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Ettore Majorana Lectures 2017

Challenges in Early & Late-Time Cosmology

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III. Inhomogeneities and precision cosmology

INTRODUCTION

- Most of cosmology is based on observing & interpreting light (or light-like) signals.

- Such signals travel on null geodesics lying on our past light cone (PLC).

- In a FLRW space-time it's easy to define our PLC and describe geodesics therein.

- In the presence of inhomogeneities our PLC and its null geodesics become messy.

- Q: Can we simplify our life by a suitable choice of coordinates?

- And, if yes: What can we do with them?

OUTLINE

- The GLC gauge & its properties
- Light-cone averaging in GLC coordinates
- Average (and dispersion in the) Hubble diagram for a "realistic" Universe

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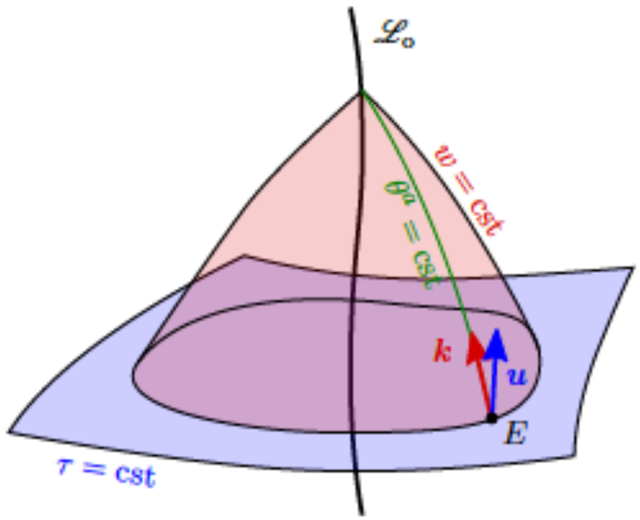
- Other applications:
  - ✳ Determination of local H_0
  - ✳ Number counts
  - ✳ Higher order deflection & CMB lensing
  - ✳ TOF of UR particles

The geodetic light cone (GLC) gauge

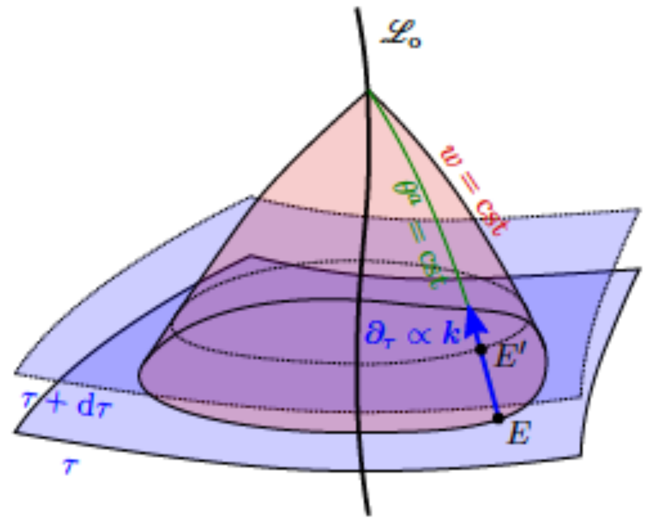
(Gasperini, Marozzi, Nugier & GV, 1104.1167)

A more “constructive” approach

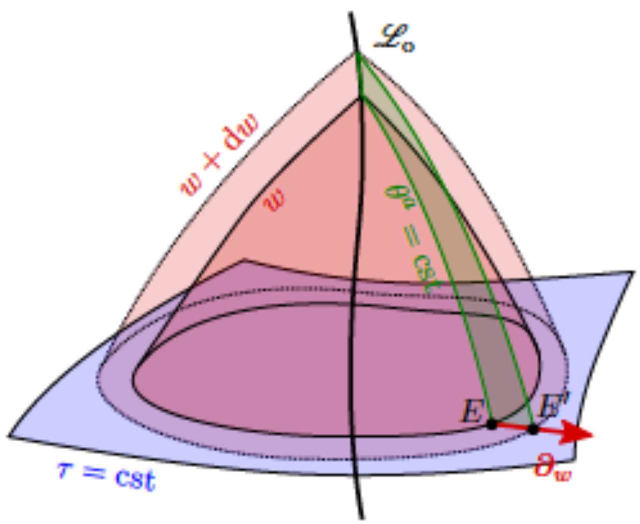
(P. Fleury, F. Nugier, G. Fanizza, 1602.04461)



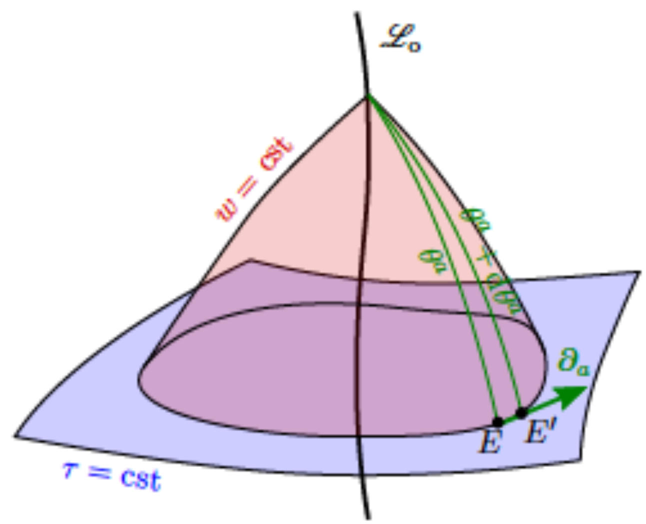
(a) Definition of the GLC coordinates τ, w, θ^a. The curve $\mathscr{L}_o$ is the observer's worldline.



(b) Basis vector $\boldsymbol{\partial}_\tau$ is tangent to the $w, \theta^a = \mathrm{cst}$ lines, hence parallel to $\boldsymbol{k}$.



(c) Basis vector $\boldsymbol{\partial}_w$ is tangent to the $\tau, \theta^a = \mathrm{cst}$ lines. It defines a notion of radial direction.



(d) Basis vectors $\boldsymbol{\partial}_a$ are tangent to the $\tau, w, \theta^{b \neq a} = \mathrm{cst}$ lines.

The geodetic light cone (GLC) gauge

(Gasperini, Marozzi, Nugier & GV, 1104.1167)

An almost fully gauge-fixed variant of the "observational coordinates" of G. Ellis et al.
The metric w.r.t. the coordinates (τ, w, θ^a):

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\theta^a - U^a dw)(d\theta^b - U^b dw) \quad ; \quad a,b = 1,2$$

$$g_{\mu\nu} = \begin{pmatrix} 0 & -\Upsilon & \vec{0} \\ -\Upsilon & \Upsilon^2 + U^2 & -U_b \\ \vec{0}^T & -U_a^T & \gamma_{ab} \end{pmatrix} \quad , \quad g^{\mu\nu} = \begin{pmatrix} -1 & -\Upsilon^{-1} & -U^b/\Upsilon \\ -\Upsilon^{-1} & 0 & \vec{0} \\ -(U^a)^T/\Upsilon & \vec{0}^T & \gamma^{ab} \end{pmatrix}$$

Flat-FRW limit ($a(\eta)d\eta = dt$, η = conformal time):

$$\tau = t, \qquad w = r + \eta, \qquad \Upsilon = a(t)$$

$$U^a = 0, \qquad \gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2).$$

Two residual gauge freedoms

- $w \to w'(w)$ allows to relabel the light cones and to set $Y = 1$ along L_0

- $\theta^a \to \theta'^a(w, \theta^a)$ allows to relabel the angles and to set $U_a = 1$ along L_0

Generic properties of GLC coordinates

- w =(<) w_0 defines our past light cone (causal past)
- w = constant hypers. provide a null-foliation
- τ can be identified with synchronous-gauge time
- Static geodetic observers in SG have $u_\mu = \partial_\mu \tau$
- Photons travel at fixed w and θ^a :

$$k_\mu = \partial_\mu w \Rightarrow \dot{x}^\mu \sim \delta^\mu_\tau$$

Other nice properties of the GLCG

1. A simple, exact expression for the redshift

In FRW cosmology z is simple (& factorizes) in terms of entries of the standard FRW metric

$$(1+z_s) = \frac{a(\eta_o)}{a(\eta_s)}$$

In the GLC gauge this property remains true:

$$(1+z_s) = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{\Upsilon_o}{\Upsilon_s}\frac{(u_\tau)_s}{(u_\tau)_o} \to \frac{\Upsilon_o}{\Upsilon_s}$$

Ratio depends in general from the θ^a coordinates

2. An exact & factorized expression for the Jacobi Map

(Fanizza, Gasperini, Marozzi, GV, 1308.4935)

Recall deviation equation for null geodesics:

$$\nabla_\lambda^2 \xi^\mu = R_{\alpha\beta\nu}{}^\mu k^\alpha k^\nu \xi^\beta \quad ; \quad \nabla_\lambda \equiv k^\alpha \nabla_\alpha$$

projected along the Sachs basis:

$$s_A^\mu (A = 1, 2) \; ; \; s_A^\mu u_\mu = s_A^\mu k_\mu = 0 \; ; \; g_{\mu\nu} s_A^\mu s_B^\nu = \delta_{AB} \; ; \; \xi^A = \xi^\mu s_\mu^A$$

$$\Pi_\nu^\mu \nabla_\lambda s_A^\nu = 0 \quad ; \quad \Pi_\nu^\mu = \delta_\nu^\mu - \frac{k^\mu k_\nu}{(u^\alpha k_\alpha)^2} - \frac{k^\mu u_\nu + u^\mu k_\nu}{u^\alpha k_\alpha}$$

$$\frac{d^2 \xi^A}{d\lambda^2} = R_B^A \xi^B \; ; \; \frac{d}{d\lambda} \equiv k^\mu \partial_\mu \; ; \; R_B^A \equiv R_{\alpha\beta\nu\mu} k^\alpha k^\nu s_B^\beta s_A^\mu$$

$$\frac{d^2\xi^A}{d\lambda^2} = R^A_B \xi^B \; ; \; \frac{d}{d\lambda} \equiv k^\mu \partial_\mu \; ; \; R^A_B \equiv R_{\alpha\beta\nu\mu} k^\alpha k^\nu s^\beta_B s^\mu_A$$

Def. of J: $$\xi^A(\lambda_s) = J^A_B(\lambda_s, \lambda_o) \left(\frac{k^\mu \partial_\mu \xi^B}{k^\nu u_\nu} \right)_o$$ $J^A{}_B$ obeys:

$$\frac{d^2}{d\lambda^2} J^A_B(\lambda, \lambda_o) = R^A_C J^C_B \; ; \; J^A_B(\lambda_o, \lambda_o) = 0 \; ; \; \frac{d}{d\lambda} J^A_B(\lambda_o, \lambda_o) = \delta^A_B \left(k^\nu u_\nu\right)_o$$

FGMV: exact expression for J in GLCG!

$$J^A_B(\lambda, \lambda_o) = s^A_a(\lambda) \left\{ \left[\left(\frac{k^\mu \partial_\mu s}{k^\mu u_\mu} \right)^{-1} \right]^a_B \right\}_{\lambda=\lambda_o} \; ; \quad s^A_a s^A_b = \gamma_{ab}$$

Again (bi)local and factorized ($s_a{}^A$ = zweibeins for γ_{ab}) in this gauge (NB: expression is NOT covariant!)

3. Area & luminosity distance (d_A, d_L)

(Ben-Dayan, Gasperini, Marozzi, Nugier & GV, 1202.1247 & FGMV 1308.4935)

Much easier if one has the Jacobi map!

$$d_A^2 = \det\left(J_B^A(\lambda_s, \lambda_o)\right) = \frac{\sqrt{\gamma(\lambda_s)}}{\det\left(u_\tau^{-1}\partial_\tau s_b^B\right)_{\lambda=\lambda_o}} \; ; \; \gamma \equiv \det \gamma_{ab}$$

$$\det\left(u_\tau^{-1}\partial_\tau s_b^B\right)_{\lambda=\lambda_o} = \frac{1}{4}\left[\det\left(u_\tau^{-1}\partial_\tau \gamma^{ab}\right)\gamma^{3/2}\right]_o$$

Using residual gauge freedom in GLCG:

$$d_A^2 = \frac{\sqrt{\gamma}}{\sin\theta}$$

& finally: $d_L = (1+z)^2 d_A$

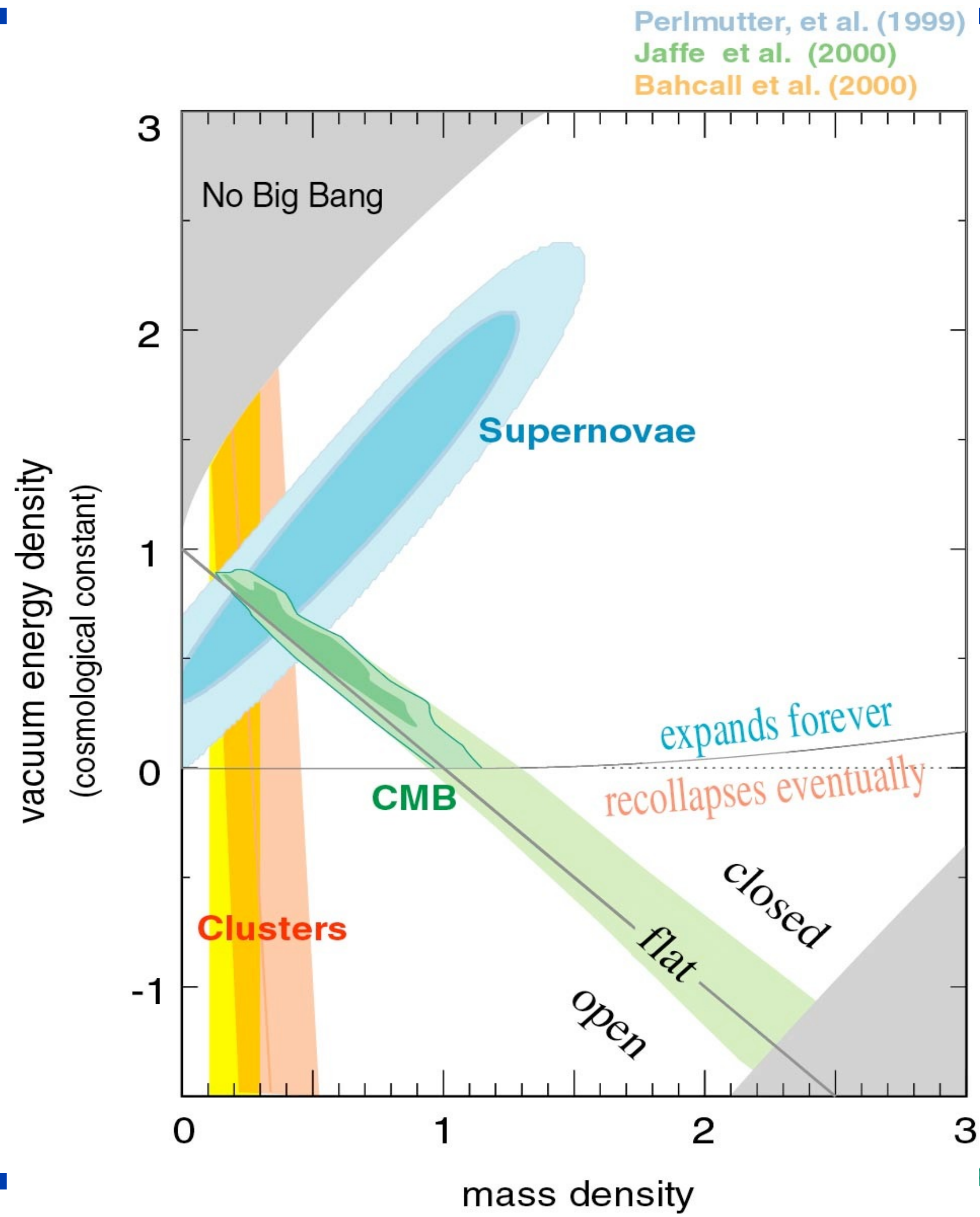
I: The inhomogeneous Hubble diagram in a realistic cosmology

For a complete summary of our (and related) work see: F. Nugier's thesis: 1309.65420

The concordance model:

3 sets of data pointing at Dark Energy

Cosmic Concordance



Two arguments for DE are based on inhomogeneities/structures
The 3rd (SNIa) ignores them completely!

Basic tool: the famous Hubble diagram of redshift vs. luminosity-distance

A short reminder (for FLRW)

Definition of luminosity distance d_L:

$$\Phi = \frac{L}{4\pi d_L^2}$$

where L is the absolute luminosity and Φ the flux.

For FLRW: $1 + z(t) = \frac{a_0}{a(t)}$ $\qquad q_0 \equiv -\frac{a\ddot{a}}{\dot{a}^2}(t = t_0)$

For a spatially flat ΛCDM Universe (for simplicity):

$$d_L^{FLRW}(z) = \frac{1+z}{H_0}\int_0^z \frac{dz'}{[\Omega_{\Lambda 0} + \Omega_{m0}(1+z')^3]^{1/2}}$$

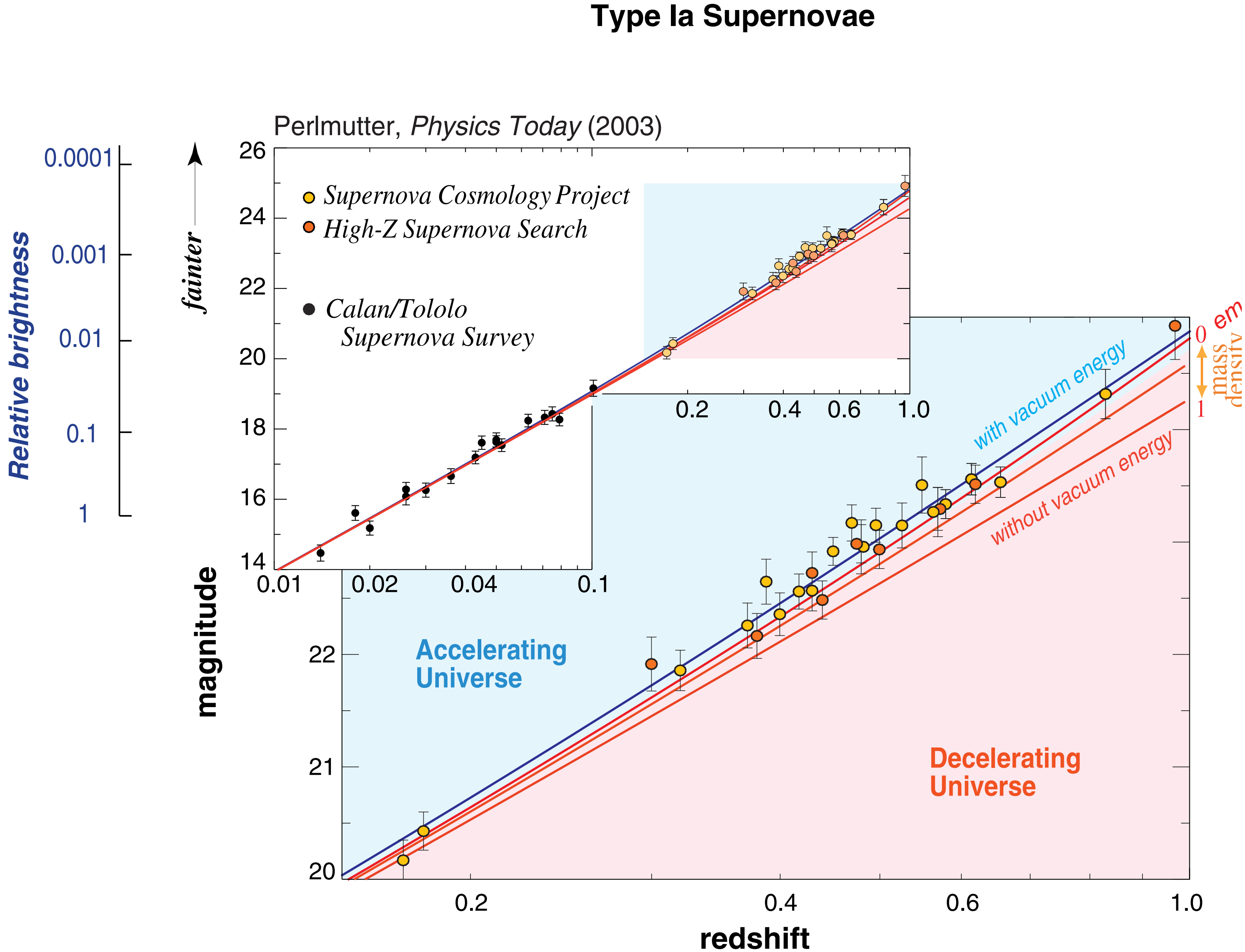
If expanded to 2^{nd} order in z:

$$d_L(z) = H_0^{-1}\left[z + \frac{1}{2}(1 - q_0)z^2 + O(z^3)\right]$$

In FLRW cosmology: $q_0 = \frac{4\pi G(\rho_0 + 3p_0)}{3H_0^2} = \frac{1}{2}(\Omega_{m,0} - 2\Omega_{\Lambda,0})$

Hubble law beyond linear order => information about eq. of state!

Using Type Ia supernovae as standard candles: evidence for negative q_0, DE...



The Universe is fairly homogeneous only on very large scales (> few 100 Mpc?).

Q: What's the effect of smaller scale inhomogeneities?

A. Not obvious! Averages of physical quantities do not obey the homogeneous EEs (Buchert & Ehlers, Buchert,...).

There are extra, so-called "backreaction", terms. This "averaging problem" has been a rather hot topic in recent years.

Hopes have been raised that inhomogeneities might "explain" cosmic acceleration and give a natural resolution of the famous coincidence (why now?) problem (Buchert, Rasanen, Kolb-Matarrese-Riotto...)

Too optimistic (given other evidence for DE)? Yet still important to take inhomogeneities into account for (future) precision cosmology and/or for testing the concordance model itself.

Most of previous work deals with spatial averages and with formal definitions of acceleration.

Not clear what's the relation between such averages and the averaged d_L-z relation (Hubble diagram)

We therefore looked at how to average directly that relation.

Gauge-invariant light-cone averages

(Gasperini, Marozzi, Nugier & GV, 1104.1167)

WHAT'S THE CORRECT MEASURE?

(G. Marozzi, G. Veneziano et al, unpublished,
P. Fleury, C. Clarkson, R. Maartens, 1612.03726)

An important issue is whether one should average the physical quantity (e.g. d_L^{-2}) with a non trivial measure.

In our SNe papers we took as measure the proper area of the 2-D (fixed-z) surface element. If the proper number density of SNe is constant on a fixed-z hyper surface our procedure gives the measured average!

Kaiser & Hudson discuss other measures as well (e.g. a "galaxy-averaged bias").

FCM argue in favor of a number-count weighting.

For CMB averaging on the last-scattering surface one may argue that the correct measure is yet different.

Averaging the flux at 2nd-order

(BGMNV,1207.1286, 1302.0740; BGNV,1209.4326)

Considering < Φ > ~ < d_L^{-2} > (not < d_L > $^{-2}$) simplifies life further. In GLCG (w/ our measure):

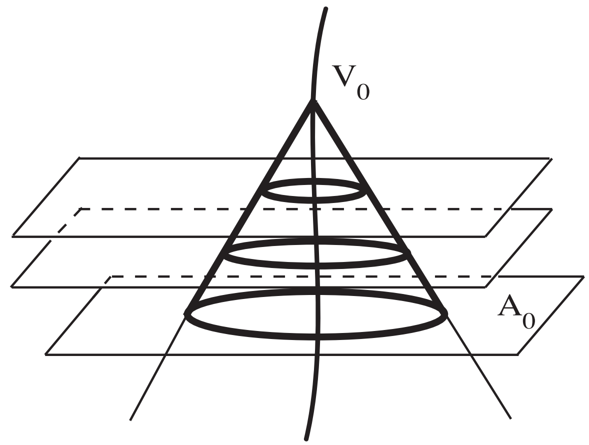
$$\langle d_L^{-2}\rangle(z_s, w_0) = (1+z_s)^{-4}\left[\int \frac{d^2\theta}{4\pi}\gamma^{\frac{1}{2}}(w_0, \tau_s(z_s, \theta^a), \theta^b)\right]^{-1}$$

where $\tau_s(z_s, \theta^a)$ is the solution of:

$$(1+z_s) = \frac{\Upsilon(w_0, \tau_0, \theta^a)}{\Upsilon(w_0, \tau_s, \theta^a)}$$

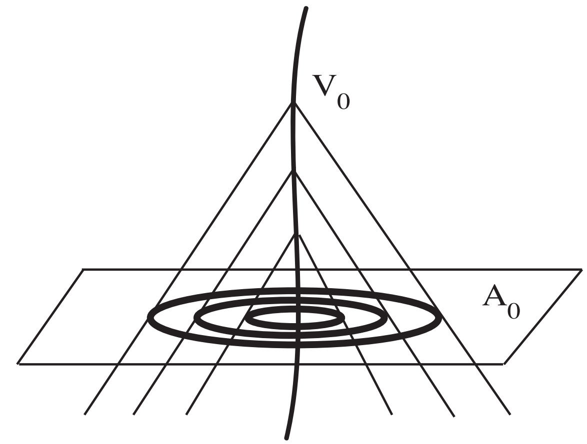
Intersection of $w = w_0$ and $z = z_s$ hypersurfaces is a 2-surface (topologically a sphere) on which SNe of given redshift z_s are located.

truncated light cone



(a) $I(1;V_0;A_0)$

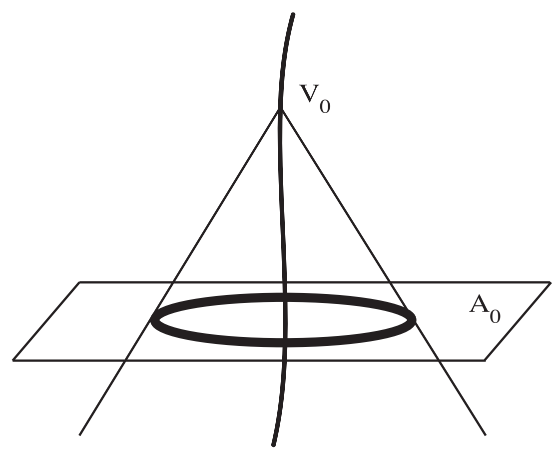
causally connected sphere



(b) $I(1;A_0;V_0)$

2-sphere embedded in the light cone

Relevant for this talk



(c) $I(1;V_0,A_0;-)$

This is exact: can be used for any specific (fixed-geometry) inhomogeneous model (e.g. LTB with us at center)

A more realistic (and Copernican) model is the one produced by inflation: a stochastic background of perturbations with statistical isotropy and homogeneity.
Vanishing effects at 1st order, need 2nd order (at least)

Unfortunately, perturbations are normally studied in other gauges (e.g. Newtonian or Poisson, synchronous): we need to find the coordinate transformation taking us from that gauge to the GLC up 2nd order.

Quite a lot of work, see F. Nugier's thesis, yet easier than starting directly in the Poisson Gauge (see e.g. Bernardeau, Bonvin, Vernizzi 0911.2244).

The calculation proceeds in two steps:

1. Calculation of d_L^{-2} to 2nd order in the Poisson gauge (BGNV,1209.4326) via coordinate transformation. Independent result by Umeh, Clarkson & Maartens (1207.2109, 1402.1933) being compared to ours (G. Marozzi, 1406.1135 and in progress).

2. Performing the appropriate LC integrals to compute the effect on different averages and on the corresponding dispersions. Part of the calculation is analytic, part is numerical using realistic power spectra (BGMNV,1302.0740).

See BGMNV 1207.1286 (prl) for a summary of both

$$\bar{\delta}^{(2)}_S(z_s,\tilde{\theta}^a)=\bar{\delta}^{(2)}_{path}+\bar{\delta}^{(2)}_{pos}+\bar{\delta}^{(2)}_{mixed}$$

$$\begin{aligned}
\bar{\delta}^{(2)}_{path} &= \Xi_s\Bigg\{-\frac{1}{4}\left(\phi^{(2)}_s-\phi^{(2)}_o\right)+\frac{1}{4}\left(\psi^{(2)}_s-\psi^{(2)}_o\right)+\frac{1}{2}\psi_s^2-\frac{1}{2}\psi_o^2-(\psi_s+J_2^{(1)})\partial_+Q_s \\
&+ \frac{1}{4}(\gamma_0^{ab})_s\partial_aQ_s\partial_bQ_s+Q_s\left(-\partial_+^2Q_s+\partial_+\psi_s\right)+\frac{1}{\mathcal{H}_s}\partial_+Q_s\,\partial_\eta\psi_s \\
&+ \frac{1}{4}\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\partial_+\left[\phi^{(2)}+\psi^{(2)}+4\psi\;\partial_+Q+\gamma_0^{ab}\;\partial_aQ\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a) \\
&- \frac{1}{2}\partial_a(\partial_+Q_s)\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\gamma_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\Bigg\} \\
&- \frac{1}{2}\psi^{(2)}_s-\frac{1}{2}\psi_s^2-K_2+\psi_sJ_2^{(1)}+\frac{1}{2}(J_2^{(1)})^2+J_2^{(1)}\frac{Q_s}{\Delta\eta}-\frac{1}{\mathcal{H}_s\Delta\eta}\left(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{1}{2}(\partial_+Q_s)^2 \\
&- \frac{2}{\mathcal{H}_s\Delta\eta}\psi_s\partial_+Q_s+\frac{1}{2}\partial_a\left(\psi_s+J_2^{(1)}+\frac{Q_s}{\Delta\eta}\right)\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\gamma_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right) \\
&+ \frac{1}{4}\partial_aQ_s\partial_+\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\gamma_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right) \\
&+ \frac{1}{16}\partial_a\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\gamma_0^{bc}\;\partial_cQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\partial_b\left(\int_{\eta_o}^{\eta_s^{(0)-}}d\bar{x}\;\left[\gamma_0^{ad}\;\partial_dQ\right](\eta_s^{(0)+},\bar{x},\tilde{\theta}^a)\right) \\
&- \frac{1}{4\Delta\eta}\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\phi^{(2)}+\psi^{(2)}+4\psi\;\partial_+Q+\gamma_0^{ab}\;\partial_aQ\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a) \\
&+ \frac{1}{\mathcal{H}_s}\partial_+Q_s\left\{-\partial_\eta\psi_s+\partial_r\psi_s+\frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\right\} \\
&+ Q_s\Bigg\{\partial_r\psi_s+\partial_+\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\frac{1}{(\eta_s^{(0)+}-x)^2}\int_{\eta_o}^{x}dy\Delta_2\psi(\eta_s^{(0)+},y,\tilde{\theta}^a)\right) \\
&+ \frac{1}{2\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\Bigg\} \\
&+ \frac{1}{16\sin^2\tilde{\theta}}\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\;\left[\gamma_0^{1b}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)^2, \qquad \text{(B.18)}
\end{aligned}$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{pos} &= \frac{\Xi_s}{2}\Bigg\{ (\partial_r P_s)^2 + (\gamma_0^{ab})_s \partial_a P_s \, \partial_b P_s - \frac{2}{\mathcal{H}_s}\left(\partial_r P_s - \partial_r P_o\right)\left(\mathcal{H}_s \partial_r P_s + \partial_r^2 P_s\right) \\
&- \int_{\eta_{in}}^{\eta_s^{(0)}} d\eta' \frac{a(\eta')}{a(\eta_s^{(0)})} \partial_r \left[\phi^{(2)} - \psi^2 + (\partial_r P)^2 + \gamma_0^{ab}\partial_a P \partial_b P\right](\eta', \Delta\eta, \tilde{\theta}^a) \Bigg\} \\
&+ \frac{1}{2\mathcal{H}_s \Delta\eta}\Bigg\{ (\partial_r P_o)^2 + \lim_{r\to 0}\left[\gamma_0^{ab}\partial_a P \partial_b P\right] \\
&- \int_{\eta_{in}}^{\eta_o} d\eta' \frac{a(\eta')}{a(\eta_o)} \partial_r \left[\phi^{(2)} - \psi^2 + (\partial_r P)^2 + \gamma_0^{ab}\partial_a P \partial_b P\right](\eta', 0, \tilde{\theta}^a) \Bigg\} \\
&- \frac{1}{2\mathcal{H}_s \Delta\eta}\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(\partial_r P_s - \partial_r P_o\right)^2 \ ,
\end{aligned} \tag{B.19}
$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{mixed} &= \Xi_s \Bigg\{ \partial_r P_s J_2^{(1)} - \left(\partial_r P_s - \partial_r P_o\right)\frac{1}{\mathcal{H}_s}\partial_\eta \psi_s - (\gamma_0^{ab})_s \partial_a Q_s \partial_b P_s \\
&+ \frac{1}{\mathcal{H}_s}\partial_+ Q_s \partial_r^2 P_s + Q_s \partial_r^2 P_s \\
&+ \frac{1}{2}\partial_a\left(\partial_r P_s - \partial_r P_o\right)\left(\int_{\eta_o}^{\eta_s^{(0)-}} dx \ \left[\gamma_0^{ab} \ \partial_b Q\right](\eta_s^{(0)+}, x, \tilde{\theta}^a)\right) \Bigg\} \\
&- \frac{1}{\mathcal{H}_s \Delta\eta}\left(\psi_o - \psi_s - J_2^{(1)}\right)\partial_r P_o + \frac{Q_s}{\Delta\eta}\partial_r P_s \\
&+ \frac{1}{\Delta\eta}\left(\partial_r P_s - \partial_r P_o\right)\left\{\frac{1}{\mathcal{H}_s}\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_+ Q_s + \frac{2}{\mathcal{H}_s}\psi_s\right\} \\
&+ \frac{1}{\mathcal{H}_s}\left(\partial_r P_s - \partial_r P_o\right)\left\{\partial_\eta \psi_s - \partial_r \psi_s - \frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \Delta_2 \psi(\eta', \eta_o - \eta', \tilde{\theta}^a)\right\}.
\end{aligned} \tag{B.20}
$$

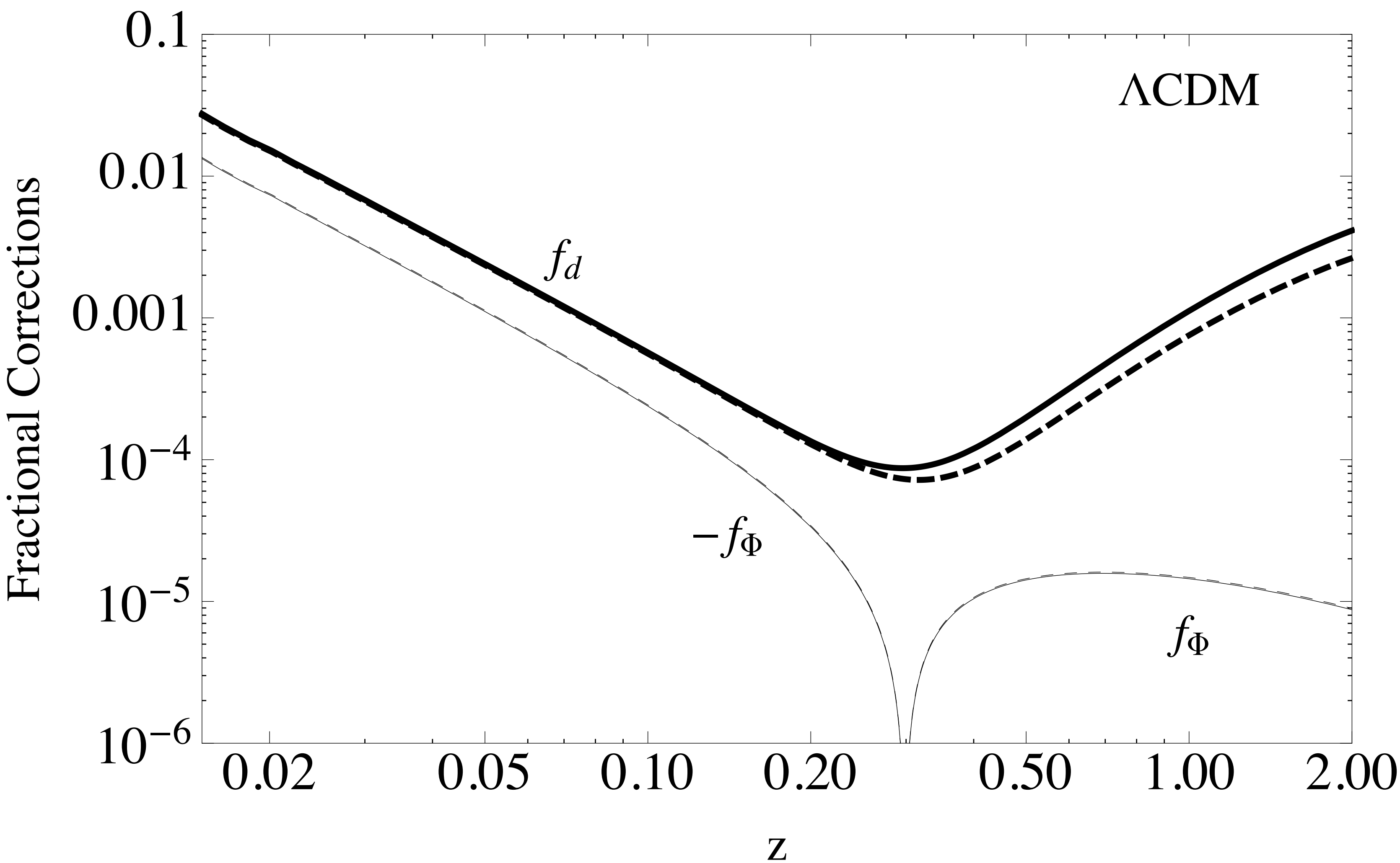
$$
P(\eta, r, \theta^a) = \int_{\eta_{in}}^{\eta} d\eta' \frac{a(\eta')}{a(\eta)} \phi(\eta', r, \theta^a) \quad , \qquad Q(\eta_+, \eta_-, \theta^a) = \int_{\eta_o}^{\eta_-} dx \ \frac{1}{2}\left(\psi + \phi\right)(\eta_+, x, \theta^a)
$$

Fortunately, many terms are very small/negligible. The most important ones pick up some moments (2nd and 3rd at most) of the power spectrum.

Their contribution is enhanced, relative to a very naive estimate of 10^{-10}, by powers of k^*/H_0, where k^* is a characteristic scale of the power spectrum.

Yet the overall effect is small...

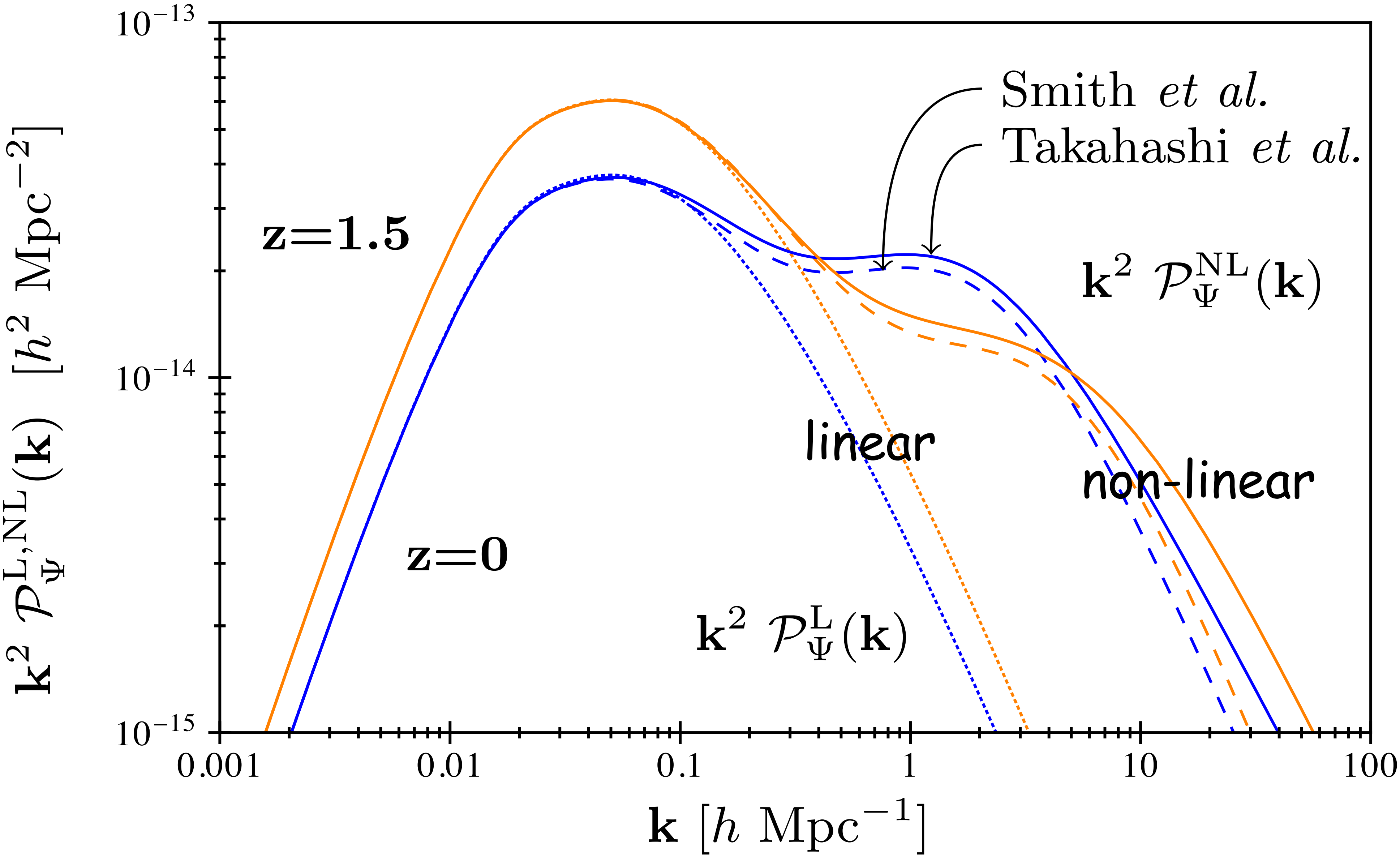
Different observables suffer different corrections (here with area measure)

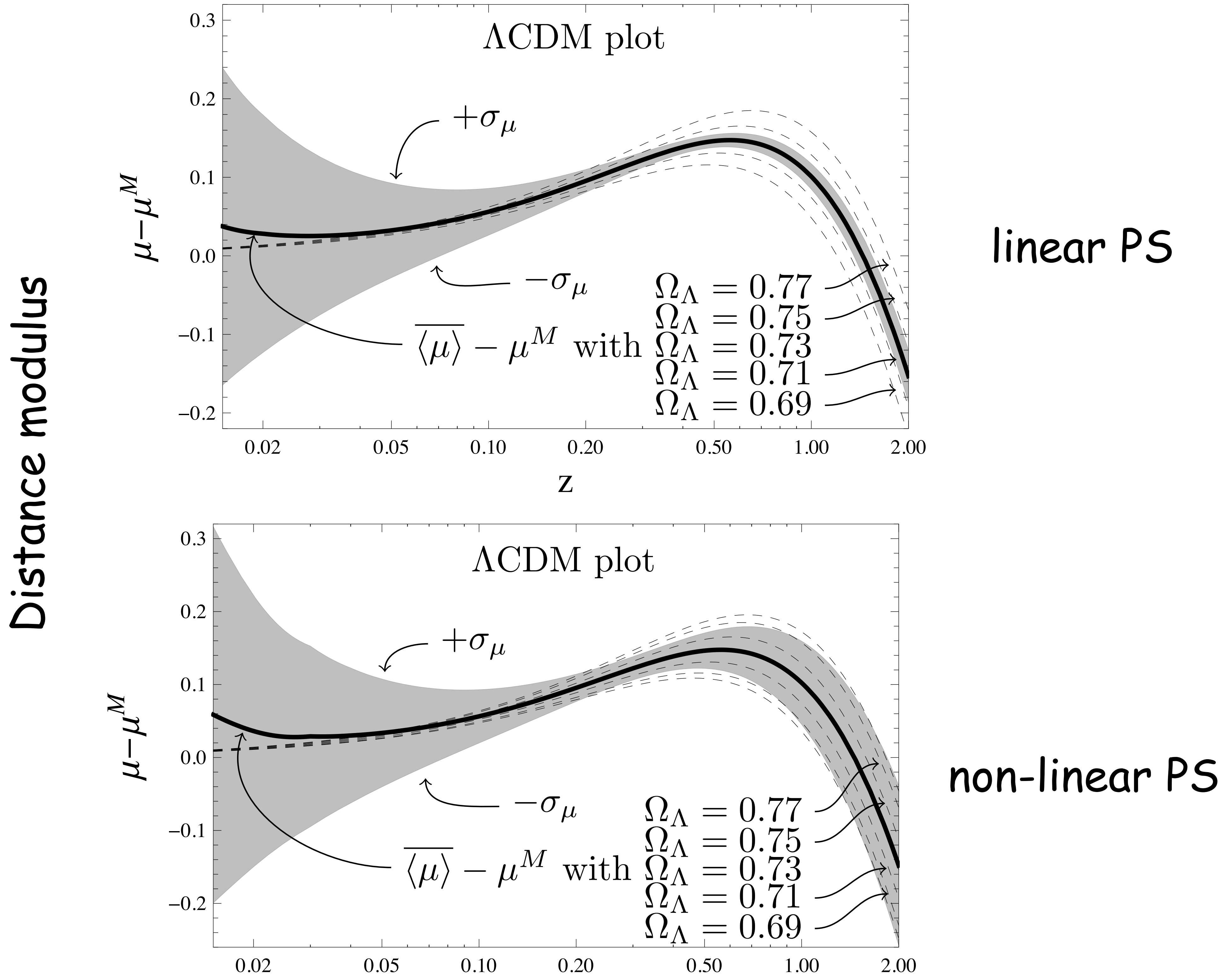


Flux turns out to be the least affected observable

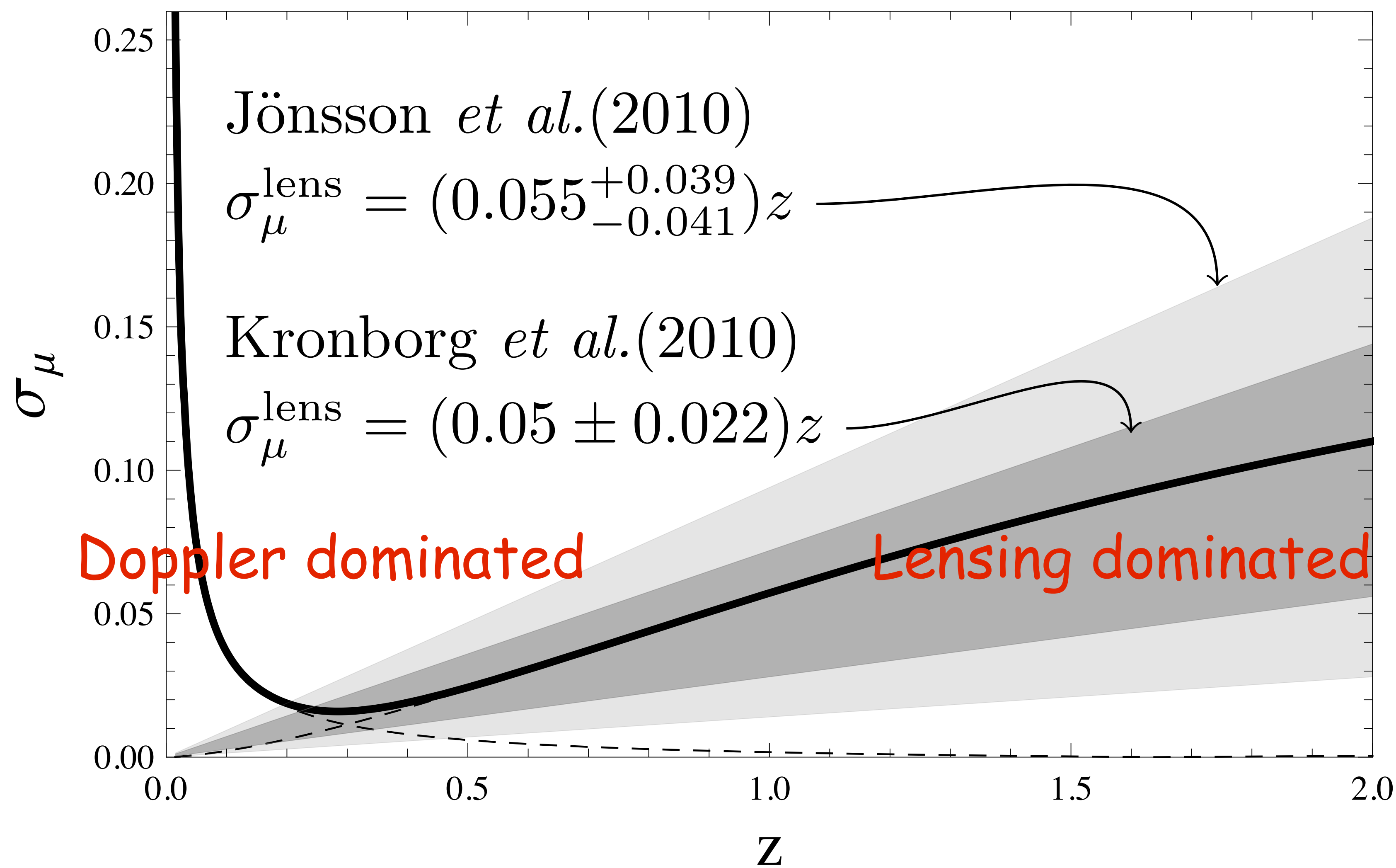
No leading lensing contribution for our average!

Results somewhat sensitive to the power spectrum used (but no IR or UV divergence)




Distance modulus
ΛCDM plot
$+\sigma_\mu$
$-\sigma_\mu$
$\overline{\langle\mu\rangle} - \mu^M$ with $\Omega_\Lambda = 0.73$
$\Omega_\Lambda = 0.77$
$\Omega_\Lambda = 0.75$
$\Omega_\Lambda = 0.71$
$\Omega_\Lambda = 0.69$
$\mu - \mu^M$
z
linear PS
ΛCDM plot
$+\sigma_\mu$
$-\sigma_\mu$
$\overline{\langle\mu\rangle} - \mu^M$ with $\Omega_\Lambda = 0.73$
$\Omega_\Lambda = 0.77$
$\Omega_\Lambda = 0.75$
$\Omega_\Lambda = 0.71$
$\Omega_\Lambda = 0.69$
$\mu - \mu^M$
non-linear PS

$$(\sigma_\mu^{\rm obs})^2 = (\sigma_\mu^{\rm fit})^2 + (\sigma_\mu^{z})^2 + \left[(\widehat{\sigma_\mu^{\rm int}})^2 + (\sigma_\mu^{\rm lens})^2\right]$$



Conclusions on DE application

Inhomogeneities (of a stochastic inflationary type) cannot mimic DE.

Averaging gives negligible corrections to the FLRW results. Particularly true for the flux.

In principle 10^{-4} precision attainable, however...

Effects on the variance/dispersion are much larger and may limit the determination of DE parameters (via SNIa data) to the few % level because of limited statistics.

Short Break

Trying to make use of our simple, exact result on the Jacobi Map for gravitational lensing

(G. Fanizza and F. Nugier, 1408.1604 & work in progress)

The Jacobi map is a basic ingredient in gr. lensing (see "Gravitational Lensing" by Schneider, Ehlers & Falco). By its definition, J(s,o) connects lengths at the source to angles at the observer:

$$\xi_s^A = J_B^A(s,o)\left(\frac{k^\mu \partial_\mu \xi^B}{k^\nu u_\nu}\right)_o = J_a^A(s,o)\theta_o^a$$

Its determinant gives the so-called area distance:

$d_A{}^2 = dA_s/d\Omega_o = \det J.$

Another map, J(o,s), connects angles at the source to lengths at the observer:

$$\xi_o^A = J_B^A(o,s) \left(\frac{k^\mu \partial_\mu \xi^B}{k^\nu u_\nu} \right)_s = J_a^A(o,s)\theta_s^a$$

Its determinant gives the so-called corrected luminosity distance d'_L.

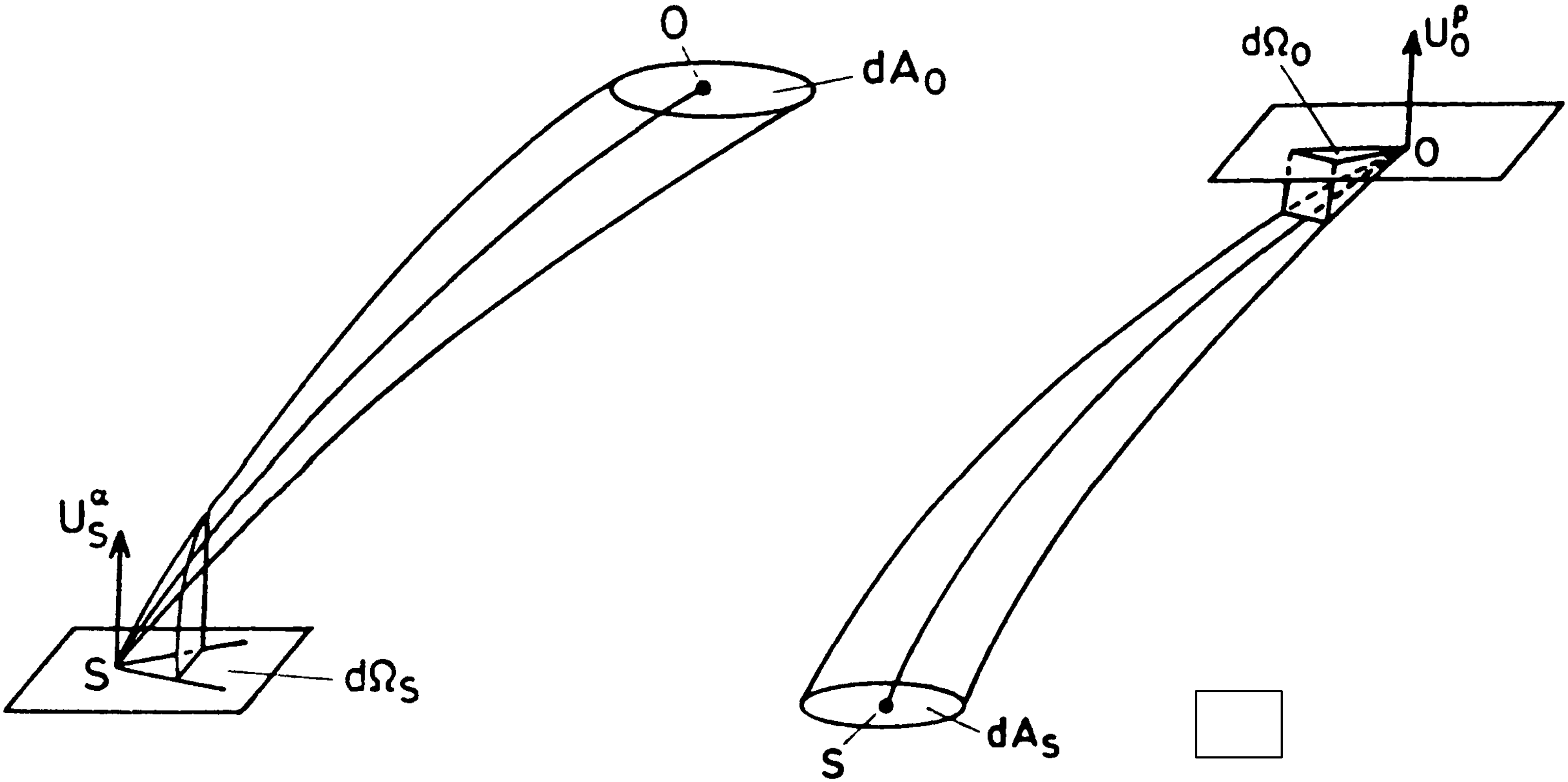
The two Jacobi maps (hence the two distances) are related by Etherington's (exact) reciprocity relation:

$$J(o,s) = -(1+z)\, J(s,o).$$

The (uncorrected) luminosity distance is given by:

$$d_L = (1+z)\, d'_L = (1+z)^2\, d_A$$

From Schneider, Ehlers & Falco

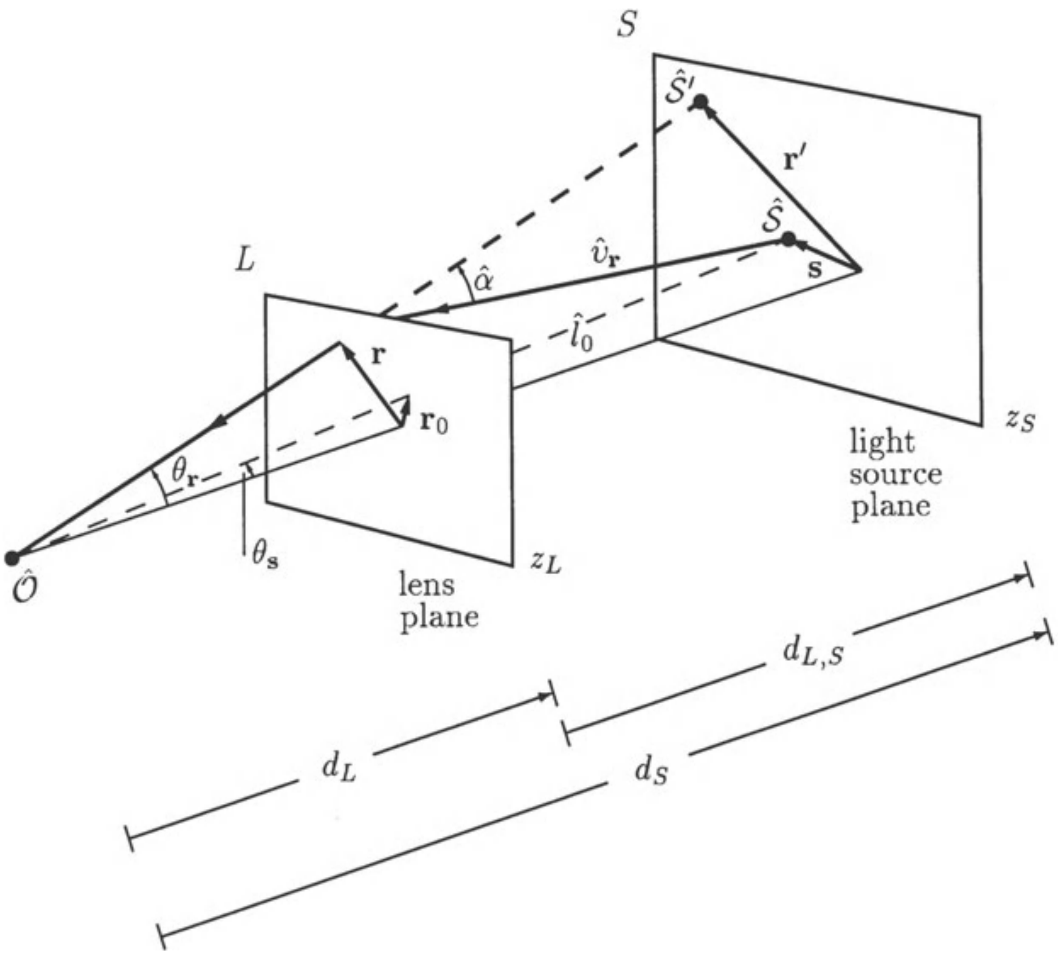


In the lensing literature one relates more often angles at the observers to angles at the source through the so-called (2x2) amplification matrix, containing both convergence κ and shear γ.

$$\mathcal{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

The total magnification μ is related to its determinant:

$$\mu^{-1} = \det \mathcal{A} = (1-\kappa)^2 - \gamma^2$$



Thin-lens example

$$\mathcal{A}_{ab} = \frac{\partial \beta^a}{\partial \theta^b} \to \frac{J^A_B(\lambda_s, \lambda_o)}{\bar{d}_A(\lambda_s)}$$

FLRW area distance

$$\mathcal{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

1. In the GLCG it should be possible to give the amplification matrix in a compact non-perturbative form directly from the known Jacobi map.

In particular, once J is divided by d_A, the resulting unimodular matrix turns out to be a group element of SU(1,1) with the 3 parameters denoting rotation and shear associated with lensing.

Another important issue

GLC coordinates become very tricky in the presence of caustics (points where rank (γ_{ab}) < 2).

One project is to improve treatment of gravitational lensing when a perturbative approach is inadequate, e.g. in the presence of caustics. This can only be done by putting together different sets of GLC coordinates which join each other along the caustic itself.
Remains to be properly done!

Other applications of GLC

- Determination of local H_0
- Number counts @ 2nd order
- 3rd order deflection & application to CMB lensing
- TOF of UR particles

Determination of local H_0 (BMDS, 1401.7973)

Fluctuations introduce some uncertainty in the extraction of the local Hubble expansion rate H_0. Indeed, H_0 is not very well known and different observations (e.g. SN and CMB) tend to give different values for it.

It seems that taking into account perturbations helps understanding the origin of the discrepancy.

Number counts @ 2nd order (DDMM, 1407.0376)

Galaxy Number Counts = # N of galaxies per solid angle and redshift.
This quantity fluctuates and its fluctuations have been computed up to second order in cosmological perturbation theory.
This also allows to compute the so-called bispectrum (i.e. the 3-point correlation)

Deflection @ 3rd order and application to CMB lensing (FGMV, 1506.02003)

We have studied the deflection of light-like geodesics up to 3rd order in the approximation in which some leading terms (in terms of the moment k^n of the power spectrum P(k) that contributes) are kept. Validity of a lens equation?

This is relevant (MFDD, 1612.07263) for CMB lensing and also for the accurate calculation of the B-polarization induced by lensing (which has to be subtracted in order to expose a primordial contribution from Tensor perturbations)

TOF of UR particles
(FMGV 1512.08489, P. Fleury 1604.03543)

In a homogeneous Universe it is easy to compute the TOF difference between two UR particles

$$\Delta\tau = \tau_1 - \tau_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right)\int_{\tau_s}^{\tau_o} \frac{d\tau}{1+z(\tau)}$$

As pointed out by Zatsepin (1968) and later by Stodolsky (2000) this can be used to measure either cosmological or particle phys. parameters

$$\Delta\tau = \tau_1 - \tau_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right)\int_{\tau_s}^{\tau_o} \frac{d\tau}{1+z(\tau)}$$

We have studied the TOF difference between UR particles in a non-homogeneous Universe.

Working in the GLC gauge simplifies things (null geodesics are very simple, almost null ones are simple). The result (to leading order in m/E but exact in cosm. pert. th.) is a simple generalization of the homogeneous one:

$$\tau_1 - \tau_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right)\int_{\tau_s}^{\tau_o} \frac{d\tau}{1+z(\tau, w_o, \tilde{\theta}_o^a)}$$

$$(1+z_s) = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{\Upsilon_o}{\Upsilon_s}\frac{(u_\tau)_s}{(u_\tau)_o} \to \frac{\Upsilon_o}{\Upsilon_s}$$

We have also studied the magnitude of TOF fluctuations since these represent a limit with which we can reconstruct either cosmological or particle properties from measurements of TOF differences.

All this, of course, assuming that we can find suitable sources of such particles with very small uncertainty on the relative emission times…

THANK YOU!