MATCHING PURSUIT ALGORITHM – AN OUTLINE OF POSSIBLE USAGE FOR GRAVITATIONAL WAVES' DATA ANALYSIS

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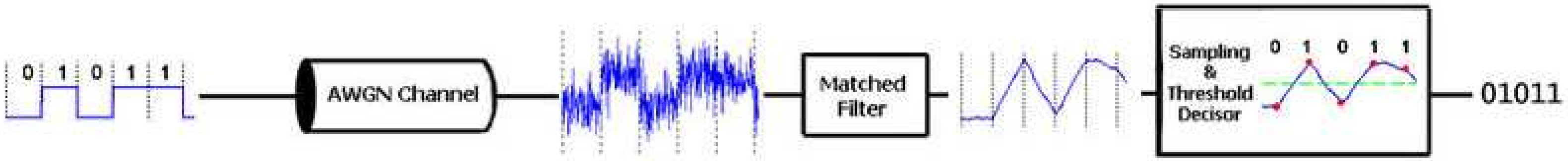
AGENDA

- Introduction
- Standard methods
- Matching Pursuit
- Results
- Summary

Standard data analysis

Matched Filter

- If we know what to expect we can just correlate data with the template …
- … and check if there is something or not



- In other words we try to answer the question:
  - Do we have a **<u>specific signal</u>** hidden in the noise?

Signal-to-Noise Ratio

Input SNR

- P_s – the power of a signal
- P_N – the power of the noise

$$SNR_{input} = \sqrt{\frac{P_s}{P_N}}$$

Optimal SNR

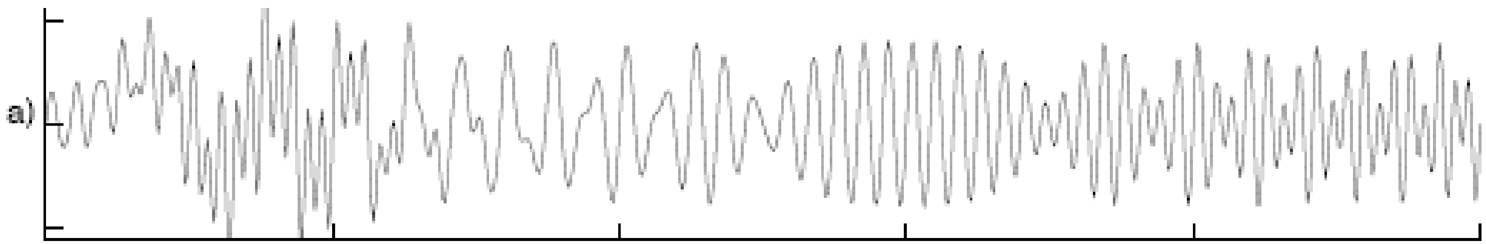
- $\tilde{s}(f)$ – fourier transform of a signal
- g(f) – spectral density of the noise

$$SNR_{opt} = 2\sqrt{\int_0^{\infty} df \frac{|\tilde{s}(f)|^2}{g(f)}}$$

Matching Pursuit

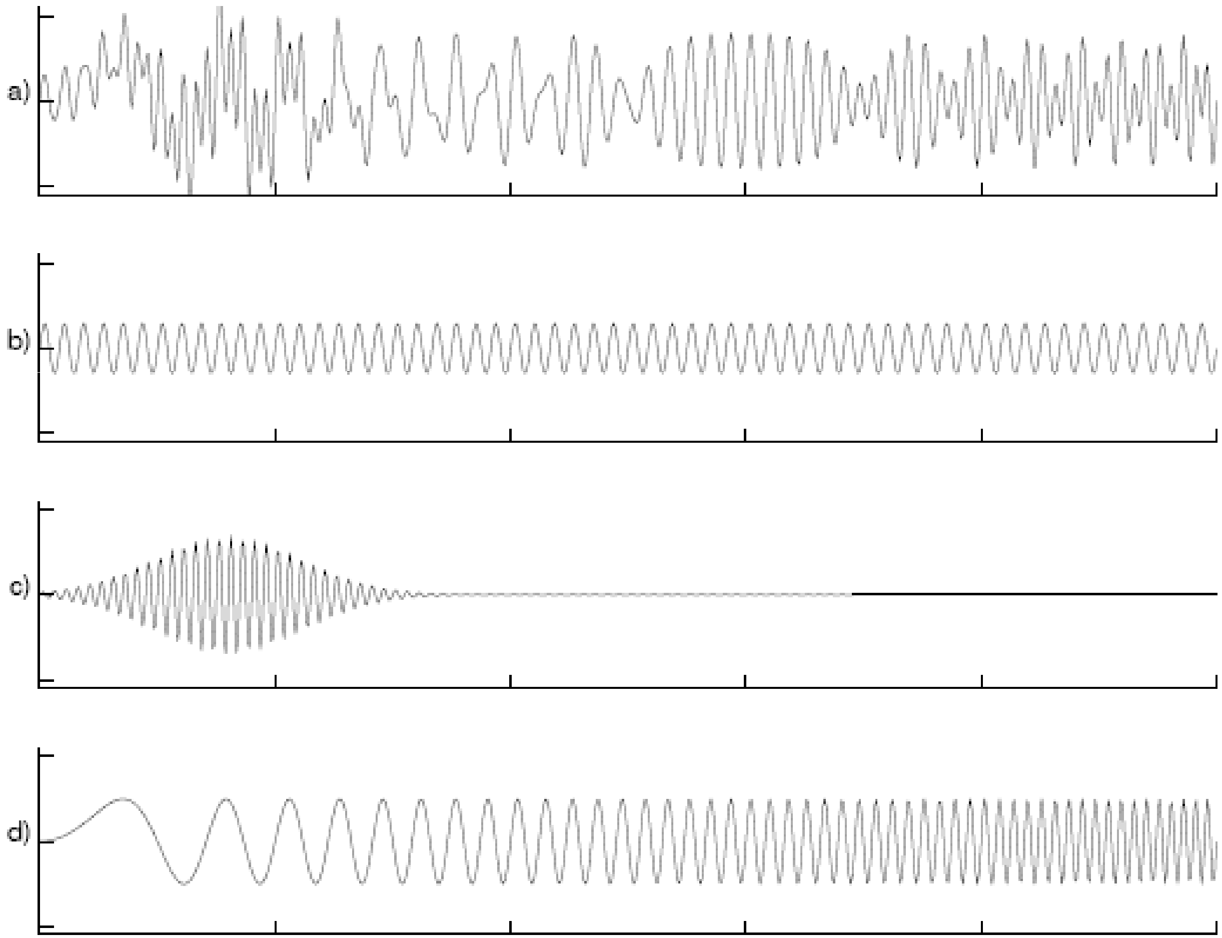
Matching pursuit – Core idea

- Can we decompose a signal directly into base functions?
- Is it possible to obtain the time-frequency picture without cross terms?



- Introduced by Mallat and Zhang in 1993 in a paper *Matching Pursuits with time-frequency dictionaries*

DECOMPOSITION OF THE CASE



DICTIONARY – PROPERTIES

- Overcomplete
- Let us have a huge dictionary consisting of
  - Gabor Functions

$$g_\gamma(t) = K(\gamma) e^{-\left(\frac{t-u}{s}\right)^2} \cos(\omega(\mathrm{t} - \mathrm{u}) + \phi)$$

  - Sine-cosine

$$g_\gamma(t) = K(\gamma) \sin(\omega(\mathrm{t} - \mathrm{u}) + \phi)$$

  - Gaussian Functions

$$g_\gamma(t) = K(\gamma) e^{-\left(\frac{t-u}{s}\right)^2}$$

  - Dirac's deltas

$$g_{t_0}(t) = \delta(t - t_0)$$

- Functions that are taken from dictionary are called atoms

GABOR DICTIONARIES – AND HOW TO CONSTRUCT THEM?

- Gabor functions have 3 main parameters:
  - Position in time - u
  - Frequency - ω
  - Scale - s

$$\gamma = (u, \omega, s)$$

$$\|g_\gamma\| = 1$$

$$g_\gamma(t) = K(\gamma) e^{-\left(\frac{t-u}{s}\right)^2} \cos(\omega(t-u) + \phi)$$

- Dilatation factor – a
- Frequency interval – Δω

$$\Delta u = \frac{\Delta\omega}{2\pi} < 1$$

- Now we can construct a parameter space of Gabor dictionary D_a using the formula:

$$\gamma = \left(p a^j \Delta u, k \frac{\Delta\omega}{a^j}, a^j \right)$$

$$j, k, p \in \mathbb{Z}$$

MATCHING PURSUIT ALGORITHM – MATHEMATICAL DESCRIPTION 1/2

- Dictionary $D_a = \{g_{\gamma_1}, g_{\gamma_2}, \ldots, g_{\gamma_K}\}$ (K – size of the dictonary)
- Signal x
- Residuum in mth step $R^m x$
- Starting point

$$R^0 x = x$$

- Procedure in each step

$$\begin{cases} g_{\gamma_m} = \arg\max_{g_{\gamma_m} \in D_a} \left|\left(R^m x, g_{\gamma_m}\right)\right| \\ R^m x = \left(R^m x, g_{\gamma_m}\right) g_{\gamma_m} + R^{m+1} x \end{cases}$$

- Reconstruction (M – numer of steps)

$$x \approx \sum_{m=1}^{M} a_m g_{\gamma_m} + R^{M+1} x$$

Matching pursuit algorithm – Mathematical description 2/2

- Stopping criteria
  - Number of steps

$$x \approx \sum_{m=0}^{M-1} a_m g_{\gamma_m}$$

  - Error minimalization

$$\varepsilon > \left\| x - \sum_{m=0}^{M-1} a_m g_{\gamma_m} \right\|$$

- The fact that $(g_{\gamma_m}, R^{m+1}x) = 0$ implies energy conservation

$$\|x\|^2 = \sum_{m=0}^{M-1} \left|\left(R^m x, g_{\gamma_m}\right)\right|^2 \|g_{\gamma_m}\|^2 + \|R^{M+1}x\|$$

Computation complexity

- Depends on implementation
- N – numer of points in signal
- In used software (MP.V – done by Medical Physics of Warsaw University)

$$\mathrm{O}\left(N^2 \ln^2 N\right)$$

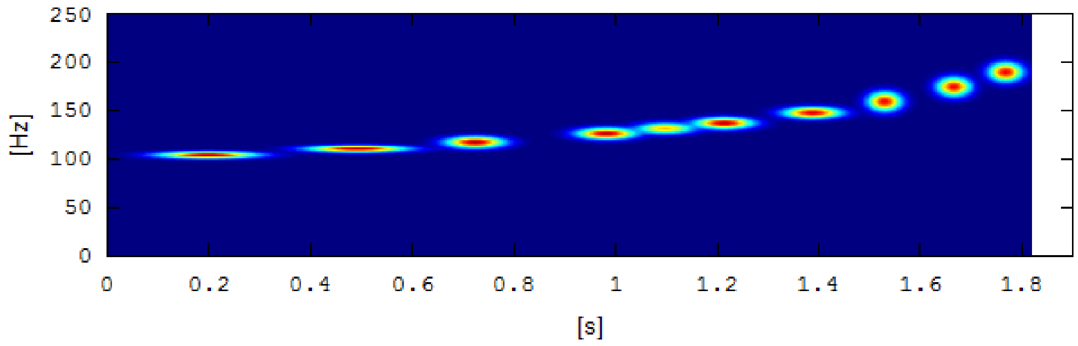
- Typical complexity

$$\mathrm{O}\left(N^3\right)$$

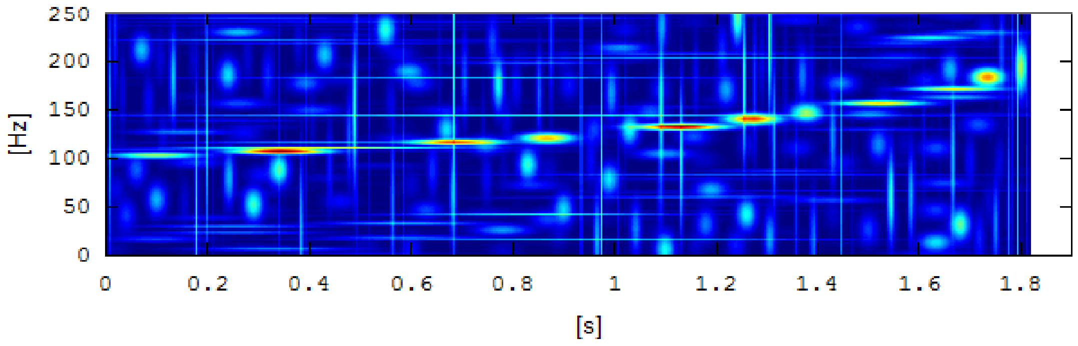
Results

Preliminary outline

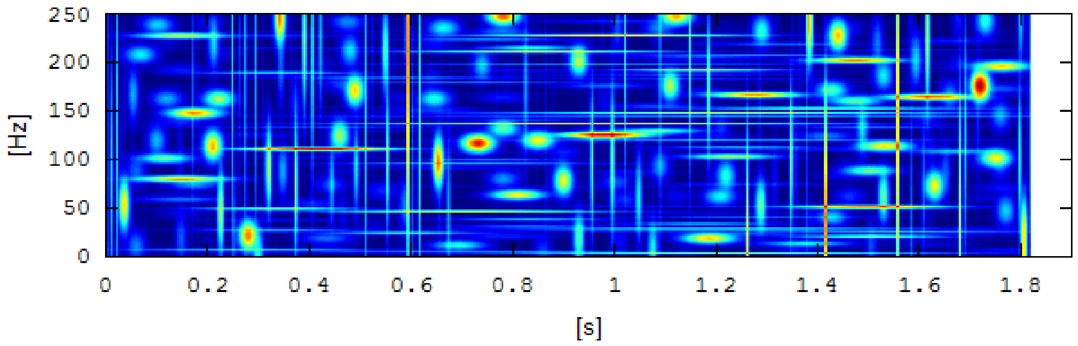
DECOMPOSITION OF NS-NS CHIRP (100-200 [Hz])



A pure signal,
10 strongest atoms

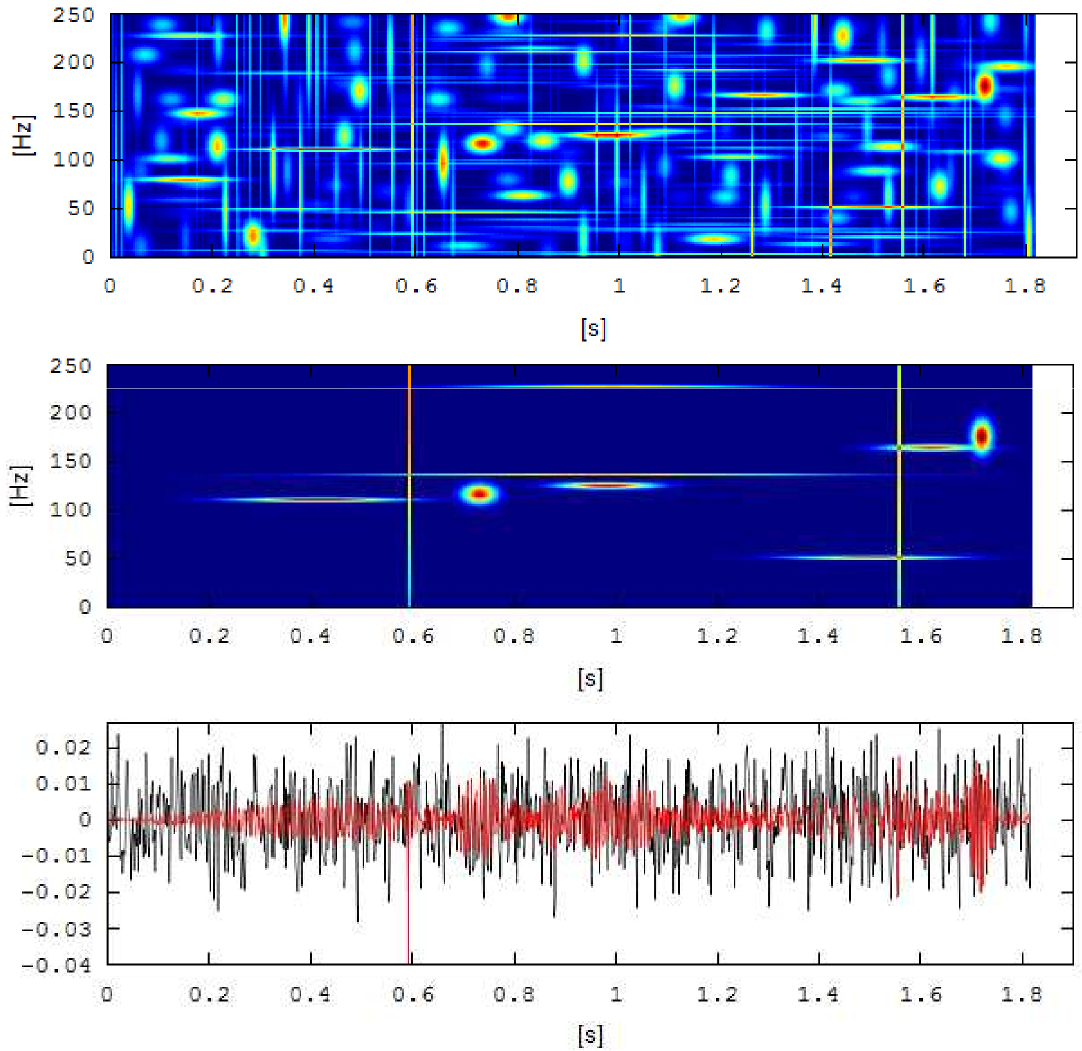


SNR = 10.0



SNR = 7.0

Simple scheme for signal searched for in strong noise (SNR = 7.0)

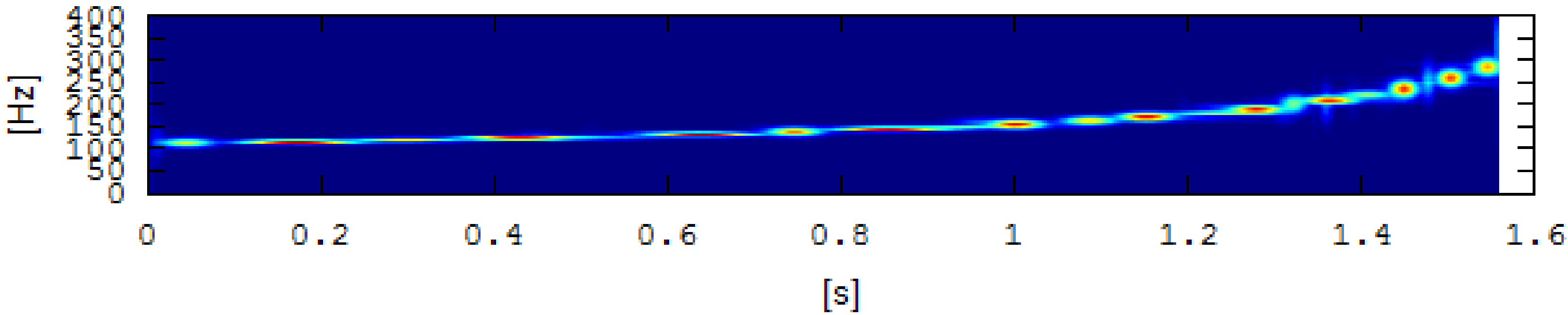


Criteria taken for statistics

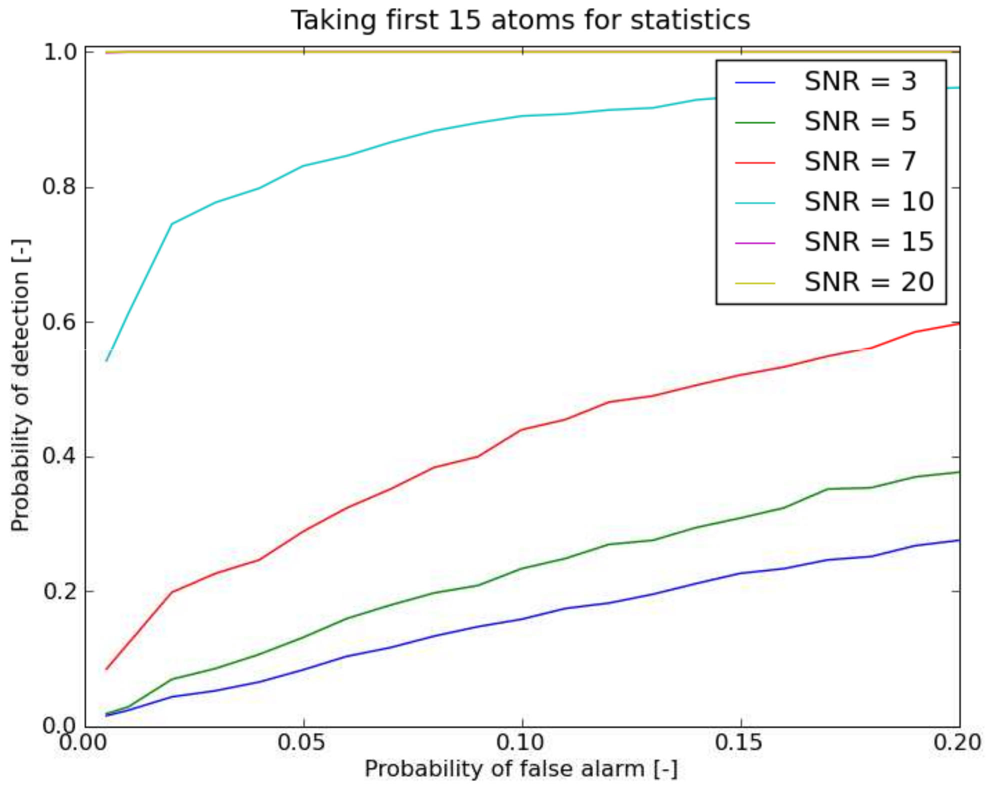
- Statistics

$$s = \sum_{i=1}^{L} \left| \left(R^i x, g_{\gamma_i} \right) \right|^2 \qquad L \in \{1, \ldots, M\}$$

- L – number of atoms taken
- M – total number of atoms

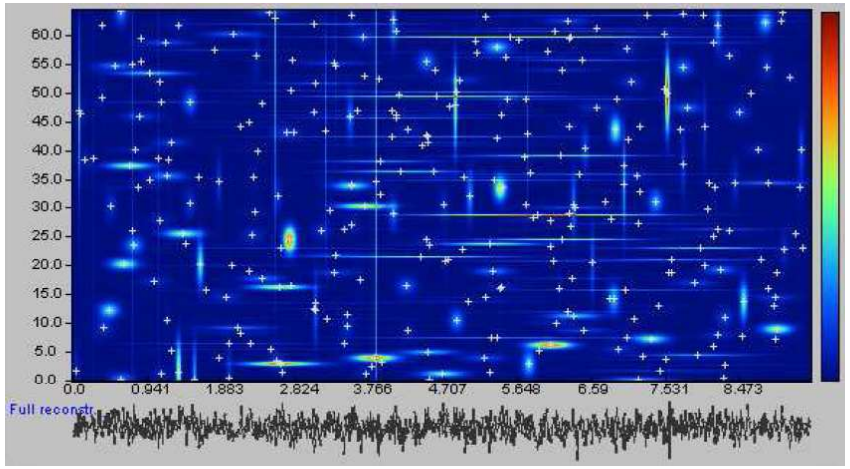


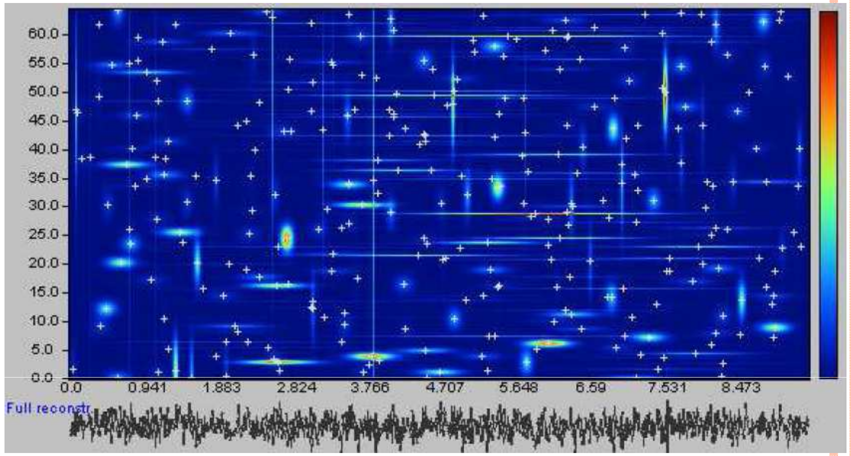
Detector curves
Gabor dictionary (Dilatation Factor = 2.0)

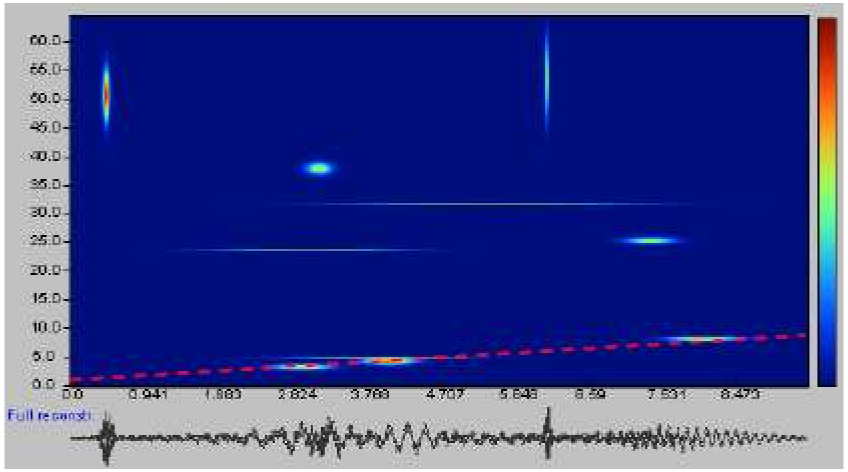


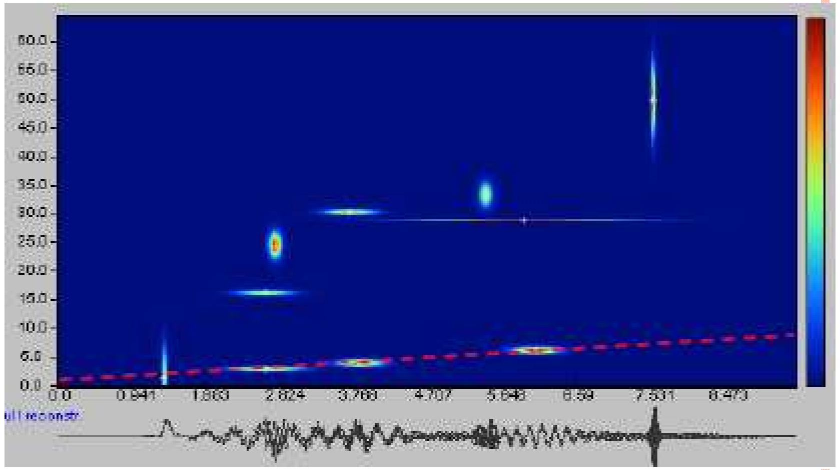
- The statistics has been calculated on 1000 samples

Fitting signal in many data streams with single channel matching pursuit





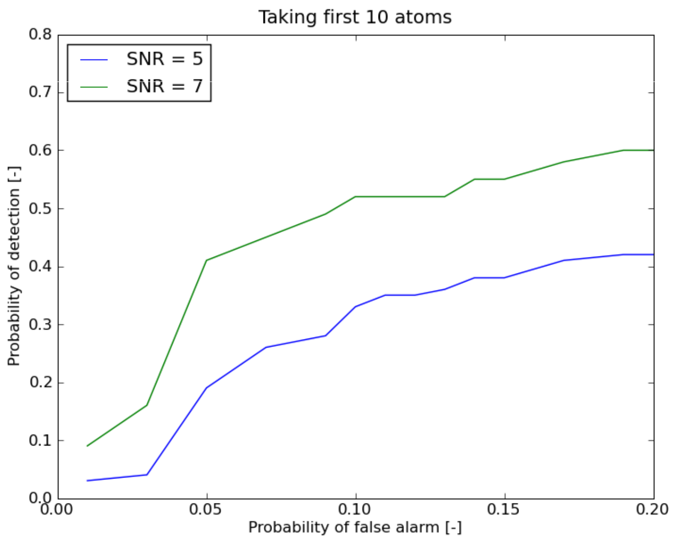


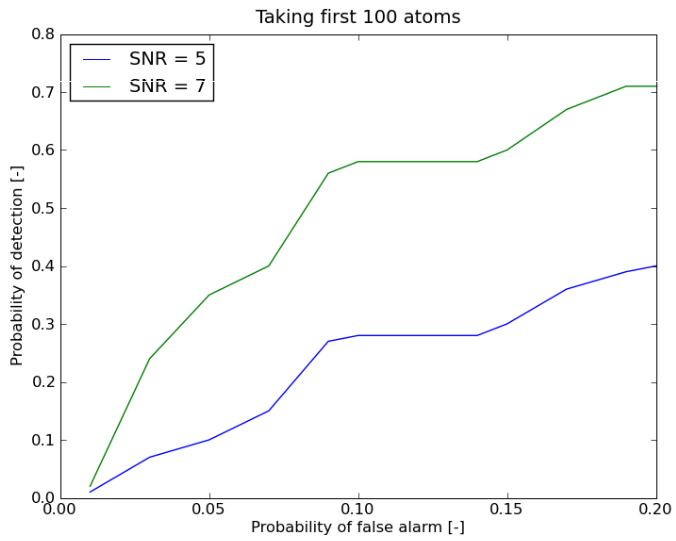


Dilatation factor and detector curves

Dilatation Factor – 2.0

Dilatation Factor – 1.6

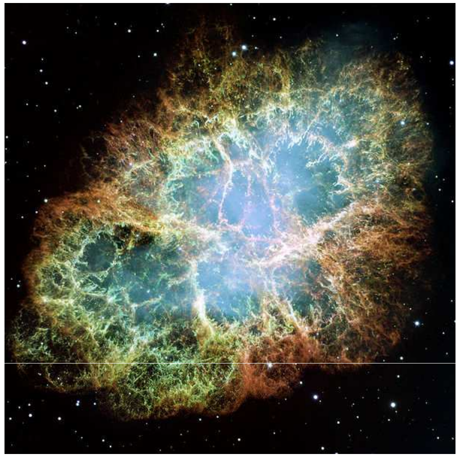




The statics were calculated on 100 samples

Target Signals

- Unmodeled gravitational wave signals



- Supernova signals

- Ringdown phase of merger of compact binary coalescence signals

Further work

- Detailed detector curves for target signals

- Heuristics algorithms for the detecting signals
  - Density of atoms in regions
  - Fitting expected signals' curves

- Efficient algorithms for signal reconstruction

Summary

- It might be a good tool for
  - Unmodelled signals
  - A signal for which it is hard to construct a matched filter
- There is a chance that the matching pursuit algorithm might be an additional analysis tool for gravitational wave data
- The matching pursuit algorithm has not been tested for gravitational data yet
  - This is only an initial study

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Backup

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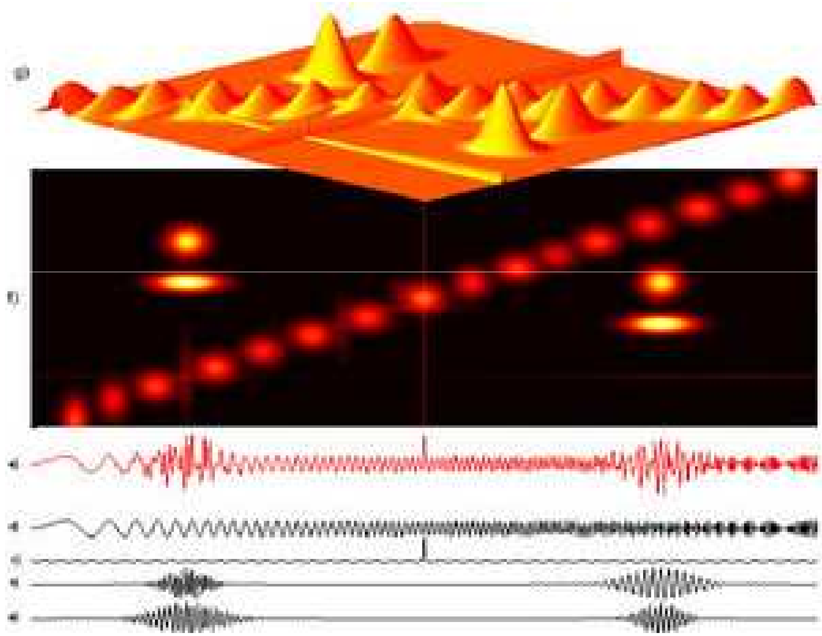
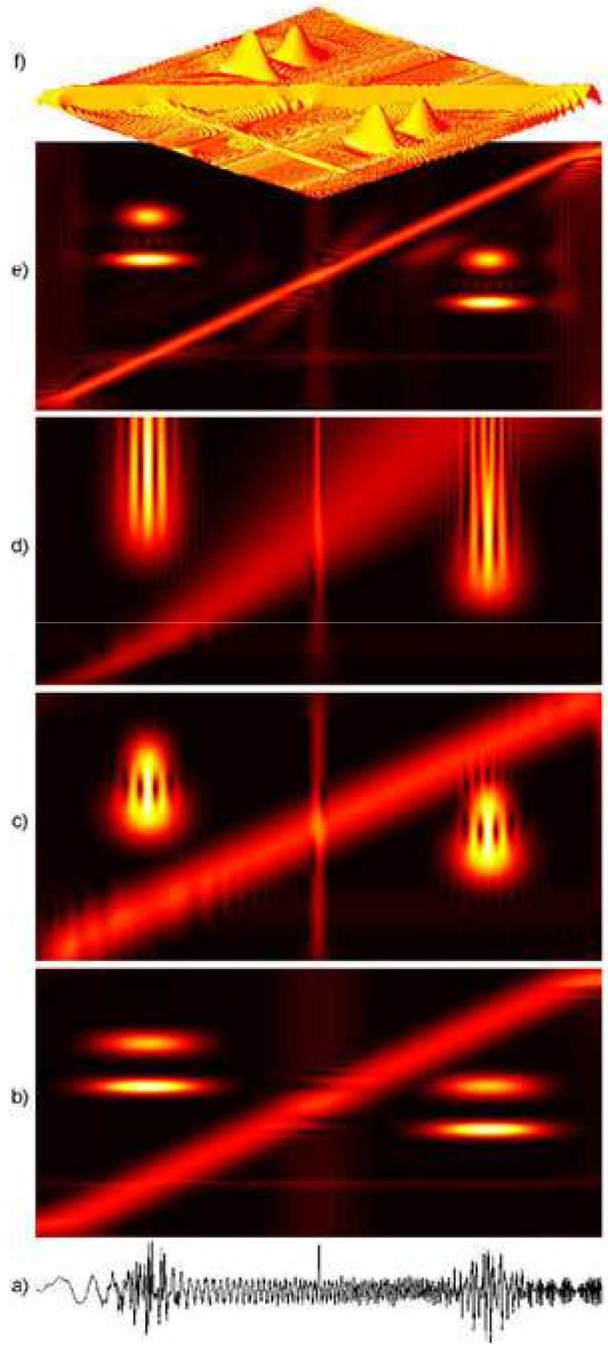
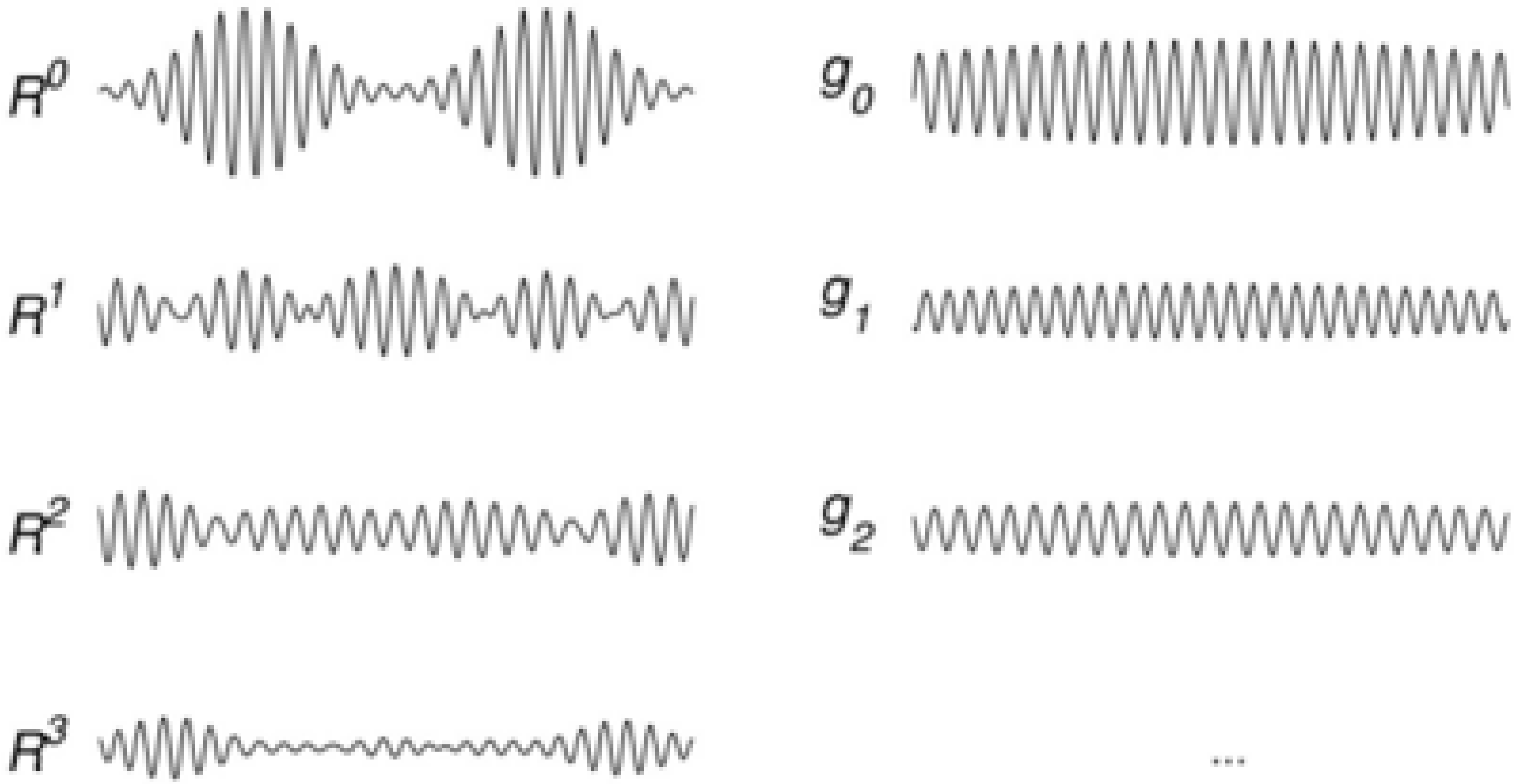
Comparison with Wigner-Weyl transformation

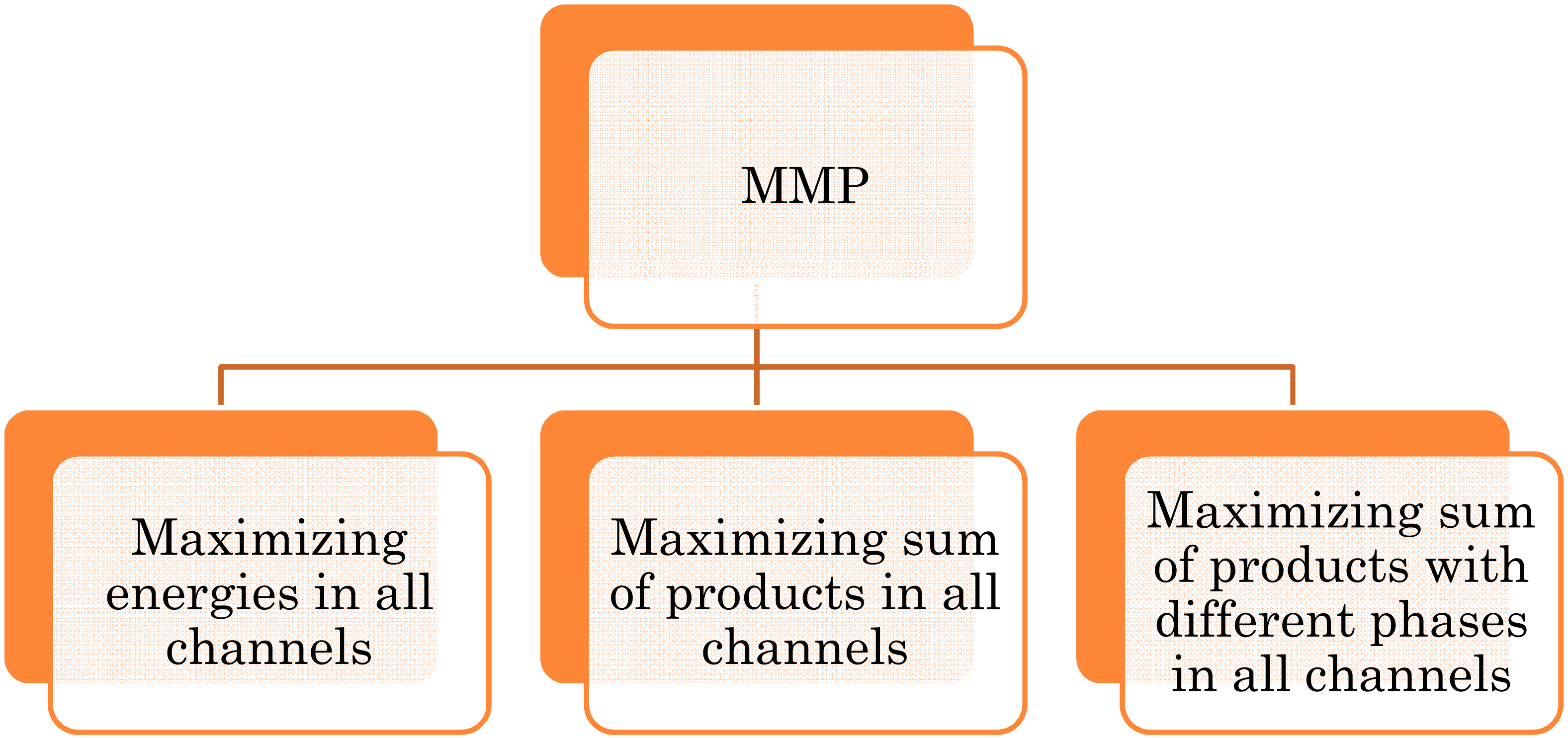




Failure of Matching Pursuit algorithm

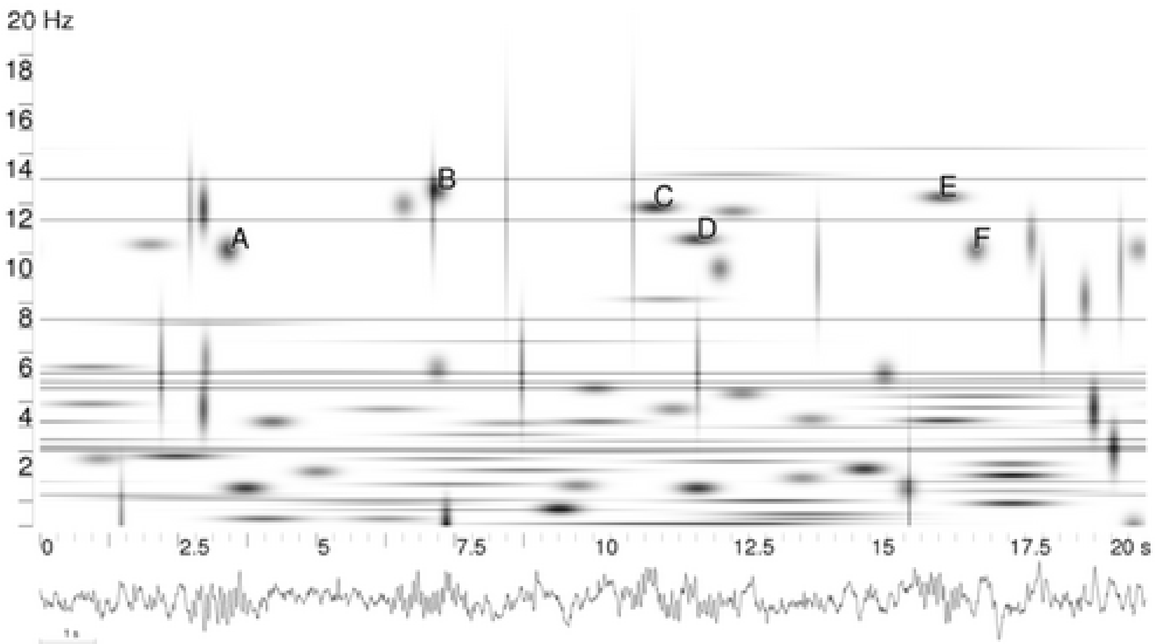


Versions of multichannel matching pursuit

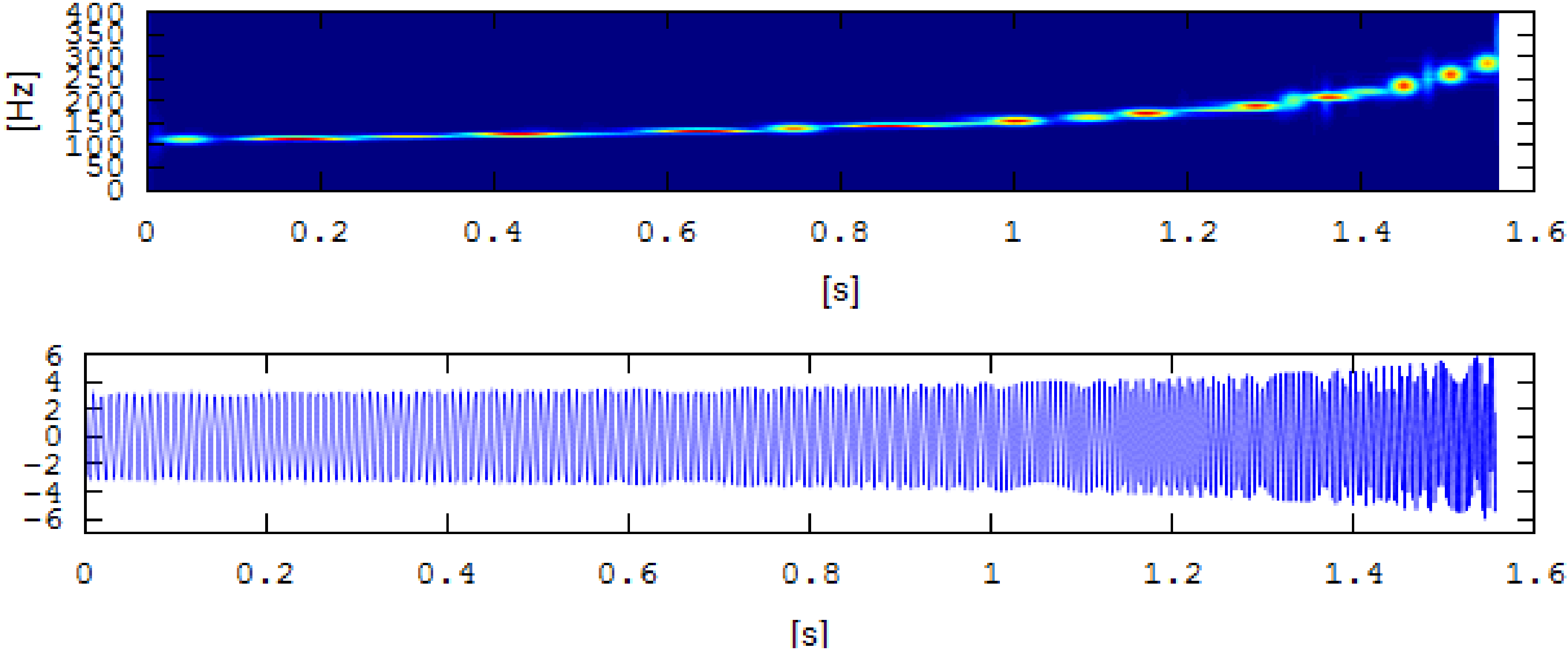


WHERE IS IT USED?

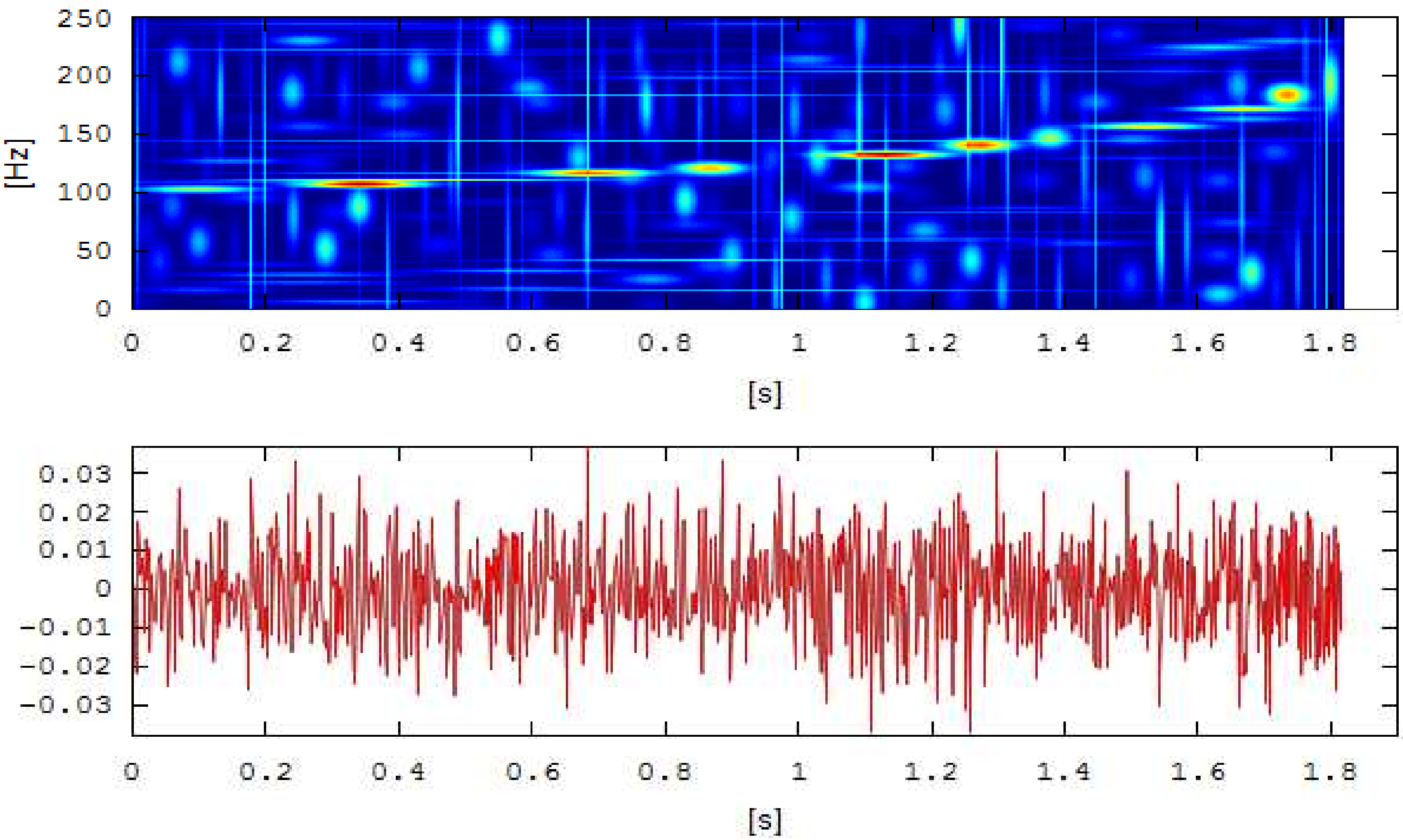
- EEG analysis
  - example sleep spindles
- Curently there is undergoing work on using it for Brain Computer Interface



Decomposition of gravitational signals



CHIRP AND NOISE
SNR = 10



Chirp and noise SNR = 7