# Probing the physics of neutron stars with gravitational waves

#### Valeria Ferrari



#### GWADAW, Roma, January 2010

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- Rotating deformed stars
- Stellar oscillations
- Coalescing binaries

Modeling the different processes is far from easy.

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- Rotating deformed stars
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Modeling the different processes is far from easy.

What can we learn from future GW observations?

$$h_{0} = 4.21 \cdot 10^{-24} \left[\frac{ms}{T}\right]^{2} \left[\frac{K\rho c}{r}\right] \left[\frac{I_{zz}}{10^{38} Kg m^{2}}\right] \left[\frac{\epsilon}{10^{-6}}\right] .$$
  
$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}} \qquad \text{quadrupole ellipticity}$$



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Haskell, Jones, Andersson, MNRAS (2006)

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Recent molecular dynamics simulations indicates  $u_{\rm break} \sim 0.1$ , i.e. the crust would break if the deformation were to exceed about 20 cm. (For terrestrial materials  $u_{\rm break} \approx 10^{-4} - 10^{-2}$ )

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Horowitz, Kadau, PRL (2009)
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Solid phases may also be present at higher densities, in which case stars with larger deformations may be possible.

Based on a model of a solid strange quark star, Owen estimates

$$\epsilon < 6 \times 10^{-4} (u_{\rm break}/10^{-2}),$$

Owen, PRL (2005)

while, based on a crystalline colour superconducting quark phase, Haskell et al estimate

$$\epsilon < 10^{-3} (u_{
m break}/10^{-2})$$

Haskell et al, PRL (2007)

The degree of admissible asimmetry in a neutron star depends upon the high density equation of state.

The magnetic field will also tends to deform the star.

It is the *internal*, rather than the external field strength that counts. Assuming a normal fluid core

$$\epsilon \approx 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}}\right)^2$$

Haskell,Samuelsson, Glampedakis, Andersson MNRAS (2008); Colaiuda, Ferrari, Gualtieri, MNRAS (2008); Lander, Jones, MNRAS (2009)

a superconducting core could produce larger asymmetries:

$$\epsilon \approx 10^{-9} \left(\frac{B}{10^{12}\,\mathrm{G}}\right) \left(\frac{H_{\mathrm{crit}}}{10^{15}\,\mathrm{G}}\right)$$

Cutler, PRD (2002); Akgün, Wasserman, MNRAS (2008)

(assuming a type II superconducting core)

For typical pulsars field strenght, the deformation is small

If the magnetic field reaches a stationary configuration before the crust is formed, the induced stellar deformation could persist after the crust forms. It would subsequently evolve on a timescale of  $\sim 10^3 - 10^5$  years.



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#### Twisted torus configuration:

- poloidal component: extends throughout the star and in the exterior
- toroidal component: is confined into a torus-shaped region inside the star, where the field lines are closed.

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For these configurations:

$$\epsilon \lesssim k \left(\frac{B[G]}{10^{16}}\right)^2 \cdot 10^{-4} ,$$

 $k \sim (5-10)$  depending on the stellar compactness.

For sufficiently strong magnetic fields,  $\epsilon$  could be large, and the star could be a strong source of gravitational waves.

Ciolfi, Ferrari, Gualtieri, MNRAS (2009), Ciolfi, Ferrari, Gualtieri, (2010)

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S5 run found that, for the Crab pulsar, no more than 2% of the spin-down energy was being emitted in the GW channel, corresponding to  $\epsilon < 10^{-4}$ .

This result shows that GW emission does not dominate the spin-down of the Crab.

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This result tells us, for instance, that the Crab cannot be a maximally strained quark star.

Different families of modes can be directly associated with different core physics.

- f(fundamental)-mode (which should be the most efficient GW emitter) scales with average density
- **p**(pressure)-modes overtones probe the sound speed throughout the star
- g(gravity)-modes are sensitive to thermal/composition gradients
- w-modes represent oscillations of spacetime

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#### There is a lot of physics to explore.

It relies on how well future GW detectors will be able to detect the various pulsation modes  $% \left( {{{\left[ {{{\rm{T}}_{\rm{T}}} \right]}}} \right)$ 

## The frequency of NS modes would provide information on the EOS in the inner core of a neutron star, which is unknown



 $\nu_{f}$  is versus the mass of the star, for neutron/hybrid stars (continuous lines) and for strange stars modeled using the MIT bag model (blue region) Benhar, Ferrari, Gualtieri, Phys. Rev. D (2004), Benhar, Ferrari, Gualtieri, Marassi, Gen. Rel. Grav. (2007)



- Strange stars cannot emit GWs with  $\nu_f \lesssim 1.7$  kHz, for any values of the mass in the range we consider.
- There is a small range of frequency where neutron/hybrid stars are indistinguishable from strange stars. However, there is a large frequency region where only strange stars can emit.



Since  $\nu_f$  is an increasing function of the bag constant *B*, detecting a GW from a strange star would allow to set constraints on *B* much more stringent than those provided by the available experimental data ( $B \in (57 - 95)$  MeV/fm<sup>3</sup>).

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$$h(t) = \mathcal{A}e^{-(t-t_0)/\tau_d} \sin[2\pi f(t-t_0)]$$
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Assuming  $f \sim 1500$  Hz,  $\tau_d \sim 0.1$  s, d = 1 kpc,  $\mathcal{A} \approx 5 \times 10^{-24}$ 3rd generation detectors are needed to detect signals from old NS. More promising are oscillations from newly-born neutron stars: more energy can be stored in the modes The oscillation spectrum evolves during the observation:

v (Hz)

The frequencies of the fundamental mode, and of the first g-, p- and w-modes of an evolving proto-neutron star are plotted as functions of the time elapsed from the gravitational collapse, during the first 5 seconds.

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Ferrari, Miniutti, Pons, MNRAS (2003)
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Having  $\nu(t)$  and  $\tau(t)$ , we can estimate the amount of energy  $\Delta E_{GW}$  that should be stored in a given mode for the signal to be detectable with an assigned SNR by a given detector.

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SNR = 2.7by Virgo+SNR = 8by Advanced Virgo.

#### If the newly born star is oscillating in the g-mode:



#### modes evolution



#### If the newly born star is oscillating in the g-mode:



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## NS binary inspiral and merger

The inspiral "chirp" is well modelled by post-Newtonian methods and much of the signal is adequately described by a point-mass approximation.

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From an observed chirp one would infer: the individual masses, the spin rate of the objects, the source distance.

• The final stages of inspiral, and the merger, are very interesting. As the binary tightens the equation of state of matter in the stellar interior begins to influence the GW signal.

## The merger signal is rich in information l = m = 2 component of $\tilde{h}(f)f^{1/2}$



Baiotti, Giacomazzo, Rezzolla, PRD (2008), Andersson et al. arXiv-0912.0384v, GRG to appear.

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blue solid line: binary when evolved with a "cold" EOS. red dashed line: "hot" EOS. dashed-dot line: inspiral phase. black solid line: nonspinning black-hole binary with total mass  $M = 200 M_{\odot}$ .

#### The merger signal is rich in information



#### Blue solid line (cold EOS)

• The merging terminates abruptly with a prompt collapse to black hole. The peak at  $f \approx 4$  kHz is associated to the BH formation.

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• The GW signal terminates at a cut-off frequency  $f_{\rm QNM} \simeq 6.7$  kHz which corresponds to the fundamental quasi-normal mode of the BH.

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#### Red dashed line (hot EOS)

- the two stars merge and "bounce" several times, emitting a number of peaks, having almost comparable amplitude. Then a hypermassive NS is fomed which eventually collapses to a black hole.
- The cutoff frequency corresponds to the QNM frequency of the BH QNM.

If the coalescing bodies are a BH and a NS, depending on the mass ratio, on the NS EOS and on the BH angular momentum, there are two possibilities:

• the NS is swallowed by the Black Hole (waveform similar to the NS-NS case)



astrogravs.nasa.gov/images/catalog

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#### $\bullet$ the NS is destroyed by the BH tidal field before reaching the $\ensuremath{\mathsf{ISCO}}$



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In this case the GW-signal is very different: it shuts off at the disruption point and it has a frequency cutoff, corresponding to the orbit where the star is disrupted. A **SGRB** can be powered.

#### Neutron star disruption in a NS-BH binary coalescence

 $r_{tide} > r_{ISCO}$  .

 $r_{tide}$  = radial distance at which the NS is torn apart by the BH tidal field. ( $r_{tide} > r_{ISCO}$  is a necessary conditions in order to power a short GRB.)

 $r_{tide} \longrightarrow [\nu_{GW \ tide}] = \text{cutoff frequency.}$ 

The detection of a tidal disruption signal will offer the possibility to gain some insight on the EOS of matter in the NS interior.



We have studied the tidal disruption of neutron stars in BH-NS coalescing binaries calculating  $r_{tide}$  and  $\nu_{GW \ tide}$  for several equations of state describing the matter inside the neutron star, and for a large set of the binary parameters.

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*Ferrari, Gualtieri, Pannarale, CQG (2009), Ferrari, Gualtieri, Pannarale, arXiv:0912.3692* 

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The parameters of our simulations are:

- The neutron star mass  $M_{NS}$  ,
- The black hole angular momentum a

(normalized to the BH mass).

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• The mass ratio  $q = M_{BH}/M_{NS}$ 

## Suppose that we detect a gravitational signal emitted by a coalescing binary.

From the chirp waveform we can estimate the values of the chirp mass and of the simmetric mass ratio:

$$\mathcal{M}_{chirp} = \frac{(M_{NS}M_{BH})^{3/5}}{(M_{NS}+M_{BH})^{1/5}}, \qquad \eta = \frac{M_{NS}M_{BH}}{(M_{NS}+M_{BH})^2}$$

with some errors. For instance, for a system with total mass  $M_{tot} \in$  5 – 26  $M_{\odot}$ ,

$$rac{\Delta\eta}{\eta} \sim 1 - 3 \cdot 10^{-2}, \qquad rac{\Delta \mathcal{M}_{chirp}}{\mathcal{M}_{chirp}} \lesssim 10^{-3}$$

Sathyaprakash, Schutz, http://www.livingreviews.org/Irr-2009-2.

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Sathyaprakash, Schutz, http://www.livingreviews.org/lrr-2009-2. We convert these errors into errors on  $M_{NS}$  and on  $q = M_{BH}/M_{NS}$  For instance, for  $M_{NS} = 1.4 M_{\odot}$  and q = 5 (i.e.  $M_{BH} = 7 M_{\odot}$ ), we find

$$\Delta M_{NS} = 0.02 \ M_{\odot}, \qquad \Delta q = 0.15$$

We also assume that a can be measured with a 10% accuracy.

The errors on  $M_{NS}$ , q, a affect the evaluation of the  $R_{NS}$  and of  $\nu_{GWtide}$ :



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The data identify a region (in yellow) in which, for any value of  $\nu_{GWtide}$ , the neutron star radius would lay.

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- Waves emitted by mature NSs oscillating in their modes, would provide information on their internal structure and EOS, those emitted by newly born NS would tell us about their evolution and about dissipative processes driven by neutrinos.
- Waves emitted in the merging of NS-NS binaries would clarify the dinamics of the merging and of the compact object which forms.

• Observing the wave emitted when a NS is disrupted by a tidal interaction with a black hole, would allow to measure the stellar radius with a very good accuracy

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• If a SGRB would be observed in coincidence, this would allow to validate the model of SGRBs as coalescing binary systems, clarifying one of the most interesting open issues in astrophysics.