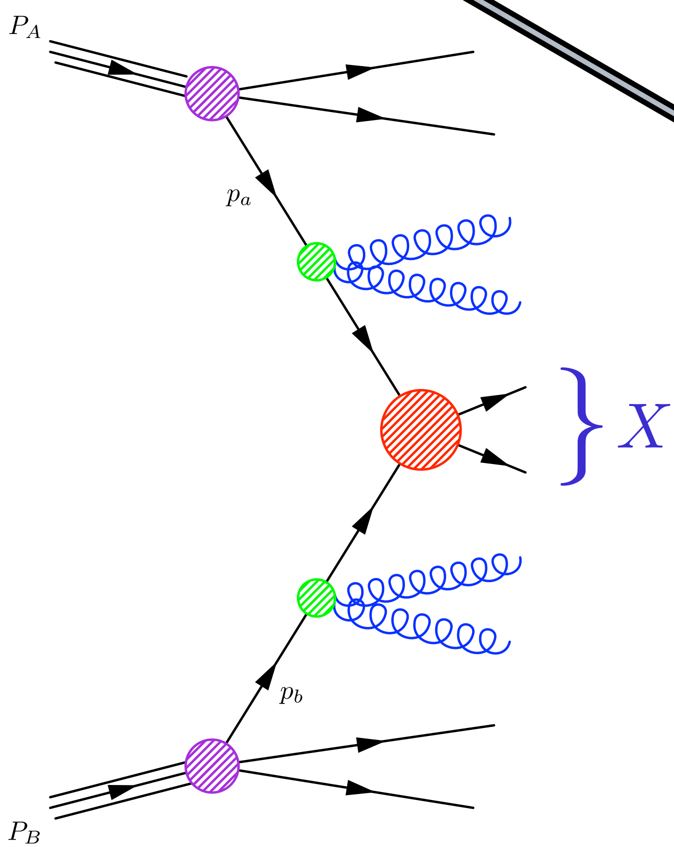
- Parton model
- Perturbative QCD
  - factorisation
  - universality of IR behaviour
  - cancellation of IR singularities
  - IR safe observables: inclusive rates
    - jets
    - event shapes

Factorisation

computed in pQCD

extracted from data evolved through DGLAP

is the separation between the short- and the long-range interactions



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1, \mu_F^2)\, f_{b/B}(x_2, \mu_F^2)$$
$$\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n(1 + c_1\alpha_S + c_2\alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\qquad c_2 =$ NNLO

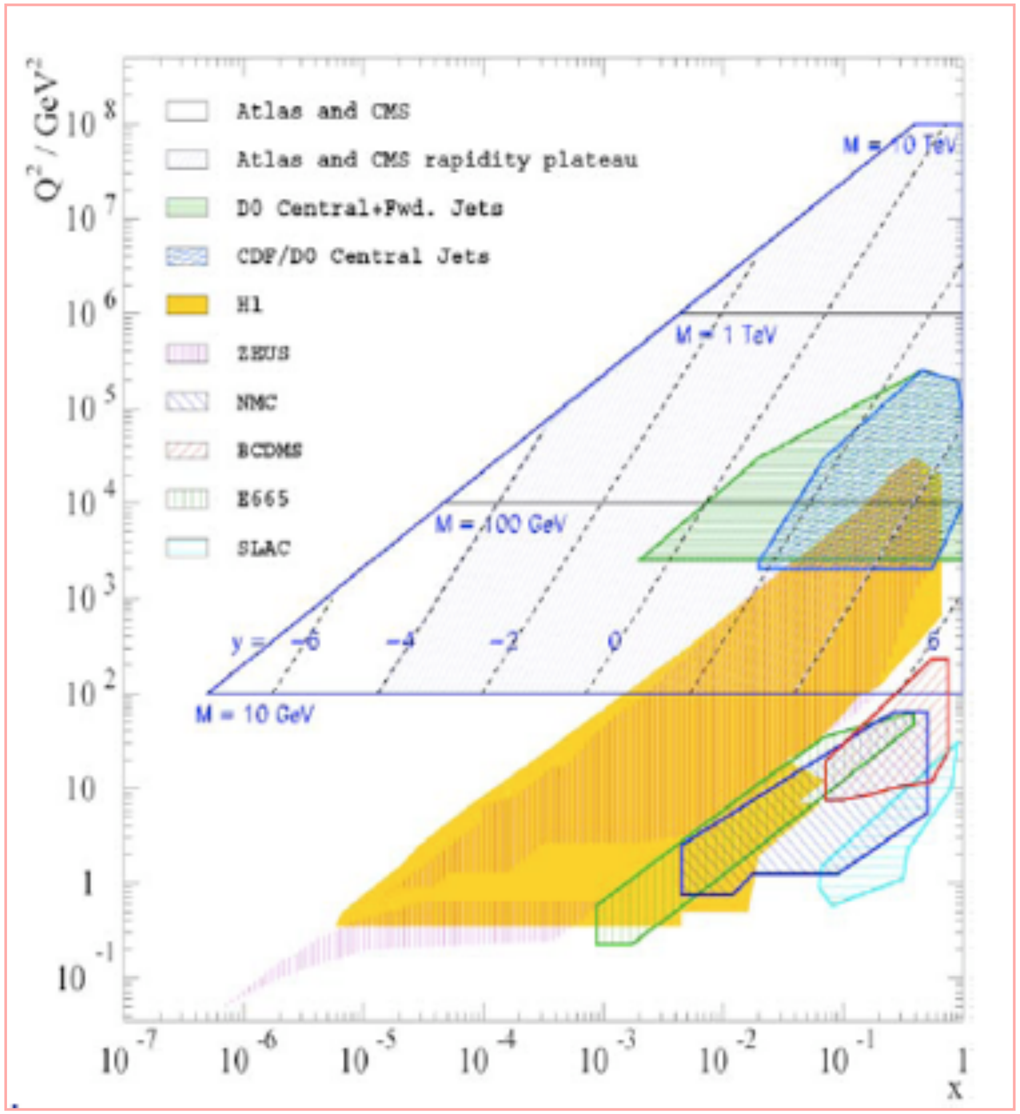
or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n[1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\qquad c_{10}, c_{21} =$ NLL $\qquad c_{20} =$ NNLL

LHC kinematic reach



LHC opens up a new kinematic range

x range covered by HERA but Q^2 range must be provided by DGLAP evolution

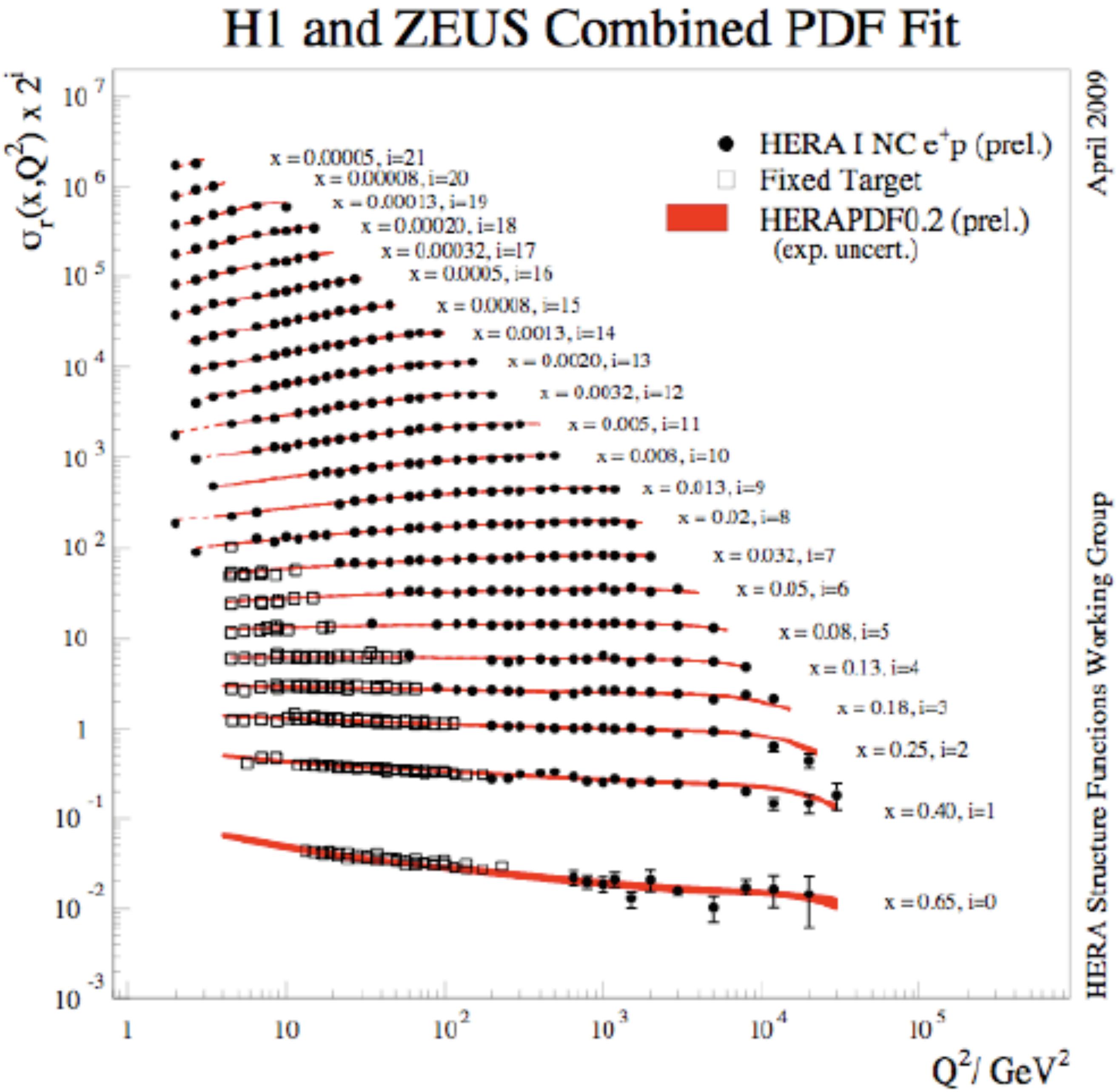
100-200 GeV physics is large x physics (valence quarks) at Tevatron, but smaller x physics (gluons & sea quarks) at the LHC

rapidity distributions span widest x range

Feynman x's for the production of a particle of mass M $x_{1,2} = \frac{M}{14\,\mathrm{TeV}} e^{\pm y}$

HERA F_2

DIS09



large violations at small x

small violations at large x

horizontal lines ⇒ Bjorken scaling

straight (non-horizontal) lines ⇒ scaling violations, logarithmic in Q^2

PDF global fits

J. Stirling, KITP collider conf 2004

global fits

MRST, MSTW: Martin *et al.*

CTEQ: Pumplin *et al.*

Alekhin (DIS data only)

method

Perform fit by minimising χ^2 to all data, including both statistical and systematic errors

Start evolution at some Q_0^2, where PDF's are parametrised with functional form, e.g.
$xf(x,Q_0^2) = (1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$

Cut data at $Q^2 > Q^2_{\min}$ and at $W^2 > W^2_{\min}$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

accuracy

NNLO evolution

H1, ZEUS $F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$

SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

CCFR $F_2^{\nu(\bar\nu)p}(x,Q^2), F_3^{\nu(\bar\nu)p}(x,Q^2)$

$\rightarrow q, \bar{q}$ at all x and g at medium, small x

H1, ZEUS $F_{2,c}^{e^+p}(x,Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar\mu + X \rightarrow \bar{q}\ (g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar{u}, \bar{d}$

CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high x

CDF, D0 Inclusive jet data $\rightarrow g$ at high x

CCFR, NuTeV Dimuon data constrains strange sea $s, \bar{s}$

- no prompt photon data included in the fits

PDF: recent developments

- more systematic treatment of uncertainties in global fits

  MSTW, CTEQ, Alekhin

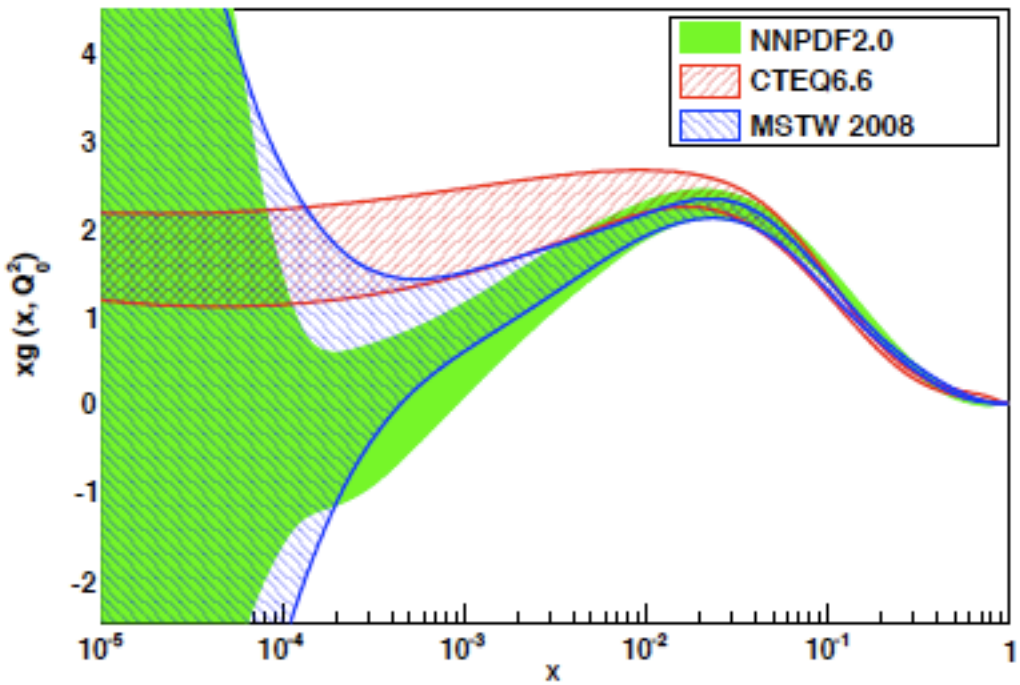
- more accurate treatment of heavy flavours in the vicinity of the quark mass (few % effect on Drell-Yan at the LHC)

- PDF's from neural network global fit (NNPDF), based on unbiased priors

  NNPDF2.0 arXiv:1002.4407

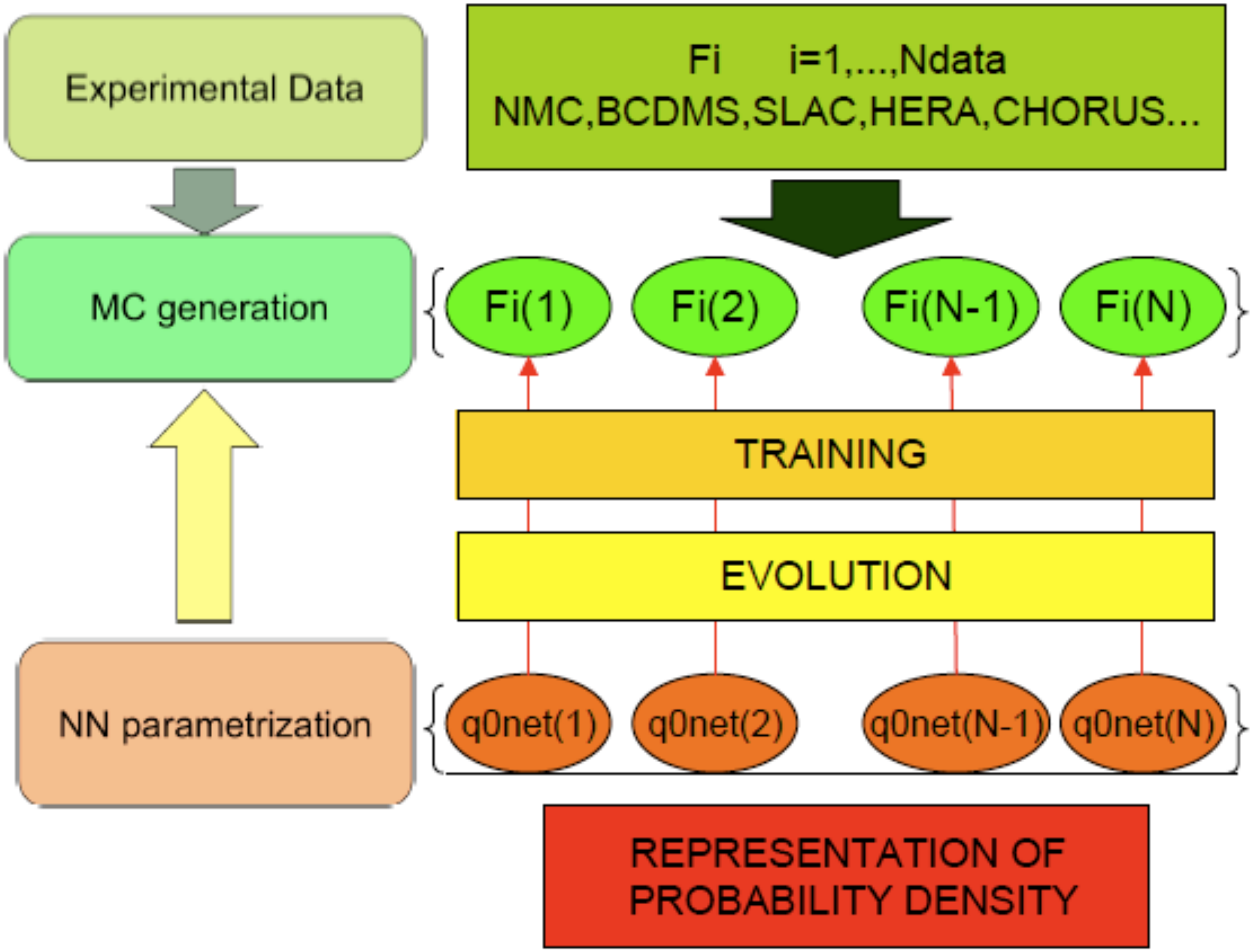
gluon distribution

note the larger uncertainty in NNPDF at small *x*



NNPDF

Ubiali (NNPDF Coll.) DIS09



stop before over-training: NN fitting of statistical fluctuations

NNPDF

Ubiali (NNPDF Coll.) DIS09

Determination of unbiased PDFs with faithful estimation of their uncertainties.

$$\langle \mathcal{F}[f_i(x)] \rangle = \int [\mathcal{D}f_i]\, \mathcal{F}[f_i(x)] \mathcal{P}[f_i(x)] \quad \rightarrow \quad \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{F}[f_i^{(k)(\rm net)}(x)]$$

$$\sigma_{\mathcal{F}[f(x)]} \quad = \quad \sqrt{\langle \mathcal{F}[f(x)]^2 \rangle - \langle \mathcal{F}[f(x)] \rangle^2}$$

* The measure $\mathcal{P}[f_i(x)]$ in space of PDFs is determined with MC method.
* Use all information contained in experiments.
* Redundant parametrization of PDFs: reduce bias.
* Statistic estimators to assess errors, correlations, stability and size of systematics.

sort of path-integral method in the space of PDFs

Parton cross section

3 complementary approaches to $\hat{\sigma}$

	matrix-elem MC's	fixed-order x-sect	shower MC's
final-state description	hard-parton jets. Describes geometry, correlations, ...	limited access to final-state structure	full information available at the hadron level
higher-order effects: loop corrections	hard to implement: must introduce negative probabilities	straightforward to implement (when available)	included as vertex corrections (Sudakov FF's)
higher-order effects: hard emissions	included, up to high orders (multijets)	straightforward to implement (when available)	approximate, incomplete phase space at large angles
resummation of large logs	?	feasible (when available)	unitarity implementation (i.e. correct shapes but not total rates)

M.L. Mangano KITP collider conf 2004

Parton shower MonteCarlo generators

- HERWIG B. Webber *et al.* 1992

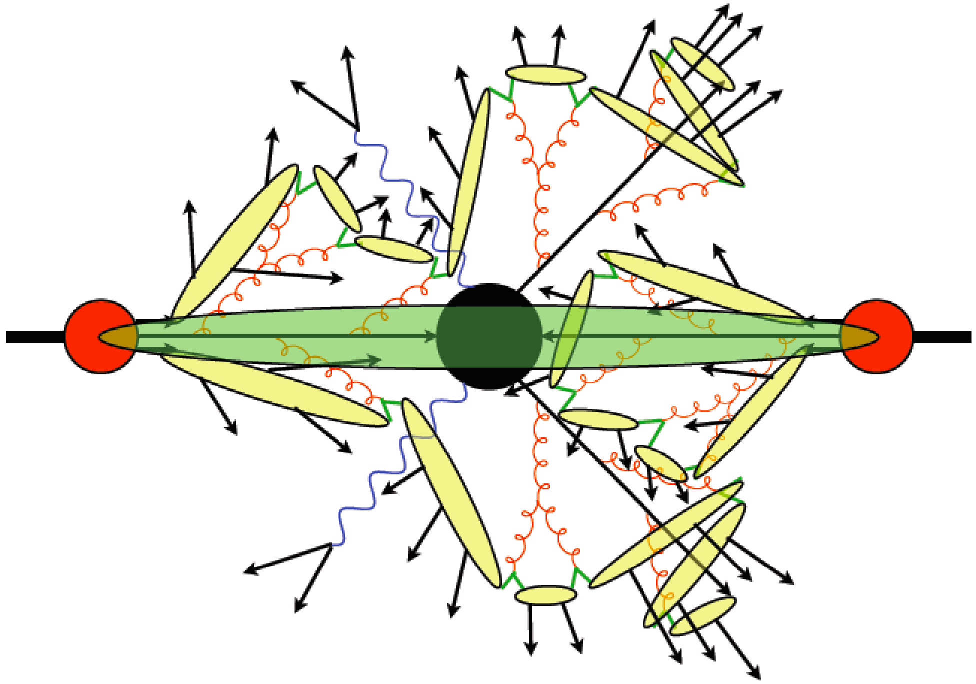
  re-written as a C++ code (HERWIG++)

- PYTHIA T. Sjostrand 1994 (also re-written as a C++ code)

- SHERPA F. Krauss *et al.* 2003

model parton showering and hadronisation

LHC Event Simulation



Matrix-element MonteCarlo generators

↑ several automated codes to yield large number of (up to 8-9) final-state partons

↑ can be straightforwardly interfaced to parton-shower MC's

↑ ideal to scout new territory

↓ large dependence on ren/fact scales
example: Higgs (via gluon fusion) + 2 jets is $\alpha_s^4(Q^2)$

⇨ unreliable for precision calculations

Matrix-element MonteCarlo generators

- multi-parton LO generation: processes with many jets (or V/H bosons)
  - ALPGEN M.L.Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002
  - MADGRAPH/MADEVENT W.F. Long F. Maltoni T. Stelzer 1994/2003
  - COMPHEP A. Pukhov et al. 1999
  - GRACE/GR@PPA T. Ishikawa *et al.* K. Sato *et al.* 1992/2001
  - HELAC C. Papadopoulos *et al.* 2000
- processes with 6 final-state fermions
  - PHASE E. Accomando A. Ballestrero E. Maina 2004
- merged with parton showers
  - all of the above, merged with HERWIG or PYTHIA

MonteCarlo interfaces

- CKKW S. Catani F. Krauss R. Kuhn B. Webber 2001

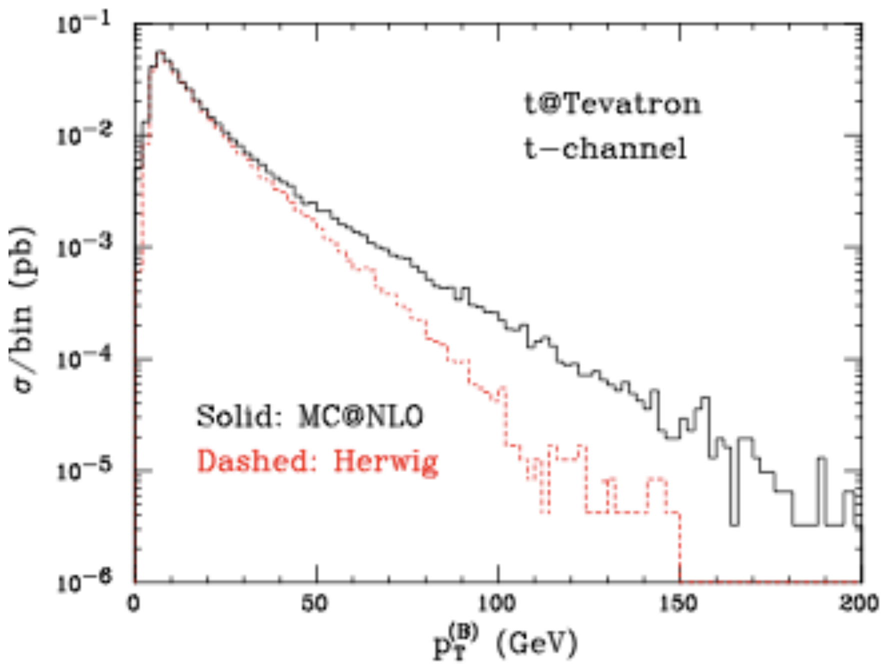
  MLM L. Lonnblad 2002 M.L. Mangano 2005

  procedures to interface parton subprocesses with a different number of final states to parton-shower MC's

- MC@NLO S. Frixione B. Webber 2002

  POWHEG P. Nason 2004

  procedures to interface NLO computations to parton-shower MC's



Single top in MC@NLO

Frixione Laenen Motylinski Webber 2005

at low p_T, parton shower models collinear radiation

at high p_T, NLO models hard radiation

Matrix-element MonteCarlo generator at NLO

- desirable to have a multi-parton NLO generator
  interfaced to a parton shower:
  a sort of MadGraph cum MC@NLO

- a step in this direction: automation of subtraction of IR divergences

  Frederix Frixione Maltoni Stelzer 2009

  MadGraph provides real amplitude
  user inputs virtual amplitude
  procedure provides subtraction counterterms

Next to Leading Order

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's:
  MC@NLO POWHEG

NLO cross sections: experimenter's wishlist

2005 Les Houches

QCD, EW & Higgs working group hep-ph/0604210

Table 42: The LHC "priority" wishlist for which a NLO computation seems now feasible.

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

High (EXP) demand for cross sections of $Y + n$ jets
Y = vector boson(s), Higgs, heavy quark(s), ...
Big TH community effort

To compute the cross section of $Y + n$ jets, we need:

1) tree-level amplitude for $Y + (n+3)$ partons
2) one-loop amplitude for $Y + (n+2)$ partons
3) a method to cancel the IR divergences
   and so to compute the cross section

3: until the mid 90's, we did not have systematic methods
   to cancel the IR divergences
2: until 2007-8, we did not have systematic methods
   to compute the one-loop amplitudes

NLO history of final-state distributions

- $e^+e^- \to 3$ jets — K. Ellis Ross Terrano 1981
  $e^+e^- \to 4$ jets — Bern *et al.*; Glover *et al.*; Nagy Trocsanyi 1996-97
- $e^+e^- \to 4$ fermions — Denner Dittmaier Roth Wieders 2005
- $pp \to 1, 2$ jets — K. Ellis Sexton 1986 Giele Glover Kosower 1993
  $pp \to 3$ jets — Bern Dixon Kosower; Kunszt Signer Trocsanyi 1993-1995 Nagy 2001
- $pp \to V + 1$ jet — Giele Glover Kosower 1993
  $pp \to V + 2$ jet — Bern Dixon Kosower Weinzierl; Campbell Glover Miller 1996-97 Campbell K. Ellis 2003
  $pp \to V + 3$ jet — Berger *et al.* (BlackHat); K. Ellis Melnikov Zanderighi 2009
- $pp \to Vb\bar{b}$ — Campbell K. Ellis 2003
- $pp \to VV$ — Ohnemus Owens, Baur *et al.* 1991-96; Dixon *et al.* 2000
  $pp \to VV +$ jet — Dittmaier Kallweit Uwer; Campbell K. Ellis Zanderighi 2007
- $pp \to VVV$ — Lazopoulos Melnikov Petriello; Hankele Zeppenfeld 2007 Binoth Ossola Papadopoulos Petriello 2008
- $pp \to \gamma\gamma$ — Bailey *et al.* 1992; Binoth *et al.* 1999
  $pp \to \gamma\gamma + 1$ jet — Bern *et al.* 1994 Del Duca Maltoni Nagy Trocsanyi 2003

NLO history of final-state distributions

- $pp \to Q\bar{Q}$ — Dawson K. Ellis Nason 1989 Mangano Nason Ridolfi 1992

  $pp \to Q\bar{Q} + 1$ jet — Brandenburg Dittmaier Uwer Weinzierl 2005-7; Dittmaier Kallweit Uwer 2007-8

  $pp \to Q\bar{Q} + 2$ jets — van Hameren Papadopoulos Pittau 2009; Bevilacqua Czakon Papadopoulos Worek 2010

- $pp \to t\,\bar{t}\,Z$ — Lazopoulos McElmurry Melnikov Petriello 2008
- $pp \to t\,\bar{t}\,b\,\bar{b}$ — Bredenstein Denner Dittmaier Pozzorini 2009
- $pp \to t\,(+\;W)$ — Harris Laenen Phaf Sullivan Weinzierl 2002; Campbell K. Ellis Tramontano 2004; Cao Yuan et al. 2004-5; *W*: Campbell Tramontano 2005
- $pp \to H + 1$ jet — (GGF; ∞ m_t) C. Schmidt 1997 De Florian Grazzini Kunszt 1999

  $pp \to H + 2$ jets — (GGF; ∞ m_t) Campbell K. Ellis Zanderighi 2006; (WBF) Campbell K. Ellis; Figy Oleari Zeppenfeld 2003; Ciccolini Denner Dittmaier 2007 (includes s channel)

  $pp \to H + 3$ jets — (WBF) Figy Hankele Zeppenfeld 2007

- $pp \to H Q\bar{Q}$ — Beenakker *et al.*; Dawson *et al.* 2001
- $pp \to H\,Q$ — *t*: Maltoni Paul Stelzer Willenbrock 2001; *b*: Campbell, K. Ellis Maltoni Willenbrock 2002

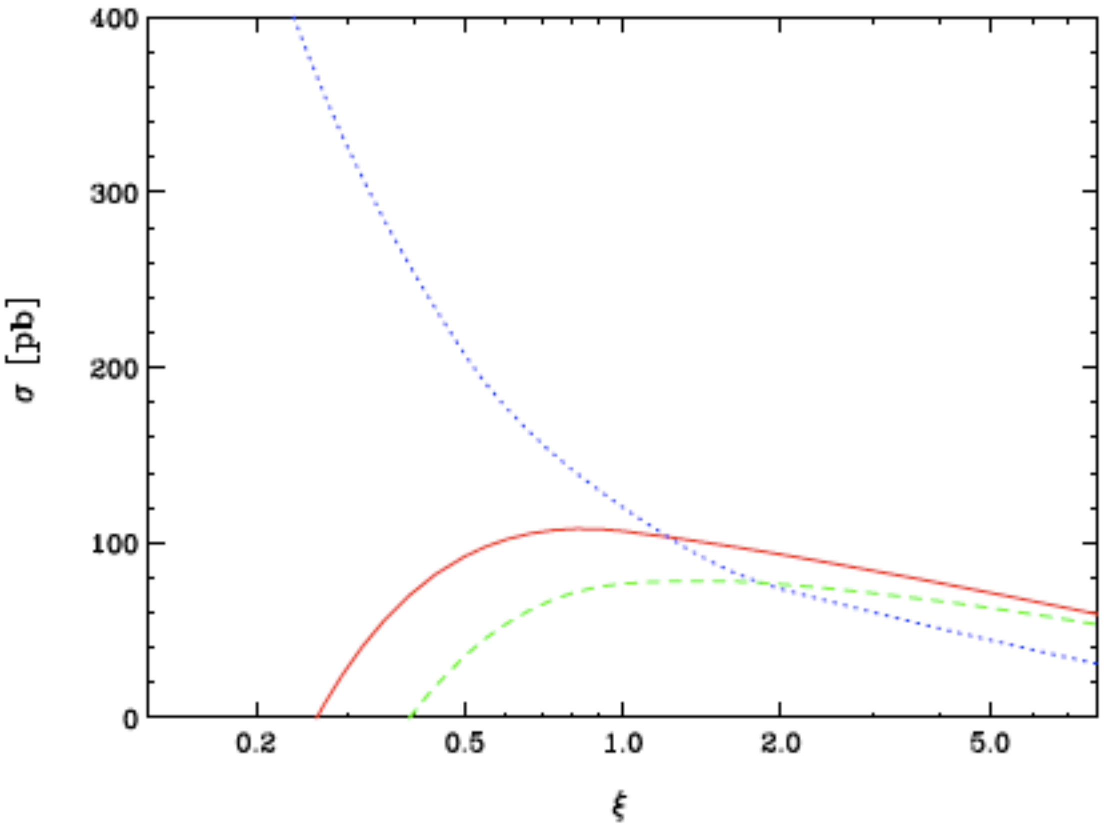
... summarising

- in the past, long time span to add one more jet to a x-section
- $2 \to 2$ and $2 \to 3$ processes:
  almost all computed and included into NLO packages
- $2 \to 4$ processes: very few computed
  $e^+e^- \to 4$ fermions — Denner Dittmaier Roth Wieders 2005
  $pp \to t\,\bar{t}\,b\,\bar{b}$ — Bredenstein Denner Dittmaier Pozzorini 2009
  $pp \to V + 3$ jet — Berger *et al.* (BlackHat); K. Ellis Melnikov Zanderighi 2009
  $pp \to Q\bar{Q} + 2\,\text{jets}$ — Bevilacqua Czakon Papadopoulos Worek 2010
- $2 \to 5$ processes: none

$pp \to t\bar{t} + 2\,\mathrm{jets}$ at NLO

Scale dependence of total x-sect

Bevilacqua Czakon Papadopoulos Worek 2010



$\mu_R = \mu_F = \xi\,\mu_0$ with $\mu_0 = m_t$

dots: LO
solid: NLO
dash: NLO with jet veto of 50 GeV

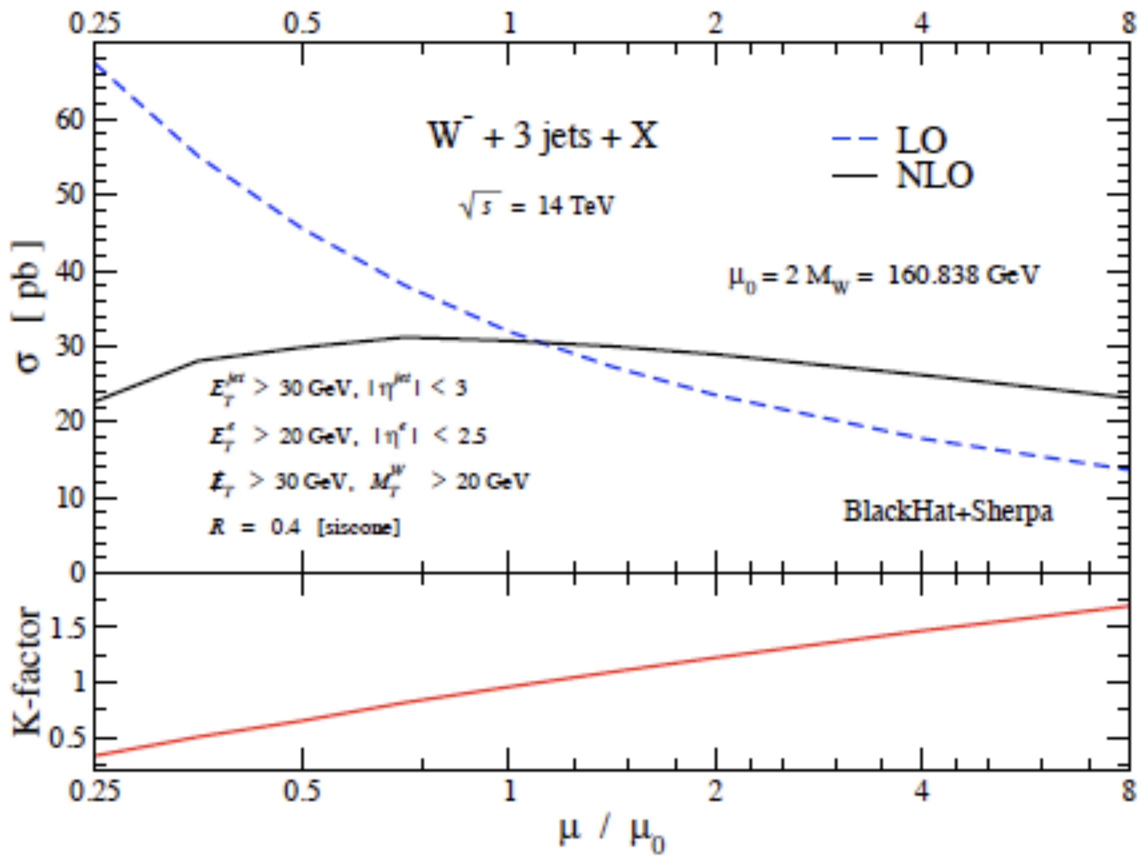
- reducible background to $pp \to H\,t\,\bar{t}$
- NLO/LO ≡ *K* factor = 0.89
- Reduced theoretical error: 40-70% at LO; 12-13% at NLO

W + 3-jet cross section at NLO

BlackHat: Berger et al. 2008

A C++ code based on generalised unitarity,
and on-shell recursion for the rational parts

computes
- one-loop 6-gluon (and MHV 7- and 8-gluon) amplitudes
- one-loop W + 5-parton amplitudes
- NLO W + 3-jet cross section



one-loop amplitudes

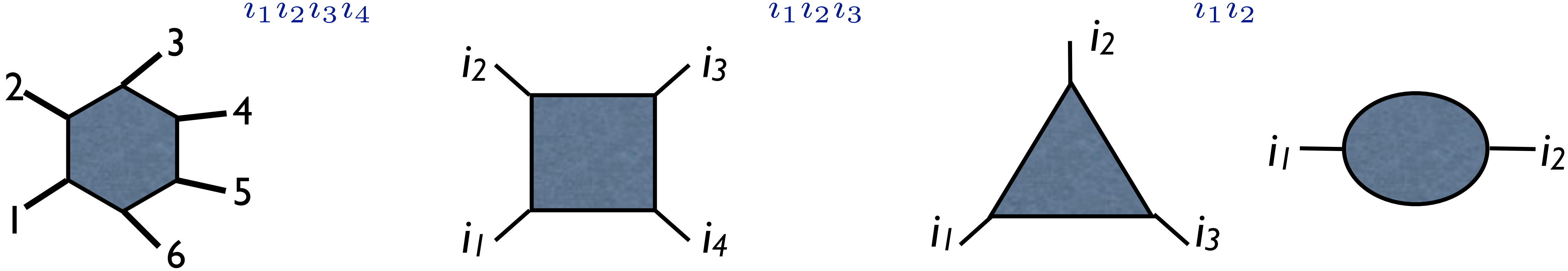
- one-loop *n*-point amplitudes A_n are IR divergent

↑ IR divergences are universal — Kunszt Signer Trocsanyi 1994; Catani 1998

↓ IR finite terms are process dependent:
many final-state particles → many scales → lengthy expressions

- A_n can be reduced to boxes, triangles and bubbles with rational coefficients

$$A_n = \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4}\, I^D_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3}\, I^D_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2}\, I^D_{i_1 i_2}$$



I: master integrals

b, c, d: rational functions of kinematic variables

- higher polygons contribute only to $O(\varepsilon)$

NLO progress: unitarity method

- use unitarity cuts: Cutkovsky rule $\frac{1}{p^2+i0} \to 2\pi i\,\delta^+(p^2)$

  to factorise coefficients of A_n into products of tree amplitudes

- compute b, c, d with $D=4$: cut-constructible terms
  $O(\varepsilon)$ part of b, c, d: rational term R_n Bern Dixon Dunbar Kosower 1994

$$A_n = \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4}\, I^4_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3}\, I^4_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2}\, I^4_{i_1 i_2} + R_n$$

- R_n computable through on-shell recursion

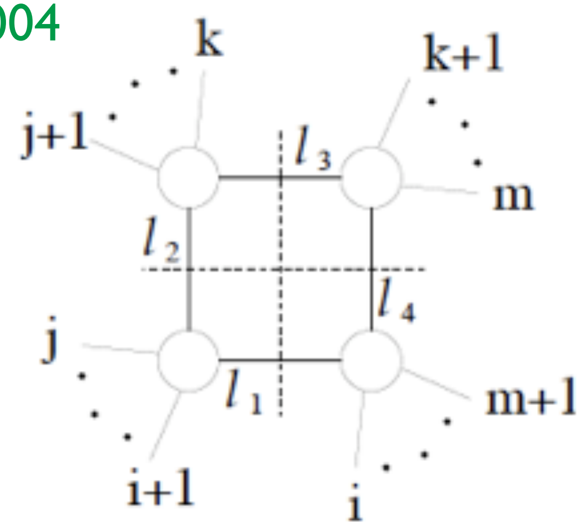
  Berger Bern Dixon Forde Kosower 2006

- generalized unitarity: Britto Cachazo Feng 2004

  quadruple cuts with complex momenta

  box coefficient d_i determined by
  product of 4 tree amplitudes

  however, c_i, b_i still difficult to extract from triple and double cuts because terms already included in d_i must be subtracted



NLO progress: OPP method

Ossola Papadopoulos Pittau 2006

- reduction of one-loop amplitude at integrand level

$$A(q') = \frac{N(q)}{\bar{D}_1 \cdots \bar{D}_m} \qquad \bar{D}_i = (q' + p_i)^2 - m_i^2 \qquad q' = q + \tilde{q} \qquad q \cdot \tilde{q} = 0$$

q' lives in *D* dimensions; *q* lives in 4 dimensions
ε part of numerator treated separately

- partial fraction the numerator into terms with 4, 3, 2, 1 denominator factors

$$N(q) = \sum_{i_1 i_2 i_3 i_4} (d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}) \prod_{i \neq i_1 i_2 i_3 i_4} D_i + \sum_{i_1 i_2 i_3} (c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}) \prod_{i \neq i_1 i_2 i_3} D_i + \sum_{i_1 i_2} (b_{i_1 i_2} + \tilde{b}_{i_1 i_2}) \prod_{i \neq i_1 i_2} D_i$$

$\tilde{d}, \tilde{c}, \tilde{b}$ vanish upon integration

- reduced to problem of fitting *d, c, b* by evaluating *N(q)* at different values of *q*, e.g. singling out choices of *q* such that 4, 3, 2, 1 among all D_i vanish, and then inverting the system. First find all possible 4-pt functions, then 3-pt functions, etc.
- for quadruple cuts, similar to BCF, but algorithmic: can be automated

NLO progress: integer dimensions

Giele Kunszt Melnikov 2008

- in D dimensions the one-loop amplitude A_n can be reduced to pentagons, boxes, triangles and bubbles

$$A_n = \sum_{i_1 i_2 i_3 i_4 i_5} e_{i_1 i_2 i_3 i_4 i_5} I^D_{i_1 i_2 i_3 i_4 i_5} + \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4} I^D_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3} I^D_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2} I^D_{i_1 i_2}$$

take D_s = # of spin states of internal particles

in D_s dimensions, gluons have D_s-2 spin states, quarks have $2^{(D_s-2)/2}$ spin states

$$A_n^{(D,D_s)} = \int d^D q \frac{N^{(D_s)}(q)}{d_1 \cdots d_n} \qquad \text{with } D \leq D_s$$

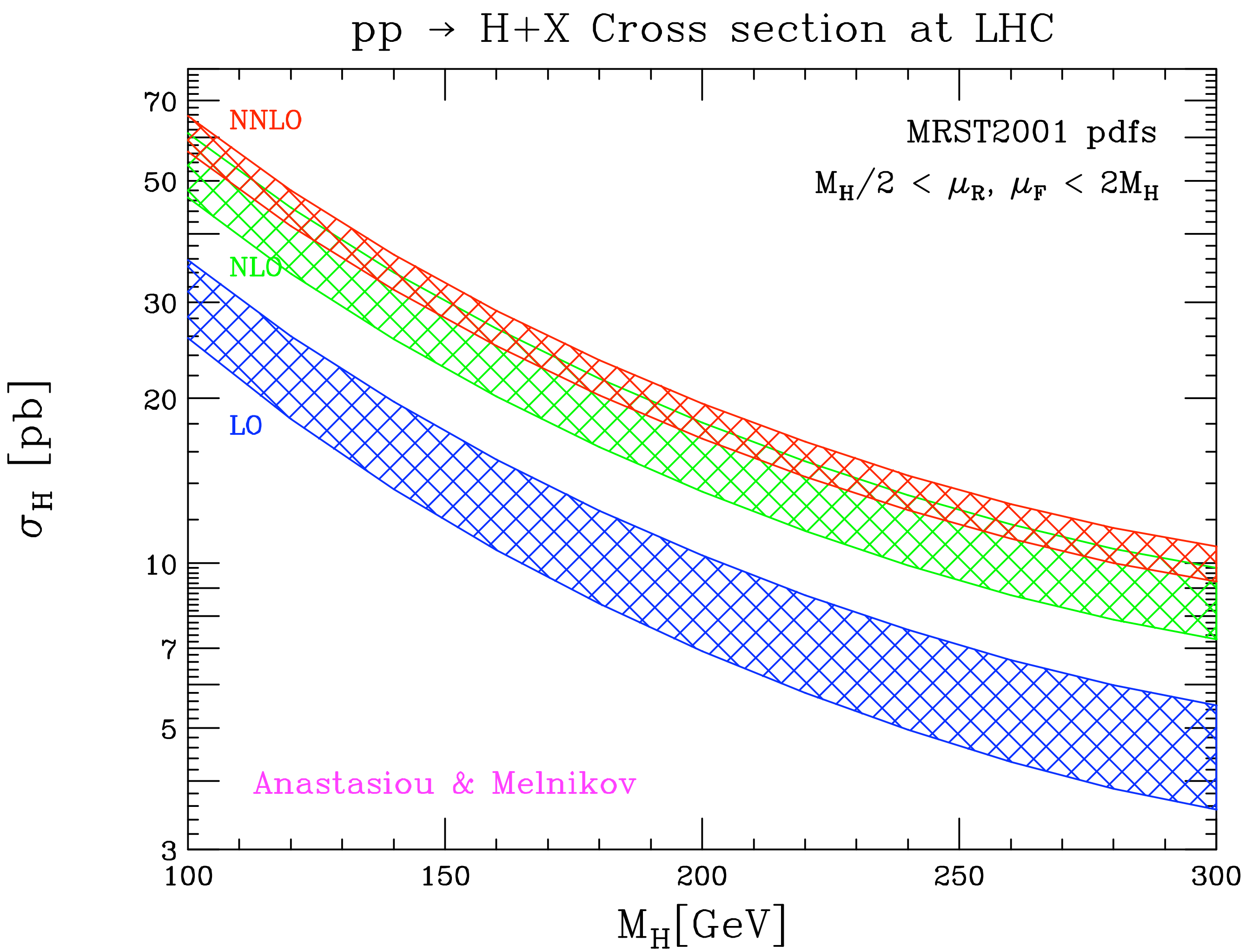
- dependence of A on D_s is linear $\quad N^{(D_s)}(q) = N_0(q) + (D_s - 4) N_1(q)$

  - compute N_0, N_1 numerically separately through 2 different integer values of D_s
  - on D_s-dimension cuts, spin density matrix is well defined
  - choose basis of master integrals with no explicit D dependence in the coefficients
  - after reduction to master integrals, continue D to $4-2\varepsilon$

- procedure can be completely automated

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

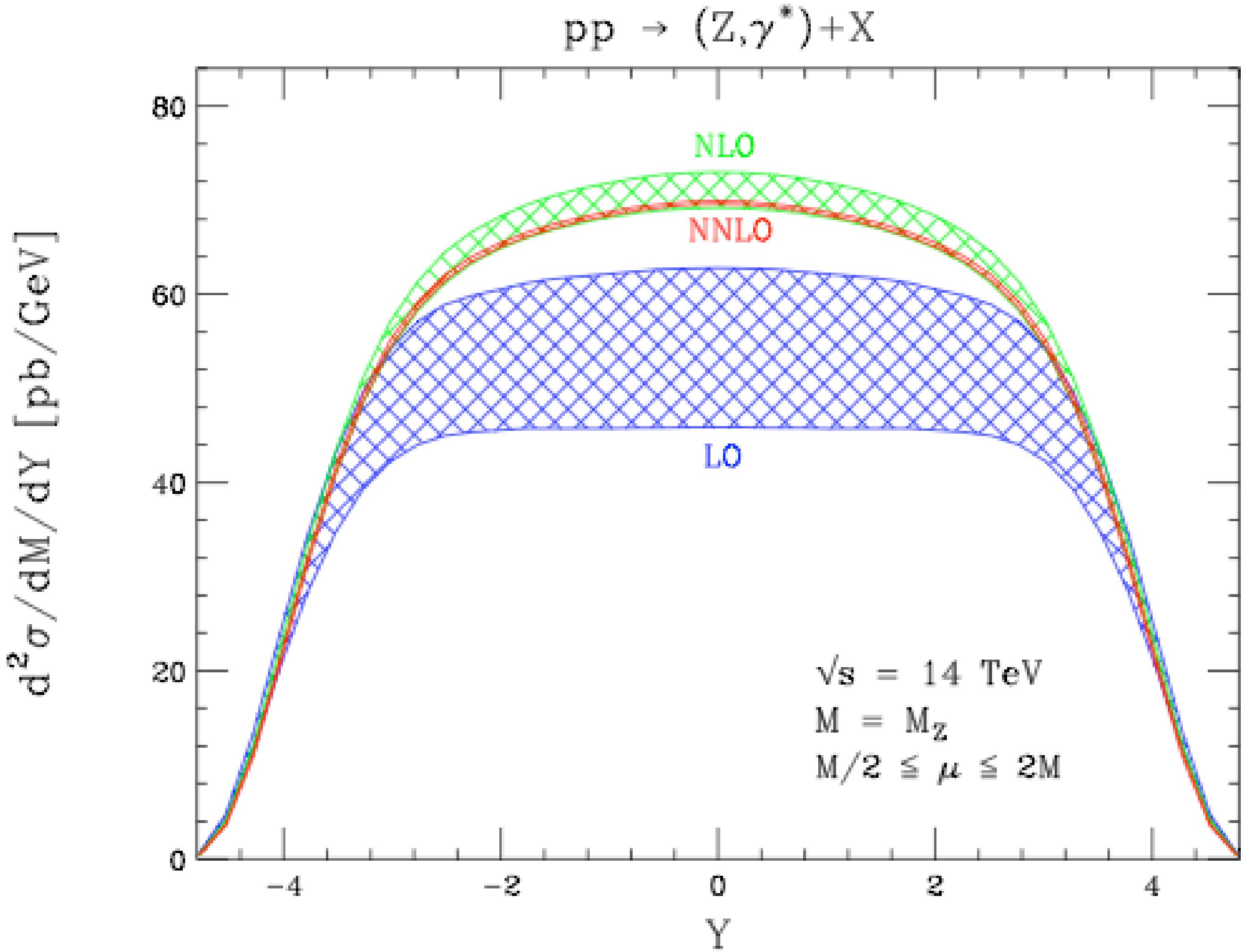
upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

Drell-Yan Z production at LHC

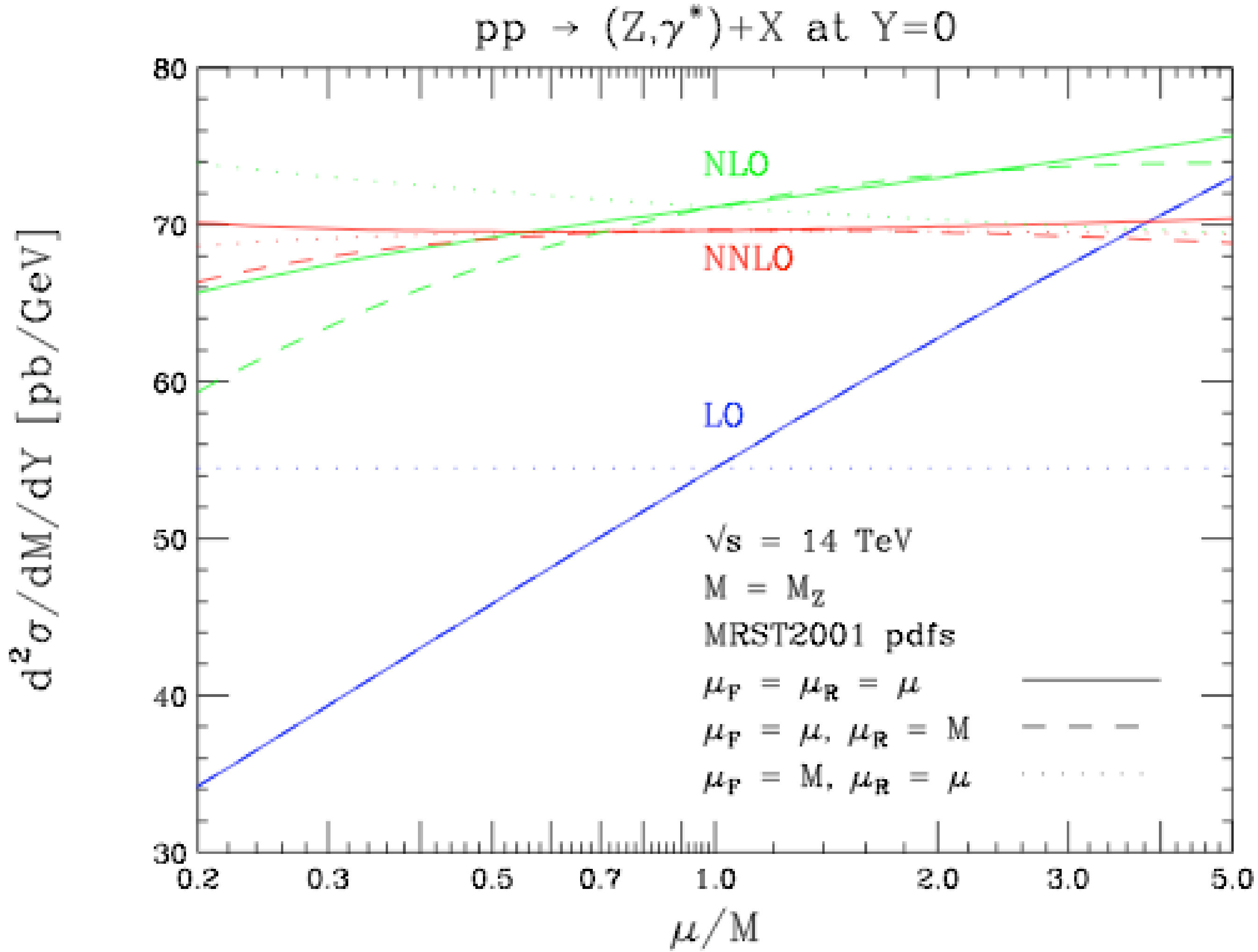


Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's)
  NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Scale variations in Drell-Yan Z production



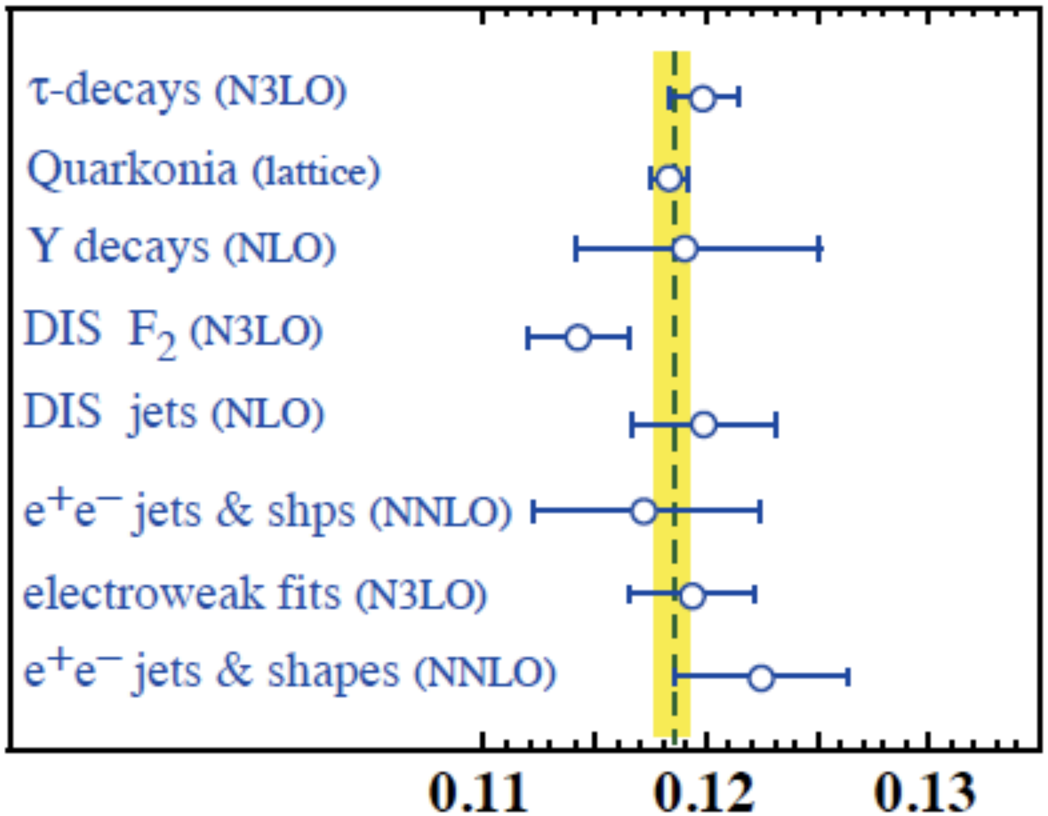
- solid: vary μ_R and μ_F together
- dashed: vary μ_F only
- dotted: vary μ_R only

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

World average of $\alpha_S(M_Z)$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$

S. Bethke arXiv:0908.1135



- vertical line and shaded band mark the world average
- first time that shapes are included at NNLO

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_S measurements
- NLO corrections are large: Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order: energy distributions in jet cones

NNLO state of the art

- Drell-Yan W, Z production

  total cross section — Hamberg van Neerven Matsuura 1990; Harlander Kilgore 2002

  fully differential x-section — Melnikov Petriello 2006; Catani Cieri Ferrera de Florian Grazzini 2009

- Higgs production

  total cross section — Harlander Kilgore; Anastasiou Melnikov 2002; Ravindran Smith van Neerven 2003

  fully differential x-section — Anastasiou Melnikov Petriello 2004; Catani de Florian Grazzini 2007

- $e^+e^- \to 3 \text{ jets}$

  event shapes, α_s — de Ridder Gehrmann Glover Heinrich 2007; Weinzierl 2008

$$\alpha_s(M_Z^2) = 0.1224 \pm 0.0009\,(\text{stat}) \pm 0.0009\,(\text{exp}) \pm 0.0012\,(\text{had}) \pm 0.0035\,(\text{theo})$$

Dissertori et al. 2009

$\alpha_s(M_Z^2) = 0.1224 \pm 0.0039$ combined in quadrature

NNLO + NLL accuracy

TH uncertainty much reduced

NNLO cross sections

- **Analytic integration** — Hamberg, van Neerven, Matsuura 1990; Anastasiou Dixon Melnikov Petriello 2003
  - ⇧ first method
  - ⇨ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators”

- **Sector decomposition** — Denner Roth 1996; Binoth Heinrich 2000; Anastasiou, Melnikov, Petriello 2004
  - ⇧ flexible enough to include any acceptance cuts
  - ⇧ cancellation of divergences is performed numerically
  - ⇨ can it handle many final-state partons ?

- **Subtraction** — Kosower 1998, 2003; Weinzierl 2003; Frixione Grazzini 2004; de Ridder Gehrmann Glover 2004-5; Somogyi Trocsanyi VDD 2005-6
  - ⇧ process independent
  - ⇨ cancellation of divergences is semi-analytic can it be fully automatised ?

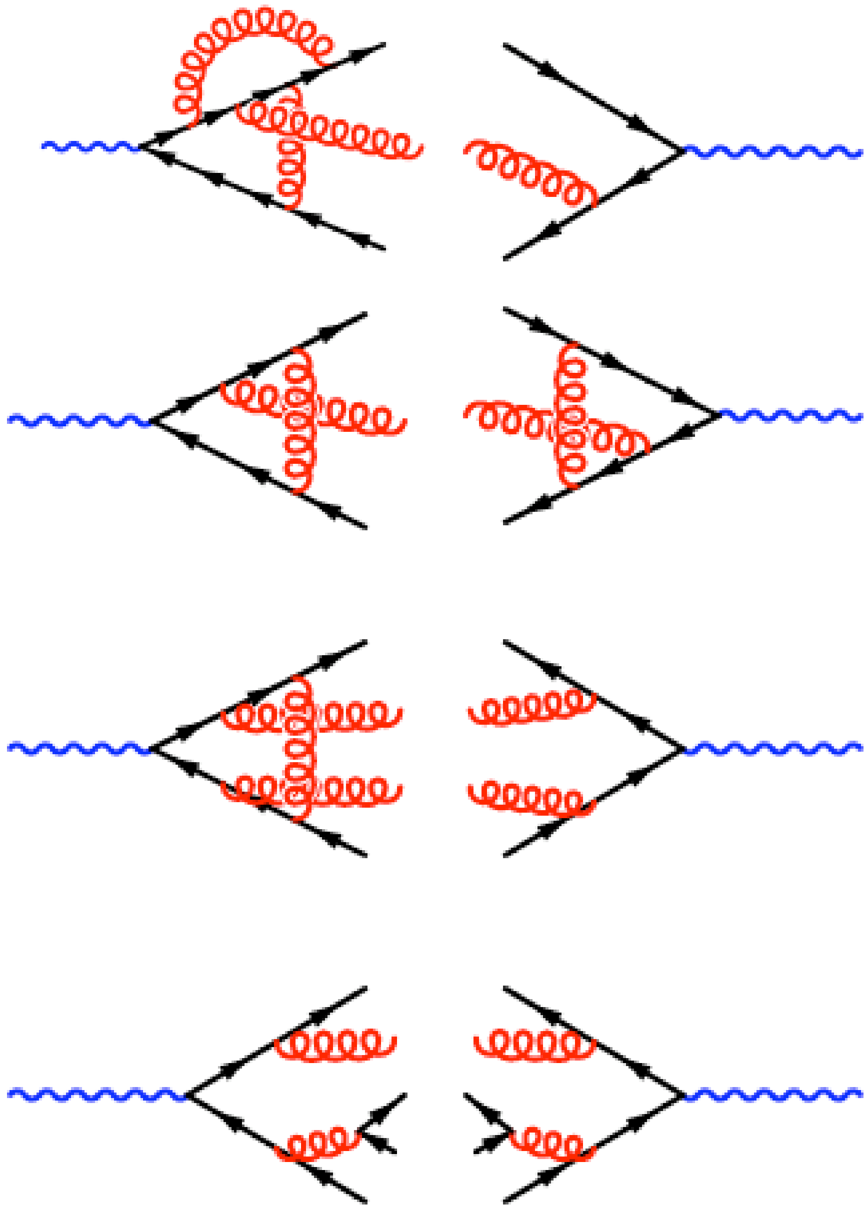
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual

real-virtual

double real



Two-loop matrix elements

- two-jet production $qq' \to qq',\ q\bar{q} \to q\bar{q},\ q\bar{q} \to gg,\ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma,\ gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

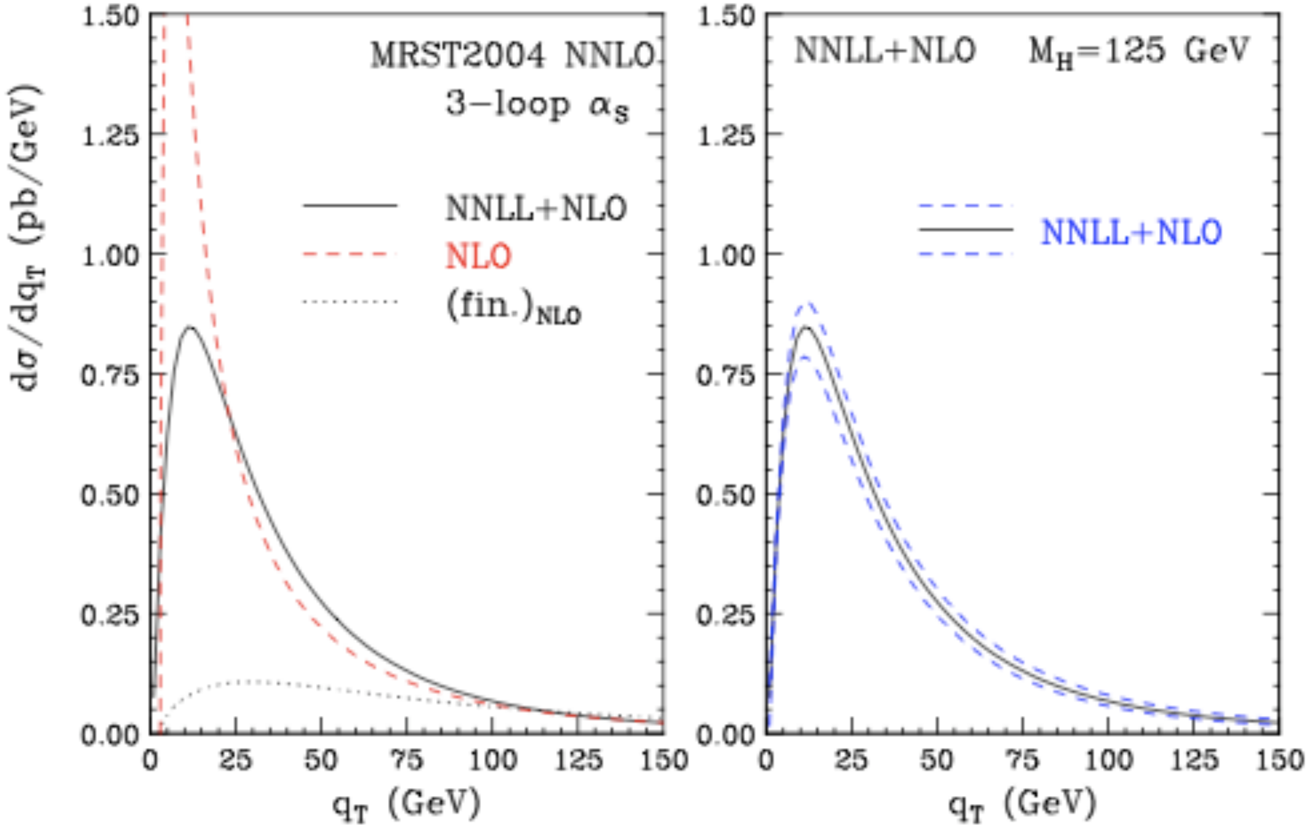
- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

Resummations: Higgs production from gluon fusion

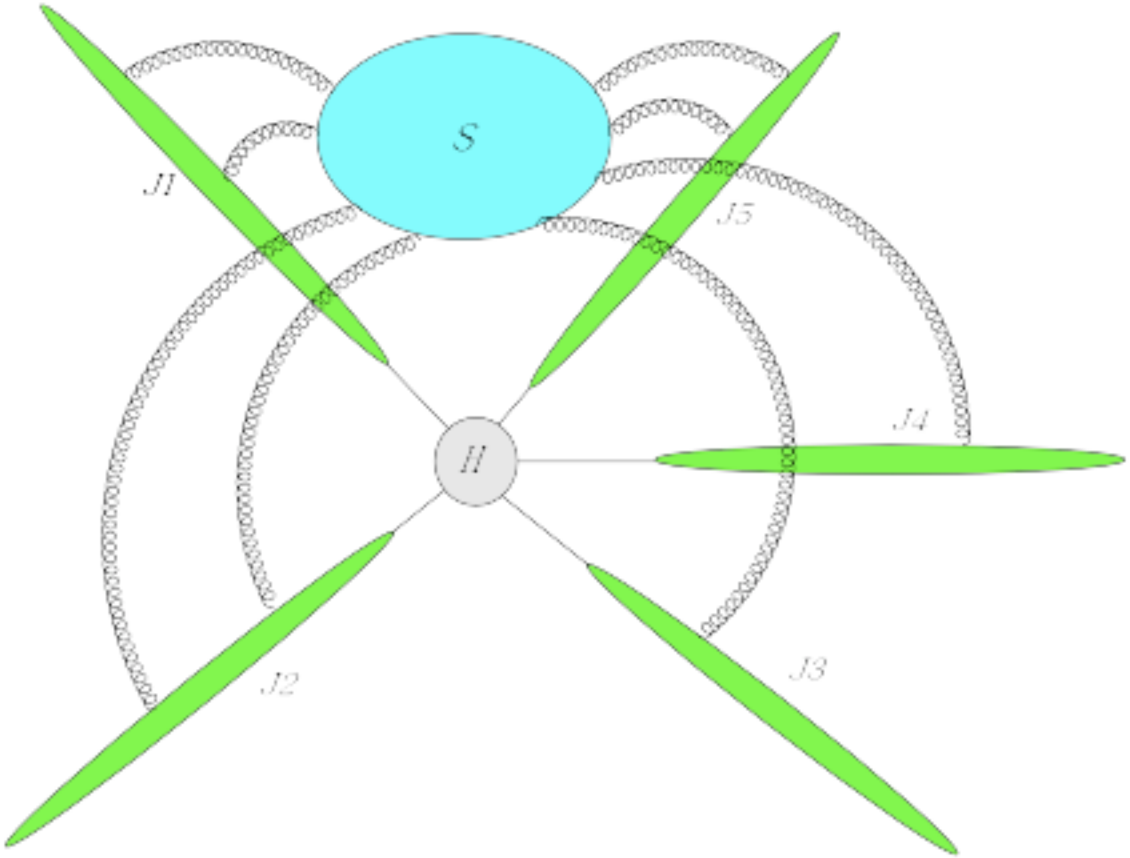
- gluon density is rapidly increasing as $x \to 0$
  $\Rightarrow$ Higgs production occurs near partonic threshold
- total energy of gluons in the final state is $E_X = \dfrac{\hat{s} - m_H^2}{2m_H} \to 0$
  need to resum soft-gluon emission

Higgs q_T distribution   Bozzi Catani de Florian Grazzini 2005



At small q_T, NLO blows up

Factorisation of a multi-leg amplitude



Mueller 1981
Sen 1983
Botts Sterman 1987
Kidonakis Oderda Sterman 1998
Catani 1998
Tejeda-Yeomans Sterman 2002
Kosower 2003
Aybat Dixon Sterman 2006
Becher Neubert 2009
Gardi Magnea 2009

$$\mathcal{M}_N(p_i/\mu,\epsilon) = \sum_L \mathcal{S}_{NL}(\beta_i\cdot\beta_j,\epsilon)\, H_L\left(\frac{2p_i\cdot p_j}{\mu^2},\frac{(2p_i\cdot n_i)^2}{n_i^2\mu^2}\right)\prod_i \frac{J_i\left(\frac{(2p_i\cdot n_i)^2}{n_i^2\mu^2},\epsilon\right)}{\mathcal{J}_i\left(\frac{2(\beta_i\cdot n_i)^2}{n_i^2},\epsilon\right)}$$

$p_i = \beta_i Q_0/\sqrt{2}$ value of Q_0 is immaterial in *S*, *J*

to avoid double counting of soft-collinear region (IR double poles),
$\mathcal{J}_i$ removes eikonal part from J_i, which is already in *S*
$J_i/\mathcal{J}_i$ contains only single collinear poles

Soft anomalous dimension

evolution equation for reduced soft anomalous dimension

$$\sum_{j\neq i} \frac{\partial}{\partial \ln \rho_{ij}} \Gamma^{\bar{S}}(\rho_{ij}, \alpha_s) = \frac{1}{4}\gamma_K^{(i)}(\alpha_s)$$

Becher Neubert; Gardi Magnea 2009

solution

$$\Gamma^{\bar{S}}(\rho_{ij}, \alpha_s) = -\frac{1}{8}\hat{\gamma}_K(\alpha_s) \sum_{i\neq j} \ln(\rho_{ij})\, T_i \cdot T_j + \frac{1}{2}\,\hat{\delta}_{\bar{S}}(\alpha_s) \sum_{i=1}^{n} C_i$$

with

$$\gamma_K^{(i)} = C_i \hat{\gamma}_K(\alpha_s) \qquad \hat{\gamma}_K = 2\frac{\alpha_s(\mu^2)}{\pi} + K\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2 + K^{(2)}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^3 + \cdots$$

- only 2-eikonal-line correlations
- generalises 2-loop solution

Are there any 3(or more)-line correlations ?
Gardi Magnea say maybe; Becher Neubert say no

Conclusions

- QCD is an extensively developed and tested gauge theory
- a lot of progress in the last few years in
  - MonteCarlo generators
  - NLO computations with many jets
  - NNLO computations
  - resummations
- better and better approximations of signal and background for Higgs and New Physics