Entanglement generation and bound states in one-dimensional QED

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San Rufo, 27 June 2018

quantum physics in 1D

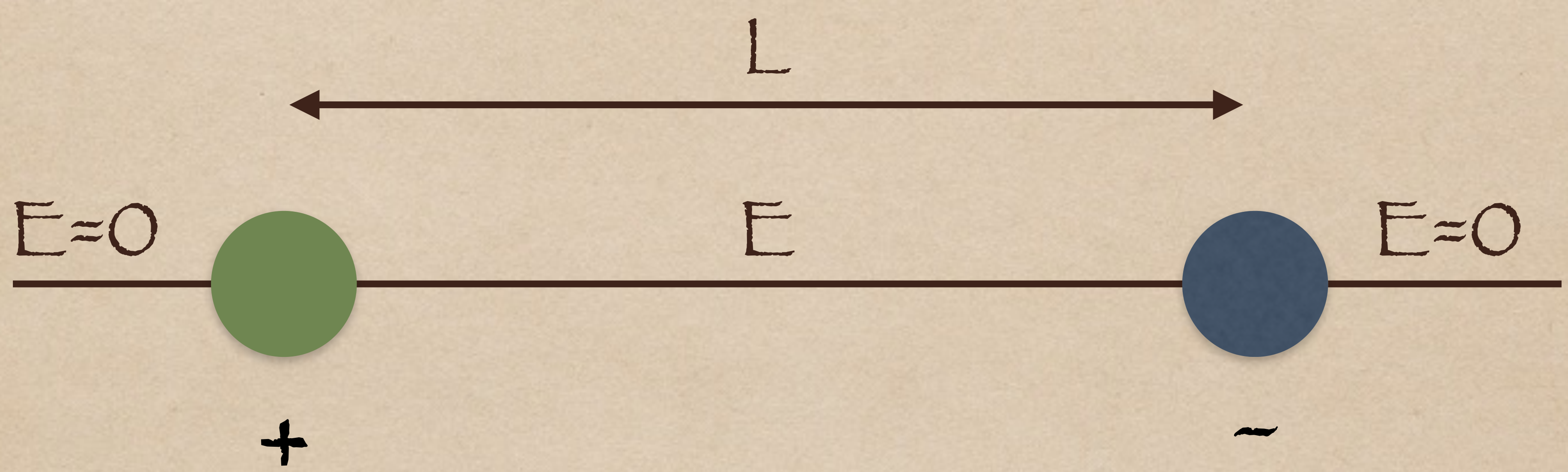
- quantum emitters in 1D waveguides
- simulations of quantum field theories
- 1D effects: slow light, ultra-strong coupling

T. Giamarchi, Quantum Physics in One Dimension, 2004

Y. Kuramoto and Y. Kato, Dynamics of One-Dimensional Quantum Systems: Inverse-Square Interaction Models, 2009

preliminaries: (Q)ED in 1D

(electric field E confined here)



$$\text{energy} = \frac{1}{2} L \times E^2 \propto \text{volume}$$

Schwinger, Coleman, Kogut, Susskind, Casher, 't Hooft, Parisi thinking about confinement mechanisms

Gauss' law in 1D

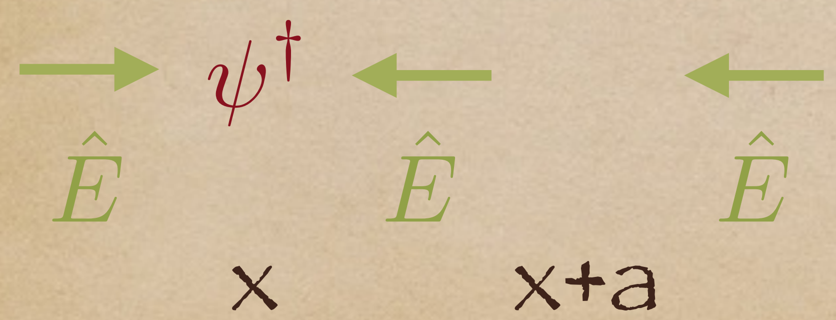
Theories with local symmetries (to be satisfied at every point)

CLASSICAL (electrodynamics)

$$\rho = \vec{\nabla} \cdot \vec{E}$$

Gauss' law

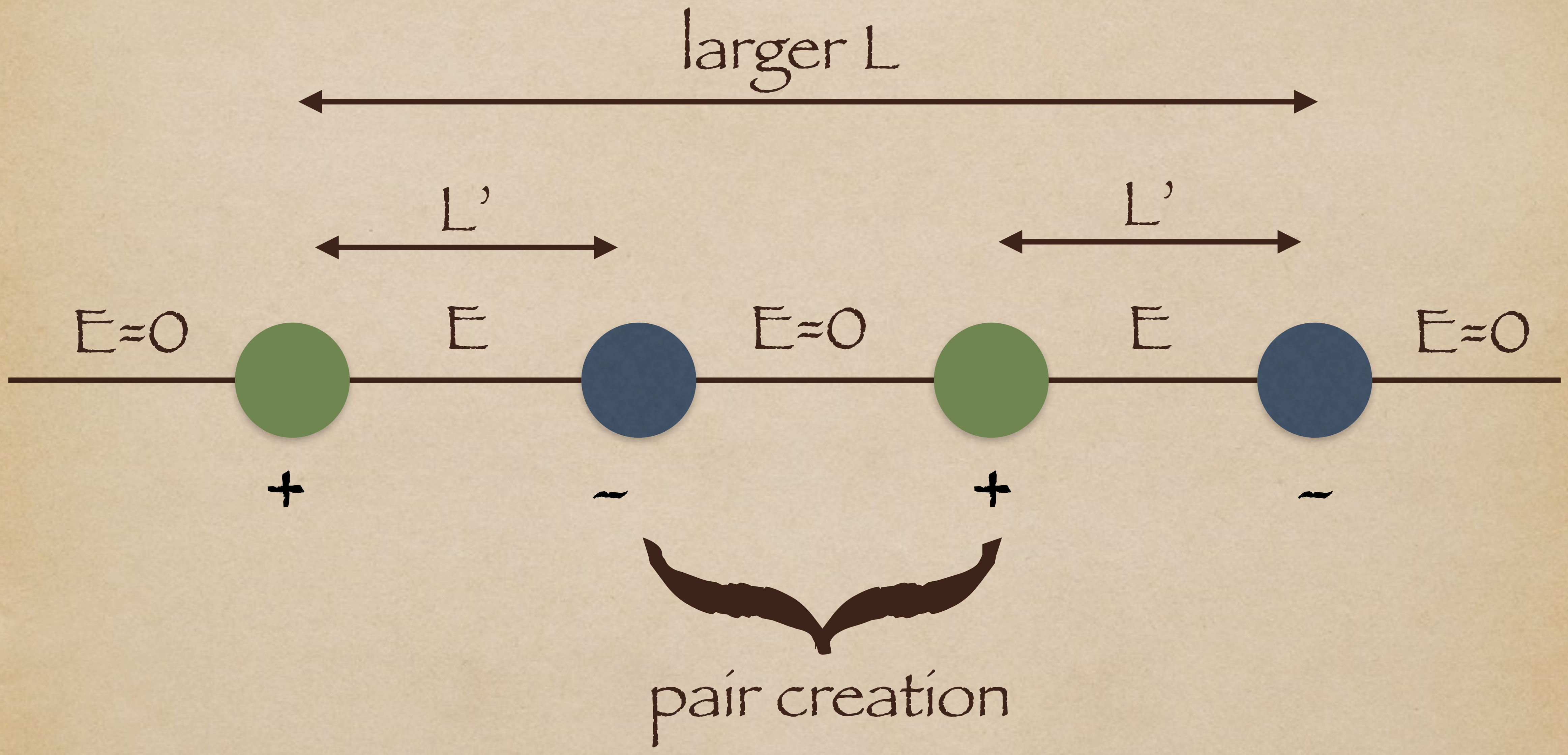
QUANTUM (QED)



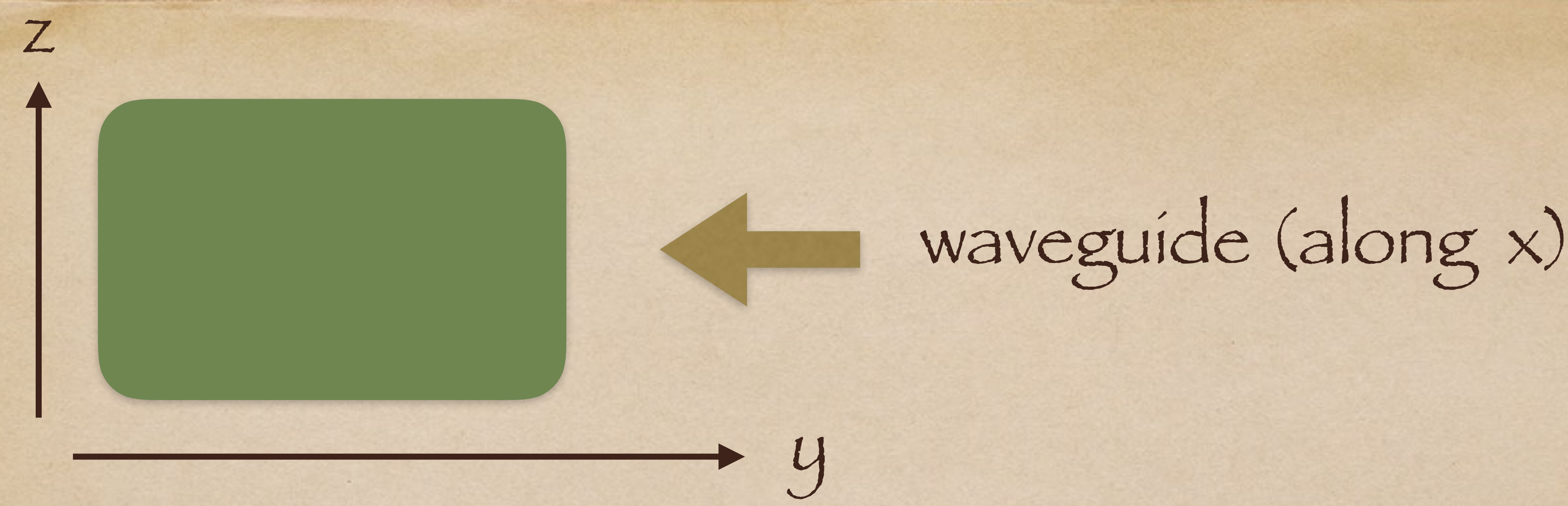
$$\psi_x^\dagger \psi_x |\Psi\rangle = \Delta E_{x,x+a} |\Psi\rangle$$

(site) (links)

QED in 1D: string breaking



$$\text{energy} = \frac{1}{2} 2L' \times E^2 \propto \text{volume}$$



$$E_x|_S = 0 \qquad \text{and} \qquad \left.\frac{\partial B_x}{\partial n}\right|_S = 0$$

surface S
normal n

TE$_{1,0}$ mode

$$B_x = B_0 \cos\left(\frac{\pi y}{L_y}\right) e^{i(kx-\omega_{1,0}(k)t)},$$

$$B_y = -i\frac{kL_yB_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx-\omega_{1,0}(k)t)},$$

$$E_z = i\frac{\omega_{1,0}(k)L_yB_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx-\omega_{1,0}(k)t)}$$

$$A_z = \frac{L_y B_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx - \omega_{1,0}(k)t)}$$

vector potential

$$\omega_{m,n}(k) = \sqrt{(vk)^2 + \omega_{m,n}(0)^2}$$

$$\omega_{m,n}(0) = v\left[\left(\frac{m\pi y}{L_y}\right)^2 + \left(\frac{n\pi z}{L_z}\right)^2\right]^{\frac{1}{2}}$$

$$\begin{aligned}
H^{(1,0)} &= \mathcal{E}_{el}^{(1,0)} + \mathcal{E}_{mag}^{(1,0)} \\
&= \int dk\, \hbar\omega_{1,0}(k) a^\dagger(k) a(k) \\
&= \hbar v \int dk \sqrt{k^2 + \left(\frac{\pi}{L_y}\right)^2}\, a^\dagger(k) a(k)
\end{aligned}$$

free Hamiltonian

atom + interaction

$$H_{\rm at} = \frac{1}{2m_e}\left(\boldsymbol{p} - e\boldsymbol{A}^{(1,0)}(\boldsymbol{r})\right)^2 + V(\boldsymbol{r})$$

$$= H^0_{\rm at} - \frac{e}{m_e}\boldsymbol{p}\cdot\boldsymbol{A}^{(1,0)}(\boldsymbol{r}) + \frac{e^2}{2m_e}\left(\boldsymbol{A}^{(1,0)}(\boldsymbol{r})\right)^2$$

interaction

$$H^{(dip,RW)}_{\rm int} = \omega_0 D_{eg}\left(\frac{\hbar}{2\pi\epsilon v L_y L_z}\right)^{\frac{1}{2}} \int \frac{dk}{(k^2 + (vM/\hbar)^2)^{1/4}}$$

$$\times \left[b(k)|e\rangle\langle g|e^{ikx_0} + b^\dagger(k)|g\rangle\langle e|e^{-ikx_0}\right].$$

atom A in $x_0 = 0$ and atom B in $x_0 = d$

First principles

an initially factorized atomic state can spontaneously relax towards a long-lived entangled state. By analyzing the poles of the resolvent operator, we have shown how to quantify the robustness of the entangled bound state to small variations in the model parameters, and how to identify the time scales that are crucial for the preparation of an entangled state by relaxation.

While it has been pointed out that quantum computation may be achievable in waveguide-QED trough effective photon-photon interactions [49], focusing on the atomic degrees of freedom may also hold significant potential for applications in quantum information [50]. Further investigation will thus be devoted to the analysis of many-atom systems [51–54], in which photon-mediated interactions could possibly produce stable configurations such as W states or cluster states.

ACKNOWLEDGMENTS

We thank F. Ciccarello for useful discussions. M.S.K. thanks the UK EPSRC, the Royal Society, and the FP7 Marie Curie programme (Grant No. 317232). P.F. was partially supported by the Italian National Group of Mathematical Physics (GNFM-INdAM). P.F., S.P., and F.V.P. are partially supported by Istituto Nazionale di Fisica Nucleare (INFN) through the project "QUANTUM."

APPENDIX A: DERIVATION OF THE QUASI-1D FREE FIELD HAMILTONIAN

We derive here the Hamiltonian in Eq. (1) of the main text from first principles. Let us consider a waveguide of infinite length, parallel to the x axis, characterized by a rectangular cross section with $y \in [0,L_y]$ and $z \in [0,L_z]$. We conventionally assume that $L_y > L_z$. A common choice is $L_y/L_z = 2$. In a generic guide made of a linear dielectric with uniform density and coated by a conducting material, the boundary conditions for the electric and magnetic fields on the surface S read

$$E_x|_S = 0 \qquad \text{and} \qquad \left.\frac{\partial B_x}{\partial n}\right|_S = 0, \tag{A1}$$

with $\partial/\partial n$ denoting the normal derivative with respect to the surface. Transverse electric (TE) modes are characterized by $E_x = 0$ everywhere in the guide and obtained by imposing $\partial B_x/\partial n = 0$ on the surface. On the other hand, transverse magnetic (TM) modes have $B_x = 0$ identically. If the waveguide is rectangular, the boundary conditions for TE modes reduce to

$$\left.\frac{\partial B_x}{\partial y}\right|_{y=0} = \left.\frac{\partial B_x}{\partial y}\right|_{y=L_y} = \left.\frac{\partial B_x}{\partial z}\right|_{z=0} = \left.\frac{\partial B_x}{\partial z}\right|_{z=L_z} = 0, \tag{A2}$$

which limits the form of the longitudinal magnetic field to the real part of

$$B_x = B_0 \cos\left(\frac{m\pi y}{L_y}\right)\cos\left(\frac{n\pi z}{L_z}\right)e^{i(kx-\omega_{m,n}(k)t)}, \tag{A3}$$

with $m,n \in \mathbb{N}^2\backslash\{(0,0)\}$ and B_0 a constant.

The integers m and n label the mode $\mathrm{TE}_{m,n}$. The dispersion relation with respect to the longitudinal momentum has the same form as a massive relativistic particle,

$$\omega_{m,n}(k) = \sqrt{(vk)^2 + \omega_{m,n}(0)^2}, \tag{A4}$$

with $\omega_{m,n}(0) = v[(\frac{m\pi y}{L_y})^2 + (\frac{n\pi z}{L_z})^2]^{\frac{1}{2}}$, where the mass term is called the *cutoff frequency* of the mode, and $v = (\mu\epsilon)^{-1/2}$ is the phase velocity in the waveguide, assumed isotropic and nondispersive with magnetic permeability μ and dielectric constant ϵ. Since $L_y < L_z$, the $\mathrm{TE}_{1,0}$ mode has the lowest cutoff frequency. It can be proved [46] that $\omega_{1,0}(0)$ is also lower than the cutoffs of all TM modes. Thus, at sufficiently low energy the contribution of the higher energy modes can be neglected, and propagation occurs effectively in one dimension.

The $\mathrm{TE}_{1,0}$ mode is characterized by the following behavior of the fields:

$$B_x = B_0 \cos\left(\frac{\pi y}{L_y}\right)e^{i(kx-\omega_{1,0}(k)t)}, \tag{A5}$$

$$B_y = -i\frac{kL_yB_0}{\pi}\sin\left(\frac{\pi y}{L_y}\right)e^{i(kx-\omega_{1,0}(k)t)}, \tag{A6}$$

$$E_z = i\frac{\omega_{1,0}(k)L_yB_0}{\pi}\sin\left(\frac{\pi y}{L_y}\right)e^{i(kx-\omega_{1,0}(k)t)}, \tag{A7}$$

with the other three components vanishing. These fields can be derived from the (transverse) vector potential,

$$A_z = \frac{L_yB_0}{\pi}\sin\left(\frac{\pi y}{L_y}\right)e^{i(kx-\omega_{1,0}(k)t)}. \tag{A8}$$

The mode can be quantized by introducing the time-0 field operators,

$$\begin{aligned}\boldsymbol{A}^{(1,0)}(\boldsymbol{r}) = \int dk\left(\frac{\hbar}{2\pi\epsilon\omega_{1,0}(k)L_yL_z}\right)^{\frac{1}{2}}\sin\left(\frac{\pi y}{L_y}\right)\\ \times[a(k)e^{ikx} + a^\dagger(k)e^{-ikx}]\hat{u}_z,\end{aligned} \tag{A9}$$

$$\begin{aligned}\boldsymbol{E}^{(1,0)}(\boldsymbol{r}) = i\int dk\left(\frac{\hbar\omega_{1,0}(k)}{2\pi\epsilon L_yL_z}\right)^{\frac{1}{2}}\sin\left(\frac{\pi y}{L_y}\right)\\ \times[a(k)e^{ikx} - a^\dagger(k)e^{-ikx}]\hat{u}_z,\end{aligned} \tag{A10}$$

with $[a(k),a^\dagger(k')] = \delta(k-k')$ and $\hat{u}_z = (0,0,1)$. The electric field energy operator associated with the mode thus reads

$$\begin{aligned}\mathcal{E}_{el}^{(1,0)} &= \frac{\epsilon}{2}\int d\boldsymbol{r} :\left(E_z^{(1,0)}(\boldsymbol{r})\right)^2:\\ &= \frac{1}{2}\int dk\,\hbar\omega_{1,0}(k)\bigg[a^\dagger(k)a(k)\\ &\quad - \frac{a(k)a(-k) + a^\dagger(k)a^\dagger(-k)}{2}\bigg],\end{aligned} \tag{A11}$$

with : (...) : denoting normal ordering, while the magnetic field energy can be evaluated using the relation $\boldsymbol{B}^{(1,0)} = \nabla\times\boldsymbol{A}^{(1,0)}$:

$$\begin{aligned}\mathcal{E}_{\mathrm{mag}}^{(1,0)} &= \frac{\epsilon}{2}\int d\boldsymbol{r} :\left(\partial_y A_z^{(1,0)}(\boldsymbol{r})\right)^2 + \left(-\partial_x A_z^{(1,0)}(\boldsymbol{r})\right)^2:\\ &= \frac{1}{2}\int dk\,\hbar\omega_{1,0}(k)\bigg[a^\dagger(k)a(k)\\ &\quad + \frac{a(k)a(-k) + a^\dagger(k)a^\dagger(-k)}{2}\bigg].\end{aligned} \tag{A12}$$

Thus, the free Hamiltonian for the electromagnetic field takes the diagonal form,

$$\begin{aligned}H^{(1,0)} &= \mathcal{E}_{el}^{(1,0)} + \mathcal{E}_{\mathrm{mag}}^{(1,0)}\\ &= \int dk\,\hbar\omega_{1,0}(k)a^\dagger(k)a(k)\\ &= \hbar v\int dk\sqrt{k^2 + \left(\frac{\pi}{L_y}\right)^2}\,a^\dagger(k)a(k).\end{aligned} \tag{A13}$$

It is worth noticing that the analogy with a massive boson is not limited to the dispersion relation. Indeed, the quantum theory of the mode can be mapped onto a real scalar theory in one dimension, by introducing the operators,

$$\begin{aligned}\alpha(x) &= \int dx\sqrt{\frac{\hbar}{2(2\pi)\omega_{1,0}(k)}}[a(k)e^{ikx} + a^\dagger(k)e^{-ikx}],\\ \Pi(x) &= -i\int dx\sqrt{\frac{\hbar\omega_{1,0}(k)}{2(2\pi)}}[a(k)e^{ikx} - a^\dagger(k)e^{-ikx}],\end{aligned} \tag{A14}$$

satisfying

$$[\alpha(x),\Pi(x')] = i\hbar\delta(x-x'), \tag{A15}$$

and related to the vector potential and the electric field by multiplication. The Hamiltonian can be expressed in terms of the field operator $\alpha(x)$ and its canonically conjugated momentum $\Pi(x')$ as

$$\begin{aligned}H^{(1,0)} = \frac{1}{2}\int dx :\bigg[[\Pi(x)]^2 + v^2[\partial_x\alpha(x)]^2\\ + v^4\left(\frac{M}{\hbar}\right)^2[\partial_x\alpha(x)]^2\bigg]:\end{aligned} \tag{A16}$$

with $M := \frac{\pi\hbar}{vL_y}$, which also allows one to identify a linear Hamiltonian density $\mathcal{H}(x)$ such that $H^{(1,0)} = \int dx\mathcal{H}$.

APPENDIX B: INTERACTION HAMILTONIAN

The interaction between the field and an artificial atom, made up of a particle trapped in a potential $V(\boldsymbol{r})$, can be obtained by the minimal coupling prescription:

$$\begin{aligned}H_{\mathrm{at}} &= \frac{1}{2m_e}(\boldsymbol{p} - e\boldsymbol{A}^{(1,0)}(\boldsymbol{r}))^2 + V(\boldsymbol{r})\\ &= H_{\mathrm{at}}^0 - \frac{e}{m_e}\boldsymbol{p}\cdot\boldsymbol{A}^{(1,0)}(\boldsymbol{r}) + \frac{e^2}{2m_e}(\boldsymbol{A}^{(1,0)}(\boldsymbol{r}))^2,\end{aligned} \tag{B1}$$

with $\boldsymbol{r}$ and $\boldsymbol{p}$ the canonically conjugated position and momentum of the artificial "electron." The transverse choice $\nabla\cdot\boldsymbol{A} = 0$ for the vector potential makes the ordering with respect to $\boldsymbol{p}$ immaterial. We adopt a two-level approximation for the atom, retaining only the ground state $|g\rangle$ and the first excited state $|e\rangle$, satisfying

$$H_{\mathrm{at}}^0|g\rangle = 0, \qquad H_{\mathrm{at}}^0|e\rangle = \hbar\omega_0|e\rangle. \tag{B2}$$

Furthermore, we apply long-wavelength approximations to the interaction terms, which enable one to neglect the $O(e^2)$ contribution, whose relevance is suppressed like the ratio of the photon momentum to the particle momentum [44], and to apply a dipolar approximation to the $O(e)$ term. The position operator $\boldsymbol{r}$ is replaced by a nondynamical center-of-mass position $\boldsymbol{r}_0$. The interaction Hamiltonian thus reads

$$\begin{aligned}H_{\mathrm{int}}^{(\mathrm{dip})} = -\frac{e}{m_e}A_z^{(1,0)}(\boldsymbol{r}_0)[\langle g|p_z|g\rangle|g\rangle\langle g| + \langle e|p_z|e\rangle|e\rangle\langle e|\\ + \langle g|p_z|e\rangle|g\rangle\langle e| + \langle e|p_z|g\rangle|e\rangle\langle g|].\end{aligned} \tag{B3}$$

The assumption that the expectation value of momentum vanishes in the eigenstates of the free Hamiltonian simplifies the interaction. Moreover, the canonical commutation relation can be used to obtain

$$\begin{aligned}\langle e|p_z|g\rangle &= \frac{im}{\hbar}\langle e|\left[H_{\mathrm{at}}^0,z\right]|g\rangle = im\omega_0\langle e|z|g\rangle =: im\omega z_{eg}\\ &= im\omega_0|z_{eg}|e^{i\theta_{eg}},\end{aligned} \tag{B4}$$

by which the mass m_e disappears from the theory, and the Hamiltonian takes the form of a coupling between the dipole moment $D_{eg} = e|z_{eg}|$ and the electric field. Finally, we can define new canonically conjugated field operators $b(k) := e^{-i(\theta_{eg}+\pi/2)}a(k)$ and retain only the rotating-wave terms $b(k)|e\rangle\langle g|$ and $b^\dagger(k)|g\rangle\langle e|$, to obtain the interaction operator,

$$\begin{aligned}H_{\mathrm{int}}^{(\mathrm{dip,RW})} = \omega_0 D_{eg}\left(\frac{\hbar}{2\pi\epsilon vL_yL_z}\right)^{\frac{1}{2}}\int\frac{dk}{(k^2 + (vM/\hbar)^2)^{1/4}}\\ \times[b(k)|e\rangle\langle g|e^{ikx_0} + b^\dagger(k)|g\rangle\langle e|e^{-ikx_0}].\end{aligned} \tag{B5}$$

Notice that $y_0 = L_y/2$ has been used. The dynamics for the atom pair is thus determined by

$$H = H_{\mathrm{at},A}^0 + H_{\mathrm{at},B}^0 + H^{(1,0)} + H_{\mathrm{int},A}^{(\mathrm{dip,RW})} + H_{\mathrm{int},B}^{(\mathrm{dip,RW})}, \tag{B6}$$

with atom A in $x_0 = 0$ and atom B in $x_0 = d$.

APPENDIX C: ENERGY DENSITY

The study in the main text has been focused on the $\mathcal{N} = 1$ sector, spanned by the wave functions,

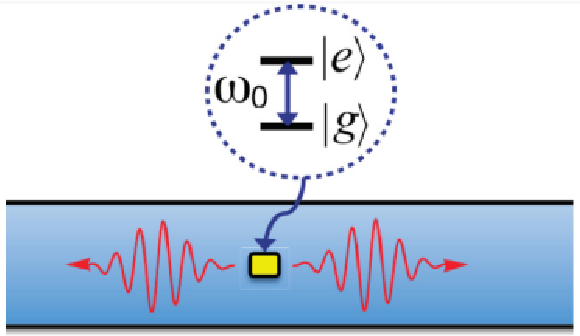
$$\begin{aligned}|\psi_1\rangle = c_A|e_A,g_B;\mathrm{vac}\rangle + c_B|g_A,e_B;\mathrm{vac}\rangle\\ + \int dkF(k)|g_A,g_B;k\rangle.\end{aligned} \tag{C1}$$

Using the scalar Hamiltonian density defined in Sec. A, one can compute the energy density,

$$\begin{aligned}\langle\psi_1|\mathcal{H}(x)|\psi_1\rangle = \frac{1}{2}\bigg[\langle\psi_1| : (\Pi(x))^2 : |\psi_1\rangle\\ + v^2\langle\psi_1| : (\partial_x\alpha(x))^2 : |\psi_1\rangle\\ + v^4\left(\frac{M}{\hbar}\right)^2\langle\psi_1| : (\partial_x\alpha(x))^2 : |\psi_1\rangle\bigg]\end{aligned}$$

one atom in a (1D) waveguide

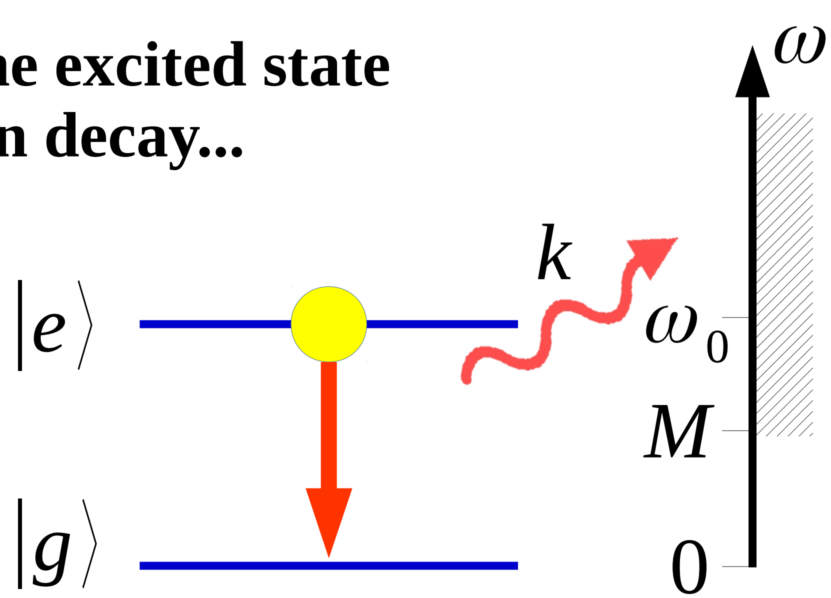
Coupling with the lowest-energy mode in a linear waveguide



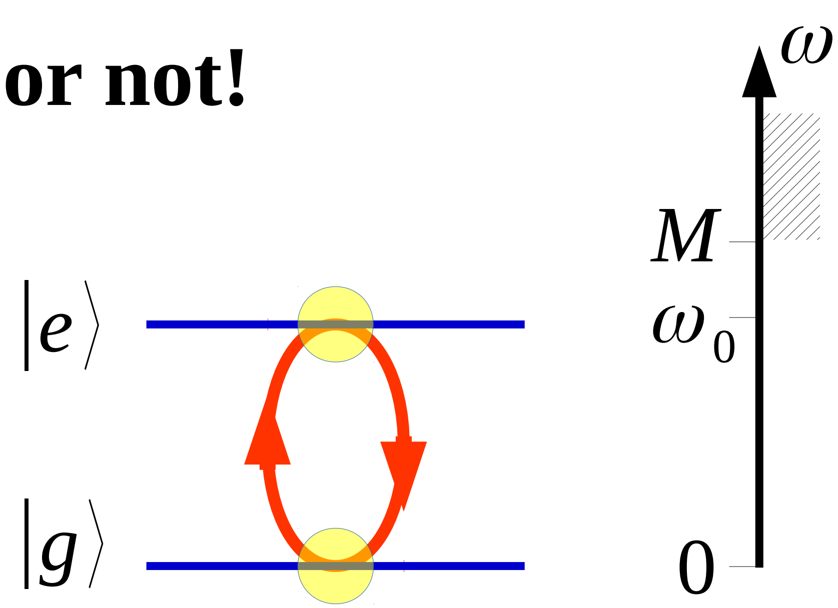
Dispersion relation (massive)

$$\omega(k) = \sqrt{k^2 + M^2}$$

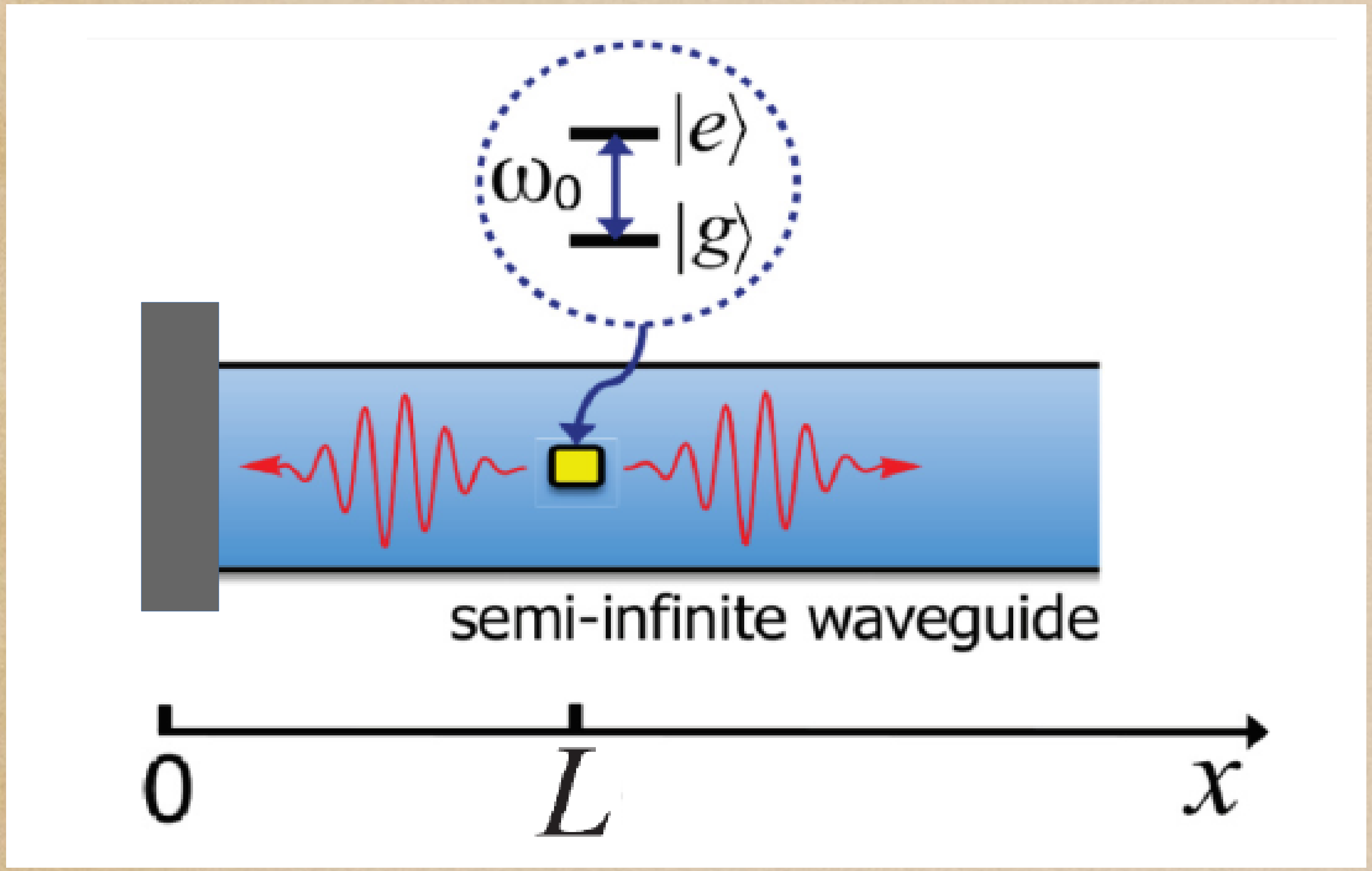
The excited state can decay...



...or not!



add a mirror

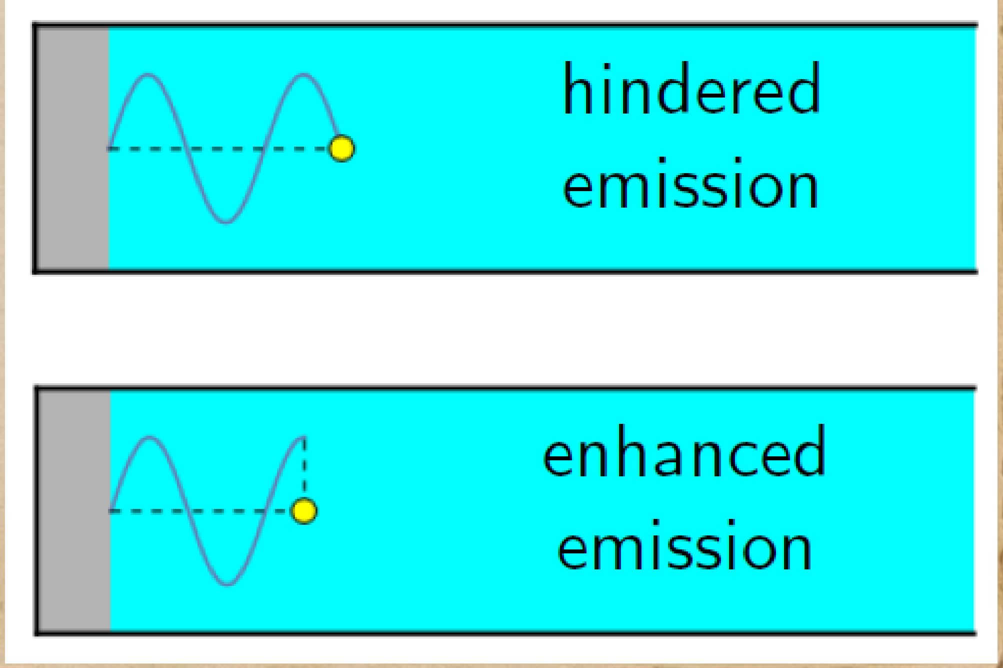


main ingredient:

E=0 at mirror

Fermi golden rule:

$$\gamma \propto \left|\langle g;\bar{k}|H_{\text{int}}|e\rangle\right|^2 \propto \sin^2 \bar{k} L$$

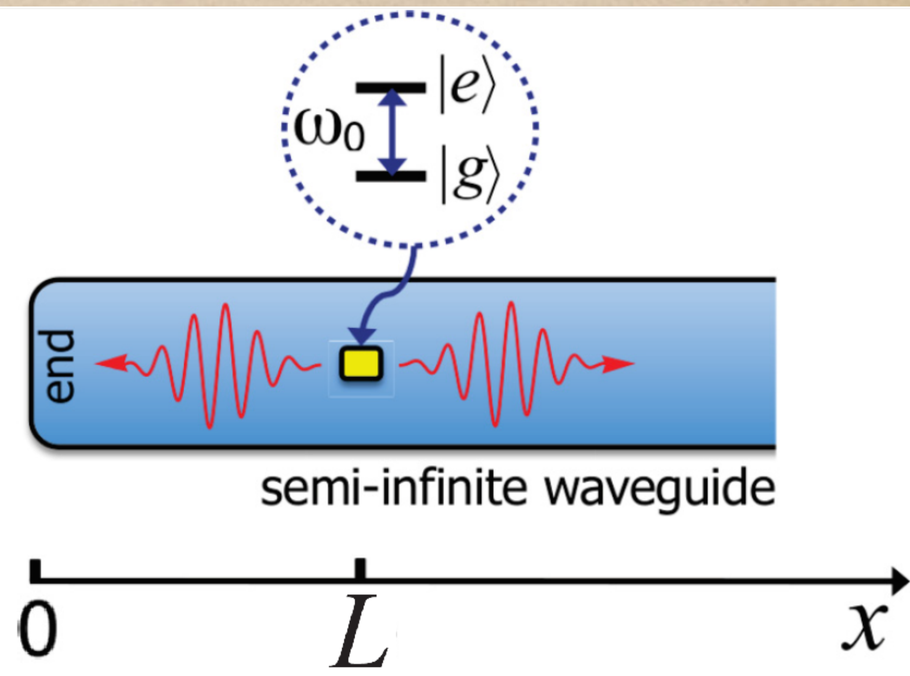


- Dorner & Zoller 2002
- Shen & Fan 2005
- Gonzales-Tudela, Martin-Cano, Moreno, Martin-Moreno, Tejedor & Garcia-Ripoll, Vidal 2011
- Tufarello, Ciccarello & Kim 2013

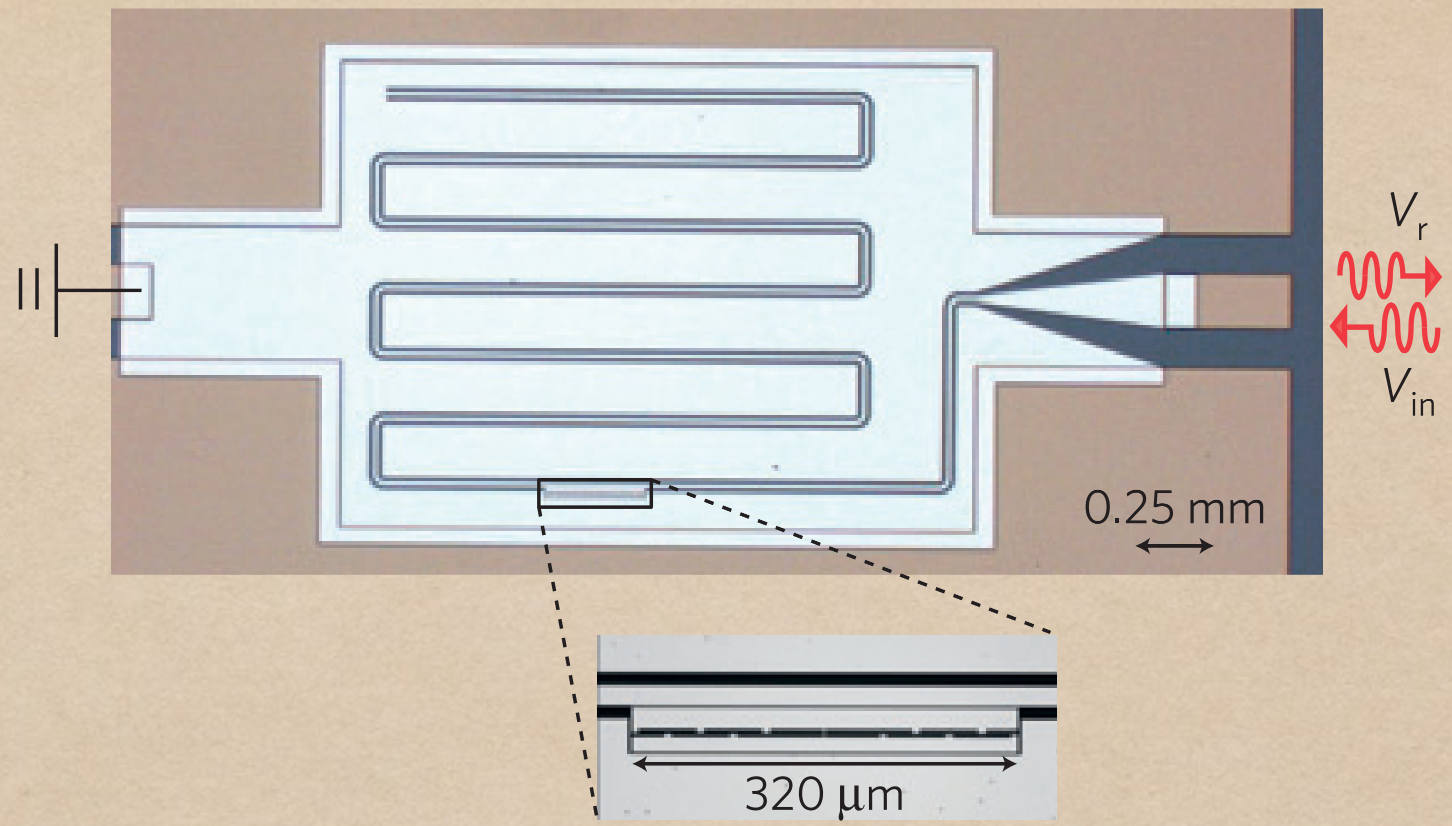
$$|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

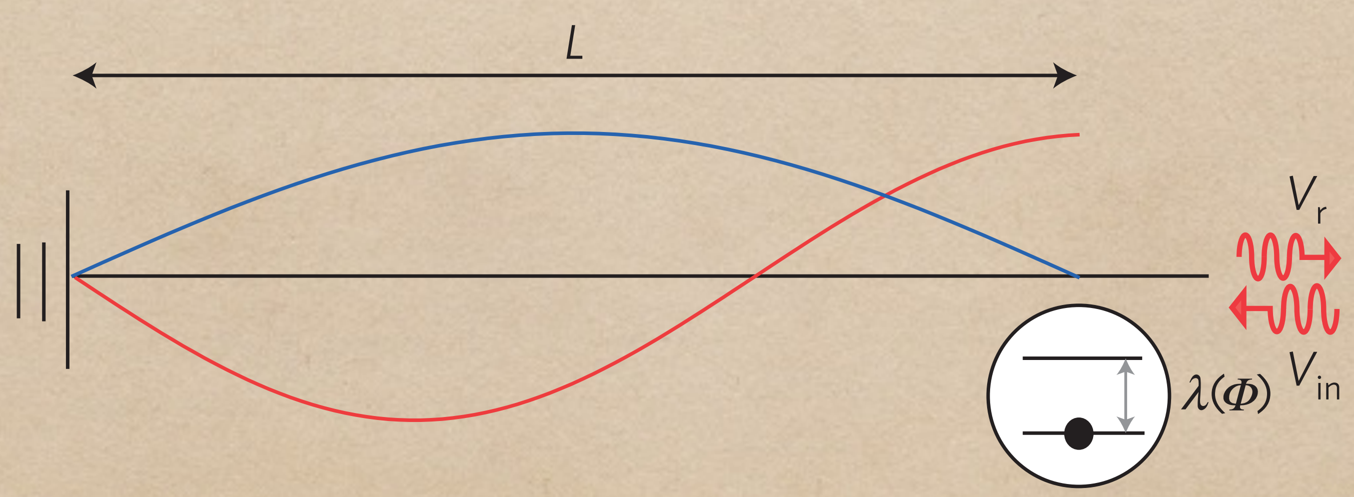
by a Rotating Wave Hamiltonian:

$$\begin{aligned}\hat{\mathcal{H}} &= \hat{\mathcal{H}}_0 + d\left(\hat{E}^\dagger(L)\hat{\sigma}_- + \hat{\sigma}_+\hat{E}(L)\right)\\ &= \hat{\mathcal{H}}_0 + \int dk\; g(k)(\hat{a}_k^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_k)\end{aligned}$$



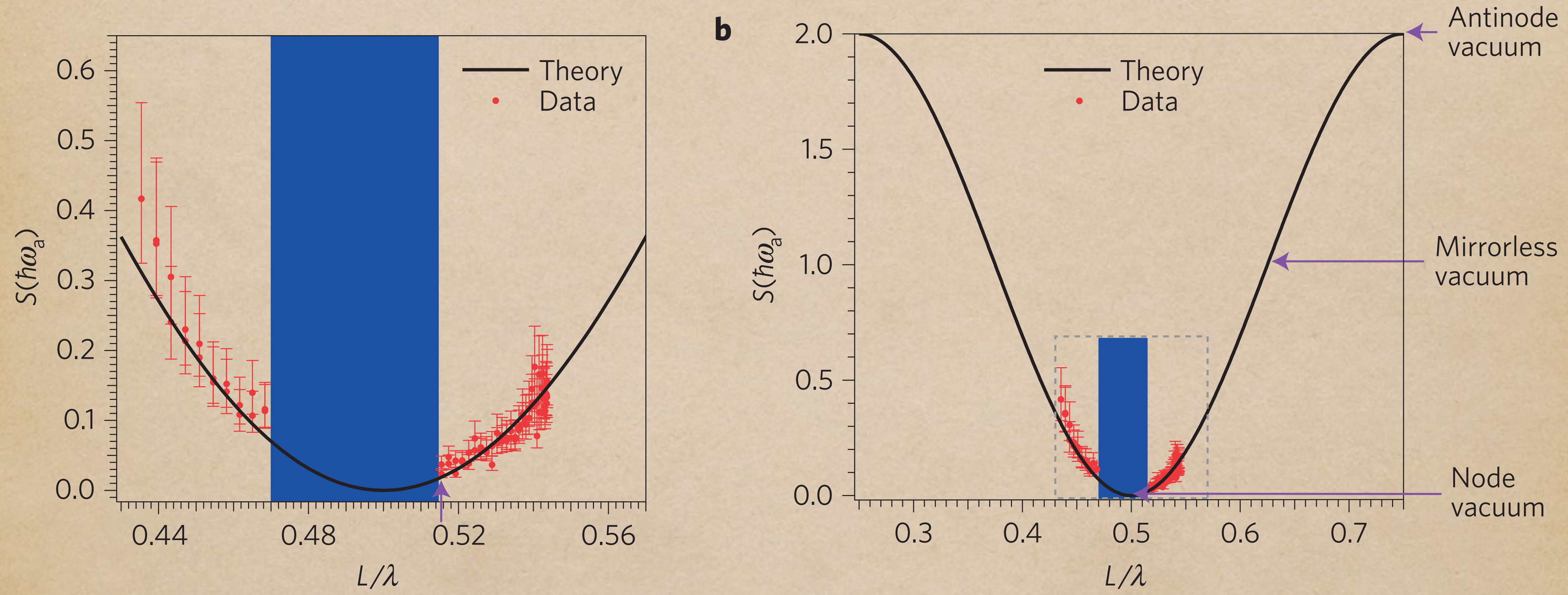
Hoi, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson 2015





(Probing the quantum vacuum with an artificial atom in front of a mirror)

Hoi, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson 2015

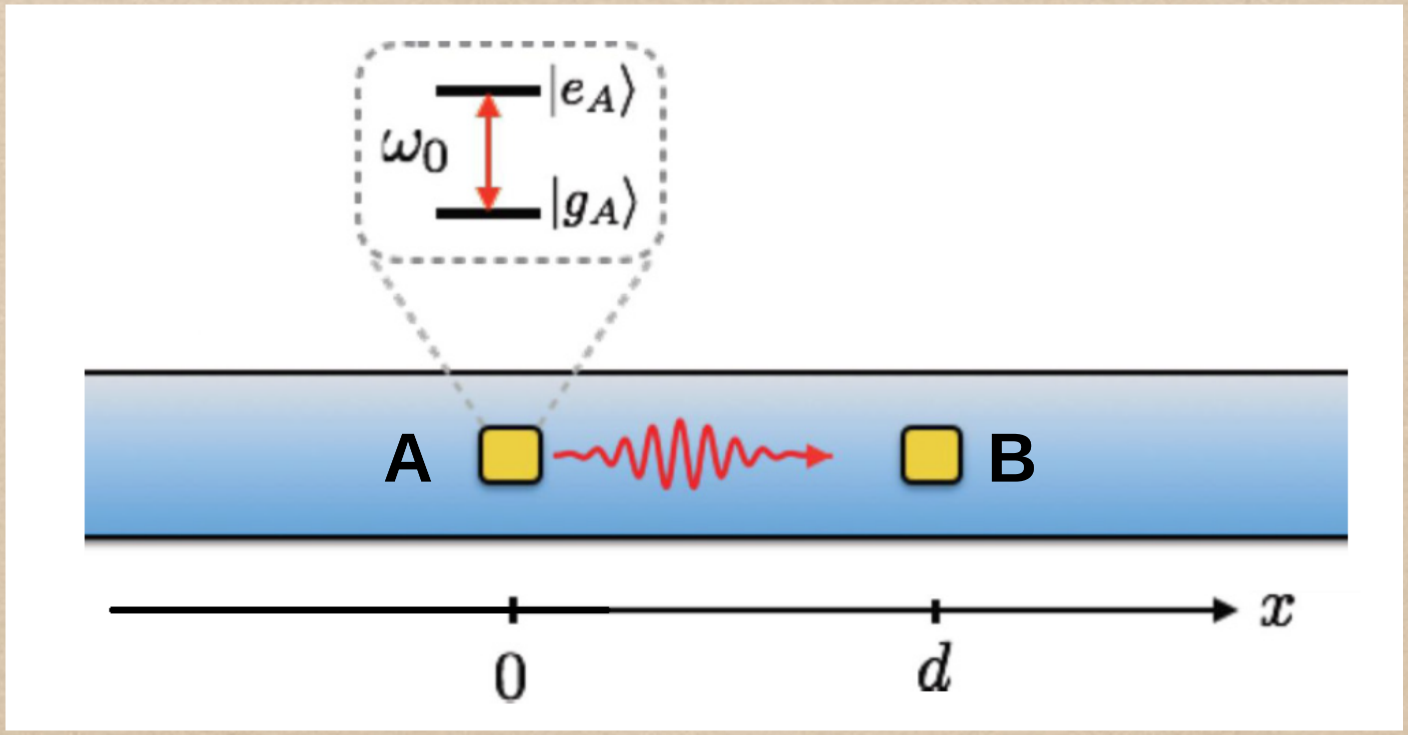


atom becomes "invisible" at 5.4 GHz

Objective: threefold

- extend to TWO emitters
- remove mirror
- (use resolvent formalism)
- Shen, Fan (2005)
  Gonzales-Tudela et al (2011)
  ...

a pair of two-level (artificial) atoms in a waveguide

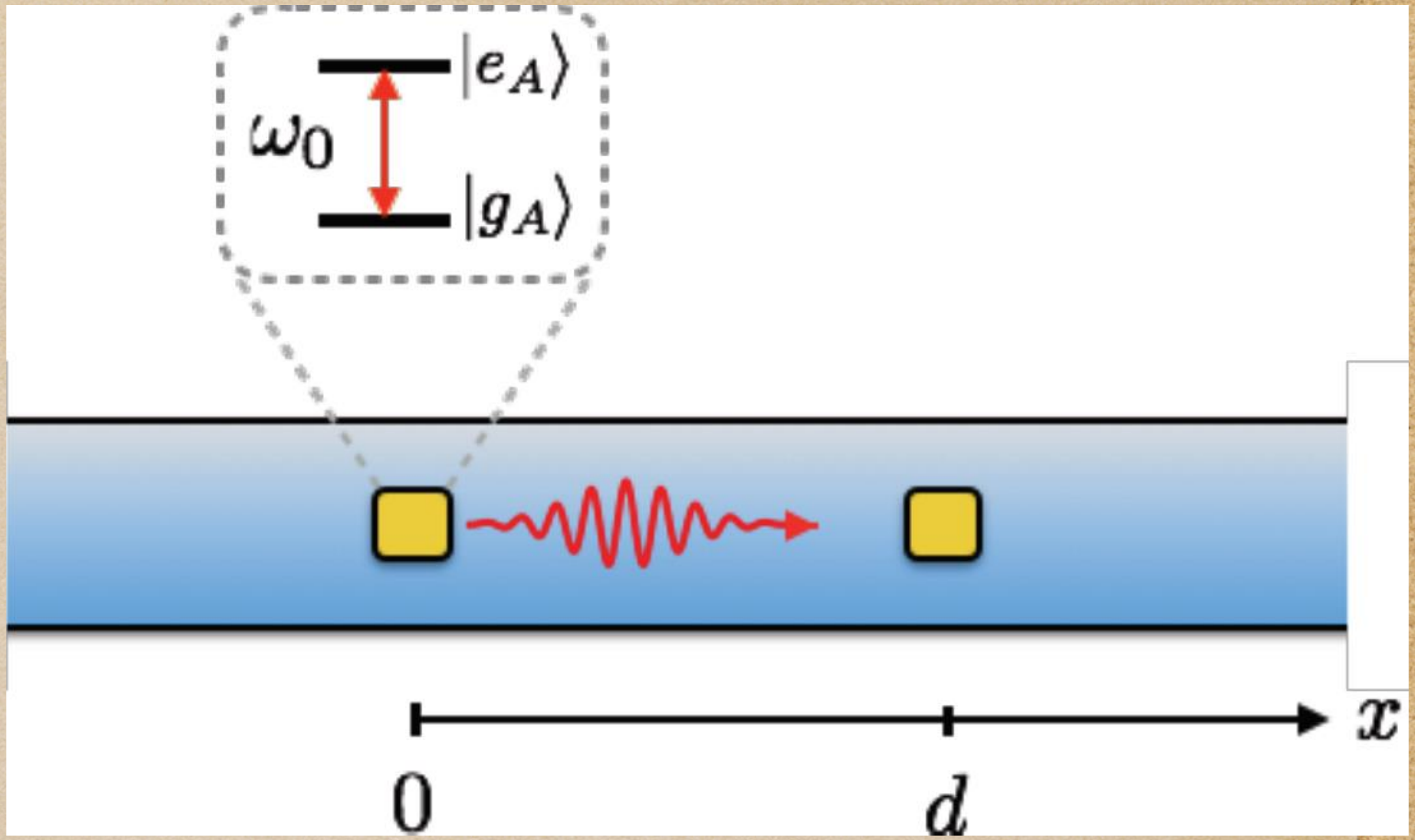


lowest energy mode, one-excitation sector

System and Hamiltonian

No need to have mirror!

Atoms behave like "mirrors"



$$
\begin{aligned}
H &= H_0 + \lambda V \\
&= \omega_0(|e_A\rangle\langle e_A| + |e_B\rangle\langle e_B|) + \int \mathrm{d}k\, \omega(k) b^\dagger(k) b(k) \\
&\quad + \lambda \int \frac{\mathrm{d}k}{\omega(k)^{1/2}} \Big[|e_A\rangle\langle g_A| b(k) + |g_A\rangle\langle e_A| b^\dagger(k) \\
&\qquad + |e_B\rangle\langle g_B| b(k) \mathrm{e}^{\mathrm{i}kd} + |g_B\rangle\langle e_B| b^\dagger(k) \mathrm{e}^{-\mathrm{i}kd} \Big],
\end{aligned}
$$

$$\omega(k) = \sqrt{k^2 + M^2}$$

$$M \propto L_y^{-1}$$

Rotating Wave Approximation

The total number of excitations is a constant of motion

$$N = N_{\mathrm{at}} + N_{\mathrm{field}} = |e_A\rangle\langle e_A| + |e_B\rangle\langle e_B| + \int \mathrm{d}k\, b^\dagger(k)\, b(k)$$

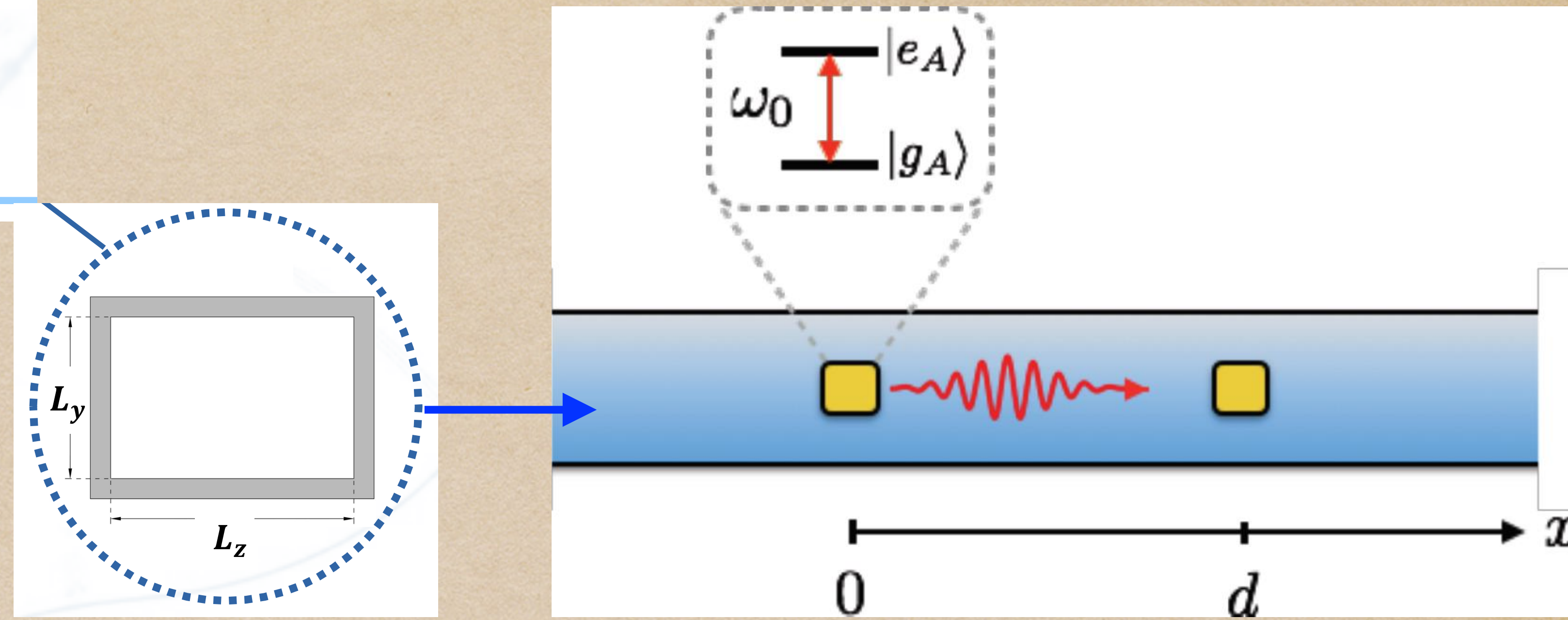
let N=1 (one-excitation sector)

General wavefunction in the sector

$$|\psi\rangle = \left(c_A |e_A, g_B\rangle + c_B |g_A, e_B\rangle\right)|\mathrm{vac}\rangle + |g_A, g_b\rangle |1\ \mathrm{photon}\rangle$$

Bound states $H|\psi\rangle = E|\psi\rangle$ with $\langle\psi|\psi\rangle = 1$

$$\omega_{nm}(k) = \sqrt{\frac{k^2}{\mu\epsilon} + M_{nm}^2}$$

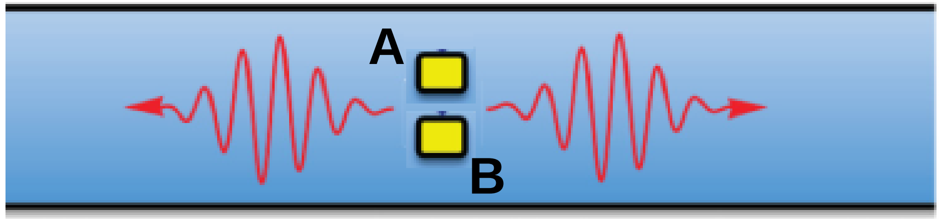


$\mathrm{TE}_{1,0}$ mode; role of boundary conditions

$$|\psi\rangle = \left(c_A|e_A, g_B\rangle + c_B|g_A, e_B\rangle\right) \otimes |\mathrm{vac}\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle$$

$$d_n = \frac{n\pi}{\bar{k}}, \quad \text{with} \quad \bar{k} := \sqrt{\left(\omega_0 + \frac{2\lambda^2}{M}\right)^2 - M^2}.$$

observation: dark state of an atomic pair (identical but distinguishable atoms)



The one-excitation antisymmetric state

$$|\Psi^{(-)}\rangle = \frac{|e_A, g_B\rangle - |g_A, e_B\rangle}{\sqrt{2}}$$

decouples from the interaction

$$H_{\text{int}}|\Psi^{(-)}\rangle = 0$$

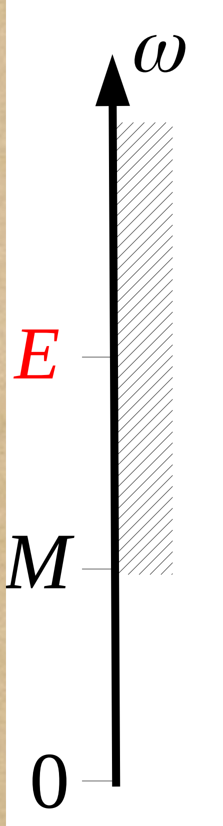
The one-excitation symmetric state

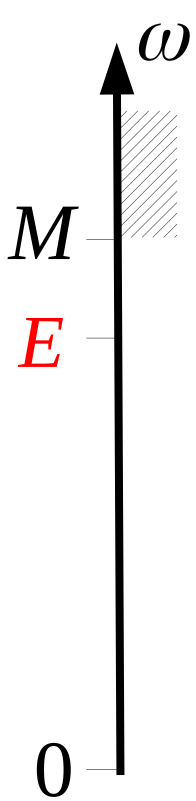
$$|\Psi^{(+)}\rangle = \frac{|e_A, g_B\rangle + |g_A, e_B\rangle}{\sqrt{2}}$$

decays faster than a free atom

$$\gamma^{(+)} = 2\gamma_{\text{free}}$$

another observation





Bound states below the threshold for photon propagation are expected:

- Effective interatomic interaction mediated by evanescent waves
- Symmetric and antisymmetric eigenstates for any interatomic distance d :

$$|\psi\rangle \simeq \left[\frac{|e_A, g_B\rangle \pm |g_A, e_B\rangle}{\sqrt{2}}\right] |\mathrm{vac}\rangle$$

- $E = \omega_0 + O(\lambda^2)$, level splitting $\sim \exp\left(-\sqrt{E^2 - M^2}\, d\right)$

Shahmoon & Kurizki 2013

above threshold?
much less obvious

General wavefunction in the sector

$$|\psi\rangle = \left(c_A|e_A, g_B\rangle + c_B|g_A, e_B\rangle\right)|\text{vac}\rangle + |g_A, g_b\rangle|1\text{ photon}\rangle$$

Bound states $\quad H|\psi\rangle = E|\psi\rangle \quad$ with $\langle\psi|\psi\rangle = 1$

The eigenvalue equation $\quad H|\psi\rangle = E|\psi\rangle = \sqrt{\bar{k}^2 + M^2}|\psi\rangle$

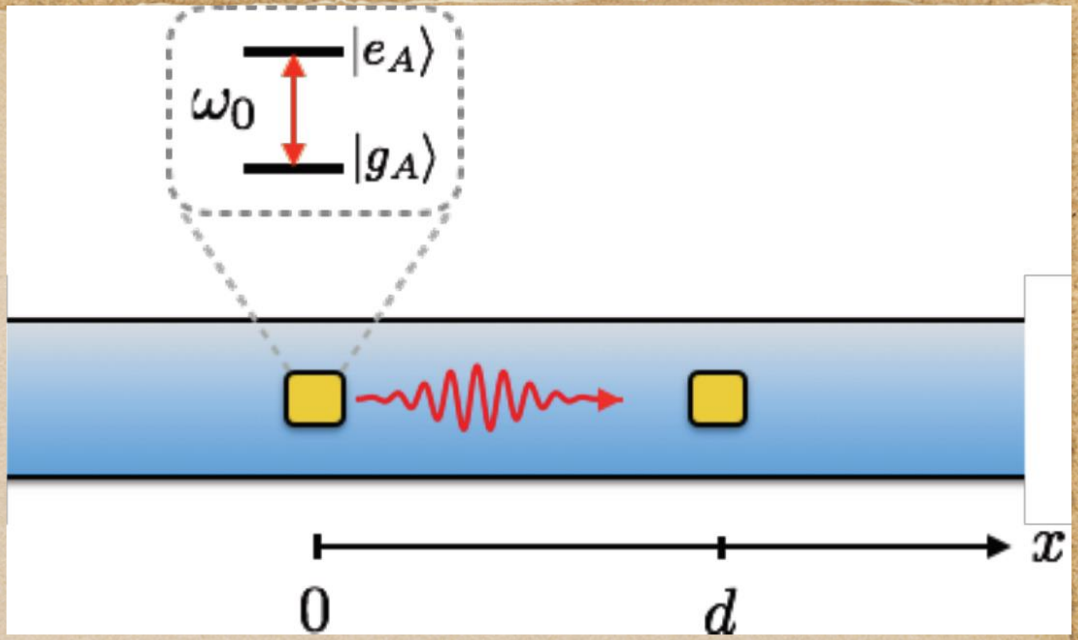
can be satisfied by a normalizable state only if:

i) $\quad c_A + e^{\pm i\bar{k}d} c_B = 0$

ii) $\quad E = \omega_o + \int dk \frac{\lambda^2}{\sqrt{k^2+M^2}} \frac{1 - e^{i(\bar{k}-k)d}}{E - \sqrt{k^2+M^2}}$ $\quad$ has real solutions

iff

$$\boxed{c_A = (-1)^{n+1} c_B, \qquad d = d_n = \frac{n\pi}{\bar{k}} \quad (n \in \mathbb{Z}_+)}$$



structure of bound state

$$|\psi_n\rangle = \sqrt{p_n}|\Psi^s\rangle \otimes |\text{vac}\rangle + |g_A, g_B\rangle \otimes |\varphi_n\rangle$$

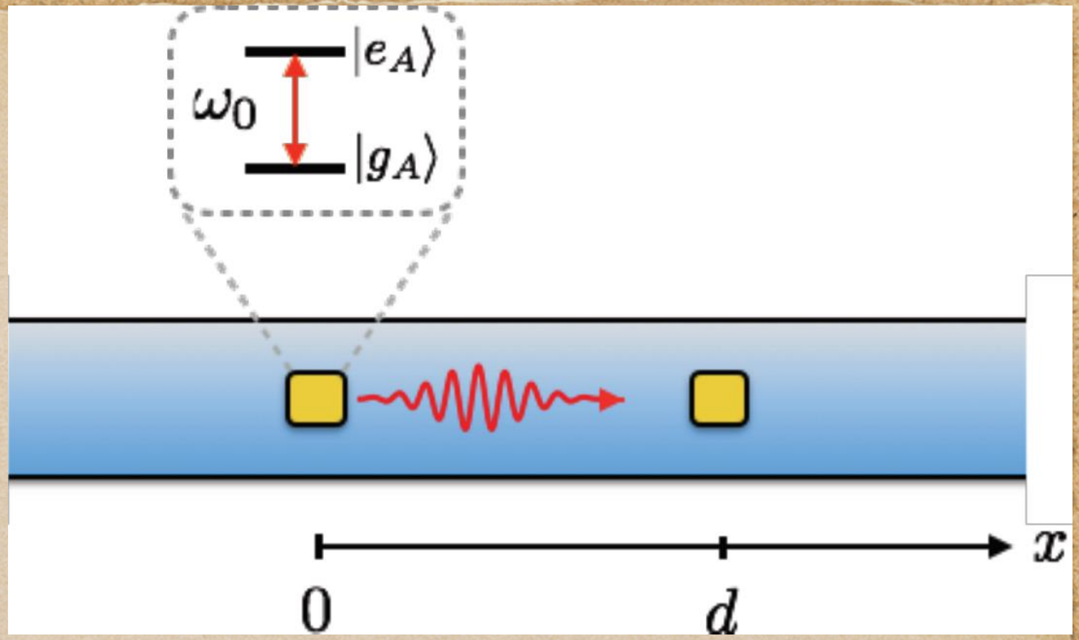
$$|\Psi^{\pm}\rangle = (|e_A, g_B\rangle \pm |g_A, e_B\rangle)/\sqrt{2}$$

if state factorized at t=0
atomic density matrix

$$p_n = \left(1 + n\frac{2\pi^2\lambda^2}{\bar{k}^2}\right)^{-1}$$

$$\rho_{\text{at}}(\infty) = \frac{p_n^2}{2}|\Psi^s\rangle\langle\Psi^s| + \left(1 - \frac{p_n^2}{2}\right)|g_A, g_B\rangle\langle g_A, g_B|$$

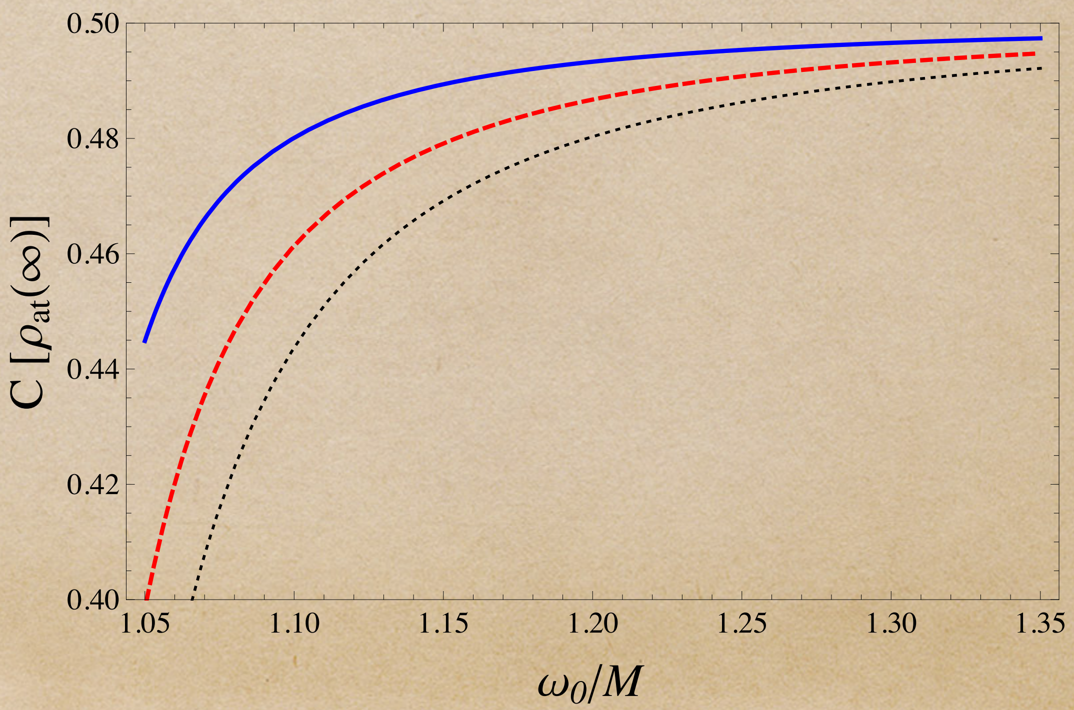
Facchi, Kim, P, Pepe,
Pomarico, Tufarelli 2016



$$\rho_{\rm at}(\infty) = \frac{p_n^2}{2}|\Psi^s\rangle\langle\Psi^s| + \left(1 - \frac{p_n^2}{2}\right)|g_A, g_B\rangle\langle g_A, g_B|$$

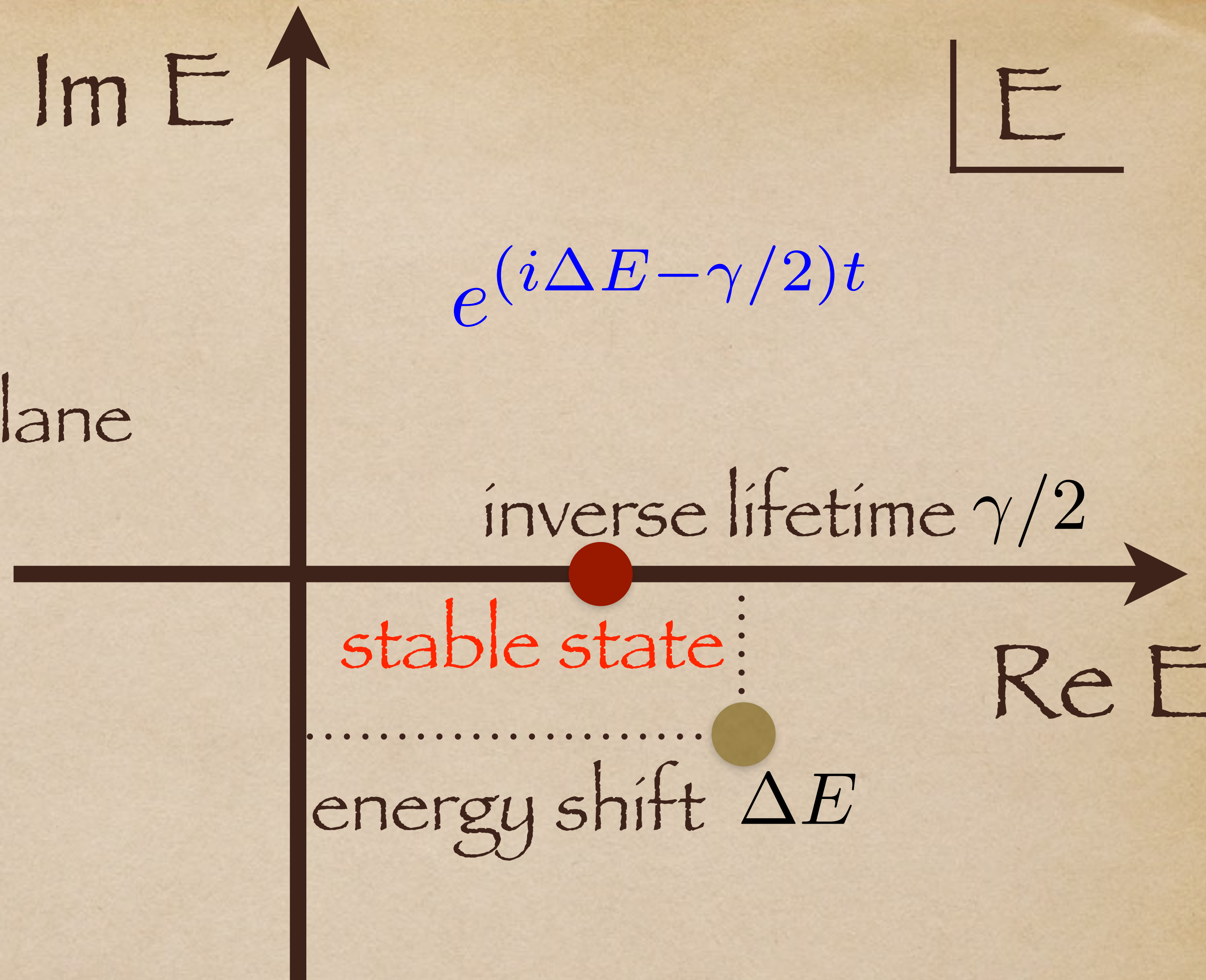
$$p_n = \left(1 + n\frac{2\pi^2\lambda^2}{\bar{k}^2}\right)^{-1}$$

concurrence @ long t



fast tutorial

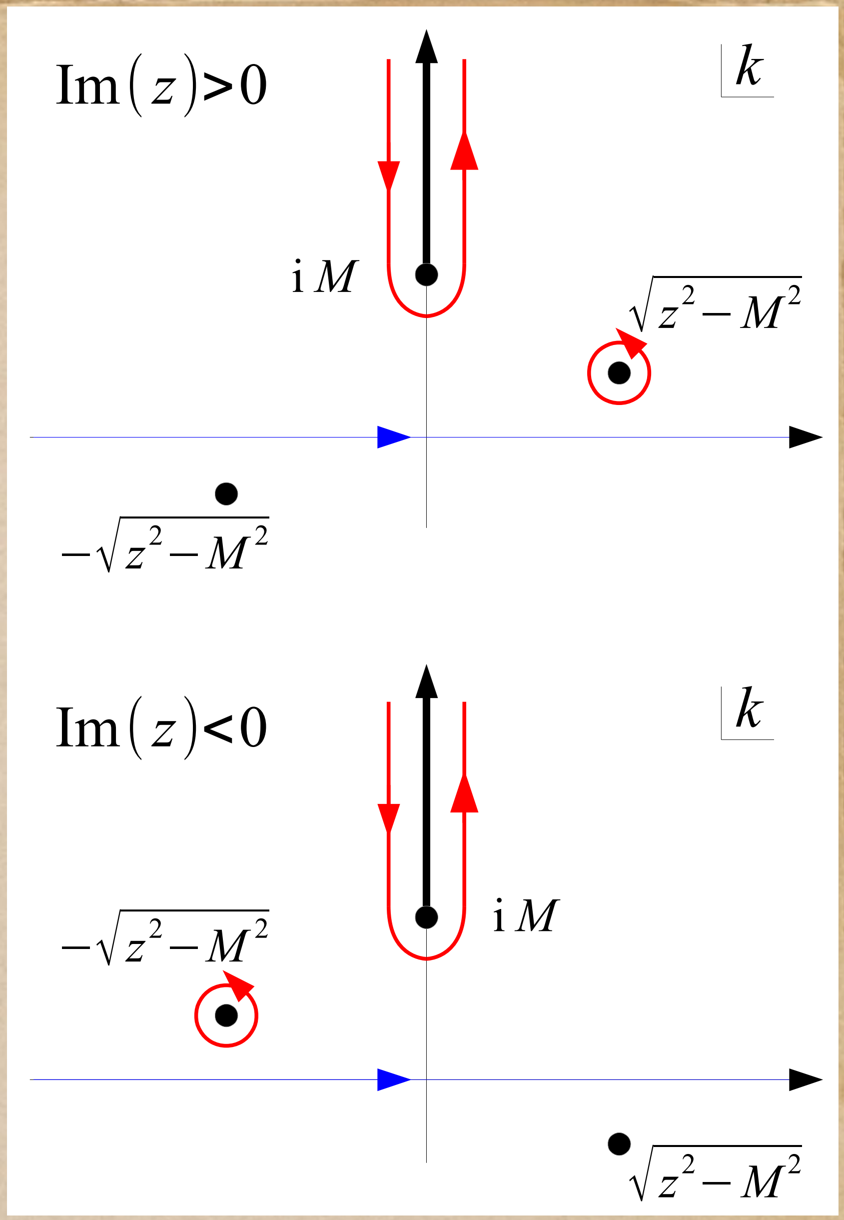
complex energy plane



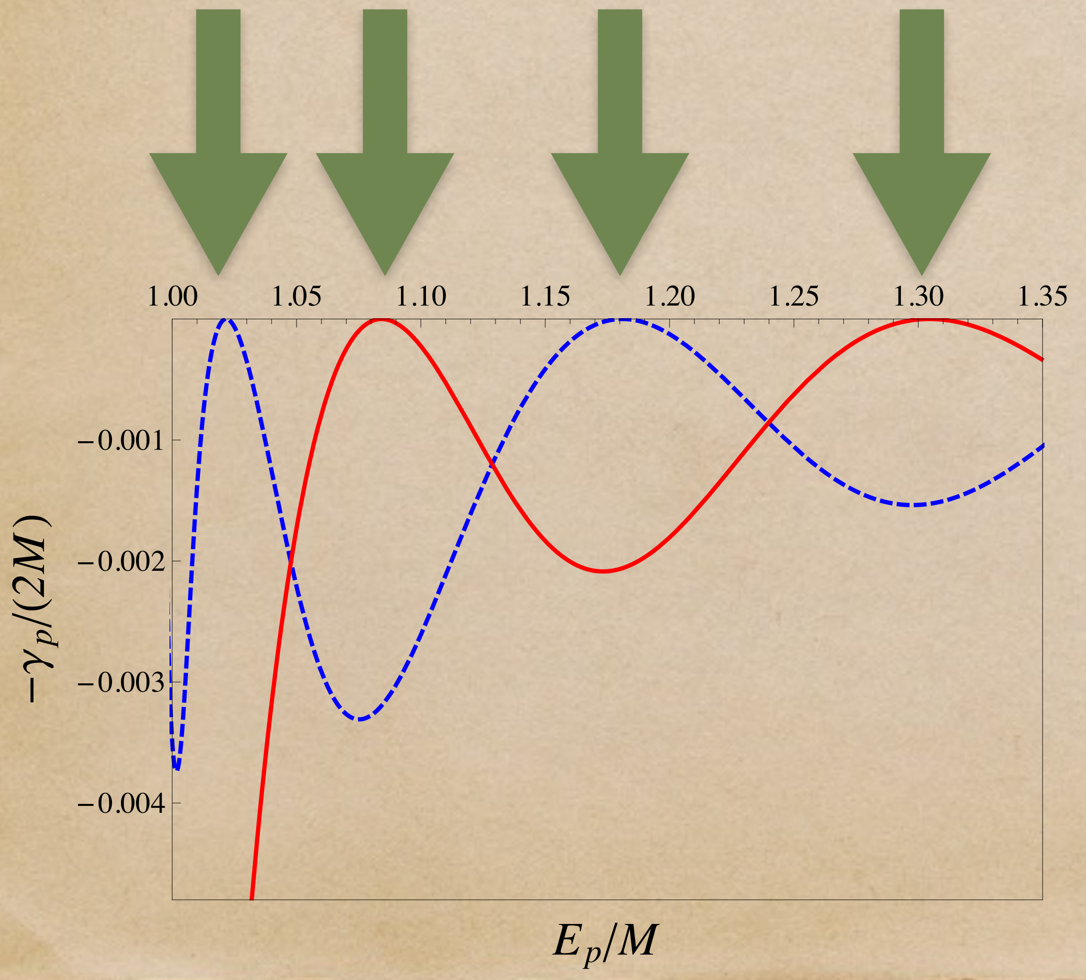
Schwinger (simple poles);

Araki et al (proof of Fermi "Golden rule")

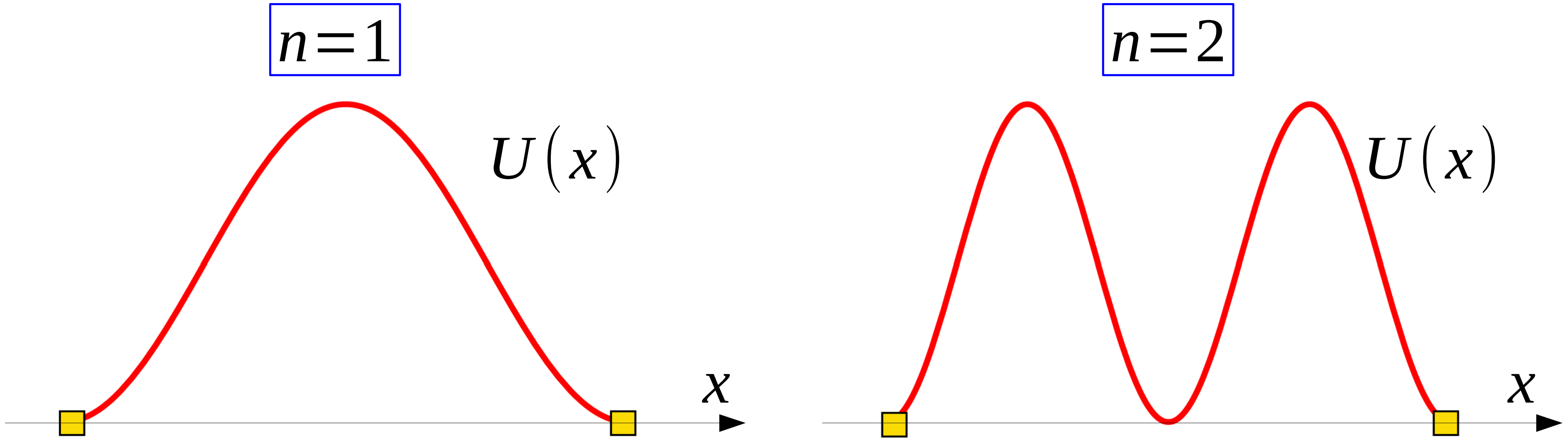
resolvent: poles in the complex energy plane; each pole associated to (unstable) state;
imaginary part = (inverse) lifetime



"bound" states



poles: trajectories (robust)



Field energy density

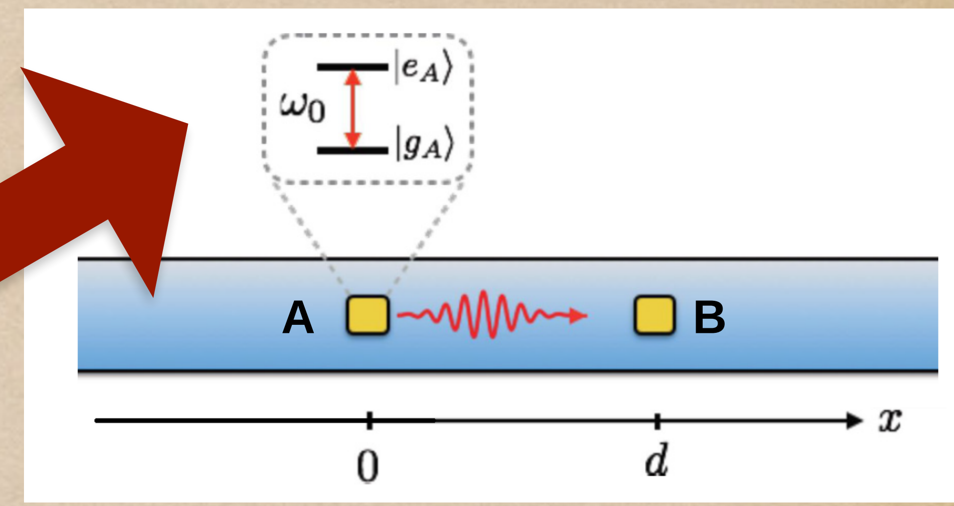
$$U(x) \propto \sin^2(\bar{k}\, x) \quad \text{if } x \in [0, d_n]$$

$$U(x) \simeq 0 \quad \text{elsewhere}$$

The atoms behave as dynamical mirrors

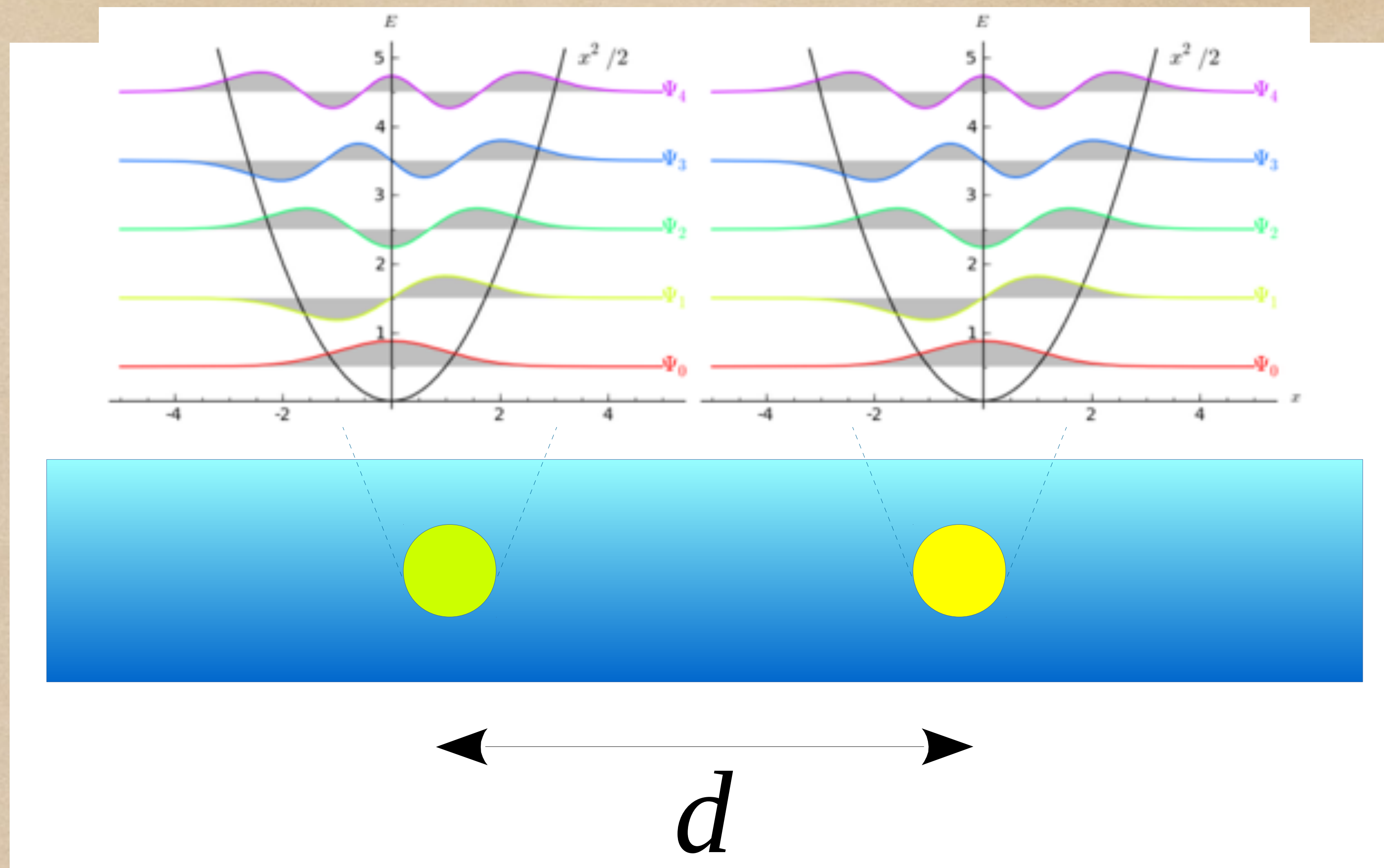
extension

atom/oscillator



sector

dynamics will preserve sector
(number of excitation conserved)
+ robustness + entanglement

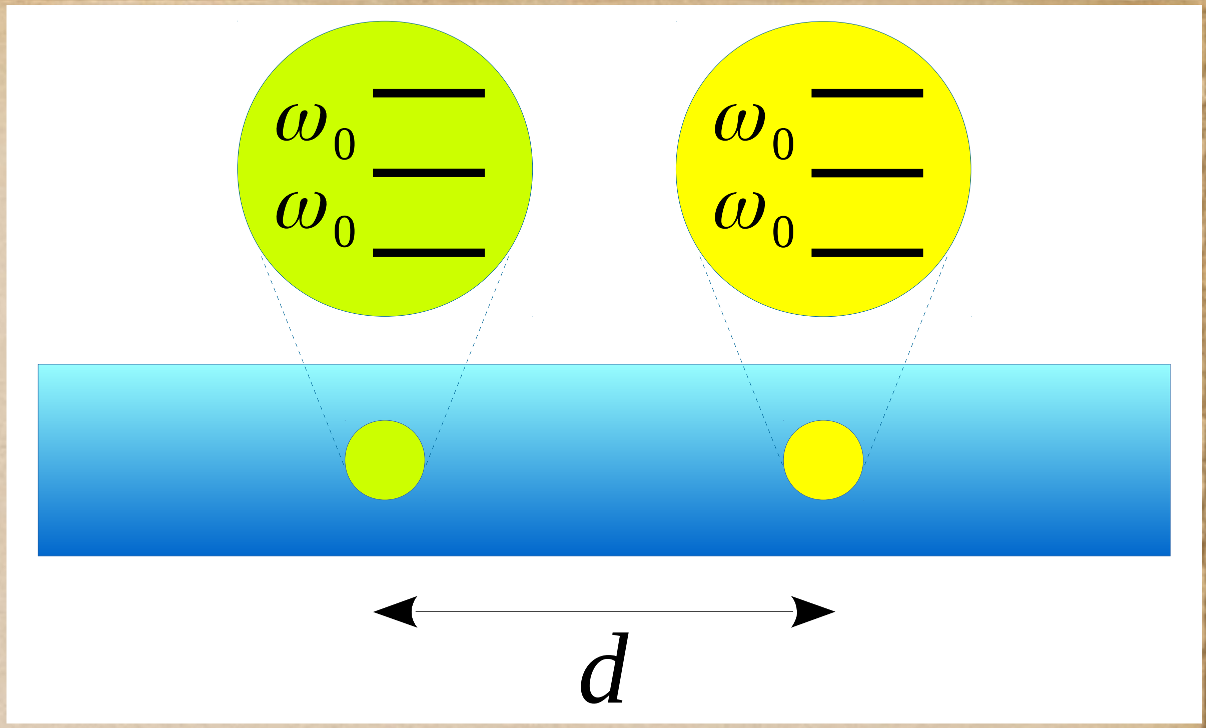


harmonic oscillators (e.g. optical cavities)

Long-lived entanglement of two ~~multilevel atoms~~ in a waveguide

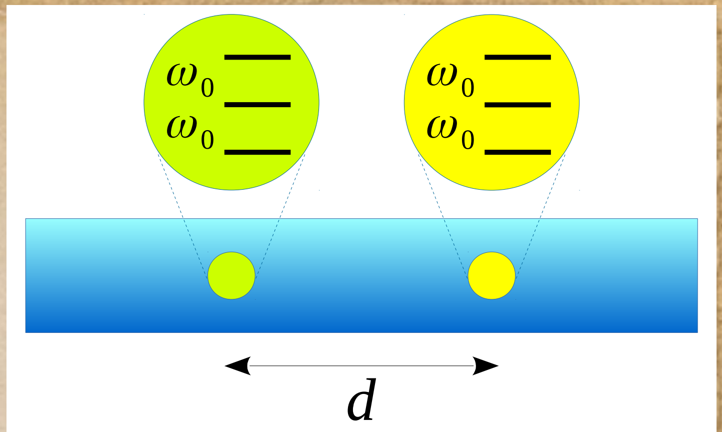
Paolo Facchi, SP, Francesco V. Pepe, Kazuya Yuasa 2018

(assumption equal spacings to be relaxed)



$$H = \omega_0 (b_A^\dagger b_A + b_B^\dagger b_B) + \int dk\, \omega(k) b^\dagger(k) b(k)$$
$$+ \int dk\, g(k) \left[(b_A^\dagger + b_B^\dagger e^{ikd}) b(k) + \text{H.c.} \right]$$

A, B = harmonic oscillators OR N-level atoms



N+1 levels

$$|N\rangle = \frac{(b_\phi^\dagger)^N}{\sqrt{N!}}|0\rangle = \sum_{m=0}^{N}\binom{N}{m}^{\frac{1}{2}} p_{\rm at}^{\frac{m}{2}}(1-p_{\rm at})^{\frac{N-m}{2}}|\psi^{(m)}\rangle_{AB}\otimes|\phi^{(N-m)}\rangle$$

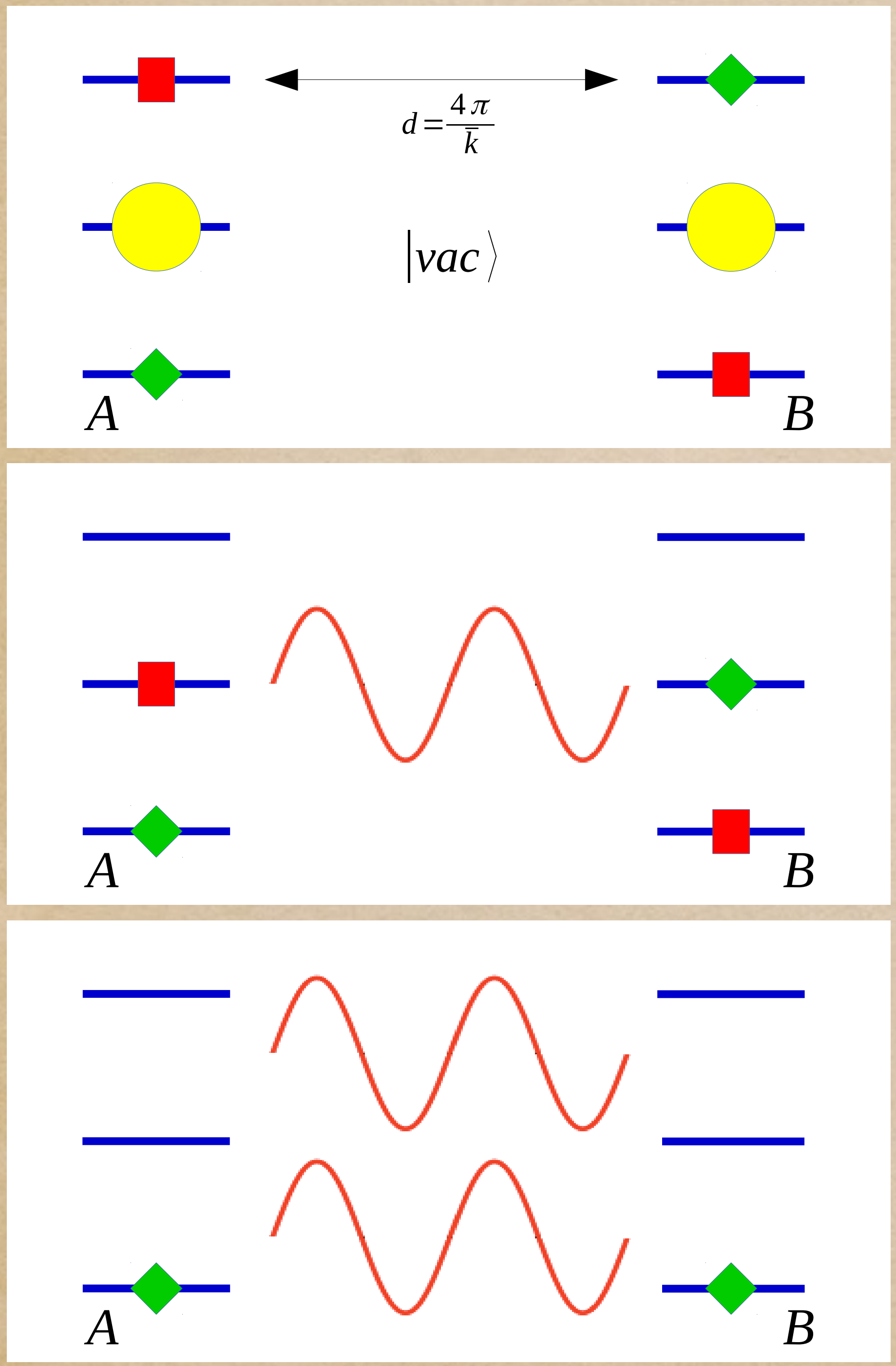
$$|\psi^{(m)}\rangle_{AB} = 2^{-\frac{m}{2}}\sum_{\ell=0}^{m}\binom{m}{\ell}^{\frac{1}{2}}(-1)^{\ell(n+1)}|\ell_A,(m-\ell)_B\rangle$$

$$|\phi^{(m)}\rangle = \frac{1}{\sqrt{m!}}\left[\sqrt{\frac{p_{\rm at}}{2(1-p_{\rm at})}}\int dk\, g(k)\frac{1+(-1)^{n+1}e^{ikd}}{\omega(n\pi/d)-\omega(k)}b^\dagger(k)\right]^m|{\rm vac}\rangle$$

3 levels

$$\begin{aligned}|2\rangle = {} & \frac{p_{\rm at}}{\sqrt{6}}\Big(|0_A,2_B\rangle - 2|1_A,1_B\rangle + |2_A,0_B\rangle\Big)\otimes|{\rm vac}\rangle \\ & + \sqrt{2p_{\rm at}(1-p_{\rm at})}\Big(|0_A,1_B\rangle - |1_A,0_B\rangle\Big)\otimes|\phi^{(1)}\rangle \\ & + (1-p_{\rm at})|0_A,0_B\rangle\otimes|\phi^{(2)}\rangle.\end{aligned}$$

entanglement !



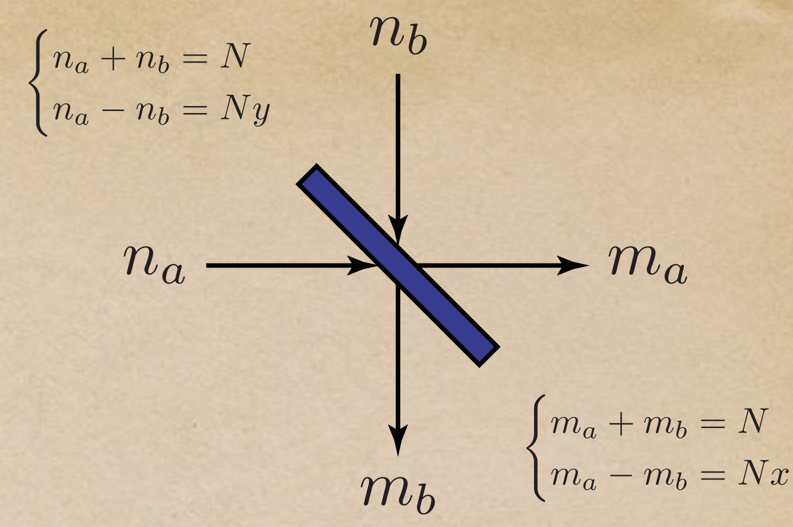
3 levels

$$|2\rangle = \frac{p_{\rm at}}{\sqrt{6}}\Big(|0_A,2_B\rangle - 2|1_A,1_B\rangle + |2_A,0_B\rangle\Big)\otimes|{\rm vac}\rangle$$
$$+\ \sqrt{2p_{\rm at}(1-p_{\rm at})}\Big(|0_A,1_B\rangle - |1_A,0_B\rangle\Big)\otimes|\phi^{(1)}\rangle$$
$$+\ (1-p_{\rm at})|0_A,0_B\rangle\otimes|\phi^{(2)}\rangle.$$

bound state in N=2 sector,
for d=4 pi/k

many nice options to
create robust entangled states

single-emitter density matrix



$$\rho_A^{(N)} = \mathrm{tr}_B\, \rho_{AB}^{(N)} = \sum_{\ell=0}^{N} C_\ell^{(N)}(p_{\mathrm{at}}) |\ell_A\rangle\langle\ell_A|$$

$$C_\ell^{(N)}(p_{\mathrm{at}}) := \sum_{m=0}^{N} \frac{1}{2^m} \binom{N}{m} \binom{m}{\ell} p_{\mathrm{at}}^m (1 - p_{\mathrm{at}})^{N-m}$$

binomial: beam splitter, generalizes NOON

(Nakazato, P, Stobinska, Yuasa, 2016)

(vector model in QFT - de Prunelé J. Math. Phys. 1988)

purity

$$\pi_A^{(N)} = \sum_{\ell=0}^{N} \left(C_\ell^{(N)}(p_{\mathrm{at}})\right)^2 = \frac{\Gamma\left(N + \frac{1}{2}\right)}{\sqrt{\pi} N!} \left(1 + O[(N\lambda^2)^2]\right) \sim \left[\frac{1}{\sqrt{\pi N}} + O(N^{-3/2})\right] \left(1 + O[(N\lambda^2)^2]\right)$$

ideas

- set of N two-level atoms in optical waveguide: presence of bound states affects the interactions among atoms
  (Calajo, Ciccarello, Chang, Rabl, PRA 2016)
  (Notice: interaction is waveguide-mediated; slow light)

- moving atoms in 1D photonic waveguide
  (Calajo, Rabl, PRA 2017) (strong coupling, slow light)

- circuit QED with single LC resonator: very strong interactions decouples photon mode and projects qubits into entangled gs
  (Jaako, Xiang, Garcia-Ripoll, Rabl, PRA 2016) (ultra-strong coupling)

- Scattering effects in a waveguide
  (Calajo, Fang, Baranger, Ciccarello)

perspectives/expts

- Quantum computation trough effective photon-photon interactions in waveguide-QED
  (Zheng, Gauthier, Baranger, PRL 2013)
- But atomic degrees of freedom have significant potential for applications
  (Paulisch, Kimble, Gonzalez-Tudela, NJP 2016)
- Probing vacuum with artificial atom in front of mirror
  (Hoi1, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson Nat. Phys. 2015)

comment on interdisciplinarity

Quantum technologies (in general) and one-dimensional QED
blend different physical disciplines
(high-energy physics, QED, gauge theories
vs solid state, low energy, circuit QED, optics)