Parameter estimation of massive black hole binaries with pulsar timing arrays

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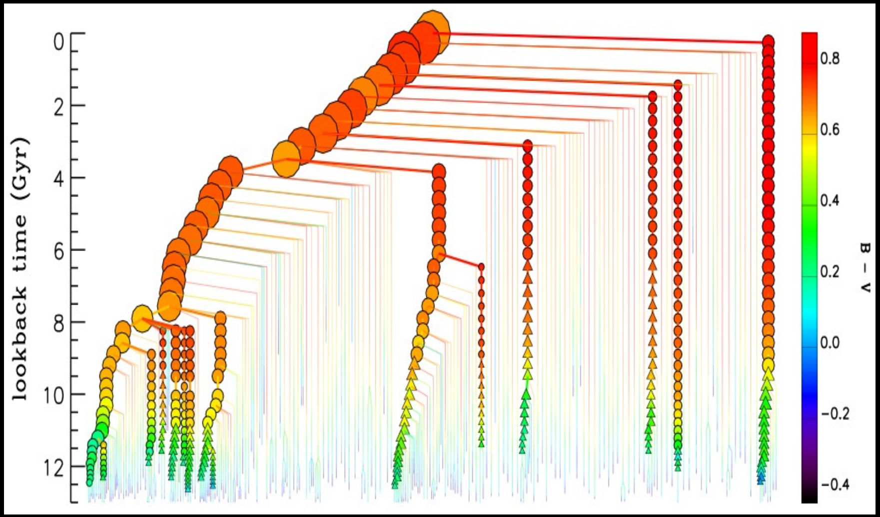
OUTLINE

- > PTAs as MBHB detectors
- > unresolved background and resolvable sources
- > Parameter estimation: preliminary results

Structure formation in a nutshell



From De Lucia et al 2006

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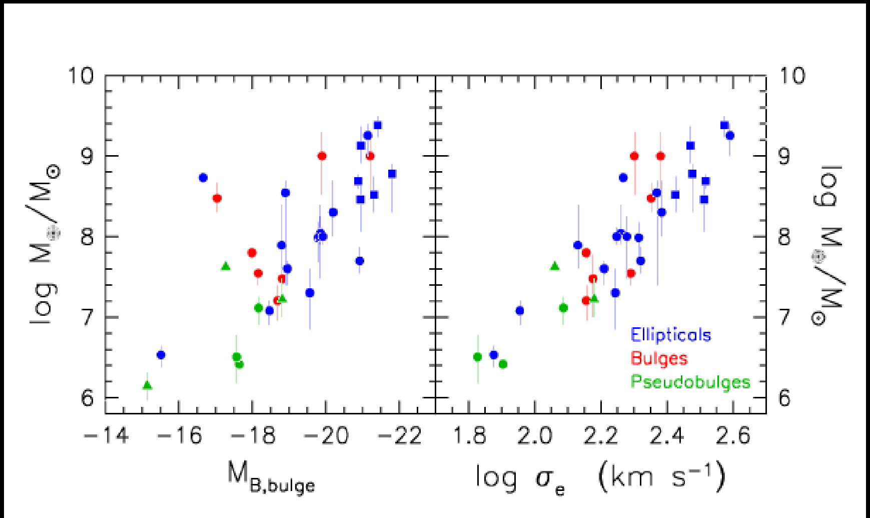

Ferrarese & Merritt 2000, Gebhardt et al. 2000

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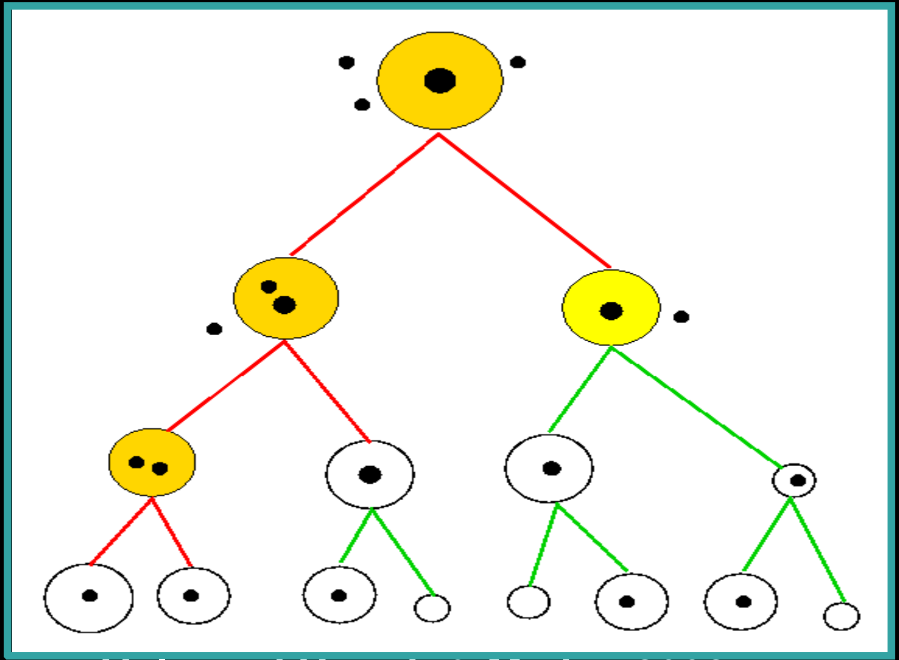
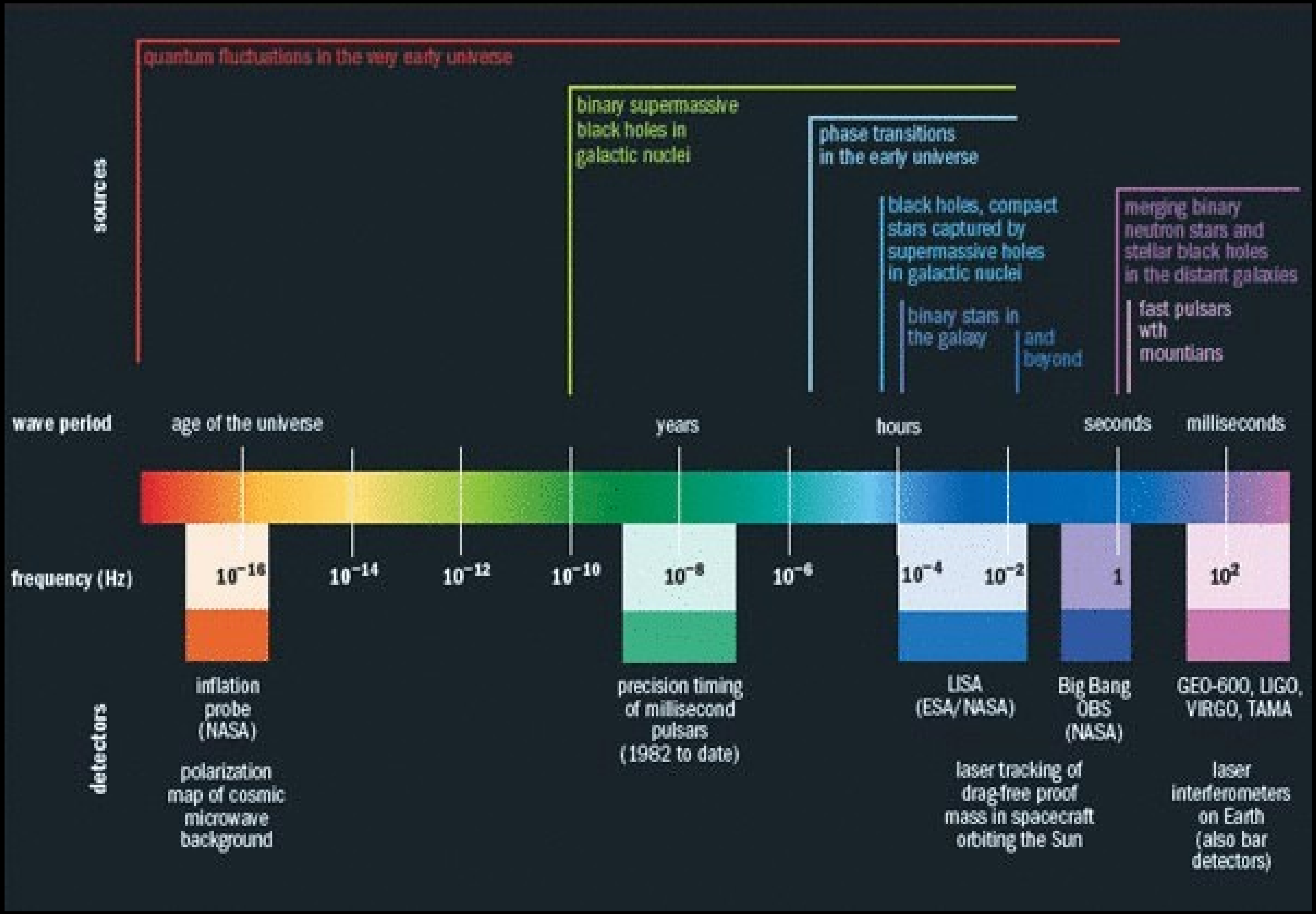


Volonteri Haardt & Madau 2003

THE GRAVITATIONAL WAVE SPECTRUM



PPTA (Parkes pulsar timing array)

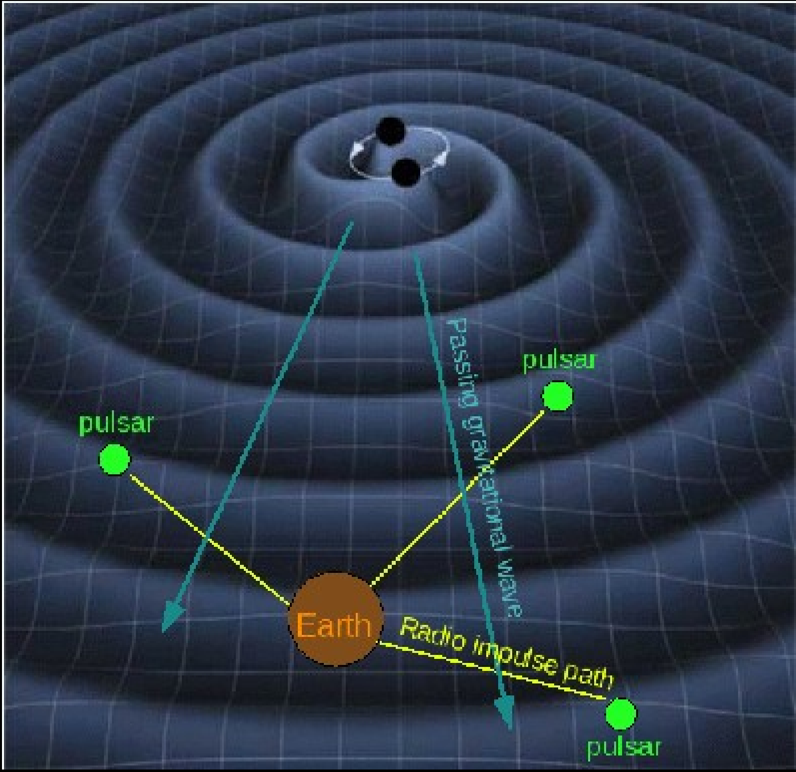
NanoGrav (north American nHz observatory for gravitational waves)

LEAP (large European array for pulsars)

The timing residual R

The GW passage cause a modulation of the MSP frequency

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv h_{ab}(t_{\rm p},\hat{\Omega}) - h_{ab}(t_{\rm ssb},\hat{\Omega})$$

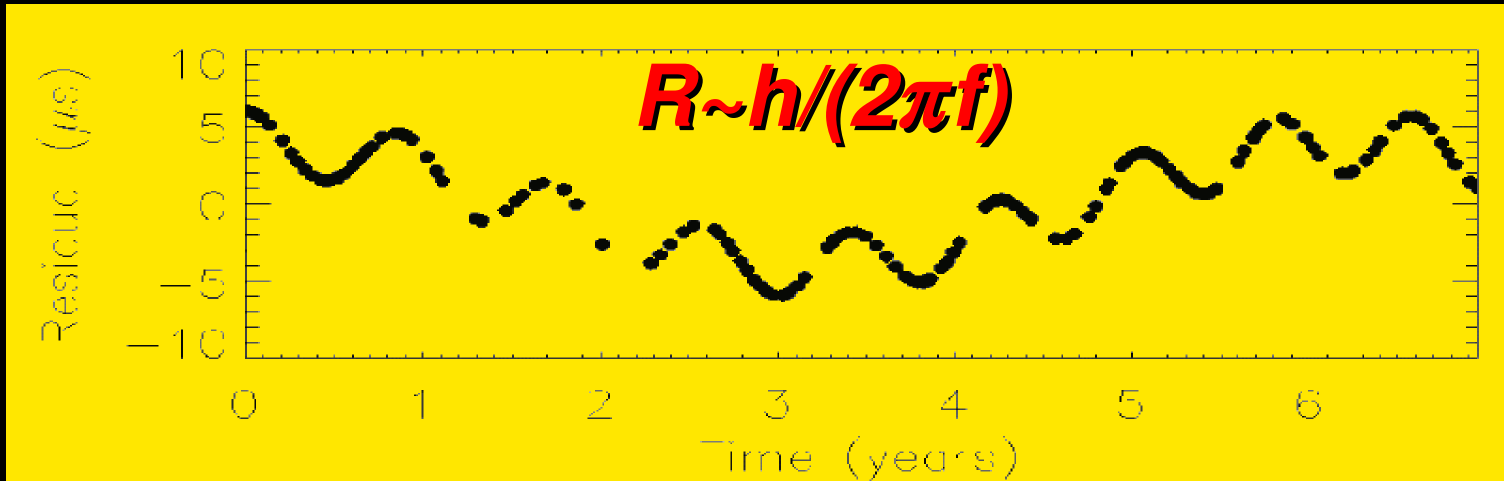


The ***residual*** in the time of arrival of the pulse is the integral of the frequency modulation over time

$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0}\,dt$$

Construct a model for the emission-propagation-detection of your pulse. If your model is perfect, then *R*=0. *R* contains all the uncertainties related to the signal propagation and detection plus the effect of unmodelled physics, like -possibly- ***gravitational waves***

$$R = \mathrm{TOA} - \mathrm{TOA}_m$$



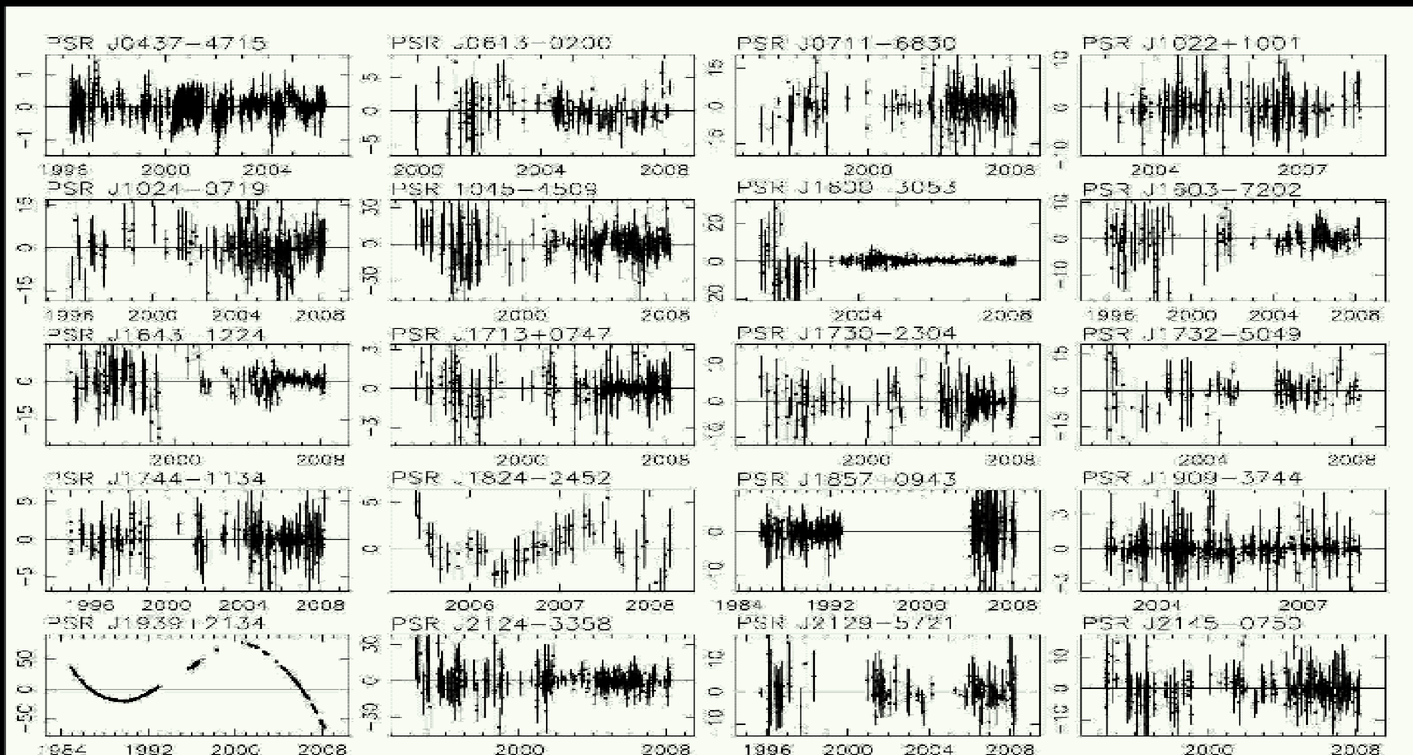


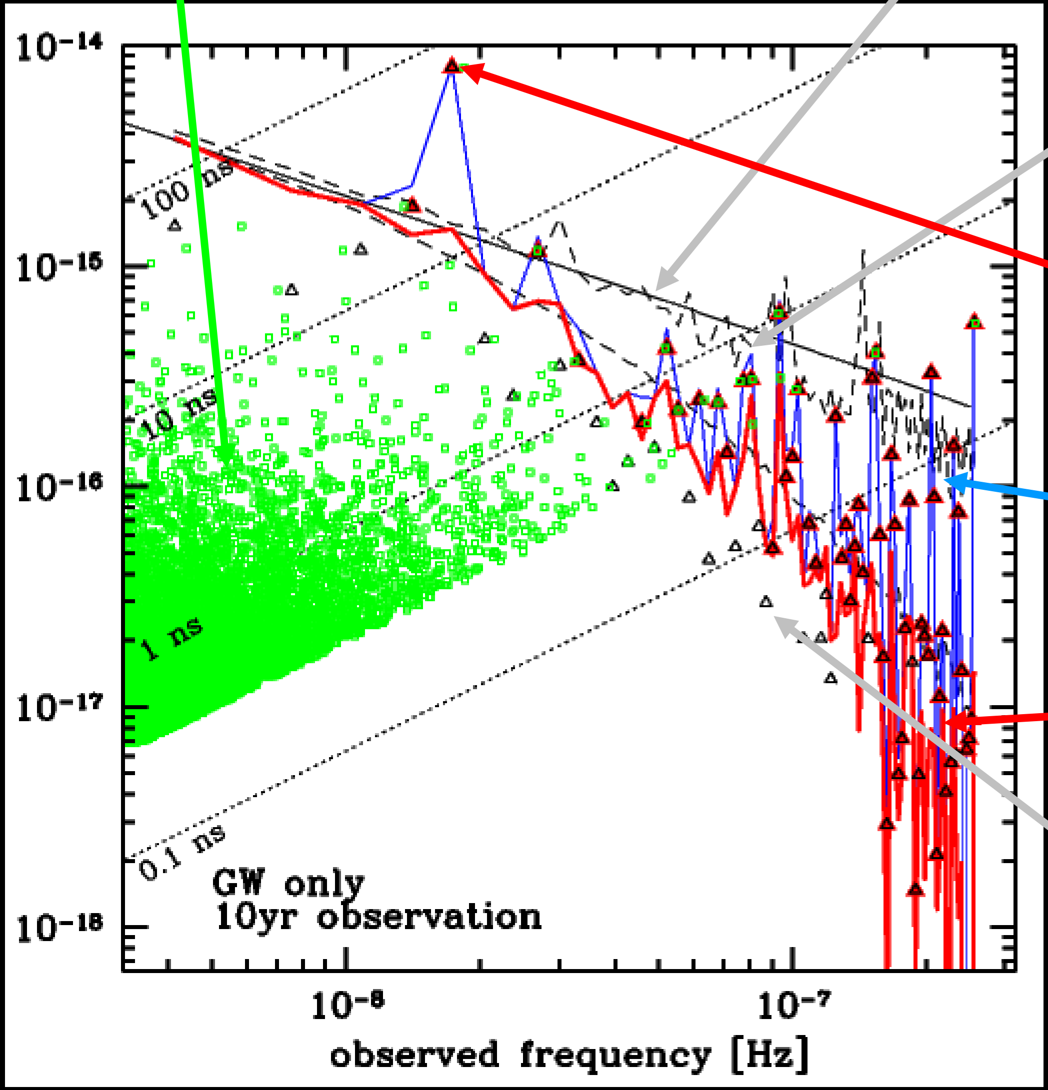
Figure 1. Timing residuals of the 20 pulsars in our sample. Scaling on the x-axis is in years and on the y-axis in μs. For PSRs J1857+0943 and J1939+2134, these plots include the Arecibo data made publically available by Kaspi et al. (1994); all other data are from the Parkes telescope, as described in §2. Sudden changes in white noise levels are due to changes in pulsar backend set-up - see §2 for more details.

Verbiest et al 2009

Signal from a MBHB population

Theoretical 'average' spectrum

Contribution of individual sources



Spectrum averaged over 1000 Monte Carlo realizations

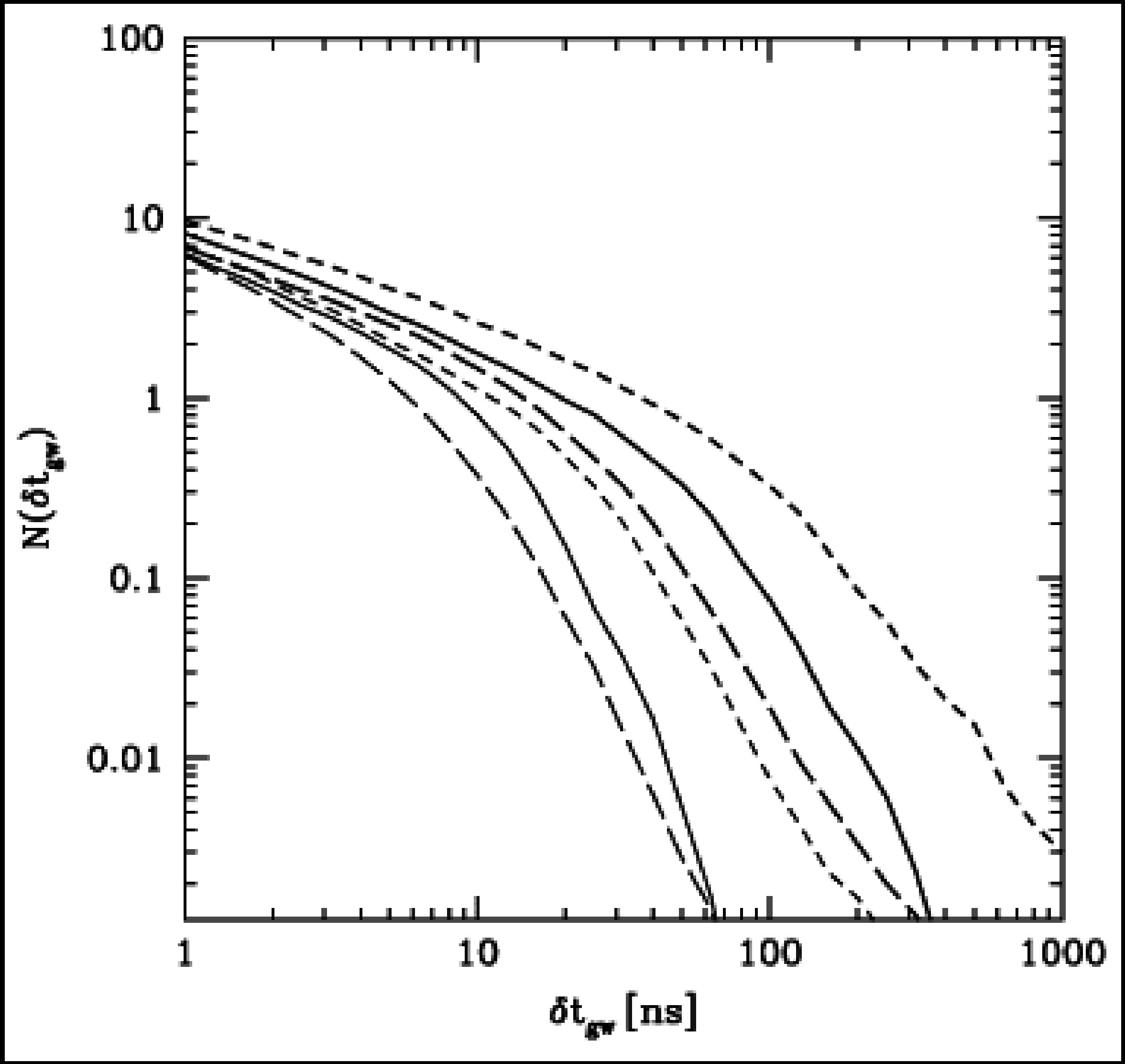
Resolvable systems: i.e. *systems whose signal is larger than the sum of all the other signals falling in their frequency bin*

Total signal

Unresolved background

Brightest sources in each frequency bin

Cumulative number of resolvable sources...

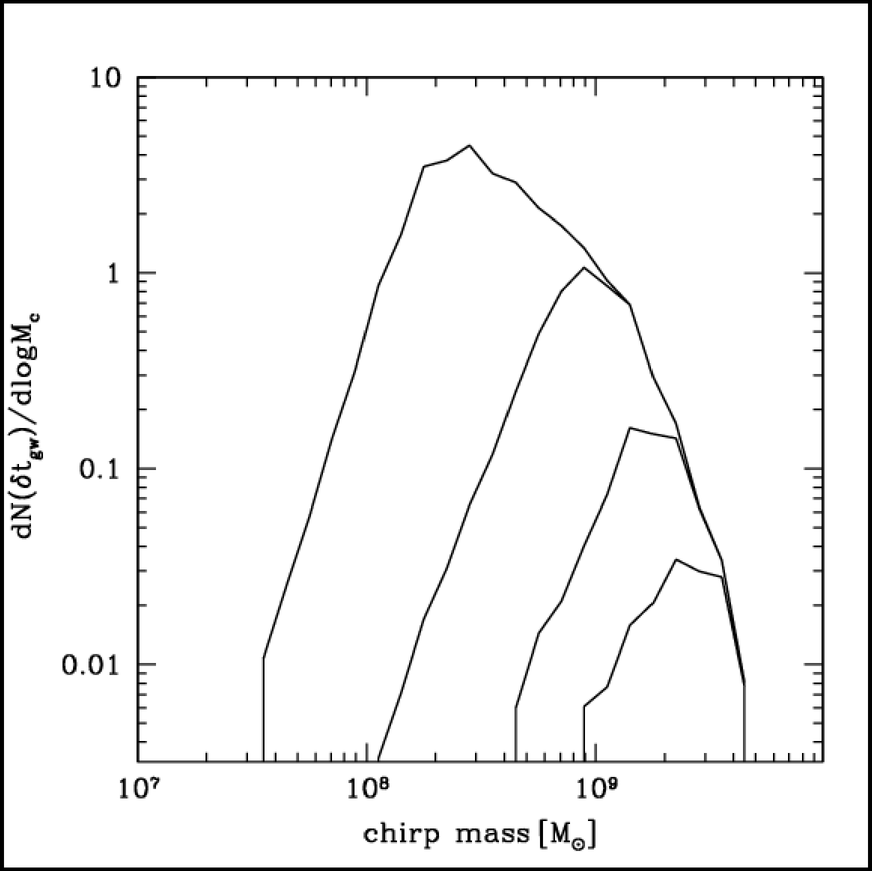


(Sesana, Vecchio & Volonteri, 2009, MNRAS, 394, 2255)

>a total timing precision of ***5-50 ns*** is required to detect an individual resolvable MBHB

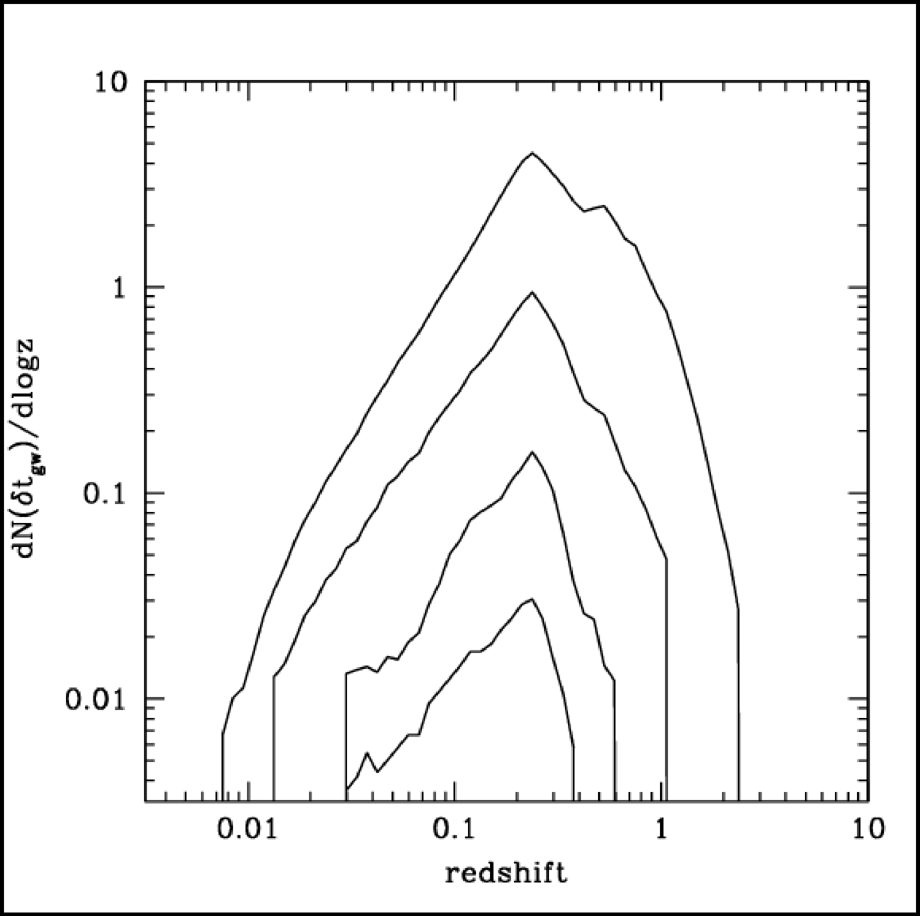
>Uncertainties depend on the ***MBH-host*** relation and MBH ***accretion route*** during mergers

Source distributions



Probe very massive systems
mass > *10^8* solar masses

Probe MBHs at
low-to-medium redshift *$0.1<z<1.5$*



Parameter estimation

(Sesana & Vecchio, submitted to PRD)

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv \sum_A F^A(\hat{\Omega}) \Delta h_A(t;\hat{\Omega})$$

$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0} dt$$

The signal depends on the 'antenna beam pattern' which is a function of the relative source-pulsar position in the sky and of the source polarization angle

$$F^+(\hat{\Omega}) = \frac{1}{2}\frac{(\hat{m}\cdot\hat{p})^2 - (\hat{n}\cdot\hat{p})^2}{1+\hat{\Omega}\cdot\hat{p}}$$

$$F^\times(\hat{\Omega}) = \frac{(\hat{m}\cdot\hat{p})(\hat{n}\cdot\hat{p})}{1+\hat{\Omega}\cdot\hat{p}}$$

We restrict our analysis to *circular non evolving sources*, we consider the *Earth term only* in the residual computation

$$r_\alpha(t) = R\left[a\,F_\alpha^{+}\left(\sin\Phi(t) - \sin\Phi_0\right) - b\,F_\alpha^{\times}\left(\cos\Phi(t) - \cos\Phi_0\right)\right],$$

$$R = \frac{A_{\rm gw}}{2\pi f}$$

$$\Phi(t) = 2\pi f t + \Phi_0$$

$$A_{\rm gw} = 2\frac{\mathcal{M}^{5/3}}{D}\left(\pi f\right)^{2/3}$$

The signal depends on *seven unknown parameters*

$$\vec{\lambda} = \{R, \theta, \phi, \psi, \iota, f, \Phi_0\}$$

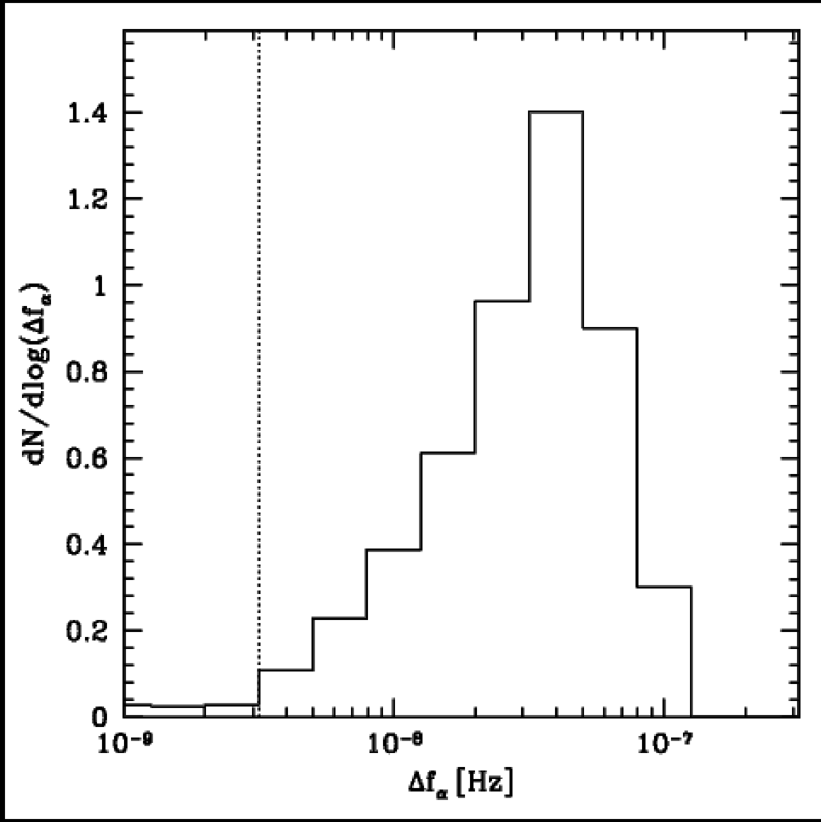
The sky errorbox is computed as

$$\Delta\Omega = 2\pi\sqrt{(\sin\theta\Delta\theta\Delta\phi)^2 - (\sin\theta c^{\theta\phi})^2}$$

Why the Earth term only

The signal consists of *two terms*: the *perturbation at the pulsar* and the *perturbation at the Earth*
The time delay among the two is

$$\tau_\alpha = L_\alpha(1+\hat{\Omega}\cdot\hat{p}_\alpha) \simeq 1.1\times10^{11}\,\frac{L_\alpha}{1\,\mathrm{kpc}}\,(1+\hat{\Omega}\cdot\hat{p}_\alpha)\,\mathrm{s}$$



And the frequency difference between the two terms will be

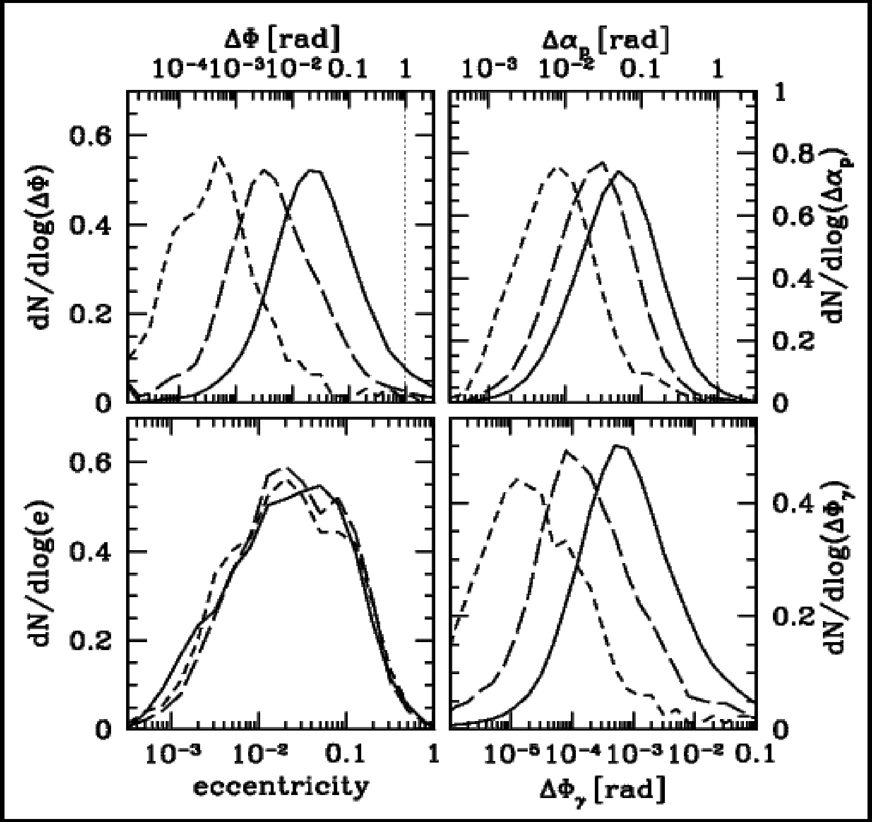
$$\Delta f_\alpha = \int_{t-\tau_\alpha}^{t}\frac{df}{dt}dt \sim \frac{df}{dt}\tau_\alpha \approx 15\,\mathcal{M}_{8.5}^{5/3} f_{50}^{11/3}\tau_{\alpha,1}\ \ \mathrm{nHz}$$

All the *Earth terms will sum coherently*, while the pulsar terms will be spread over different frequencies, with different phases.

Why circular, non evolving binaries

>Dynamical models for MBHB evolution in stellar environments predict *mild eccentricities* in the PTA window, if binaries were circular at the moment of pairing (Sesana, private communication)

>Frequency drift, mild eccentricities and spin orbit coupling have a *negligible effect* on the orbital evolution and on the waveform



$$\Delta\Phi \approx \pi \dot{f} T^2 \approx 0.04\,\mathcal{M}_{8.5}^{5/3}\, f_{50}^{11/3}\, T_{10}^2 \ \ \mathrm{rad}$$

$$\Delta\Phi_\gamma \approx \frac{d^2\gamma}{dt^2}T^2 = \frac{96\pi^{13/3}}{(1-e^2)} M^{2/3}\mathcal{M}^{5/3} f^{13/3} T^2 \approx 2\times10^{-3}\left(1-e^2\right)^{-1} M_9^{2/3}\mathcal{M}_{8.5}^{5/3}\, f_{50}^{13/3}\, T_{10}^2\,\mathrm{rad}$$

$$\Delta\alpha_p \approx 2\pi^{5/3}\left(1+\frac{3m_2}{4m_1}\right)\mu M^{-1/3} f^{5/3} T \approx 0.8\left(1+\frac{3m_2}{4m_1}\right)\left(\frac{\mu}{M}\right) M_9^{2/3} f_{50}^{5/3} T_{10}\,\mathrm{rad}$$

But binaries may as well be eccentric (Sesana Haardt & Amaro-Seoane, in prep.)
Eccentric waveform deferred to future work.

The Monte Carlo simulations

We consider several distribution of pulsars:

1- Isotropic distribution

(varying M=3, 5, 10, 20, 50, 100, 200, 500, 1000)

2-Anisotropic (polar) distribution

(varying the sky coverage=0.21, 0.84, 1.84, π, 2π, 4π srad)

3-The Parkes sample of 20 MSP

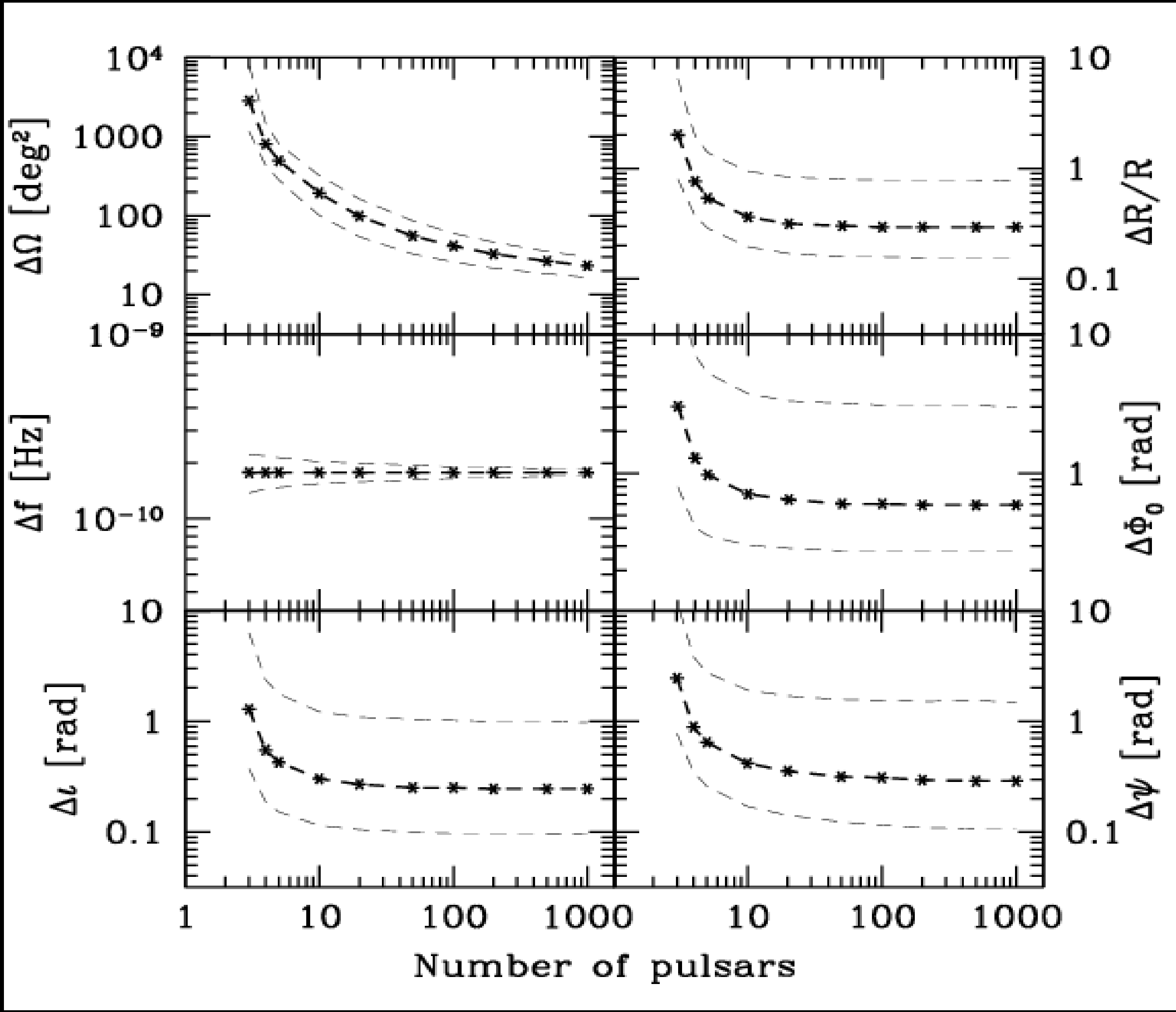
For any distribution of pulsar we generate a random sample of GW sorces in the sky:

- **-amplitude normalized to provide a given SNR in the array**
- **-isotropic sky position**
- **-random phase and polarization**
- **-inclination according to an anisotropic distribution of the orbital L**
- **-random frequency in the range 10^{-8}-10^{-7} Hz**

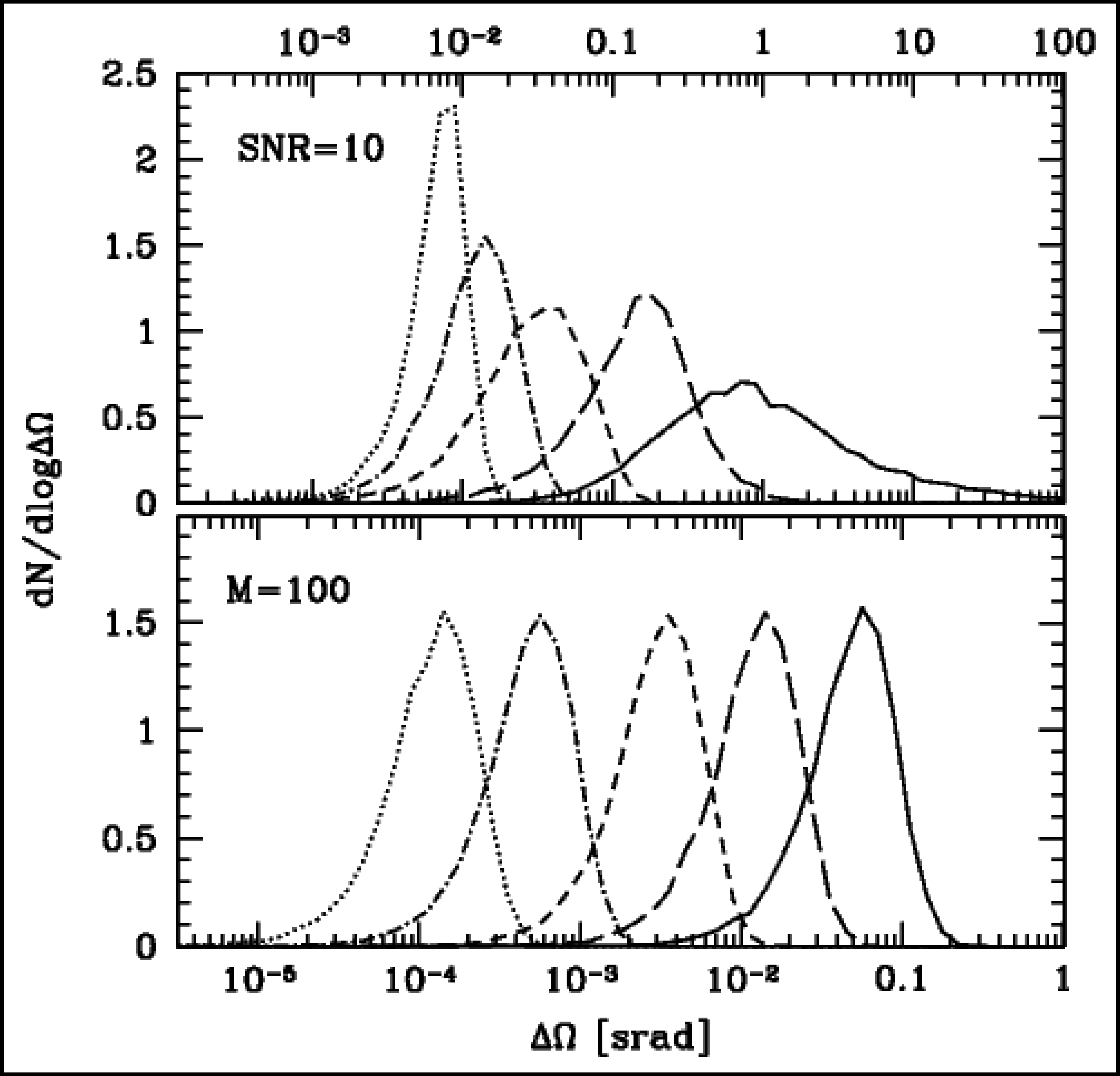
We compute the errors for each source using the Fisher information matrix formalism and we evaluate median values, etc etc...

Median statistical errors

As a function of the number of pulsars

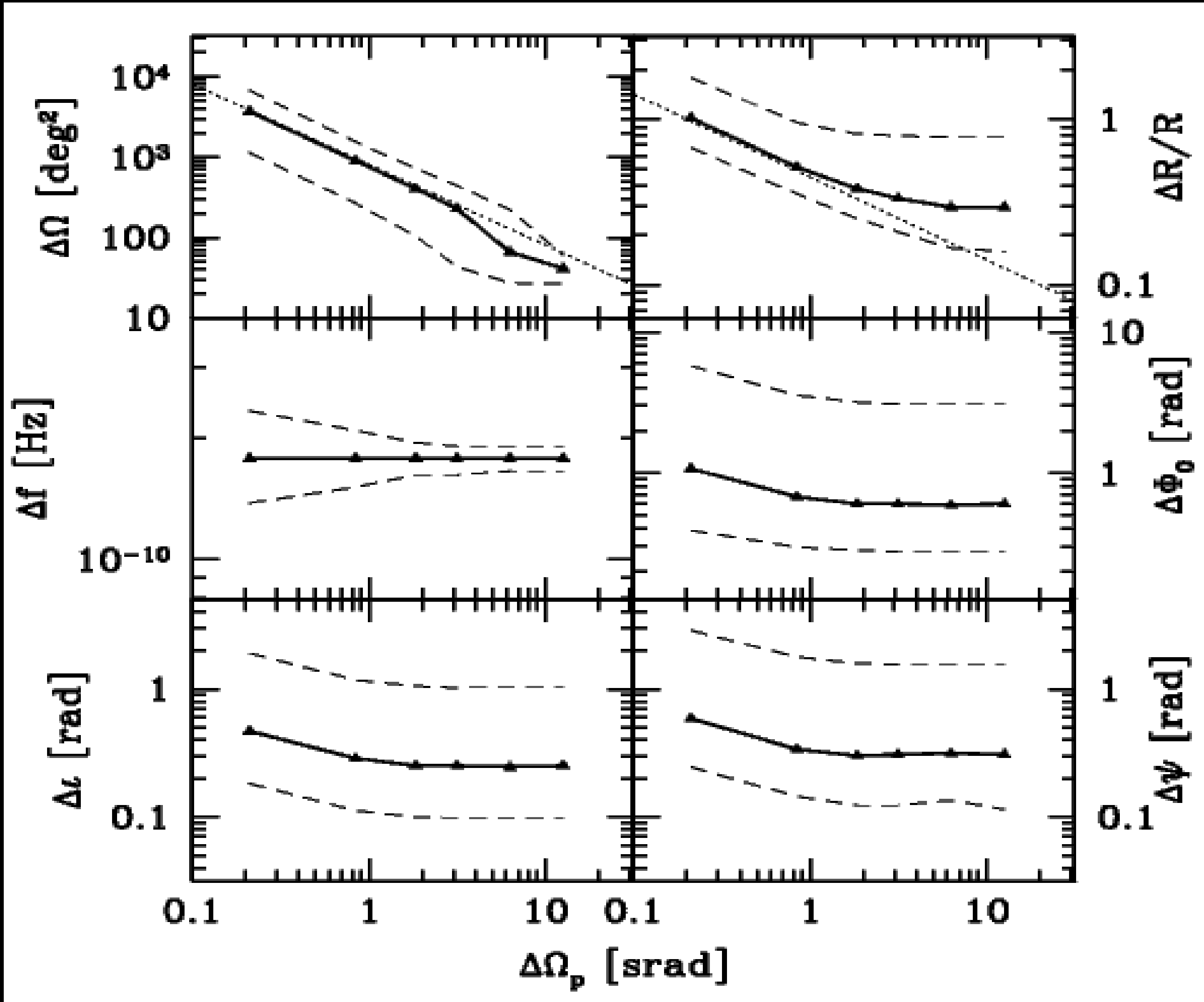


Typical ΔΩ distributions

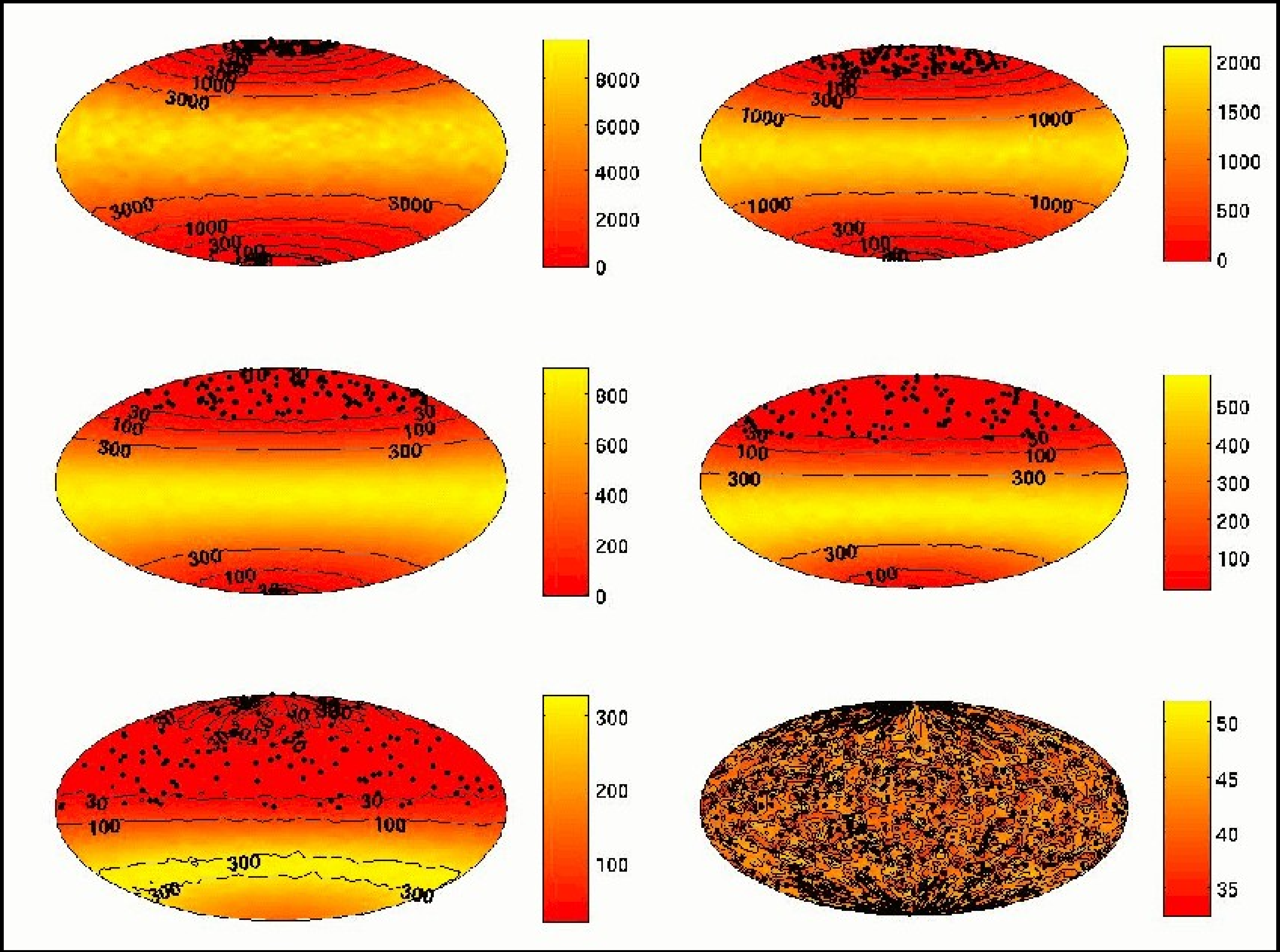


Median statistical errors

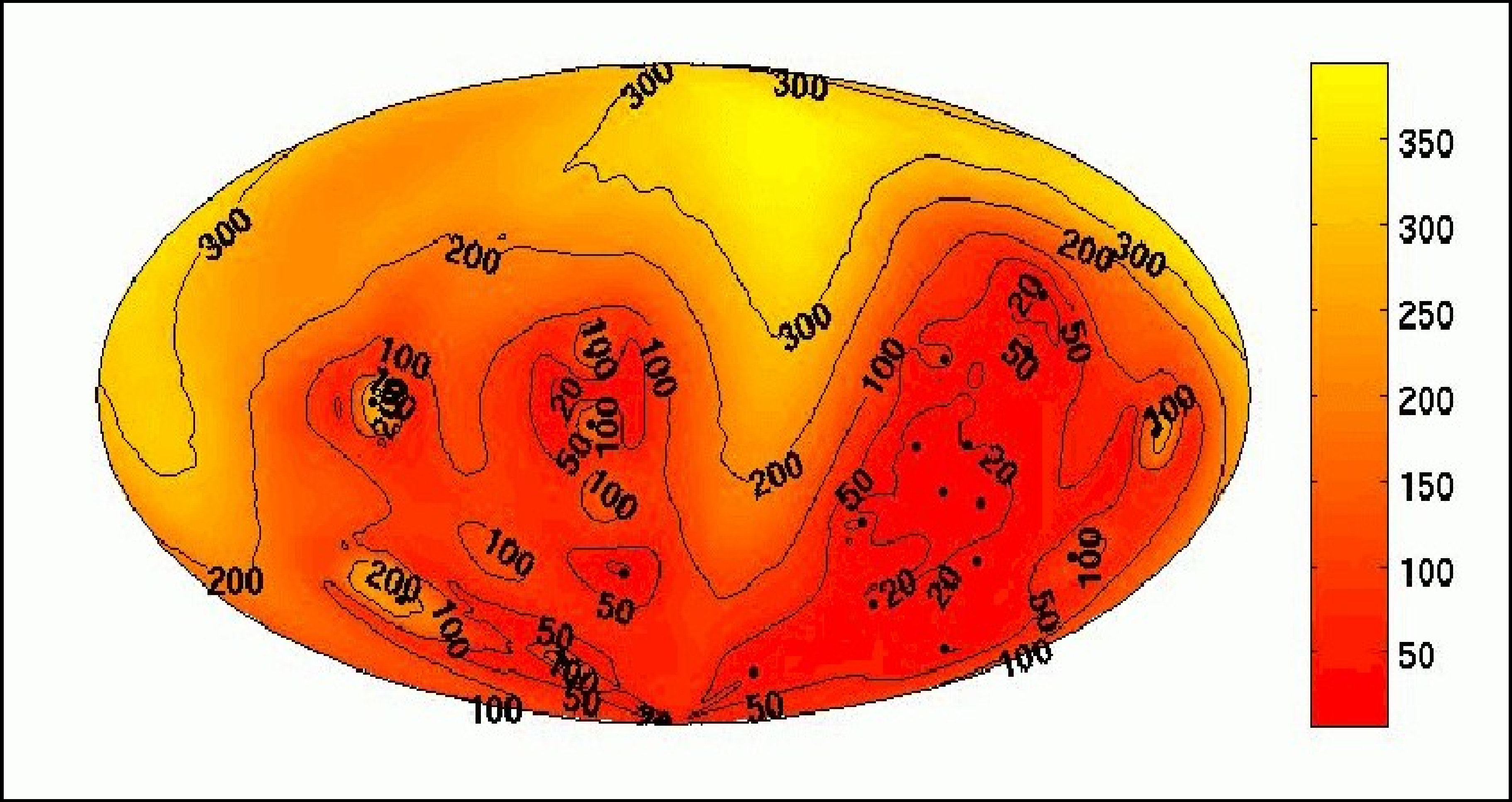
As a function of the sky coverage of the array of pulsars



Anisotropic distribution of pulsars



Sky position accuracy in the Parkes PTA



Summary

> Future PTAs will have the capability of detecting signals from genuine MBHBs

> A timing precision between ***5-50 ns*** should guarantee the detection of at least one individual MBHB

> Resolvable sources will be ***very massive*** ($M>10^8$ solar masses) and ***cosmologically nearby*** ($z<2$)

> Hard to tell the source mass and distance (but pulsar term may help)

> Sources can be locate in the sky within ***40 deg²*** considering an isotropic distribution of ***100 pulsars*** and ***SNR=10***.

Future work

> How beneficial would be to include the pulsar term? Would that be feasible?

> Extend the analysis to eccentric systems

> Electromagnetic counterparts? (break the degeneracy between the chirp mass and the luminosity distance)

> Go beyond the FIM formalism: realistic data sets, signal extraction, MCMC.....etc etc...

The Fisher information matrix formalism

For a given signal which has any functional dependence on N parameters, the FIM is defined as

$$\Gamma_{jk}^{(\alpha)} = (\partial_j r_\alpha | \partial_k r_\alpha)$$

Where the inner product is

$$(u|v) = \frac{2}{S_0} \int_0^T u(t)v(t)dt$$

Considering detection using M pulsars, the total FIM is the sum of the FIMs of each single pulsar

$$\Gamma_{jk} = \sum_{\alpha=1}^{M} \Gamma_{jk}^{(\alpha)}$$

1sigma errors are given by inverting the diagonal elements of the matrix

$$\sigma_k^2 = (\Gamma^{-1})_{kk}$$

Summary

> Future PTAs will detect the unresolved MBHB background

> The background spectrum can be fitted by a ***double power law*** with a break around 10^{-8} Hz

> A timing precision between ***5-50 ns*** should guarantee the detection of at least one individual MBHB

> Efficient ***gas dynamics*** may significantly ***lower the background*** level leaving the statistics of resolvable sources unaffected

> If triplets are common, ***1-100 eccentric bursts*** may be observable in the PTA data stream

> Sources can be locate in the sky within ***40 deg²*** considering an isotropic distribution of ***100 pulsars*** and ***SNR=10***.