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Ettore Majorana Lectures 2017

Challenges in Early & Late-Time Cosmology

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I. The standard (“concordance”) model of cosmology after PLANCK

Outline

- Old (< 1980) Hot Big Bang cosmology
- Puzzles of HBB cosmology and inflation
- The importance of QM in inflation
- Which Big Bang are we talking about?

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- Classical vs. quantum perturbations
- The example of tensor perturbations
- Perturbations in slow-roll inflationary models.

Hot Big Bang cosmology (a reminder)

Einstein's equations, together with the cosmological principle (assumption of a homogeneous, isotropic Universe at large scales) and present observations (e.g. the redshift), lead to a very simple model known as Hot Big Bang (HBB) cosmology.

Its geometry is described by the Friedman-Lemaître-Robertson-Walker (FLRW) metric:

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right] \\
d\Omega^2 &= d\theta^2 + sin^2\theta d\phi^2 \quad ; \quad K = 0, \pm 1
\end{aligned}
$$

It contains a scale-factor a(t), telling us how physical distances depend on (cosmic-proper) time, and a discrete parameter (K = 0, 1, -1) giving, at any given time, a constant curvature (flat, closed, open) spatial geometry:

$^{(3)}R \sim K/a^2(t)$.

a(t) is related to the redshift by $(1+z) = a(t_0)/a(t_s)$. Its evolution is determined by the energy & pressure content of the universe via the two Friedman equations:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho \ ; \ \ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad H(t) \equiv \frac{\dot{a}}{a}$$

implying: $\dot{\rho} = -3H(\rho + p) = -3H\rho(1 + w) \ \ ; \ \ w \equiv \frac{p}{\rho}$

For standard matter with $\rho + 3p > 0$ this leads to a scale factor that goes to zero at a finite time in our past, conventionally called t=0.

At t=0, curvature and energy density diverge, forcing the physical interpretation of t=0 as the beginning of time. This singular event has been dubbed the BIG BANG

Critical density and fractions

Introducing $\rho^{(cr)} \equiv \frac{3H^2}{8\pi G} = \sum_i \rho_i + \rho_K \; ; \; \rho_K = -\frac{3K}{8\pi G a^2}$

$$\Omega_i \equiv \frac{\rho_i}{\rho^{(cr)}} \qquad \dot{\rho}_i = -3H\rho_i(1+w_i)$$

NB. Non rel. matter (dust) has w=0, radiation has w = +1/3; Spatial curvature behaves as a fluid with w = -1/3

The 1st Friedman equation: $H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\sum_i \rho_i$

can be rewritten in the simple form:

$$\Omega \equiv \sum_{i \neq K} \Omega_i = 1 - \Omega_K$$

NB: A spatially flat Universe is equivalent to Ω = 1.

Successes of HBB cosmology

1. The cosmic microwave background

(Penzias and Wilson 1965)

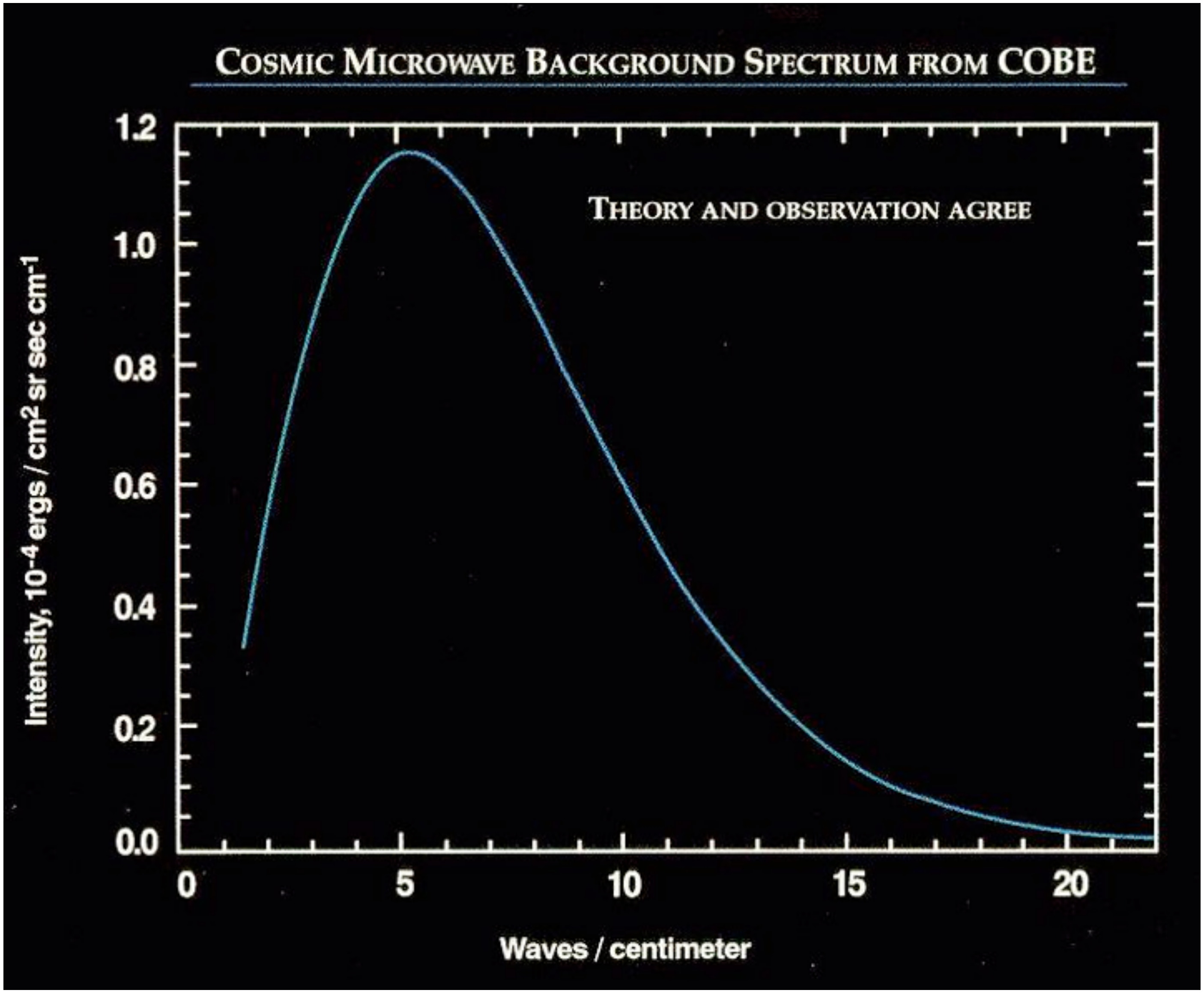
Since the 1940s, Gamow and coll. realized that the Universe should now be filled with a black-body spectrum of electromagnetic radiation.

The first theoretical estimate (~1950) for the present temperature was 5K in quite good agreement with the first determination of 3.5±1.0 K.

Today, the CMB spectrum is the best Planck spectrum known in Nature. Its average temperature is 2.725±0.002K.

Predicting the CMBR and its temperature was the first clear success of HBB cosmology!

$$dn(\nu) = \frac{8\pi\nu^2 d\nu}{\exp(h\nu/k_B T) - 1}$$

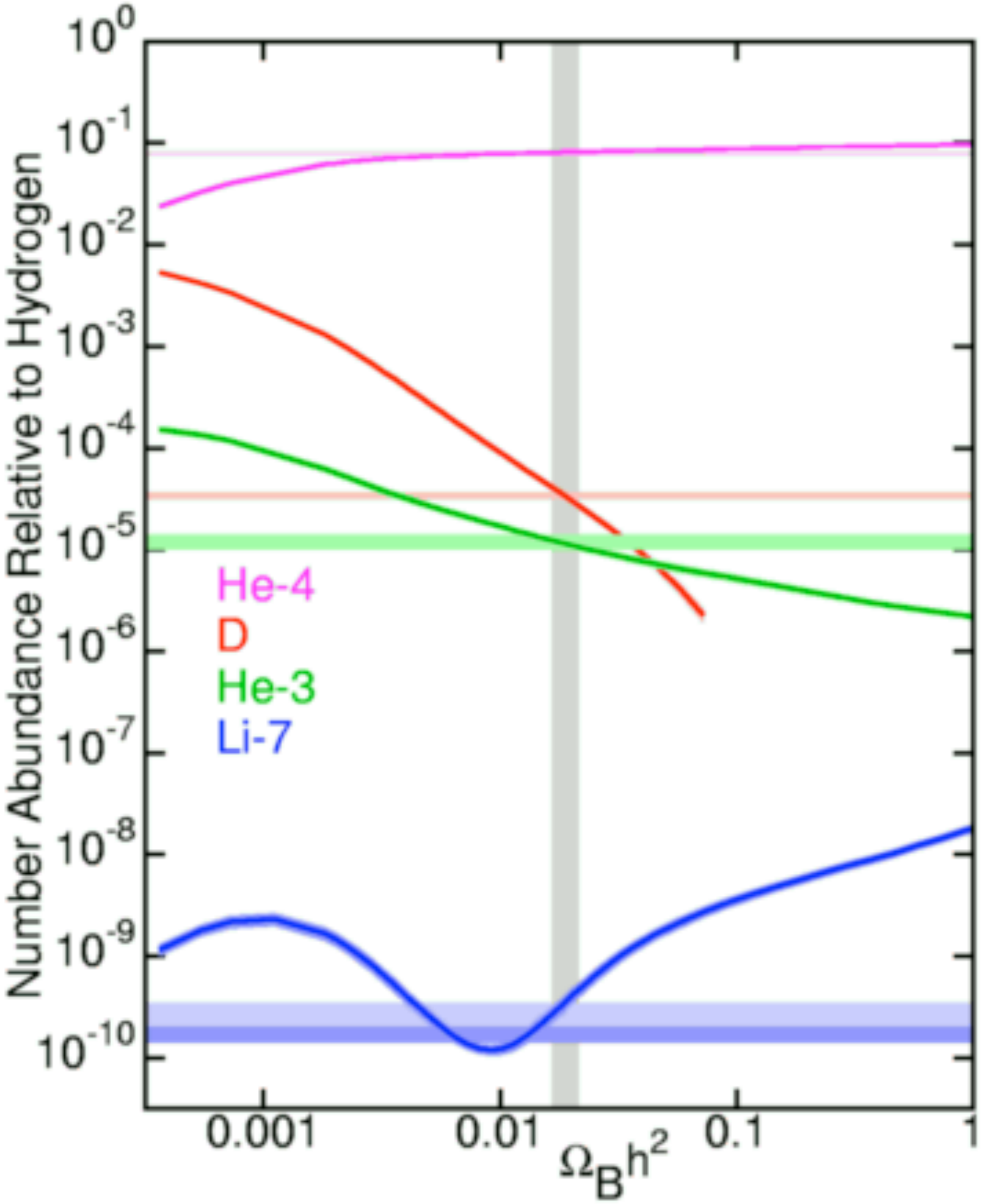


2. Primordial (BB) nucleosynthesis

A second big success of HBB cosmology is that it provides a mechanism (BBN) for producing light nuclei*) (d, He, Li, ..) out of protons and neutrons.

Temperatures of order 10^{10}K (~ 1 MeV) are needed for this to happen. The success of BBN is not just qualitative: we know the physics of the underlying processes, we can calculate the relative abundances of those light elements and compare them with the data.

*) Heavy elements are believed to be produced much later in very hot and dense stars, like supernovae.



Comparison with data

Horizontal bands correspond to experimental bounds;

Vertical band to allowed range for

$\Omega_B \sim 0.021\ h^{-2}$

$$H(t_0) \equiv H_0 = 100\ h\ \mathrm{km}\ s^{-1}\mathrm{Mpc}^{-1} \quad ; \quad h \sim 0.72 \pm 0.05$$

Shortcomings of HBB cosmology

1. Flatness problem

We know that, today, $|\Omega_K|$ cannot exceed 0.1. On the other hand Ω_K evolves in time according to:

$$\Omega_K(t) = \Omega_{K,0}\ \frac{a_0^2}{a^2}\ \frac{H_0^2}{H^2} = \Omega_{K,0}\ \left(\frac{\dot{a}_0}{\dot{a}(t)}\right)^2 \sim \Omega_{K,0}\ \left(\frac{t}{t_0}\right)^{\frac{2(1+3w)}{3(1+w)}}$$

and increases with t for a decelerated expansion (w > -1/3).

=> $|\Omega_K| < 10^{-32}$ at BBN & < 10^{-60} at $t = t_P \sim 10^{-43}$ sec.

Q: Why should the Universe start with such a small spatial curvature w.r.t. the total space-time curvature?

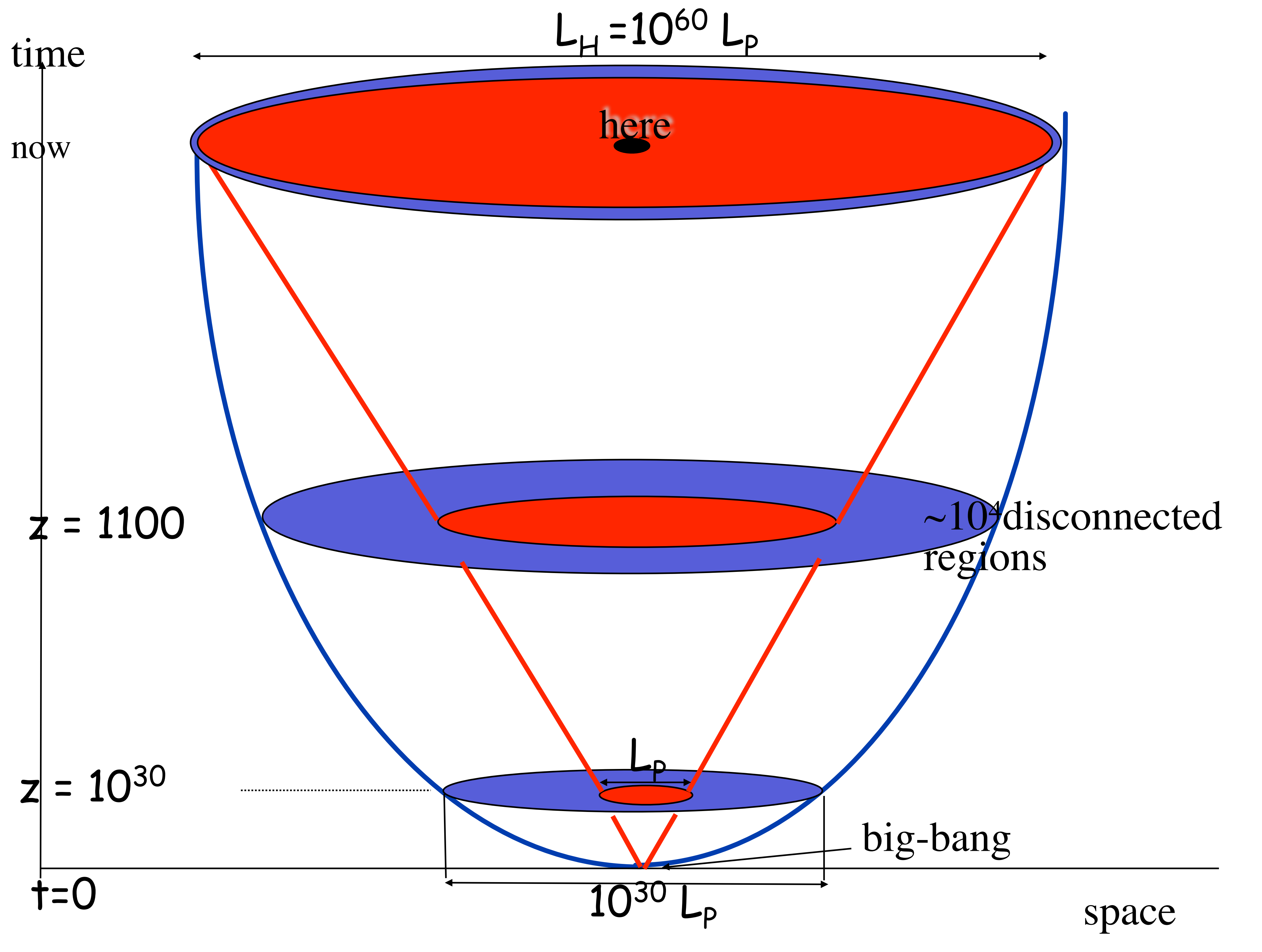
NB: A similar result holds for the contribution of spatial gradients. It had to be infinitesimal in the early Universe in order not to dominate today.

2. Homogeneity problem

The CMB comes to us today, basically undisturbed (just redshifted) from the time of recombination (or last scattering, when atoms formed and the Universe became transparent to photons). This happened at $z = z_{rec} \sim 1100$ i.e. when the Universe we can observe today was 1100 times smaller.

This size should be compared with another scale, the horizon, which is the distance traveled by light from $t=0$ till t_{rec}.

For standard HBB cosmology this second length scale is much smaller than the former one. The ratio is about 30 at recombination and can be as large as 10^{30} if we go back to $t = t_P \sim 10^{-43}$ sec (see picture).



$L_H = 10^{60}\ L_P$
time
now
here
z = 1100
~10^4 disconnected regions
L_P
$z = 10^{30}$
big-bang
t=0
$10^{30}\ L_P$
space

By causality (finite c), primordial inhomogeneities can only be washed out over distances shorter than the horizon.

Thus, at recombination our Universe consisted of about 10^4-10^5 causally disconnected regions.

The puzzle is that the CMB temperature was(is) the same in each one of those causally disconnected region (directions).

The reason why in the past the Universe was (much) larger than the horizon is, again, that w > -1/3:

$$\frac{(a/a_0)}{(t/t_0)} = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}-1} = \left(\frac{t}{t_0}\right)^{-\frac{1+3w}{3(1+w)}}$$

3. Origin of large-scale structure (LSS)

The Universe, even if homogeneous on very large scales, has large (and to an even larger extent small) scale structures: clusters of galaxies, galaxies, stars, ...

In HBB cosmology there is no explanation for LSS. In order to explain today's structures one has to start with some tiny inhomogeneities to be put by hand on top of the LFRW Universe.

In other words the HBB model tends to give either too much or too little LSS. Another fine-tuning problem.

The obvious solution: acceleration!

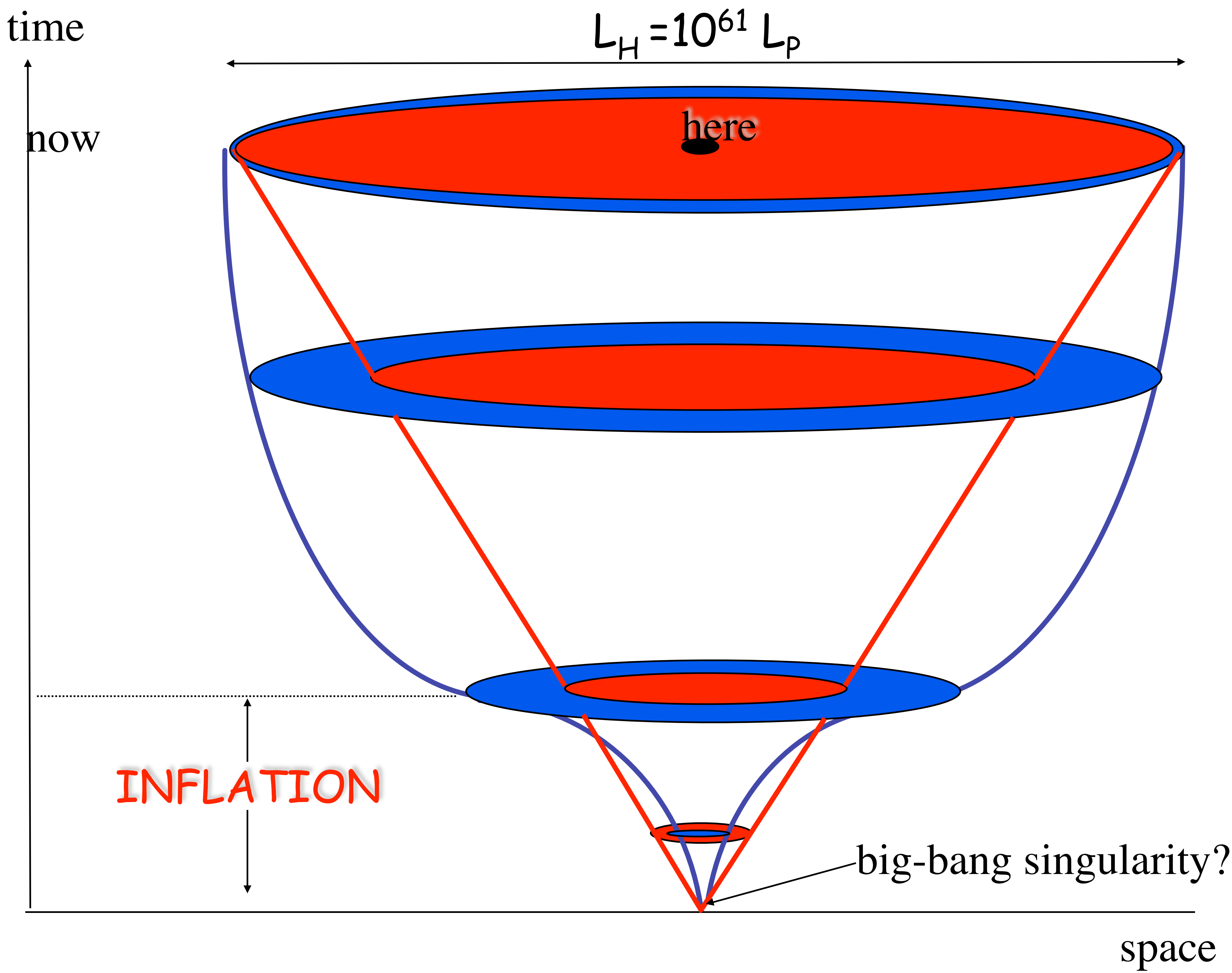
From the preceding discussion it is clear that an obvious solution to our puzzles is to insert a sufficiently long period of accelerated expansion, called inflation. One demands:

$$\frac{(a_f H_f)}{(a_i H_i)} = \frac{\dot{a}_f}{\dot{a}_i} \geq e^{N_{\rm min}}$$

If $N > N_{min} \sim 60$ inflation turns a generic initial Universe into a very (spatially) flat one since a^{-2} goes down faster than H^2.

Thus, $\Omega = 1$ is a generic prediction of inflation. Also, initial inhomogeneities are stretched to scales larger than our present Horizon.

The homogeneity problem is also solved since, in the far past, our visible Universe was inside a single Hubble patch (picture).



time
$L_H = 10^{61} L_P$
now
here
INFLATION
big-bang singularity?
space

Who provides the acceleration?

Ordinary matter, thanks to gravitational attraction, resists the expansion, decelerates it. In order to accelerate the expansion we need a "fluid" with $\rho + 3p < 0$ (negative enough pressure).

Quite amazingly it is relatively easy to "invent" such fluids. A positive cosmological constant is the simplest example (in fact was invented by Einstein for a similar purpose) but it's hard to get rid of. A more interesting choice is the potential energy of a nearly homogeneous and constant scalar field, called the inflaton. It has almost the same equation of state as a cosmological constant: $w \sim -1$ ($p \sim -\rho$).

At some point the inflaton starts changing rapidly in time and inflation stops. The inflaton's potential energy has to be dissipated, heating up the Universe (otherwise no BBN!) and w becomes positive ($w \sim +1/3$ presumably).

Inflation's bonus: a quantum origin of LSS

One of the greatest bonuses of inflation is that, besides providing a mechanism for erasing initial inhomogeneities and spatial curvature, it can also generate a calculable (within a given inflationary model) amount of primordial perturbations.

As we shall discuss the reason for this "miracle" is quantum mechanics. Indeed, while the wavelength of any primordial classical perturbation gets stretched beyond our horizon by inflation, quantum mechanics keeps acting throughout inflation continuously generating new short-scale perturbations. When amplified and stretched to present cosmological scales by inflation they may well give rise to all the structures we see in the sky.

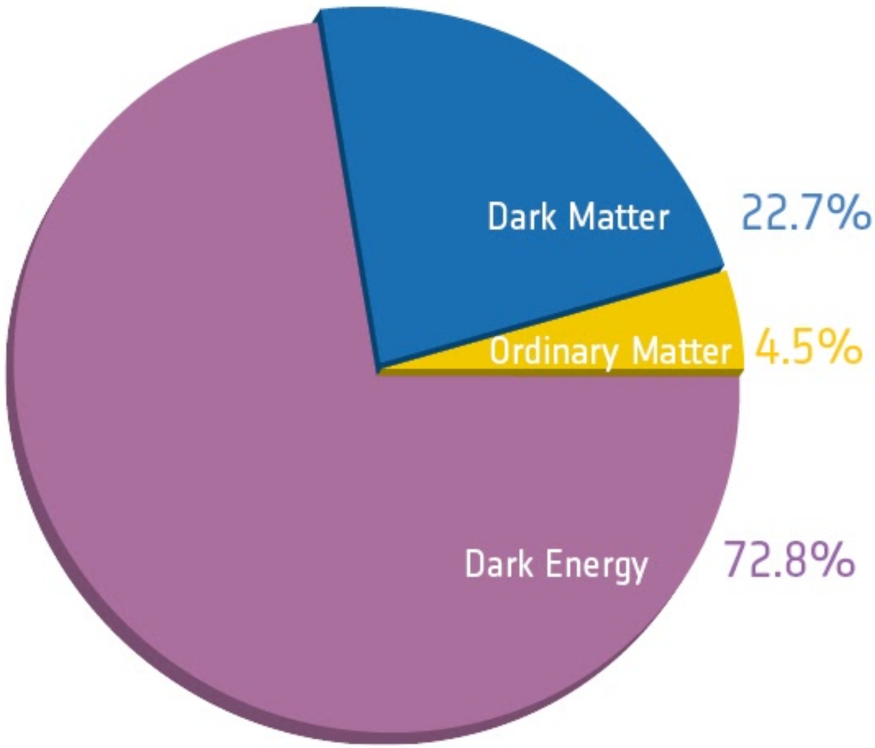
The “concordance model”

Since a few decades we have a good cosmological model combining the inflationary paradigm with that of a “dark“ sector in the energy budget of the Universe.

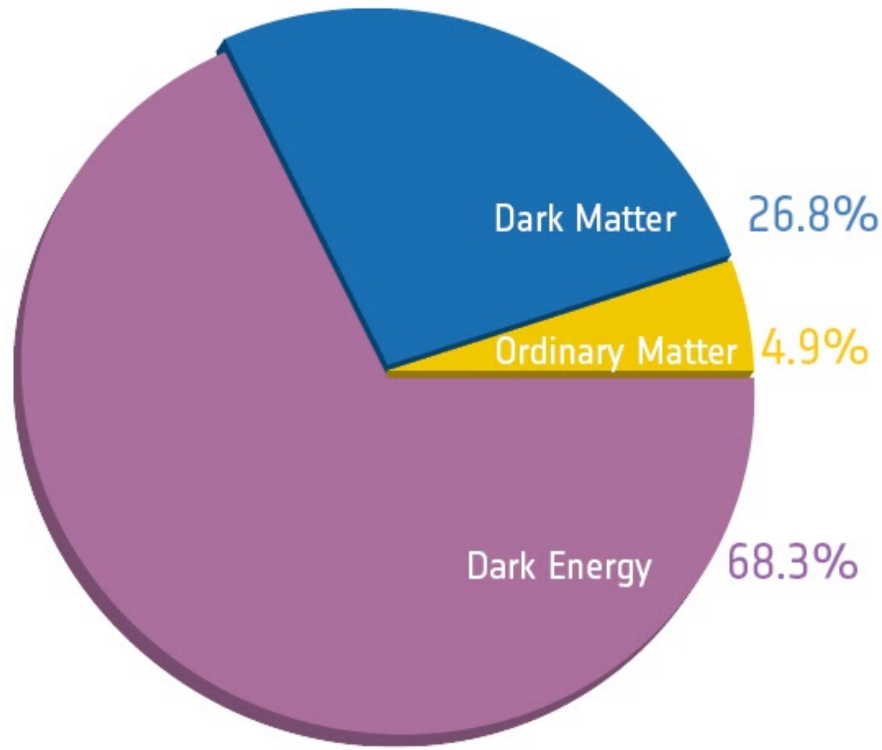

spiral
galaxy M74
Dark Matter


From
supernovae
Dark Energy

Portions in cosmic composition pie…
somewhat redistributed after PLANCK



Before Planck



After Planck

Importance of QM in cosmology:
1. Origin of structure

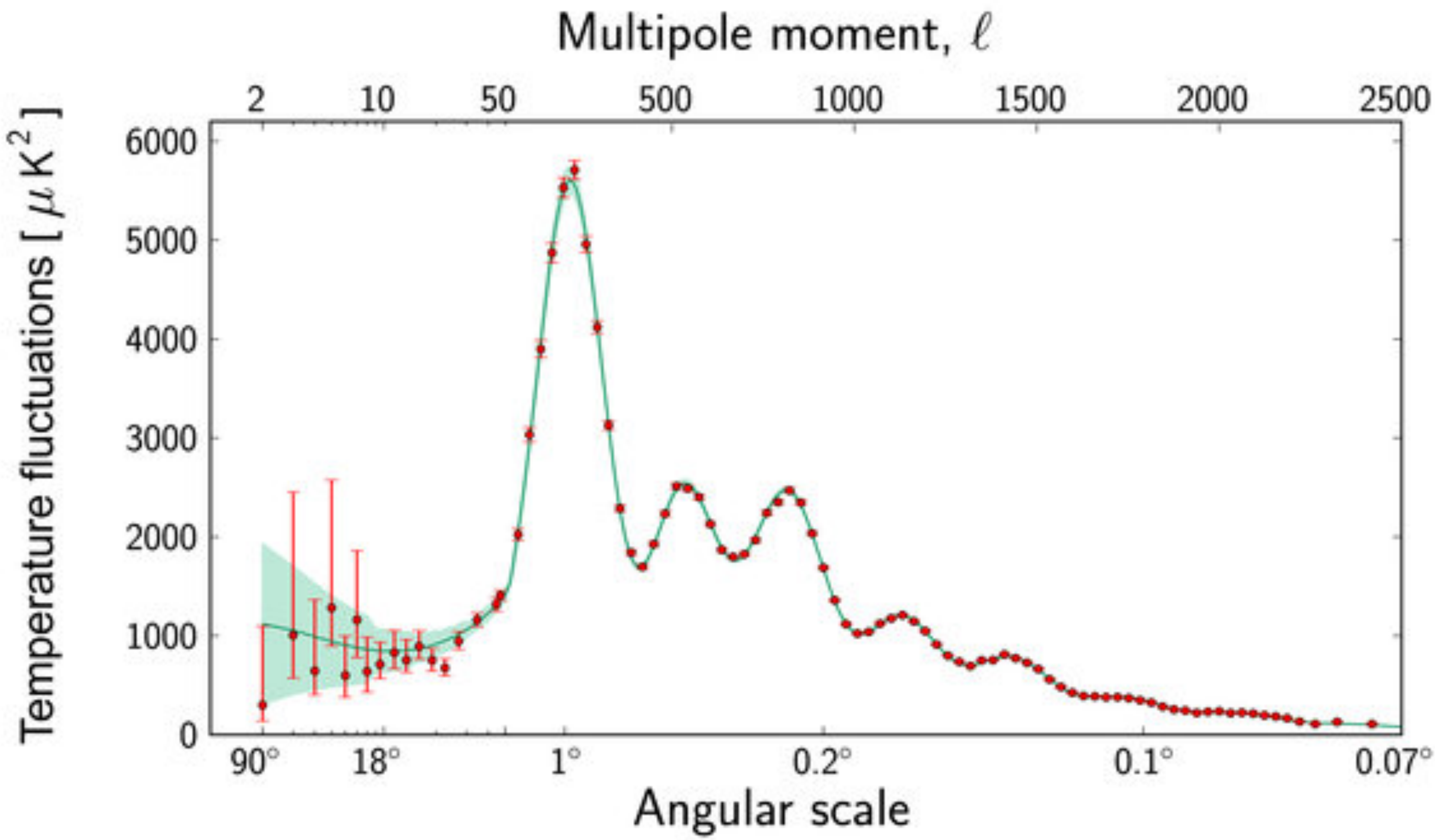
Without QM: inflation produces a perfectly homogeneous Universe: initial inhomogeneities are erased and the structures we see in the sky could not have formed.

With QM: classical initial inhomogeneities are erased, but they replaced by calculable quantum fluctuations produced, amplified and stretched throughout the inflationary epoch.

The CMB we observe today carries an imprint of those primordial fluctuations.

PLANCK POWER SPECTRUM

PLANCK (ESA, 2013)



2. Reheating after inflation

Inflation cools the Universe practically down to zero temperature. How can we generate a hot Universe after inflation? If not, no CMB, BBN!!

NB: after inflation the U is still expanding!

Second intervention of QM: dissipative, non adiabatic conversion of potential energy into a hot thermal soup of elementary particles.

This reheating plays the role of the old hot BB!

...but: 1. has no associated singularity and

2. It is certainly NOT the beginning of time!

Which Big Bang are we talking about?

- For many decades we have taught the general public (and not only!) a simple equation:

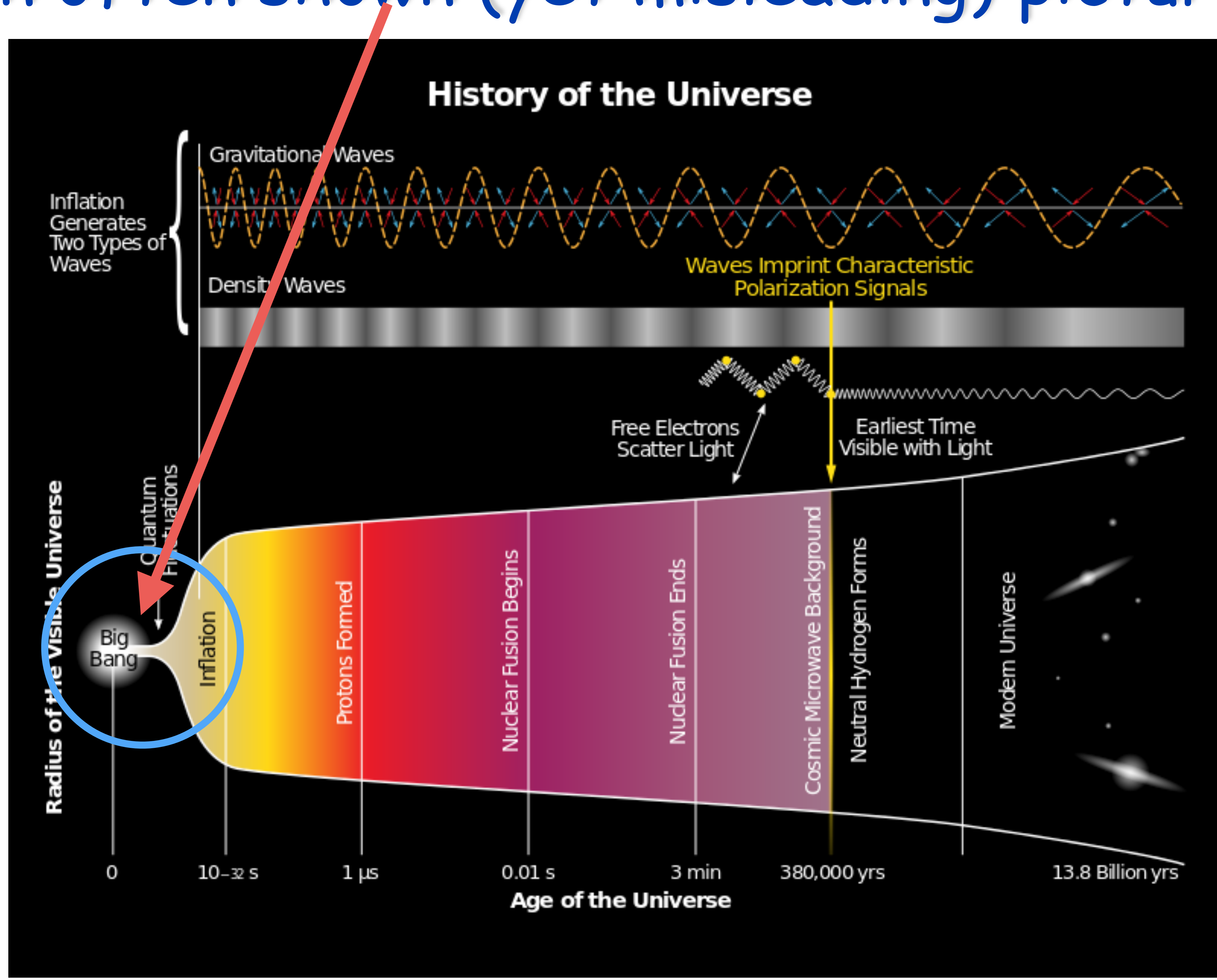
Big Bang = Beginning of Time

But, according to modern cosmology, we must distinguish:

1. The "physical", non singular BB, at the end of inflation, that left measurable relics (CMB, BBN…) and:
2. A "theoretical" singular(?) BB, that **could have preceded** inflation (without leaving relics?).

   In any case: the BB we know something about has nothing to do with a beginning of time.

An often shown (yet misleading) picture



Short Break

Comparing cosmological perturbations

One of the most important virtues of inflation is that it provides a mechanism for generating an interesting spectrum of cosmological perturbations.

This is also regarded as the best way to test the inflationary paradigm, to select among its many different realizations, and to compare it against alternative cosmologies.

We shall first review how this amazing phenomenon comes about and which are the characteristic properties of the perturbations produced by standard slow-roll inflation.

Tomorrow we will compare these predictions with those obtained in the context of (a particular example of) string cosmology.

Theory of cosmological perturbations

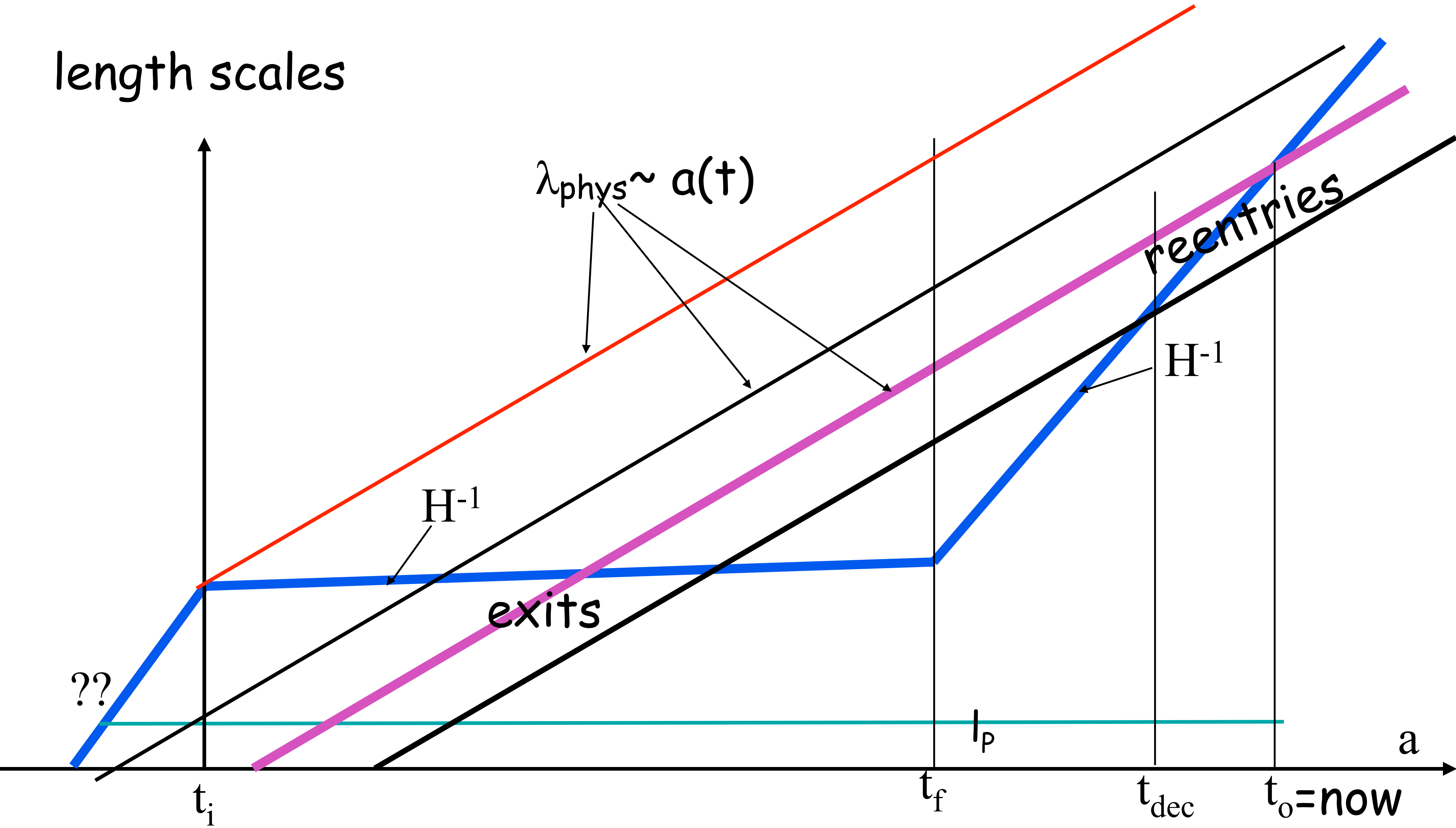
A distinctive property of any inflationary epoch is that, all along its duration, physical scales are continuously pushed outside the horizon. This a direct consequence of the growth of the ratio

$$\frac{\lambda_{phys}}{H^{-1}} \sim aH = \dot{a}$$

during inflation.

During a decelerated expansion, physical scales "re-enter the horizon". The amplification of fluctuations has a lot to do with this basic kinematical fact: scales initially inside the horizon go out during inflation and reenter after inflation.

Kinematics of slow-roll inflation



Classical considerations

Let's assume that we have a homogeneous solution of the classical cosmological field equations. Let us look for the a general solution describing non-homogeneous small perturbations by expanding every field (the metric as well as the matter fields) around their homogeneous values.

The action describing the dynamics of these perturbations will be quadratic in them (since the action is stationary on the unperturbed solution) but in general is not diagonal. One can diagonalize the kinetic terms of the perturbations and make them canonically normalized.

At lowest order in the derivatives a generic perturbation ψ will enter the action in the form:

$$S_{eff} = -\frac{1}{2}\int d^4x\sqrt{-g}\; Q_\psi(x)\left[\partial_\mu\psi\partial^\mu\psi + m^2(x)\psi^2\right]$$

where Q_ψ is a ψ-dependent scalar field. If the background metric is conformally flat (it soon becomes in ordinary inflation) we can go over to conformal time and the action takes an even simpler form:

$$S_{eff} = \frac{1}{2}\int d^3x d\eta\; P_\psi(\eta)\left[(\psi')^2 - (\partial_i\psi)^2 - m^2a^2\psi^2\right]\;;\; \psi' \equiv \partial_\eta\psi = a\partial_t\psi$$

$P_\psi(\eta) = a^2\, Q_\psi$ is called the "pump field" for ψ. Introducing Fourier modes ψ_k wrt the space coordinates different modes decouple and each mode obeys a very simple linear dynamics. We will be mainly concerned with massless perturbations for which:

$$S_{eff} = \frac{1}{2}\int d^3x d\eta\; P_\psi(\eta)\left[(\psi')^2 - (\partial_i\psi)^2\right] = \frac{1}{2}\sum_{\vec{k}} P(\eta)\left[|\psi'_k|^2 - |k\psi_k|^2\right]$$

Comment: the comoving wave vector k is related to the physical wave vector p and wavelength λ_{phys} by:

$$p = \frac{k}{a} \; ; \; \lambda_{phys} = \frac{1}{p} = \frac{a}{k}$$

k is constant in time and has to be compared to aH (the comoving Hubble parameter). A perturbation is inside the horizon if k > (>>) aH = a'/a and is outside if k < (<<) aH. During inflation aH grows, while it decreases afterwards => this is how exits & reentries are seen in comoving variables.

The time evolution of each mode depends crucially on its relation (in or out) wrt the horizon.

It is convenient (also in view of discussing the quantum case) to go over to a Hamiltonian formalism:

$$\mathcal{H} = \frac{1}{2} \sum_{\vec{k}} \left[P^{-1}(\eta)|\Pi_k|^2 + P(\eta)|k\psi_k|^2 \right] \; ; \; \Pi_k = P\psi_k'$$

The one-mode Hamiltonian

$$\mathcal{H}_k = \frac{1}{2}\left[P^{-1}(\eta)|\Pi_k|^2 + P(\eta)|k\psi_k|^2\right] \;;\; \Pi_k = P\psi_k'$$

Hamilton's equations:

$$\psi_k' = P^{-1}\Pi_k \;;\; \Pi_k' = -Pk^2\psi_k$$

can be rewritten in terms of some rescaled "canonical variables" as Schroedinger-like equations:

$$\hat{\psi}_k = P^{1/2}\psi_k \;;\; \hat{\Pi}_k = P^{-1/2}\Pi_k$$

$$\hat{\psi}_k'' + \left(k^2 - \frac{(\sqrt{P})''}{\sqrt{P}}\right)\hat{\psi}_k = 0 \;;\; \hat{\Pi}_k'' + \left(k^2 - \frac{(\sqrt{1/P})''}{\sqrt{1/P}}\right)\hat{\Pi}_k = 0$$

and we can distinguish two opposite regimes:

1. When the perturbation is deeply inside the horizon its evolution corresponds to an adiabatically damped oscillator and to a conserved Hamiltonian:

$$\hat{\psi}_k \sim \text{const} \ ; \ \hat{\Pi}_k \sim \text{const}$$

$$\psi_k \sim P^{-1/2} \ ; \ \Pi_k = P^{1/2} \ ; \ \mathcal{H} \sim \text{const.}$$

2. When the perturbation is deeply outside the horizon its evolution corresponds to an over-damped oscillator and the amplitude freezes. This corresponds to an increase in H which can be due either to the freezing of the perturbation or of its conjugate momentum. In both cases H grows.

$$\psi_k \sim \text{const.} \ ; \ \Pi_k =\sim \text{const.} \ ; \ \mathcal{H} \sim \text{Max}(P, P^{-1}) \ ; \ \dot{\mathcal{H}} > 0$$

Turning on Quantum Mechanics

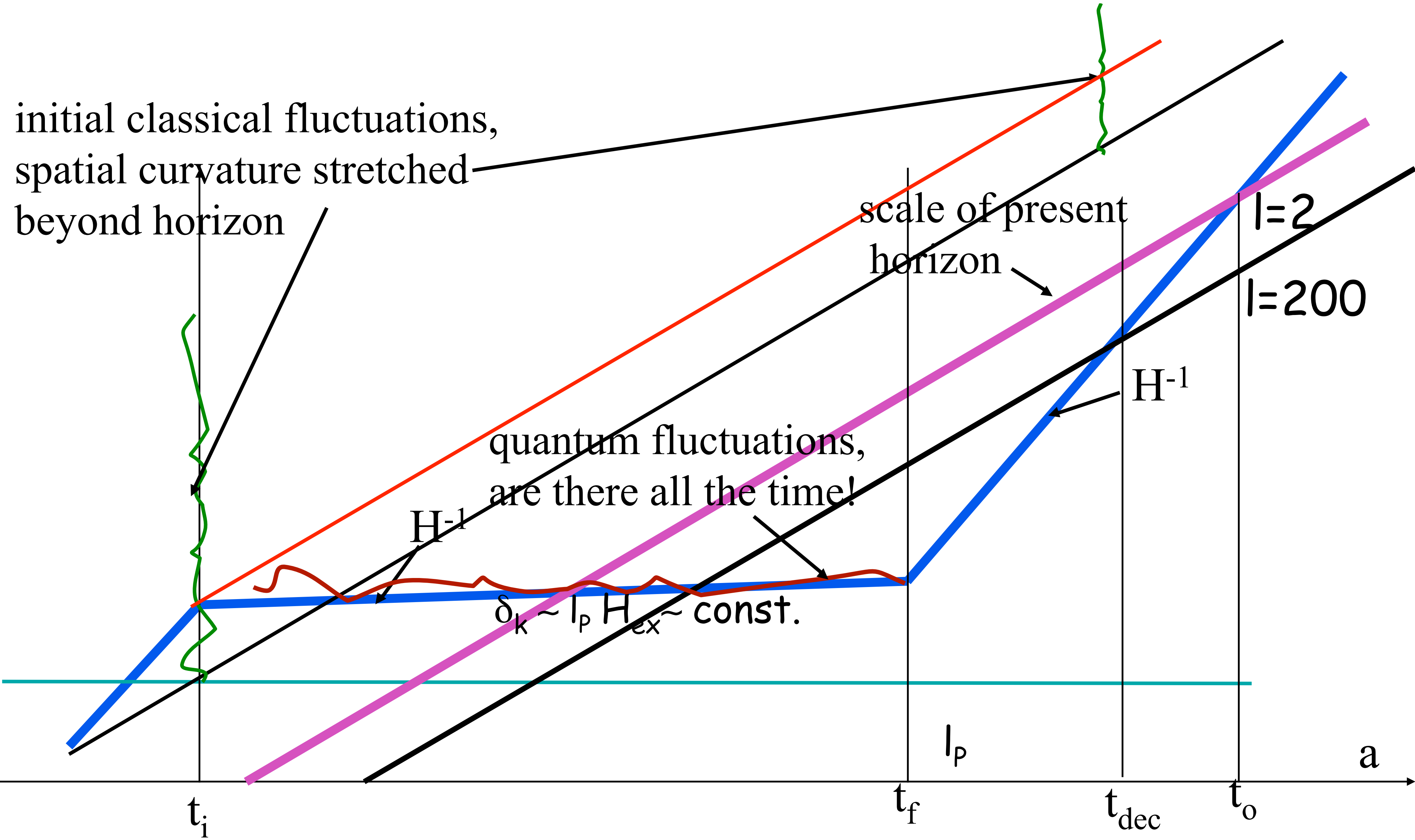
The evolution of perturbations is basically the same in the classical and quantum theory.

What really makes the difference is that classical perturbations are given "initially" and then evolve deterministically. In order to be called classical they involve physical lengths initially larger than l_P.

If inflation lasts long enough such initial perturbations have been stretched way beyond our present horizon H_0^{-1}.

Instead, quantum fluctuations are produced all the time (we cannot turn off h!).

Since they can be generated much later than the classical ones they can be still inside our horizon today.



We now turn to a specific example

Tensor perturbations in slow-roll inflation

This is one of the most robust predictions of inflation.

Consider a tensor perturbation of the FLRW metric:

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x})) \; ; \; \partial^\nu h_{\mu\nu} = h^\mu_\mu = 0$$

The associated "pump field" turns out to be $a^2(\eta)$ so that the Fourier modes of ah satisfy:

$$\hat{h}_k'' + \left(k^2 - \frac{a''}{a}\right)\hat{h}_k = 0 \; ; \; \hat{h}_k = a(\eta) h_k$$

At early enough times the scale 1/k is inside the horizon. (ah) oscillates like exp(i k η) with constant amplitude. In the ground state of this harmonic oscillator QM gives:

$$\hat{h}_k = l_P k^{-1/2} \Rightarrow \delta h(\lambda) = k^{3/2} a^{-1} \hat{h}_k = \frac{l_P}{\lambda} \; ; \; \lambda = a k^{-1} \; ; \; l_P = \sqrt{\frac{8\pi G\hbar}{c^2}}$$

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At a later time the scale 1/k goes out of the horizon and h itself freezes. By matching the solution at exit we find:

$$\delta h(\lambda) = \frac{l_P}{\lambda(\eta_{ex})} \sim \frac{H(t_{ex}(\lambda))}{M_P} \ ; \ \eta > \eta_{ex}$$

Smaller wavelengths have a larger initial amplitude but they exit later and therefore are not amplified as much as longer wavelengths. These two competing effects produce a spectrum that depends on how H changes in time. For slow-roll inflation H(t) is a slowly decreasing function of t and therefore the resulting spectrum is expected to be slightly red-tilted. The amplitude is fixed in terms of H/M_P.

An almost scale-invariant (Harrison-Zel'dovich) spectrum of tensor perturbations from slow-roll inflation!

In slow-roll inflation this calculation can be repeated for "scalar" perturbations. These are coupled perturbations of the inflaton and of the metric (curvature perturbations).

Because of this "mixing" the calculation is more complicated and one has to get rid of possible gauge artifacts.

After the pioneering work of Bardeen several gauge-inv. scalar perturbations have been used. Besides the Bardeen potentials, popular ones are R / ζ, the curvature perturbation on comoving/constant density hypersurfaces.

The end result is that also scalar perturbations have a flattish (and typically red-tilted) spectrum ($n_s \sim 1$).

Their amplitude is not fixed by H/M_P since it is enhanced by 1/(slow-roll parameters).

=> The S contribution to CMB anisotropies dominates over T, but its properties are model-dependent.

We can only put some upper bound on the tensor perturbations in order not to exceed the observed $\Delta T/T$.

If large enough T (for tensor!) can be seen in the B-mode of the polarization of the CMB (after subtracting other possible sources). For the moment we only know (BICEP + PLANCK) that $T/S < 0.12$ @ 90 % c.l.

An important comment.

The detection of primordial tensor perturbations would be a "prima facie" evidence for gravity to be quantized.

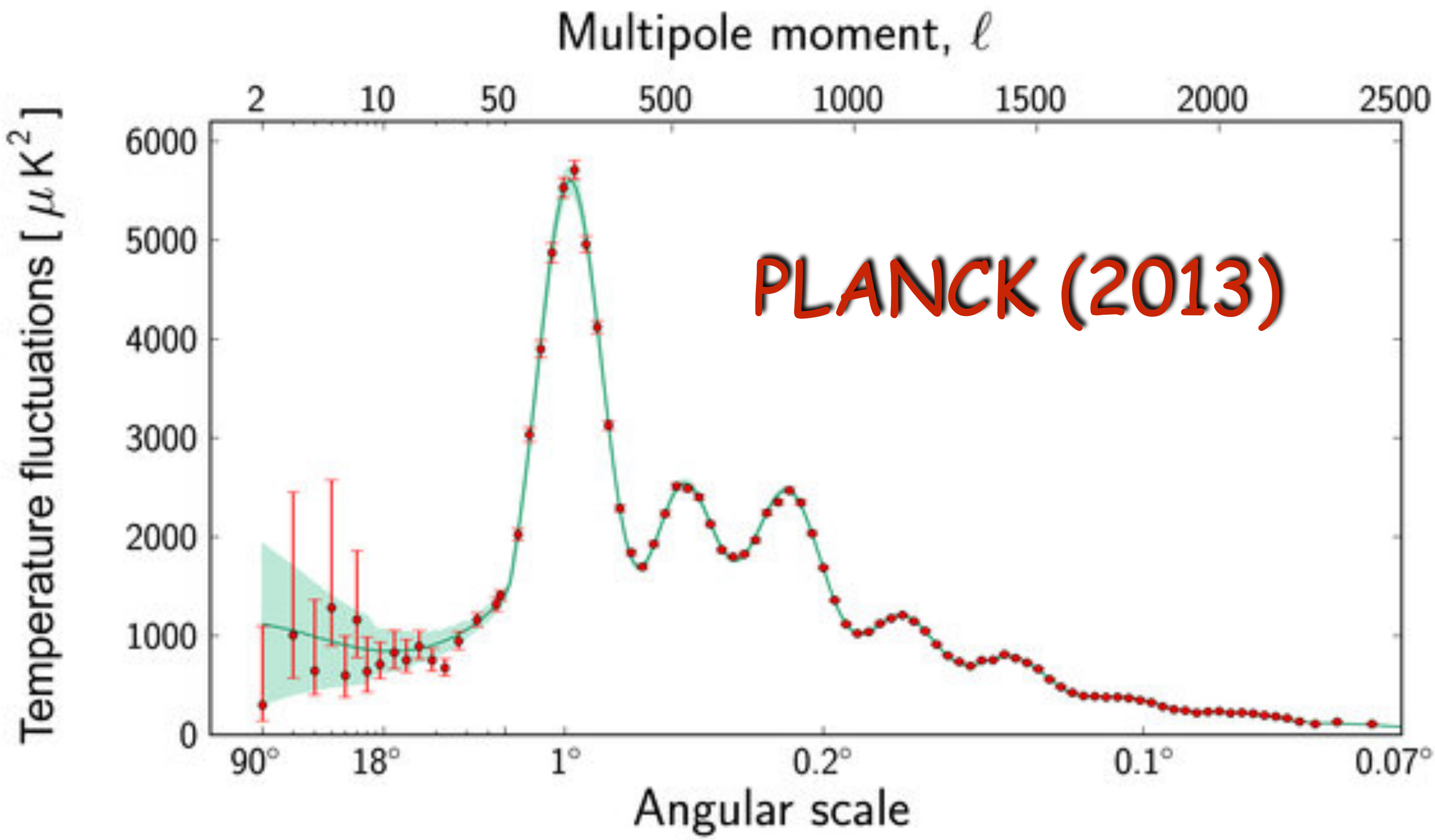
However, given that gauge invariant scalar perturbations mix matter and metric perturbations, I claim that even the success of inflation in predicting scalar density/curvature perturbations strongly supports the claim that gravity, like all other interactions, ought to be quantized.

Summarizing:
Perturbations in Conventional Inflation

- Tensor perturbations (GW) generated with $n_T \sim 0$ (approximately scale-invariant);
- Scalar (Density/curvature) perturbations w/ $n_S \sim 1$ (also approx. scale-invariant); PLANCK: $n_S \sim 0.96$
- T/S = $O(n_T)$, smallish but perhaps observable in CMB polarization (PLANCK?), too small for direct GW searches;
- Non Gaussian, isocurvature components: small, at least in single-field models;
- EM perturbations: absent since, In D=4, an inflationary metric couples trivially to Maxwell's term and α is constant (running is not enough).

PLANCK POWER SPECTRUM

A typical inflationary prediction for temperature fluctuations



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