Precision Measurement of Complete Black Hole Binary Inspiral-Merger-Ringdown Signals with LISA

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Until recently, only the inspiral and ringdown phases of black hole binary (BHB) coalescences had been modeled. The merger signals, which were expected to be the most luminous portion of the total signal, were unavailable due to the technical difficulty of calculating the behavior of a BHB in this highly dynamical and non-linear regime. Advancements in the field of numerical relativity now make it possible to include the merger segment of BHB coalescence in the search for and characterization of gravitational wave signals. The implications for LISA include an increase in the event rate due to the increase in achievable signal-to-noise ratio, as well as potentially improved accuracy regarding the extraction of the source parameters. We investigate the degree to which mergers improve parameter estimation, by studying the impact of including mergers on achievable parameter accuracy over a significant range of masses and mass ratios for nonspinning systems.

By using the EOB-IRS model introduced in [1], which is tuned to available numerical relativity results, we can calculate nonspinning waveforms with arbitrary mass ratio, as shown in Fig. 1. To assess the accuracy with which we can extract parameters from a complete coalescence waveform, we can numerically calculate the Fisher information matrix, $\Gamma_{ij}(\lambda) = (h_{,i}|h_{,j})$, for variations of the extrinsic parameters, those being the **total *redshifted mass***, the ***luminosity distance***, the ***arrival time***, the ***source inclination*** and ***polarization***, the ***initial orbital phase***, and the ***ecliptic latitude*** and ***longitude***. We note that we exclude the mass ratio and other intrinsic parameters, as they would require additional runs of the IRS-EOB code, rather than simple rescalings of our baseline waveform. As evidenced by Fig. 1b and Fig. 2, these excluded parameters may have significant degeneracy with our extrinsic parameters, so we are implementing an efficient method to include them in future work. For large SNR, inverting the Fisher matrix yields the covariance matrix,

$$\Delta\lambda^i \, \Delta\lambda^j = \Gamma^{ij}(\lambda)^{-1}[1 + O(\mathrm{SNR}^{-1})],$$

so in this way we can measure the standard deviation corresponding to any given set of extrinsic parameter sets. The inner product is weighted by the LISA noise spectrum, and we use *Synthetic LISA* [2] to apply the detector's signal response. By performing a number of trials, we can generate a distribution of uncertainties. The results in Fig. 3 are for a subset of the parameters. Both Fig. 3 and Table I correspond to a 1.33 x 10^6 $M_\odot$ (redshifted) binary at z=1, and have been included in our publication of this work [3]. Table I shows the relative improvement in the uncertainty of each extrinsic parameter by including the merger and higher harmonics.

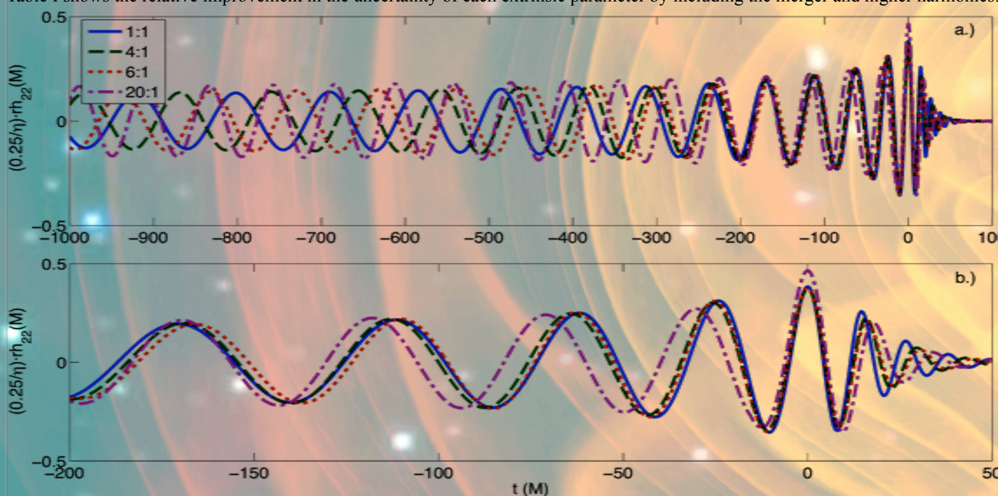

FIG. 1: a.) 1:1, 4:1, 6:1, and 20:1 mass ratio waveforms, with amplitude scaled by the reduced mass ratio η.
b.) same as a., but zoomed to emphasize the merger signal.

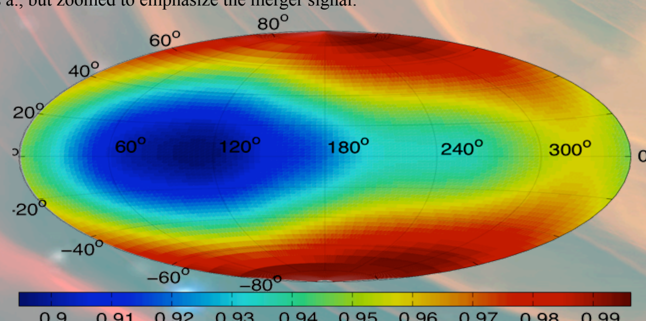

FIG. 2: Sky map of the "match" [4] between the full 1:1 and 4:1 waveforms with harmonics, as a function of location on the source sky. Close to the orbital axis, where the quadrupolar content shown in Fig. 1 is most emphasized, the mass ratio degeneracy is apparent in the large match exceeding 0.97, which means the 1:1 waveform could be used as a template to detect the 4:1 signal for this portion of the source sky.

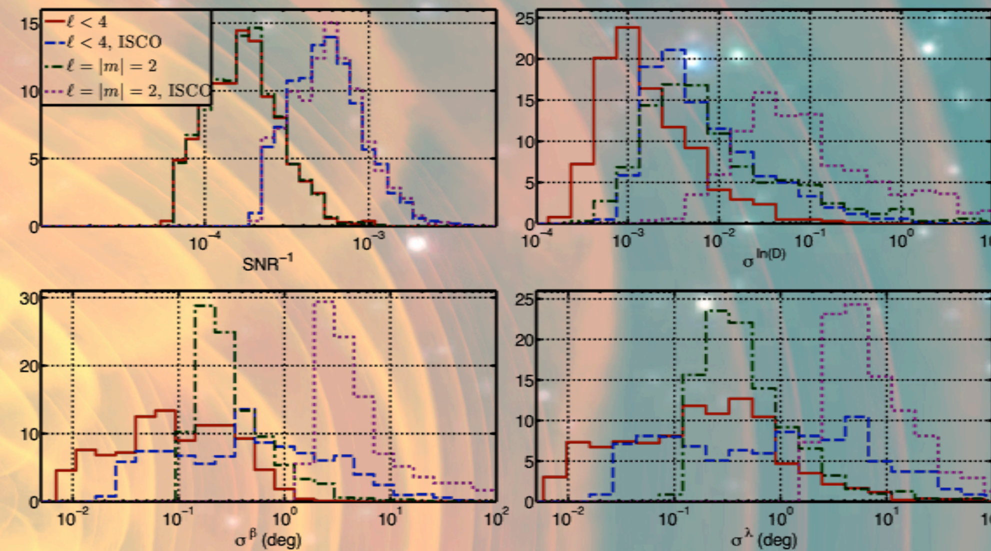

FIG. 3: Histograms of inverse SNR, fractional distance uncertainty, and ecliptic latitude and longitude uncertainties for our 2:1 mass ratio waveform. The waveform is scaled for a redshift $z = 1$ and a total system mass $M = 1.33 \times 10^6 M_\odot$. The histograms are calculated using a full inspiral-merger-ringdown waveform with harmonics $\ell \leq 4$ (solid), an inspiral waveform truncated at the ISCO frequency (dashed), a full waveform including only quadrupole ($\ell = 2$, $m = \pm 2$) modes (dash-dotted), and an inspiral waveform including only quadrupole modes (dotted). All histogram bins are normalized by the total number of cases and are expressed as percentages.

$(1+z)M$	m_1/m_2	Numerator	Denominator	σ^M/M	σ^{D_L}/D_L	σ^β	σ^λ	Ω	σ^ι	σ^{ϕ_o}	σ^ψ	σ^{t_c}	SNR^{-1}
1.33e6	1/1	$\ell \leq 4$, ISCO	$\ell \leq 4$, full	1.2	3.6	4.8	6.4	27	2.3	2.7	5.0	13	3.1
-	-	$\ell = \|m\| = 2$, full	$\ell \leq 4$, full	1.0	2.3	1.8	1.5	2.7	1.8	1.7	2.5	1.2	1.0
-	-	$\ell = \|m\| = 2$, ISCO	$\ell \leq 4$, ISCO	1.0	9.1	6.1	4.0	28	8.3	6.8	7.1	1.2	1.0
1.33e6	1/2	$\ell \leq 4$,ISCO	$\ell \leq 4$, full	1.2	3.5	5.5	5.6	29	2.9	2.7	4.8	15	3.2
-	-	$\ell = \|m\| = 2$, full	$\ell \leq 4$, full	1.0	4.5	3.1	2.3	8.2	3.8	3.9	4.8	1.3	1.0
-	-	$\ell = \|m\| = 2$, ISCO	$\ell \leq 4$, ISCO	1.0	16	8.1	6.1	63	16	17	14	1.3	1.0

TABLE I: Ratio of the waveform-model results for median variance of all the extrinsic parameters for two sets of comparable-mass physical systems. The "Numerator" and "Denominator" columns indicate the models compared in constructing the ratios for that row. The models vary by the harmonic content of the waveforms and by whether the merger is included (full) or not (ISCO). For the systems considered here, the fractional loss in estimated precision from ignoring the final merger is comparable to the corresponding loss in SNR and has a greater impact than ignoring higher harmonics. The significance of the higher harmonics is lower when full models are considered, as compared with ISCO-terminated models.

References

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[4] B. J. Owen, Phys. Rev. D **55**, 6749 (1996).