Chances for SUSY-GUT in the LHC Epoch

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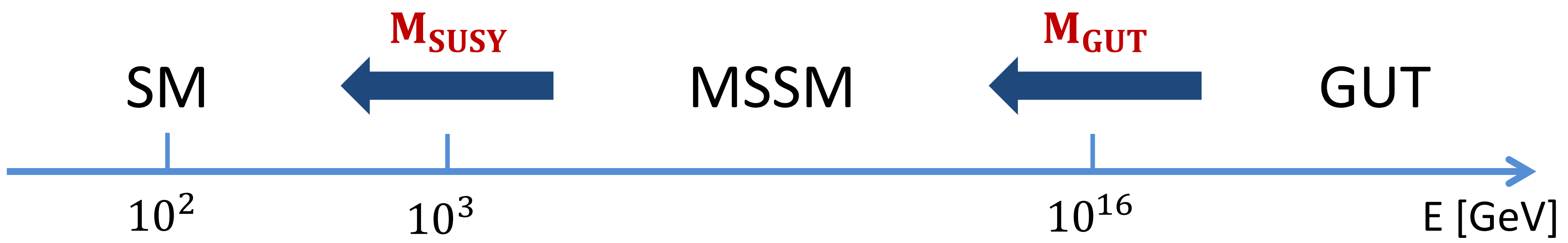
Beyond Standard Model

Several phenomena or open problems suggest the presence of physics beyond Standard Model (SM):

- hint of gauge unification
- structure of fermion masses
- hierarchy problem
- baryonic/leptonic number violation processes necessary at scale larger than EW for baryogenesis

Grand Unified Theories (GUT) address some of these issues and Supersymmetry (SUSY) improves the scheme solving the remaining problems.



Beyond Standard Model

The missing evidence for a SUSY phenomenology at LHC run-I has caused a pessimistic attitude of the scientific community toward SUSY and SUSY-GUT paradigms.

Hence the question:

After the 8 TeV LHC run-I, do they still represent an accepatable pardigm?

We have tried:

- to reanalyze the room still remaining for SUSY-GUT inspired models
- to study the limits on the mass spectrum of SUSY particles by spanning on the *compatible* SUSY-GUT
- to determine the most pessimistic case where the SUSY spectrum is the heaviest one. It results $\mathbf{M_{UB} = 20\ TeV}$ for the lightest SUSY particle.

Naturality requirements

According to the **naturalness principle** we require:

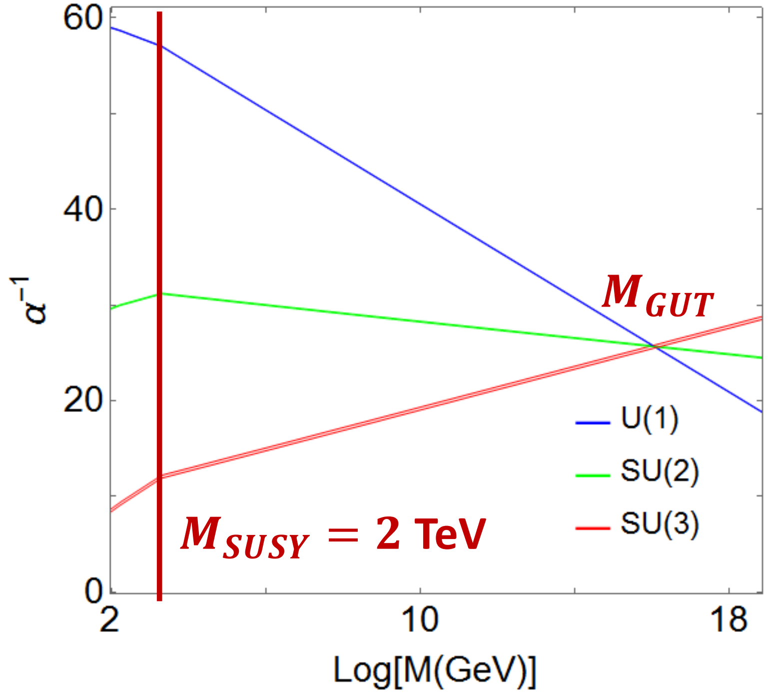
- all couplings, including Yukawa, must be order one in the “mother” GUT scenario above $\mathrm{M_{GUT}}$
- there are no ad hoc small parameter and no ad hoc fine tuning among parameters
- the mass parameters involved in the SUSY breaking are supposed to be of the same order of magnitude (modulo possible differences between F- and D-terms)

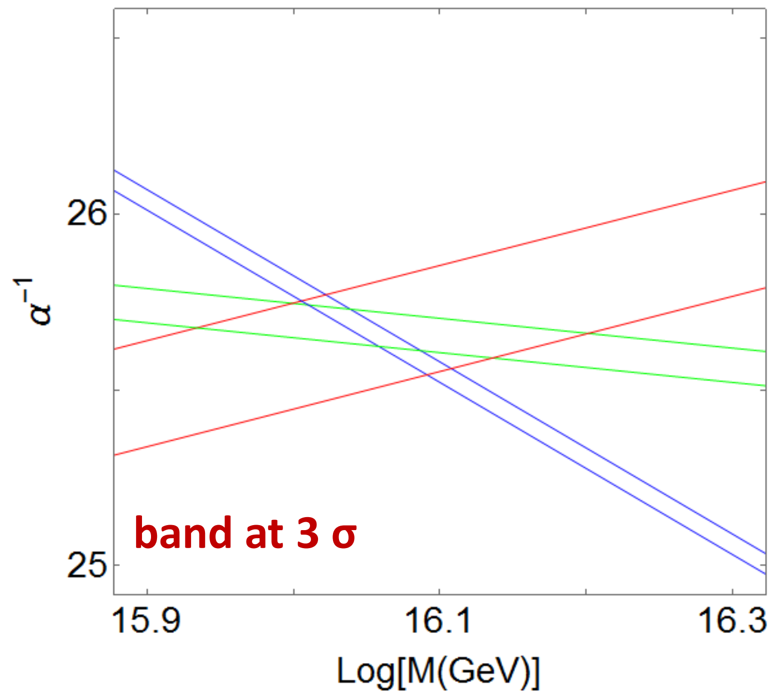
Requirements

In the present *model-independent* analysis we also require:

- one step gauge unification at a single energy scale $\mathbf{M_{GUT}}$, without intermediate symmetry scales

→ **SU(5) Bottleneck**





Requirements

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- one step gauge unification at a single energy scale $\mathbf{M_{GUT}}$, without intermediate symmetry scales

→ **SU(5) Bottleneck**

- consistency of third family fermion masses and possible Yukawa b-τ unification

$$y_{\mathrm{b}}(M_{\mathrm{GUT}}) = y_\tau(M_{\mathrm{GUT}}) \left(1 + \mathcal{O}\left(\frac{y_\mu(M_{\mathrm{GUT}})}{y_\tau(M_{\mathrm{GUT}})}\right)\right)$$

We do not consider the possible limits on SUSY-GUT coming from the assumption that one of the SUSY particles is the DM candidate (strongly model-depend).

Renormalization Group Equations

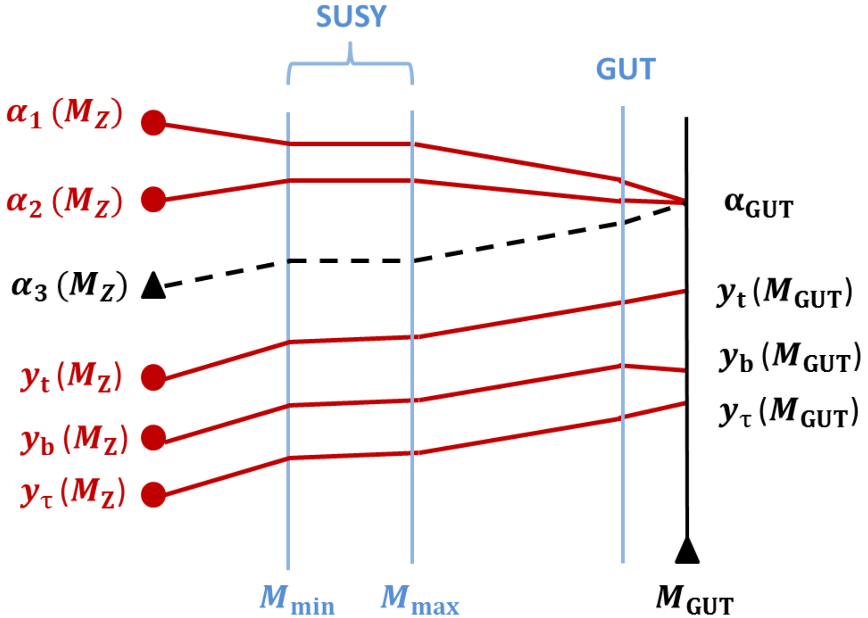
We have developed a *Mathematica* code, which resolves all the RGEs up to **2-loop order** with numerical iterative method.

$$\frac{d}{dt}X_i = \frac{1}{16\pi^2}\beta^{(1)}_{X_i}(X_j) + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{X_i}(X_j)$$

The analysis takes into account all the matching and threshold relations at 1-loop level.

We consider the general possibility of:

- several SUSY thresholds (**multi-scale approach**), namely different masses for SUSY particles;
- GUT threshold related to GUT multiplet fragments that can be lighter than $\mathrm{M_{GUT}}$.

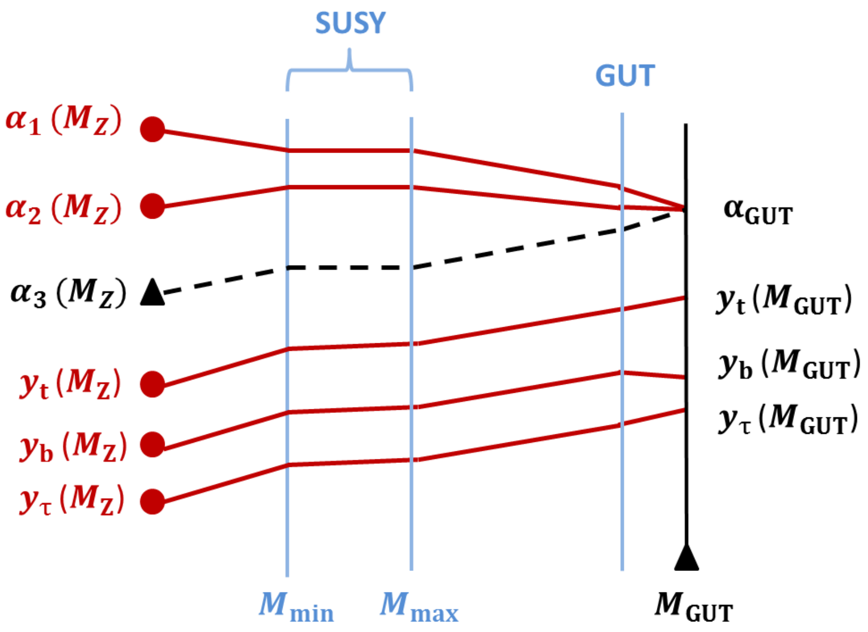


Renormalization Group Equations

In order to impose the gauge unification at M_{GUT}, starting from the known values

$$M_Z,\ M_h,\ M_t,\ M_b,\ M_\tau,$$
$$\alpha_1^{-1}(M_Z) \text{ and } \alpha_2^{-1}(M_Z)$$

we iteratively



- evolve α_1 and α_2, from M_Z to higher energy scales;
- find their intersection point that defines the values of α_{GUT} and its corresponding unification scale M_{GUT};
- evolve backward α_3 using (α_{GUT}, M_{GUT}) as initial point, obtaining $\alpha_3(M_Z)$;
- starting from this solution for running gauge coupling constants we introduce the contributions of y_t, y_b and y_τ in such a way that they consistently reproduce the known values of M_t, M_b and M_τ. This fixes at the end the corresponding values for $y_t(M_{GUT})$, $y_b(M_{GUT})$ and $y_\tau(M_{GUT})$. Note that the other Yukawa couplings are not considered since they give subdominant contributions.

A general SUSY-GUT model

A renormalizable Lagrangian for a vector superfield V (adjoint rep.) and for a chiral superfield ϕ (IRR) reads

$$\mathcal{L}_{\rm SUSY} = \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger e^V \Phi + \left[\int d^2\theta\, \mathcal{W}\mathcal{W} + \underbrace{\int d^2\theta\, W(\Phi) + \text{h.c.}}_{\text{Mass and interaction terms between fermion and scalars}}\right]$$

$$W = \underbrace{W_{\rm Higgs}}_{\text{Responsible for the vevs}} + W_{\rm Yukawa}$$

The soft supersymmetry breaking terms (SSB) have a similar form

$$\mathcal{L}_{\rm SSB} = \int d^2\theta d^2\bar{\theta}\, \rho\, \Phi^\dagger e^V \Phi + \left[\int d^2\theta\, \eta\, \mathcal{W}\mathcal{W} + \int d^2\theta\, \eta\, W'(\Phi) + \text{h.c.}\right]$$

where

$$\eta = M_F\, \theta^2 \qquad \textbf{F-term}$$

$$\rho = M_D^2\, \theta^2\bar{\theta}^2 \qquad \textbf{D-term}$$

A general SUSY-GUT model

In MSSM the superpotential W reads

$$W_{\text{MSSM}} = Y^u_{ij} Q_i u^c_j H_u + Y^d_{ij} Q_i d^c_j H_d + Y^e_{ij} e^c_i L_j H_d + \mu H_u H_d$$

The SSB F-terms $\mathcal{O}(\mathrm{M_F})$ has the same structure

$$\mathcal{L}_{\text{F}} = A^u_{ij} \tilde{Q}_i \tilde{u}^c_j H_u + A^d_{ij} \tilde{Q}_i \tilde{d}^c_i H_d + A^e_{ij} \tilde{e}^c_i \tilde{L}_j H_d + \mu B_\mu H_u H_d + \tilde{m}^a_{\text{G}} \lambda_a \lambda_a$$

The soft masses of all scalars including Higgses are then given by D-terms $\mathcal{O}(\mathrm{M_D})$

$$\mathcal{L}_{\text{D}} = \tilde{m}^2_{Qij} \tilde{Q}^\dagger_i \tilde{Q}_j + \tilde{m}^2_{uij} \tilde{u}^{c\dagger}_i \tilde{u}^c_j + \tilde{m}^2_{dij} \tilde{d}^{c\dagger}_i \tilde{d}^c_j + \tilde{m}^2_{Lij} \tilde{L}^\dagger_i \tilde{L}_j + \tilde{m}^2_{eij} \tilde{e}^{c\dagger}_i \tilde{e}^c_j + \tilde{M}^2_u H^*_u H_u + \tilde{M}^2_d H^*_d H_d$$

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SUSY μ-term

The SSB F-terms $\mathcal{O}(\mathrm{M_F})$ has the same structure

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F-term

The soft masses of all scalars including Higgses are then given by D-terms $\mathcal{O}(\mathrm{M_D})$

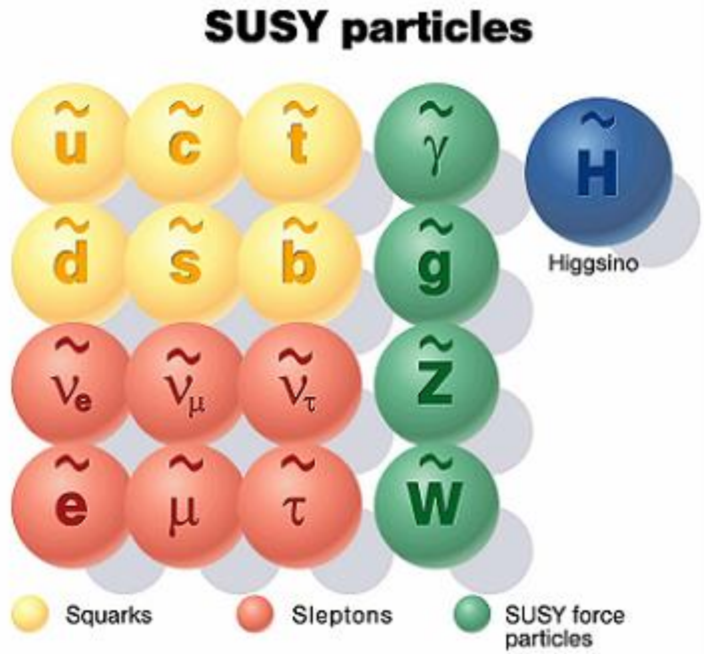
$$\mathcal{L}_{\mathrm{D}} = \boxed{\tilde{m}^2_{Qij} \tilde{Q}^\dagger_i \tilde{Q}_j} + \tilde{m}^2_{uij} \tilde{u}^{c\dagger}_i \tilde{u}^c_j + \tilde{m}^2_{dij} \tilde{d}^{c\dagger}_i \tilde{d}^c_j + \tilde{m}^2_{Lij} \tilde{L}^\dagger_i \tilde{L}_j + \tilde{m}^2_{eij} \tilde{e}^{c\dagger}_i \tilde{e}^c_j + \tilde{M}^2_u H^*_u H_u + \tilde{M}^2_d H^*_d H_d$$

D-term

SUSY threshold

Three classes of soft parameters:

- the so-called μ-term that determines the Higgsino masses $\widetilde{m}_h$ and contributes to masses of scalar doublets H_u e H_d;
- the soft Majorana gaugino masses $\widetilde{m}_g$ and $\widetilde{m}_W$, $\mathcal{O}(\mathrm{M_F})$;
- the soft masses of scalars as squarks $\widetilde{m}_{sq}$ and sleptons $\widetilde{m}_{sl}$, $\mathcal{O}(\mathrm{M_D})$.



GUT implies that all gauginos must have the same mass at the GUT scale, and the similar mass unification can be assumed for masses of squarks and sleptons entering in the same GUT multiplet.

At $\mathbf{M_{GUT}}$: $\dfrac{\widetilde{m}_g}{\widetilde{m}_W} = 1 \quad \dfrac{\widetilde{m}_{sq}}{\widetilde{m}_{sl}} = 1$ → $\boxed{\textbf{SUSY thresholds} \; \widetilde{m}_h, \widetilde{m}_g, \widetilde{m}_{sq}}$

Higgs sector

The mass matrix of the Higgs scalars H_u and H_d involves mass parameters of different origin.

$$\mathcal{M}^2 = \begin{pmatrix} \tilde{M}_u^2 + \mu^2 & \mu B_\mu \\ \mu B_\mu & \tilde{M}_d^2 + \mu^2 \end{pmatrix}$$

SUSY μ-term

$$\mu^2 = \mathcal{O}\left(\tilde{m}_h\right)$$

F-term

$$\mu B_\mu = \mathcal{O}\left(\tilde{m}_g\right)$$

D-term

$$\tilde{M}_{u,d}^2 = \mathcal{O}\left(\tilde{m}_{sq}\right)$$

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SUSY μ-term | **F-term** | **D-term**

$$\mu^2 = \mathcal{O}\left(\tilde{m}_h\right) \qquad \mu B_\mu = \mathcal{O}\left(\tilde{m}_g\right) \qquad \tilde{M}^2_{u,d} = \mathcal{O}\left(\tilde{m}_{sq}\right)$$

A fine-tuning condition has to be imposed in order to get the SM Higgs.

$$-m^2 = \frac{1}{2}\left(2\mu^2 + \tilde{M}_u^2 + \tilde{M}_d^2 - \sqrt{4\mu^2 B_\mu^2 + (\tilde{M}_u^2 - \tilde{M}_d^2)^2}\right) \sim -(100\ \text{GeV})^2$$

$$\Longrightarrow \quad \tilde{m}_h \sim \tilde{m}_g \sim \tilde{m}_{sq}$$

same order of magnitude

Higgs sector

In MSSM the two Higgses H_u and H_d take different vevs.

$$v_u = \langle H_u^0 \rangle \qquad v_d = \langle H_d^0 \rangle \longrightarrow v_u^2 + v_d^2 = v_{\text{SM}}^2$$

This relation is tipically expressed in terms of the quantity $\tan\beta \equiv \frac{v_u}{v_d}$ where

$$\tan 2\beta = 2\mu B_\mu / (\tilde{M}_u^2 - \tilde{M}_d^2)$$

In the transition between SM and MSSM we have to impose

$$m_t = y_t v \sin\beta$$
$$m_b = y_b v \cos\beta$$
$$m_\tau = y_\tau v \cos\beta$$

$\longrightarrow$ **Yukawa matching conditions** ***tan β***

A general SUSY-GUT model

In minimal SUSY SU(5) the Higgs superpotential W reads

$$W_{\rm Higgs} = \frac{M_\Sigma}{2}\,\Sigma^2 + \frac{\lambda_\Sigma}{3}\,\Sigma^3 + M_H\, H\overline{H} + \xi\, H\Sigma\overline{H}$$

IRR: Σ = **24** and $H, \overline{H}$ = **5**, $\overline{\mathbf{5}}$

When the Higgs Σ breaks SU(5) down to the MSSM getting the vev, one obtains

$$\chi_\Sigma \equiv \frac{M_{\rm GUT}}{\tilde{M}_\Sigma} = \frac{\sqrt{2\pi\alpha_{\rm GUT}}}{\lambda_\Sigma} \qquad \chi_T \equiv \frac{M_{\rm GUT}}{\tilde{M}_T} = \frac{\sqrt{2\pi\alpha_{\rm GUT}}}{\xi}$$

where $\tilde{M}_\Sigma \equiv (\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \subset \mathbf{24}$ and $\tilde{M}_T \equiv (\mathbf{3},\mathbf{1}) \subset \mathbf{5}$

$\Rightarrow$ $\boxed{\textbf{GUT thresholds} \quad \chi_\Sigma \text{ and } \chi_\xi}$

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$\Rightarrow$ **GUT thresholds**

χ_Σ and $\cancel{\chi_\xi}$

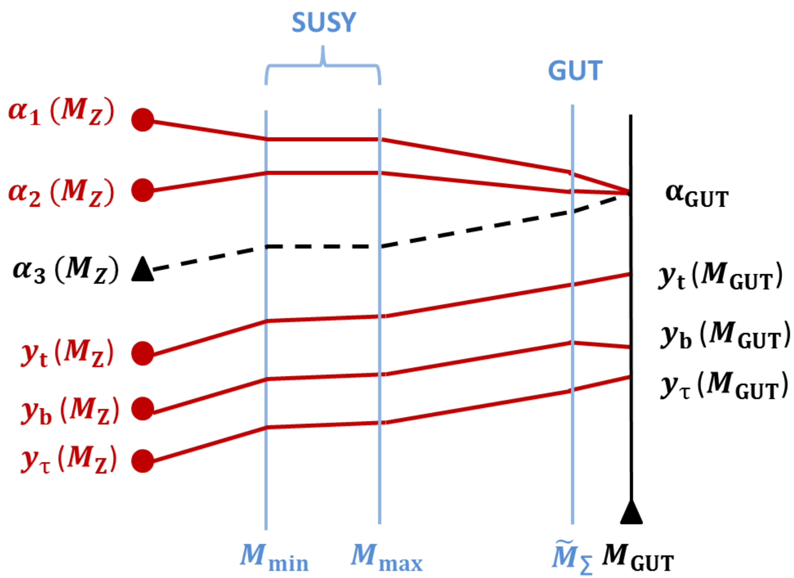
but... $\boxed{\chi_\xi < 1}$

otherwise too fast proton decay via 5d operators

Compatible models

For a given choice of input parameters (*model*)

$$\tilde{m}_h, \tilde{m}_g, \tilde{m}_{sq}, \chi_\Sigma, \tan\beta$$



one obtains

$$\boxed{\alpha_3(M_Z)}, \boxed{M_{GUT}, \alpha_{GUT}}$$
$$y_t(M_{GUT}), y_b(M_{GUT}), y_\tau(M_{GUT})$$

These two values must be compatible with the experimental measurements.

EW measurement

$$\alpha_3^{\rm exp}(M_Z) = 0.1184 \pm 0.0007$$

proton decay

$$\frac{M_{\rm GUT}}{\sqrt{\alpha_{\rm GUT}}} \gtrsim 3 \cdot 10^{16}\ {\rm GeV}$$

compatibility constraints

Proton decay

We take into account the **6d** operators describing the proton decay mediated by leptoquarks.

$$\Gamma\left(p \to e^{+}\pi^{0}\right) = \frac{\pi}{4}\frac{\alpha_{\mathrm{GUT}}^{2}}{M_{\mathrm{GUT}}^{4}}\frac{m_p}{f_\pi^2}\alpha_H^2|1+D+F|^2\left(1-\frac{m_\pi^2}{m_p^2}\right)^2\left[\left(A_{\mathrm{R}}^{(1)}\right)+\left(A_{\mathrm{R}}^{(2)}\right)\left(1+|V_{ud}|^2\right)^2\right]$$

Hisano, Kobayashi, Nagata, PL B716 (2012)

$$\tau_p/\mathrm{Br}\left(p \to e^{+}\pi^{0}\right) > 1.29 \cdot 10^{34}\ \mathrm{yr} \quad \Rightarrow \quad \boxed{\frac{M_{\mathrm{GUT}}}{\sqrt{\alpha_{\mathrm{GUT}}}} \gtrsim 3 \cdot 10^{16}\ \mathrm{GeV}}$$

Super-Kamiokande, PR D85:112001 (2012)

We do not consider the **5d** operators since they are strongly model dependent.

$$\Rightarrow \quad \chi_\xi < 1 \quad \Rightarrow \quad \widetilde{M}_T > M_{GUT}$$

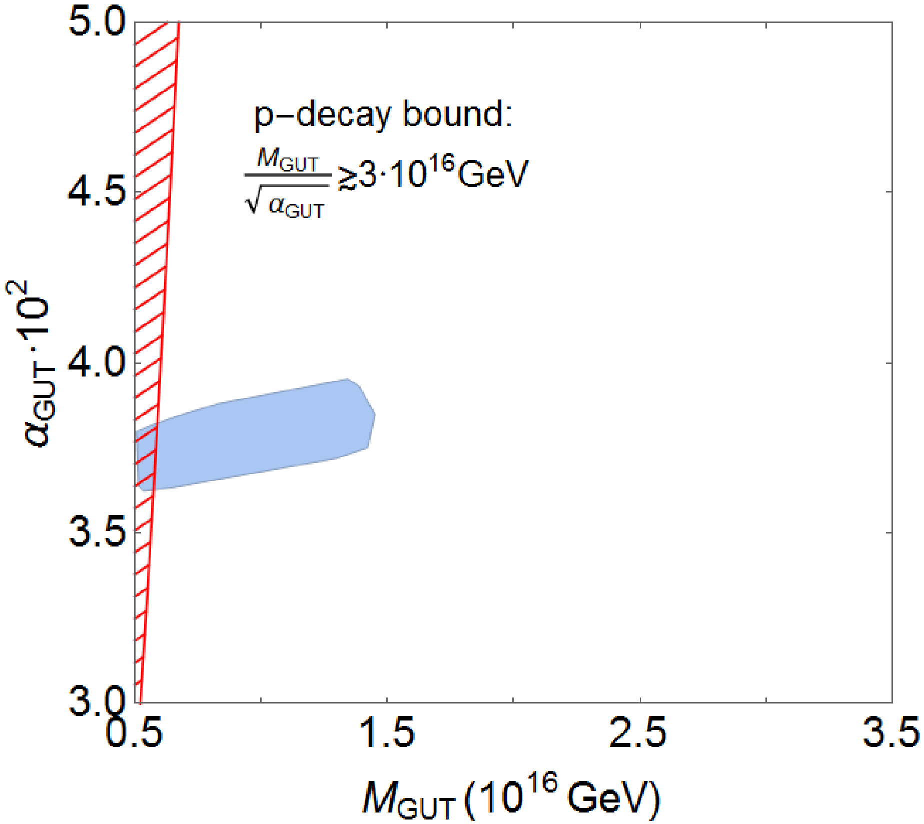
Conspirancy among parameters

Level of dependence of $\alpha_3(M_Z)$ and M_{GUT} on the main parameters related to SUSY and GUT thresholds. The dependence on $\tan\beta$ results to be negligible at this level.

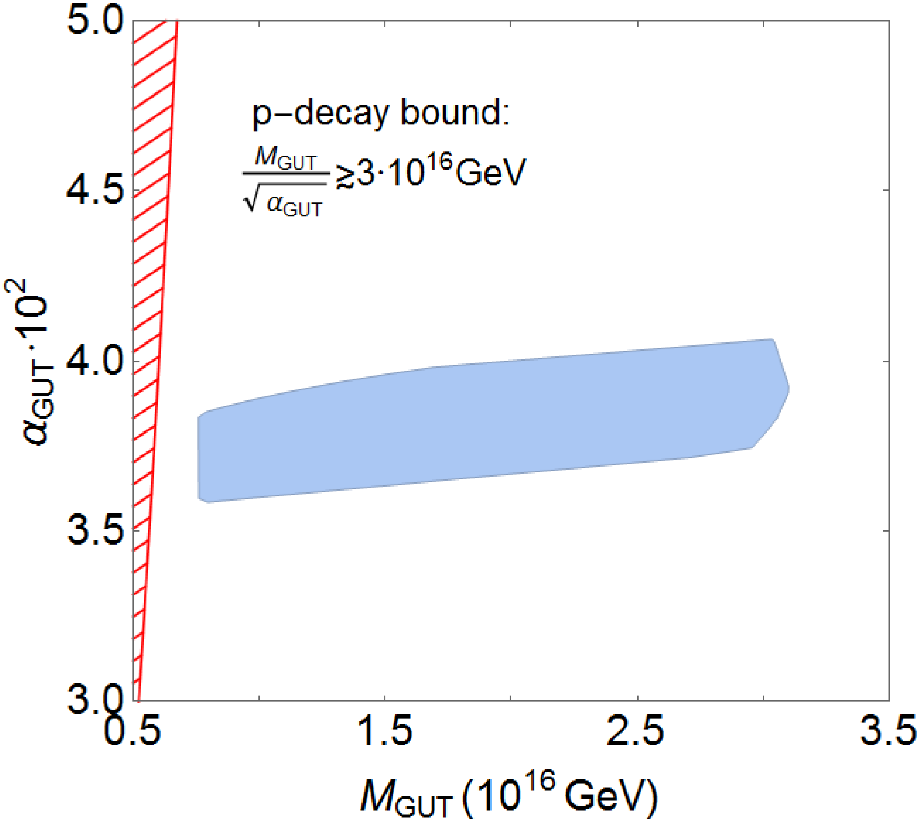
Parameters	$\alpha_3(M_Z)$	M_{GUT}
χ_Σ: $1 \to 10$	+2.0 %	+106 %
$\tilde{m}_g$: $1 \to 10$ TeV	-2.2 %	-47 %
$\tilde{m}_{sq}$: $1 \to 10$ TeV	+0.2 %	-2.6 %
$\tilde{m}_h$: $1 \to 10$ TeV	-4.5 %	-16 %

GUT region

$\chi_\Sigma = 1$

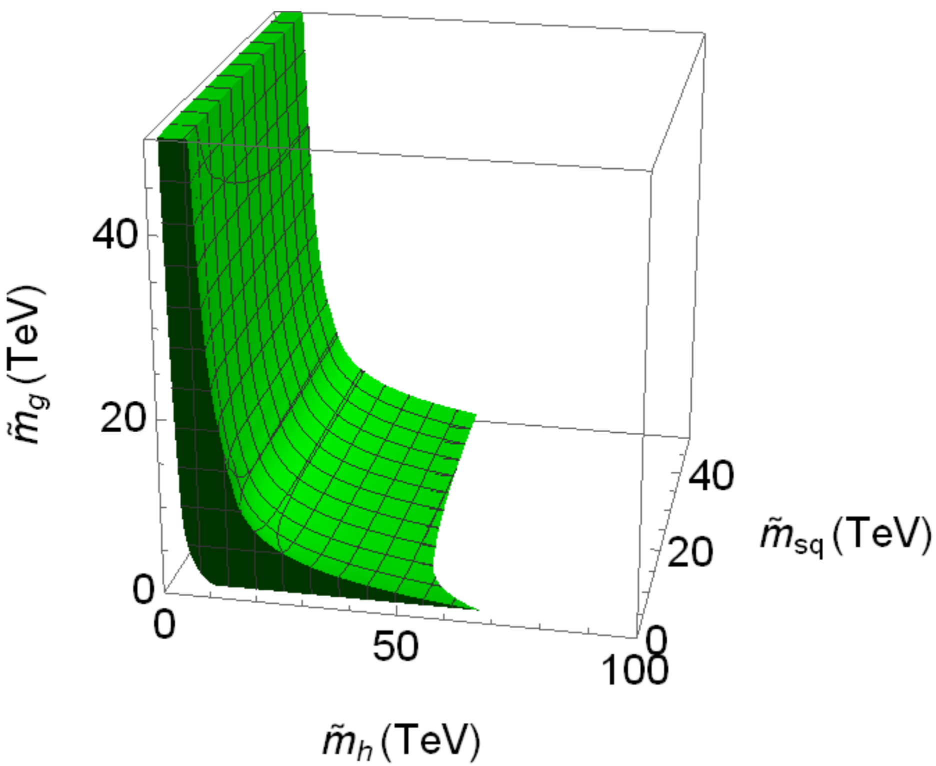


$\chi_\Sigma = 10$



SUSY mass spectrum

$\chi_\Sigma = 1$



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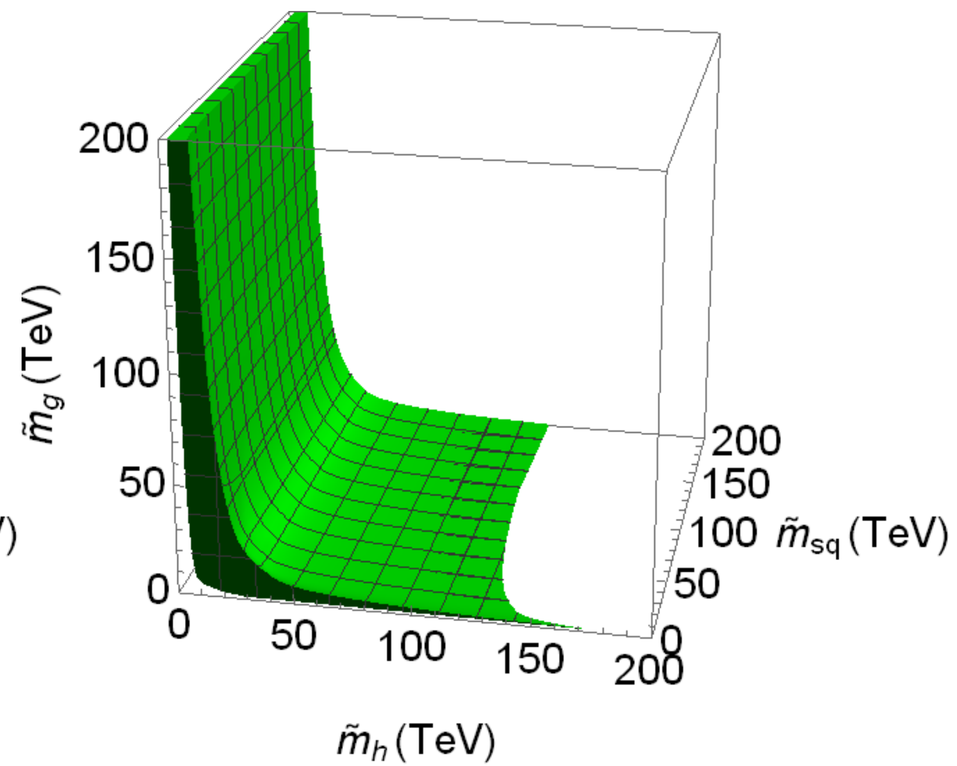
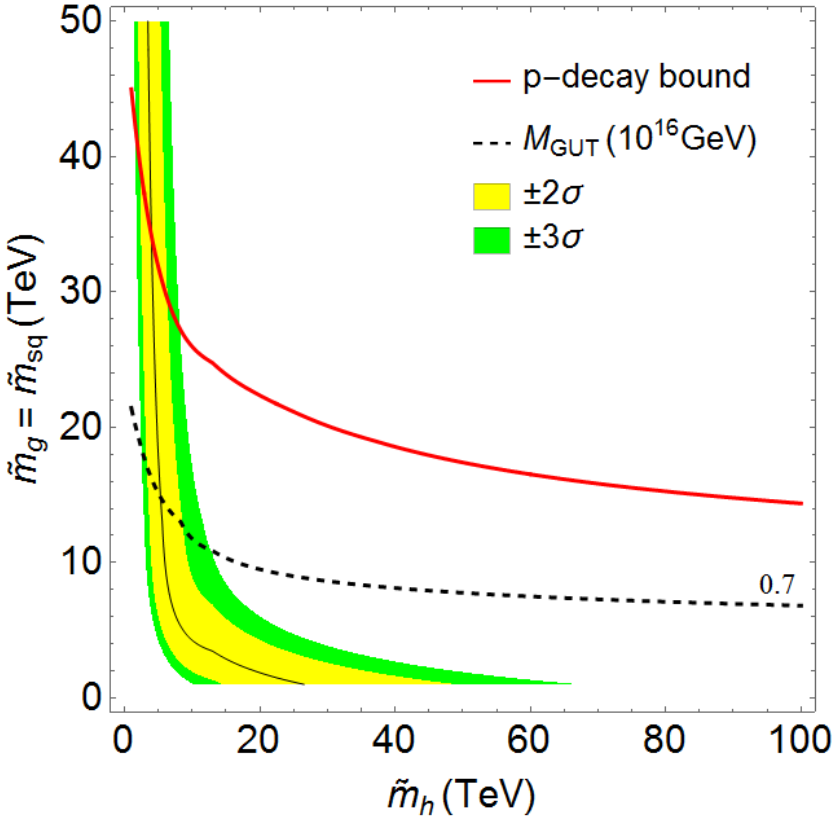


Surface $\tilde{m}_g = \tilde{m}_{sq}$

$\chi_\Sigma = 1$



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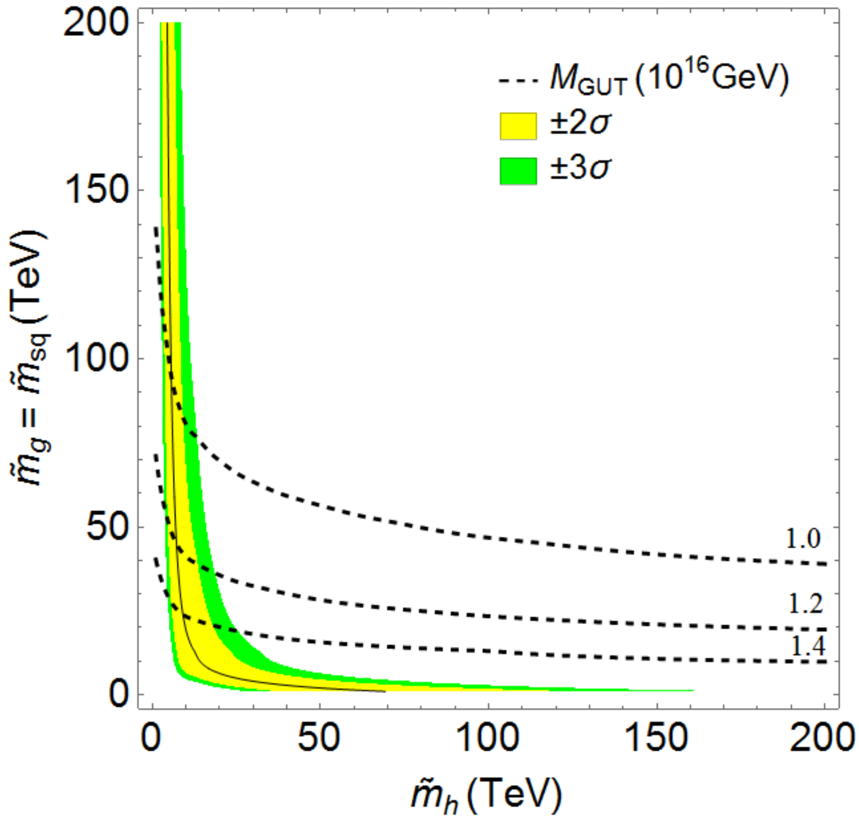

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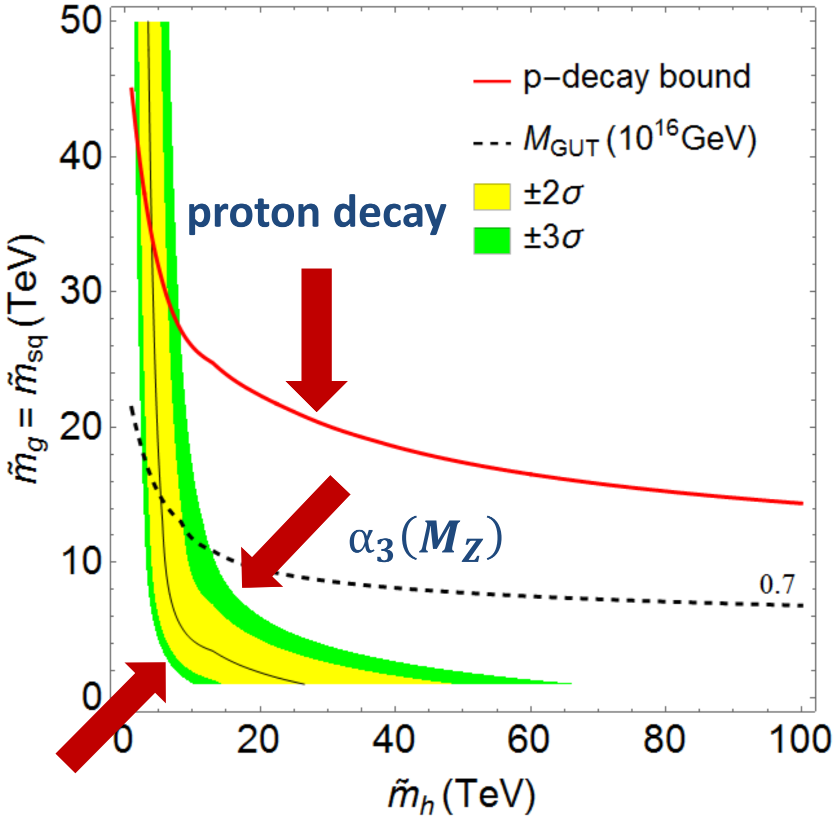

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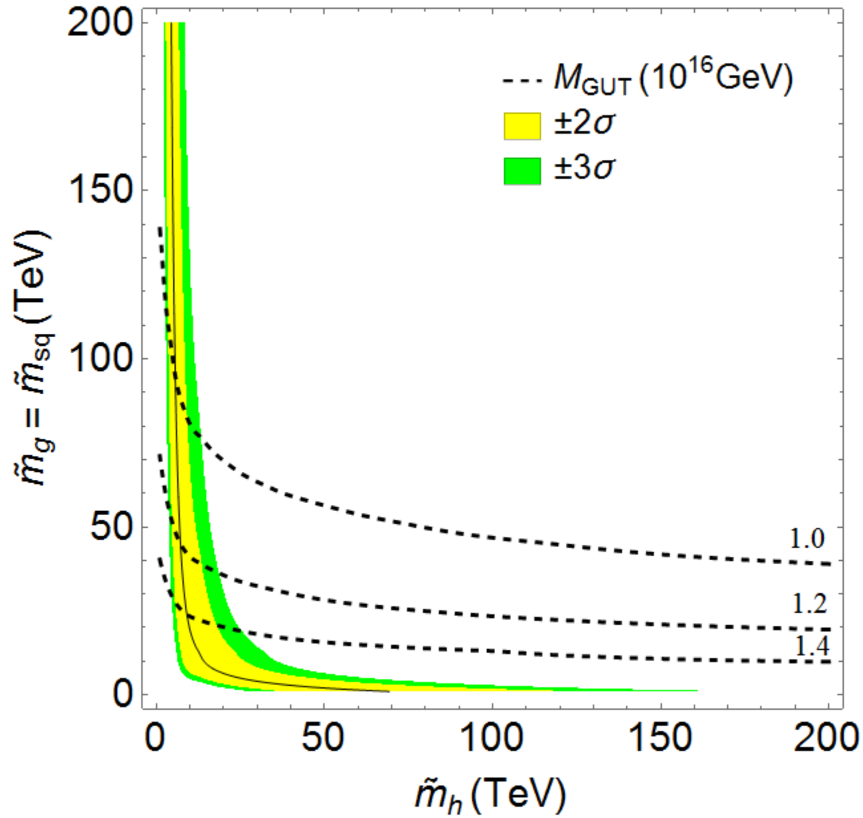
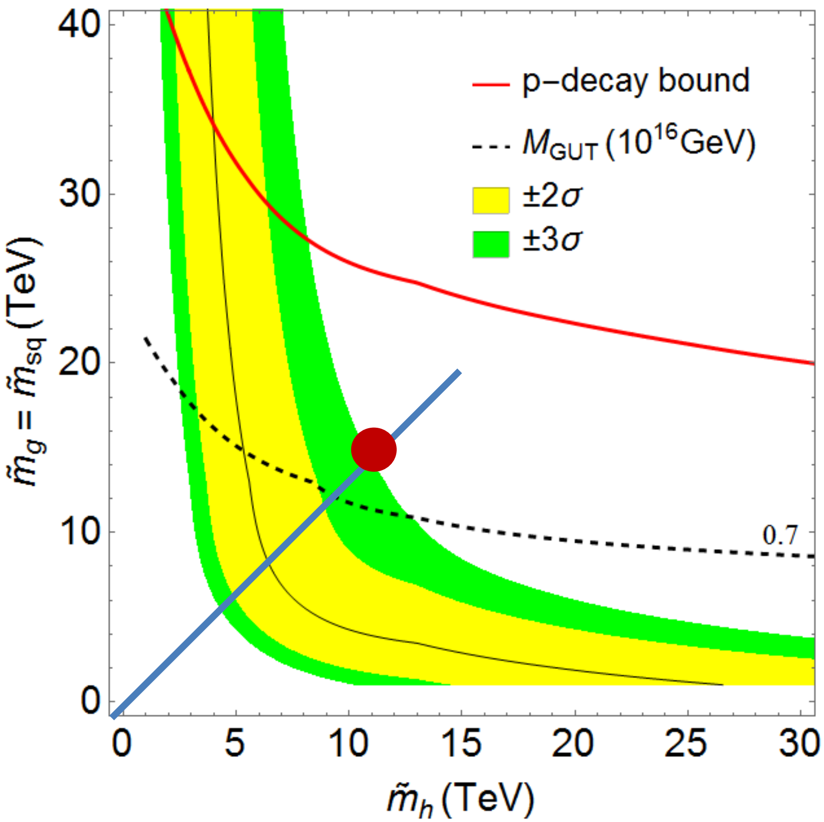
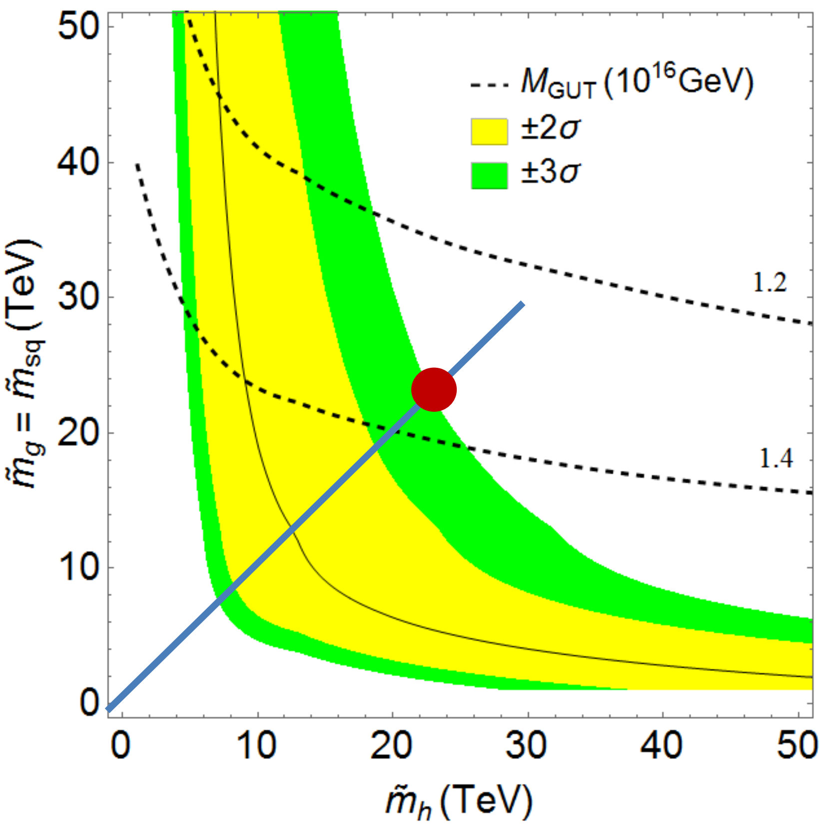


Surface $\tilde{m}_g = \tilde{m}_{sq}$

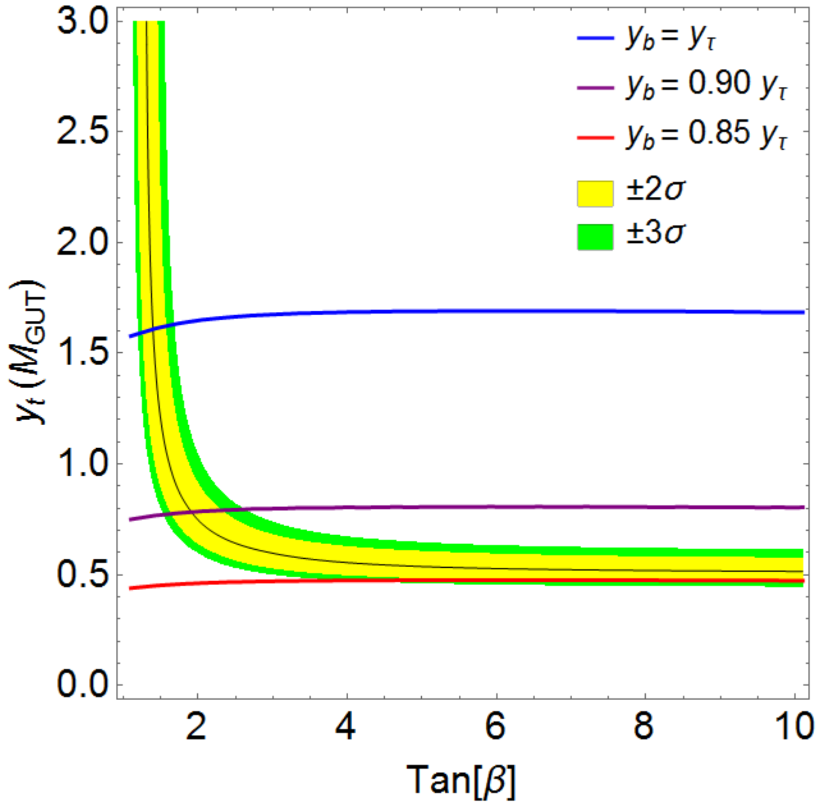
$\chi_\Sigma = \mathbf{1}$

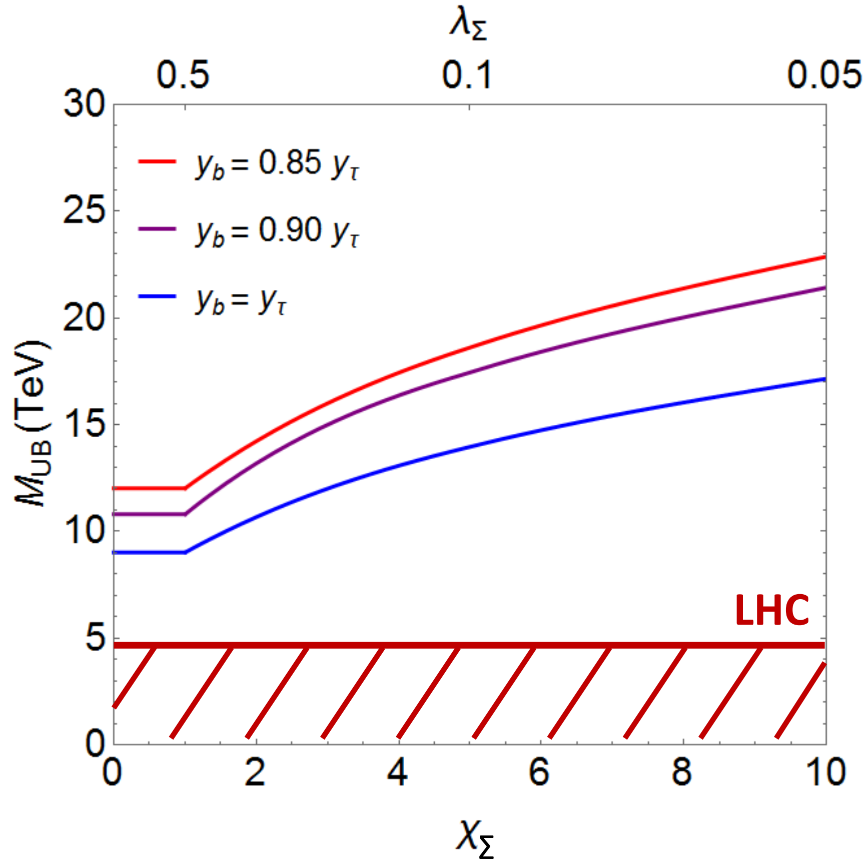


$\chi_\Sigma = \mathbf{10}$



Upper bound for SUSY physics





Larger values for GUT threshold χ_Σ are not allowed since:

- naturalness requirement implies $\lambda_\Sigma \sim \mathcal{O}(1)$;
- M_{GUT} unnaturally approaches M_{Plack}.

$\boxed{M_{UB} \sim 20\ \text{TeV}}$

Conclusions

After LHC run-I (8 TeV), for planning new colliders it is of interest:

- to reanalyze the room still remaining for SUSY-GUT inspired models;
- to determine the **upper bound M_{UB}** for the energy below which SUSY signatures have to show up.

Assuming one step unification (SU(5) bottleneck), under natural assumptions we have obtained general bounds on SUSY mass spectrum.

We claim that if a SUSY-GUT model is the proper way to describe physics beyond the SM, the lighest gluino or higgsino cannot have a mass larger than

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Thanks for your attention