Systematic errors in the construction of hybrid waveforms for binary black holes

Frank Ohme[1], Lucía Santamaría[1], Mark Hannam[2], Sascha Husa[3], Badri Krishnan[1], Philipp Mösta[1], Denis Pollney[1,3], Christian Reisswig[1]

[1]Albert-Einstein-Institut Potsdam, Germany [2]Universität Wien Vienna, Austria [3]Universitat de les Illes Balears Palma, Spain

MAX-PLANCK-GESELLSCHAFT

Motivation

The detection of gravitational waves (GWs) from a coalescing black-hole binary and the extraction of astrophysical information requires accurate template waveforms that model this process.

To construct complete waveforms, post-Newtonian (PN) results for the inspiral part can be combined with data from non-perturbative numerical relativity (NR) simulations that model the late inspiral, merger and ringdown. These simulations are computationally expensive and there are various approaches to eventually obtain an analytic description of the entire signal [5, 6].

However, before such an analytic fitting can take place, PN and NR results have to be combined in a hybrid waveform – the accuracy of this procedure limits the reliability of all further results.

Here, we are concerned with the basic errors in the construction of hybrid waveforms: matching error, model errors and combination error, as explained in the following sections.

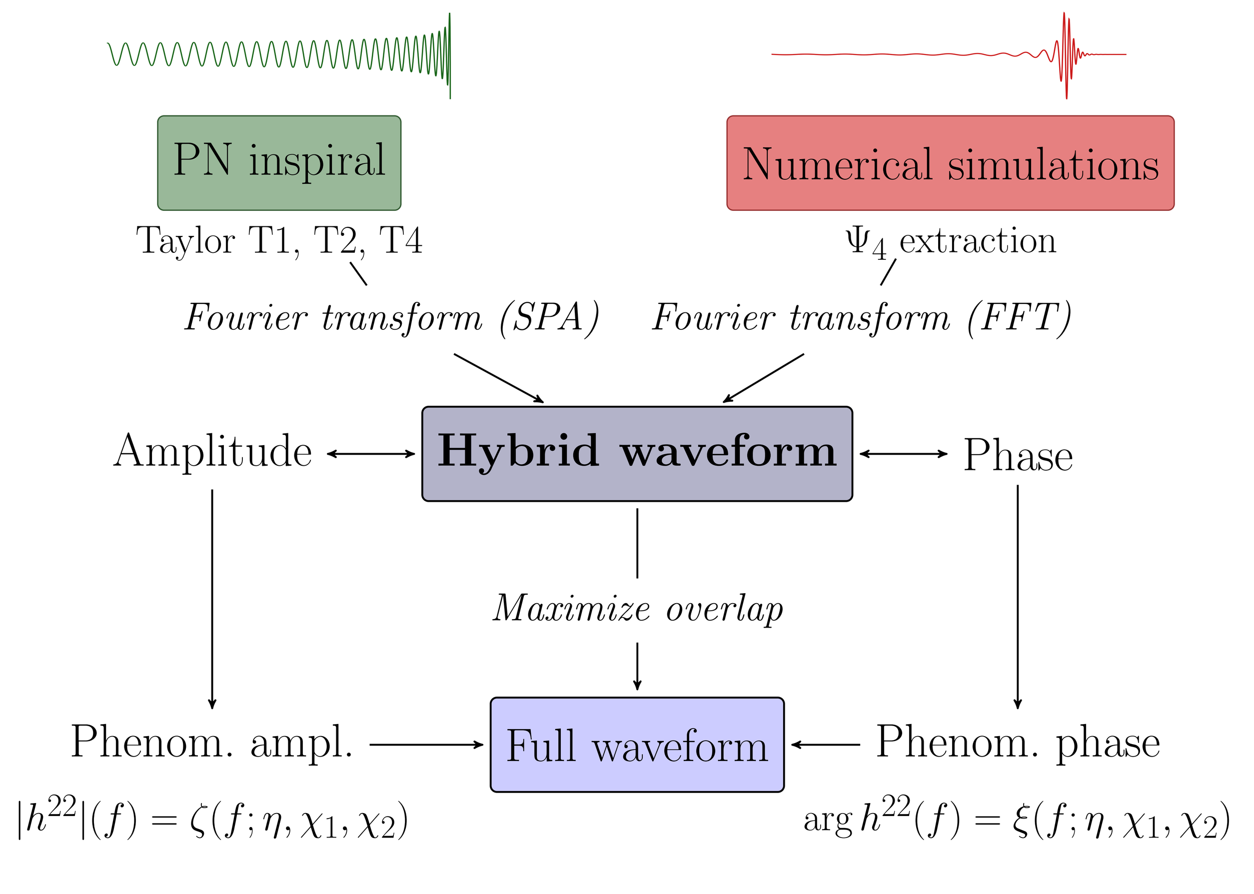

Figure 1: Principal strategy of combining PN and NR results, here presented for our Fourier-domain procedure to obtain a phenomenological model of the entire waveform.

Analysis tools

The distance of two waveforms $h_1(f)$ and $h_2(f)$ is given by

$$\|\Delta h\|^2 = 4\int_0^\infty \frac{|h_1(f)-h_2(f)|^2}{S_n(f)}\,df\,, \qquad (1)$$

where f is the Fourier frequency and S_n is the noise spectral density of the detector. The global phase and time-shift of h_2 is chosen to minimize $\|\Delta h\|$. We further normalize by the square of the signal-to-noise ratio of h_1, which reads

$$\rho^2 = 4\int_0^\infty \frac{|h_1|^2}{S_n(f)}\,df\,. \qquad (2)$$

Following [1] we can conclude

- h_1 and h_2 are indistinguishable if $\|\Delta h\| < 1$.
- $\|\Delta h\|^2 < 2\rho^2\,\epsilon$ is accurate enough for detection purposes with a maximal mismatch of ϵ.

The NR waveforms used throughout this poster are simulations for mass-ratio 1:1 and 1:2 performed with a full general relativistic multipatch code [2].

Matching Error

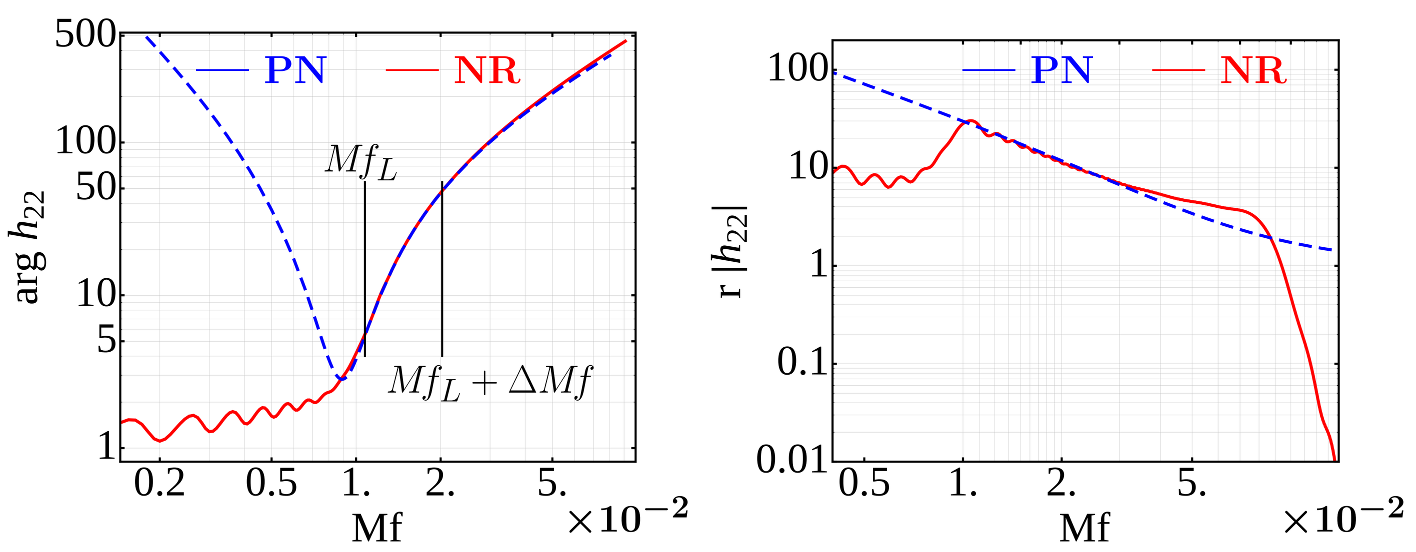

Figure 2: Aligning the PN and NR data in Fourier space. The 3.5PN (Fourier) phase needs to be fitted to fix the global phase and time. There is no freedom in aligning the 3PN [8] and NR amplitude.

Aligning the PN and NR part of the waveform usually involves some sort of fitting procedure to fix the global phase and time. In our Fourier domain example, two free parameters (t_0 and ϕ_0) of the *TaylorF2 phase* [9] have to be determined by a least-square-fit to the NR phase. The location of the fitting window depends on the length and quality of the NR waveform as well as on the statistical uncertainty in determining the fitting parameters. Analyzing these fitting errors of the parameters allows us to find a preferred window which is a compromise between

- fitting/matching at early frequencies (times) where PN is more reliable.
- fitting in a range where a considerable evolution of the orbital frequency of the binary allows an accurate determination of free parameters.

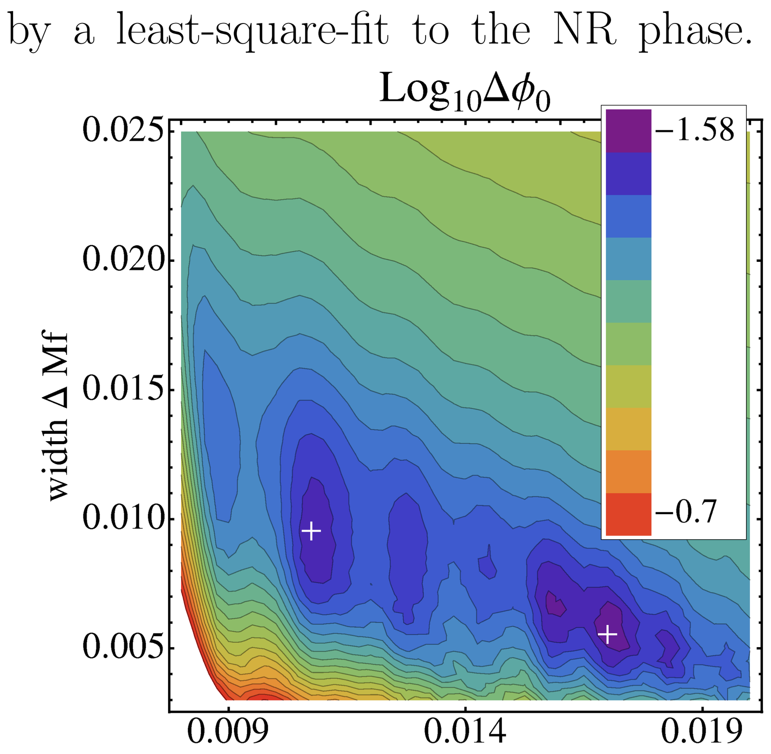

Figure 3: The statistical fitting error of the phase parameter ϕ_0 for different fitting windows.

Fig. 3 illustrates how the fitting uncertainty of ϕ_0 behaves as a function of the fitting window location and width. The minimal errors we find are of the order $\Delta\phi_0 \approx 0.03$ and $\Delta t_0 \approx 0.3$. However, even with such small parameter errors there are several possible fitting windows which lead to slightly different hybrid waveforms. How different these waveforms are is shown in Fig. 4, where an "optimal" early and long window is compared against a late and short one (indicated in Fig. 3).

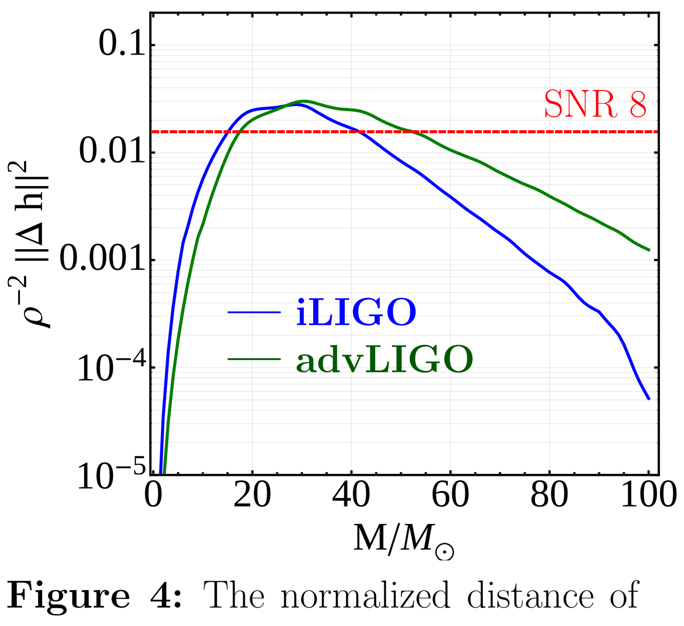

Figure 4: The normalized distance of two hybrid waveforms that differ in the fitting window for the phase.

Model Error

Both the PN and NR description of the wave signal are subject to errors of different kinds. Although there are estimates of amplitude and phase errors for NR waveforms (see e.g. [2, 3]), we estimate them here by comparing different resolutions of an equal mass [2], Cauchy-characteristic extracted waveform [4] as ingredients for the hybrid waveform.

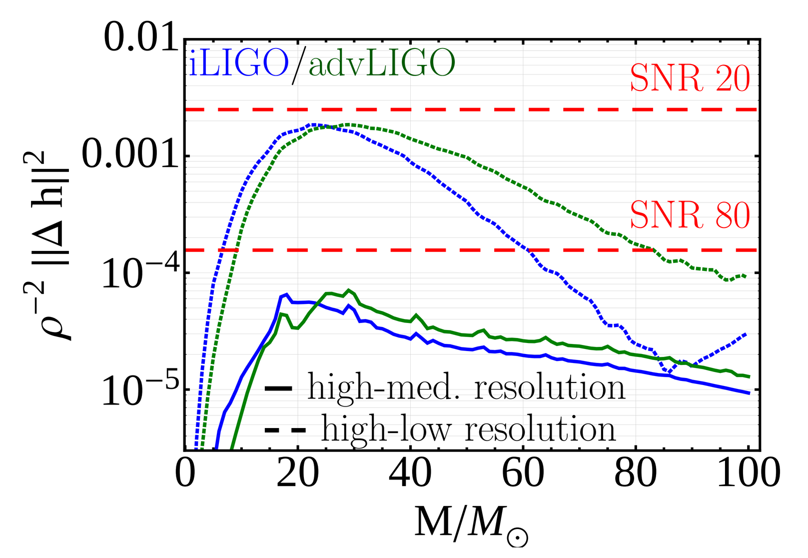

Figure 5: Distance of hybrids that differ only in the NR part. The grid spacings of the different resolutions are $h = 0.64$ (high), 0.80 (med), 0.96 (low).

The errors introduced by different PN descriptions (especially in the matching region) are hard to quantify a priori. We estimate them here by keeping the NR waveform (mass-ratio 1:2) and the matching region fixed and varying the PN order of the TaylorF2 phase.

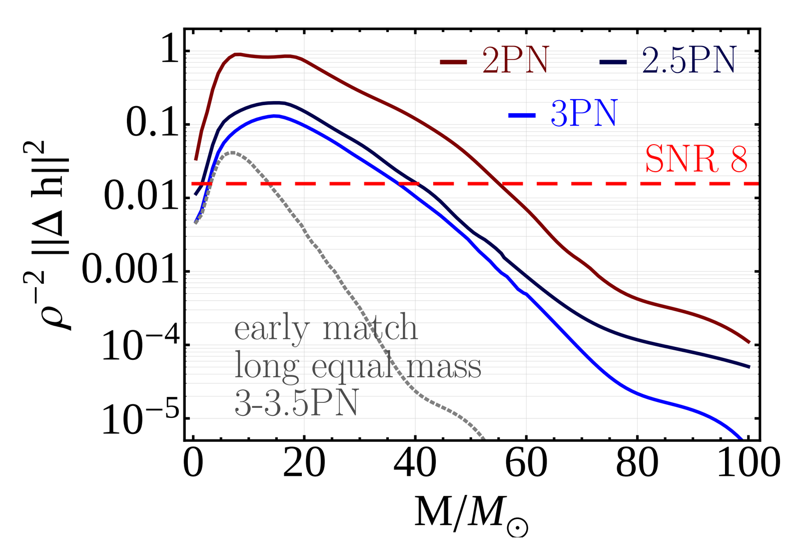

Figure 6: Distance of hybrids that differ only in the PN part. Different orders are compared to the 3.5PN phase hybrid. As a lower limit, the 3-3.5PN distance of matching to the currently longest equal mass waveform [7] is included.

⇒ Figs. 5 and 6 suggest that errors introduced by different models in the applied procedure are dominated by PN uncertainties.

Combination Error

Combining parts of the wave signal involves identifying physical quantities such as the symmetric mass ratio η or the total mass M both in PN and NR. These quantities are difficult to compare and it is not obvious that various definitions lead to the same results in dynamical spacetimes.

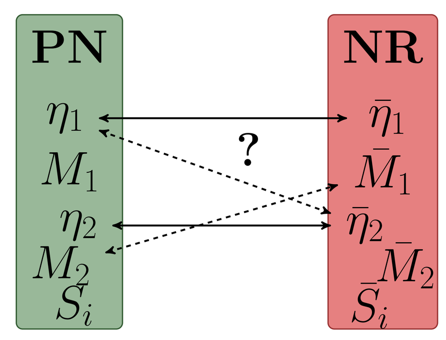

Figure 7: Identifying physical parameters.

Instead of using the horizon-based measure for η that NR predicts as input for PN, one may as well fit for the corresponding value, similarly to what is done for ϕ_0 and t_0.

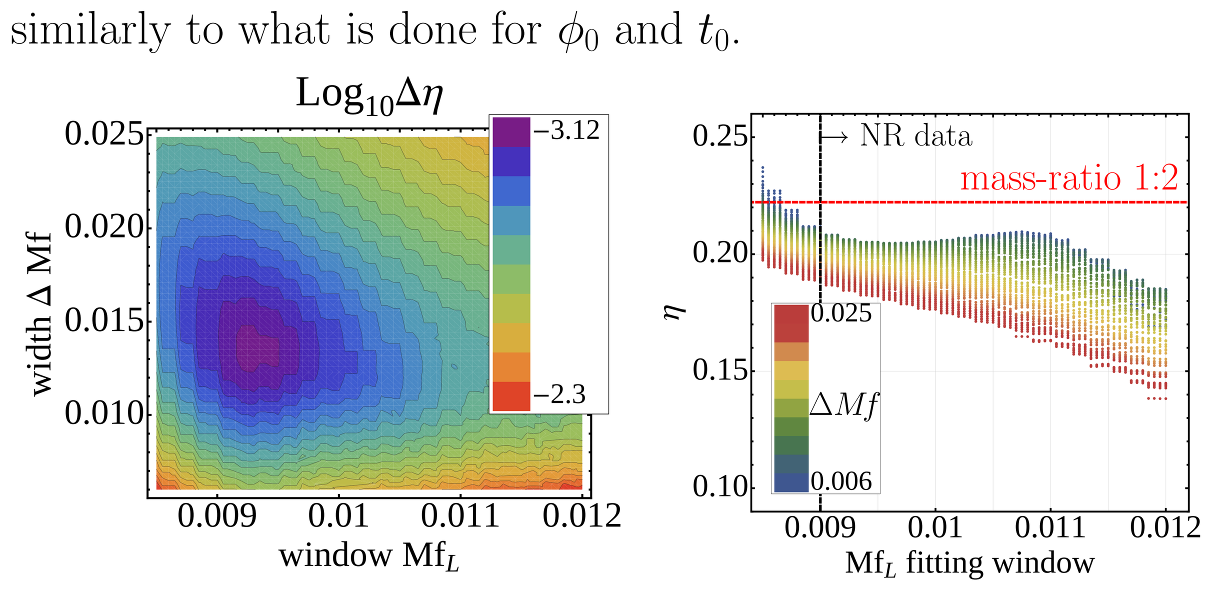

Figure 8 : Determining η by fitting TaylorF2 to the NR phase.

The best-fitted value we find is $\eta \approx 0.2045$ which is 8% smaller than expected from the NR simulation. This deviation may well be mostly due to PN uncertainties, see section Model Error and [10]. However, if fundamental differences in the definitions lead to different values, this has to be considered in the construction of hybrid waveforms. We (over)estimate this effect by the distance of the conventional hybrid (physical quantities not fitted) to a hybrid where

- the PN value for η is fitted to NR
- the quotient of the total masses is fitted for given η.

In the second case we find $M_{\rm NR}/M_{\rm PN} \approx 0.97$. The optimal fitting window is always defined by the minimum of the statistical parameter error, see the left panel of Fig. 8.

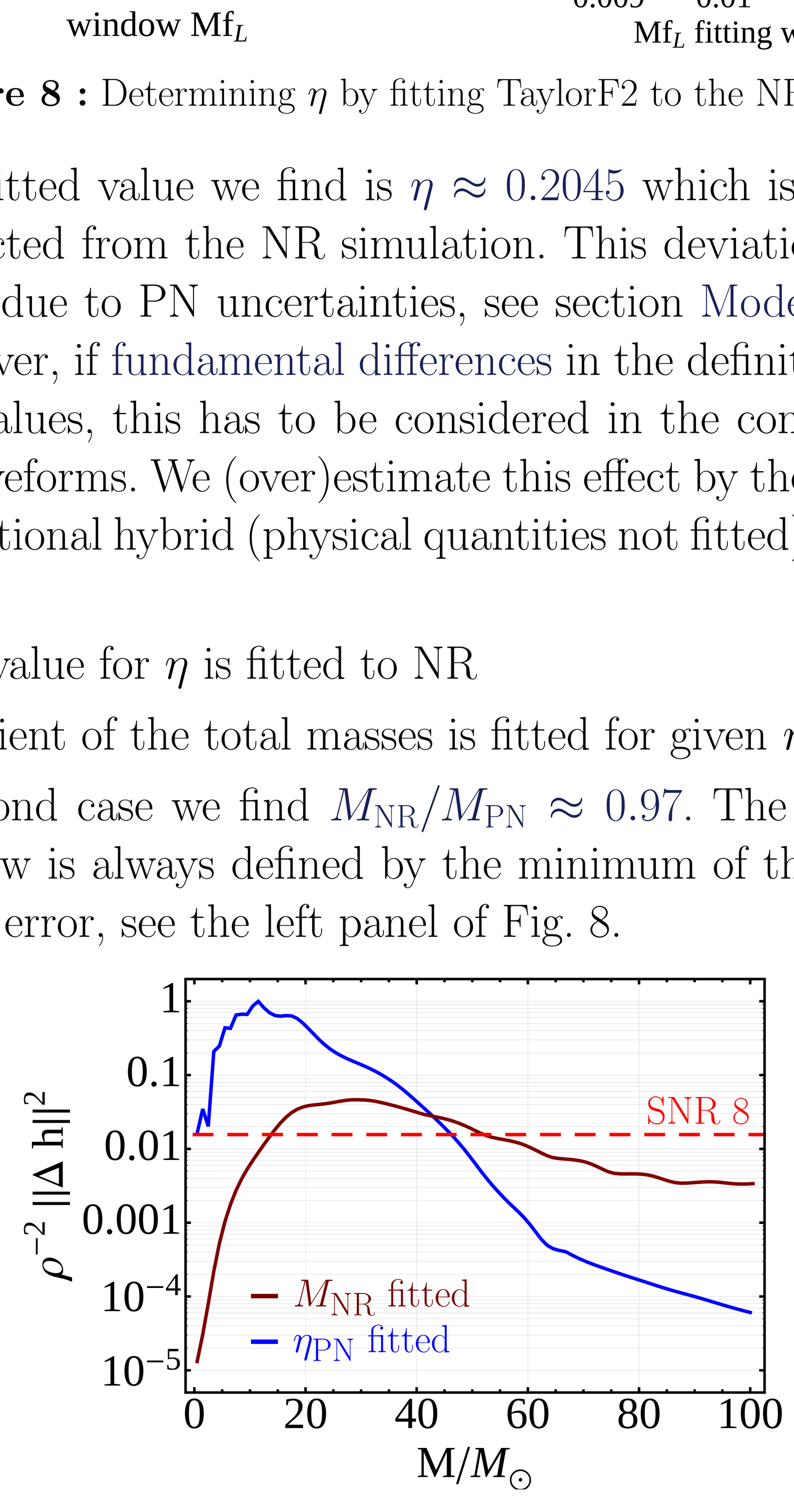

Figure 9: Distance of the conventional hybrid (physical parameters not fitted) to hybrids with additionally fitted values.

References

[1] L. Lindblom, B. Owen, D. Brown, Phys. Rev. D **78**, 124020 (2008)

[2] D. Pollney *et al.*, arXiv: 0910.3803 [gr-qc] (2009)

[3] M. Hannam *et al.*, Phys. Rev. D **79**, 084025 (2009)

[4] C. Reisswig C., N. Bishop, D. Pollney, B. Szilagyi, arXiv: 0912.1285 [gr-qc] (2009)

[5] P. Ajith *et al.*, Phys. Rev. D **77**, 104017 (2008)

[6] P. Ajith *et al.*, arXiv: 0909.2867 [gr-qc] (2009)

[7] M. Scheel *et al.*, Phys. Rev. D **79**, 024003 (2009)

[8] L. Blanchet, G. Faye, B. R. Iyer, S. Sinha, *Class. Quant. Grav.* **25**, 165003 (2008)

[9] K. G. Arun, B. R. Iyer, B. S. Sathyaprakash, P. A. Sundararajan, Phys. Rev. D **71**, 084008 (2005) [Erratum-ibid. D **72**, 069903 (2005)]

[10] A. Buonanno *et al.*, Phys. Rev. D **80**, 84043 (2009)

Acknowledgements

It is a pleasure to thank Ian Hinder for many useful discussions. LS acknowledges support from DAAD. MH was supported by FWF Lise Meitner fellowship (M1178-N16). S. Husa has been supported in part as a VESF fellow of EGO, by the grants SFB/Transregio 7 (DFG), D/07/13385 (DAAD) and FPA-2007-60220 (Ministerio de Educación y Ciencia, Spain).

Contact

frank.ohme@aei.mpg.de|lucia.santamaria@aei.mpg.de