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Ettore Majorana Lectures 2017

Challenges in Early & Late-Time Cosmology

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II. Big Bang or Big Bounce?

Outline

- Shortcomings of conventional inflation
- String Theory's effective action
- Homogeneous string cosmology SFD, PBB
- Initial conditions and fine-tuning issues

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- Perturbations in PBB cosmology
- The examples of tensor & EM perturbations
- Where does LSS come from?
- The KR axion as curvaton
- Advantages and disadvantages wrt SRI.

Shortcomings of standard inflation

Inflation is a successful paradigm but looks short of a truly satisfactory theory:

1. One needs a special kind of potentials in order to keep the inflaton nearly constant for a long time (slow-roll conditions);
2. Initially, the inflaton has to be away from the minimum of its potential and has to be "fairly homogeneous" (i.e. over several Hubble lengths);
3. It is difficult (but perhaps not impossible, cf. Higgs inflation) to identify the inflaton with some (fundamental or effective) scalar field already present in models of particle physics.
4. Initial conditions in a highly quantum (hence out of control) regime. Need a theory of Quantum Gravity?

Can String Theory help?

There have been several attempts to incorporate standard (i.e. slow-roll) inflation in QST. It seems that slow-roll inflation is not a natural outcome of string theory. We shall instead ask the question:

What is the most natural cosmology that emerges from QST?

Let us start from the field equations that follow from the effective action of string theory at tree level (small g_s) and small curvature (i.e. neglecting higher-derivative terms).

First a brief reminder of how such an effective action is defined and of its main properties.

The effective action of String Theory

String theory has a length/mass scale (l_s/M_s) and contains a bunch of massless fields together with infinite number of heavy ($M \gg M_s$) and high spin fields. At $E \ll M_s$ one can talk about an effective action for the massless fields. Keeping only 3 (universal) ones it reads:

$$\Gamma_{eff} = -\int \frac{d^D x}{l_s^{D-2}} \sqrt{-G} e^{-\phi} \left[\frac{4(D-10)}{3 l_s^2} + R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

Here $G_{\mu\nu}$ is the (string-frame) metric, $B_{\mu\nu}=-B_{\nu\mu}$ is the KR field, $H = dB$ and ϕ is the dilaton. If $D \neq 10$ we cannot get a low-curvature solution. We shall then limit ourselves to $D = 10$:

$$\Gamma_{eff} = -\int \frac{d^{10} x}{l_s^{8}} \sqrt{-G} e^{-\phi} \left[R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

$$\Gamma_{eff} = -\int \frac{d^{10}x}{l_s^8}\sqrt{-G}e^{-\phi}\left[R(G) - \partial_\mu\phi\partial^\mu\phi + \frac{1}{12}H^2\right]$$

These massless fields represent a universal sector in all string theories (we shall briefly discuss later what happens if we add other backgrounds).

We allow the extra dimensions to be dynamical (unfrozen).

We work in the "string frame" (fixed l_s, varying l_P, see below for how they are related) but physical consequences are frame-independent.

A theory of gravity but not Einstein's!

In D dimensions, the analogue of the Einstein-Hilbert action takes the form:

$$\frac{1}{\hbar} S_{EH} = \left(\frac{1}{l_P}\right)^{D-2} \int d^D x \sqrt{-g(x)} \left(\Lambda - \frac{1}{2} R(g)\right) \quad ; \quad 8\pi G_N \hbar \equiv l_P^{D-2}$$

while in QST we found (NB: $2\Phi = \phi$):

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D-D_c)}{3 l_s^2} + R(G) - 4\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H^2 + \dots\right]$$

Are they equivalent up to some field redefinition? The answer is obviously no, even if we set H=0. QST gives a scalar-tensor theory of the Jordan-Brans-Dicke kind!

$$\frac{1}{\hbar}S_{EH} = \left(\frac{1}{l_P}\right)^{D-2}\int d^Dx\sqrt{-g(x)}\left(\Lambda - \frac{1}{2}R(g)\right) \quad ; \quad 8\pi G_N\hbar \equiv l_P^{D-2}$$

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{12}H^2 + \dots\right]$$

For a constant Φ we can identify l_P^{D-2} with $\exp(2\,\Phi)\,l_s^{D-2}$ but a massless dilaton still induces long-range interactions that violate the equivalence principle: the dilaton, having spin zero, couples (non universally!) to mass rather than to energy and produces violations of UFF (see e.g. T. R. Taylor and GV, PLB 213 (1988) 450). A mass > 10^{-4} eV needed to be safe (or invent a decoupling mechanism: DP, DPV).

A real threat to QST, making it vulnerable even to long-distance/low-energy experiments.

At tree-level, string theory is already ruled out by precision tests of the EP (cf. JBD with a small ω).

The two meanings of Γ_{eff}

The effective action actually has two distinct meanings. The first is the one we have just said and is a familiar one for an effective action: Γ_{eff} can be used to compute the couplings of various massless particles and their scattering amplitudes as an expansion in powers of energy (Cf. zero-slope limit).

The second meaning is more unconventional: Γ_{eff} generates (as its eom) the conditions to be satisfied by the background fields in order to preserve the crucial 2D local (gauge) symmetries of string theory.

It is amusing that these two concepts get interconnected in string theory.

The effective action is the result of a two-dimensional QFT but can be used for D-dimensional physics!

The two expansions of Γ_{eff}

Quantization of (integrating over) the string coordinates produces potential gauge anomalies that have a natural expansion in powers of l_s (times derivatives of the fields).

Integrating over the 2D metric produces another expansion in powers of exp(2Φ).

Therefore Γ_{eff} has a double perturbative expansion:

$$\begin{aligned}\Gamma_{eff} &= -\left(\frac{1}{l_s}\right)^{D-2}\int d^D x\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2}+R(G)-4\partial_\mu\Phi\partial^\mu\Phi+\frac{1}{12}H^2+O(l_s^2)\right]\\ &+ \left(\frac{1}{l_s}\right)^{D-2}\int d^D x\sqrt{-G}\,[\ldots]+O(e^{2\Phi})\end{aligned}$$

The latter expansion has a QFT analogue (Cf. loop expansion in QED). The former does not!

This effective action modifies gravity at large distances (which is dangerous, but hopefully cured by loop & non-perturbative corrections) and also, of course, at short distances $O(l_s)$.

These latter modifications make loop corrections well defined in the UV. Indeed, one gets their correct order of magnitude, $\exp(2\Phi)$, by computing loops as in a QFT but with a short distance cutoff given by the string length.

Here is a prototypical quantum-gravity loop correction:

$$\left(\frac{\text{loop}}{\text{tree}}\right) \sim G_N \Lambda_{UV}^{D-2} \rightarrow \left(\frac{l_P}{l_s}\right)^{D-2} = exp(2\Phi)$$

which is of the same order as a gauge-loop correction. Unification survives loop corrections at $E \sim M_s$. However, at $E \ll M_s$ gauge and gravity "run" differently.

Homogeneous (Bianchi I) equations

It is straightforward to write down the field equations for a homogeneous (for simplicity Bianchi I) universe:

$$ds^2 = -dt^2 + \sum_i a_i^2(t)(dx^i)^2 \quad ; \quad \phi = \phi(t) \quad ; \quad B = 0$$

They take the simple form:

$$(\dot{\bar{\phi}})^2 - \sum_i H_i^2 = 0 \quad ; \quad \dot{H}_i - \dot{\bar{\phi}} H_i = 0 \quad ; \quad H_i \equiv \frac{\dot{a}_i}{a_i} \quad ; \quad \dot{\bar{\phi}} \equiv \dot{\phi} - \sum_i H_i$$

where the so-called shifted dilaton is defined by:

$\bar{\phi} = \phi - \frac{1}{2}\log(\det G_{ij})$ and satisfies, as a consequence,

$$\ddot{\bar{\phi}} - (\dot{\bar{\phi}})^2 = 0 \Rightarrow \frac{d}{dt} e^{-\bar{\phi}} = \text{constant}$$

Generalized Kasner solutions

In the absence of other sources the equations of Bianchi I string cosmology can be easily solved. One finds:

$$a_i(t) = (\pm t)^{p_i} \quad ; \quad \phi(t) = -(1 - \sum_i p_i) \log(\pm t) + \text{const.} \quad ; \quad \sum_i p_i^2 = 1$$

These reduce to the usual Kasner cosmology if we impose a constant dilaton. Note however that, unlike for pure Kasner, one can have a perfectly isotropic cosmology for a non-trivial dilaton:

$$a_i(t) = t^{\pm \frac{1}{\sqrt{d}}} \quad ; \quad \phi(t) = -(1 \mp \sqrt{d}) \log t \quad ; \quad t > 0 \quad ; \quad d \equiv D - 1 = 9$$

and similarly for $t < 0$.

Also note the interesting possibility of flipping arbitrarily the signs of the Kasner exponents if we do not freeze the dilaton.

Scale-factor duality

This last feature is related to an interesting symmetry of the string-cosmology equations under inversion of any individual scale factor $a_i(t)$, provided we keep the shifted dilaton invariant.

Indeed, under $a_i(t) \to 1/a_i(t)$, $H_i(t) \to - H_i(t)$, but our two independent equations go into themselves under this change.

This symmetry, mapping solutions into new (and generically inequivalent) ones has been called scale-factor duality (SFD) and is closely connected to T-duality (although the latter is a true symmetry of the theory). It also holds if we add stringy matter.

If the $B_{\mu\nu}$ field is turned on, the discrete (Z_2^9) SFD symmetry becomes a continuous O(9,9;R) symmetry closely connected to Narain's O(n,n;R) group of (generically inequivalent) compactifications of n space dimensions.

NB. Only a discrete subgroup leaves the physics invariant.

The pre-big bang scenario

The so-called pre big bang scenario in string cosmology is deeply rooted on SFD (a stringy symmetry) combined with the (more standard) invariance of the cosmological equations under T, the time reversal operation t -> -t. The combination SFDxT clearly acts on an individual scale factor as follows:

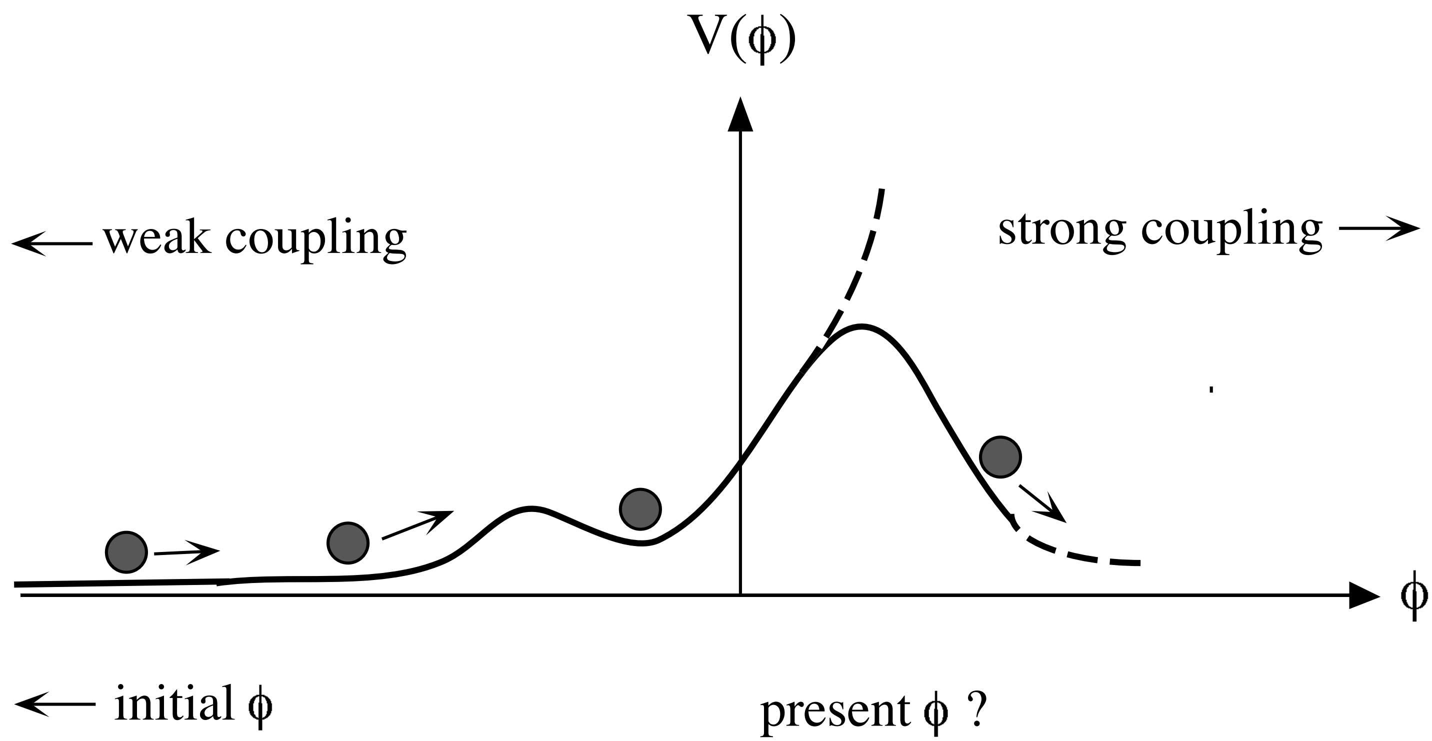
$$a_i(t) \to \tilde{a}_i(t) \equiv a_i^{-1}(-t) \Rightarrow \tilde{H}_i(-t) = H_i(t) \ ; \ \dot{\tilde{H}}_i(-t) = -\dot{H}_i(t)$$

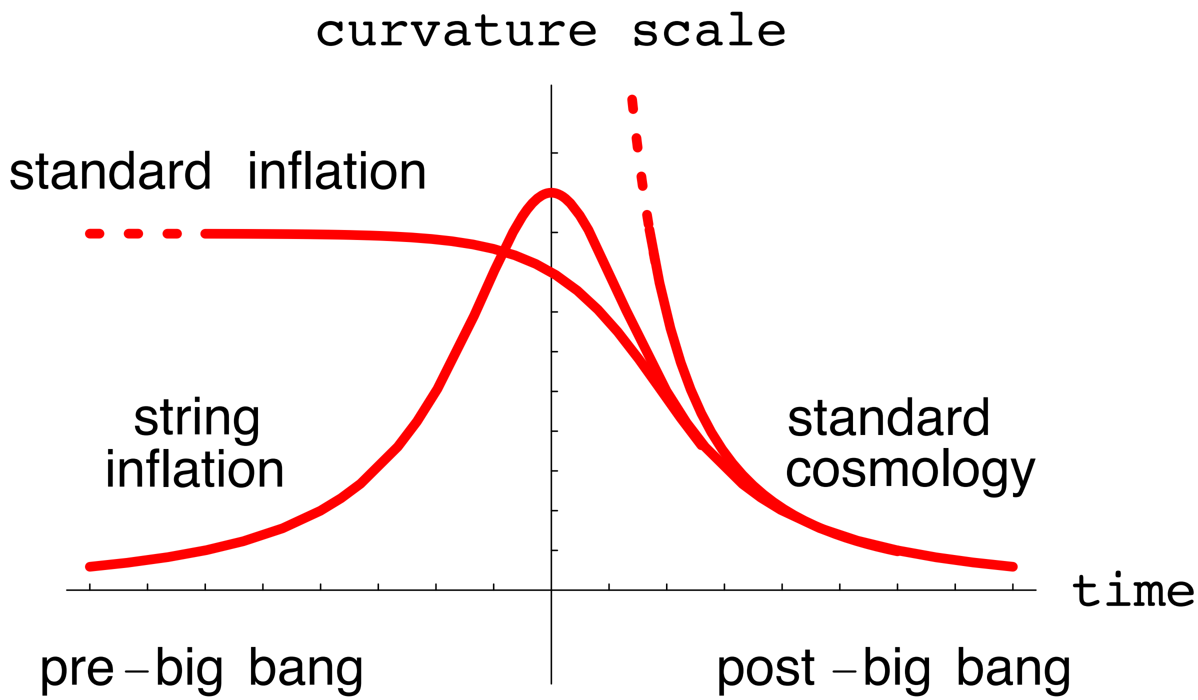
Therefore, given a standard FLRW cosmology (an expanding & decelerating Universe at t > 0), SFDxT associates with it another expanding, but now accelerating, cosmology at t < 0. Can we put together these two SFDxT-related cosmologies?

If the answer is yes we may have a new scenario in which a long "dual" inflationary phase at t < 0 preceded the standard FLRW phase possibly solving the shortcomings of the latter.

Diagrams illustrating PBB idea

(GV '91, Gasperini & GV '93)




curvature scale
standard inflation
string
inflation
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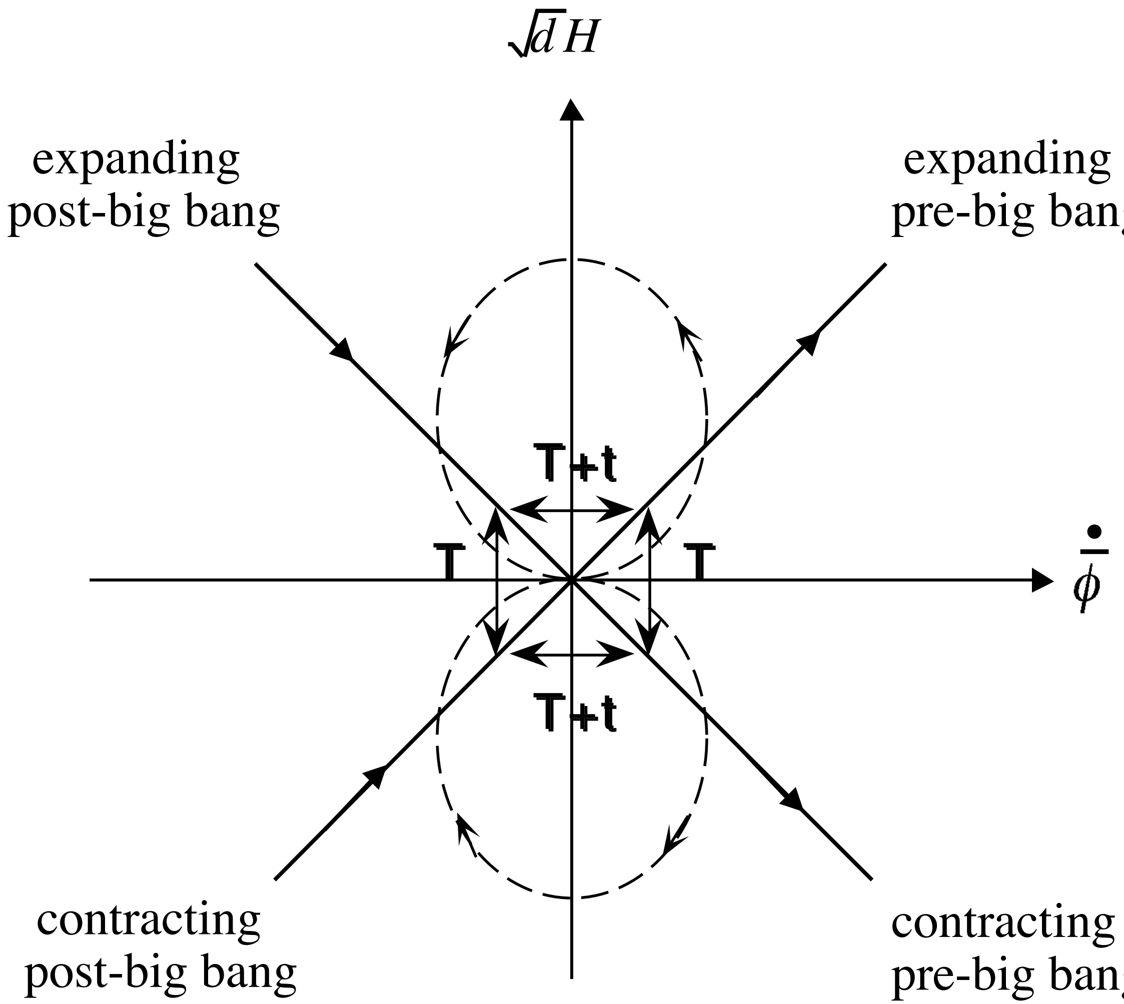
cosmology
time
pre – big bang
post – big bang


$\sqrt{d}H$
expanding
post-big bang
expanding
pre-big bang
T+t
T
T
$\dot{\phi}$
T+t
contracting
post-big bang
contracting
pre-big bang

The pre-big bang accelerator

It looks as if we have obtained inflation for free in string theory! How is that possible with just a scalar field with vanishing potential?

The answer to this question lies in the peculiar way the dilaton appears in string theory. Recall that the exponential of the dilaton controls g_s and the ratio l_P/l_s.

Consider a post-big bang solution describing a decelerating expansion with a constant dilaton. Under SFDxT this solution goes into one describing a pre-big bang accelerating universe with a growing dilaton, hence a growing g_s and l_P/l_s.

NB. The accelerated expansion is present in the string frame, i.e. if we measure distances in l_s-units. But the growth of l_P/l_s is so fast that the universe contracts if, instead, we measure distances in l_P-units.

Thus the answer to the question:

Is PBB a bouncing cosmology?

depends on the frame, i.e. on the meter we use to measure distances. The scale-factor may or may not bounce.

However, independently of the frame, PBB cosmology corresponds to a "curvature bounce" in that it has a phase of growing curvature turning into one of decreasing curvature through an intermediate "string phase" during which the curvature is of order l_s^{-2}.

Actually, an accelerated contraction can also help solving the HBB puzzles and indeed the physical predictions are identical in the two frames.

Initial conditions & fine-tuning

So far we have assumed a "cosmological principle" for string cosmology, like it's done for the HBB scenario.
We would like instead PBB cosmology to emerge from generic (i.e. non fine-tuned) initial conditions. This is possible if we make an assumption (BDV) of "Asymptotic Past Triviality".
This is just the opposite of what is assumed in HBB cosmology (where everything started at a singularity).
In fact, the need for a beginning of time, is now completely removed provided we can smoothly join the pre and post bang phases.

Asymptotic Past Triviality (APT)

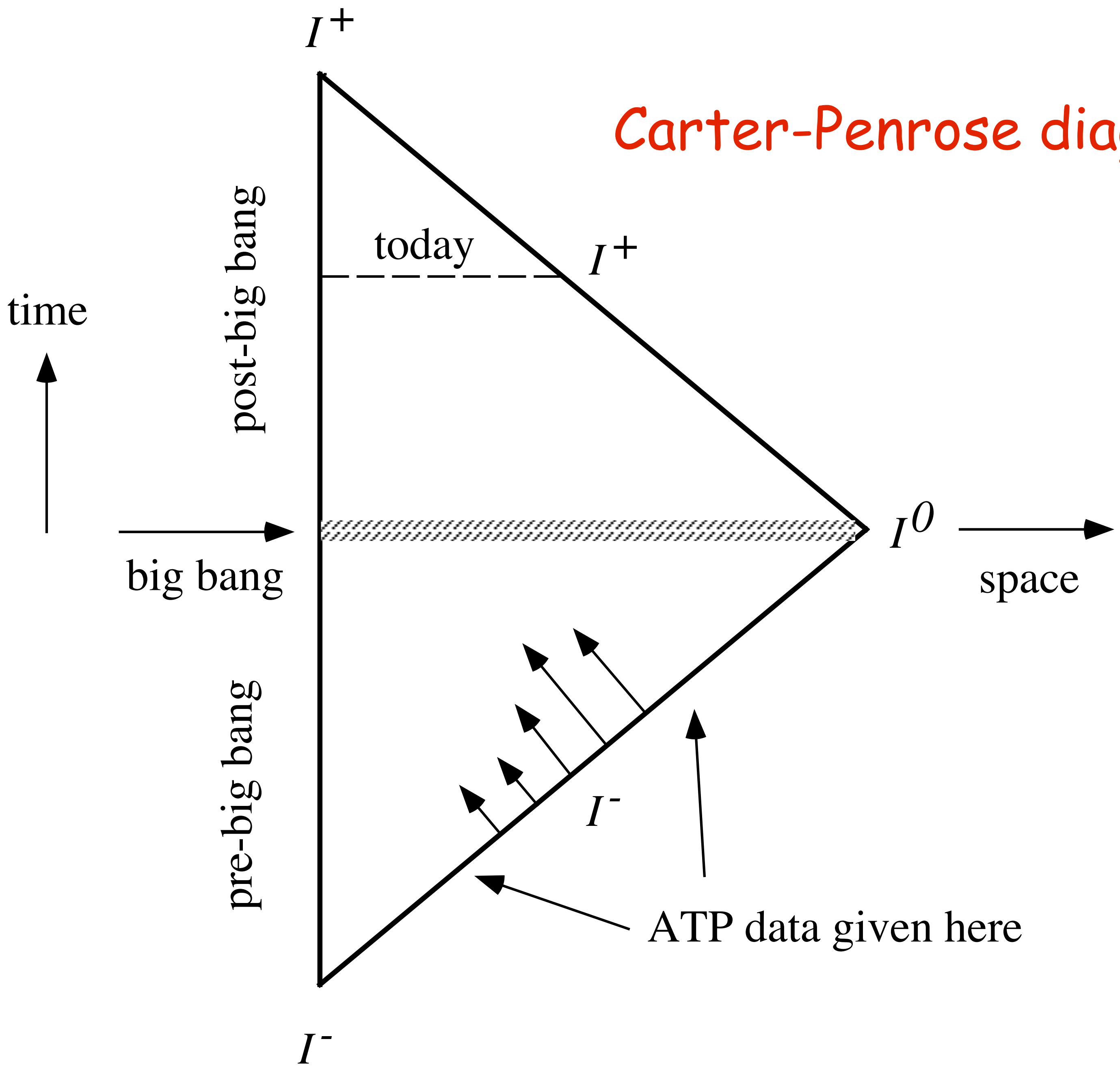
APT: As we go towards t = -∞ the Universe gets closer and closer to the trivial vacuum of superstring theory (nearly flat D=10 spacetime and nearly vanishing string coupling, $e^{\phi} \ll 1$) but is otherwise generic (in a precise math. sense).

Because of SUSY, a dilatonic potential cannot be produced at any finite loop order and is therefore completely negligible as long as g_s is very small ($\phi \ll -1$).

Thanks to APT we can thus use the effective action of QST at lowest order both in the genus and in the derivative expansion:

$$\Gamma_{eff} = -\int \frac{d^{10}x}{l_s^8}\sqrt{-G}e^{-\phi}\left[R(G) - \partial_\mu\phi\partial^\mu\phi + \frac{1}{12}H^2\right]$$

Carter-Penrose diagram

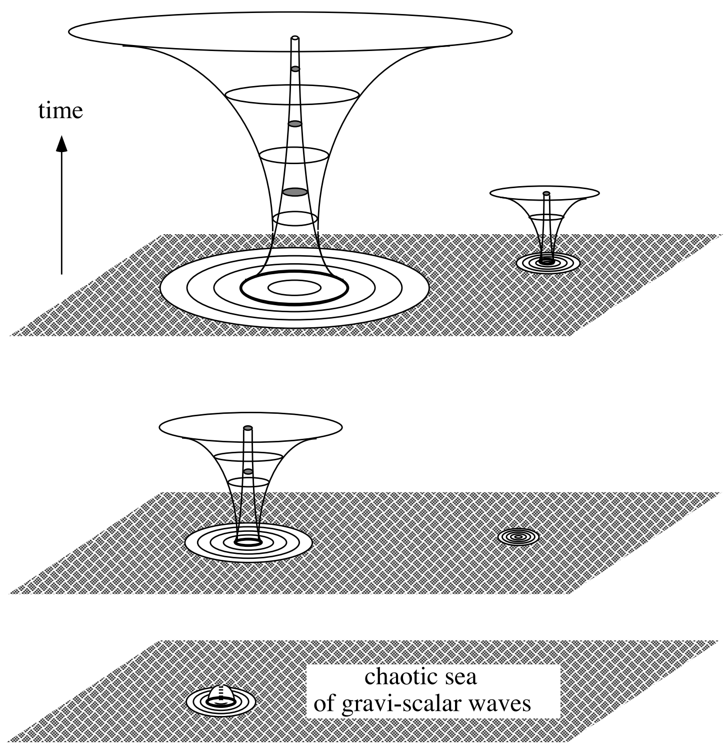


$$\Gamma_{eff} = -\int \frac{d^{10}x}{l_s^8}\sqrt{-G}e^{-\phi}\left[R(G) - \partial_\mu\phi\partial^\mu\phi + \frac{1}{12}H^2\right]$$

We can write down a generic solution in the far past and check that it contains the appropriate number of arbitrary functions to be called generic. It describes, physically, a chaotic superposition of gravitational and dilatonic waves.
In the APT regime the field equations are invariant under a constant shift of ϕ and under a global rescaling of x. As a result, the generic initial data include, as free parameters, the initial value of the dilaton ϕ_{in} and the initial curvature scale.
Solutions that go to the trivial vacuum in the infinite past, become increasingly complicated, curved and coupled as one moves forward in time.

As a consequence of singularity theorems (Hawking, Penrose), the evolution generically brings about the formation of Closed Trapped Surfaces (CTS), i.e. of black holes in different spacetime locations with arbitrary (and randomly distributed?) values for ϕ_{CTS} and for the horizon radius R_{CTS}.

A PBB cosmology then takes place inside the CTS. How?



Models for the onset of PBB cosmology

I. Spherical symmetry (BDV 1999)

Consider spacetime around one of the CTS and approximate it by a spherically symmetric one.
By going to the Einstein frame we map the problem into one extensively studied in the GR literature, in particular by D. Christodoulou (1991, ...), the collapse of a minimally coupled massless scalar field in the case of spherical symmetry.
DC has given a quantitative criterion for the formation of a CTS and has studied the solution inside the horizon. As expected, DC's criterion is scale and dilaton-shift invariant: as usual, it has to do with some critical dimensionless ratio of incoming energy per unit advanced time (in $G = c = 1$ units).

The solution near the singularity is dominated by time derivatives with spatial gradients becoming more and more subdominant as the singularity is approached. One obtains a generalized Kasner cosmology in the Einstein frame which, in the string frame, describes a quasi homogeneous PBB cosmology:

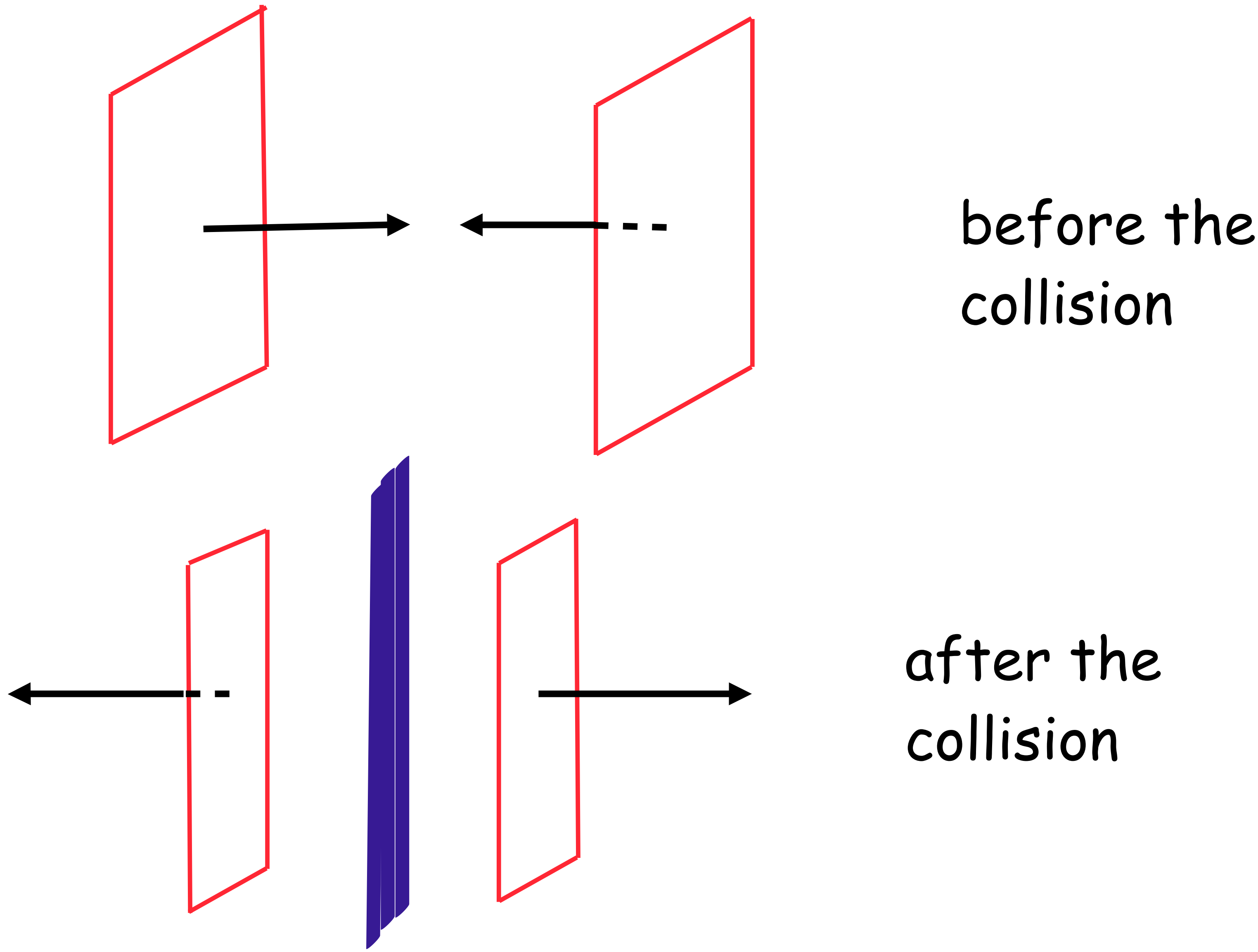
$$a_i(t) = (\pm t)^{p_i(x)} \quad ; \quad \phi(t) = -(1 - \sum_i p_i(x)) \log(\pm t) + \text{const.} \quad ; \quad \sum_i p_i^2(x) = 1$$

Near the singular hypersurface there are spatial regions that become very large (in string units) and isotropic.
In order to become sufficiently large to wash out spatial curvature we need a large-enough ratio between R_{CTS} and l_s and also a sufficiently negative ϕ_{CTS}.

II. Plane symmetry

(Feinstein, Kunze & Vasquez-Mozo '00; Bozza & GV, '00)

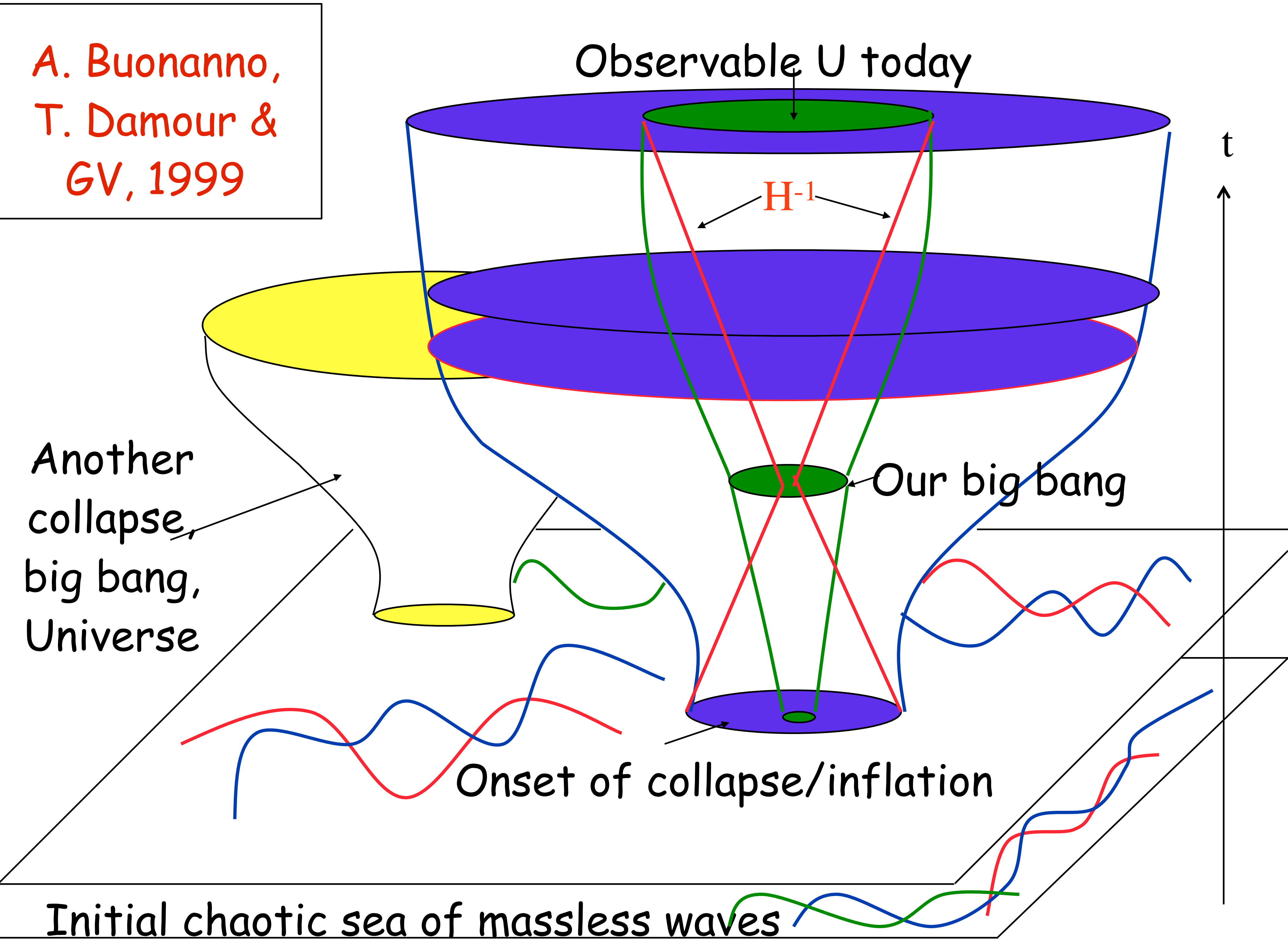
As another (opposite) limit we take the collision of two infinitely extended plane waves. If we assume translation invariance along the transverse plane the model is exactly soluble (reduced to quadratures), see Szekeres, Yurtsever. By causality, some of the results also hold for sufficiently large wavefronts in a spacetime region after the collision. The metric inside the horizon is again of the generalized Kasner type and, in the string frame, it corresponds to a quasi-homogeneous PBB cosmology.



BB-like sing. formed behind wave-fronts

The fine-tuning issue

In both examples (spherical and plane symmetry) PBB inflation starts at a curvature scale and coupling that represent two free parameters (integration constants) of the solutions.
In order to solve the HBB cosmological puzzles we need the initial coupling to be very small (say $\phi_{CTS} < -100$) and the CTS radius $R_{CTS} > 10^{-13}$ cm $\sim 10^{20}\ l_s$.
Is this fine-tuning? Given the numbers some people (like A. Linde) have answered: Yes! Personally I disagree:
because of the chaotic nature of our scenario there will be a whole distribution of possible values for ϕ_{CTS} and R_{CTS} and at least one large smooth Universe like ours will easily emerge somewhere together with many others...


A. Buonanno,
T. Damour &
GV, 1999
Observable U today
t
H-1
Another
collapse,
big bang,
Universe
Our big bang
Onset of collapse/inflation
Initial chaotic sea of massless waves

Extensions of PBB scenario

The simple PBB model can be generalized by including a non-trivial B-field and RR forms as backgrounds.
This does not seem to make the model any better.
Actually, Damour and Henneaux have shown that the addition of RR forms tends to give back the so-called BKL oscillations already known in GR as a consequence of Kasner's anisotropy.
Some spatial gradients slowly become dominant and induce a sudden jump of Kasner's $p_i(x)$. Another "velocity dominated" Kasner phase then takes place, followed by another jump of the p_i, and so on indefinitely as one approaches the singularity.
The dilaton allows for isotropic solutions without BKL oscillations, but adding other backgrounds brings them back.

Other models of a bouncing Universe

By now there are many other models of bouncing cosmology, some based on string theory, some on the brane-Universe idea, some on loop gravity. No time to review them here. Phys. Rep. of M. Gasperini & GV has some account of them.

Short Break

Testing PBB cosmology

Reminder from yesterday

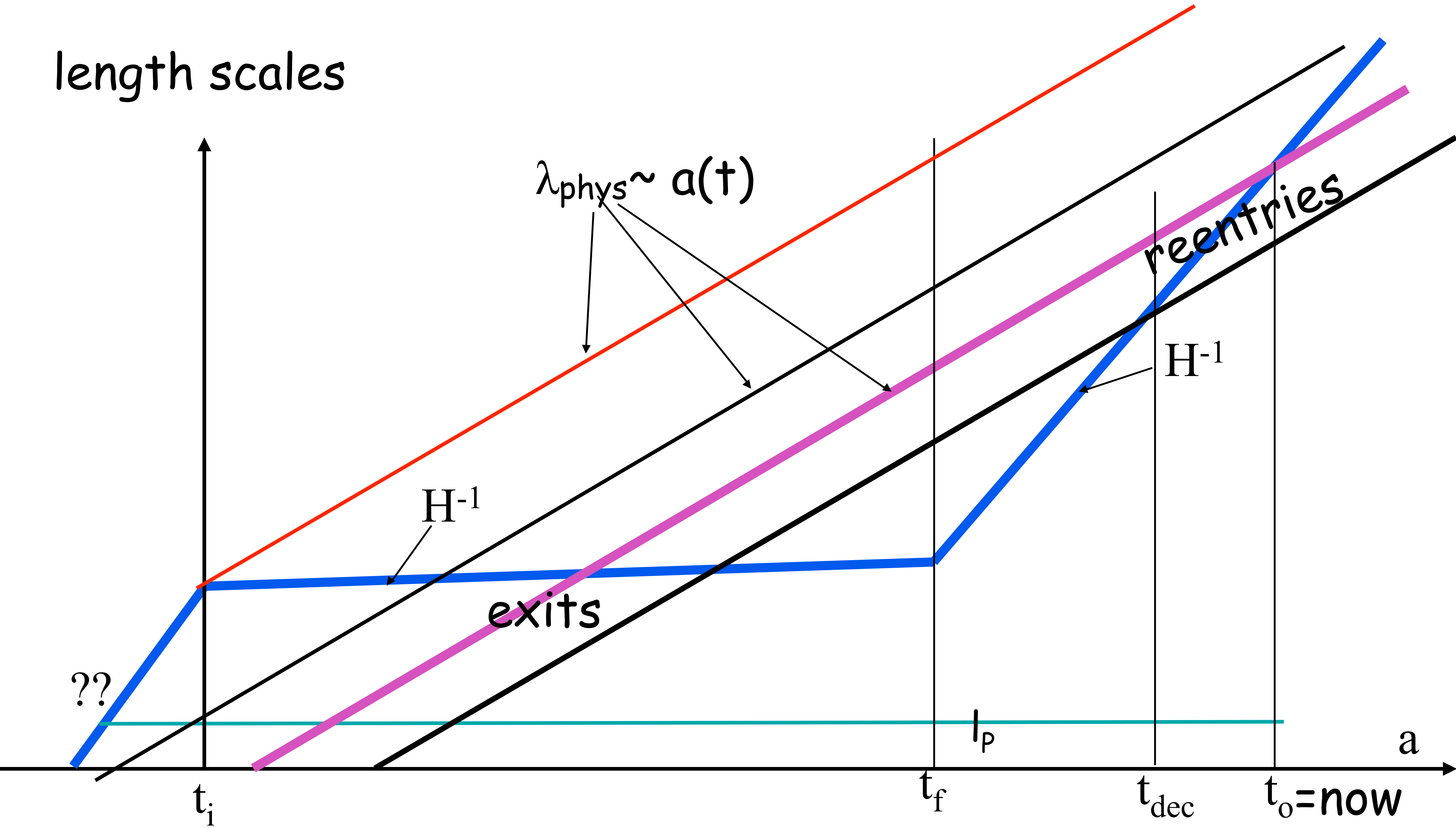
A distinctive property of any inflationary epoch is that, all along its duration, physical scales are continuously pushed outside the horizon. This a direct consequence of the growth of the ratio

$$\frac{\lambda_{phys}}{H^{-1}} \sim aH = \dot{a}$$

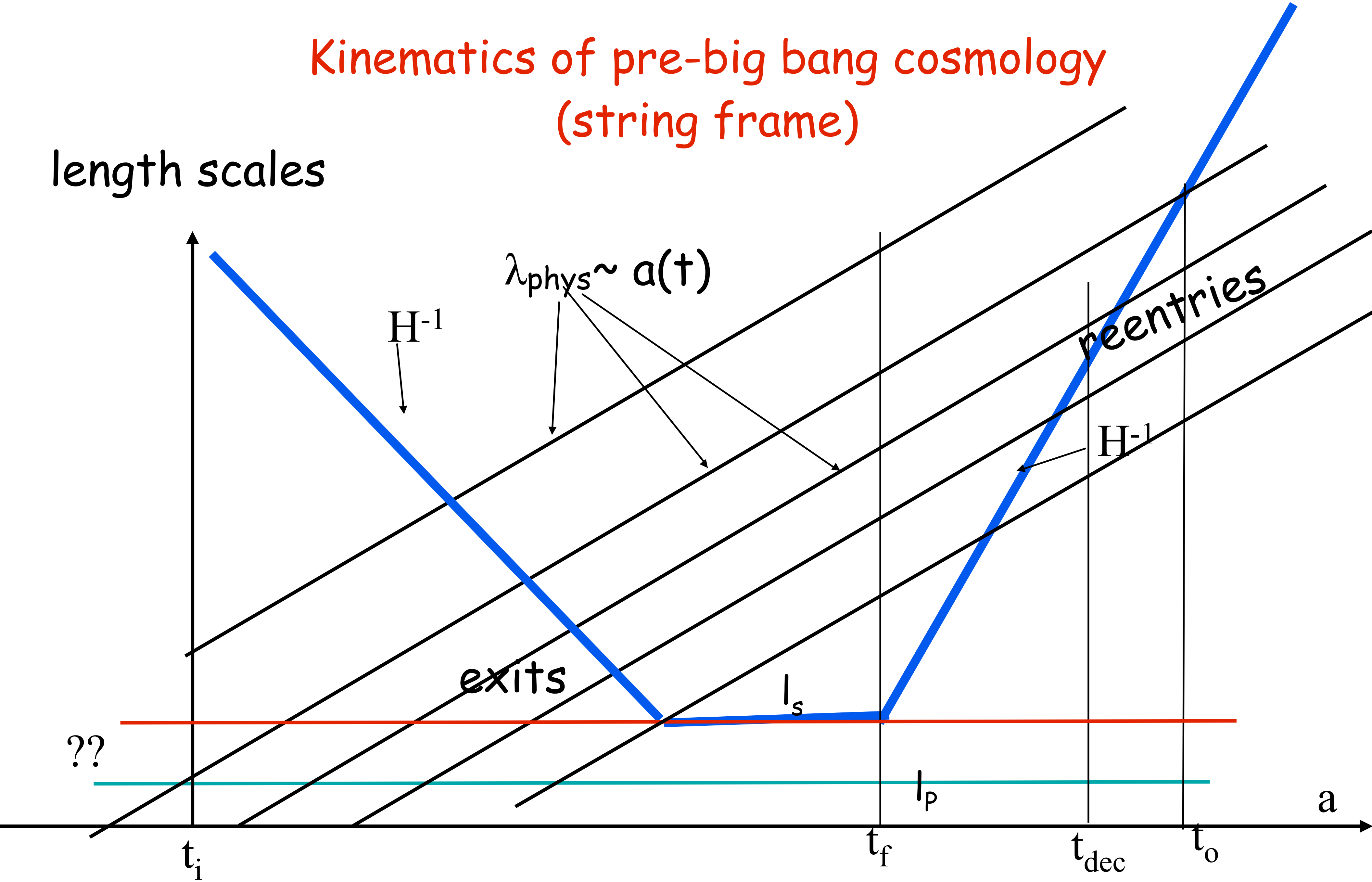
during inflation. During a decelerated expansion, physical scales "re-enter the horizon". The amplification of fluctuations has a lot to do with this basic kinematical fact: scales initially inside the horizon go out during inflation and reenter after inflation.

NB. The same happens for an accelerated contraction e.g. for the PBB in the Einstein frame. However, there are important quantitative differences…

Kinematics of slow-roll inflation



Kinematics of pre-big bang cosmology (string frame)



Tensor Perturbations in string cosmology

Unlike in slow-roll inflation H is now rapidly varying in time:

=> we expect all but a flat spectrum of tensor perturbations

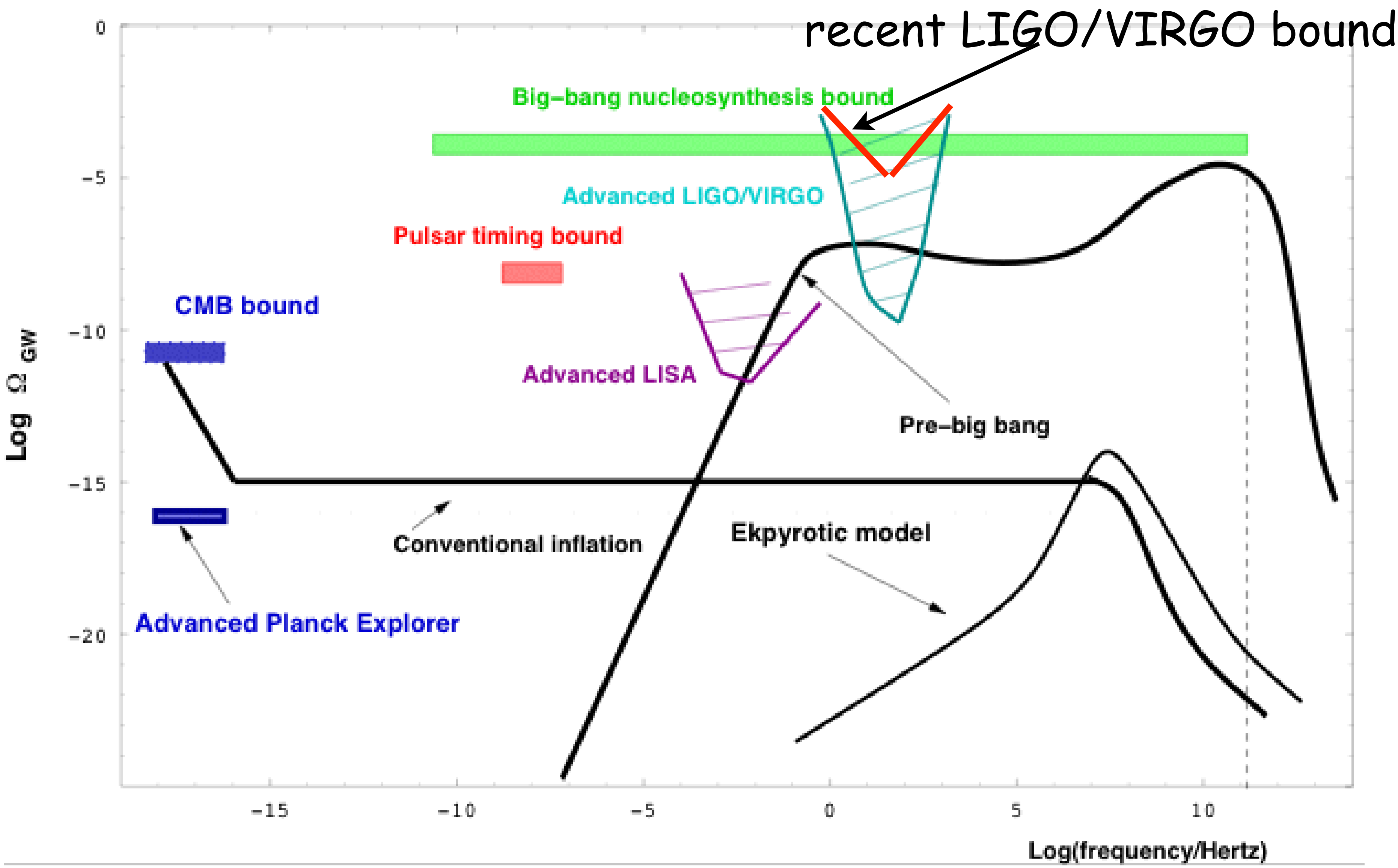
Since H grows in the pre-bang phase the spectrum is expected to be blue-tilted (more power at shorter length scales).

We have to find the "pump field" for tensor perturbations in string cosmology. Because of the way the dilaton enters the effective action one finds that the pump field is

$$P = \exp(-\phi/2)\, a = a_E \sim \eta^{1/2}$$

This implies $n_T = 3$ (as opposed to $n_T = 0$ in SRI)

=> possibly good for detection, irrelevant for CMB, LSS.


recent LIGO/VIRGO bound
Big-bang nucleosynthesis bound
Advanced LIGO/VIRGO
Pulsar timing bound
CMB bound
Advanced LISA
Pre-big bang
Conventional inflation
Ekpyrotic model
Advanced Planck Explorer
Log Ω GW
Log(frequency/Hertz)
0
-5
-10
-15
-20
-15
-10
-5
0
5
10

The reason why the spectrum is not flat in the stringy window is that we do not know the exact dynamics of the string phase. If H is constant but ϕ is not (e.g. has constant positive time derivative) the spectrum is blue-tilted.

A stochastic background of GW should be around us and can be detected, in principle, by looking for cross-correlations in two interferometers (in order to disentangle it from real noise).

Its discovery/measurement would give us a picture of the Universe when gravitons decoupled (like the CMB does for photons) i.e. of the Planck/string epoch, if it ever existed.

LIGO/VIRGO have lowered the upper bound on this stochastic background below the so-called NS bound (too many GW would have upset the successes of primordial nucleosynthesis, just like a 4th light neutrino).

Scalar perturbations in string cosmology

Like in ordinary inflation we can compute the spectrum of adiabatic curvature perturbations in string cosmology (coupled dilaton-metric perturbations).

Not surprisingly they also come out blue-tilted ($n_s = 4$) and of the same order as tensor perturbations (no slow-roll enhancement of S/T in SC, in any case they are both tiny!)

Like with tensor perturbations this result is quite insensitive to what the extra dimensions do.

These perturbations are thus irrelevant for CMB.

EM perturbations in string cosmology

Let us consider the gauge part of the string effective action

$$\Gamma^{EM} = -\frac{1}{4}\int d^4x\sqrt{-g}e^{-\phi_4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma} \to -\frac{1}{4}\int d^3x d\eta e^{-\phi_4}F_{\mu\nu}F_{\rho\sigma}\eta^{\mu\rho}\eta^{\nu\sigma}$$

where $\exp(\phi_4)$ (containing also a contribution from V_6) is the effective fine structure constant α.

In order to amplify EM perturbation we need a dynamical ϕ_4! Such a feature is absent in conventional models.

The canonical EM potential is $\exp(-\phi_4/2)A$ and satisfies:

$$\hat{A}_k'' + \left[k^2 - e^{\phi_4/2}(e^{-\phi_4/2})''\right]\hat{A}_k = 0 \; ; \; \hat{A} = e^{-\phi_4/2}A_k$$

$$\hat{A}_k'' + \left[k^2 - e^{\phi_4/2}(e^{-\phi_4/2})''\right]\hat{A}_k = 0 \; ; \; \hat{A} = e^{-\phi_4/2} A_k$$

The vacuum fluctuations of the EM field (which do exist, see Casimir effect) are amplified by a (k-dependent) factor

$$\frac{\hat{A}_k|_f}{\hat{A}_k|_i} = \sqrt{\frac{e^{\phi_{re}}}{e^{\phi_{ex}}}} = \sqrt{\frac{\alpha_{re}}{\alpha_{ex}}}$$

It is not unconceivable that these amplified vacuum fluctuations may act as seeds for the cosmic magnetic fields that are known to exist at the μ-Gauss level on galactic and intergalactic scales.

A very large increase in α is needed between exit and reentry of the relevant scales in order to have large enough seeds for a "dynamo" mechanism to work.

Putting numbers one finds that a factor of at least 10^{66} is needed between $\alpha_{(now)}$ and $\alpha_{(exit\ of\ g.s.)}$. Sounds huge but is of the same order as the increase in the scale factor one needs in order to solve the usual cosmological puzzles.

In PBB cosmology the growth of the scale factor is related to the growth of ϕ and therefore it is natural to expect the same order of magnitude for both. Comments:

a) the actual spectrum of EM perturbations is always blue-tilted and depends on the behaviour of V_6 during the pre-bang phase: a window on extra dimensions.

b) unfortunately a reliable computation of present magnetic fields from a spectrum of initial seeds is not available.

c) One of the distinctive predictions of PBB cosmology!

But then:

Where does large scale structure come from?

Axion perturbations

So far we have found only blue spectra. However:

In all known string theories there is a pseudoscalar partner to the dilaton: the universal NS-NS axion σ.

- The pump field for a massless axion is:

$$P_\sigma = \frac{a^2 e^{\phi}}{V_6} = a^2 e^{\phi_4} \quad ; \quad P_\phi = a^2 e^{-\phi_4}$$

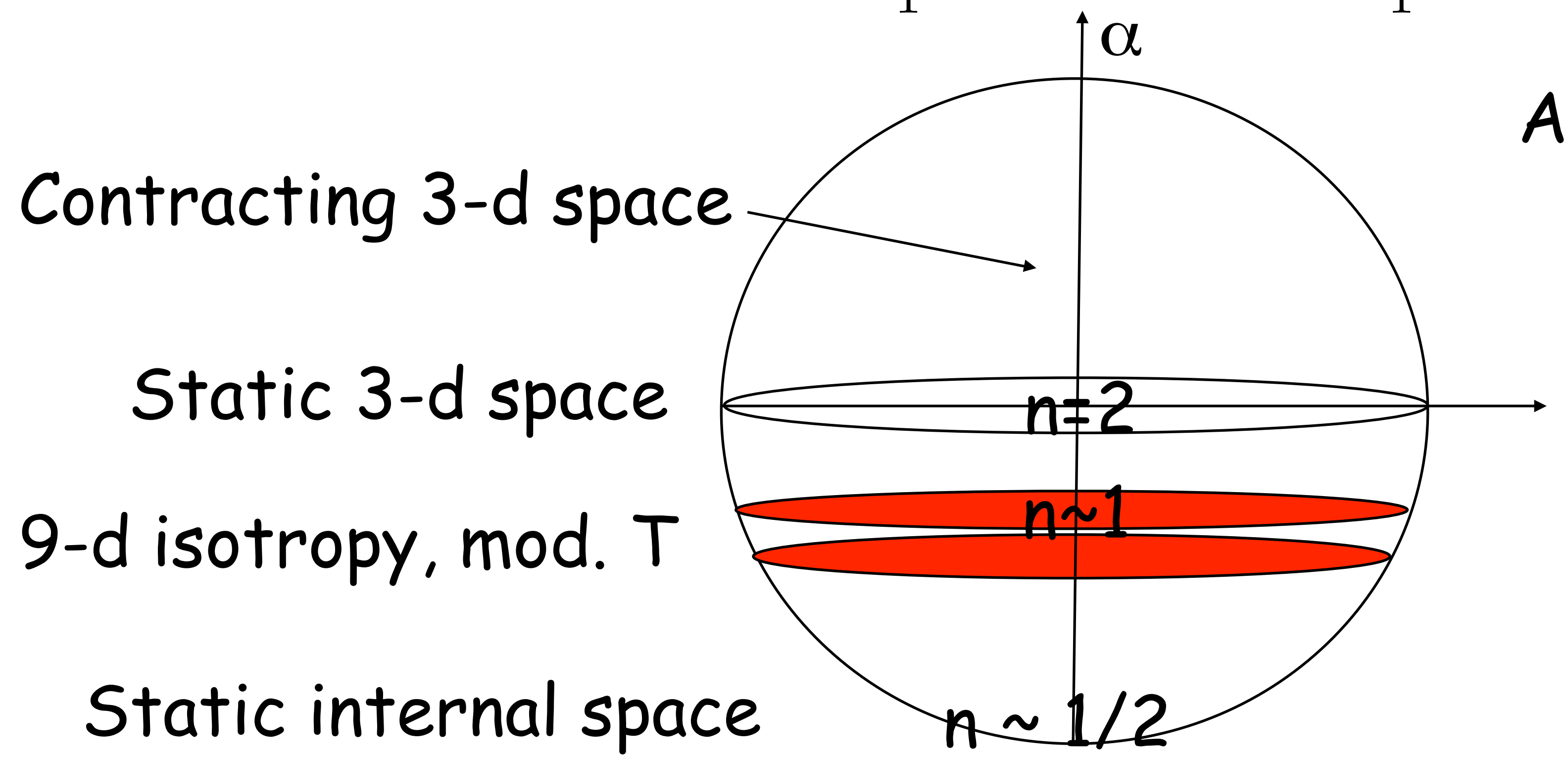
- P_σ can be of the inflationary type ($P_\sigma = 1/\eta^2$). One finds:

$$|\delta\sigma_k|^2 = \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega}{\omega^*}\right)^{n-1}, 4-2\sqrt{3} \sim 0.53 < n < 2$$

($H^* \sim M_s$, $\omega^* = H^* a^*/a_0 \sim 10^{11}$ Hz, σM_P = can.[al] axion field)

The Kasner sphere: $3\alpha^2 + \Sigma \beta_i{}^2 = 1 = \sin^2\theta + \cos^2\theta$

$$ds^2 = -dt^2 + \sum_1^3 (-t)^{2\alpha} dx_i^2 + \sum_1^6 (-t)^{2\beta_i} dy_i^2$$



Axion-spectrum can be blue, red or flat

Example:

$$n = \frac{4 + 6r^2 - 2\sqrt{3 + 6r^2}}{1 + 3r^2} \; ; \; r \equiv \frac{1}{2} \frac{\dot{V}_6}{V_6} \frac{V_3}{\dot{V}_3}$$

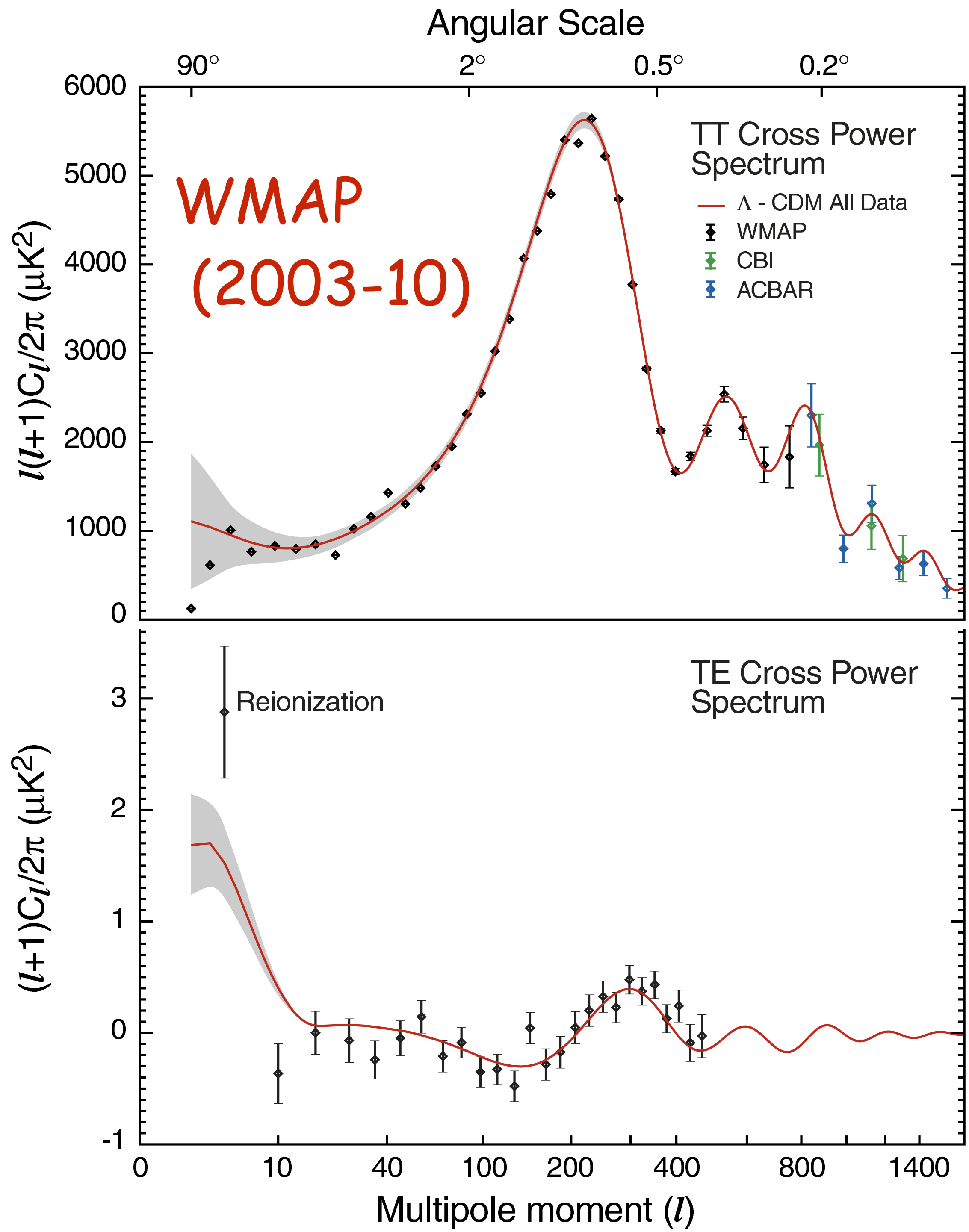
Unfortunately..

- Axion gives isocurvature (entropy) perturbations since its fluctuations (unlike the dilaton's) do not mix, to first order, with metric perturbations.
- Isocurvature perturbations feed back on curvature to 2^{nd} order but give "wrong" structure of acoustic peaks.

The situation looked quite hopeless for a while, but then a new idea, independent of string cosmology, came out...

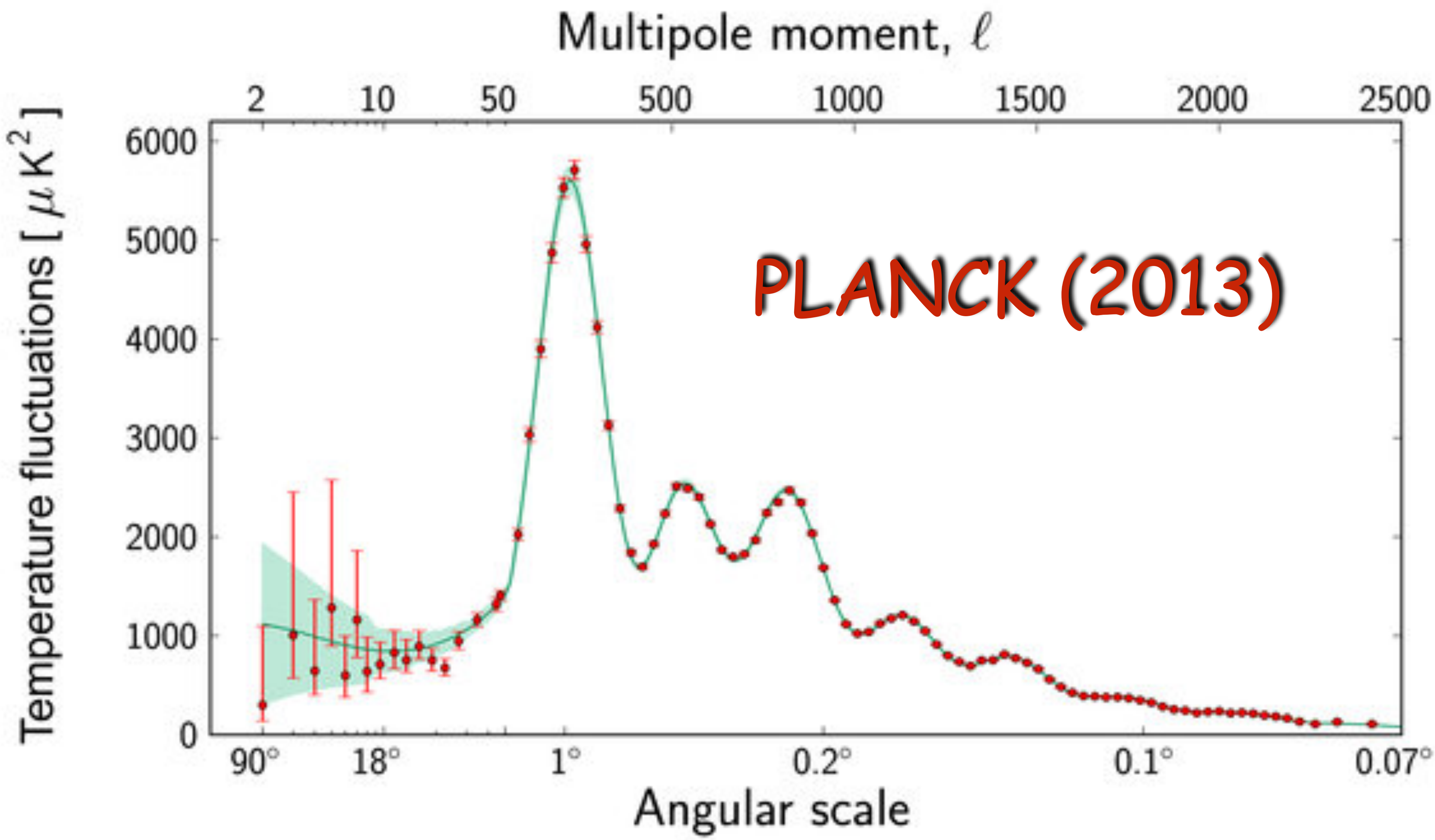
THE CURVATON

The axion can do the job by playing the curvaton's role!



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
WMAP
(2003-10)
3
2
1
0
-1
$(l+1)C_l/2\pi$ (μK^2)
Reionization
TE Cross Power Spectrum
0
10
40
100
200
400
800
1400
Multipole moment (l)

PLANCK POWER SPECTRUM



THE CURVATON MECHANISM

- If a $V(\sigma)$ is generated when the Universe cools down, and if $\langle\sigma\rangle = \sigma_i$ is not initially at its minimum, axion pert.s induce calculable curvature pert.s. This "curvaton" mechanism needs:
  1. a phase of axion relevance, dominance.
  2. the axion to decay before NS (m_σ > 10 TeV?)
- Conversion efficiency can be computed. Bardeen potential Φ_k (related to curvature pert.) after axion decay:

$$|\Phi_k|^2 = f^2(\sigma_i)\Omega_d^2|\delta\sigma_k|^2 = f^2(\sigma_i)\Omega_d^2\left(\frac{H^*}{M_P}\right)^2\left(\frac{\omega}{\omega^*}\right)^{n-1}$$

where $f(\sigma_i) \sim (4\sigma_i)^{-1}$ ($\sigma_i < 1$),

Ω_d is the fraction of critical energy in the axion at decay.

- One then computes the Sachs-Wolfe contribution to the C_l's

$$C_l^{(SW)} = \frac{1}{9\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1} \times \frac{\Gamma[l+(n-1)/2]}{\Gamma[l+2-(n-1)/2]}$$

($H^* \sim M_s$, $\omega^* \sim 10^{11}$ Hz $\sim 10^{30}\,\omega_0$, $f(\sigma_i) \sim (4\sigma_i)^{-1}$)

$$\frac{\Gamma[l+\ldots]}{\Gamma[l+\ldots]} \sim \frac{l^{n-1}}{l(l+1)} \Rightarrow l(l+1)C_l \sim l^{n-1}$$

- COBE Normalization: $C_2 = (1.09 \pm 0.23)\times 10^{-10}$ (or, from Planck, $A_s(k^*) = 3\times 10^{-10}$) gives:

$$(1.09 \pm 0.23)10^{-10} = \frac{1}{54\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1}$$

=> acoustic-peaks come out fine provided primordial axion spectrum is nearly flat (n~1). See 1606.07889 for an update.

⇒ Slightly blue spectra ($n > 1$) and/or low (H^*/M_P) preferred

Q: Can we play with $\Omega_d \sim \varepsilon^2$ to allow a higher H^*/M_P?

It turns out that one gains a factor ε^{-1} at the price of generating a $f_{NL} \sim \Omega_d^{-1} \sim \varepsilon^{-2}$

$$\frac{\Delta T}{T} = \left(\frac{\Delta T}{T}\right)_L + f_{NL}\left(\frac{\Delta T}{T}\right)_L^2$$

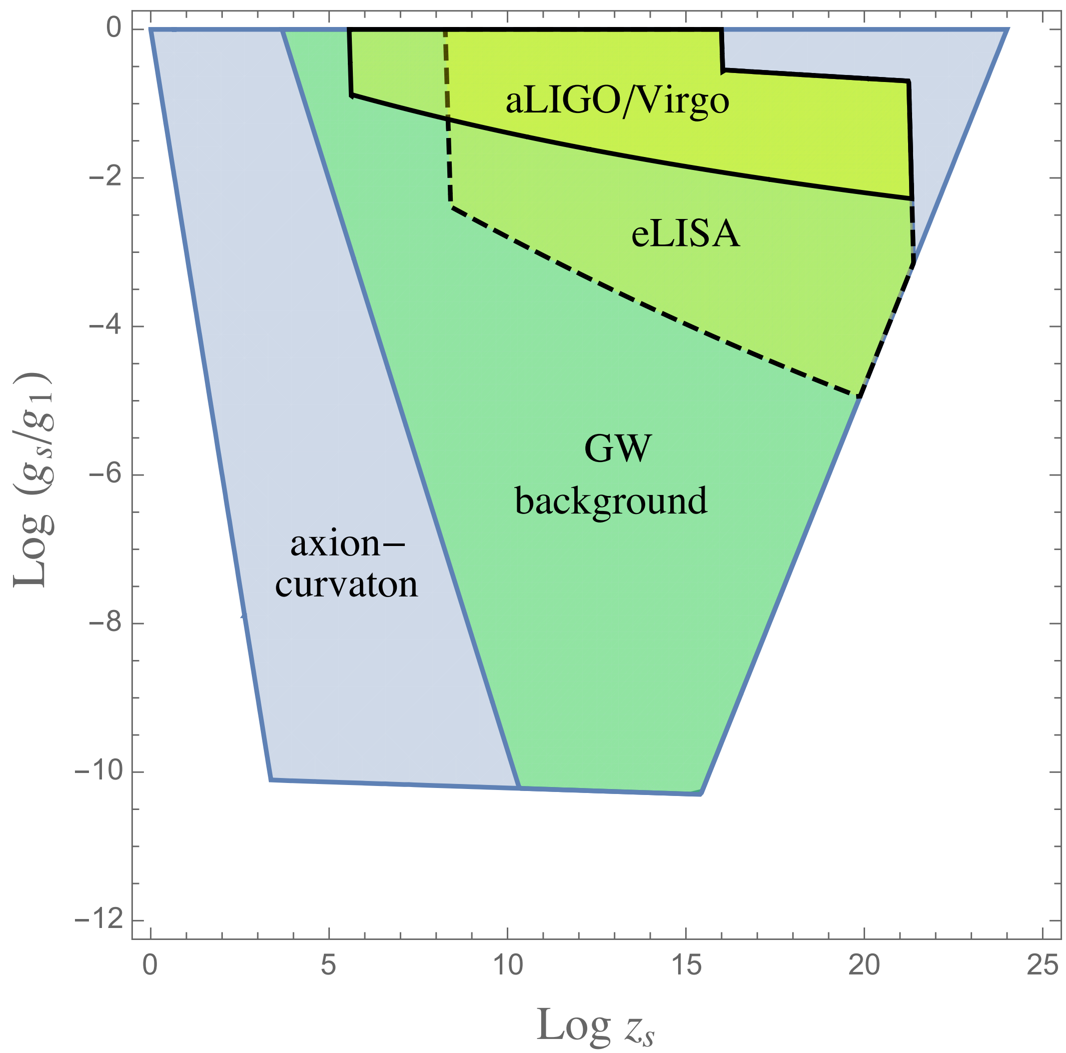
=> Given Planck's bound on f_{NL} ($O(10^2)$) we cannot gain much on normalization...

On the contrary, some non Gaussianity is all but unexpected.

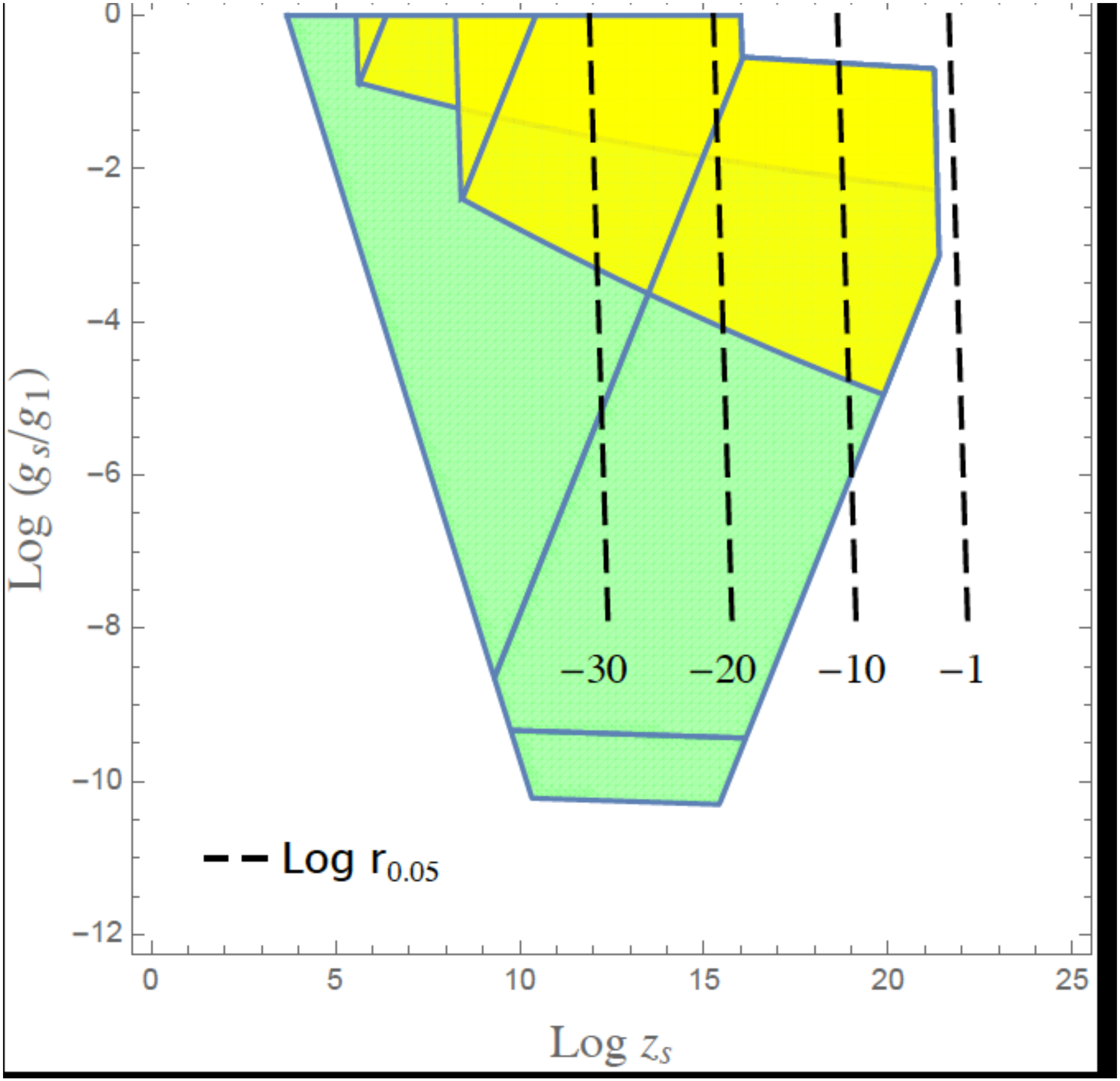
NB: Tensor contribution to CMB (B-polarization) is still completely negligible. Had BICEP's claim been confirmed our scenario would have been falsified!

In a recent paper (1606.07889) M. Gasperini has put together various constraints on PBB parameters by demanding:

1. A viable spectrum of scalar perturbations via the curvaton mechanism
2. A large-enough stochastic background of GW to be observable at Advanced LIGO/VIRGO and/or e-LISA

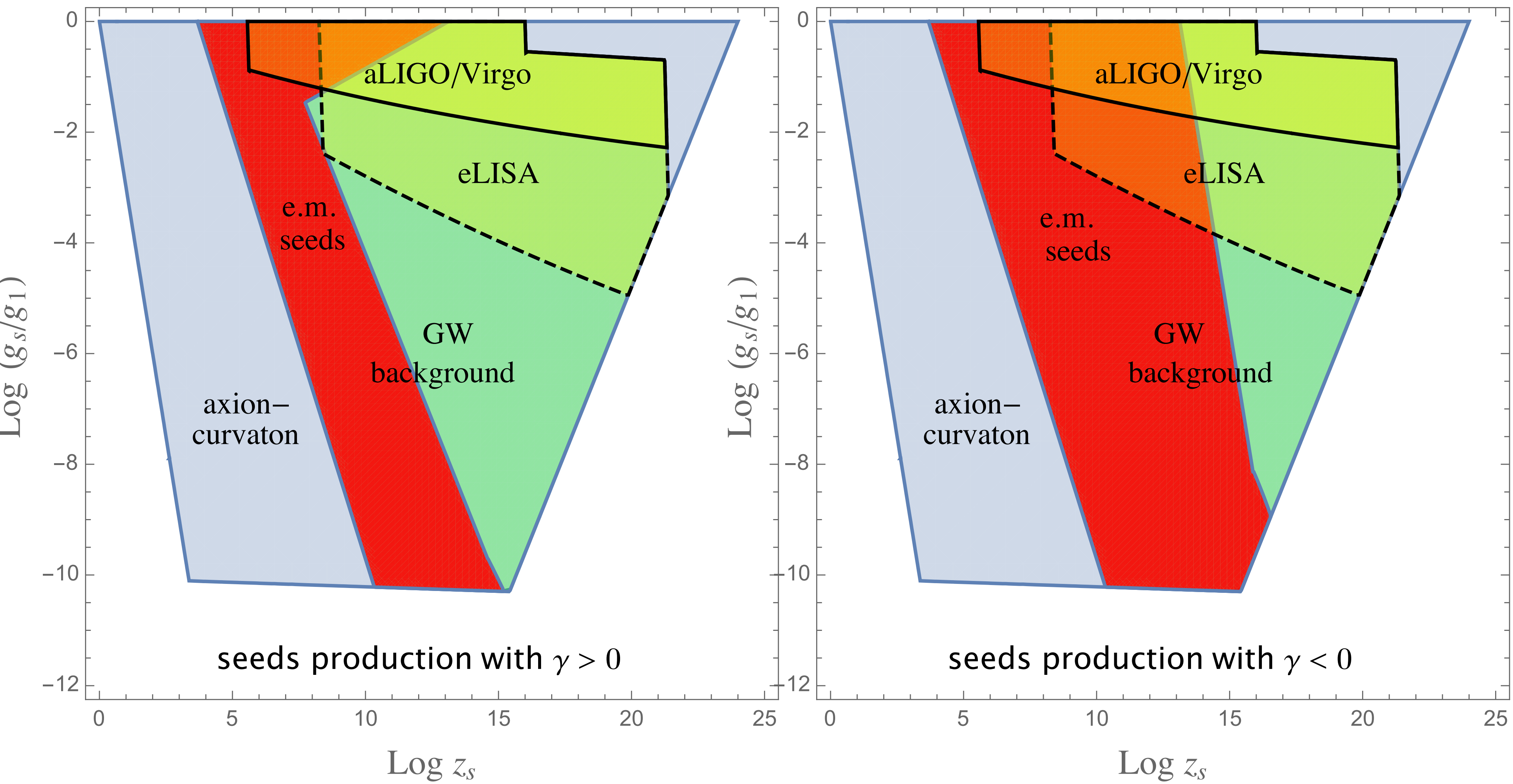

aLIGO/Virgo
eLISA
GW
background
axion–
curvaton
Log (g_s/g_1)
Log z_s
0
-2
-4
-6
-8
-10
-12
0
5
10
15
20
25

He has then looked at the ratio $r = T/S$ of tensor to scalar perturbations confirming a strong prediction of PBB cosmology: the absence of an observable primordial B-mode polarization in the CMB!



0
−2
−4
−6
−8
−10
−12
Log (g_s/g_1)
0
5
10
15
20
25
Log z_s
−30
−20
−10
−1
Log $r_{0.05}$

Finally (M. Gasperini, to appear) one can look at the parameter space where sufficiently large seeds are produced to generate cosmic magnetic fields via the dynamo mechanism


aLIGO/Virgo
eLISA
e.m. seeds
GW background
axion–curvaton
seeds production with $\gamma > 0$
seeds production with $\gamma < 0$
Log z_s
Log (g_s/g_1)

THE OUTSTANDING THEORETICAL PROBLEM OF PBB COSMOLOGY: HOW IS THE BOUNCE ACHIEVED?

There are many interesting suggestions (no time to review them) but they all fall short of showing rigorously that a bounce can be achieved...and how.

Balance Sheet

Slow-roll inflation	PBB cosmology
No obvious inflaton candidate	Dilaton as inflaton
Hard initial condition problem	Easy(er) initial condition problem
End of inflation within CGR. Reheating not a trivial problem	End of inflation in highly quantum/stringy regime. Quantum reheating?
Naturally good (SI) density perturbations	Good perturbations not automatic but possible via curvaton
T/S measurable? Unobservable GW's	T/S unobservable. Observable GW's?
No EM perturbations	EM perturbations seeding CMF?

Some bibliography

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- M. Gasperini, Elements of String Cosmology, Cambridge U. Press, 2011.
- G.V. Scientific American, April 2004 (also in: Special Number of Sci. Am.: A Matter of Time, Dec. 2014)

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