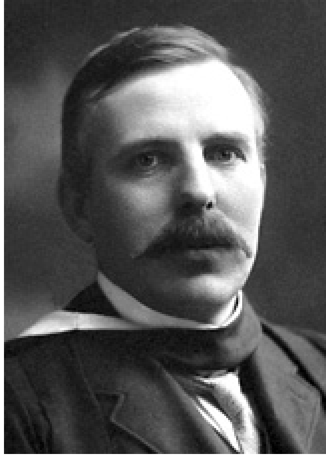
Gas detectors: general principles

- a brief history
- how they work
- ionisation
- electric field
- electron transport
- coffee
- gas gain
- gain fluctuations
- ion transport
- signals

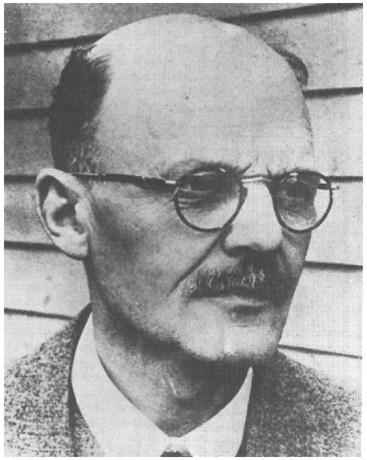
A brief history

E(a)rnest Rutherford
(1871-1937)

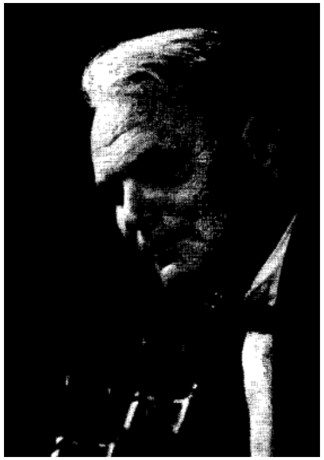


Geiger counter

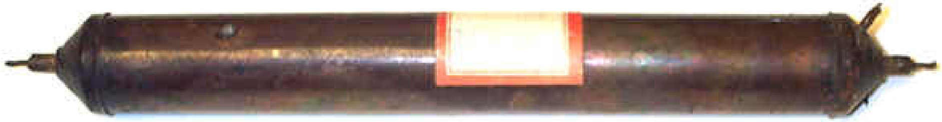
- Detects radiation by discharge;
- can count α, β and γ particles (at low rates ...);
- no tracking capability.
- 1908: Ernest Rutherford and Hans Geiger
- 1928: Hans Geiger and Walther Müller



Hans Geiger
(1882-1945)



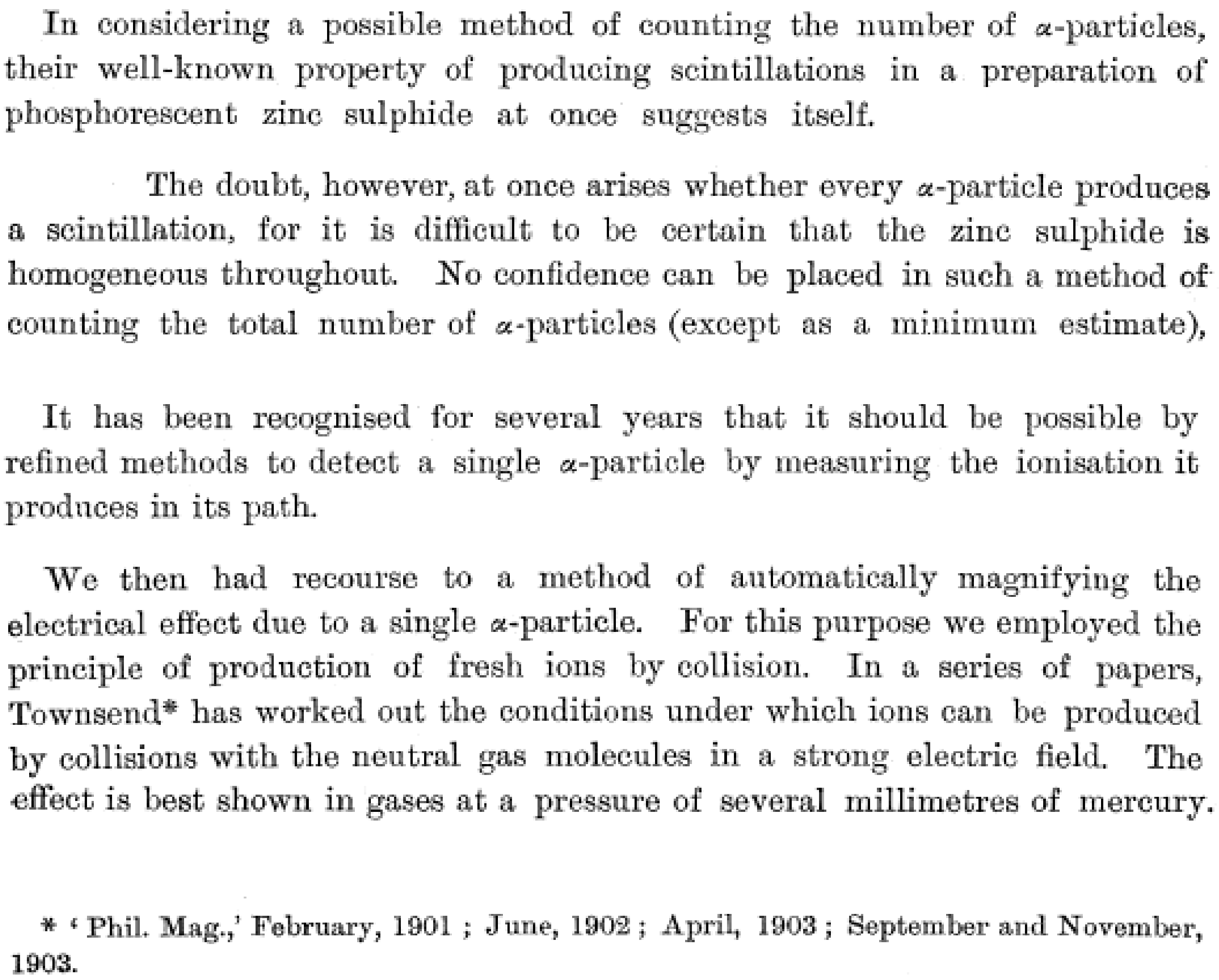
Walt(h)er Müller
(1905-1979)



A Geiger-Muller counter built in 1939 and used in the 1947-1950 for cosmic ray studies in balloons and on board B29 aircraft by Robert Millikan et al.

Made of copper, 30 cm long

Motivation for the Geiger counter



In considering a possible method of counting the number of α-particles, their well-known property of producing scintillations in a preparation of phosphorescent zinc sulphide at once suggests itself.

The doubt, however, at once arises whether every α-particle produces a scintillation, for it is difficult to be certain that the zinc sulphide is homogeneous throughout. No confidence can be placed in such a method of counting the total number of α-particles (except as a minimum estimate),

It has been recognised for several years that it should be possible by refined methods to detect a single α-particle by measuring the ionisation it produces in its path.

We then had recourse to a method of automatically magnifying the electrical effect due to a single α-particle. For this purpose we employed the principle of production of fresh ions by collision. In a series of papers, Townsend* has worked out the conditions under which ions can be produced by collisions with the neutral gas molecules in a strong electric field. The effect is best shown in gases at a pressure of several millimetres of mercury.

* 'Phil. Mag.,' February, 1901; June, 1902; April, 1903; September and November, 1903.

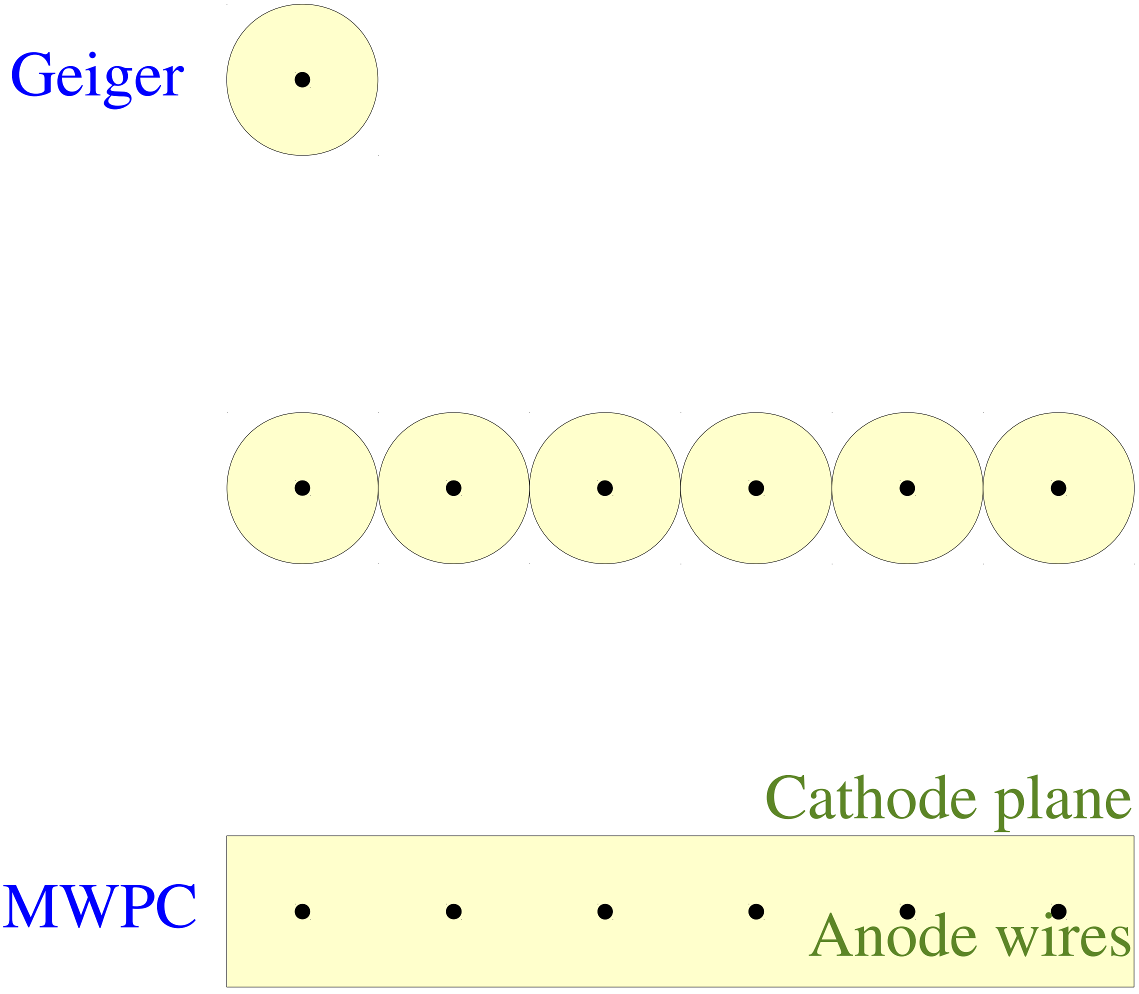
α detection by eye using ZnS scintillation suffers from efficiency losses + is laborious.

Ionisation signal is usable but small.

Use multiplication at low pressure as discovered in 1901 by JS Townsend.

[E. Rutherford and H. Geiger, *An Electrical Method of Counting the Number of α-Particles from Radio-Active Substances,* Proc. R. Soc. Lond. A **81** (1908) 141-161]

From Tube to TPC



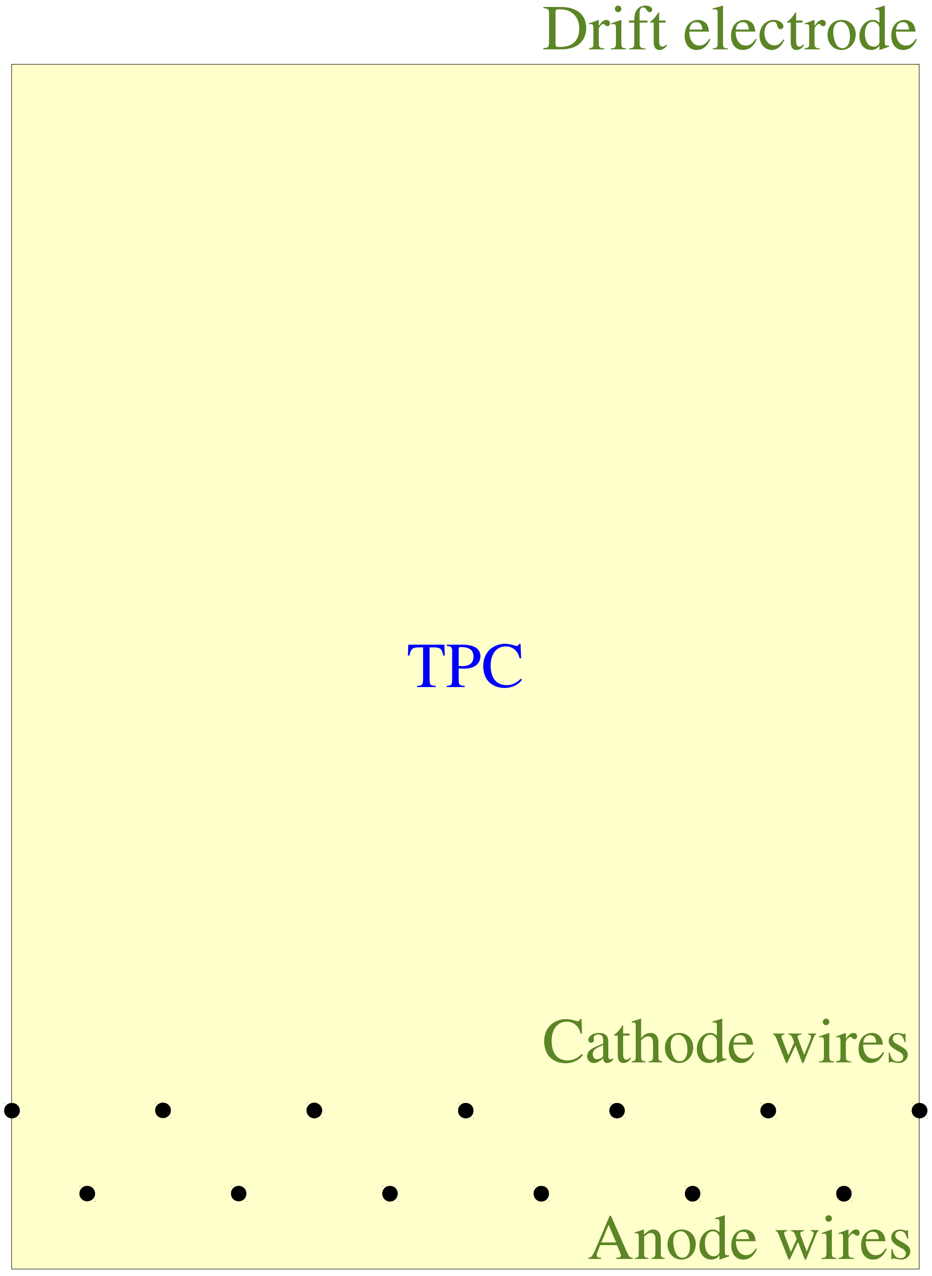
Geiger

Cathode plane

MWPC

Anode wires

Cathode plane



Drift electrode

TPC

Cathode wires

Anode wires

Read-out plane

MWPC

- First gaseous tracking device
- 1968: Georges Charpak

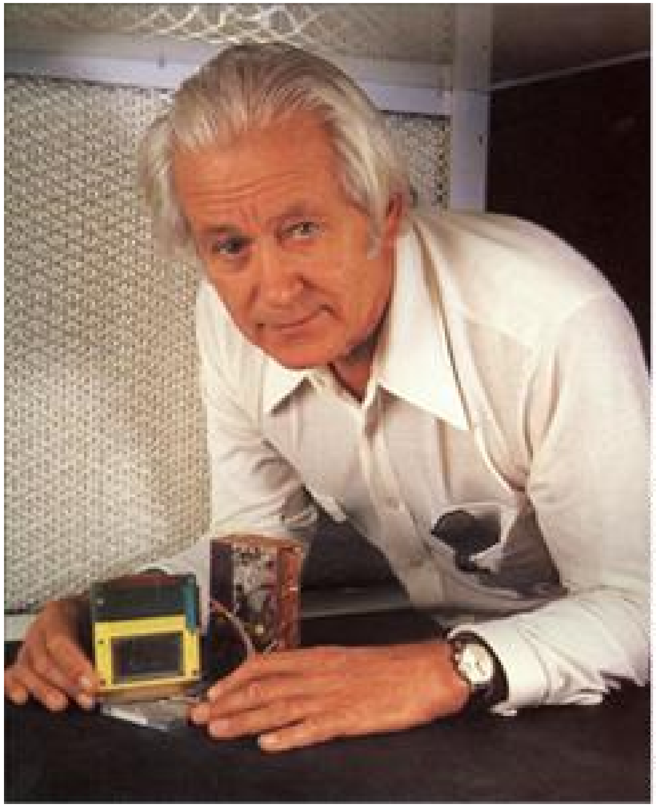
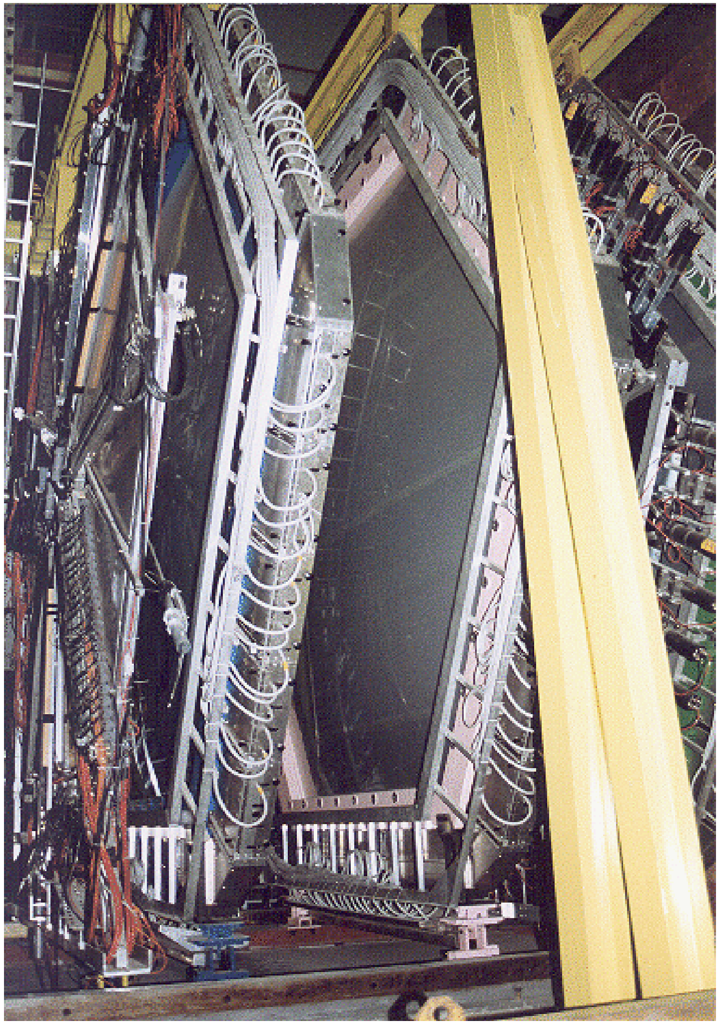


Photo: D. Parker, Science Photo Lab. UK

Georges Charpak
(1924-2010)



One of the NA60 muon chambers

TPC

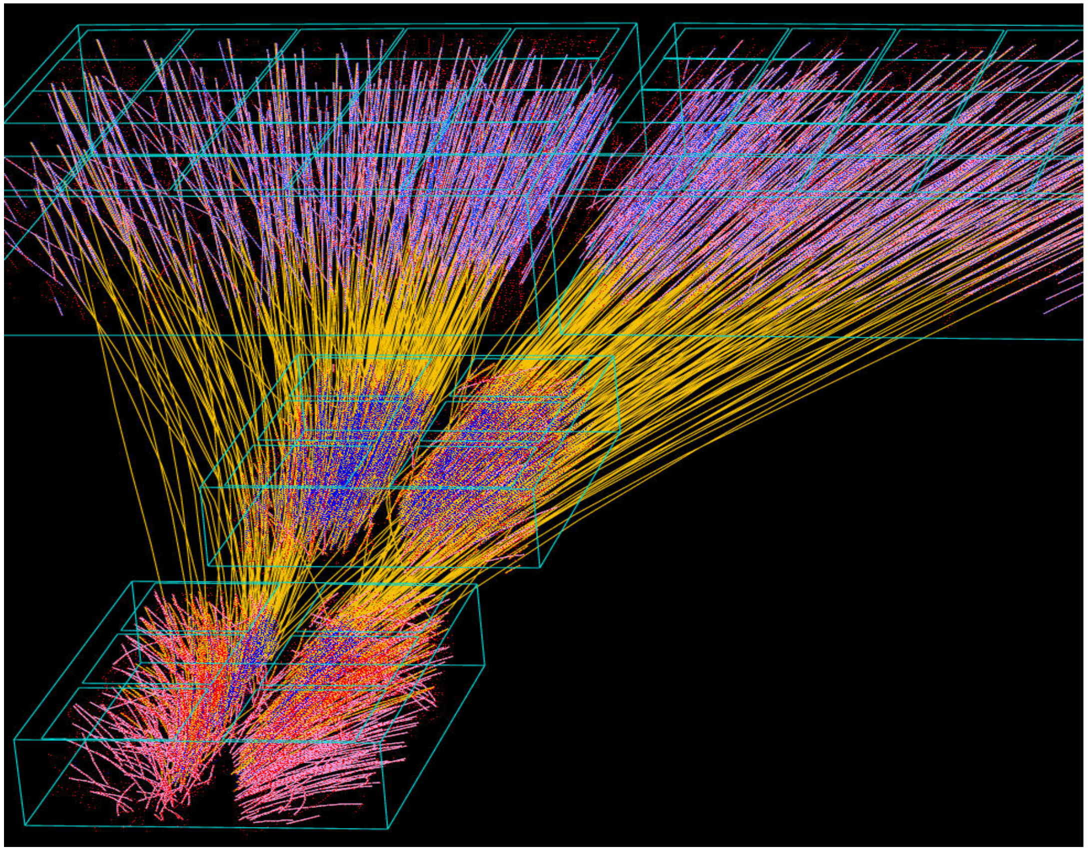
- Typically very large
- Almost empty inside
- Excellent for dealing with large numbers of tracks

- 1976: David Nygren (for PEP4)

Alice

Star

David Nygren

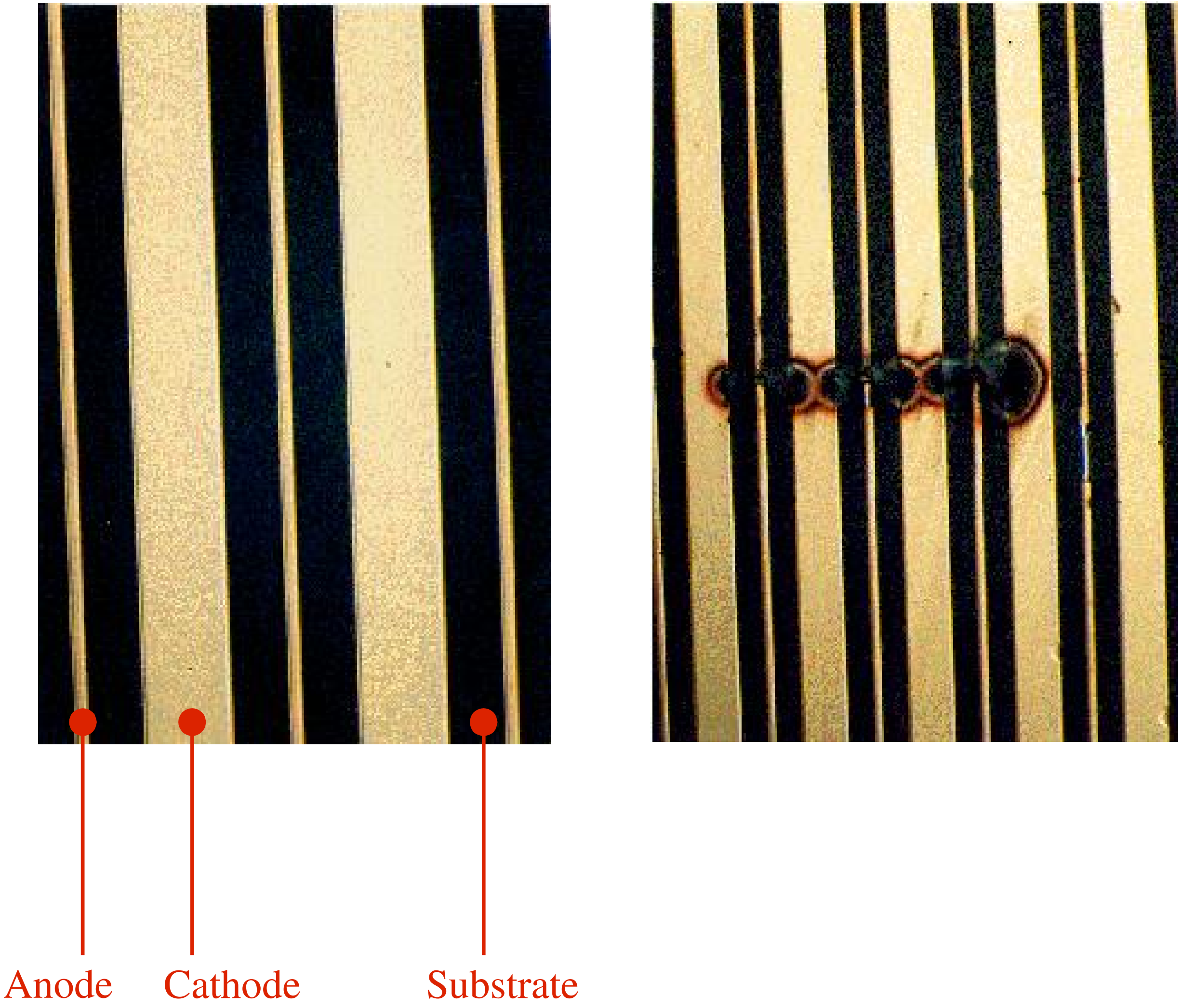


NA49

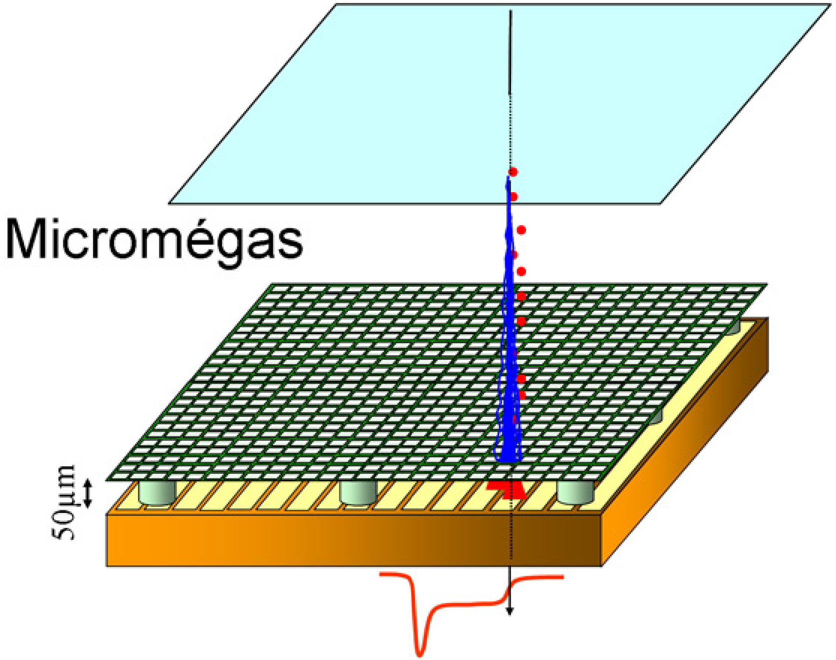
MSGC: an early MPGD

- Built using solid-state techniques;
- good resolution;
- poor resistance to high rates.

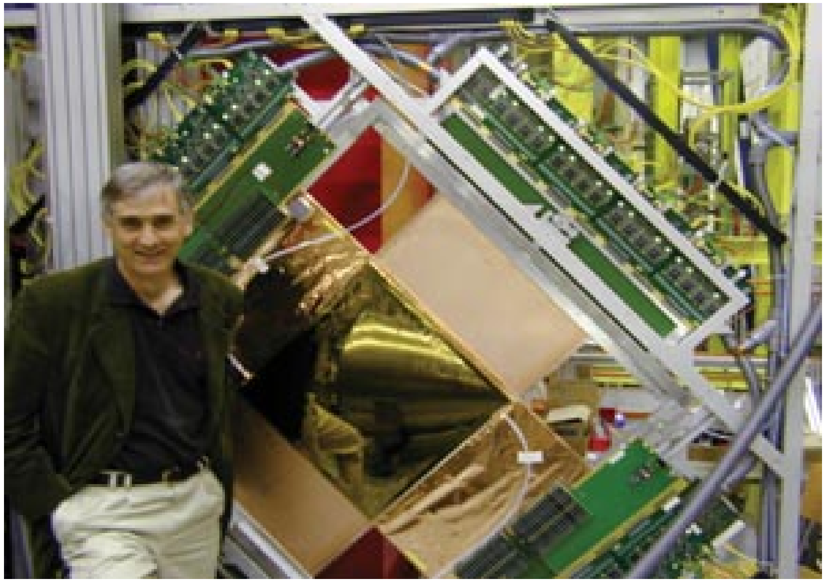
- 1988: Anton Oed

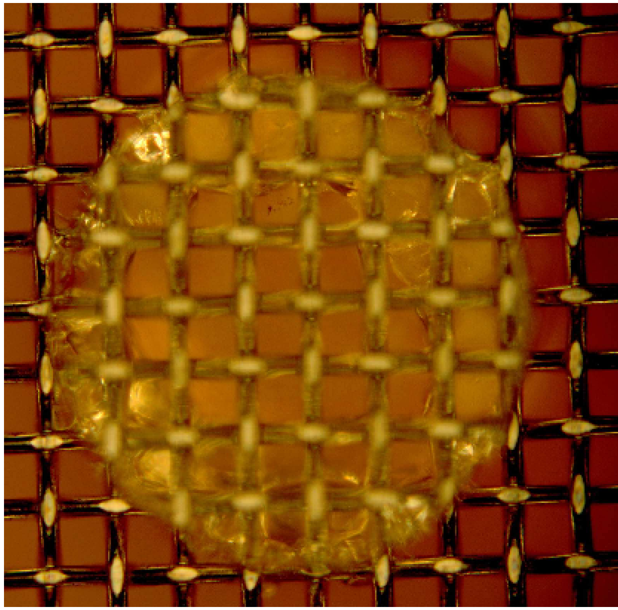


Micromégas



- Fast, rate tolerant tracking device
- 1994: Yannis Giomataris and Georges Charpak

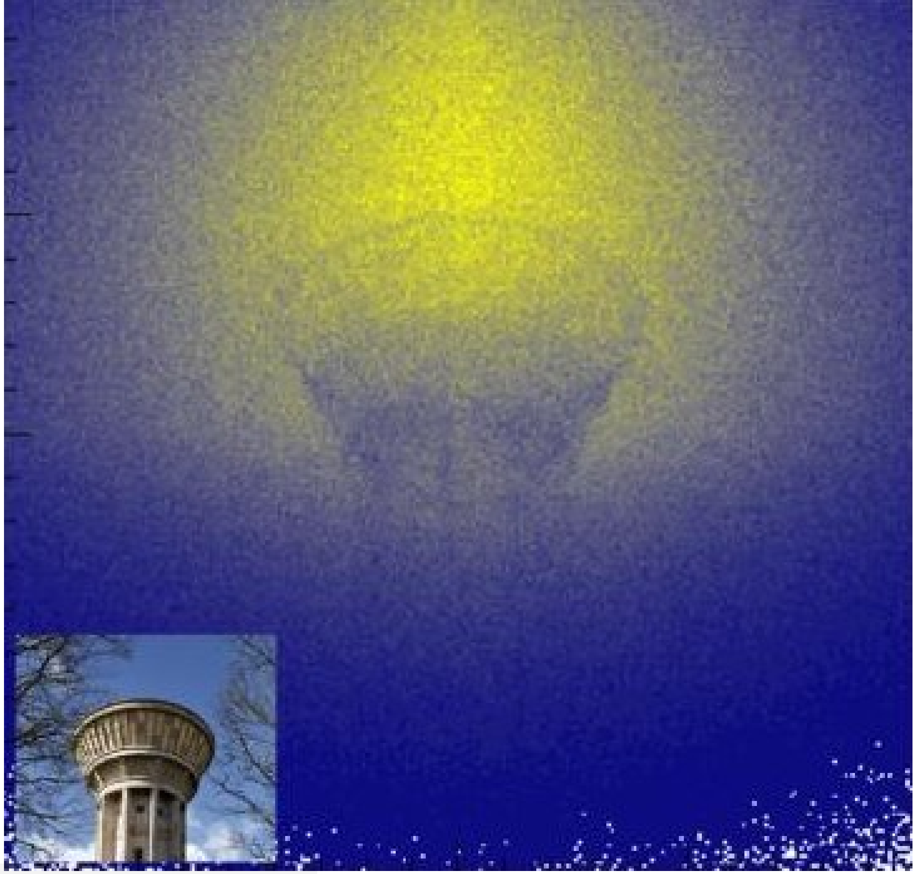

Yannis Giomataris

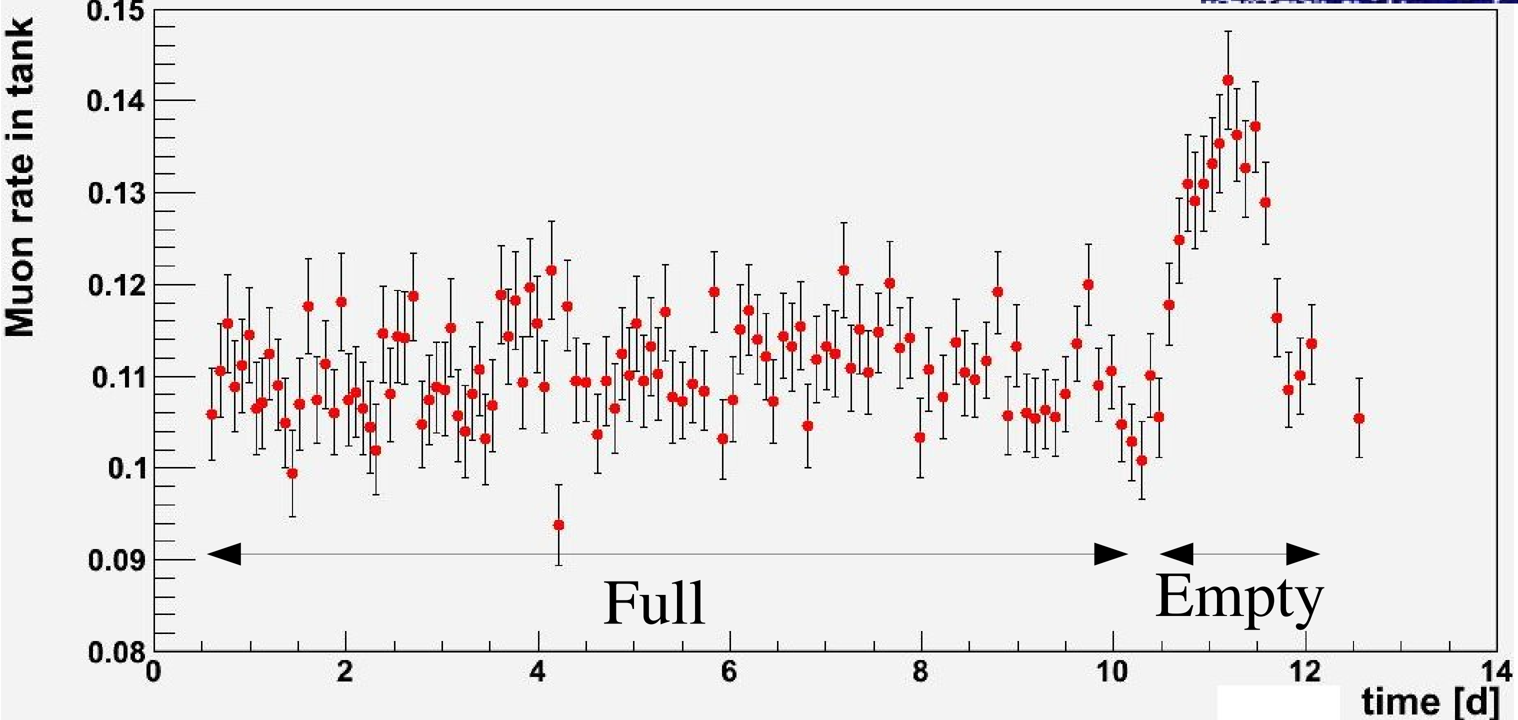

Wire diameter: 18 µm,
Pitch: 63 µm, Gap: 192 µm

[Purba Bhattacharya et al., 10.1016/j.nima.2013.07.086; ILC NewsLine]

Muon tomography

- Tower structure and water level are visible.
- Portable system (30 W).

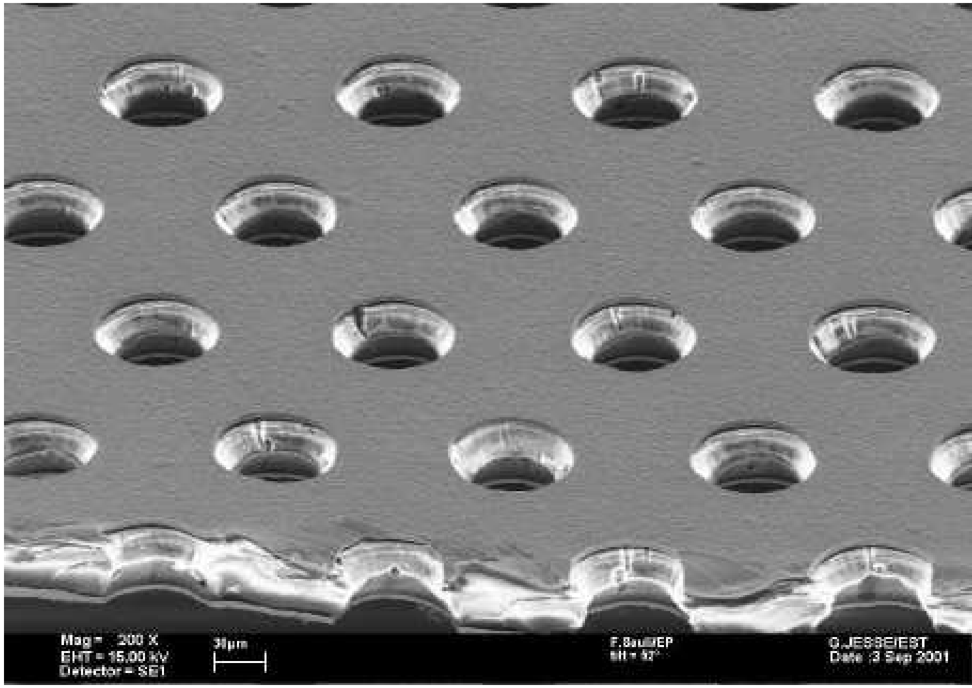




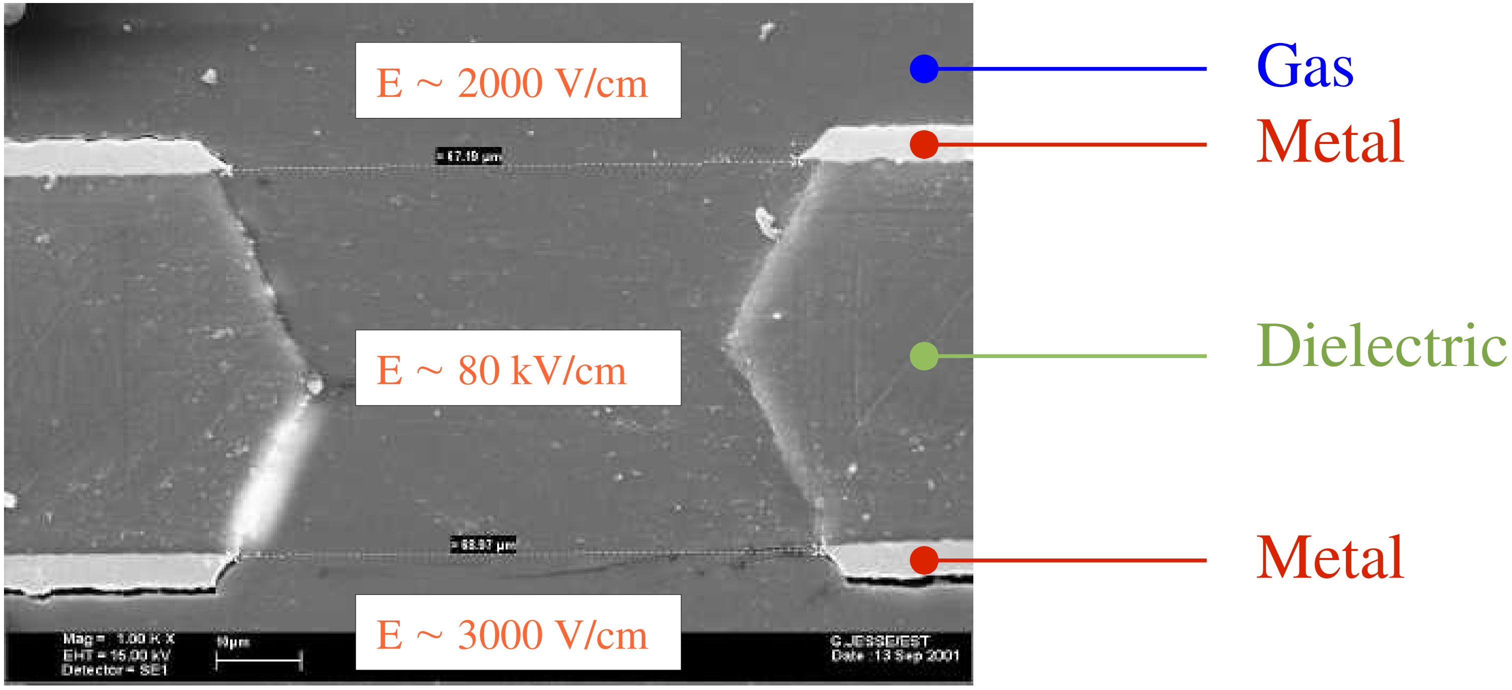
[Sébastien Procureur, Simon Bouteille, David Attié]

GEMs

- Acts as a “pre-amplifier”
- 1996: Fabio Sauli



A few electrons enter here



E ~ 2000 V/cm

E ~ 80 kV/cm

E ~ 3000 V/cm

Gas

Metal

Dielectric

Metal

Many electrons exit here

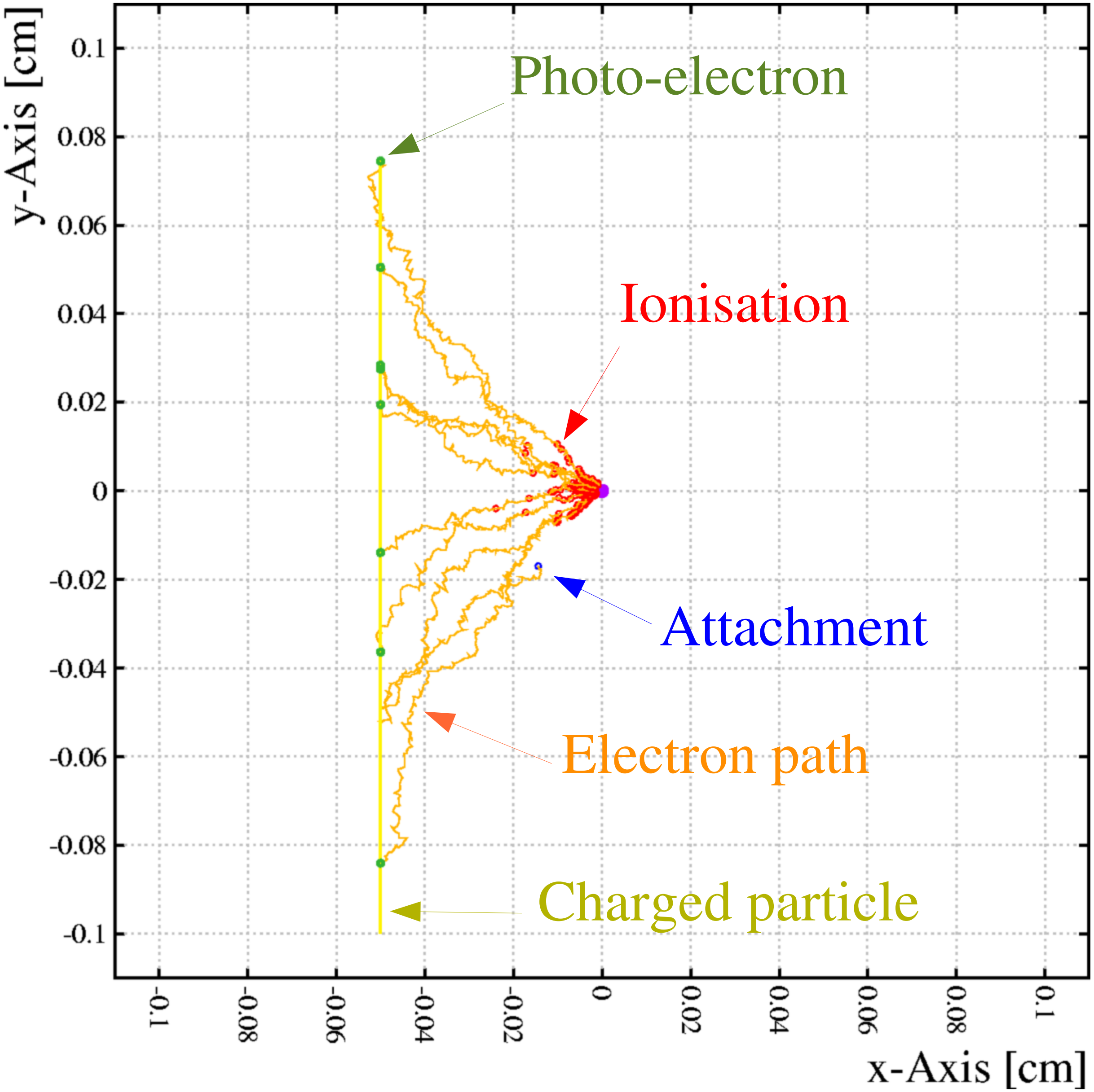
Fabio Sauli

How they work

- Perhaps surprisingly, they all work according to much the same principles:
  - a charged particle passing through the gas ionises a few gas molecules;
  - the electric field in the gas volume transports the ionisation electrons and provokes multiplication;
  - the movement of electrons and ions leads to induced currents in electrodes;
  - the signals are processed and recorded.

At the 100 μm – 1 mm scale

- Example:
  - CSC-like structure,
  - Ar 80 % CO_2 20 %,
  - 10 GeV μ.
- Electron are shown every 100 collisions, but have been tracked rigorously.
- Ions are not shown.



Ionisation

[Four Curies: Pierre, Marie, Irène and Pierre's father, around 1904 at the BIPM]

1896: Ionisation by radiation

- Early in the study of radioactivity, ionisation by radiation was recognised:

” Becquerel discovered in 1896 the special radiating properties of uranium and its compounds. Uranium emits very weak rays which leave an impression on photographic plates. These rays pass through black paper and metals; they make air electrically conductive. “

[Pierre Curie, Nobel Lecture, June 6th 1905]

“A sphere of charged uranium, which discharges spontaneously in the air under the influence of its own radiation, retains its charge in an absolute vacuum. The exchanges of electrical charges that take place between charged bodies under the influence of the new rays, are the result of a special conductivity imparted to the surrounding gases, a conductivity that persists for several moments after the radiation has ceased to act.”

[Antoine Henri Becquerel, Nobel Lecture, December 11th 1903]

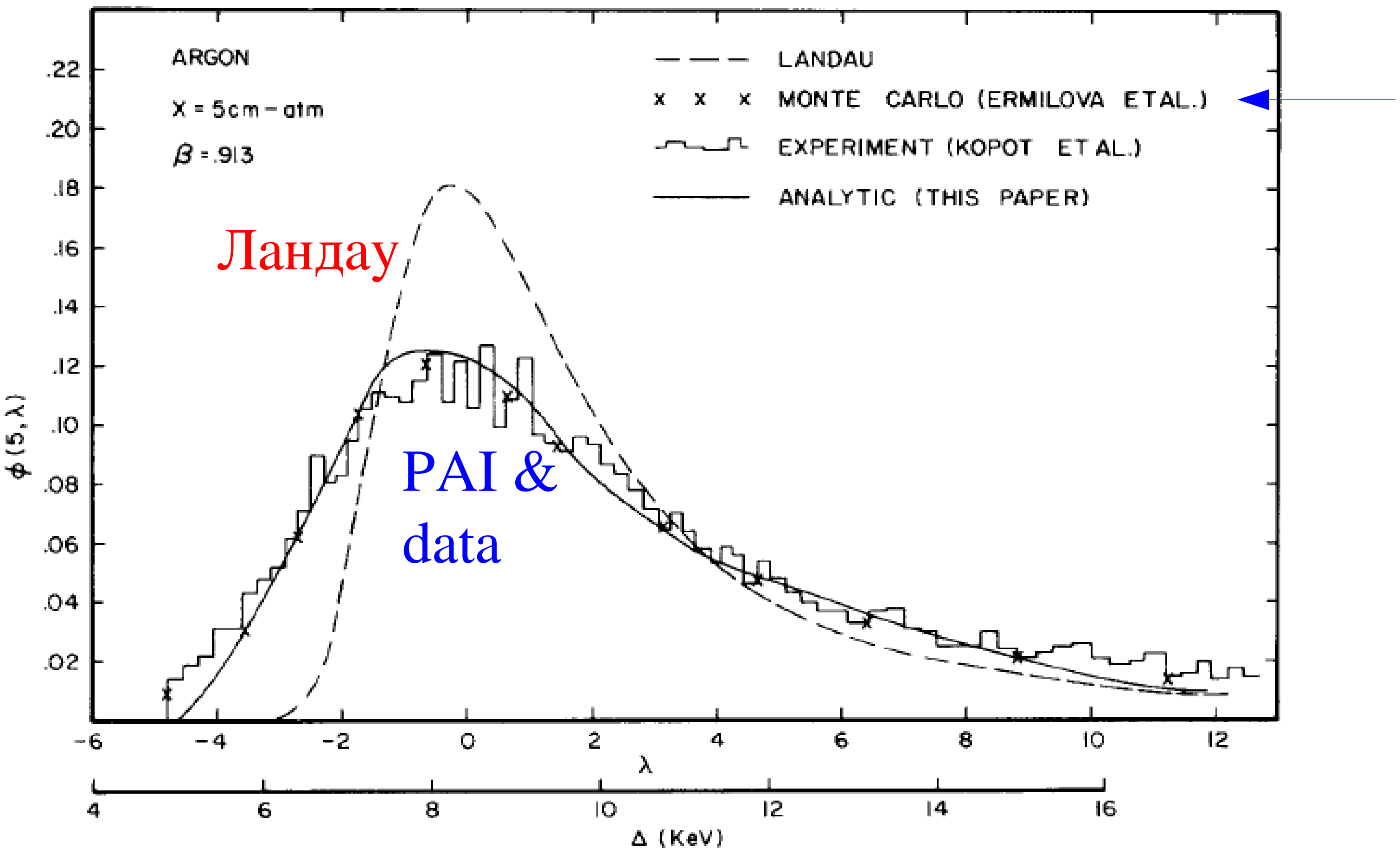
Ionisation electrons per cm in pure Ar

- Ionisation electrons deposited in Ar by a minimum ionising particle:
  - d*E*/d*x* / IP: 160 e^-/cm (from PDG, 2.5 keV/cm)
  - Heed: 41 e^-/cm (most probable)
  - Degrad: 50 e^-/cm (most probable)

- Apparently, ionising takes more than the binding energy:
  - not all energy is used (→ Fano factor),
  - some energy goes into excitations (→ Work function),
  - or there may be errors in the d*E*/d*x* tables ?

Energy loss fluctuations

- 2 GeV protons on an (only !) 5 cm thick Ar gas layer:

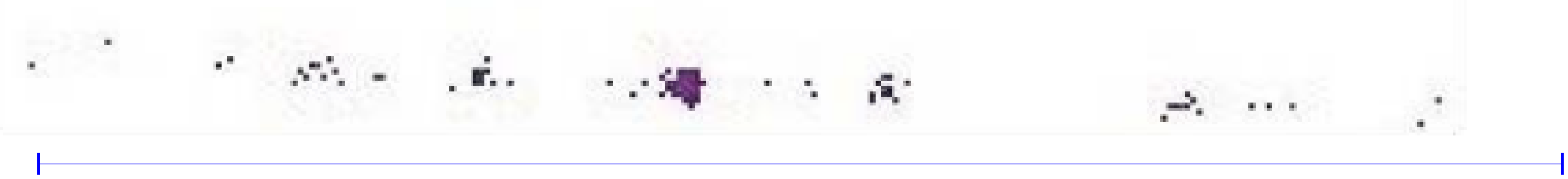


Ландау

PAI & data

[Diagram: Richard Talman, NIM A **159** (1979) 189-211]

Clustering – primary interactions

- Electrons are not evenly spaced, not even exponentially:
  - 

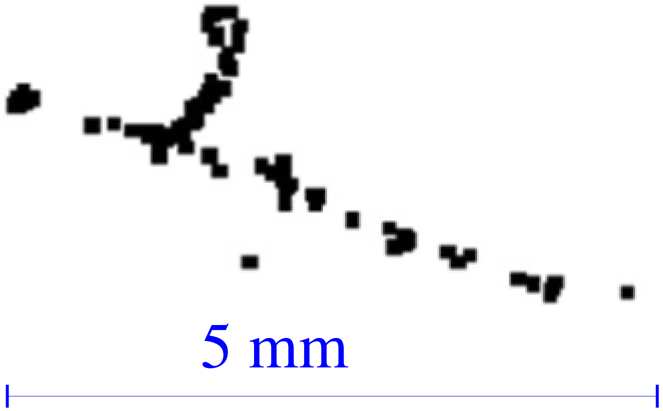
  (reported to have 25-30 clusters/cm, hard to see)

- From ILD TPC test beam studies:
  - 4 × 2 InGrid Octopuce,
  - pixels: 55 × 55 μm^2,
  - T2K gas: Ar 95 %, CF_4 3 %, iC_4H_{10} 2 %;
  - DESY II, 5 GeV e^-.

[Michael Lupberger, AIDA-PUB-2014-010,
Robert Menzen, AIDA-THESIS-2013-001]

δ-electrons

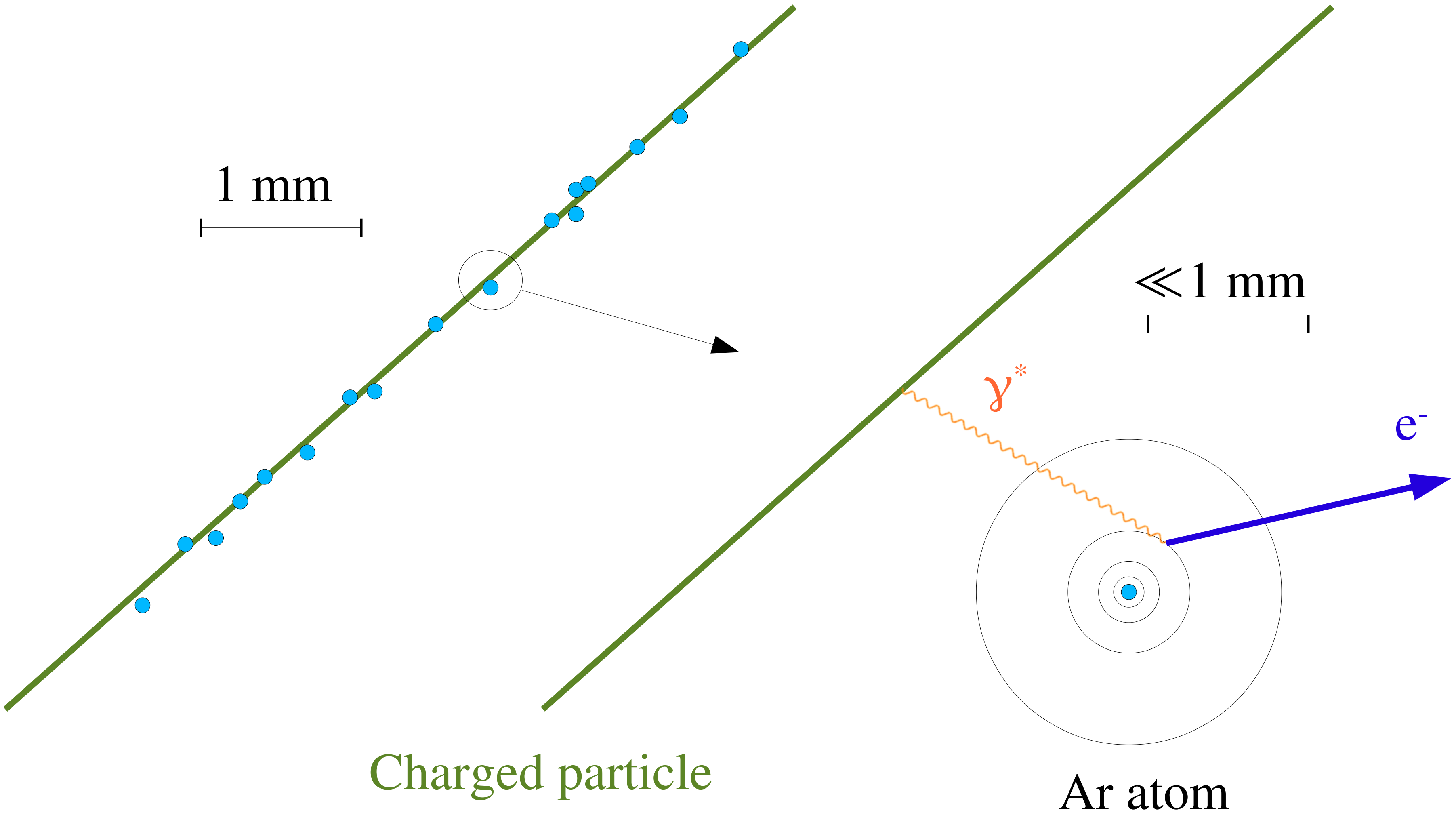
- Deposits are not always “lumps”:



- Laboratory tests:
  - modified MediPix;
  - pixels: 55 × 55 μm^2;
  - He 80 %, iC_4H_{10} 20 %.

[Harry van der Graaf and co-workers (2004)]

Virtual photon exchange



Basic formulae of the PAI model

- Key ingredient: photo-absorption cross section $\sigma_\gamma(E)$

$$\frac{\beta^2\pi}{\alpha}\frac{\mathrm{d}\sigma}{\mathrm{d}E}=\frac{\sigma_\gamma(E)}{E}\log\left(\frac{1}{\sqrt{(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2}}\right)+$$

Relativistic rise

Cross section to
transfer energy E

$$\frac{1}{N\hbar c}\left(\beta^2-\frac{\epsilon_1}{|\epsilon|^2}\right)\theta+$$

Черенков radiation

$$\frac{\sigma_\gamma(E)}{E}\log\left(\frac{2m_e c^2\beta^2}{E}\right)+$$

Resonance region

$$\frac{1}{E^2}\int_0^E\sigma_\gamma(E_1)\,dE_1$$

Rutherford scattering

With:

$$\epsilon_2(E)=\frac{N_e\hbar c}{EZ}\sigma_\gamma(E)$$

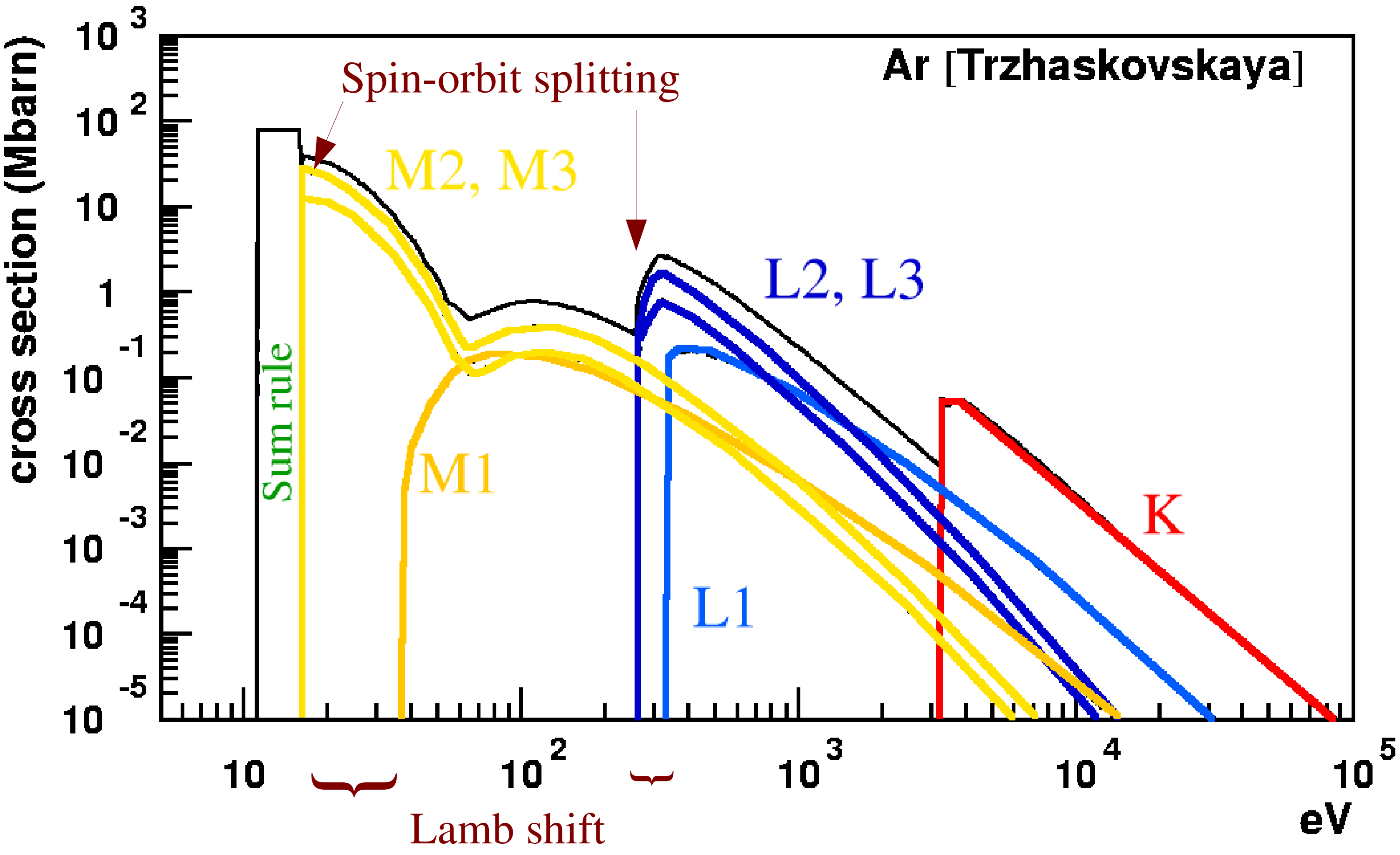
$$\epsilon_1(E)=1+\frac{2}{\pi}\mathrm{P}\int_0^\infty\frac{x\,\epsilon_2(x)}{x^2-E^2}\,\mathrm{d}x$$

$$\theta=\arg(1-\epsilon_1\beta^2+i\,\epsilon_2\beta^2)=\frac{\pi}{2}-\arctan\frac{1-\epsilon_1\beta^2}{\epsilon_2\beta^2}$$

Photo-absorption in Ar (Heed)

Igor Smirnov

- Argon has 3 shells, hence 3 groups of lines:



K = 1s

L1 = 2s
L2 = 2p 1/2
L3 = 2p 3/2

M1 = 3s
M2 = 3p 1/2
M3 = 3p 3/2

[Plot from Igor Smirnov]

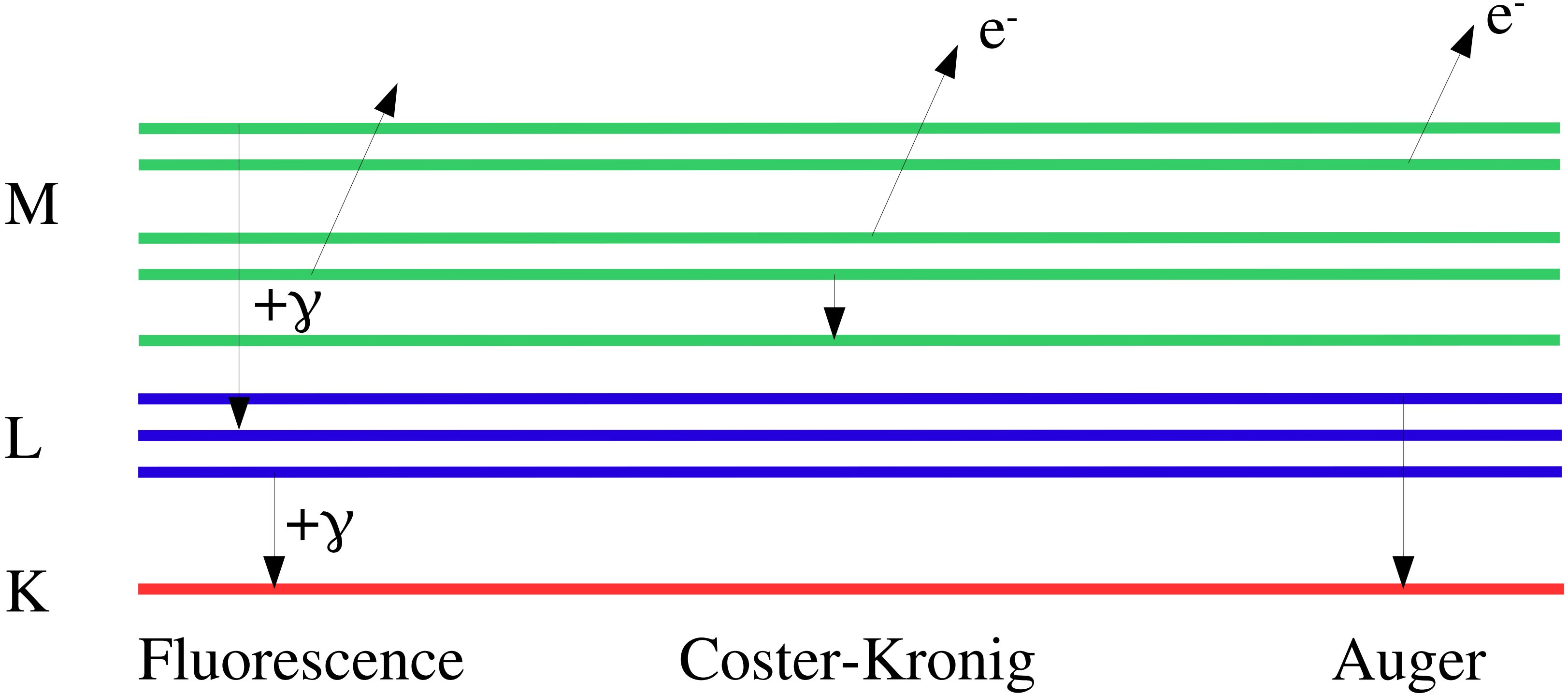
De-excitation

Ralph de Laer Kronig
(1904-1995)

Dirk Coster
(1889-1950)

Lise Meitner
(1878-1968)

Pierre Victor Auger
(1899-1993)



References:

D. Coster and R. de L. Kronig, Physica **2** (1935) 13-24.

Lise Meitner, *Über die β-Strahl-Spektra und ihren Zusammenhang mit der γ-Strahlung*, Z. Phys. **11** (1922) 35-54.

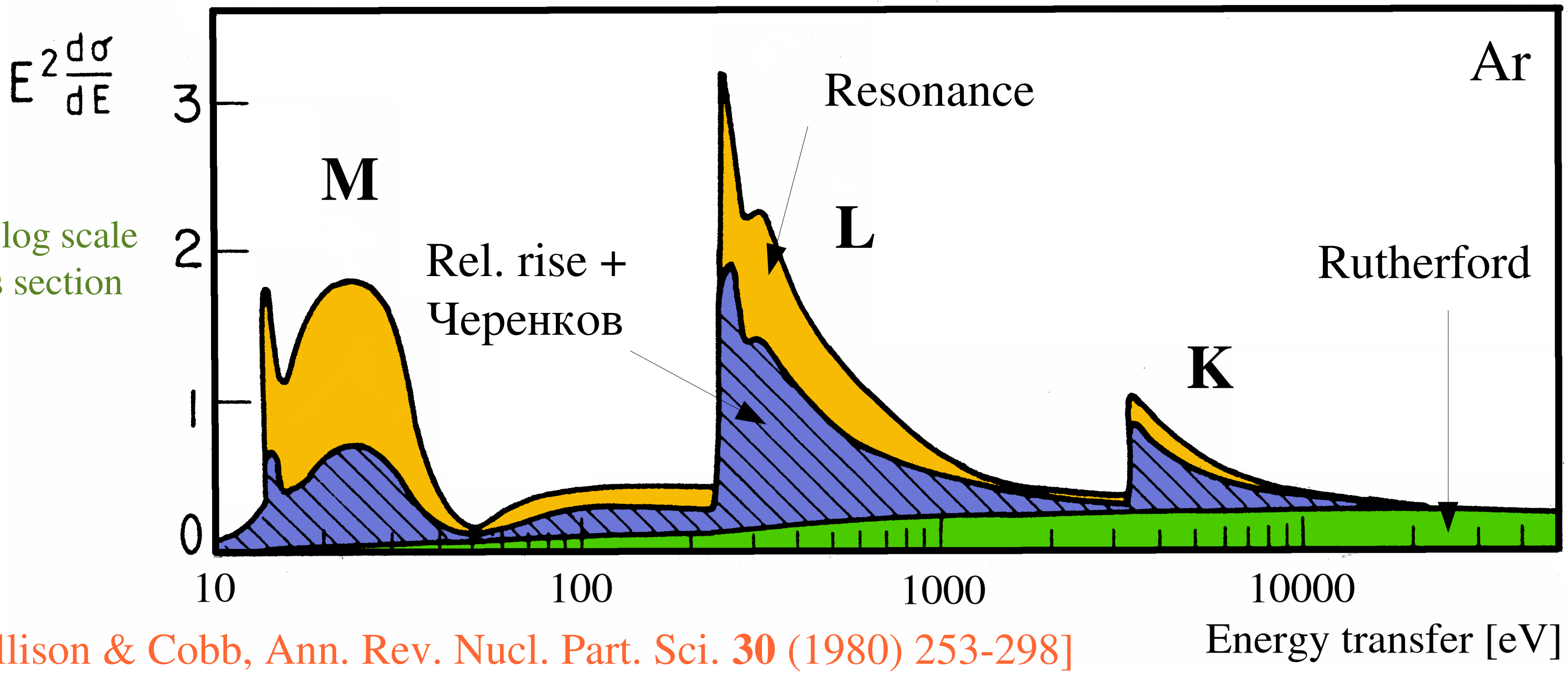
L. Meitner, *Das β-Strahlenspektrum von* UX_1 *und seine Deutung*, Z. Phys. **17** (1923) 54-66.

P. Auger, Comptes rendus hebdomadaires des séances de l'Académie des sciences, 1923/07 (T177)-1923/12, 169-171.

P. Auger, J. Phys. Radium **6** (1925) 205.

Importance of the PAI model terms

- All electron orbitals (shells) participate:
  - outer shells: frequent interactions, few electrons;
  - inner shells: few interactions, many electrons.
- All terms in the formula are important.



Scaling with E^2:
equal areas on log scale
weighing cross section

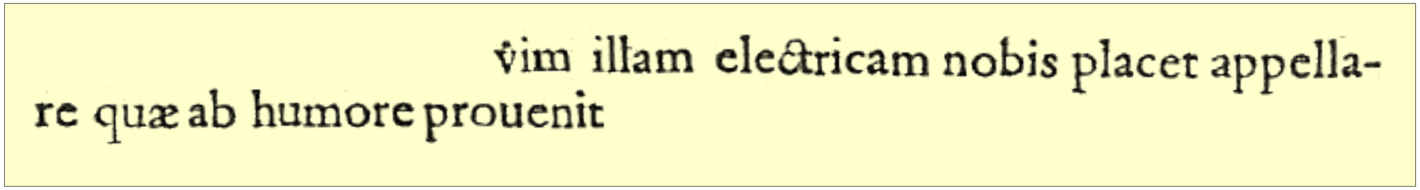
[Adapted from Allison & Cobb, Ann. Rev. Nucl. Part. Sci. **30** (1980) 253-298]

Electric fields & Electron transport

William Gilbert
(1544-1603)

1600: “Electric force”

- 1544: William Gilbert born in Colchester
- 1600: De magnete, magneticisque corporibus,
  et de magno magnete tellure.
- Concluded that the Earth is a magnet and credited with the first use of the term “electric force”:



vim illam electricam nobis placet appellare quæ ab humore prouenit

- 1601: Physician to Elizabeth I and James I.

[Guilielmi Gilberti, *De magnete* ..., excudebat Petrus Short anno MDC, Londini, courtesy Universidad Complutense de Madrid and Google books]

Mean free path in argon

- Literature will tell you
  - e^- cross section Ar atom: $\sigma \approx 1.5\ 10^{-16}$ cm^2
  - atoms per unit volume: $n_0 \approx 2.7\ 10^{19}$ atoms/cm^3

- Mean free path for an electron ?
  - An electron hits all atoms of which the centre is less than a cross section σ radius from its path;
  - over a distance L, the electron hits $n_0\ \sigma\ L$ atoms;
  - mean free path = distance over which it hits 1 atom;

$$\lambda_e = 1/(\sigma\ n_0) \approx 2.5\ \mu\text{m}$$

  - much larger than

4 nm	distance between atoms, and
140-600 pm	typical gas molecule diameters.

MPGDs and the mean free path

- Recall:
  - Mean free path of electrons in Ar: 2.5 µm,

- Compare with:
  - Micromegas mesh pitch: 63.5 µm
  - GEM polyimide thickness: 50 µm
  - Micromegas wire thickness: 18 µm
  - GEM conductor thickness: 5 µm

- Hence:
  - mean free path approaches small structural elements;
  - such devices should be treated at a molecular level.

Velocity in electric fields

- Imagine that an electron stops every time it collides with a gas molecule and then continues along *E*.

- To cover a distance λ_e, it will need a time *t*:

$$\frac{1}{2}\frac{qE}{m_e}t^2 = \lambda_e, \quad t = \sqrt{\frac{2\lambda_e m_e}{qE}}, \quad \bar{v} = \frac{\lambda_e}{t} = \sqrt{\frac{\lambda_e qE}{2m_e}}$$

- For example:

$$\bar{v} \approx 13\,\mathrm{cm}/\mu\mathrm{s} \ \text{ for } \ E = 1\,\mathrm{kV/cm}$$

Drift velocity in argon

- Compare with a Magboltz calculation for pure argon:
  - *E* dependence is not too far off (although linear is more common at low fields),

    BUT

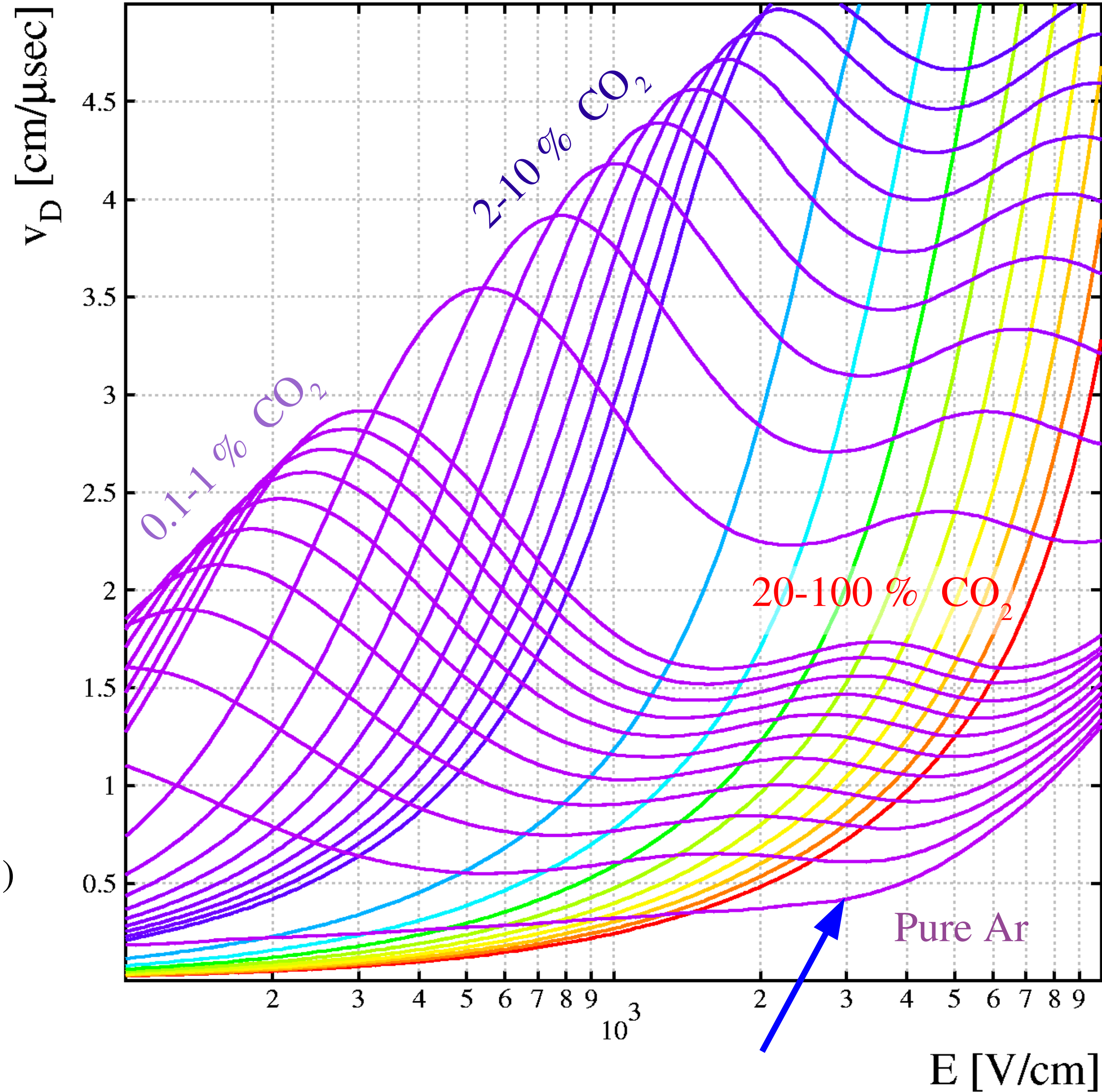
  - the velocity is *vastly* overestimated ! Magboltz finds a velocity that is *30 times* smaller ...

    WHY ?

Adding CO_2

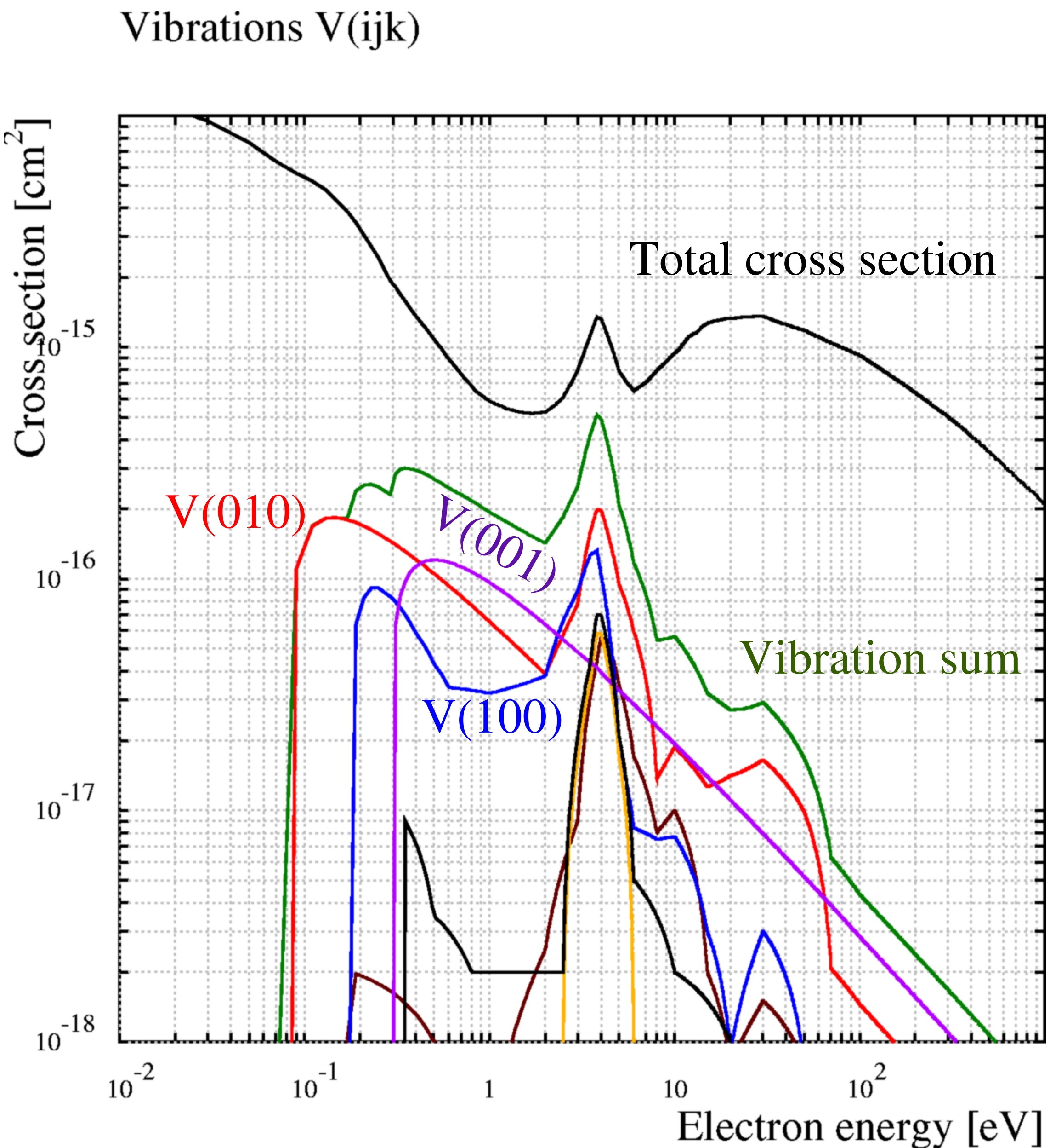
- CO_2 makes the gas faster, dramatically.
- Drift velocities calculated by Magboltz for Ar/CO_2 at 3 bar.

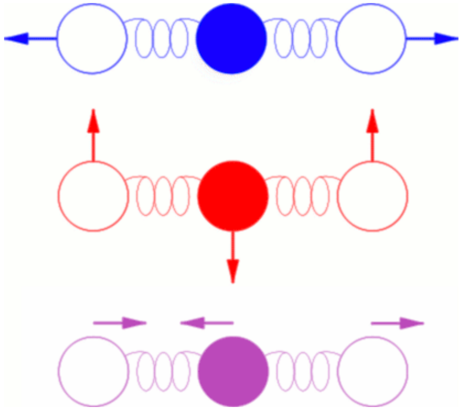
(Note where the arrow is !)



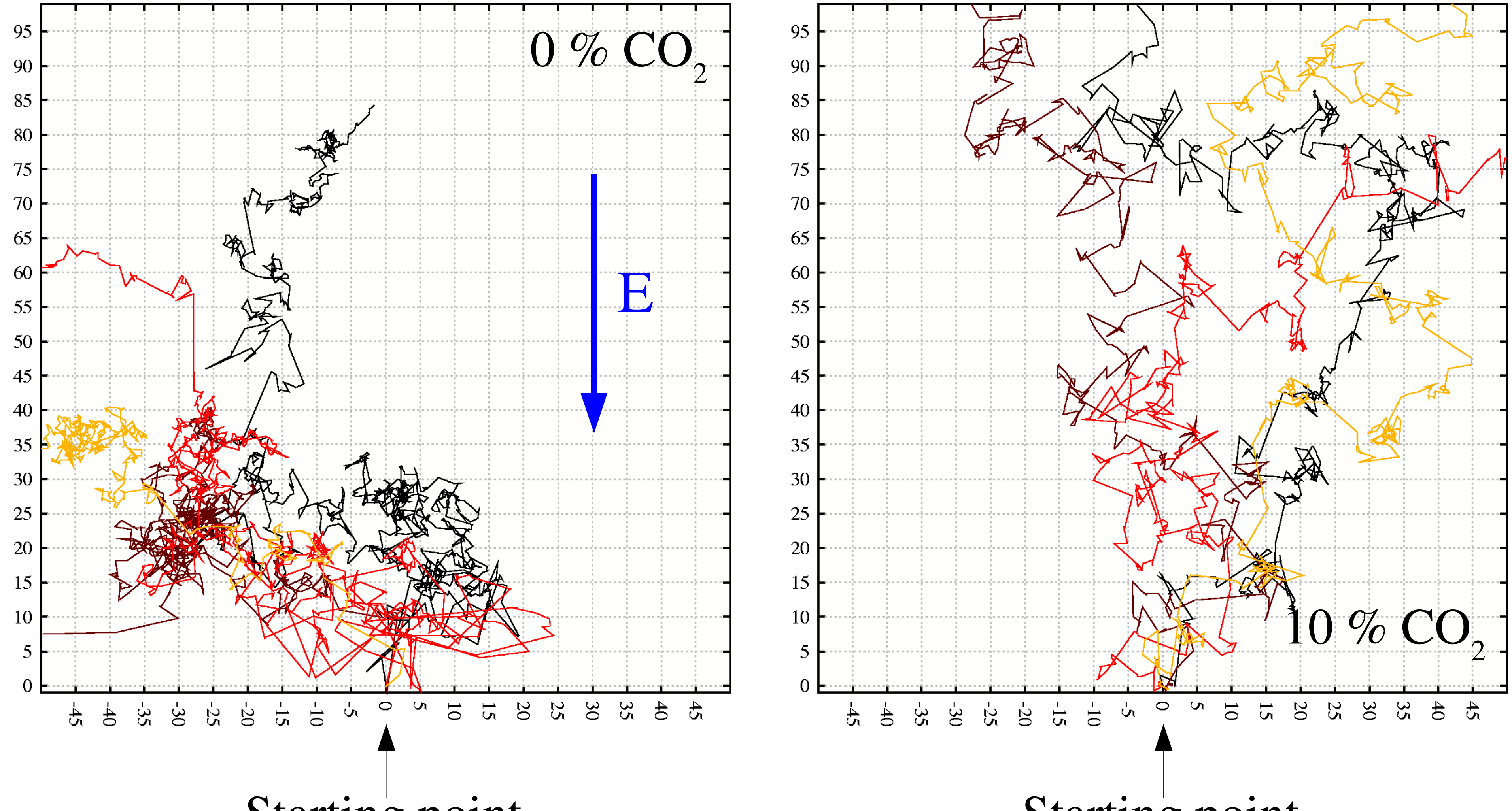
CO$_2$ – vibration modes

- CO$_2$ is linear:
  - O – C – O
- Vibration modes are numbered V(*ijk*)
  - *i*: symmetric,
  - *j*: bending,
  - *k*: anti-symmetric.





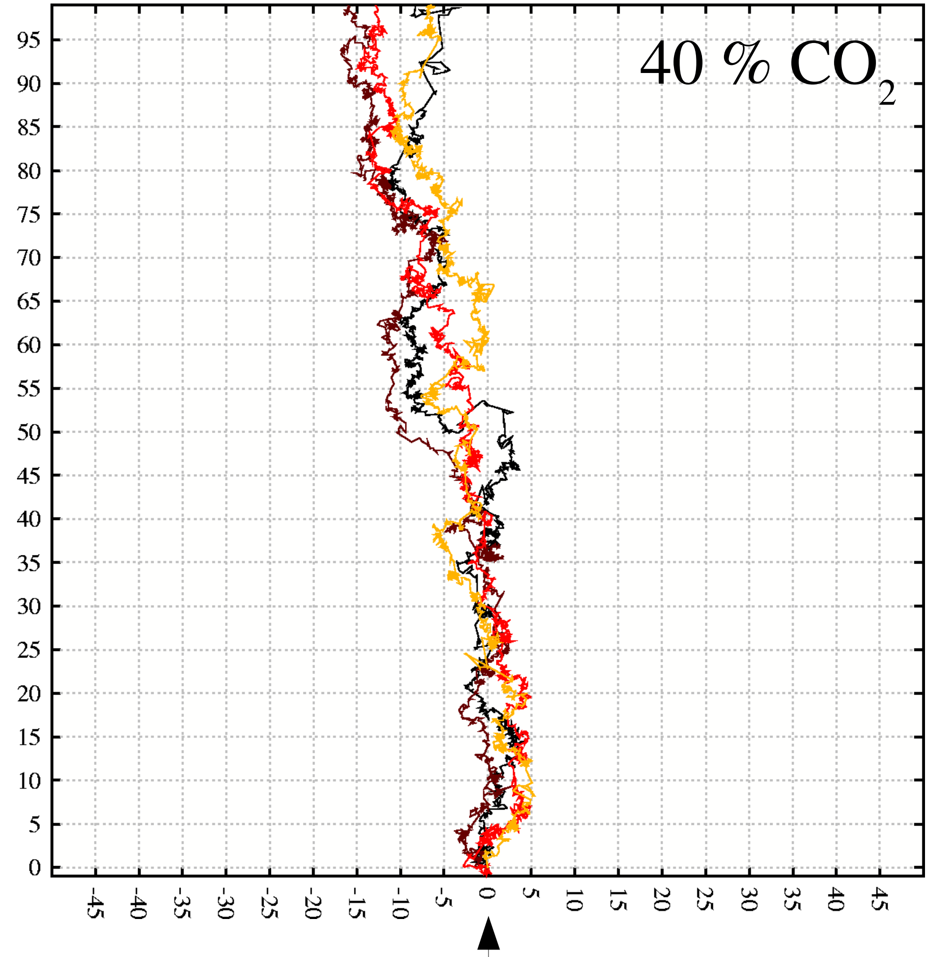
Electrons in Ar/CO_2 at E=1 kV/cm



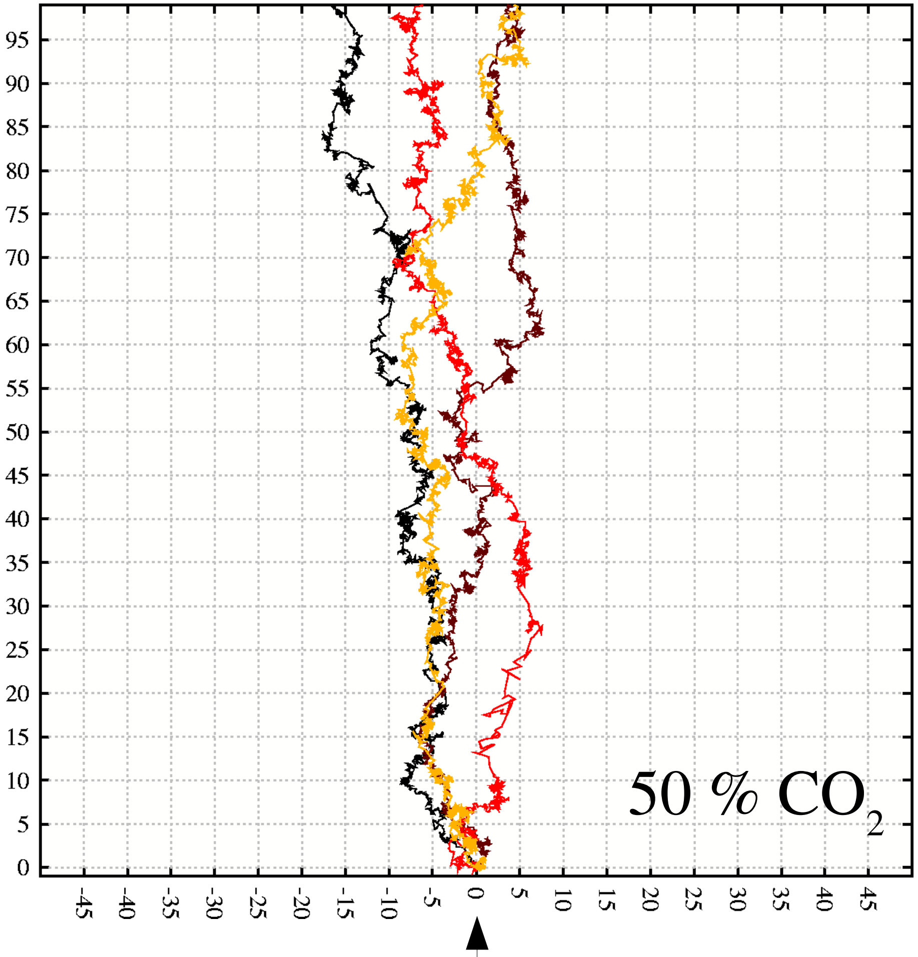
Starting point

Starting point

Electrons in Ar/CO_2 at E=1 kV/cm



Starting point



Starting point

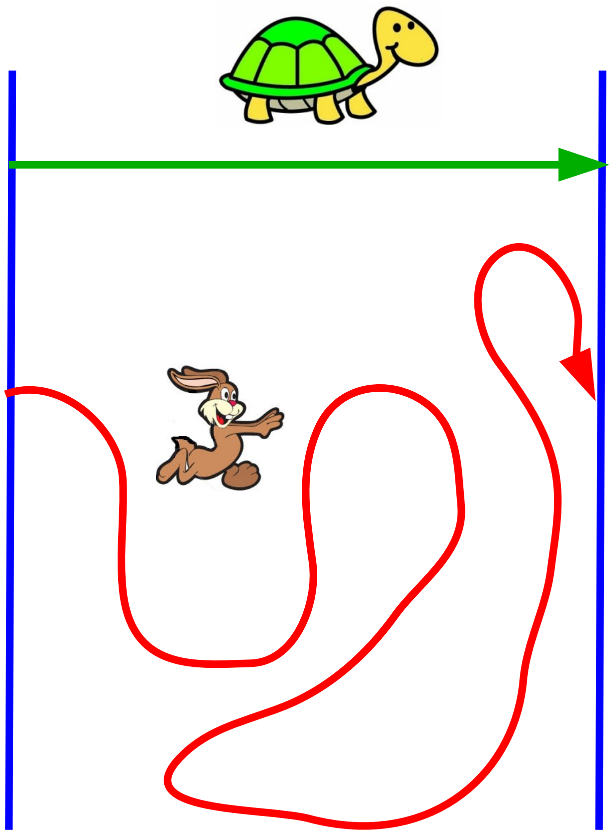
Drift velocity vs Electron velocity

- Drift velocity v_D: distance effectively travelled ÷ time needed.

- Compare rabbit and turtle:

$$v_\mathrm{D} = \bar{v}$$

$$v_\mathrm{D} \ll \bar{v}$$



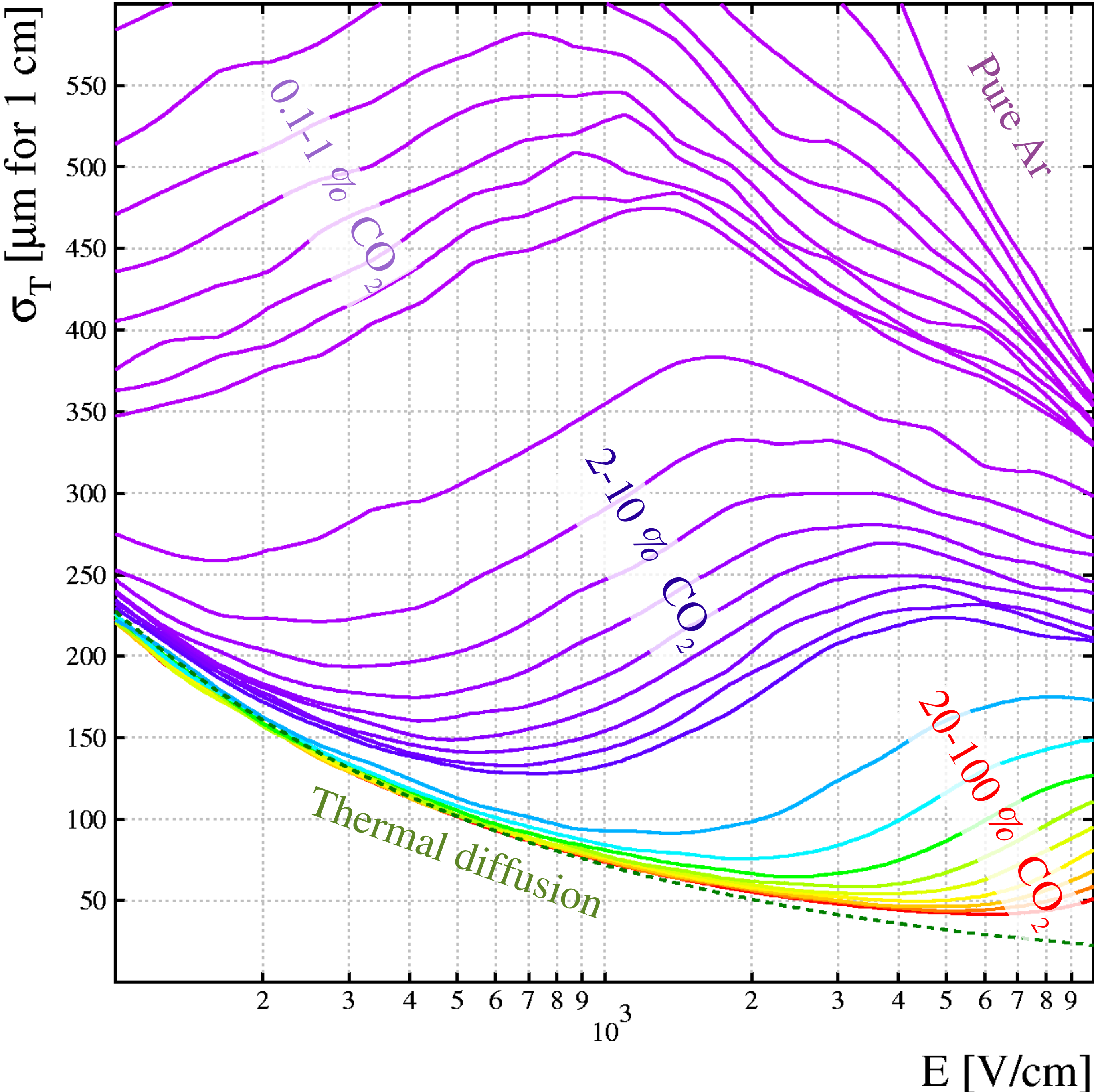
Diffusion

- The combination of a high velocity and low drift velocity implies that the electrons scatter a lot.
- Diffusion = RMS of the difference between the actual and the average movement

In a homogeneous field, if the diffusion over 1 cm of drift is 100 μm,
how large is the diffusion over 1 m ?

Adding CO_2

- Transverse diffusion is much reduced by CO_2.
- Calculated by Magboltz for Ar/CO_2 at 3 bar.

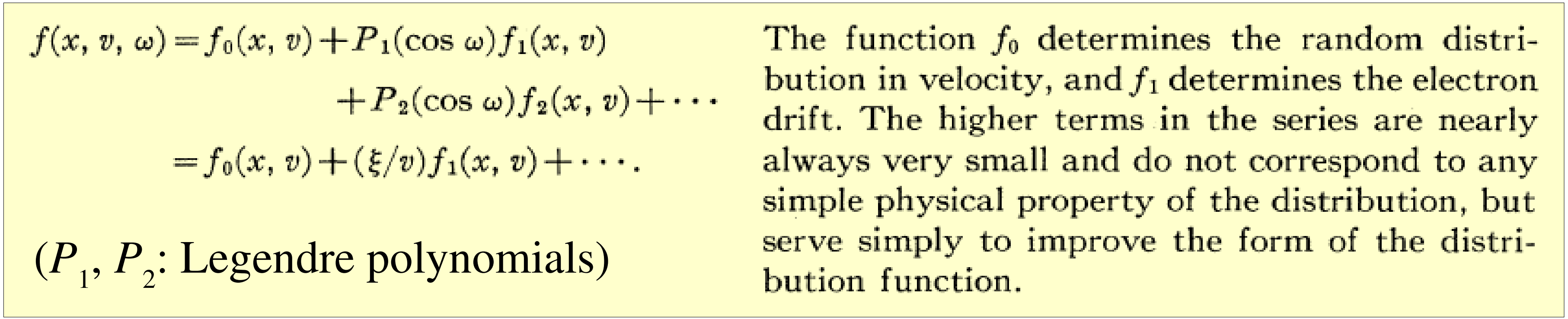


Calculating transport properties

- One can of course measure every mixture one needs …
- … but it would be far more efficient if one could compute the transport properties of arbitrary mixtures.

1935: Electron energy distribution

- Calculation of the electron energy distribution
  - allowing for energy loss in elastic collisions;
  - detailed balancing of energy and momentum gain (E-field, diffusion) and loss (elastic collision);
  - velocity dependent cross section;
  - use of Legendre expansion (crediting H.A. Lorentz, 1916):



$$\begin{aligned} f(x,v,\omega) &= f_0(x,v)+P_1(\cos\omega)f_1(x,v) \\ &\quad + P_2(\cos\omega)f_2(x,v)+\cdots \\ &= f_0(x,v)+(\xi/v)f_1(x,v)+\cdots. \end{aligned}$$

(P_1, P_2: Legendre polynomials)

The function f_0 determines the random distribution in velocity, and f_1 determines the electron drift. The higher terms in the series are nearly always very small and do not correspond to any simple physical property of the distribution, but serve simply to improve the form of the distribution function.

[Philip M. Morse, W.P. Allis and E.S. Lamar, *Velocity Distributions for Elastically Colliding Electrons*, Phys. Rev. **48** (1935) 412–419]

Arthur V. Phelps
(1923 - 2012)

1962: Numerical e$^-$ transport

- Iterative approach, allowing for inelastic cross section terms:
  - educated guess of cross sections (elastic & inelastic);
  - numerically solve the Boltzmann equation (no moments);
  - compare calculated and measured mobility and diffusion;
  - adjust cross sections.

"... more than 50,000 transistors plus extremely fast magnetic core storage. The new system can simultaneously read and write electronically at the rate of 3,000,000 bits of information a second, when eight data channels are in use. In 2.18 millionths of a second, it can locate and make ready for use any of 32,768 data or instruction numbers (each of 10 digits) in the magnetic core storage. The 7090 can perform any of the following operations in one second: 229,000 additions or subtractions, 39,500 multiplications, or 32,700 divisions. " (IBM 7090 documentation)

[L.S. Frost and A.V. Phelps, *Rotational Excitation and Momentum Transfer Cross Sections for Electrons in* H_2 *and* N_2 *from Transport Coefficients*, Phys. Rev. **127** (1962) 1621–1633.]

Magboltz: microscopic e^- transport

- A large number of cross sections for 60 molecules...
  - Numerous organic gases, additives, *e.g.* CO_2:
    - elastic scattering,
    - 44 inelastic cross sections (5 vibrations and 30 rotations + super-elastic and 9 polyads),
    - attachment,
    - 6 excited states and
    - 3 ionisations.
  - noble gases (He, Ne, Ar, Kr, Xe):
    - elastic scattering,
    - 44 excited states and
    - 7 ionisations.

LXcat

- LXcat (pronounced *elecscat*) is an open-access website for collecting, displaying, and downloading ELECtron SCATtering cross sections and swarm parameters (mobility, diffusion coefficient, reaction rates, etc.) required for modeling low temperature plasmas. [...]”

[http://www.lxcat.laplace.univ-tlse.fr/]

LXcat people

- Art Phelps,
- Leanne Pitchford – Toulouse,
- Klaus Bartschat – Iowa,
- Oleg Zatsarinny – Iowa,
- Michael Allan – Fribourg,
- Steve Biagi
- ...

Art Phelps

Leanne Pitchford

Michael Allan

Klaus Bartschat

Gas gain

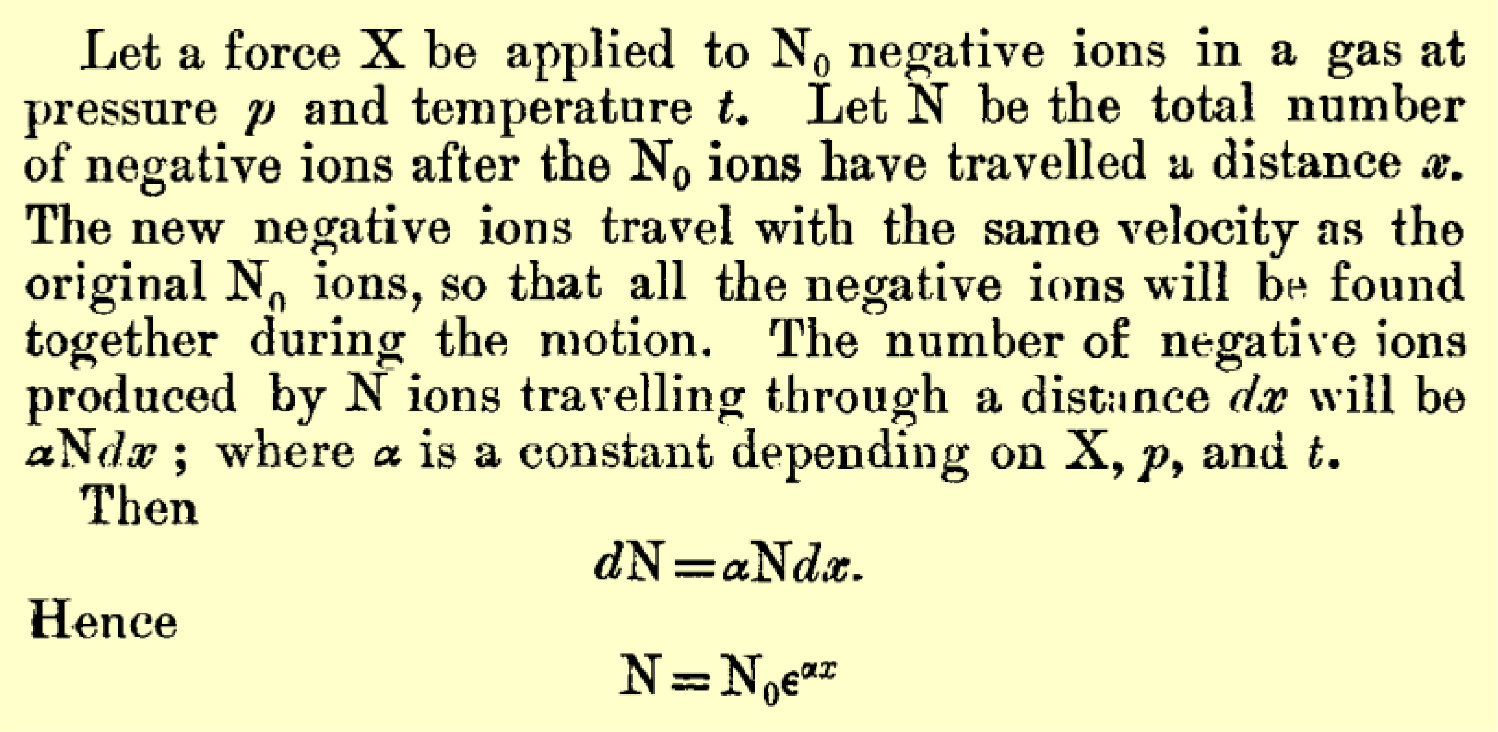
Multiplication

- Ionisation gave us just a few electrons+ions per mm of gas. We have transported them to the read-out, hopefully not losing too many.
- But ... if we collect them directly on a read-out electrode, the current will be tiny.
- We need to multiply them.
- Requires fields where the electron energy occasionally is sufficient to ionise.

Sir John Sealy Edward Townsend
(1868-1957)

1901: Gas multiplication

- John Townsend:



Let a force X be applied to N_0 negative ions in a gas at pressure p and temperature t. Let N be the total number of negative ions after the N_0 ions have travelled a distance x. The new negative ions travel with the same velocity as the original N_0 ions, so that all the negative ions will be found together during the motion. The number of negative ions produced by N ions travelling through a distance dx will be $\alpha N dx$; where α is a constant depending on X, p, and t.

Then

$$dN = \alpha N dx.$$

Hence

$$N = N_0 \epsilon^{\alpha x}$$

[J.S. Townsend, "*The conductivity produced in gases by the motion of negatively charged ions*", Phil. Mag. **6-1** (1901) 198-227. If access to the Philosophical Magazine is restricted, then consult a German-language abstract at http://jfm.sub.uni-goettingen.de/.]

Mean size of the avalanche

- Townsend coefficient α: probability per unit length that an electron creates an additional electron.

- Avalanches grow proportionally to their size:

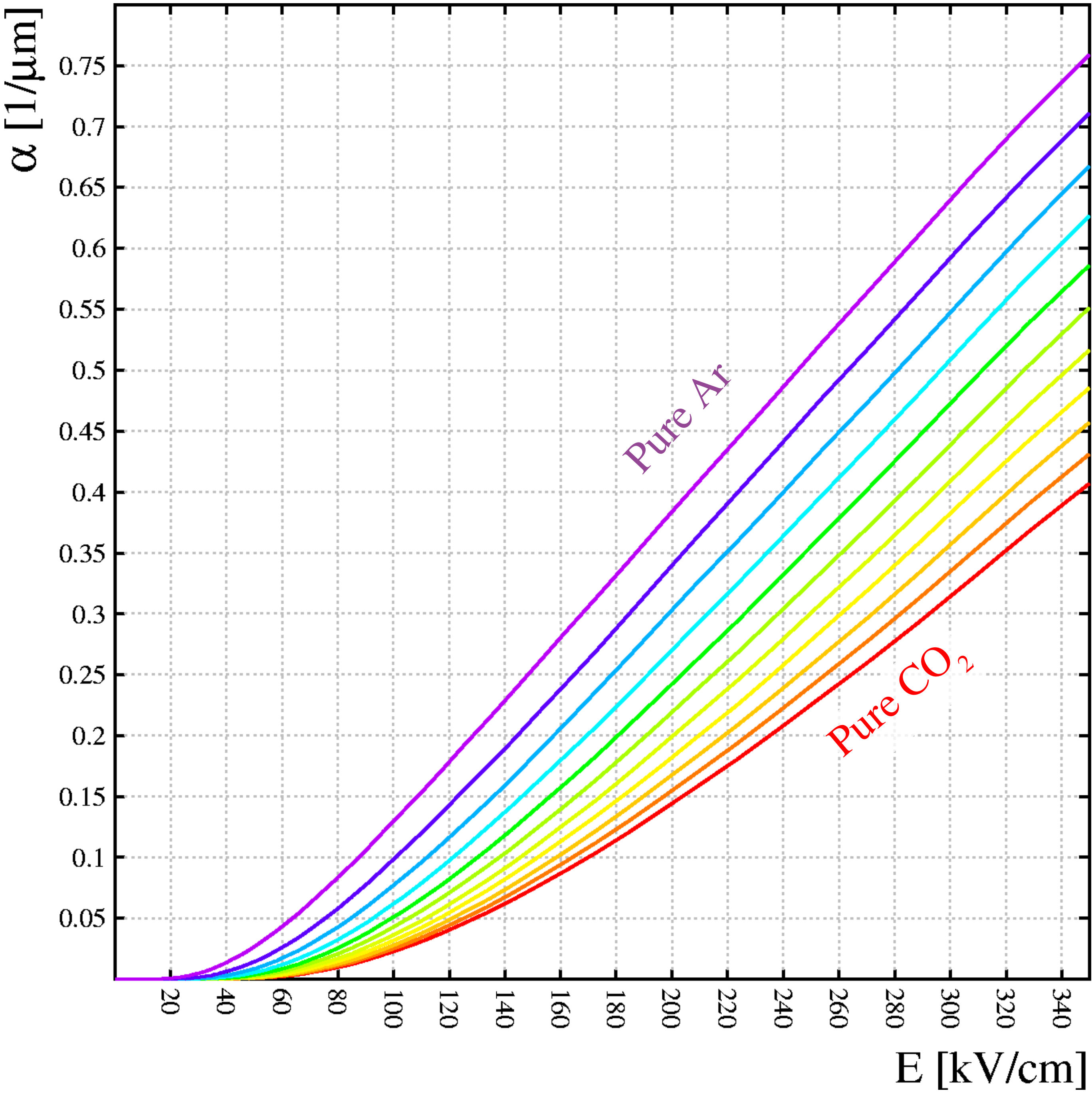
$$\mathrm{d}\,n(x) = n(x)\,\alpha(x)\;\;\mathrm{d}\,x$$

$$n(x) = n(0)\,e^{\int_0^x \alpha(y)\,\mathrm{d}\,y}$$

- Intuitively, α is merely a matter of ionisation.

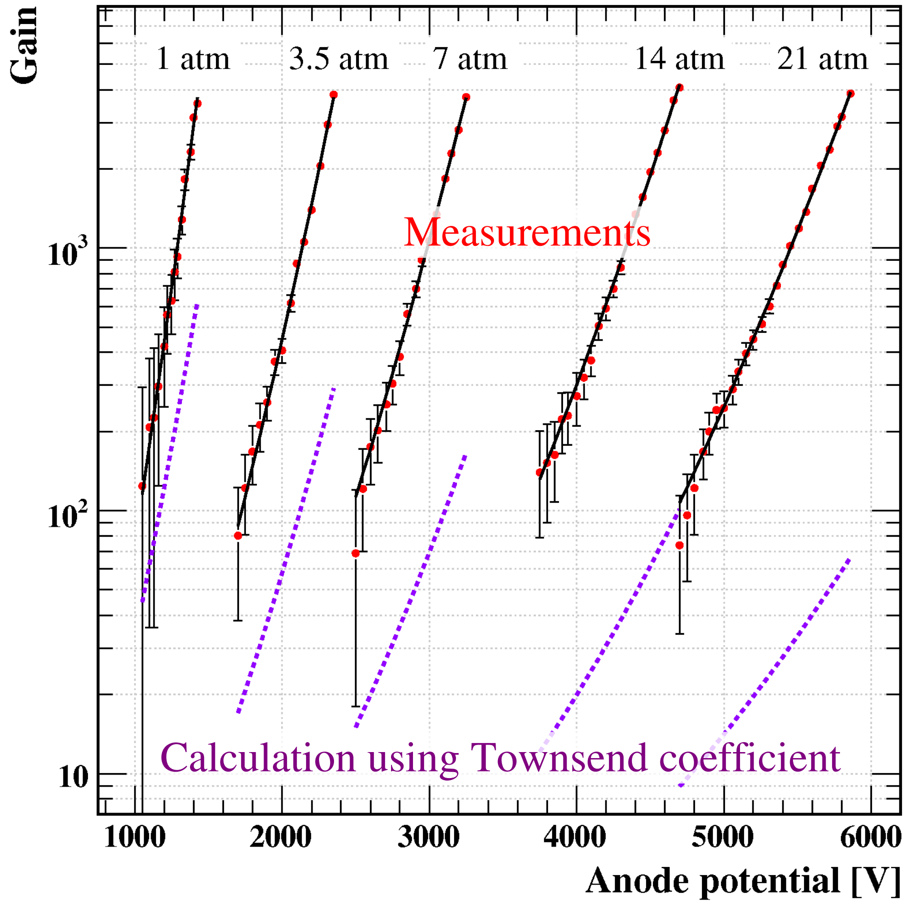
$\alpha(\mathrm{Ar\text{-}CO_2})$

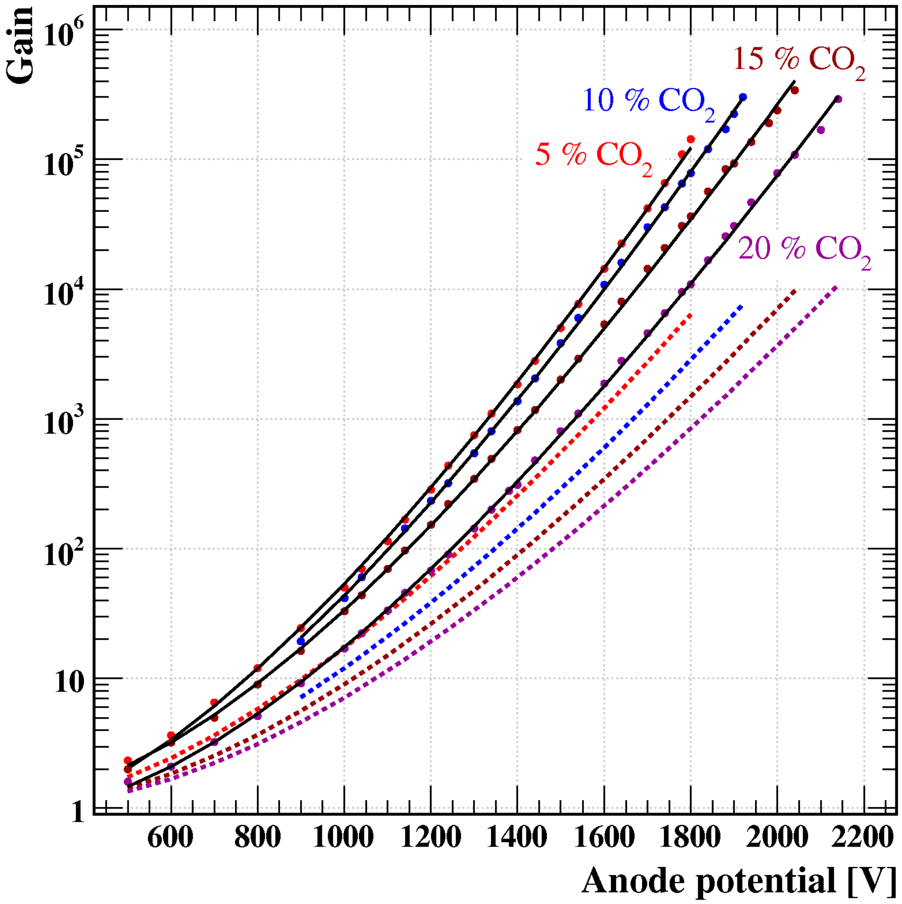
- α = number of e^- an avalanche e^- creates per cm.
- Adding CO_2 reduces the gain.
- Calculated by Magboltz for Ar/CO_2 at 3 bar.

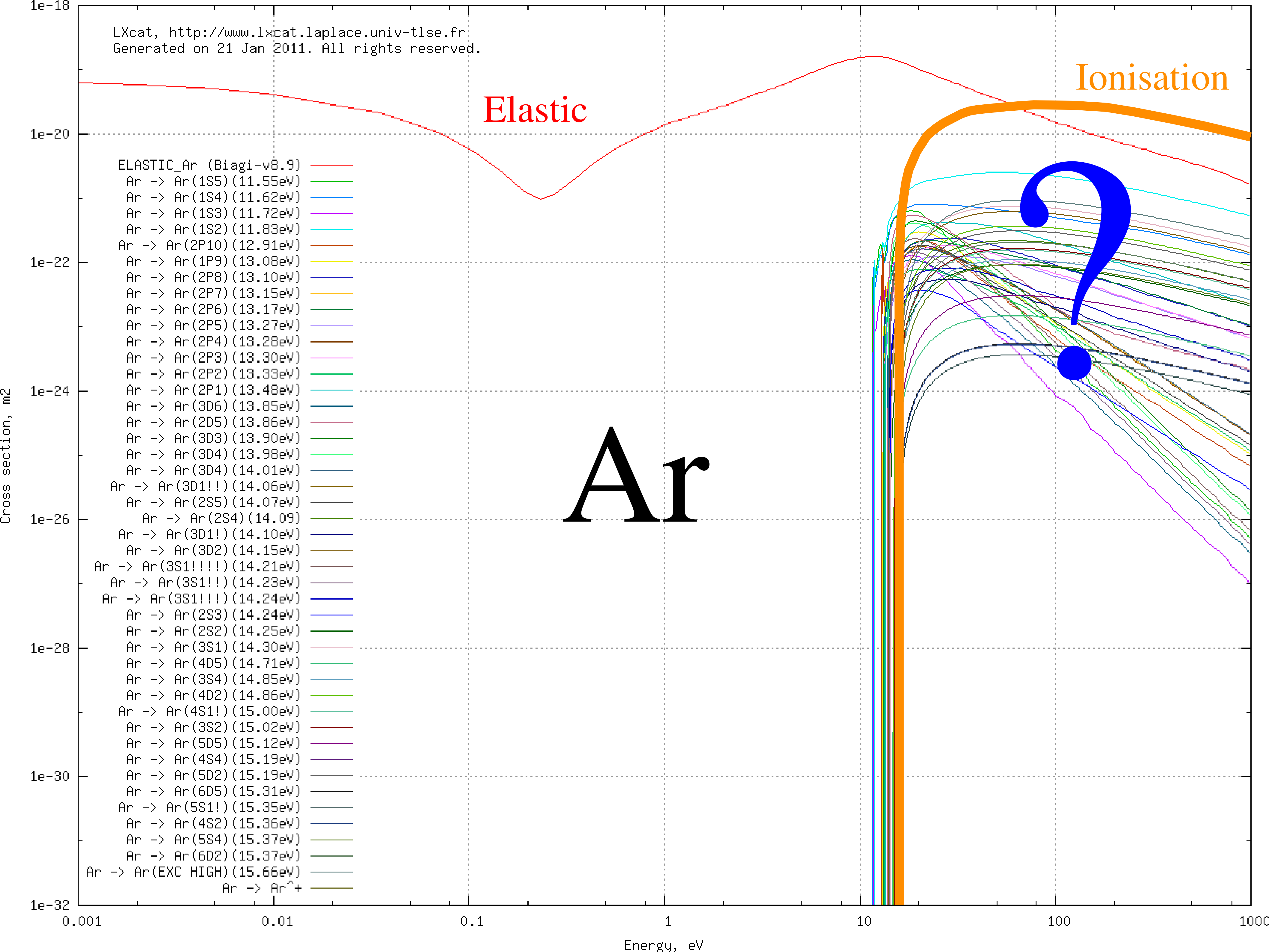


Does this reproduce the measurements ?

- Ar - CH_4
- Ar - CO_2



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Elastic
Ionisation
?
Ar
ELASTIC_Ar (Biagi-v8.9)
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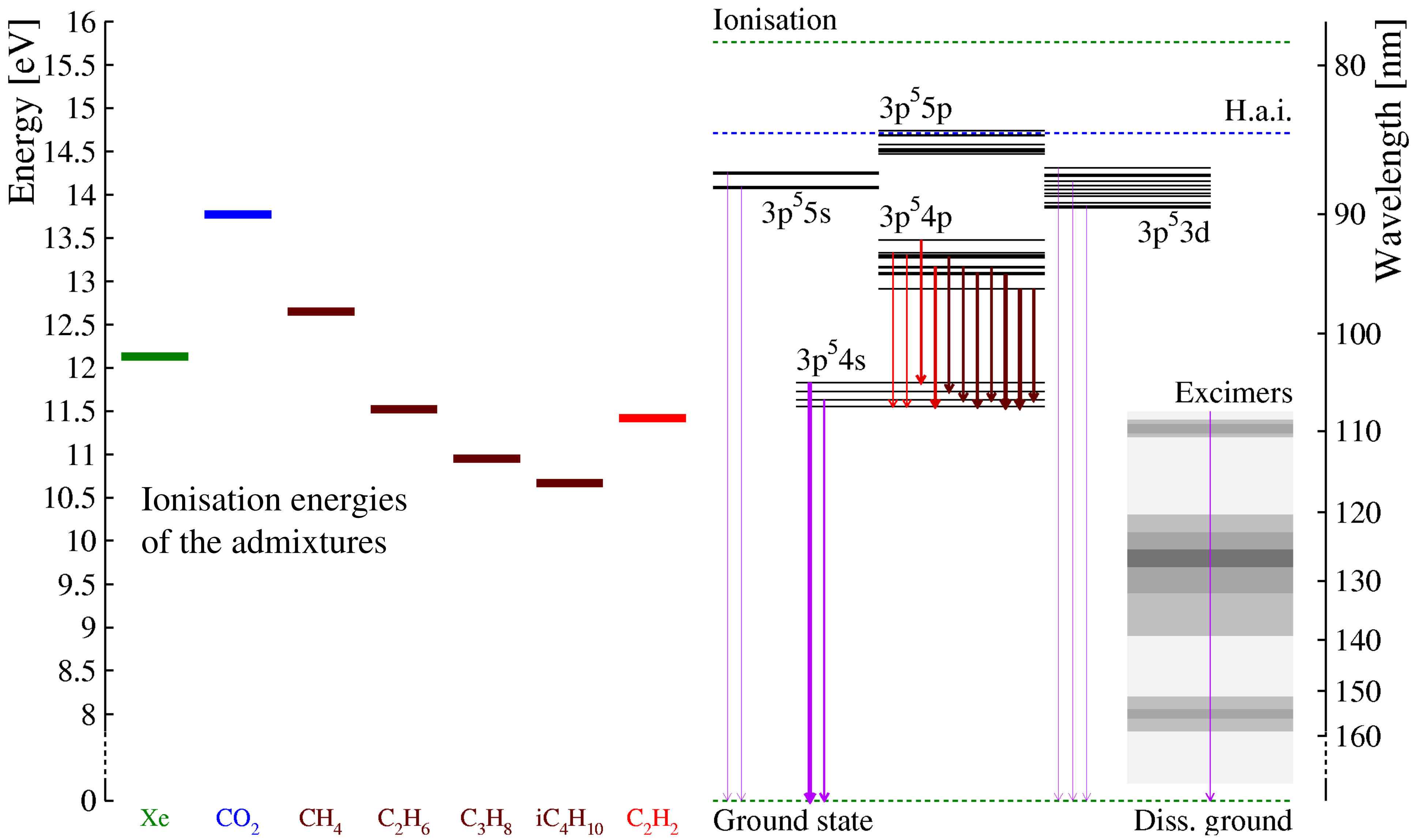
Cross section, m2
1e-18
1e-20
1e-22
1e-24
1e-26
1e-28
1e-30
1e-32
Energy, eV
0.001
0.01
0.1
1
10
100
1000

Level diagram argon and admixtures



Frans Michel Penning
(1894-1953)

Penning effect

- Ar* $3p^5 4s$ can transfer to iC_4H_{10}, C_3H_8 and C_2H_6;
  - two 4*s* are metastable, the two others live 2.6 ns and 8.6 ns;

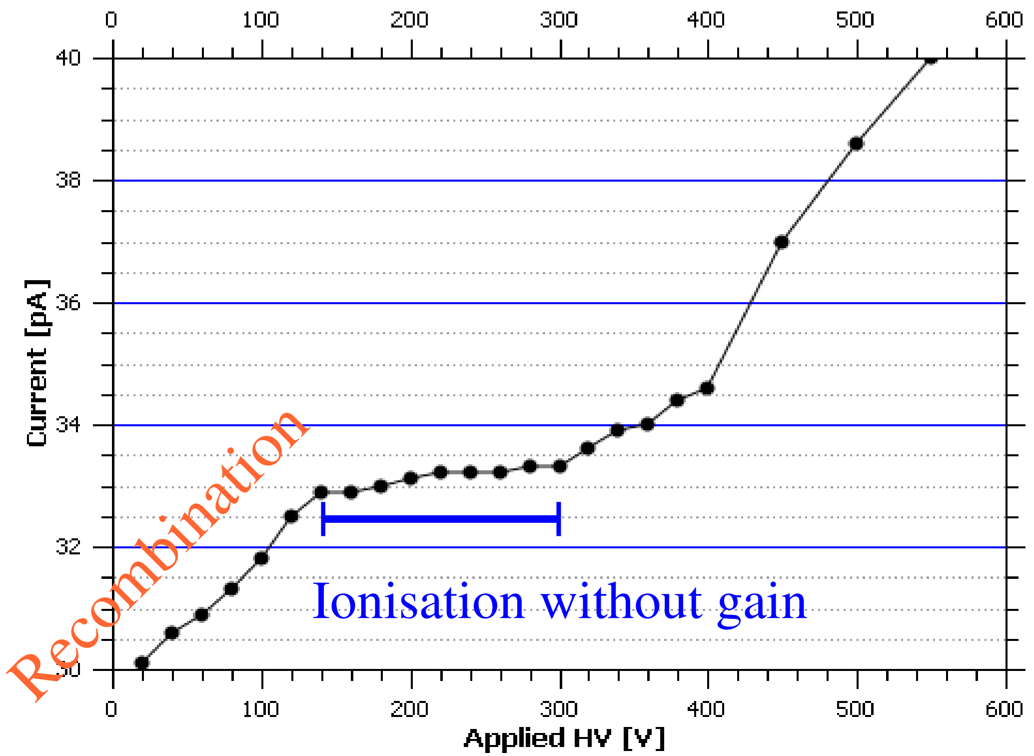
- Ar* $3p^5 4p$ can also ionise CH_4;
  - 4*p* decays to 4*s* with a lifetime of 20-40 ns;

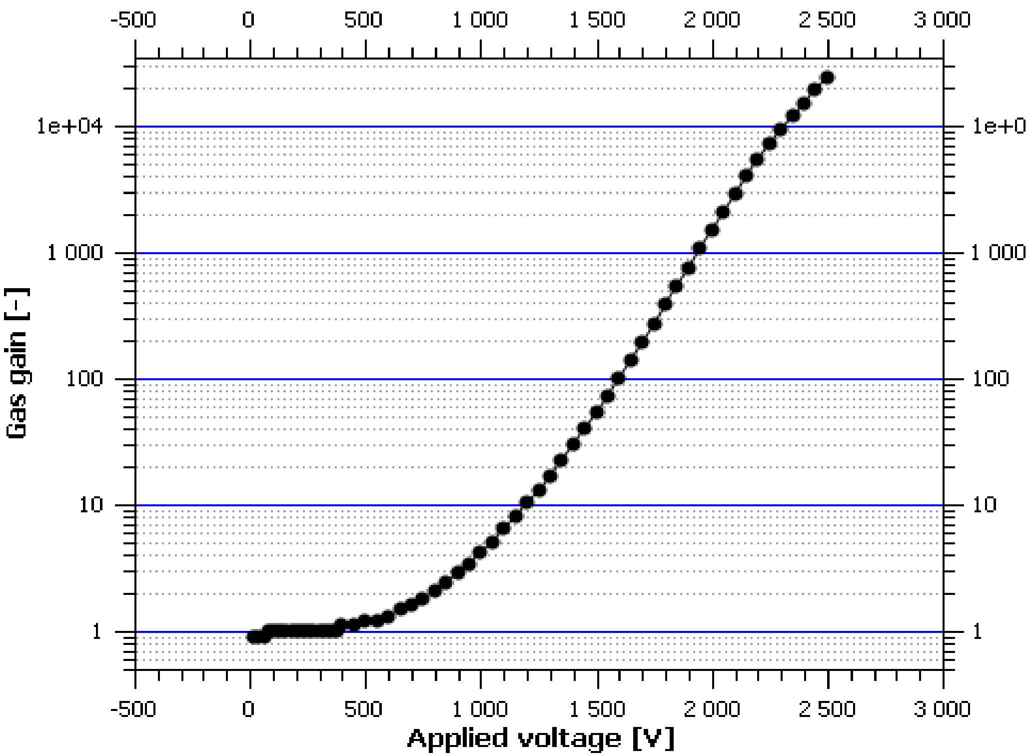
- Ar* $3p^5 3d$ can in addtion transfer to CO_2;
  - radiative 3*d* decays take ~3.5 ns, the others ~50 ns.

- For comparison, collision frequencies of Ar* in pure quencher are ~100 ps.

Data covers 5 orders of magnitude !

- Current reference is taken at the ionisation level.
- Main source of error: ~5 %.





Determining the Penning parameters

- The Penning transfer rate $r(p,c)$ is measured by finding, in experimental data, the fraction of excitations to be added to α:

$$G = \exp \int \alpha \left(1 + r(p,c) \frac{\nu_{\mathrm{exc}}}{\nu_{\mathrm{ion}}} \right)$$

- The model parameters may be found by fitting:

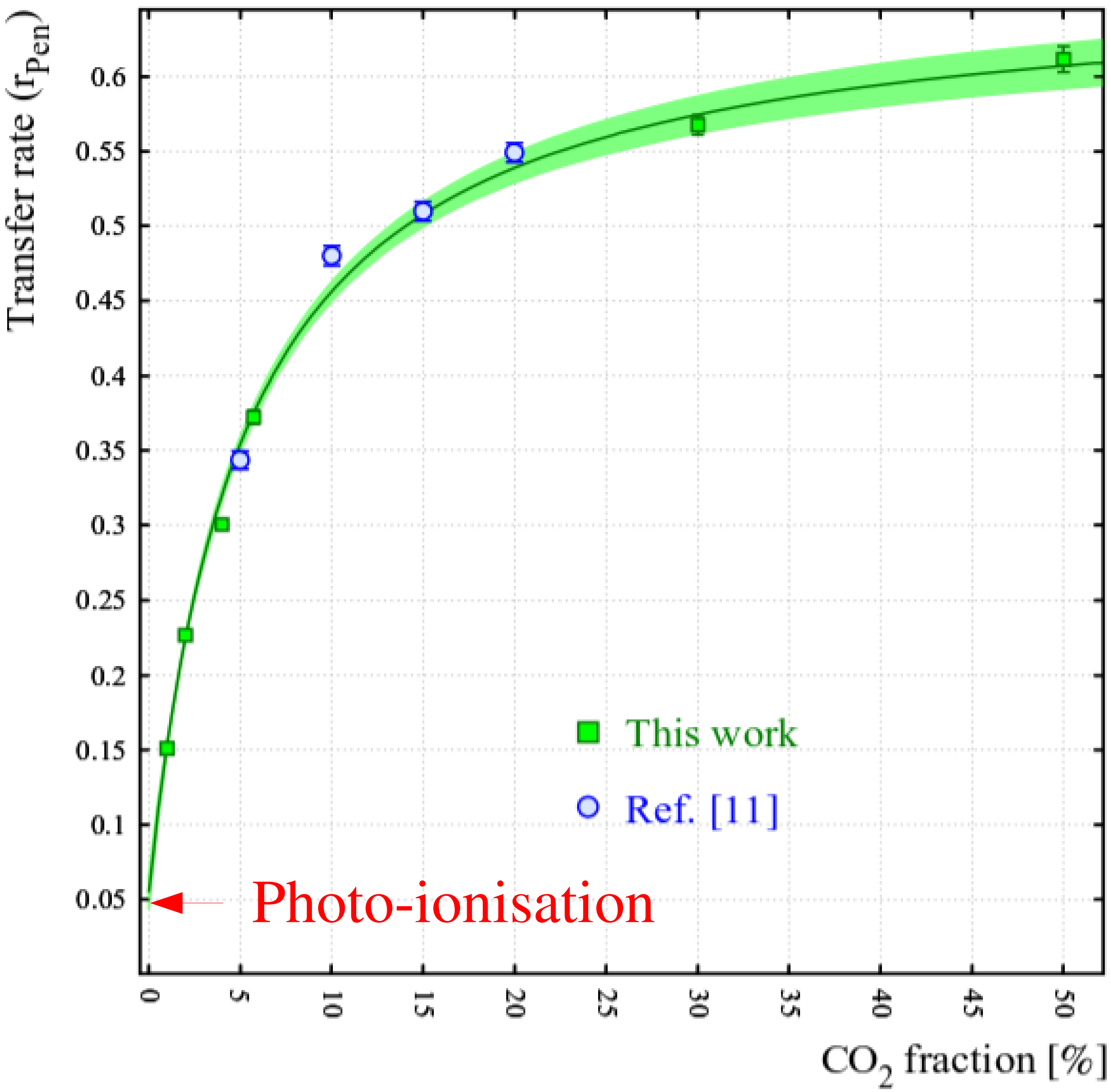
$$r(p,c) = \frac{p\,c\,f_{B^+}/\tau_{AB} + p(1-c)\,f_{A^+}/\tau_{AA} + f_{\mathrm{rad}}/\tau_{A^*}}{p\,c\,(f_{B^+} + f_{\bar{B}})/\tau_{AB} + p(1-c)(f_{A^+} + f_{\bar{A}})/\tau_{AA} + 1/\tau_{A^*}}$$

$$A^* \rightarrow B^+ \qquad\qquad A^* \rightarrow A^+ \qquad\qquad A^* \rightarrow A\,\gamma$$

Ar-CO_2 transfer rates

- Penning parameter fits with data from Tadeusz Kowalski et al. 1992 and 2013.
- At p = 1070 hPa.

[10.1016/0168-9002(92)90305-N, 10.1016/j.nima.2014.09.061]



Loss of excitation

Photo-ionisation

Gas gain fluctuations

G.U. Yule (1923), W.H. Furry (1937), R.A. Wijsman (1949) & others

- If the distance between ionisations fluctuates exponentially with a mean of 1/α (reciprocal of the Townsend coefficient),

- then, the avalanche size fluctuates (nearly) exponentially:

$$p(n) = \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1}$$

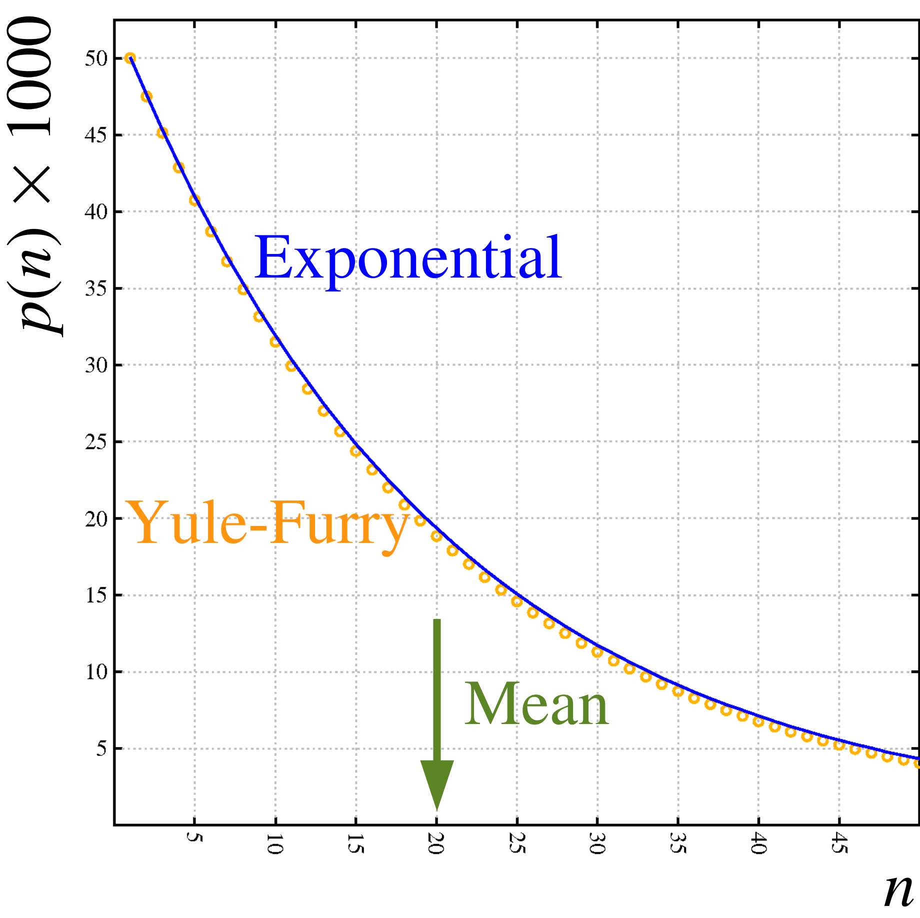
George Udny Yule
(1871-1951)

[G. Udny Yule, *A Mathematical Theory of Evolution, based on the Conclusions of Dr. J.C. Willis, F.R.S.*, Phil. Trans. Roy. Soc. London B **213** (1925) 21-87.
W.H. Furry, *On Fluctuation Phenomena in the Passage of High Energy Electrons through Lead*, Phys. Rev. **52** (1937) 569-581.
Robert A. Wijsman, *Breakdown Probability of a Low Pressure Gas Discharge*, Phys. Rev. **75** (1949) 833-838.]

Statistics Yule-Furry

- Yule-Furry is exponential for large mean avalanche sizes:

$$
\begin{aligned}
p(n) &= \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1} \\
&\approx \frac{e^{-n/\bar{n}}}{\bar{n}-1}
\end{aligned}
$$

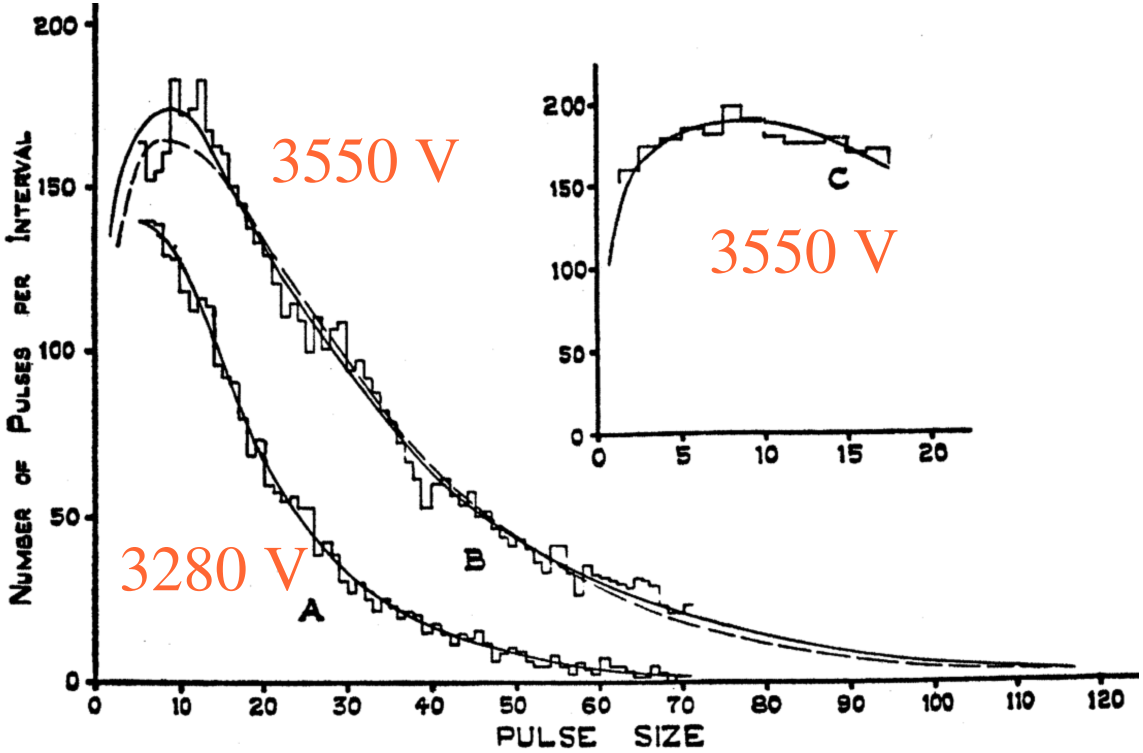


- Mean: $\bar{n}$

  RMS: $\bar{n}\sqrt{1-1/\bar{n}} \approx \bar{n}$

S.C. Curran (1949)

- S.C. Curran *et al*. measured the pulse height distribution in a cylindrical counter (d = 150 µm wire, Ar 50 % CH_4 50 %, p = 670 mbar) at $G \sim 10^4$-10^5:



$$p(n) = \sqrt{n}\, e^{-n}$$

$$f \equiv \left(\frac{\sigma}{\bar{n}}\right)^2 \approx \frac{2}{3}$$

Assumptions

- Yule-Furry follows if one assumes:
  - probability to ionise over a distance dx is α dx
    =
    distance between ionisations fluctuates exponentially with mean $1/\alpha$.
  - no history: Townsend coefficient α is constant,
  - no attachment losses.

Two schools of thought ...

- The distance between ionisations does not simply vary exponentially (e.g. the Raether group).

- The Townsend coefficient is not constant (e.g. Byrne, Lansiart & Morucci).

Mean and Minimum step size

- Mean distance between successive ionisations:

$$\bar{\lambda} = \frac{1}{\alpha}$$

- Minimum distance between successive ionisations:

$$\lambda_{\mathrm{min}} > \frac{IP}{E}$$

- Define $$\frac{\bar{\lambda}}{\lambda_{\mathrm{min}}} = \frac{E}{\alpha\, IP} = \kappa$$

large κ	minimum distance has no effect	→ exponential
$\kappa \simeq 1$	little room for fluctuations	→ hump

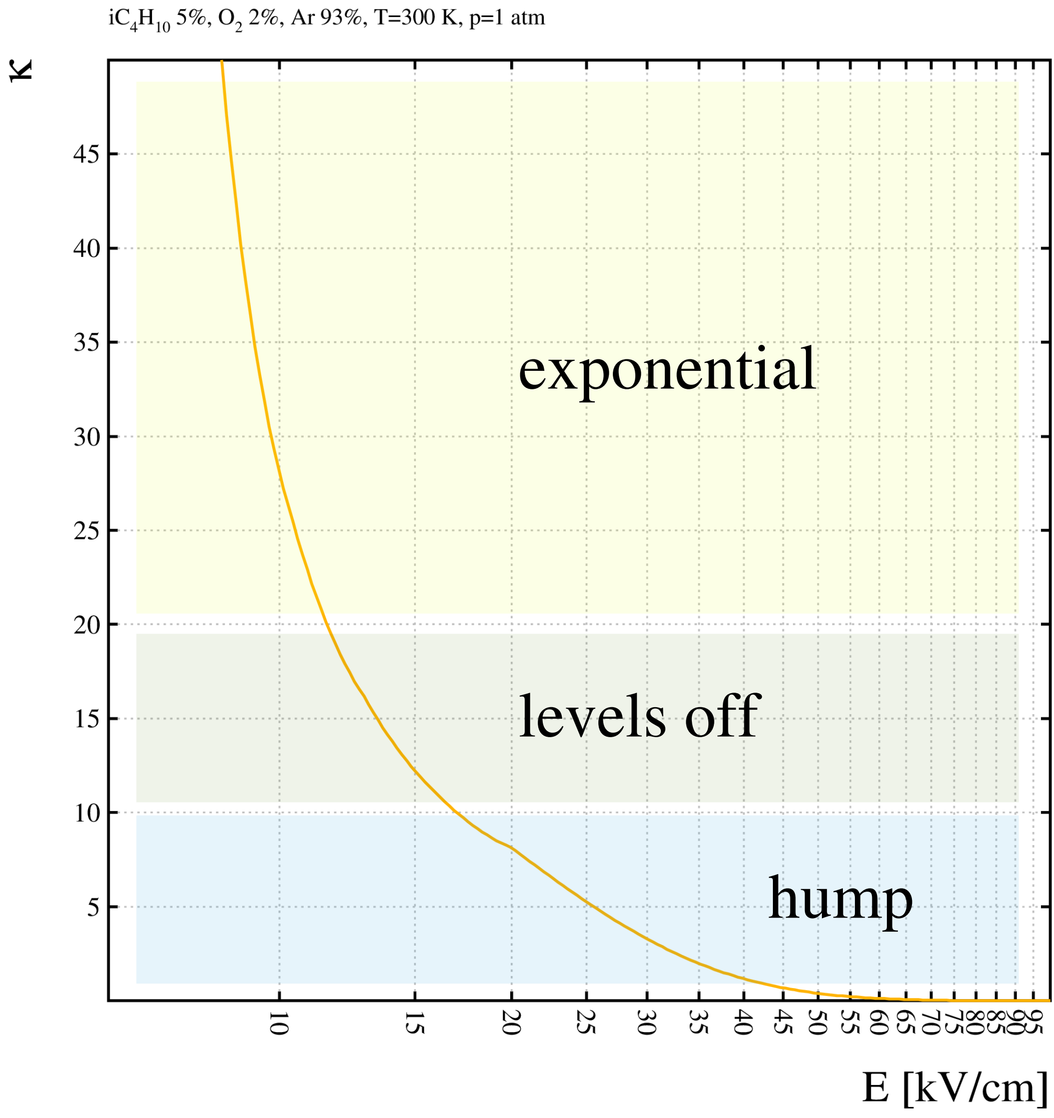
Shape vs E-field

- Mean / minimum distance:

$$\frac{\bar{\lambda}}{\lambda_{\text{min}}} = \frac{E}{\alpha\, IP} = \kappa$$

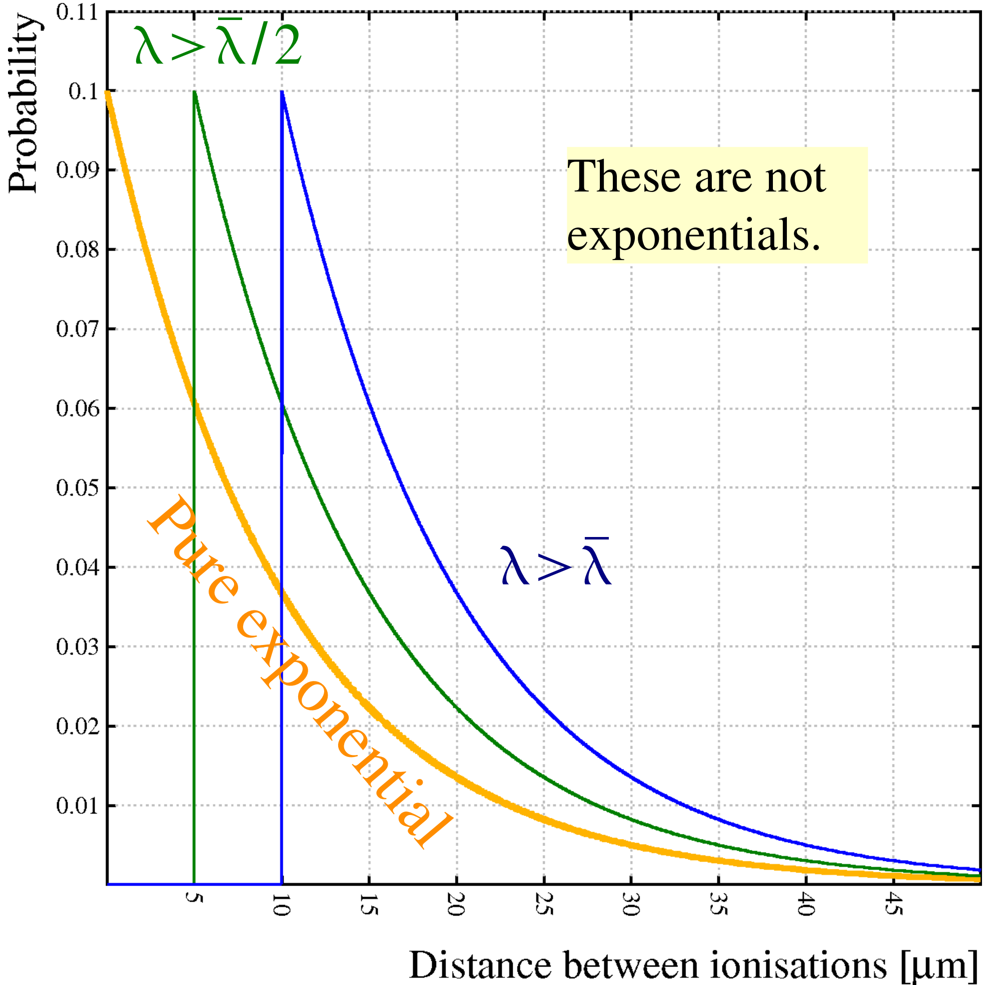
- Shape:
  - $\kappa > 20$ exponential
  - $10 < \kappa < 20$ levels off
  - $\kappa < 10$ hump

- One might think that $\kappa \propto E$ but α rises faster than E.

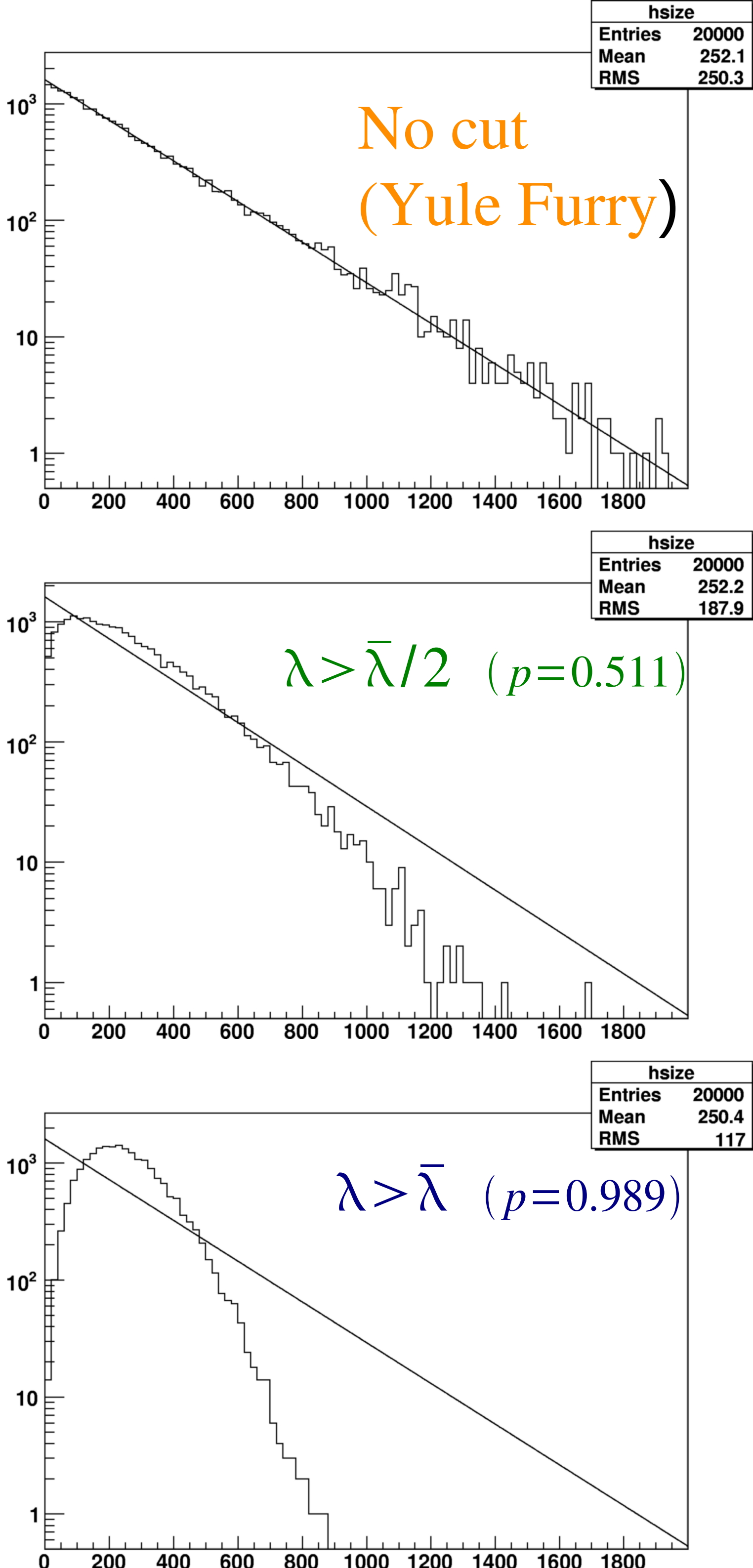


Minimum step length

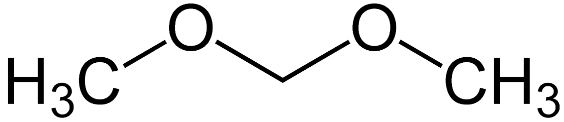
- Imposing a minimum distance between ionisations adds a hump.



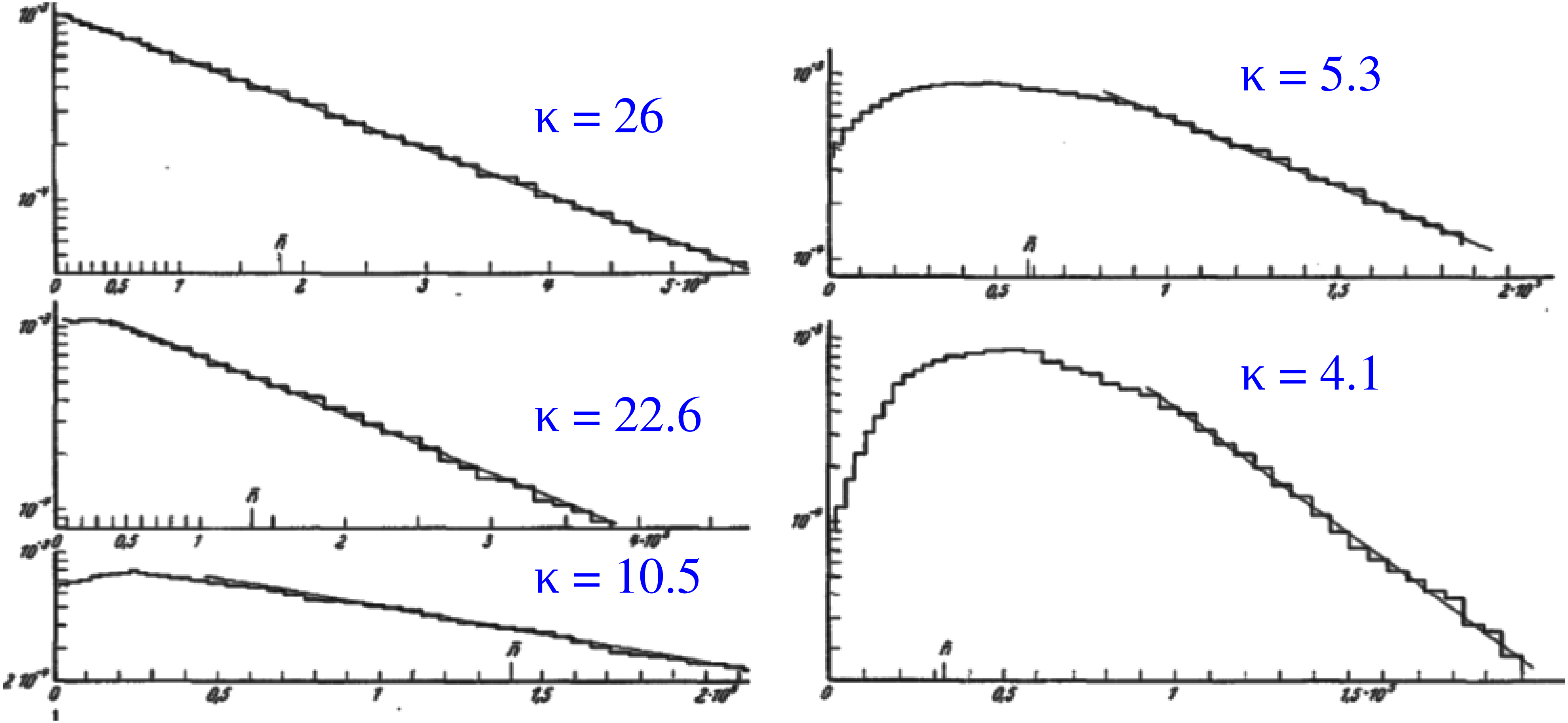
These are not exponentials.



Hans Schlumbohm (1958)



- Dimethoxymethane spectra: increasing *E*, decreasing *p d* and ~constant mean gain.



- Hans Schlumbohm, *Zur Statistik der Elektronenlawinen im ebenen Feld III*, Z. Phys. **151** (1958) 563-576.

The *Magnettrommelrechner* (1961)

- Excellent agreement ... but no closed form

Die Rechnungen wurden mit dem Magnettrommelrechner IBM 650 des Instituts für Angewandte Mathematik der Universität Hamburg durchgeführt. Den an der Rechenanlage tätigen Mitarbeitern des Instituts danke ich für die Hilfe bei der Ausführung der Rechnung.

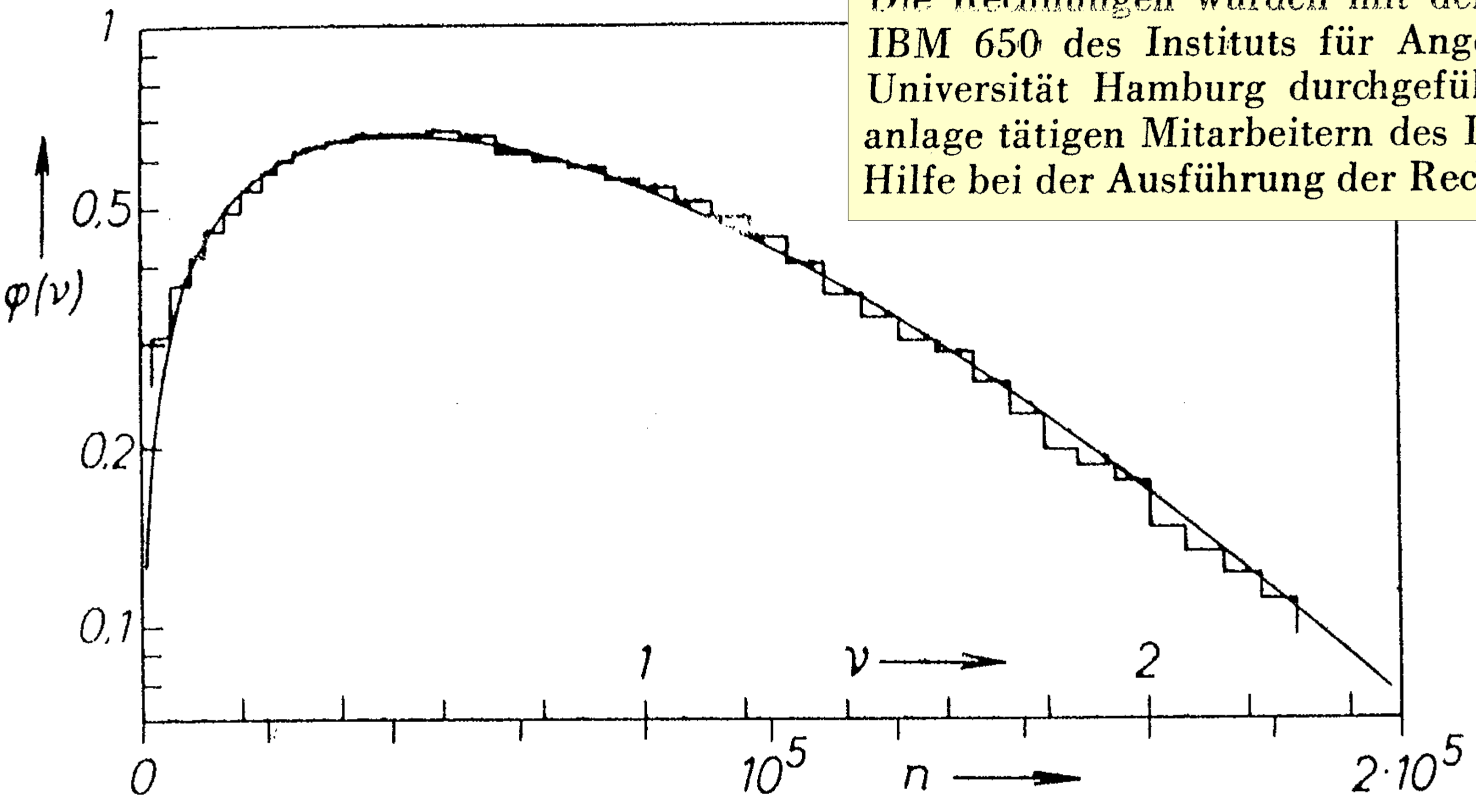


Abb. 5. Lawinenverteilung in Methylal nach SCHLUMBOHM [8]. $E/p = 186{,}5$ Volt/cm·Torr, $\alpha \cdot U_i/E = 0{,}19$. Ausgezogene Kurve: Theoretische Verteilung im Modellgas für $\alpha x_0 = 0{,}18$.

$\kappa = 5.3$

The alternative school

- Townsend coefficient not constant ...

A. Lansiart & J.P. Morucci (1962)

- Small avalanches are composed of electrons that
  - have ionised less, hence
  - have more energy, hence
  - will ionise more easily.

- They modeled this with an avalanche size-dependent α:

$$\alpha(n)=\alpha(0)\left(1+\frac{k}{n}\right)$$

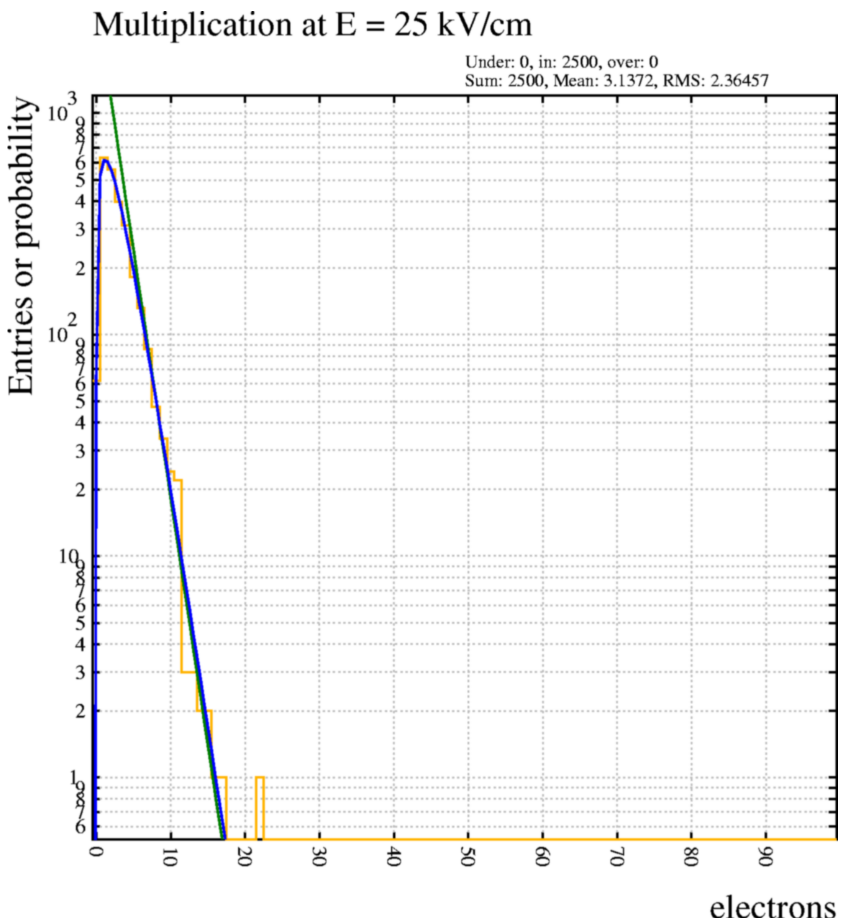
- Implies that $(\sigma/\mu)^2 = 1/(1+k) < 1$, in agreement with Curran's measurements.
- Electron energy distribution continues to decrease, without reaching an equilibrium.

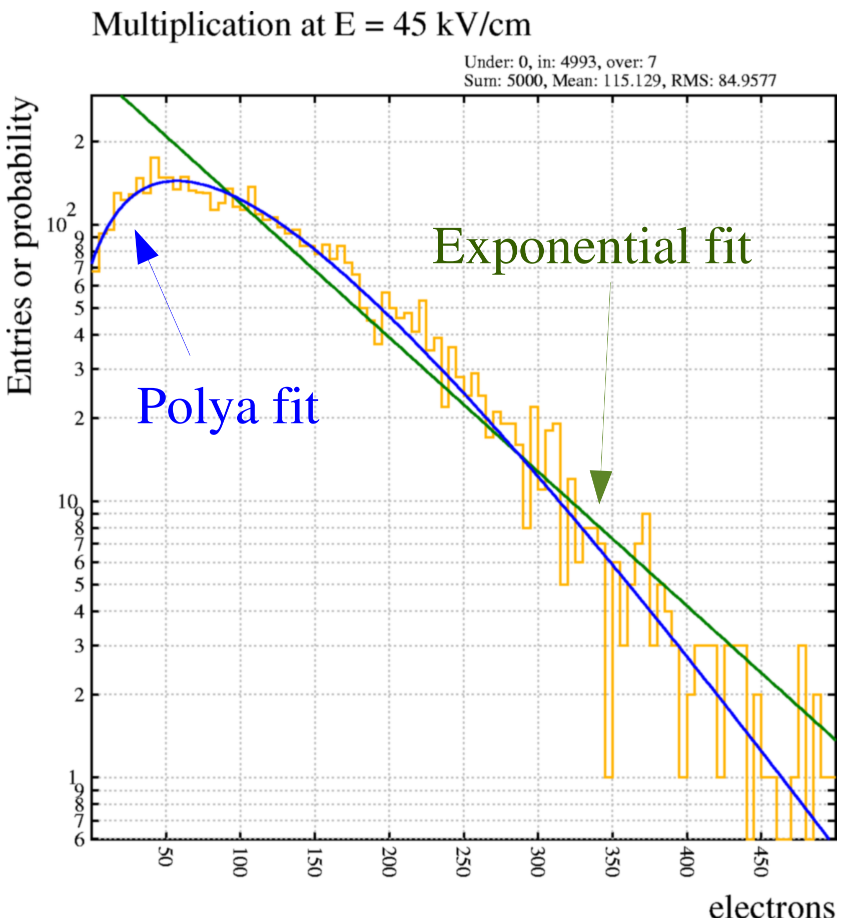
Monte Carlo approach – a way out ?

- Analytic models are precious for the insight they afford.
- But the complexity of real gases and detectors make realistic models unwieldy:
  - inelastic collisions (vibrations, rotations, polyads);
  - excitations and Penning transfers;
  - ionisation;
  - attachment;
  - intricate, position-dependent E and B fields.
- Predictions for experiments are more practical using a Monte Carlo approach, here based on Magboltz.

Ar/CO$_2$: size distribution

- Lower gain than pure Ar, but with increasing field, the size distribution still becomes more and more round:



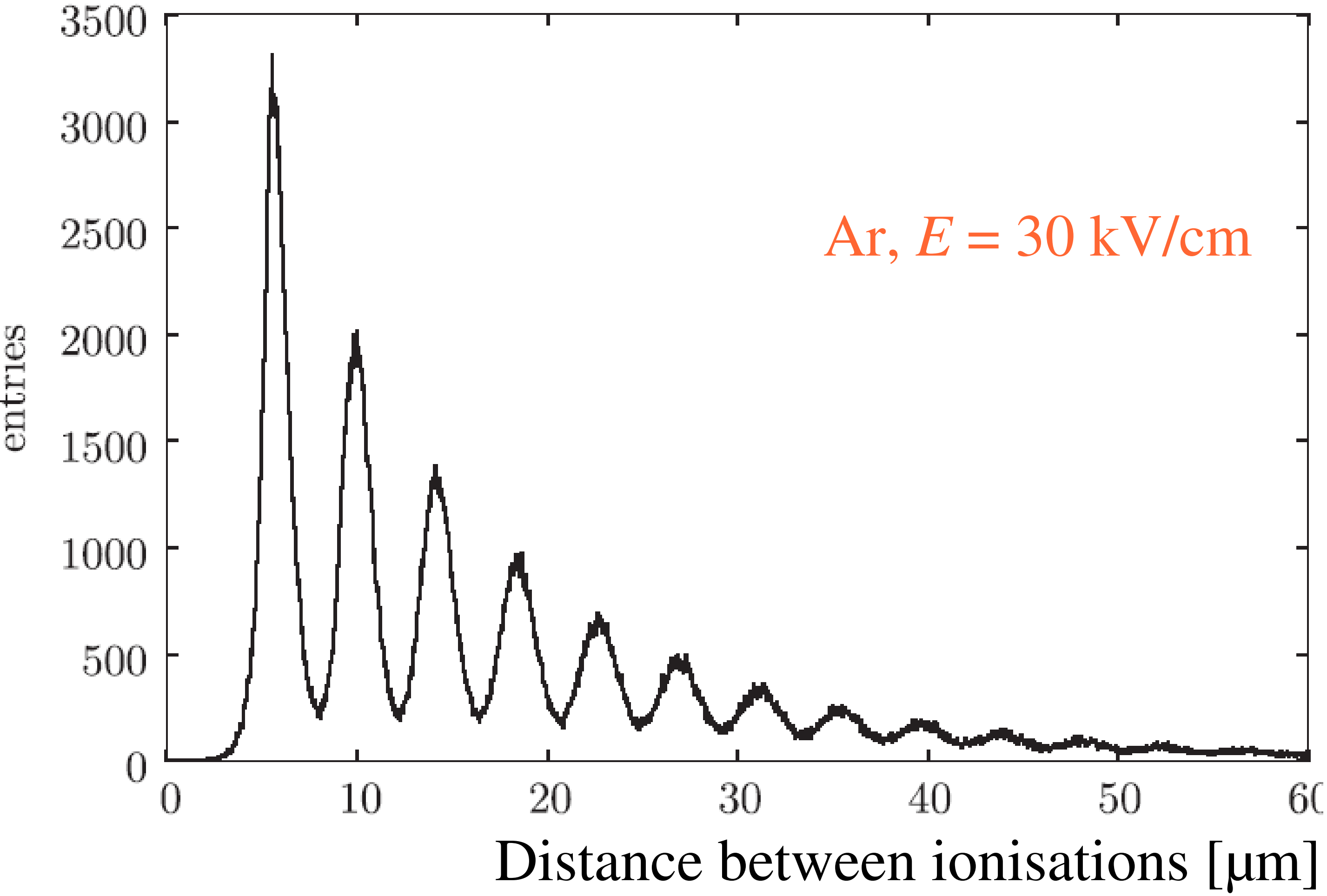


Exponential fit

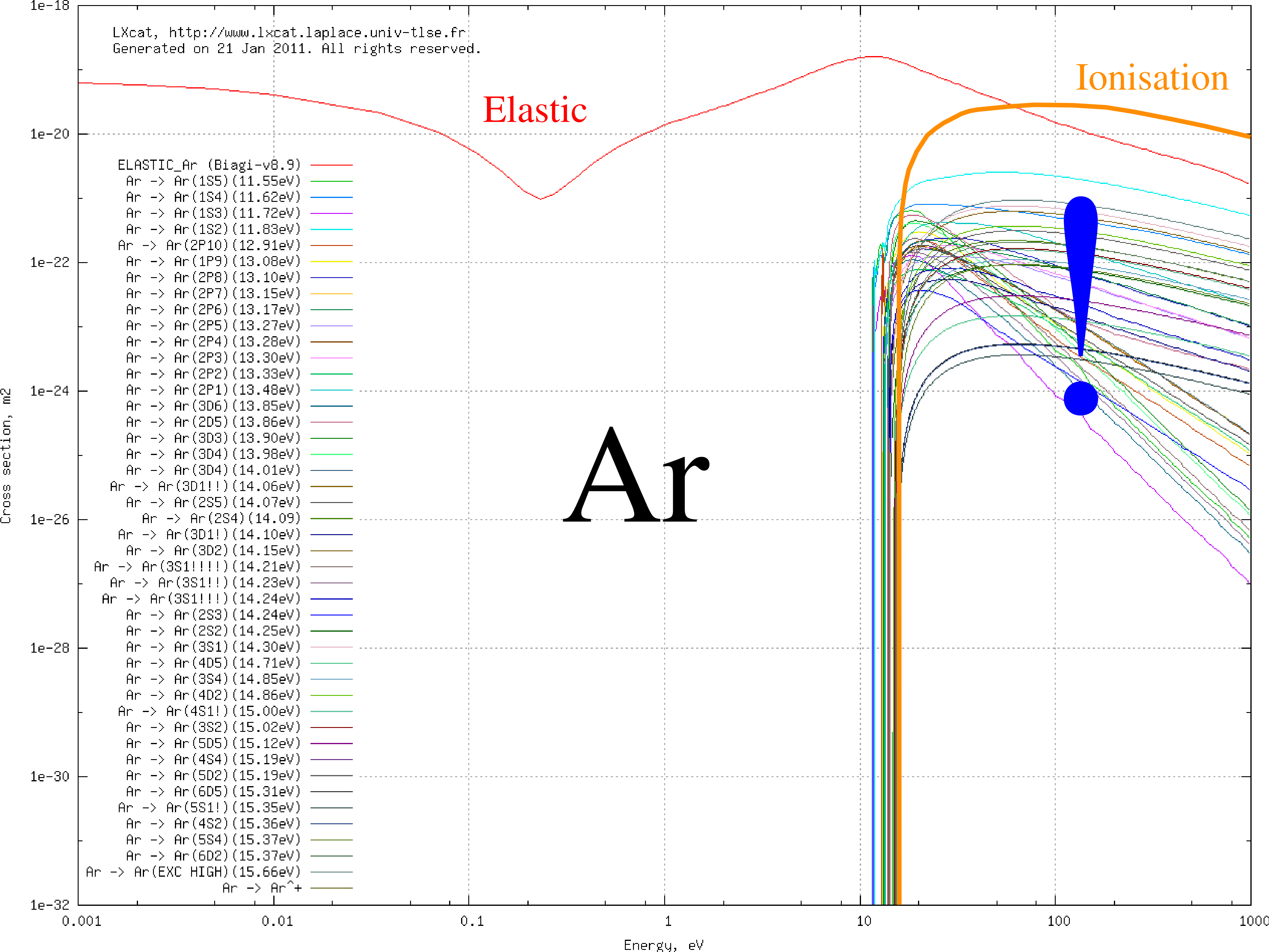
Polya fit

Distance between ionisation

- The distance between successive ionisations oscillates, shown here for Ar (also happens in CH_4 for instance).
- Why ?



[Magboltz calculations by Heinrich Schindler]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Ar -> Ar(6D2)(15.37eV)
Ar -> Ar(EXC HIGH)(15.66eV)
Ar -> Ar^+
Cross section, m2
1e-18
1e-20
1e-22
1e-24
1e-26
1e-28
1e-30
1e-32
0.001
0.01
0.1
1
10
100
1000
Energy, eV

Single-electron spectra

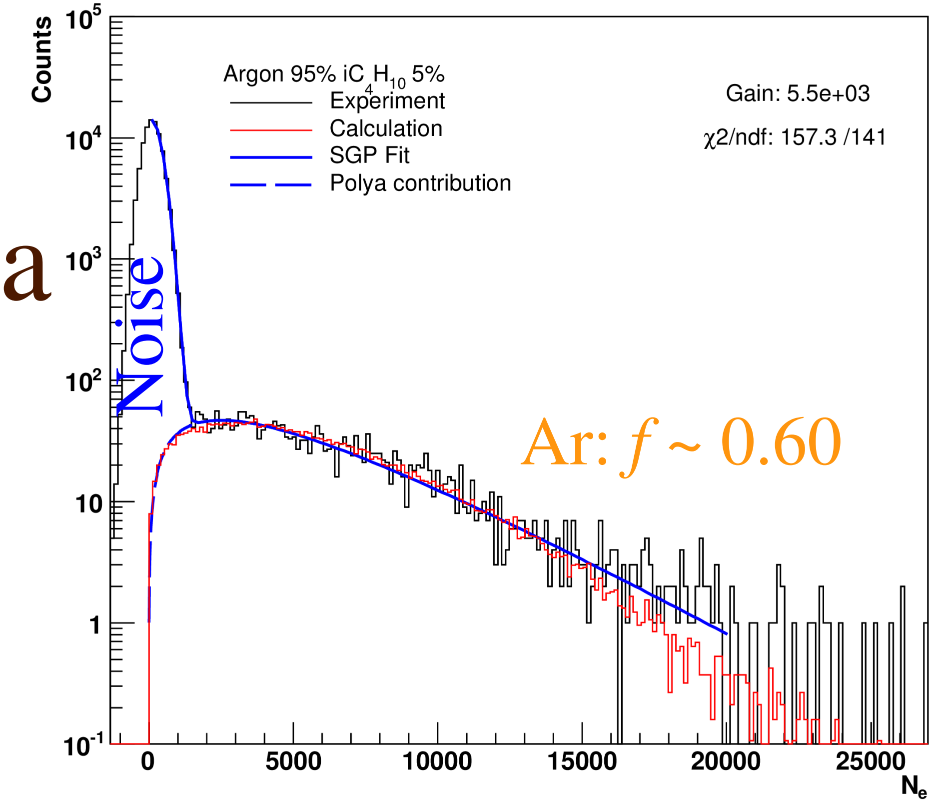
blue: Pólya signal + Gaussian noise fit;

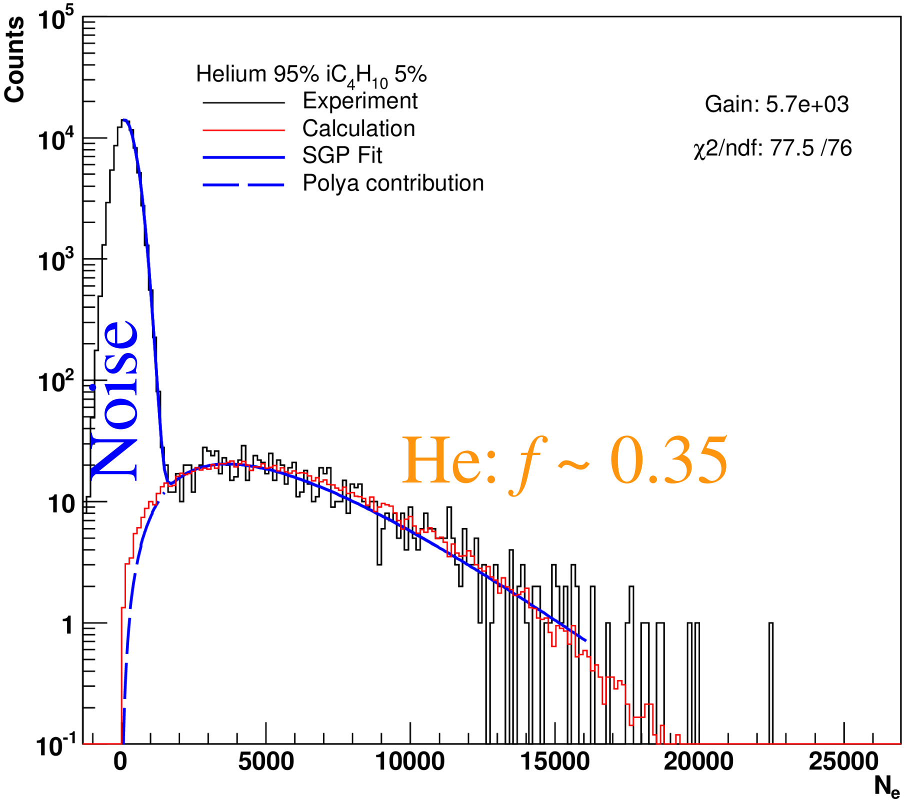
red: Monte Carlo (Magboltz), not fits !

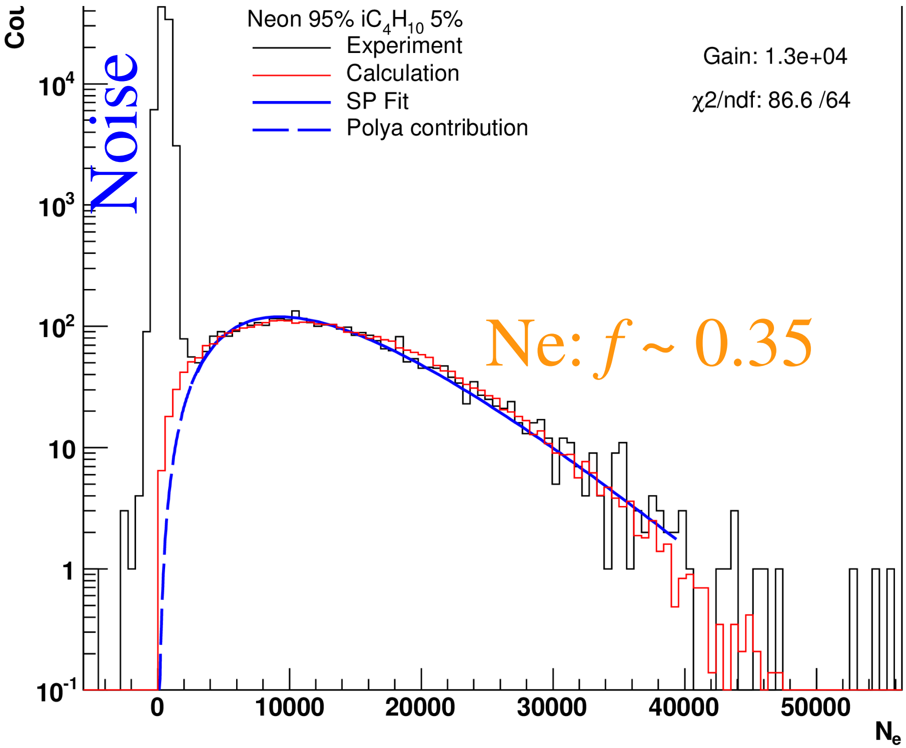
Ar 95 % iC_4H_{10} 5 %, E=28.12 kV/cm,

Ne 95 % iC_4H_{10} 5 %, E=26.25 kV/cm,

He 95 % iC_4H_{10} 5 %, E=26.25 kV/cm,







Hump – good or bad ?

- Smaller gain fluctuations:
  - better energy resolution.

- Smaller probability of small gain:
  - higher detection efficiency.

- Smaller probability of high gain:
  - fewer discharges.

Ion transport

Ions

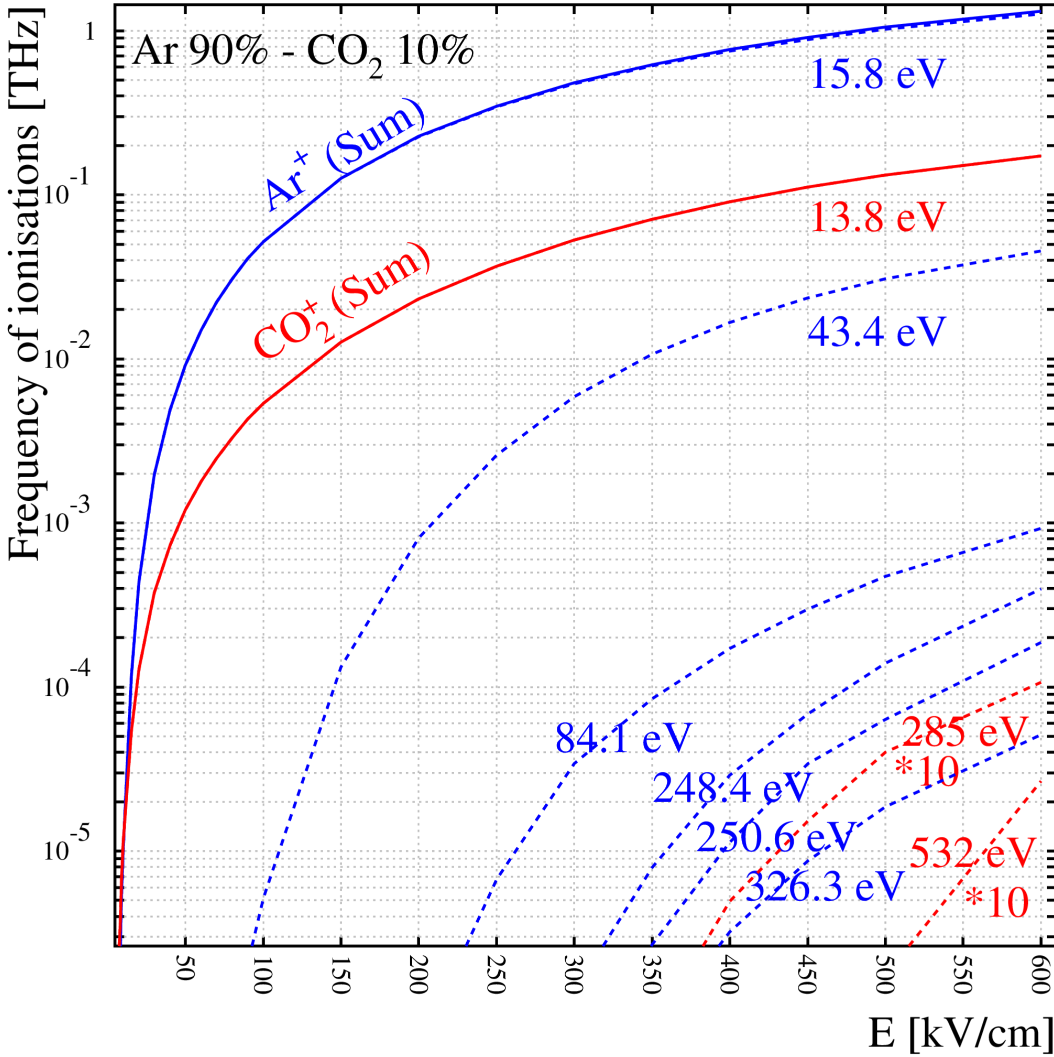
- Avalanches produce not only electrons, but also ions – at least as many.

- Detectors like Micromegas and wire chambers get their signal mostly from ion motion (→ Signals).

- Hence we better know the basics of ions:
  - *which ions* are moving ?
  - *how fast* do the ions move ?
  - are they subject to *diffusion* ?

Avalanche products

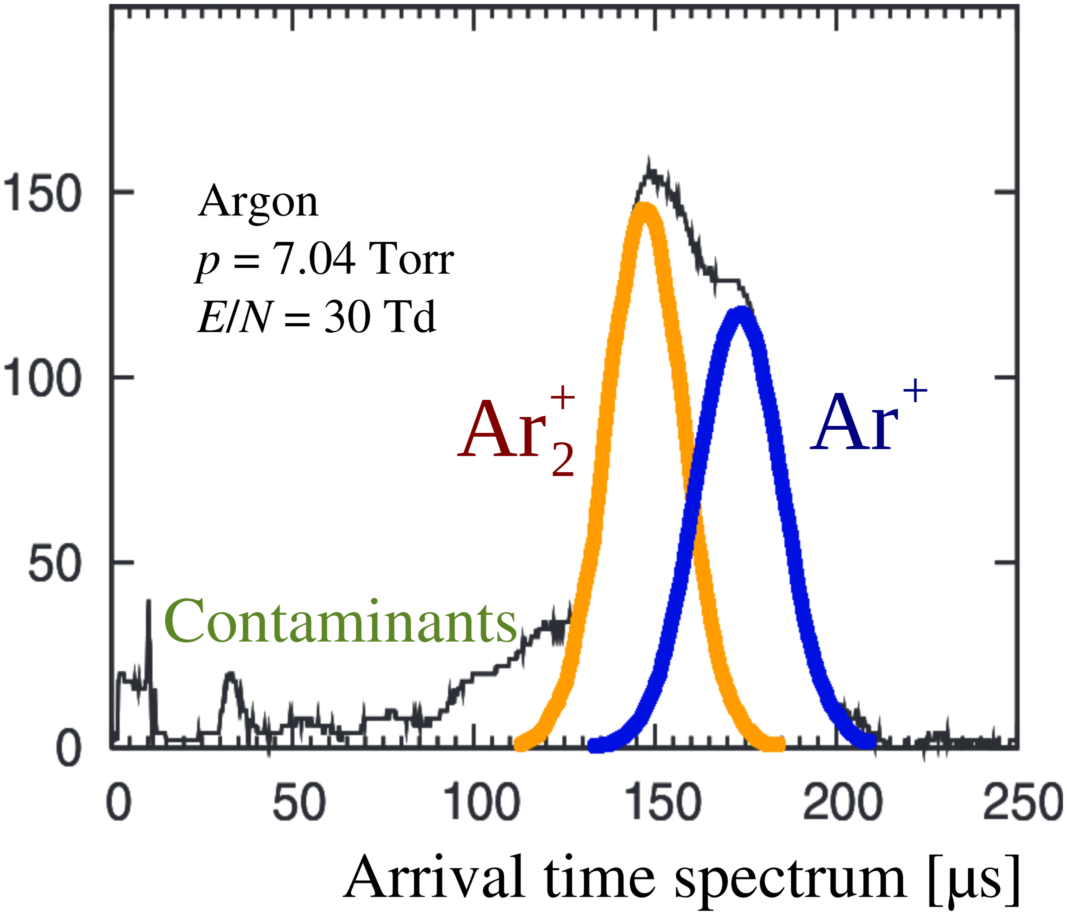
- We'll later see that ion motion is responsible for the bulk of the signal in some detectors.
- In 90/10 Ar-CO_2, the main avalanche product is Ar^+.
- Are Ar^+ ions responsible for the signal ?



Ions drifting in pure Ar

- In pure argon, dimers are formed:
  - $Ar^+(^2P^o_{3/2}) + 2Ar \rightarrow Ar^+{\bullet}Ar + Ar$ ($k = 2.3 \pm 0.1\ 10^{-31}\ cm^6/s$, 7 ns)
  - Note: dimers move *faster* than ions due to $Ar \leftrightarrow Ar^+$ resonant charge exchange

[P.N.B. Neves et al. 10.1063/1.3497651]



Thermal collision frequency

- Mean relative velocity (μ = reduced mass):

$$\bar{v}_{\mathrm{rel}} = \sqrt{\frac{8 k_{\mathrm{B}} T}{\pi \mu}} \approx 570 \ \mathrm{m/s}$$

- Multiplying with the cross section σ gives the rate constant:

$$k = \sigma \bar{v}_{\mathrm{rel}} \approx 9 \ 10^{-10} \ \mathrm{cm}^3/\mathrm{s}$$

- Combine with the number density to get collision time:

$$\tau = \frac{1}{N \sigma \bar{v}_{\mathrm{rel}}} = \frac{k_{\mathrm{B}} T}{p} \frac{1}{\sigma \bar{v}_{rel}} = \frac{1}{p \sigma} \sqrt{\frac{\pi \mu k_{\mathrm{B}} T}{8}} \approx 45 \ \mathrm{ps}$$

Principal reactions involving CO_2

- Ar^+: charge exchange, $\tau \approx 0.85$ ns
  - $Ar^+ + CO_2 \rightarrow Ar + CO_2^+$

- CO_2: 3-body association, 7-20 ps
  - $CO_2^+ + 2CO_2 \rightarrow CO_2^+ \bullet CO_2 + CO_2$

- [For 10 % CO_2, atmospheric pressure, room temperature]

Situating cluster ions

- Chemically bound molecules: 0.75 - 11.1 eV
  - covalent or ionic bond

- Cluster ions: 0.09 - 1.7 eV
  - bound by charge-induced dipole forces
  - constituents retain their identity

- van der Waals molecules: 0.0009 - 0.1 eV
  - bound by van der Waals forces
  - observed at low temperatures

[B.M. Smirnov, “Cluster Ions and Van Der Waals Molecules,” CRC press]

Binding energy of CO_2 cluster ions

- Binding energy:
  - $CO_2^+ \bullet CO_2$: 0.60 eV (16.2 kcal/mol)
  - $CO_2^+ \bullet 2CO_2$: 0.26 eV (6.0 kcal/mol)

[M. Meot-Ner and F.H. Field, J. Chem. Phys., **66** (1977) 4527]

  - $CO_2^+ \bullet CO_2$ 0.51 eV (11.8 ± 1.0 kcal/mol)
  - $CO_2^+ \bullet 2(CO_2)$ 0.14 eV (3.3 ± 1.4 kcal/mol)
  - $CO_2^+ \bullet 3(CO_2)$ 0.12 eV (2.8 ± 1.4 kcal/mol)

[S.H. Linn and C.Y. Ng, J. Chem. Phys. **75** (1981) 4921]

Conversion: 1 kcal/mole = 0.043 eV.

Thermal energy is 0.03 eV.

Life cycle of $CO_2^+ \bullet (CO_2)_n$

- $CO_2^+ \bullet CO_2$ has a dissociation energy of 0.5-0.6 eV, far above thermal energies at 1 bar. It is a so-called long-lived cluster:
  - calculated lifetime = 5 ns

  [B.M. Smirnov, “Cluster Ions and Van Der Waals Molecules,” CRC press]

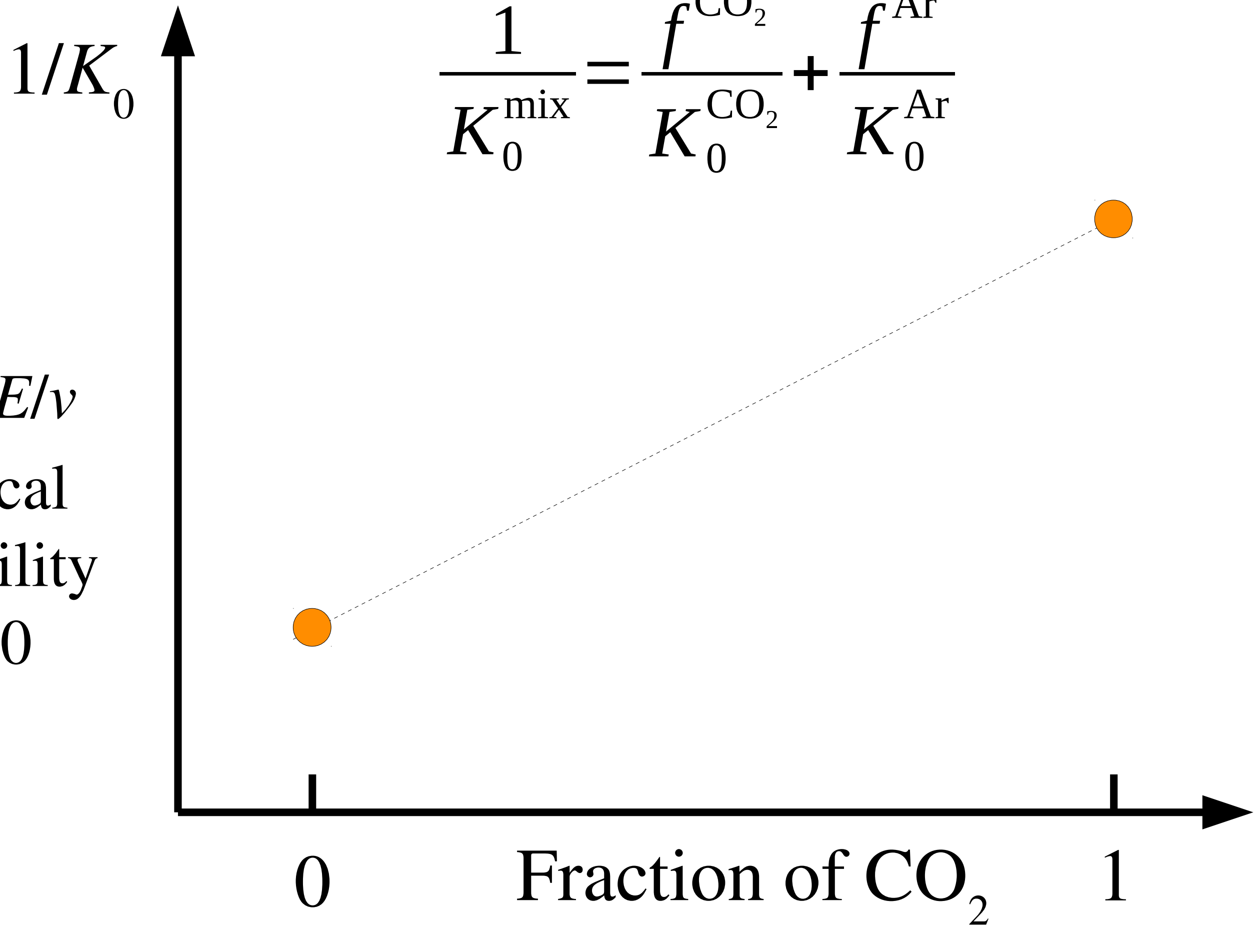
- much longer than the formation time $\tau = 7\text{-}20$ ps via 3-body association in 10 % CO_2 with Ar + CO_2 as “helpers”.

- Any isolated CO_2^+ rapidly binds again.

- $CO_2^+ \bullet (CO_2)_n$ probably lives shorter but will recombine. The cluster size *n* will therefore fluctuate at the ns time scale.

Blanc's mobility interpolation

$$\frac{1}{K_0^{\mathrm{mix}}} = \frac{f^{\mathrm{CO_2}}}{K_0^{\mathrm{CO_2}}} + \frac{f^{\mathrm{Ar}}}{K_0^{\mathrm{Ar}}}$$

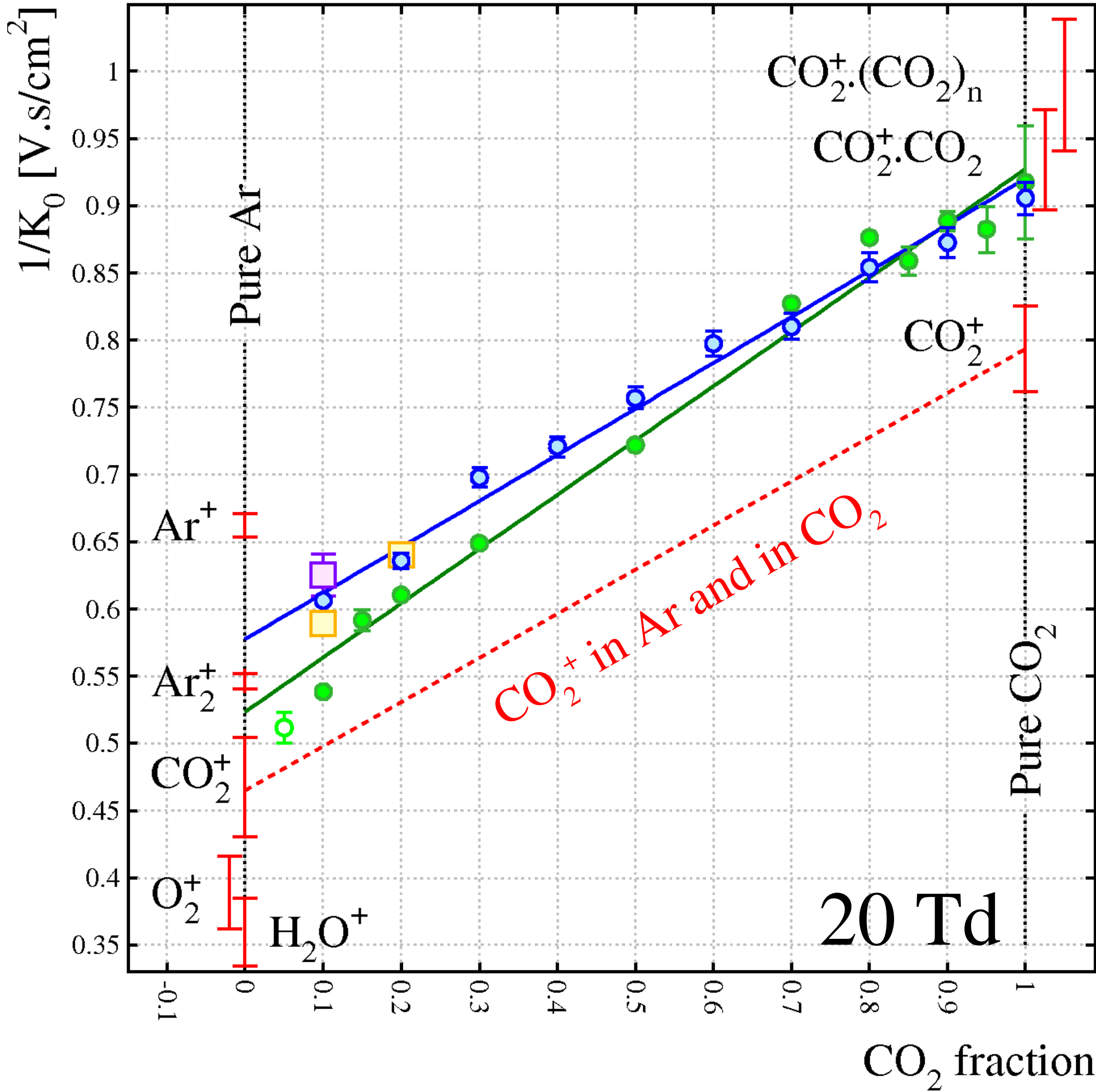


$1/K_0 = E/v$
reciprocal
of mobility
for $E \downarrow 0$

[A. Blanc, *Recherches sur les mobilités des ions dans les gaz*, J. Phys. Theor. Appl. **7** (1908) 825-839, 10.1051/jphystap:019080070082501]

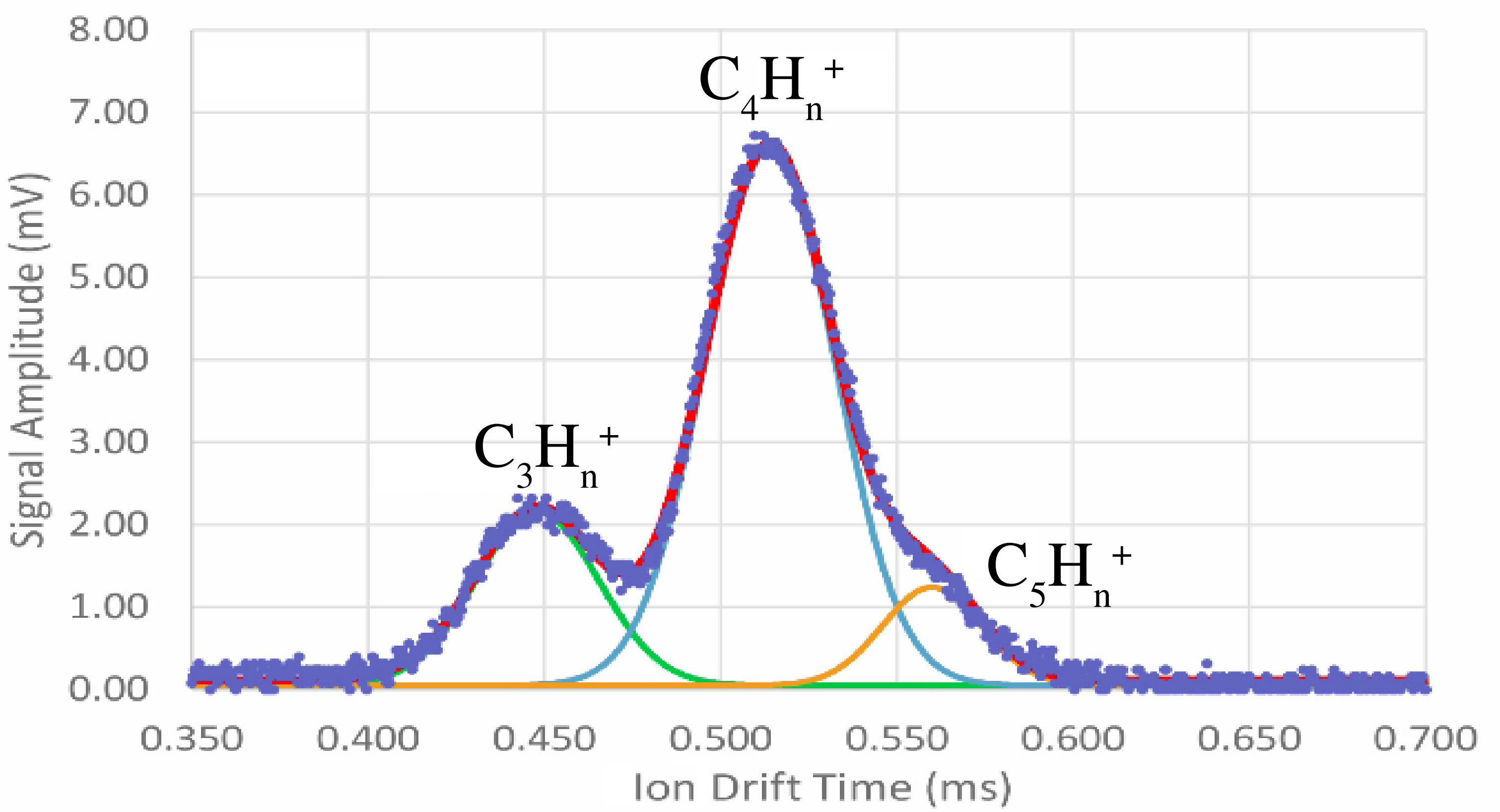
Blanc diagram for Ar-CO_2

- **Coxon**: pure CO_2
- **Schultz**: 1 atm
- **Coimbra**: 0.01 atm, smaller clusters ?
- **NA49**: 1 atm
- **ALICE:** 1 atm, water clusters ?



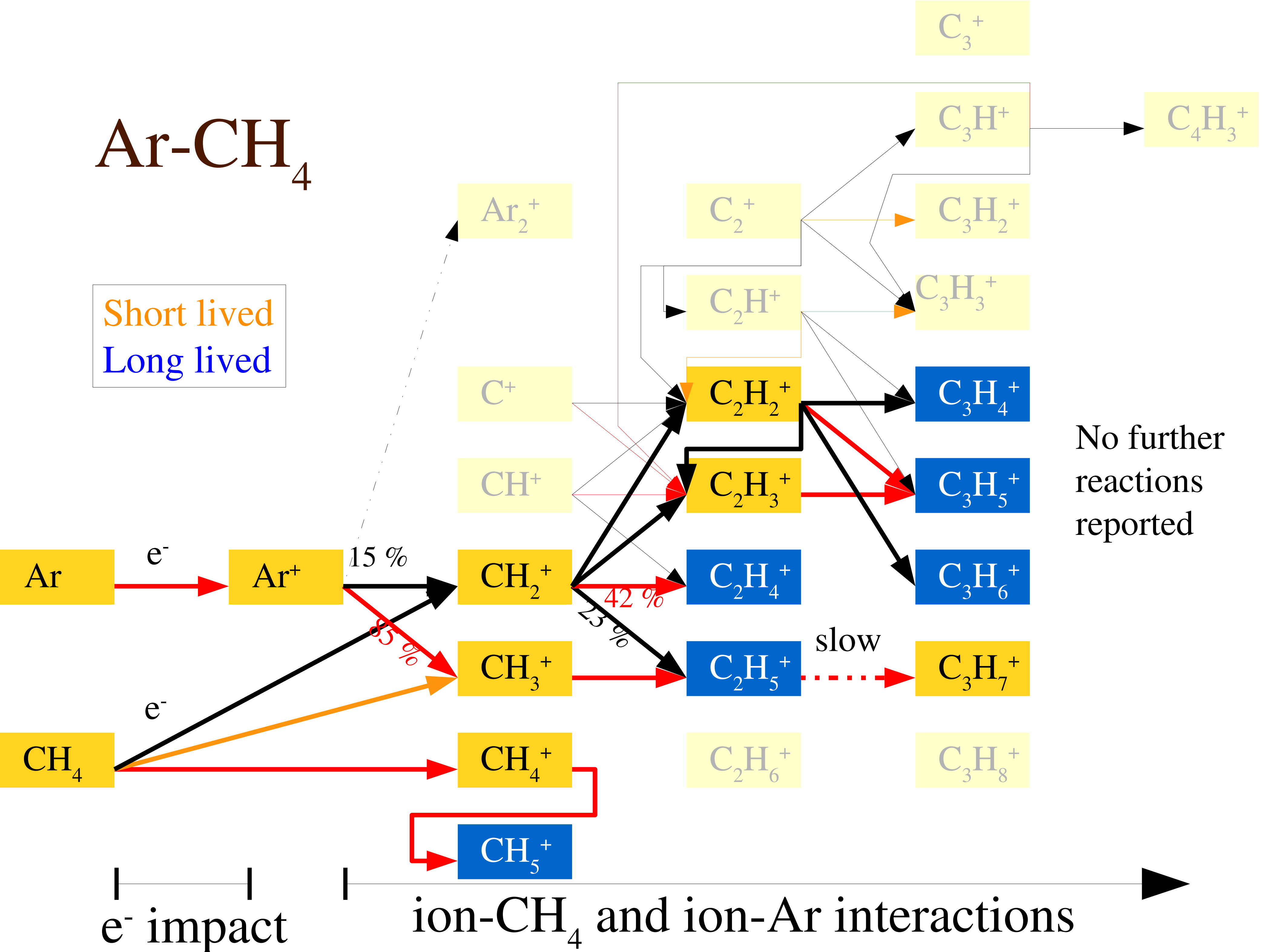
How about alkanes ?

- Ar 90 % - C_2H_6 10 %, at low pressure.
- Expect Ar^+ or C_2H_6 but ... none are seen – why ?



[André Cortez et al. 10.1088/1748-0221/8/12/P12012]

Ar-CH_4



Short lived

Long lived

Ar $\xrightarrow{e^-}$ Ar^+

CH_4 $\xrightarrow{e^-}$

15 %

85 %

42 %

23 %

slow

Ar_2^+

C^+

CH^+

CH_2^+

CH_3^+

CH_4^+

CH_5^+

C_2^+

C_2H^+

$C_2H_2^+$

$C_2H_3^+$

$C_2H_4^+$

$C_2H_5^+$

$C_2H_6^+$

C_3^+

C_3H^+

$C_3H_2^+$

$C_3H_3^+$

$C_3H_4^+$

$C_3H_5^+$

$C_3H_6^+$

$C_3H_7^+$

$C_3H_8^+$

$C_4H_3^+$

No further reactions reported

e^- impact

ion-CH_4 and ion-Ar interactions

Summary

- Quenchers play a major role in electron transport:
    - increase drift velocity
    - decrease diffusion

- Some quenchers increase the gain:
    - Penning effect

- Gain fluctuations are influenced

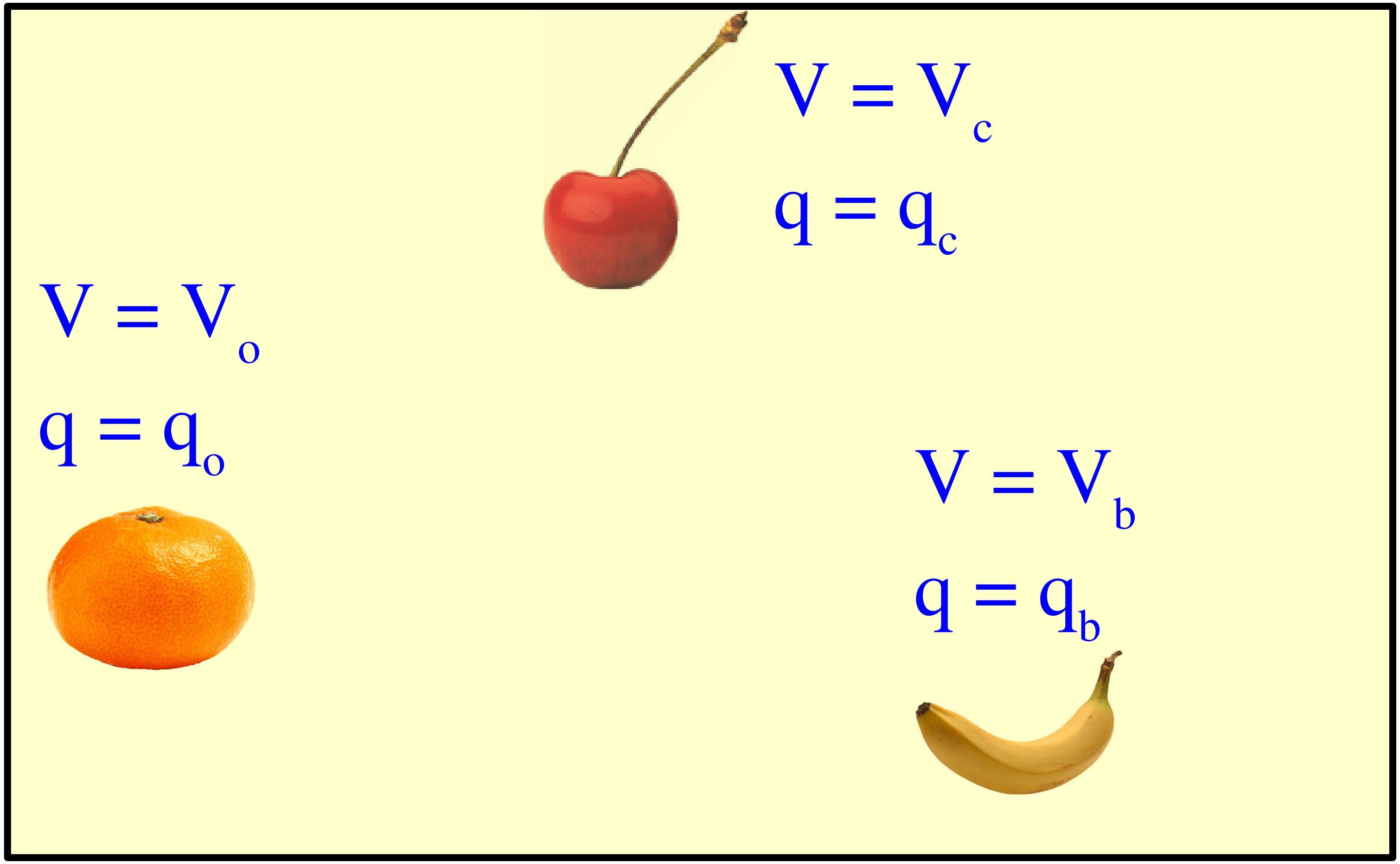
- The ions responsible for the signal are almost never noble gas ions like Ar^+, but heavy alkanes, dimers, trimers, CO_2 clusters ...

Signals

Signals

- Remains reading the signals induced by the electrons and ions moving around in the chamber.
- The charge of the electrons and ions tries to change the voltage of the electrodes.
- The electronics compensates for this by supplying charge.

Current induction



$V = V_c$

$q = q_c$

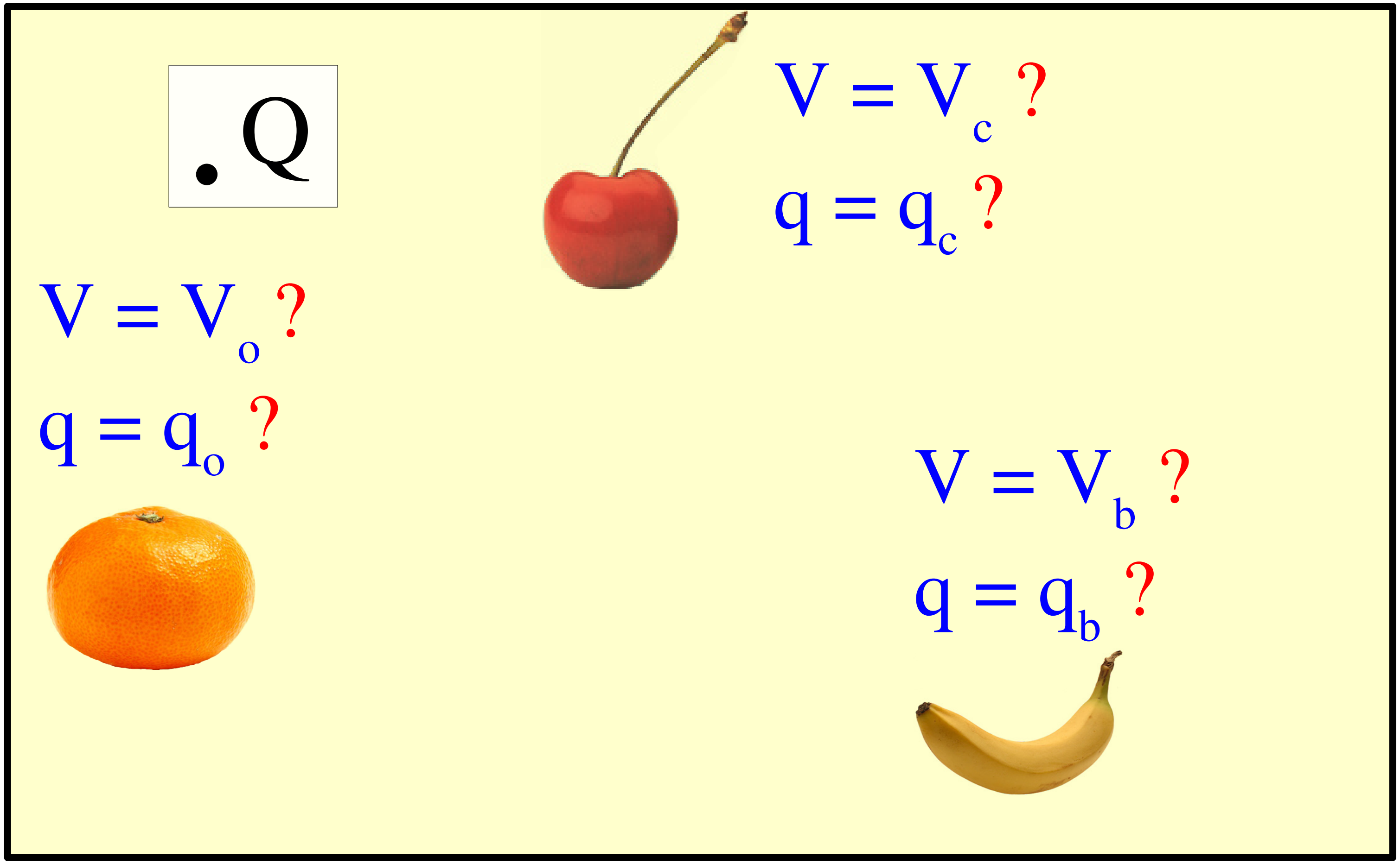
$V = V_o$

$q = q_o$

$V = V_b$

$q = q_b$

Current induction



.Q

$V = V_c$?

$q = q_c$?

$V = V_o$?

$q = q_o$?

$V = V_b$?

$q = q_b$?

Current induction

$.Q$

$V = V_c + \Delta V_c$

$q = q_c$

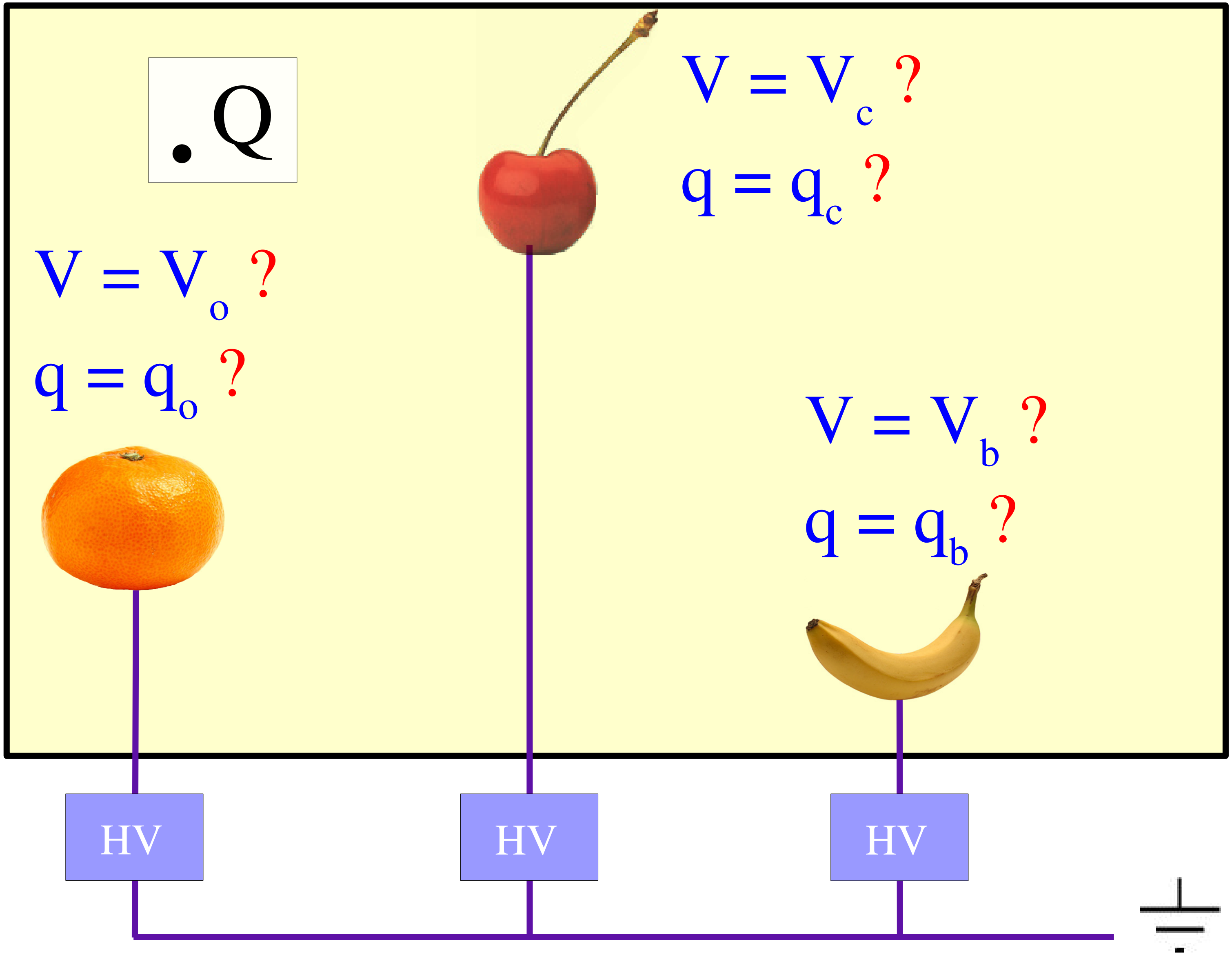
$V = V_o + \Delta V_o$

$q = q_o$

$V = V_b + \Delta V_p$

$q = q_b$

Current induction



$.Q$

$V = V_c$?

$q = q_c$?

$V = V_o$?

$q = q_o$?

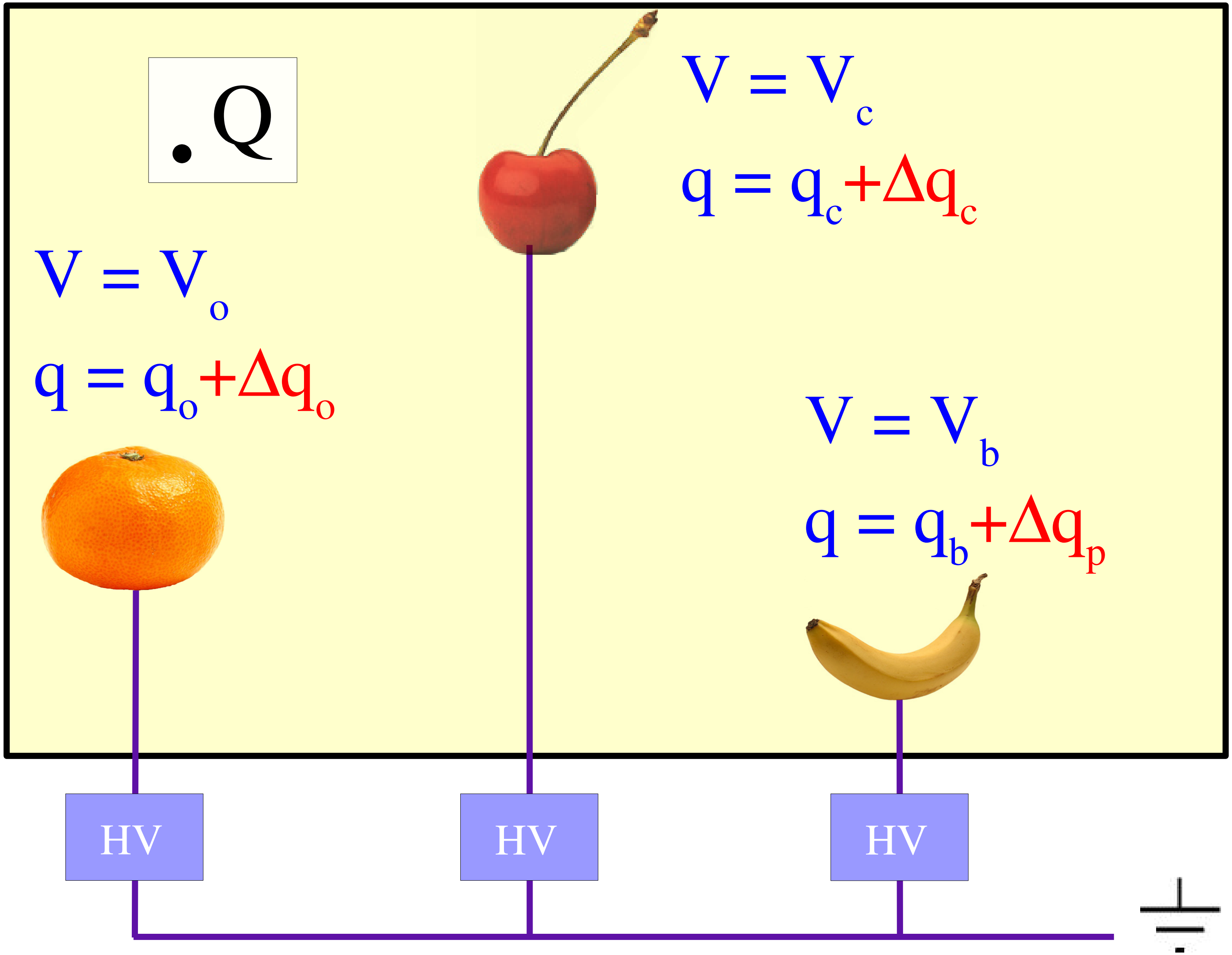
$V = V_b$?

$q = q_b$?

Current induction

No charge creation:

$\Delta q_o + \Delta q_c + \Delta q_p = 0$



.Q

$V = V_c$

$q = q_c + \Delta q_c$

$V = V_o$

$q = q_o + \Delta q_o$

$V = V_b$

$q = q_b + \Delta q_p$

Trying to guess the signals ...

- Properties of the current induced in an electrode:
  - proportional to the charge Q;
  - proportional to the velocity of the charge $\vec{v}_{\mathrm{d}}$;
  - dependent on the geometry.

- This leads to the following ansatz:

$$I = -Q\,\vec{v}_{\mathrm{d}} \cdot \vec{E}_{\mathrm{W}}$$

(the sign is mere convention, see next slide)

- The geometry is contained in $\vec{E}_{\mathrm{W}}$, necessarily a vector, the *weighting field*:
  - each electrode has its own weighting field;
  - unit of the weighting field ?

Signs of Current and Weighting field

- Sign of the current:
  - Signal current is (by convention) positive if positive charge flows from the read-out electrode to ground (via HV).

- Orientation of $\vec{E}_\mathrm{w}$:
  - place a positive charge Q on the surface of the read-out;
  - move it away from the read-out;
  - positive charge flows in to compensate: negative current;
  - to make the signs match, $\vec{v}_\mathrm{d} \cdot \vec{E}_\mathrm{w}$ needs to be positive;
  - $\vec{E}_\mathrm{w}$ points *away* from the electrodes being read out;
  - $\vec{E}_\mathrm{w}$ points *into* all other electrodes.

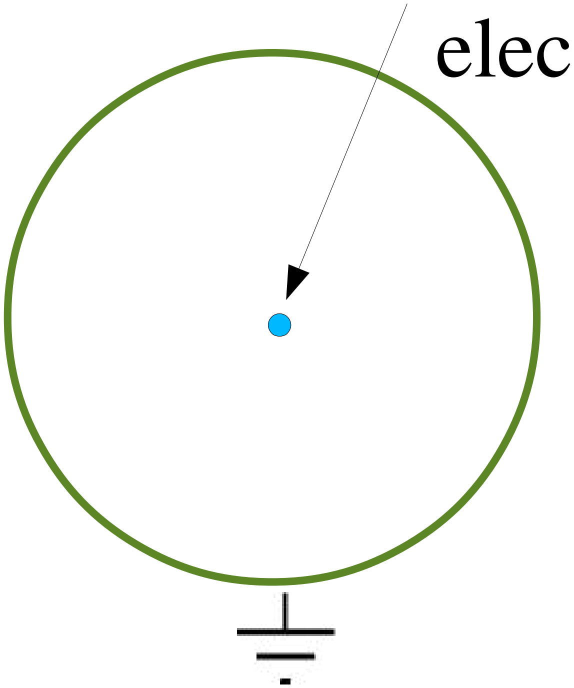
- Many people invert the signs – which is perfectly fine.

Weighting field – examples

- The weighting field is often easy to guess:

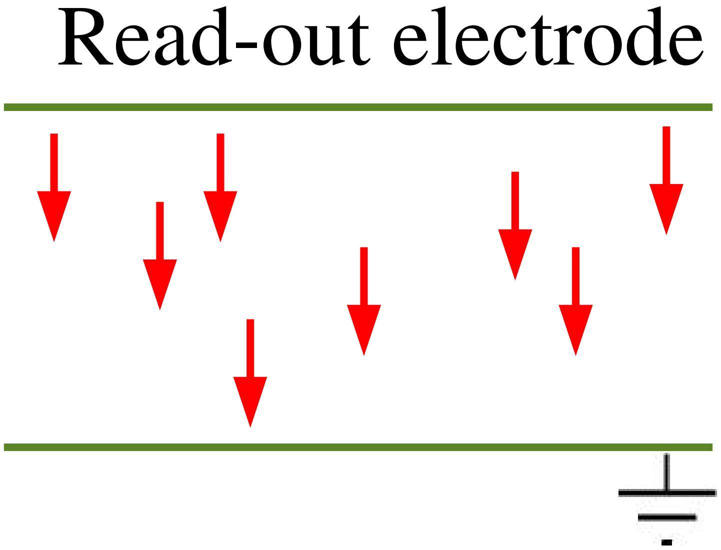
Read-out electrode

Read-out electrode

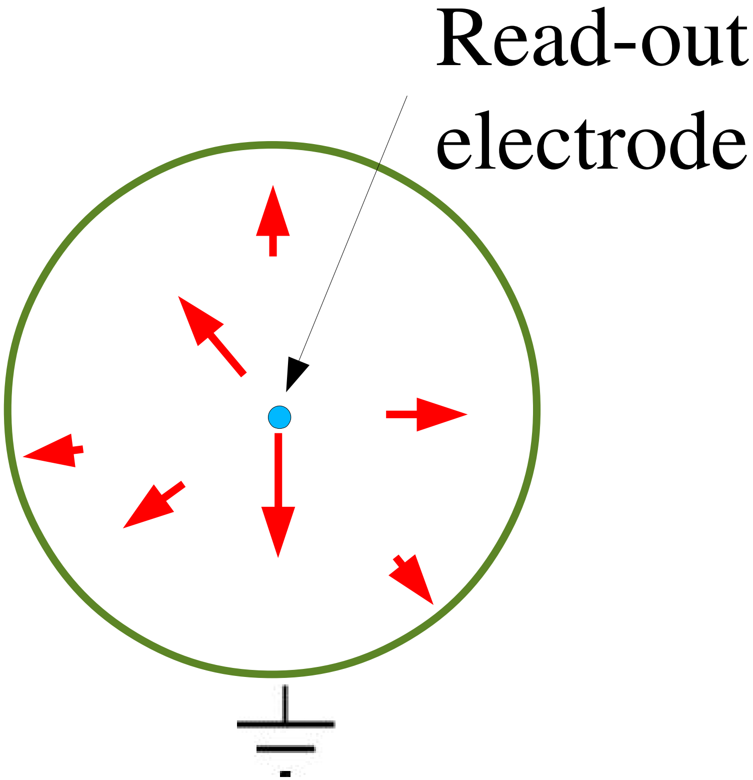


Weighting field – examples

- The weighting field is often easy to guess:



Read-out electrode



Read-out electrode

Weighting fields – more in general

- Claim: $\vec{E}_\mathrm{w}$ for a given read-out electrode can be computed the same way as a potential:
  - read-out electrode set to 1;
  - all other electrodes set to 0;
  - note ... 0 and 1, not 0 V and 1 V !

  - the resulting potential is called “weighting potential”;
- This is plausible considering examples, and is proven using Green's reciprocity.


George Green's father's mill (Nottingham)

1828: George Green's work

- The basic techniques to solve electrostatics problems, still used today, were published by George Green in:
  " An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism".

  "(...) it was written by a young man, who has been obliged to obtain the little knowledge he possesses, at such intervals and by such means, as other indispensable avocations which offer but few opportunities of mental improvement, afforded."

- Now available from http://arxiv.org/pdf/0807.0088v1, originally only 53 copies were printed, only for the subscribers.

[Original printed for the author by T Wheelhouse, Nottingham (1828). Facsimile Mayer & Müller, Berlin (1889), scanned by Google books.]

Integrating the current

- Net charge over a trajectory $z(t)$:

$$
\begin{aligned}
Q_{\text{net}} &= \int_{t_{\text{start}}}^{t_{\text{end}}} I(t) \\
&= -\int_{t_{\text{start}}}^{t_{\text{end}}} Q \; E_{\text{W}}(z(t)) \cdot v_{\text{drift}}(z(t))\,\mathrm{d}t \\
&= \int_{t_{\text{start}}}^{t_{\text{end}}} Q \; \frac{\mathrm{d}V_{\text{W}}(z)}{\mathrm{d}z} \cdot \frac{\mathrm{d}z}{\mathrm{d}t} \; \mathrm{d}t \\
&= Q \; \left(V_{\text{W}}(z_{\text{end}}) - V_{\text{W}}(z_{\text{start}})\right)
\end{aligned}
$$

$$E_{\text{W}} = -\nabla V_{\text{W}}$$

- By construction, all electrodes have $V_{\text{W}} = 0$ or $V_{\text{W}} = 1$.
- Hence, the integral of the current between electrodes can only be -1, 0 or 1.

Sum of all currents

- Summing the current on all electrodes, observe that $E_\mathrm{W}^\mathrm{tot} = \sum_i E_\mathrm{W}^i$ is computed by placing a unit weighting potential on every electrode, exactly once.

- The weighting potential is constant and the weighting field vanishes:

$$\begin{aligned} I_\mathrm{tot}(t) &= \sum_i I_i(t) = \sum_i -Q\, v_\mathrm{d} \cdot E_\mathrm{W}^i = -Q\, v_\mathrm{d} \cdot \sum_i E_\mathrm{W}^i \\ &= 0 \end{aligned}$$

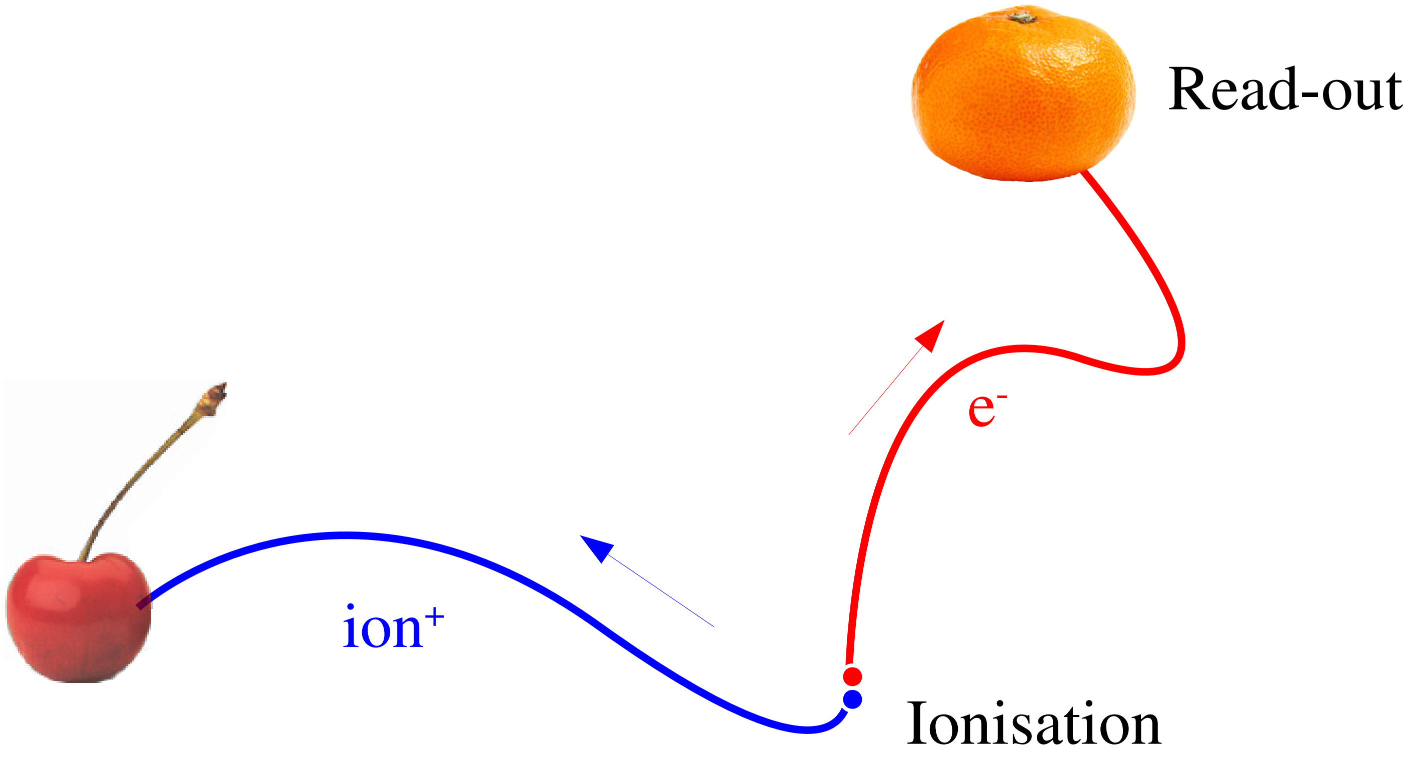
- Thus, the sum of all currents is zero at all times.

Changing sign of charge and velocity

- Observe that the following are equivalent in terms of *total* charge induced – the time dependence will differ:

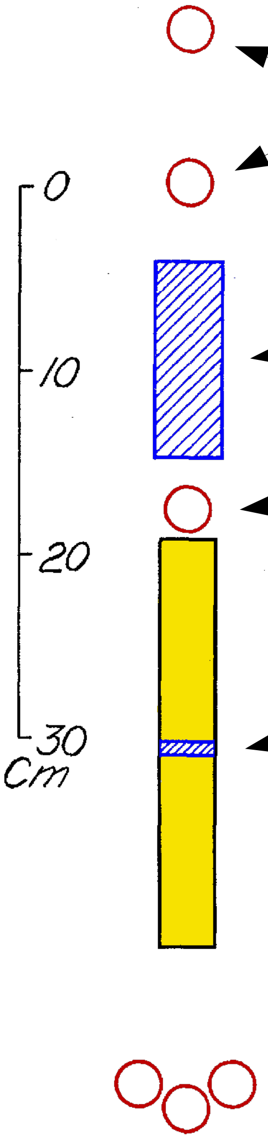
Combined e^- - ion^+ current

- How about the total current induced by an e^- - ion^+ pair ?



Not used

Muon mass estimate



Trigger counters

Lead absorber

Trigger counter

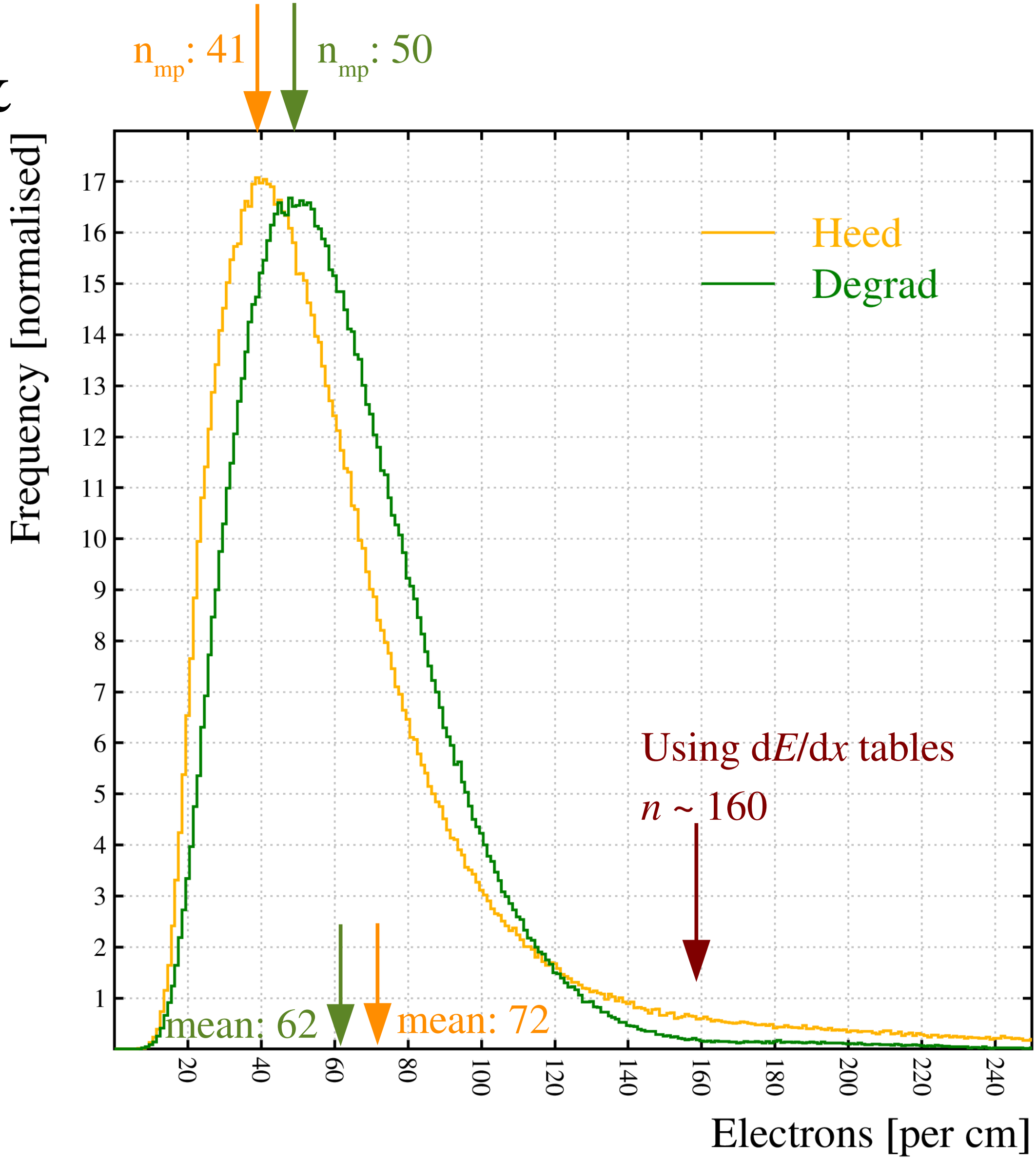
Cloud chamber,
lead filter in the middle,
0.35 T magnetic field

Veto counter

[J. C. Street and E. C. Stevenson, Phys. Rev 52 (1937) 1003]

How many ionisation e^- per cm ?

- Heed, a photo-absorption & ionisation model, finds for a minimum ionising $\mu^{\pm}$:
  - Peak: $n_e = 41/\mathrm{cm}$
  - "Mean": $n_e = 72/\mathrm{cm}$
- Degrad, an e^- transport program, finds for an e^- at the same $\beta\gamma$:
  - Peak: $n_e = 50/\mathrm{cm}$
  - "Mean": $n_e = 62/\mathrm{cm}$
- Mean is ill-defined due to rare but large deposits.

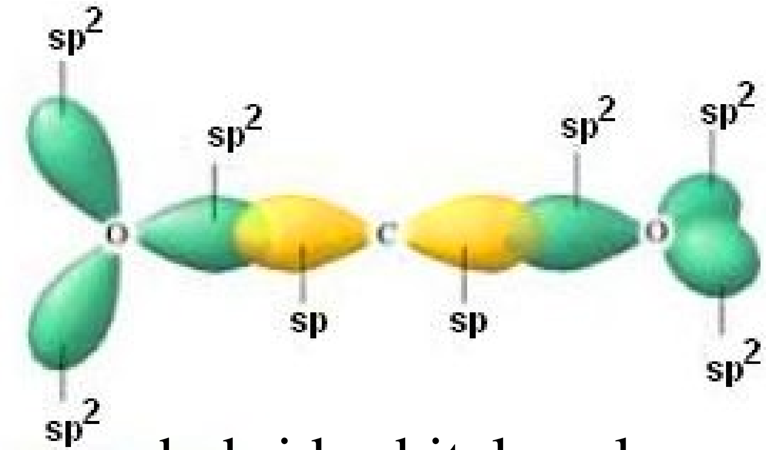


Attachment

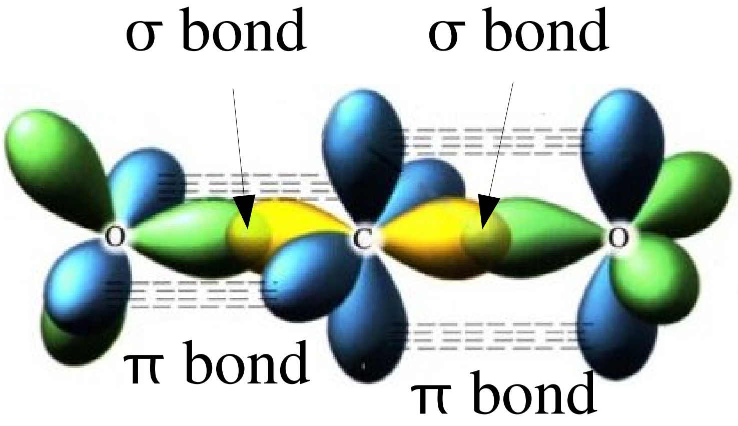
- Some of the quencher gases have the potential of attaching electrons.
- Examples include:
  - O_2: mostly 3-body O_2^- and at higher ϵ 2-body dissociative;
  - H_2O: $[H_2O]_n$ has positive electron affinity, H_2O does not;
  - CF_4: mostly dissociative $F^- + CF_3$, $F + CF_3^-$ (below 10 eV);
  - SF_6: SF_6^- up to 0.1 eV, $\sigma=10^{-18}$ cm^2, then $F^- + SF_n^-$ (n=3, 4, 5)
  - CS_2: negative ion TPC;
  - CO_2: O^-, $[CO_2]_n^-$ but no CO_2^- (4 eV and 8.2 eV).

Attachment in CO_2

- CO_2 is a linear molecule:



hybrid orbitals only,
p-orbitals not shown



[Source: presumably SS Zumdahl, Chemistry (1983) DC Heath and Company.]

Direct vs Exchange ionisation