Mapping the Galaxy with LISA

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Abstract

We have developed a Matlab Fisher Matrix code in order to study how LISA observations of Galactic compact object binaries can be used to better understand Galactic structure. The code simulates the results of data analysis and estimates the variance of recovered parameters. We will use the recovered parameters to estimate Galactic structure. Here we present first results applied to a standard Galaxy model.

1. Introduction

One of the science goals of LISA is to understand and map the structure of the Galaxy and the distribution of close white dwarf binaries (CWDBs). One way of achieving this goal is to obtain sky locations for individually resolvable CWDBs along with distances for chirping CWDBs. The Mock LISA Data Challenges (MLDCs) have shown that we can expect to resolve $\sim 20,000$ binaries. In order to estimate the degree to which LISA observations can constrain Galaxy structure models, we anticipate modeling the errors expected from typical data analyses and applying the results to a variety of Galaxy models spanning astrophysically reasonable structure parameters. We model the CWDB parameter estimation errors using a MATLAB Fisher matrix code.

2. Fisher (MATLAB Code)

The Fisher matrix is an analytical tool used in gravitational wave detection to estimate the variances of recovered source parameters, which can then provide the precision to which all source parameters can be estimated. Thus, we have developed a Matlab code to produce the Fisher matrix using the basic formulae for the Michelson signal found in Cutler (1998). This then is used to obtain its inverse in order to find the uncertainty in parameters estimated from gravitational wave observations. The project requires repairing ill-conditioned Fisher matrices, that frequently appear in estimation problems with several source parameters, by using techniques of regularization and the scaling of matrices.

The program starts by reading a data file containing the parameters of circularized binaries. Assuming the eccentricity is less than 0.02, a total of 9 parameters will be used in the signal generation code to start the data analysis. The parameters consist of two masses, the orbital period, total distance, ecliptic colatitudes and azimuth, angular momentum colatitudes and azimuth, and initial phase.

M_1	M_2	P_{orb}	d	θ	ϕ	θ_L	ϕ_L	φ_0
$M_\odot$	$M_\odot$	s	pc	Rad.	Rad.	Rad.	Rad.	Rad.

Table 1: Parameters read by the program

In this analysis we get an ill-conditioned or badly scaled matrix for the simple reason that some of the matrix elements are much larger than other elements in the matrix, causing the computing of the inverse matrix to have errors. The problem is in the inside integral of Eq. 1, the Fisher Matrix, which causes some of the elements in the matrix to gain high values with respect to the other elements in the matrix. We can fix this by correctly scaling the appropriate partial derivative. In this case the partials of $\ln A$, f_0, and $\dot{f}_0$ will be changed so they have the same order of magnitude as the parameters φ_0, θ, ϕ, θ_L and ϕ_L. We first identify the partials with respect to $\ln A$ inside the integral of Eq. 1 then multiply by a scaling factor for each partial with respect to the parameter that is being scaled, and then take the scaling factor outside the integral since we can consider this scaling number to be a constant.

$$(\partial_i h | \partial_j h) = \frac{3}{4} S_n(f_0)^{-1} \sum \int_{-\infty}^{\infty} [\partial_i A(t) \partial_j A(t) + A^2(t) \partial_i X(t) \partial_j X(t)] dt, \quad (1)$$

Working with only the diagonal elements of the matrix, and rescaling the components of the matrix, the inverse matrix can be obtained without any errors caused by an ill conditioned matrix. To find the real values of the inverse matrix, we need to divide out the scaled factors that were used to scale the matrix, from their respective parameters. Finally, we take the square root of the diagonal parameters of the inverse matrix to get the standard deviations in the distribution of the parameters. We use these to describe the expected errors or uncertainties in the recovered parameters.

References

Benacquista, M., DeGoes, J., & Lunder, D. 2004, CQG, 24, 509

Cutler, C. 1998, PRD, 57, 7089

Ruiter, A.J., Belczynski, K., Benacquista, M., Larson, S.L., & Williams, G. 2009, ApJ, Submitted, arXiv:0705:3272

Belczynski, K., Kalogera, V., Rasio, F., Taam, R., Zezas, A., Bulik, T., Maccarone, T., & Ivanova, N. 2008a, ApJS, 174, 223.

3. LISA Simulation

We obtain potentially resolved binaries from a Galaxy model by simulating the LISA data stream using the Michelson model described in Benacquista, De Goes, & Luder (2004). For this work, the Galaxy population of CWDBs is the same one used in Ruiter, et al. (2009), and is based on StarTrack (Belczynski, et al. 2008). Once we obtain the spectrum as shown in Figure 3, we first estimate the signal to noise ratio by calculating the ratio of the characteristic strain amplitude for each binary in the population with a running median of the full signal. For those CWDBs with a ratio above 5, we compute the signal to noise ratio (ρ) according to

$$\rho^2 = 4 \int_0^\infty \frac{\left| \tilde{h}(f) \right|^2}{S_n(f)} \quad (2)$$

where $\tilde{h}(f)$ is the Fourier transform of the signal and $S_n(f)$ is the one-sided power spectral density of the full data stream (calculated as a running average of the full signal). We consider resolved binaries to be those with $\rho > 7$. The parameters of the resolved binaries are then fed into the Fisher code to obtain the expected errors in the parameter estimation if a true data analysis were performed. These signals are then regressed from the initial LISA signal and the process is repeated until no binaries meet our detection threshold.
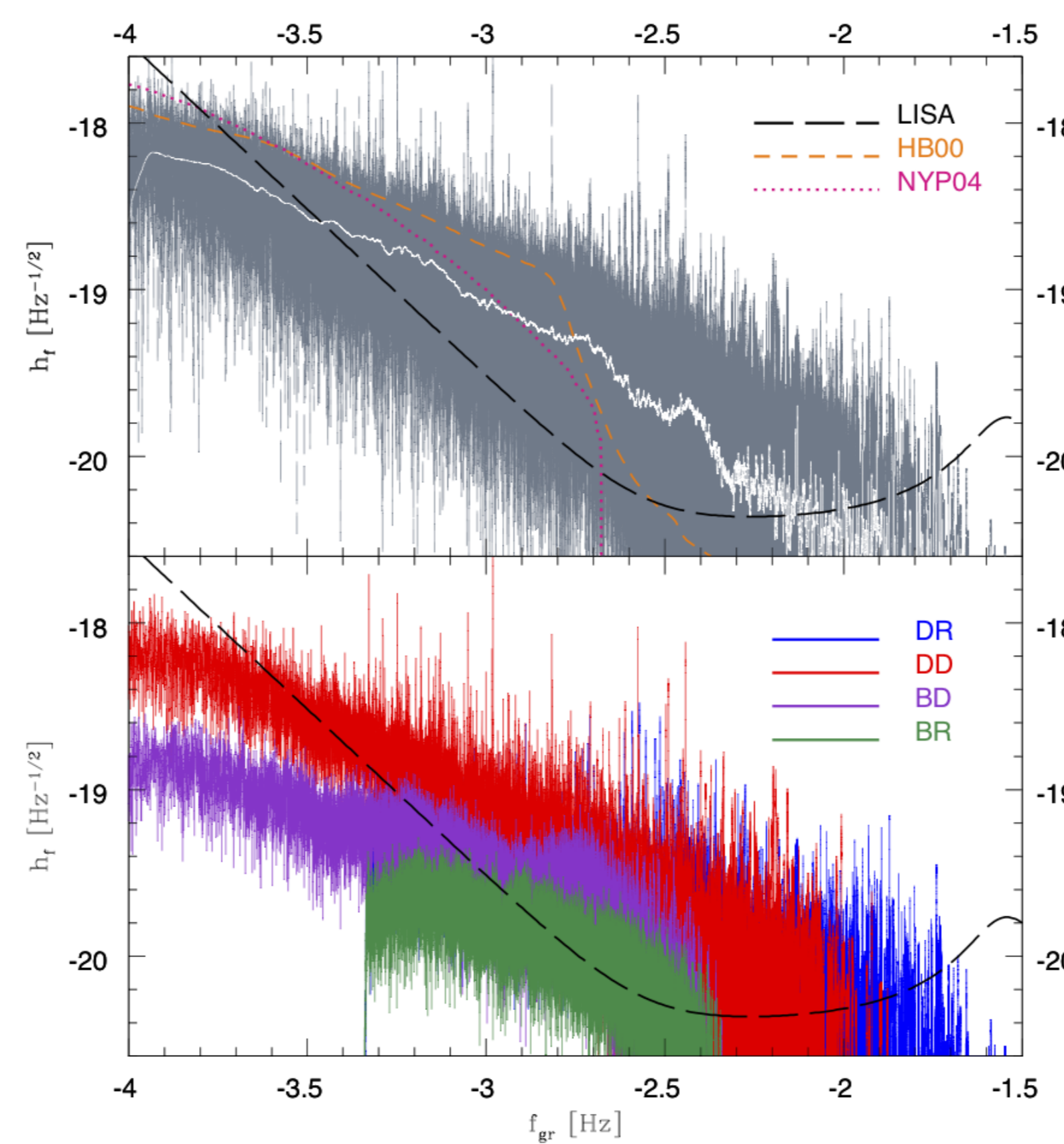

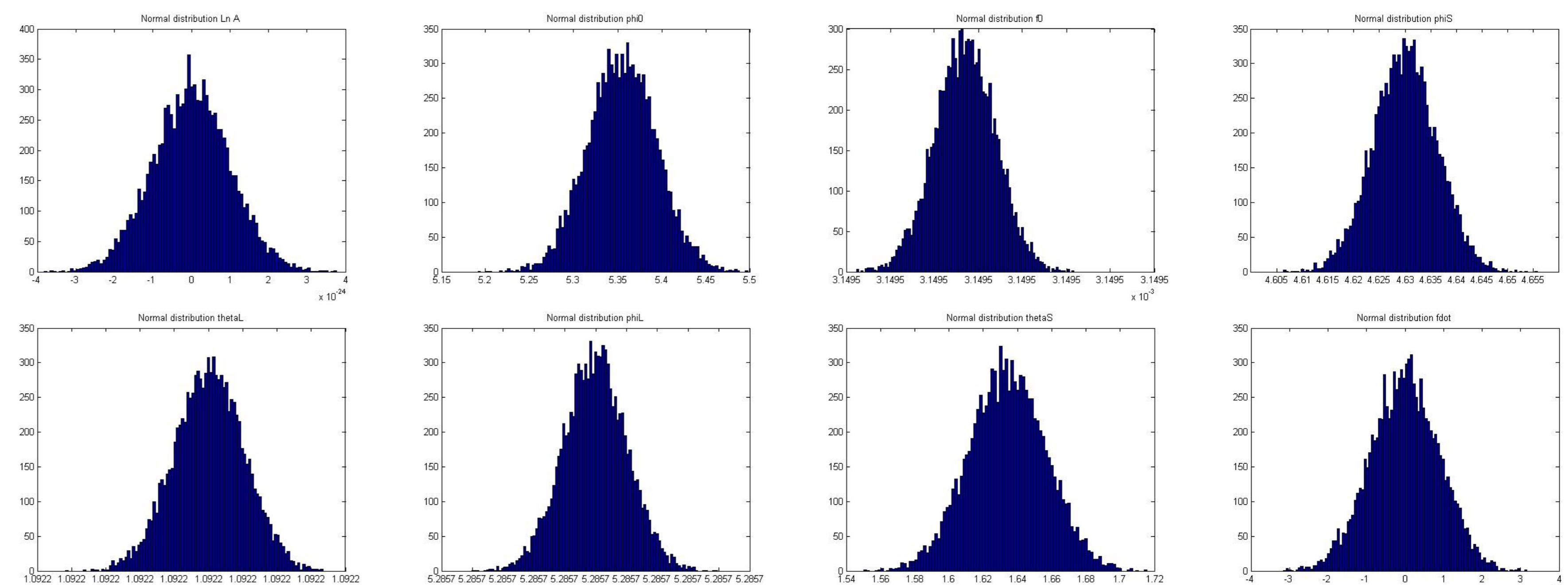Figure 3: LISA spectrum of CWDBs used in this analysis. (Ruiter et al. 2009)

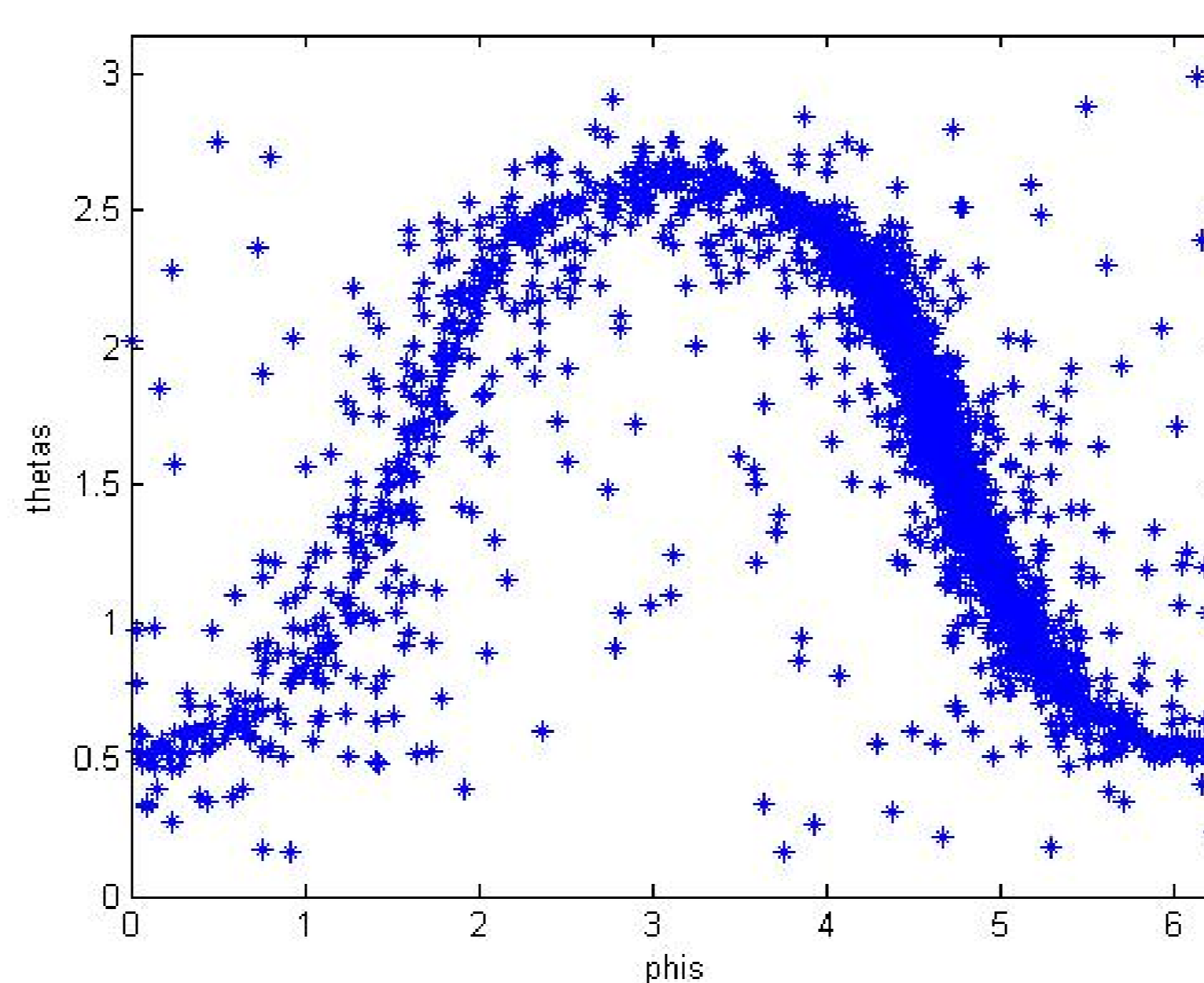Figure 2: Typical variances in parameter estimation for a single, resolved, white dwarf binary

Figure 3: Sky map of the white dwarf binaries in the Galaxy, obtained from the population synthesis.
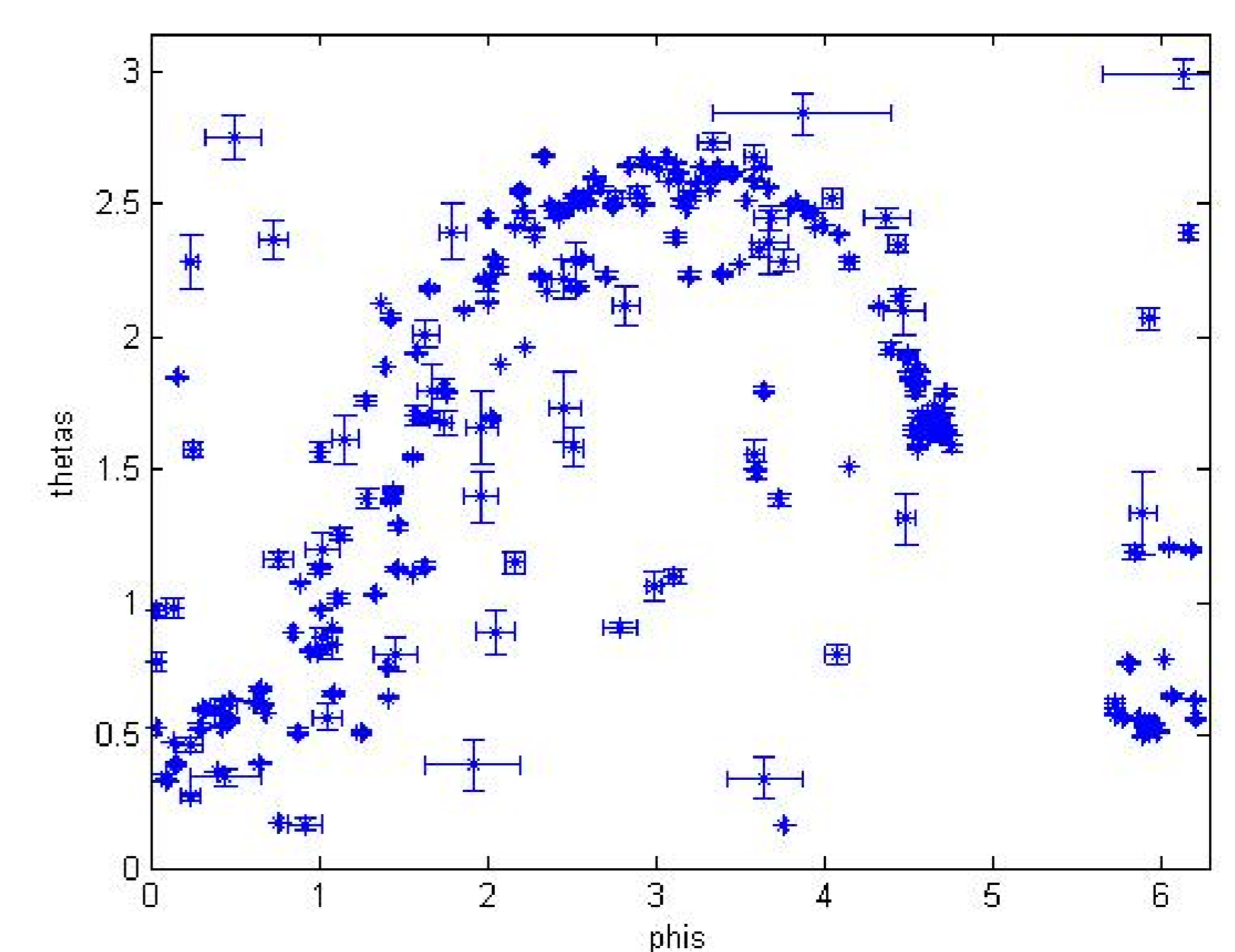

Figure 4: Sky map of the resolved binaries from the first pass through the data, with error bars obtained from the Fisher matrix analysis.

4. Conclusion

The variance obtained from the Fisher matrix is used to approximate the errors in parameter estimation. After determining the set of resolvable binaries, we use these variances to provide error bars in sky maps of the Galaxy in order to study the ability of LISA to recover Galactic structure, as shown in Figures 3 and 4. We anticipate refining this procedure to recover smooth stellar density plots and determine the structural parameters of the Galaxy. We will also refine the Fisher matrix code in order to find the variance and covariance of the sky location parameters and map error ellipses in the sky.