Detection of GW bursts with chirplet-like template families

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Abstract

GW burst detection pipelines such as the Omega pipeline searches for "locally stationary"signals (i.e, with slow variations of the frequency). This assumption translates into the use of a family of templates (sine-Gaussians) with constant frequency. However we can identify a number of scenarios in which the burst frequency evolves rapidly. For instance, this is true for the merger part of the coalescence of a black hole and/or neutron star binary. In those cases, standard sine-Gaussian families lead to losses in performance due to the mismatch between the template and the actual signal. We propose an extension of the Omega pipeline based on chirplet-like templates. Chirplet are characterized by the chirp rate, an additional parameter that controls the frequency variation. We construct a template bank which covers the new parameter space and show that the Omega pipeline can be easily adapted to chirplets. We illustrate the method with examples using simulated data.

1 Motivations

The searches of impulsive gravitational waves focus essentially on two types of waveforms: short unmodelled bursts and longer quasi-periodic signals from inspiralling black hole and/or neutron star binaries as predicted by post-Newtonian approximations. The range of possible GW signals may extend beyond those two categories. We consider here intermediate GW target signals (we refer to as "chirping bursts") that exhibit characteristics from both the above categories (i.e., short duration; "sweeping" frequency).

We propose here an extension of the Omega pipeline [1] (originally known as the Q pipeline) that searches for chirping bursts. The Omega pipeline projects the data over a family of sine-Gaussian templates (with fixed frequency). The idea is to replace these templates by frequency varying waveforms referred to as *chirplets*.

In this poster, we first define chirplets. We then build template banks of chirplets from which we obtain the *chirplet transform*. We discuss the implementation of the chirplet transform and its insertion into the Omega pipeline. We finally show few examples using simulated data.

2 From wavelets to chirplets

Chirplets are defined as follows

$$\psi(\tau+t) = A\exp\left(-\frac{(2\pi f)^2}{Q^2}\tau^2\right)\exp\left(2\pi i\left[f\tau + d/2\,\tau^2\right]\right)$$

with $A=(8\pi f^2/Q^2)^{1/4}$ so that $\int|\psi|^2=1$.

The difference wrt sine-Gaussian templates is an additional term in the phase (evidenced in red) that let the chirplet frequency vary linearly $f(\tau+t)=f+d\tau$ with slope or *chirp rate* d. Chirplets are thus associated with a four-D parameter space instead of three for sine-Gaussians.

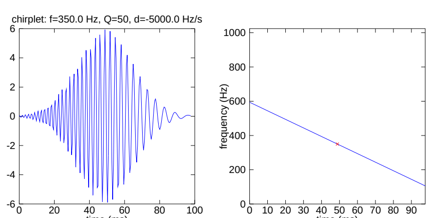

Figure 1: Example of a chirplet

3 Building template banks with chirplets

Let the *metric* μ be the second-order approximation of the mismatch variation [2] between templates for small discrepancy in the parameter value. The metric in the chirplet space reads: [a]

$$\mu = \frac{Q^4d^2+16\pi^2f^4}{4Q^2f^2}\delta t^2 + \frac{2+Q^2}{4f^2}\delta f^2 + \frac{\delta Q^2}{2Q^2} + \frac{Q^4}{128\pi^2f^4}\delta d^2 - \frac{Q^2 d}{2f^2}\delta t\delta f - \frac{\delta f\delta Q}{Qf}.$$

The extra terms (in red) are due to the non-zero chirp rate. We deduce that chirplets require a finer time sampling $\delta t \propto f/(Qd)$ for small $f \lesssim Q\sqrt{d}$ wrt sine-Gaussian where $\delta t \propto Q/f$ and that the sampling step for the chirping rate scales with $\delta d \propto (f/Q)^2$.

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[a] This calculation assumes that the detector is white. Contrarily to the sine-Gaussian case, this approximation has significant effect since the chirplet frequency varies across the detector bandwidth.

The parameter space can be covered with equispaced chirplet templates [2] (the distance being defined by the above metric). As an example, Fig. 2 shows the set of chirplets resulting from such covering.

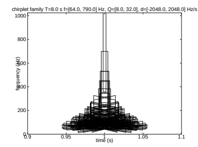

Figure 2: Example of a chirplet family resulting from the template placement procedure.

In Fig. 3, we apply the resulting template placement scheme in two different settings. About a factor of 10 more templates are needed to cover the whole parameter space for chirplets as compared to sine-Gaussians.

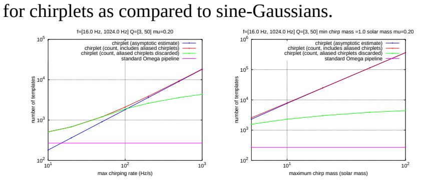

Figure 3: Size of the chirplet template bank in two cases: (*left*) assuming chirp rate limits between $\pm d_{\max}$ uniformly in frequency; (*right*) assuming frequency dependent limits consistent with the Newtonian model of the inspiralling binary chirp i.e., $CM_{\min}^{5/3}f^{11/3} \lesssim d \lesssim CM_{\max}^{5/3}f^{11/3}$.

3.1 Sine-Gaussian vs. chirplet manifolds

To understand the advantages of a chirplet-based analysis, we assume here that the GW signature is a chirplet and we correlate this signal against a sine-Gaussian template bank. Consistently to the metric estimate, the loss in SNR is $\sqrt{128\pi f^2/(dQ^2)} \sim 0.5$ in the present case and the maximum correlation is shifted to lower Q which thus yields a bias on the estimation of this parameter.

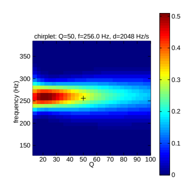

Figure 4: Correlation measurement between a chirplet and a sine-Gaussian template bank.

4 Chirplet transform

4.1 Definition

The chirplet transform T is obtained by correlating the data with the chirplet template bank which, in the frequency domain is performed as follows:

$$T(\boldsymbol{\theta}) = \int X(\xi)\Psi^*(\xi;\boldsymbol{\theta})d\xi,$$

where X and Ψ denotes the Fourier transform of the (whitened) data stream and chirplet, resp. $\boldsymbol{\theta}$ is a descriptor that contains the chirplet characteristics $\boldsymbol{\theta}=\{t,f,Q,d\}$. The chirplet Fourier transform can be expressed as

$$\Psi(\xi;\boldsymbol{\theta}) = \mathcal{A}\exp\left(-\frac{\tilde{Q}^2}{4}\frac{(\xi-f)^2}{f^2}\right),$$

where $\mathcal{A}=((\tilde{Q}^4/Q^2)/(2\pi f^2))^{1/4}$ is written in terms of a "complex-valued" quality factor $\tilde{Q}=Q\sqrt{z}/|z|$ where $z=1+id\Delta_t^2$ with the chirplet duration $\Delta_t=Q/2\sqrt{\pi}f$.

4.2 Filtering procedure

The modulus of $\Psi(\cdot)$ is a Gaussian function as in the sine-Gaussian case. This similarity allows the use of the same filtering scheme as Omega to generate the chirplet transform defined above.

In brief, Omega's scheme operates in the frequency domain and consists in multiplying the Fourier transform of the data (computed with the FFT algorithm) with that of the templates and take the inverse Fourier transform (with FFT) of the product. Omega uses a bi-square frequency window that approximates the Gaussian shape. The compact support of the bi-square window prevents aliasing.

This scheme can be applied to the chirplet case with the difference that the template Ψ is now complex and that we need to multiply the data spectrum both in modulus and phase.

4.3 Pre- and post-processing

In the single-detector network we concentrate on here, most of the pre- and post-processing can be borrowed from the standard Omega. The pre-processing of the instrumental data essentially consists in whitening the input data stream. The post-processing consists in selecting among the chirplets that partially overlap in time and frequency the one associated with maximum correlation with the data. Time-frequency tiles are associated to each chirplets. They are defined by $[t\pm\Delta_t/2, f\pm\Delta_f/2]$ where Δ_t is the chirplet duration and its frequency bandwidth is $\Delta_f = 3\sqrt{(1/\Delta_t)^2+(d\Delta_t)^2}$. Two chirplets overlap if their time-frequency tiles overlap.

5 "Chirpletized" Omega scan

As an illustration, we show here the result of the application of the pipeline to a short segment of simulated random noise (which mimics the spectral characteristics of Virgo noise) where we injected a fake gravitational wave signal (an inspiralling binary chirp). We show side by side the result of the standard Omega pipeline and that of the "chirpletized" version.

Chirplets with a positive slopes are preferred to sine-Gaussian with constant frequency. This gives indication on the frequency evolution of the event *a posteriori*. Also, the significance of the most significative chirplet is increased ($\sim 5\%$ in this example) wrt that of the most significative sine-Gaussian because of the better fit.

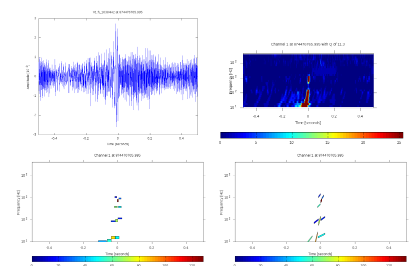

Figure 5: (*top/left*) Inspiralling binary signal in simulated noise (*top/right*) Spectrogram (*bottom/left*) Eventgram for standard Omega (using sine-Gaussian templates only) (*bottom/right*) Eventgram for "chirpletized" Omega (using chirplets)

6 Status and future plans

The single-detector network search code is ready and it can be downloaded [3] and used to produce "chirpletized" Omega scans. More work is needed to extend the method to multiple-detector networks (with e.g., Bayesian follow-up).

References

[1] S. K. Chatterji. *The search for gravitational-wave bursts in data from the second LIGO science run*. PhD thesis, MIT Dept. of Physics, 2005.

[2] B. J. Owen and B. S. Sathyaprakash. Matched filtering of gravitational waves from inspiraling compact binaries: Computational cost and template placement. *Phys. Rev.*, D60:022002, 1999. gr-qc/9808076.

[3] `https://geco.phys.columbia.edu/omega/browser/branches/chirplet`.