Measuring the precession during the inspiral of spinning Massive Black Hole binaries with LISA

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The last mock LISA data challenge has shown problems with determination of the spins and the initial orbital orientation. Those parameters enter the precession equations and lead to the modulation of the phase and the amplitude of gravitational waves emitted during the inspiral. Post-MLDC study has confirmed that there are multiple solutions which have similar values of the likelihood and are widely separated in the parameter space. We have studied this degeneracy and have identified the combinations of the parameters which could be constrained from the observations. Those combinations contain informations about relative orientation of the orbital angular momentum and spins and could be used as a constrain in the search.

Introduction

- The space-borne detector LISA, which is expected to be launched in 2018+, will return an immense amount of science knowledge: fundamental tests of General Relativity, detailed studies of black hole mergers, new insights into the formation of the giant black holes in the center of galaxies, and a detailed picture of the end-phase of binary stellar evolution.
- LISA's high sensitivity creates a new data analysis challenge for extracting astrophysics informations from the sources: the large number of parameters which defined the waveform have to be estimated taking into account the possible correlation between each other.
- New challenges require a very accurate modeling of the waveform and an appropriate parametrization in order to improve the efficiency of analysis methods.

Mock LISA Data Challenge and spinning MBH binaries

At the December 2005 meeting of the LISA International Science Team (LIST), the Data Analysis Working Group resolved to sponsor a series of Mock LISA Data Challenges (MLDCs) centered on LISA science data analysis. This had the dual purpose of encouraging early development of LISA-specific data-analysis methods and tools, and demonstrating the extent to which the gravitational-wave research community is technically ready to distill a rich science payoff from the LISA data stream.

In this work we concentrate on the result of the Challenge 3.2 which focused on the detection of the gravitational signal from spinning Massive Black Hole (MBH) binaries. These sources can be formed during the merger of two galaxies with MBHs in their nuclei.
The gravitational signal from the inspiral of two MBH was modeled as an amplitude restricted waveform (leading PN-order in the amplitude) with phase up to 2PN order [1]. This waveform is describe by 15 parameters: the two masses m_1, m_2, the time of coalescence t_c, the sky location $\hat{n}$ defined by ecliptic angles (β, λ), the two amplitudes of spins (χ_1, χ_2), and 6 precessing parameters which are the initial direction of the orbital angular momentum $\hat{L}$ and of the spins $\hat{S}_1$, $\hat{S}_2$. Two additional parameters, the luminosity distance and the phase at coalescence, are usually found via analytical maximization of the likelihood (i.e. $\mathcal{F}statistic$ for non-spinning case). The spin-spin and spin-orbital coupling cause precession of these vectors, $\hat{L}$, $\hat{S}_1$ and $\hat{S}_2$, which modulates the gravitational wave signal.

Determining the precession terms (motivation)

One of the most important result from Challenge 3.2 is the problem in determining the precessing parameters, i.e. the initial direction of $\hat{L}$, $\hat{S}_1$ and $\hat{S}_2$, whereas other parameters are reliably estimated [2]. There is a large degeneracies in these parameters, i.e. several templates with completely different initial values of $\hat{L}$, $\hat{S}_1$ and $\hat{S}_2$ have likelihood extremely close to the one corresponding to the solution. The goal of this work is to understand this feature, determine the quantities which could be well constrained and use them to improve the search algorithm and to make astrophysical statements.
We illustrate the problem and present the results using SMBH-1 of Challenge 3.2 as an example. This signal was the shortest (few months) and the strongest (SNR = 1788). In order to remove all other correlations and to isolate problem related to the precession, we fixed all other parameters (m_1, m_2, t_c, χ_1, χ_2 and $\hat{n}$). Then we conduct an extensive Monte Carlo simulations: we compute the maximized likelihood, $\mathcal{F}_i$, for randomly drawn initial values of $\hat{L}$, $\hat{S}_1$ and $\hat{S}_2$. We select only those parameters which give a ratio $\mathcal{F}_i/\mathcal{F}_{true} > 0.989$. The distribution of the selected initial orientations $\hat{L}$, $\hat{S}_1$ and $\hat{S}_2$ is shown in Fig. 1.

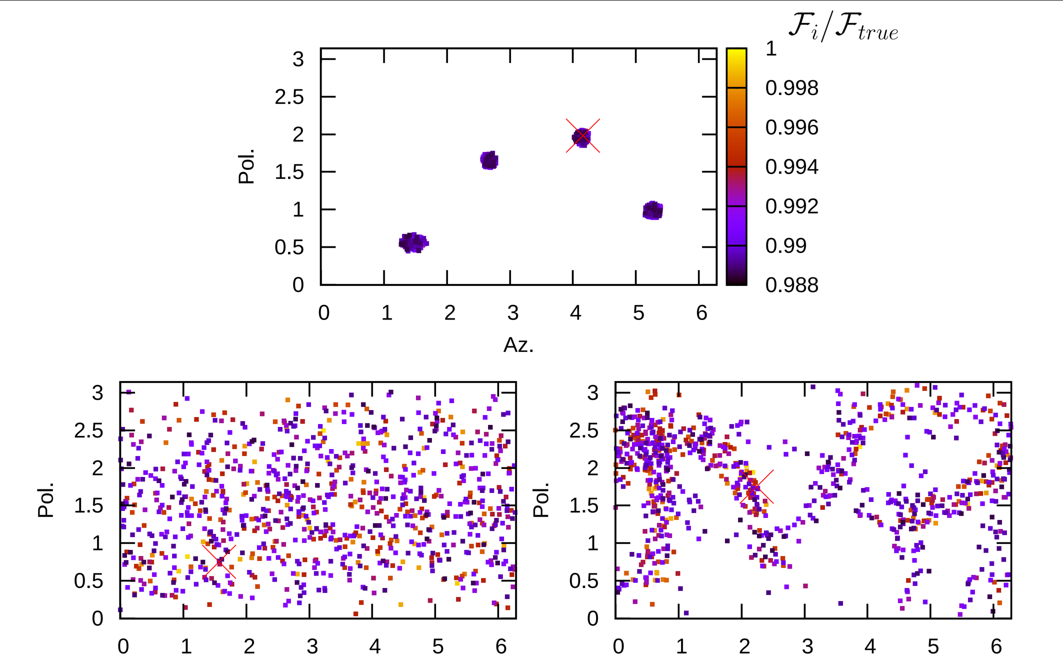

FIGURE 1: *Distribution of initial direction of $\hat{L}$ (top), $\hat{S}_1$ (bottom-left) and $\hat{S}_2$ (bottom-right) for templates with the same values for m_1, m_2, t_c, χ_1, χ_2 & sky position and $\mathcal{F}_i/\mathcal{F}_{true} > 0.989$. The red cross corresponds to the true value.*

This figure provides a good illustration of the problem. We clearly see four possible solutions in $\hat{L}$ and numerous scattered points in the initial orientation of spins.

Phase at high SNR time

The waveform was parametrized by orientations of spins and orbital angular momentum at the initial (zero) moment of time. The choice of the reference time is important as it can influence the search efficiency. As it is shown in Fig. 2, majority of SNR comes from the last few days of inspiral. Several intial configuration could produce similar precession during the last day. The reference time should be chosen at the end of inspiral.

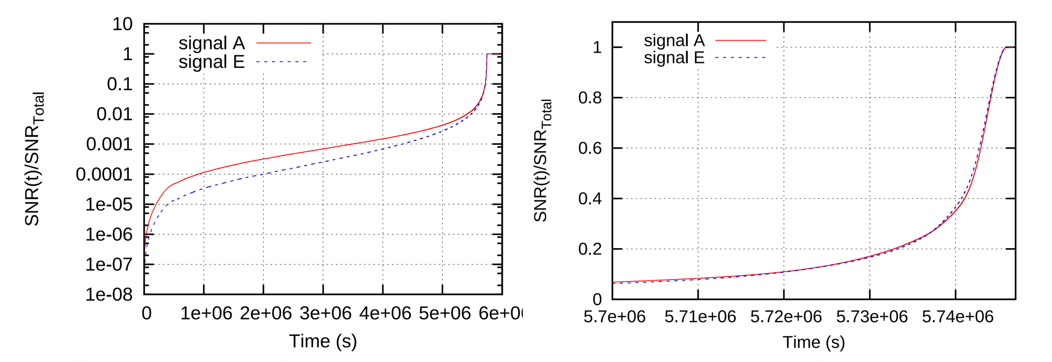

FIGURE 2: *SNR increase with the time over the total SNR, for the full waveform duration (left) and for the half of the last day (right).*

We choose as reference time $t_{ref} = 5.74 \times 10^6\ s$ which is in the last hours (fast SNR accumulation part) and not too close to end to avoid the non-physical termination of the waveform.

All waveforms with the selected parameters stay in-phase around t_{ref} with comparable amplitude. This is a necessary condition for having the $\mathcal{F}_i$ very close to $\mathcal{F}_{true}$. Since the phase is more important in correlation we focus our attention on it and try to identify the combination of parameters determining coherence. We decompose the phase as follows:

$$\Phi_{tot}(t) \approx \Theta_{LISA}(t) - 2\ \Phi_{orb}(t_k)$$

with Θ_{LISA} is the phase due to the detector response, t_k is the time in LISA frame and Φ_{orb} is the orbital phase of the binary decomposed as:

$$\Phi_{orb}(t) = \Phi_c + \Phi_{NoSpin}(t) + \Phi_\beta(t) + \Phi_\sigma(t) + \delta\Phi(t)$$

where Φ_c is the phase at coalescence, Φ_{NoSpin} is the spin-independent part of the phase, Φ_β and Φ_σ are terms corresponding to the spin-orbital β and spin-spin σ couplings, and finally $\delta\Phi$ is the precessional correction term (see [1] for more details). For the fixed sky position, the only different parts of the phase are Θ_{LISA}, Φ_β, Φ_σ and $\delta\Phi$. We studied the distribution of these terms using the selected parameter sets.

We have found that β (defined in below) which is part of Φ_β is roughly constant. We can approximate its distribution by a gaussian and estimate its value and error, as show in Fig. 3.

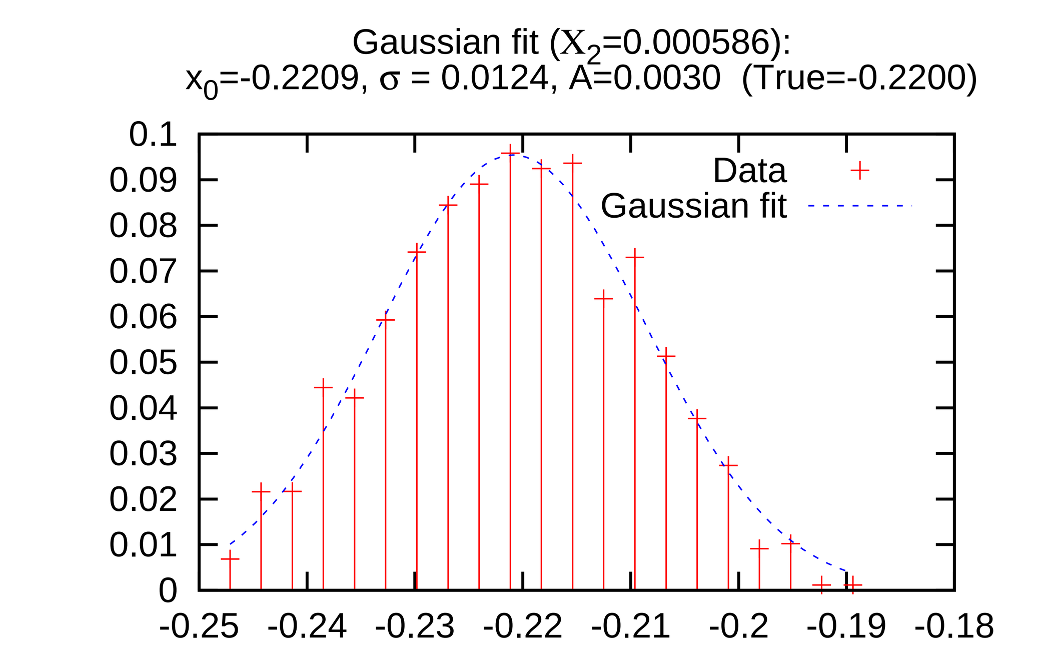

FIGURE 3: *Distribution of β for the selected templates.*

Similar analysis was performed for other terms and we found that we can determine the following combinations: $\hat{L}.\hat{n}$, a component of Θ_{LISA} including the polarization angle and $\delta\Phi$. We should use those combinations as new parameters or as constrains to improve efficiency of the search.

Constraints on relative inclination and $\hat{L}$

The spin-orbital term β is defined as

$$\beta = \frac{1}{12}\sum_{i=1,2}\left[\chi_i\left(\hat{L}\cdot\hat{S}_i\right)\left(113\frac{m_i^2}{M^2} + 75\eta\right)\right].$$

Using the estimation of β it is possible to constrained the relation between $\hat{L}.\hat{S}_1$ and $\hat{L}.\hat{S}_2$ as showed in Fig. 4.

This constraint can be used to determine the relative orientation of spins and orbital angular momentum which provides important information required to study binary formation and evolution.

The four possible orientations of $\hat{L}$ observed on top panel of Fig. 1 can be explained with help of other two constraints :

$$\begin{cases} \hat{L}.\hat{n} = constant \\ \Theta_{LISA} = constant \end{cases}$$

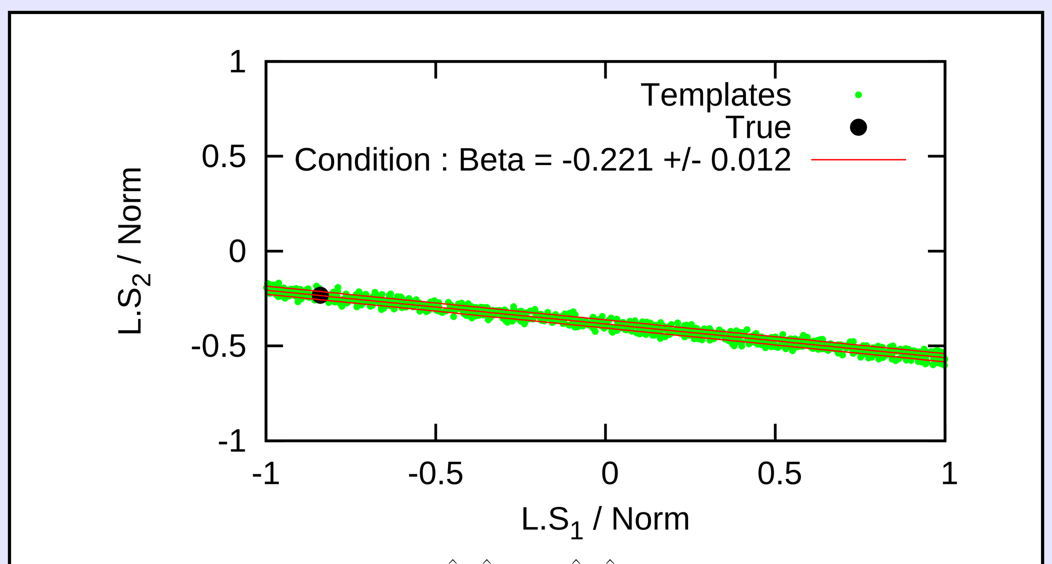

FIGURE 4: *Constraint on $\hat{L}.\hat{S}_1$ and $\hat{L}.\hat{S}_2$ from estimation of β and comparison with values of template selected with fixed sky position.*

Relaxing the sky position

We have observed that the sky location is correlated with precession parameters. Now we allow the sky location vary and perform another Monte Carlo simulation. We still keep other parameters fixed, since they can be very accurately estimated [2,3]. The sky error obtained in MLDC 3.2 for this source is around 10 degrees. The distribution of the parameters produced high $\mathcal{F}$ (above the threshold) are shown on Fig. 5. Note that we can determine the sky position better than in this simulation (black circle on top-left panet).

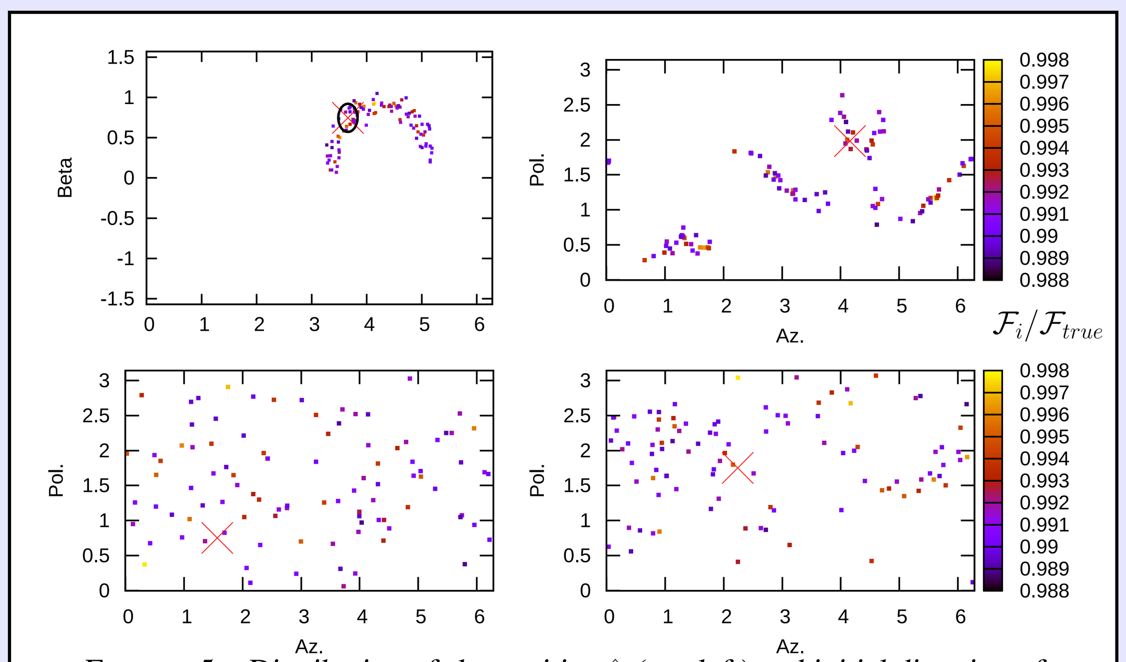

FIGURE 5: *Distribution of sky position $\hat{n}$ (top-left) and initial direction of $\hat{L}$(top-right), $\hat{S}_1$ (bottom-left) and $\hat{S}_2$ (bottom-right) for templates with the same values for m_1, m_2, t_c, χ_1 and χ_2 and $\mathcal{F}_i/\mathcal{F}_{true} > 0.989$. The circle in top-left panel corresponds to 10 degrees of error. The red cross corresponds to the true value.*

We have found that the same combinations could be again well constrained. However the dispersion in the sky location translates in larger error. As an example we show the constraint on $\hat{L}.\hat{S}_1$ and $\hat{L}.\hat{S}_2$ in the Fig. 6.

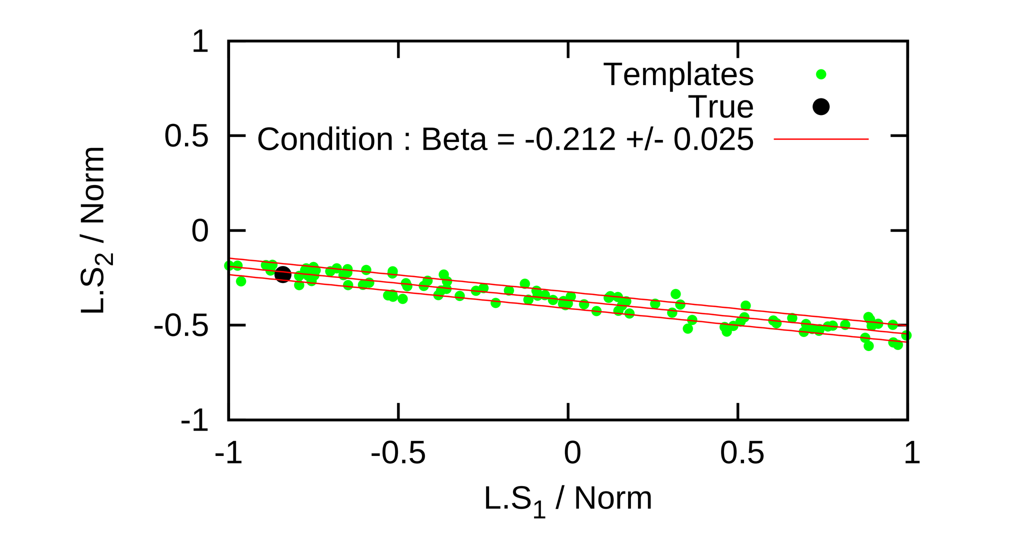

FIGURE 6: *Constraint on $\hat{L}.\hat{S}_1$ and $\hat{L}.\hat{S}_2$ from estimation of β and comparison with values of template selected with free sky position.*

This figure is to be compared to the Fig. 4: despite the larger dispersion, the constraint is still clear. Since we can determine the sky location better than in Fig.5, the situation is somewhere in between Fig.4 & Fig.5.

Conclusions

We study the degeneracy in the precession terms $\hat{L}$, $\hat{S}_1$, $\hat{S}_2$ in a particular case (preliminary studies show similar results for other cases). In the search and in the waveform parameterization we suggest the following:

- to use reference time close to the end of inspiral,
- parameter β can be well constrained and it provides relation between $\hat{L}.\hat{S}_1$ and $\hat{L}.\hat{S}_2$,
- the combinations $\hat{L}.\hat{n}$ and Θ_{LISA} (we do not give here the explicit expression) could be well constrained, and, for well determined sky location, give us several possible orientations of the vector $\hat{L}$.

These results should also be useful in sampling the parameter space with numerical waveforms.

References

1. S. Babak & al. *The MLDC: from challenge 1b to chall. 3* CQG **25**, 184026, (2008).
2. S. Babak & al.. *The MLDC: from challenge 3 to chall. 4*. gr-qc:0912.0548, (2009).
3. A. Petiteau, S. Babak, Yu Shang and F. Feroz *in preparation*