

S. Giovannella (LNF–INFN) on behalf of the KLOE Collaboration

Kaon physics Hadronic physics @ \sqrt{s} = M_{\phi} Hadronic physics @ \sqrt{s} = 1 GeV

39th LNF Scientific Committee – LNF, 26 October 2009

Kaon physics: progress since last SC



$\Gamma(\mathrm{K}{ ightarrow}e \mathbf{v}(\mathbf{\gamma}))/\Gamma(\mathrm{K}{ ightarrow}\mu \mathbf{v}(\mathbf{\gamma}))$	Accepted for publication (EPJC)
K _s lifetime	Preliminary
K _L lifetime	Preliminary
$K^{}_S \to \pi^0 \pi^0 \pi^0$	Update with all statistics in progress
$BR(\mathrm{K}^{\pm} ightarrow \pi^{\pm}\pi^{+}\pi^{-})$	In progress

K_s lifetime: measurement technique



- ★ Lifetime from fit to proper time, t_0 , distribution of $K_S \rightarrow \pi^+\pi^-$ events
- \clubsuit ϕ position event by event from PCA of K_{τ} flight direction to beamline

($\sigma(Z_{IP})=0.2$ cm while beam spread ~3 cm)

- ♦ Redundant measurement of K_s momentum (from tracks / line of flight and \sqrt{s})
- * Factor 3.5 improvement on t_0 by means of geometrical fit (K_s direction fixed, +1

IP position and decay distance free)



730 pb⁻¹ \rightarrow 25 million events after all cuts

 \succ Results as function of K_s direction (different resolution)

K_s lifetime: results

- \succ Fit range: (-2,+7) τ_{S}^{0}
- Resolution described by two gaussians > 5 parameters fit: τ_s + resolution
- 90.5 τ_{ς} (ps) preliminary preliminary 90.0 89.5 KLOE KTeV **NA48** NA31 (09)(02)(97)89.0



entries

 $[4.6 \times 10^{-4} \text{ error w.r.t. PDG08 } 5.6 \times 10^{-4}]$



K_L lifetime



The error on τ_L is the main limiting factor on V_{US} accuracy from K_L decays



- τ_{L} measurement can be improved (stat.+syst.) with whole KLOE data sample
 - \succ K_L tagged with K_S→π⁺π⁻ vertex at IP
 - K_L direction and momentum from DC measurements
 - Unique to the KLOE calorimeter:
 L_{KL} and L_γ by time measurements t_γ



$$c t_{\gamma} = L_{KL} / \beta_L + L_{\gamma}$$

K_L lifetime: results

- > K_L "photon" vertex, built with at least 3 γ 's from $3\pi^0$ decay
- > Time scale, neutral vtx resolution, γ reconstruction efficiency surveyed with K_L $\rightarrow \pi^+\pi^-\pi^0$ events





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$BR(K^+ \rightarrow \pi^+ \pi^- \pi^+ (\gamma))$



- Completes the KLOE program of precise and fully inclusive K[±] dominant BRs
- Available measurement from 70s, with no information on rad. cut-off:

CHIANG (2330 evts) : BR(K $\rightarrow \pi^+\pi^-\pi^+$) = (5.56 ± 0.20)% $\Delta BR/BR = 3.6 \times 10^{-2}$

Signal selection

- 2-tracks vertex before DC inner wall and along the K path obtained from backward extrapolation of the tagging kaon track to the I.P.
- \blacktriangleright Signal peak in the missing mass spectrum (~m_{\pi}^{-2})





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BR(K⁺ $\rightarrow \pi^{+}\pi^{-}\pi^{+}(\gamma))$

Signal selection efficiency from MC and folded with $\frac{\epsilon_{\text{single trk}} (\text{data})}{\epsilon_{\text{single trk}} (\text{MC})}$

 $K \rightarrow \pi \pi^0 \pi^0$ control sample to measure $\epsilon_{single trk}$

- Control sample selection
- K path from the tagged K track and φ kinematics
 Reconstruct neutral vertex K →π⁰π⁰X decays looking for 4 γ's with time measurements consistent with the Kaon ToF



Hadronic physics: progress since last SC

$\phi \to K_S K_S \gamma$	PLB 679 (2009) 10
$\phi \rightarrow a_0(980) \gamma$	PLB 681 (2009) 5
Gluonium content in η^\prime	JHEP 07 (2009) 105
$\eta ightarrow \pi^0 \pi^0 \pi^0$	Final, paper in preparation
$\eta ightarrow \pi^+\pi^-\gamma$	Prel. BR evaluation, $M_{\pi\pi}$ study in progress
$\eta ightarrow \pi^+\pi^-e^+e^-$	PLB 675 (2009) 283
$\eta \rightarrow e^+e^-e^+e^-$	First observation
$\eta \to \mu^+ \mu^-$	Analysis tuning
$\gamma\gamma ightarrow \pi^0\pi^0$	Analysis refined with all off-peak data
$\gamma\gamma ightarrow \eta$	New search
$\sigma(\pi\pi\gamma)$ large angle, off-peak data	Final, paper in preparation
σ(ππγ)/σ(μμγ)	In progress



- ✓ Significant contribution from the chiral anomaly responsible of $\eta \rightarrow \gamma \gamma$ decay is expected. Study of the two pion system allows for test of ChPT and its unitarized extensions (e.g. VMD or chiral unitarity approach) → M_{ππ} shape needed
- Existing data, low in statistics and not acceptance corrected, not sufficient for unambiguous theoretical interpretation
- ✓ Latest result from CLEO on $\Gamma(\eta \rightarrow \pi \pi \gamma)/\Gamma(\eta \rightarrow \pi \pi \pi)$ differs >3 σ from old measurements

$\Gamma(\eta \rightarrow \pi^+\pi\gamma)$	$\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$
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value	events	author	year
0.203 ± 0.008	PDG average		
$0.175 \pm 0.007 \pm 0.006$	859	Lopez	2007
0.209 ± 0.004	18 k	Thaler	1973
0.201 ± 0.006	7250	Gormley	1970

 $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$



- ✤ DATA SAMPLE: 1.2 fb⁻¹
- ♦ Kinematical cuts to remove all bckg but $\phi \rightarrow \pi^+ \pi^- \pi^0$:

ε = 29%, BKG/SIG=10:1

- Different topology in γγ distributions for signal and background
- Simultaneous fit to both spectra to extract signal



 $\eta \rightarrow \pi^+ \pi^- \pi^0$ selected with high efficiency (40%) and BKG/SIG=0.5%

$$\Gamma(\eta \rightarrow \pi^{+}\pi^{-}\gamma)/\Gamma(\eta \rightarrow \pi^{+}\pi^{-}\pi^{0})$$

$$\frac{\Gamma(\eta \rightarrow \pi^{+}\pi^{-}\gamma)}{\Gamma(\eta \rightarrow \pi^{+}\pi^{-}\pi^{0})} = 0.2014 \pm 0.0004_{stat}$$

Preliminary result agrees with PDG average, confirming old results from 70s

- > We are evaluating systematics, aiming to reach $\sim 1\%$
- > Plan to use full KLOE data set to investigate the $\pi^+\pi^$ invariant mass distribution: cuts on Myy and $\cos\theta(\gamma\gamma)$ in the π^0 rest frame allow to reduce the background contribution to 2% with a signal efficiency of ~25%

 $\eta \rightarrow e^+e^-e^+e^-$ analysis

- Data sample: 1.7 fb⁻¹
- MC simulation according to J.Bijnens and F. Persson, arXiv:0106130

(courtesy of J.Bijnens)

- FSR included
- *e*⁺*e*⁻ pairs from photon conversion on Beam Pipe and Drift Chamber wall rejected
- Remaining background from φ decays subtracted
- Fit to M_{eeee} distribution with MC signal + continuum background shapes yields:





The $e^+e^- \rightarrow e^+e^-X$ process





★ Γ(γγ) measurement

<u>VALUE (keV)</u> 0.510 ±0.026 OUR FI	<u>EVTS</u>	DOCUMENT ID	TECN	COMMENT	√s (GeV)
0.510±0.026 OUR AV	ERAGE				7 7 10 4
$0.51 \pm 0.12 \pm 0.05$	36	BARU	90 MD1	$e^+e^- \rightarrow e^+e^-r$	7.2-10.4
$0.490 \pm 0.010 \pm 0.048$	2287	ROE	90 ASP	$e^+e^- \rightarrow e^+e^-r$	29
$0.514 \pm 0.017 \pm 0.035$	1295	WILLIAMS	88 CBAL	$e^+e^- \rightarrow e^+e^- r$	9.4-10.0
$0.53 \pm 0.04 \pm 0.04$		BARTEL	85e JADE	$e^+e^- \rightarrow e^+e^-r$	34.0

★Transition form factors for light-by-light contributions (A. Nyffeler, J.Prades) to g-2...
→ KLOE-2 needed



$$\sigma_{\gamma\gamma \to R}(q_1, q_2) \propto \Gamma_{R \to \gamma\gamma} \frac{8\pi^2}{M_R} \delta((q_1 + q_2)^2 - M_R^2) \left| F(q_1^2, q_2^2) \right|^2$$

Selection of $e^+e^- \rightarrow e^+e^-\eta$ events

Data sample: 240 pb⁻¹ @ $\sqrt{s} = 1$ GeV Selected channel: $\eta \rightarrow \pi^+ \pi^- \pi^0$

- Two prompt neutral clusters
- Two tracks
- > Photon pairing: $\chi^2_{\pi 0} < 8$
- > Kinematic fit: $\chi^2_{\eta} < 16$
- Parabolic cut on:

$$M_{miss}^2 \approx s + M_{\eta}^2 - 2E_T \sqrt{s} - \frac{p_L^2}{E_T} \sqrt{s}$$

Signal:
$$\sigma = 0.043$$
 nb $\varepsilon_{tot} = 20\%$

Backgrounds:

	$\eta\gamma$	$\omega\pi^0$	$\pi^+\pi^-\pi^0$	K^+K^-	$K_S K_L$	$e^+e^-\gamma$
	$\eta \to \pi^+ \pi^- \pi^0$	$\omega \to \pi^+ \pi^- \pi^0$		$\mu\nu, \pi^{\pm}\pi^{0}$		
σ (nb)	0.23	5.7	30	8.6	2.0	400
ε	$9.2 imes 10^{-3}$	$6.3 imes 10^{-5}$	1.5×10^{-5}	1.9×10^{-5}	1.7×10^{-5}	$\mathcal{O}(10^{-7})$







Fit to p_L and M_{miss}² with signal and background shapes Background yields constrained by present knowledge of x-sections



Extraction of $\sigma(e^+e^- \rightarrow e^+e^-\eta)$ and $\Gamma_{\gamma\gamma}$ in progress Statistical accuracy on $\Gamma_{\gamma\gamma}$ comparable with existing measurements

Search for $\gamma\gamma \rightarrow \sigma(600) \rightarrow \pi\pi$



- > Long debate about the experimental evidence of the $\sigma(600)$ meson
- > Evidence for $\pi^+\pi^-$ bound state (E791, CLEO, BES) from Dalitz plot analyses
- Values of mass and width with large uncertainties
- **>** Indirect evidence in the $e^+e^- \rightarrow \pi^0 \pi^0 \gamma$ Dalitz plot analysis @ KLOE



Selection of $e^+e^- \rightarrow e^+e^-\pi^0\pi^0$ events



- Four prompt neutral clusters
- No tracks
- > Photon pairing: $\chi^2_{\pi\pi} < 4$
- $\succ \Sigma_{2\text{Emin}} > 60 \text{ MeV}$
- ➢ p_T < 80 MeV</p>

$$\chi_{\pi\pi}^{2} = \left(\frac{M_{ij} - m_{\pi0}}{\sigma(E_{i}, E_{j})}\right)^{2} + \left(\frac{M_{lk} - m_{\pi0}}{\sigma(E_{l}, E_{k})}\right)^{2}$$
$$\frac{\sigma(E_{i}, E_{j})}{M_{ij}} = \frac{1}{2} \left(\frac{\sigma_{E_{i}}}{E_{i}} \oplus \frac{\sigma_{E_{j}}}{E_{j}}\right)$$
$$M_{ij}^{2} = 2E_{i}E_{j}(1 - \cos\theta_{ij})$$



 $e^+e^- \rightarrow e^+e^-\pi^0\pi^0$: $M_{4\nu}$

Expected background yields:



$\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ measurement

 $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma)$ measured at fixed \sqrt{s} with high accuracy ISR used to extract $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ for \sqrt{s} from $2M_{\pi}$ to \sqrt{s}

$$s \frac{d\sigma_{\pi\pi\gamma}}{dM^2_{\pi\pi}} = \sigma_{\pi\pi}(s) \times \mathbf{H}(s)$$

FSR neglected

Requires precise calculations of the radiator function H(*s*) PHOKHARA MC NLO generator [EPJC27(2003)]

$\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma)$ measured via absolute normalization with Bhabhas:

$$\frac{d\sigma_{_{\pi\pi\gamma(\gamma)}}^{obs}}{dM_{_{\pi\pi}}^{2}} = \frac{\Delta N_{\rm Obs} - \Delta N_{\rm Bkg}}{\Delta M_{_{\pi\pi}}^{2}} \cdot \frac{1}{\varepsilon_{\rm Sel}} \cdot \frac{1}{\int Ldt}$$

- Background rejection with PID using EMC info $(ee\gamma/\mu\mu\gamma)$ and kin. cuts $(\phi \rightarrow \pi\pi\pi)$
- Efficiencies evaluated on data with independent methods
- Luminosity from large angle Bhabha scattering evts

Two samples used:

- 1) Small angle: $\theta_{\pi\pi} < 15^{\circ}$ or $\theta_{\pi\pi} > 165^{\circ}$ > Higher cross satisfies the second state of the
- 2) Large angle: $50^{\circ} < \theta_{\gamma} < 130^{\circ}$
 - Higher background
 - > All M_{$\pi\pi$} spectrum
 - ➢ ISR Photon detected

2 pion tracks at large angles $50^{\circ} < \theta_{\pi} < 130^{\circ}$ Photons at large angles $50^{\circ} < \theta_{\gamma} < 130^{\circ}$

- complementary analysis w.r.t. SA
- * threshold region $(2m_{\pi})^2$ accessible
- * γ_{ISR} photon detected (4-mom. constraints)
- Iower signal statistics
- Iarger contribution from FSR events
- **♦** larger ϕ → $\pi^+\pi^-\pi^0$ background contamination
- $\boldsymbol{\textbf{\diamond}}$ irreducible background from $\boldsymbol{\varphi}$ decays

At least 1 photon with $50^{\circ} < \theta_{\gamma} < 130^{\circ}$ and E_y > 20 MeV \Rightarrow photon detected

Threshold region non-trivial

due to irreducible FSR-effects, which have to be estimated from MC using phenomenological models (interference effects unknown)

KLOE result on Large Angle analysis

Systematic errors on $a_{\mu}^{\pi\pi}(0.1-0.85 \text{ GeV}^2)$:

Reconstruction Filter	< 0.1%
Background	0.5%
$f_0^+ \rho \pi$	0.4%
Omega	0.2%
Trackmass	0.5%
π /e-ID and TCA	< 0.1%
Tracking	0.3%
Trigger	0.2%
Acceptance	0.4%
Unfolding	negligible
Software Trigger	0.1%
Luminosity $(0.1_{th} \oplus 0.3_{exp})\%$	0.3%

Experimental fractional error on $a_{\mu} = 1.0$ %

FSR resummation	0.3%
Radiator H	0.5%
Vacuum polarization	< 0.1%

Theoretical fractional error on a_{μ} = 0.6 %

0.4%

1.0%

0.6%

 $|\mathbf{F}_{\pi}|^2$: KLOE09 vs KLOE08

Grey band: KLOE09 error

 $|F_{\pi}|^2$: KLOE09 vs SND/CMD2

Comparison on $a_{\mu}^{\pi\pi}$

$$a_{\mu}^{\text{had}} = \frac{1}{4\pi^3} \int_{x_1}^{x_2} \sigma^{\text{had}}(s) \mathbf{K}(s) \mathrm{d}s$$

<u>a_μππ(0.35-0.85GeV²):</u>

KLOE08 (small angle)

KLOE09 (large angle)

$$a_{\mu}^{\pi\pi} = (379.6 \pm 0.4_{stat} \pm 2.4_{sys} \pm 2.2_{theo}) \cdot 10^{-10}$$
$$a_{\mu}^{\pi\pi} = (376.6 \pm 0.9_{stat} \pm 2.4_{sys} \pm 2.1_{theo}) \cdot 10^{-10}$$
$$0.2\% \quad 0.6\% \quad 0.6\%$$

<u>a_μππ(0.152-0.270 GeV²):</u>

KLOE09 (large angle)

CMD-2

$$a_{\mu}^{\pi\pi} = (48.1 \pm 1.2_{stat} \pm 1.2_{sys} \pm 0.4_{theo}) \cdot 10^{-10}$$

$$a_{\mu}^{\pi\pi} = (46.2 \pm 1.0_{stat} \pm 0.3_{sys}) \cdot 10^{-10}$$

Forward-backward asymmetry

Different C parity of $\pi^+\pi^-$ for ISR and FSR+ $\phi \rightarrow S\gamma$ gives rise to a non-vanishing

PHOKHARA-MC modified by O. Shekhovtsova using scalar+VMD contribution extracted from KLOE $\pi^0\pi^0\gamma$ analysis (EPJC49(2007)473)

Next $\sigma_{\pi\pi}$ measurement: π/μ ratio

An alternative way to obtain $|F_{\pi}|^2$ is the bin-by-bin ratio of pion over muon yields (instead of using absolute normalization with Bhabhas)

Many radiative corrections drop out:

- radiator function
- int. luminosity from Bhabhas
- Vacuum polarization

Separation between pions and muons done experimentally using kinematical cuts:

- *muons:* $M_{Trk} < 115 \, MeV$
- *pions* : $M_{Trk} > 130 \, MeV$

Conclusions

- * Still a lot of relevant physics results from the analysis of the KLOE data set, both at M_{ϕ} and at 1 GeV !!!
- ***** Summary of achievements in 2009:
 - > 6 published papers
 - > 1 accepted paper
 - > 3 drafts in preparation
- Our goal is to complete all the analyses mentioned today and start addressing other selected items before the start of the upcoming KLOE-2 run

Planned analysis (2010)

- $\succ K_{S} \rightarrow \pi l \nu / K_{S} \rightarrow \pi \pi e e$
- \succ FF slopes from K₁₃ decays
- Parameters of CPT violation and Lorentz symmetry
- \succ K[±] lifetime
- \succ K[±] semileptonic decays

> η→ π^0 γγ > η'→η $\pi\pi$

From M. Davier at al, arXiv:0908:4300

From M. Davier at al, arXiv:0908:4300

Computing a_u^{ππ}

$$a^{\pi\pi(\gamma),LO}_{\mu} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} ds K(s) \sigma^0_{\pi\pi(\gamma)}(s) ,$$

where K(s) is the QED kernel,

Systematic errors

Source	fractional value ×10 ⁴
 selection cuts : 	3.3
- cos θ_{κ} FV cut	5.7
- Kaon mass :	0.4
- fit range :	5.0

 $\tau_{K_{S}} = (89.56 \pm 0.03_{stat} \pm 0.07_{syst}) \text{ ps}$ preliminary

K_L lifetime: results

al.

$\tau_L = 50.56$	± 0.14 _{stat} ±	0.21 _{syst} ns
table of systematic		
source	$\Delta \tau / \tau \%$	$\Delta \tau$ (ns)
E _{taa}	0.34	0.17
^E KI	0.16	0.08
time scale	0.12	0.06
nuclear interactions	0.16	0.08

comparison with previous KLOE measurements

 $\begin{aligned} \tau_L &= 50.92 \pm 0.17_{\text{stat}} \pm 0.13_{\text{syst uncorr}} \pm 0.27_{\text{syst corr}} \text{ ns} \\ \Delta &= 1.4 \text{ } \sigma, \text{ taking into account the correlation between syst.} \\ \tau_L &= 50.72 \pm 0.11_{\text{stat}} \pm 0.21_{\text{syst}} \text{ ns} \\ \Delta &= 0.4 \text{ } \sigma \end{aligned}$

Absolute BR($K^+ \rightarrow \pi^+ \pi^- \pi^+ (\gamma)$)

• this measurement completes the KLOE program of precise and fully inclusive K[±] dominant BR's

- needed to perform a global fit to K[±] BR's
- available measurement dates back to '72 (no information on radiation cut-off) CHIANG (2330 evts) BR(K $\rightarrow \pi^+\pi^-\pi^+$) = (5.56 ± 0.20)% $\Delta BR/BR = 3.6x10^{-2}$

~ 180 pb⁻¹ enough to reach <u>statistical relative error at few permil level</u>

KLOE measures L with Bhabha scattering

55° < θ < 125° acollinearity < 9° p ≥ 400 MeV

$$\int \mathcal{L} \, \mathrm{d}t = \frac{N_{obs} - N_{bkg}}{\sigma_{eff}}$$

F. Ambrosino et al. (KLOE Coll.) **Eur.Phys.J.C47:589-596,2006**

generator used for σ_{eff} BABAYAGA (Pavia group):

C. M.C. Calame et al., NPB584 (2000) 459

Now: C. M.C. Calame et al., NPB758 (2006) 22

newer version (BABAYAGA@NLO) gives 0.7% decrease in cross section, and better accuracy: 0.1%

Systematics on Luminosity		
Theory	0.1 %	
Experiment	0.3 %	
TOTAL 0.1 % th \oplus 0.3% exp = 0.3%		

Radiative corrections

Radiator-Function $H(s,s_{\pi})$ (ISR):

- ISR-Process calculated at NLO-level

PHOKHARA generator

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC27,2003)

Precision: 0.5%

$$s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds_{\pi}} = \sigma_{\pi\pi}(s_{\pi}) \times \mathsf{H}(\mathsf{s},\mathsf{s}_{\pi})$$

Radiative Corrections:

i) Bare Cross Section

divide by Vacuum Polarisation $\delta(s) = (\alpha(s)/\alpha(0))^2$

→ from F. Jegerlehner

ii) FSR

Cross section $\sigma_{\pi\pi}$ must be incl. for FSR for use in the dispersion integral of a_{μ}

$$\gamma$$
 $\left(\begin{array}{c} H \\ (\gamma) \\ H \\ H \end{array} \right)^{2}$

FSR corrections have to be taken into account in the efficiency eval. (Acceptance, M_{Trk}) and in the passage $s_{\pi} = M_{\pi\pi}^2 \rightarrow (M_{\pi\pi}^0)^2 = s_{\gamma*}$

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC33,2004)

Event selection

Experimental challenge: background from

 $- e^+e^- \rightarrow \mu^+\mu^- \gamma$ $- e^+e^- \rightarrow e^+e^- \gamma$ $- \phi \rightarrow \pi^+\pi^-\pi^0$

 kinematical cuts in trackmass M_{Trk} (defined by 4-momentum conservation under the hypothesis of 2 tracks with equal mass and a γ)

$$\left(\sqrt{s} - \sqrt{p_1^2 + M_{trk}^2} - \sqrt{p_2^2 + M_{trk}^2}\right)^2 - (p_1 + p_2)^2 = 0$$

- 2. angle Ω between the photon and missing momentum $\vec{p}_{miss} = -(\vec{p}_+ + \vec{p}_-)$
- 3. To further clean the samples from radiative Bhabha events, a particle ID estimator for each charged track based on Calorimeter Information and Time-of-Flight

$\eta \rightarrow \pi^+ \pi^- \gamma$: contributions

- Resonant contribution:
 - 1. ρ production with its subsequent decay to a pion pair (VDM)
 - 2. existence of a small non-VDM contribution
- Anomalous contribution:

box anomaly (similar to the classical triangle anomaly), responsible for $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays predicted by PCAC and by the Wess-Zumino-Witten chiral lagrangian