Background estimation by time slides in a network of gravitational wave detectors

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Abstract

Time shifting the outputs of Gravitational Wave detectors operating in coincidence is a convenient way to estimate the background in a search for short duration signals. We show that increasing indefinitely the number of time shifts does not provide in fact better estimates. We explicitly derive for 2 and 3 detectors how the false alarm variance saturates with the number of time shifts. We also investigate with different methods the effect of noise non stationarity. We show in particular that for mild non stationarities, time slides larger than the non stationarity time scale can be used.

Problem

- Data from GW detectors are not Gaussian

⇒ Need to measure the background coincidence rate

We place ourselves in a two detector coincident search framework. But results can be extended to more detectors and might be transposed to a coherent GW search.

Notations

FA_1	rate parameter of a Poisson process
$\widetilde{FA}_1$	rate measured in first detector
$\widetilde{FA}(\mathcal{T})$	measured coincidence rate with time shift $\mathcal{T}$
R	the number of time slides

Background estimation limited by non stationarities

- GW detectors have a non stationary trigger rate, e.g. caused by high microseismic activity.
- Problem with time slides with step larger than the non stationarity time scale T_{NS}
  - ⇒ Background estimation looks at a different overlap of glitchy periods than the one in the zero lag
  - ⇒ Limits the number of possible time shift ~ 100
- Extreme case: random circular time shifts on the whole run

⇒ Need to measure error introduced by not taking the non stationarities into account

Measure of rate non uniformity

- Cut data into P blocks of equal length $\Delta T = \frac{T}{P}$.
- Measure mean rate in each block FA_1^i
- Find the first two moments of the binned rate

$$FA_1 = \underset{i\in[\![1,P]\!]}{\text{Mean}}\, FA_1^i \qquad a_1(P) = \underset{i\in[\![1,P]\!]}{\text{Std}}\, FA_1^i / FA_1$$

⇒ Gives a measure of non stationarities with characteristic time scale ΔT

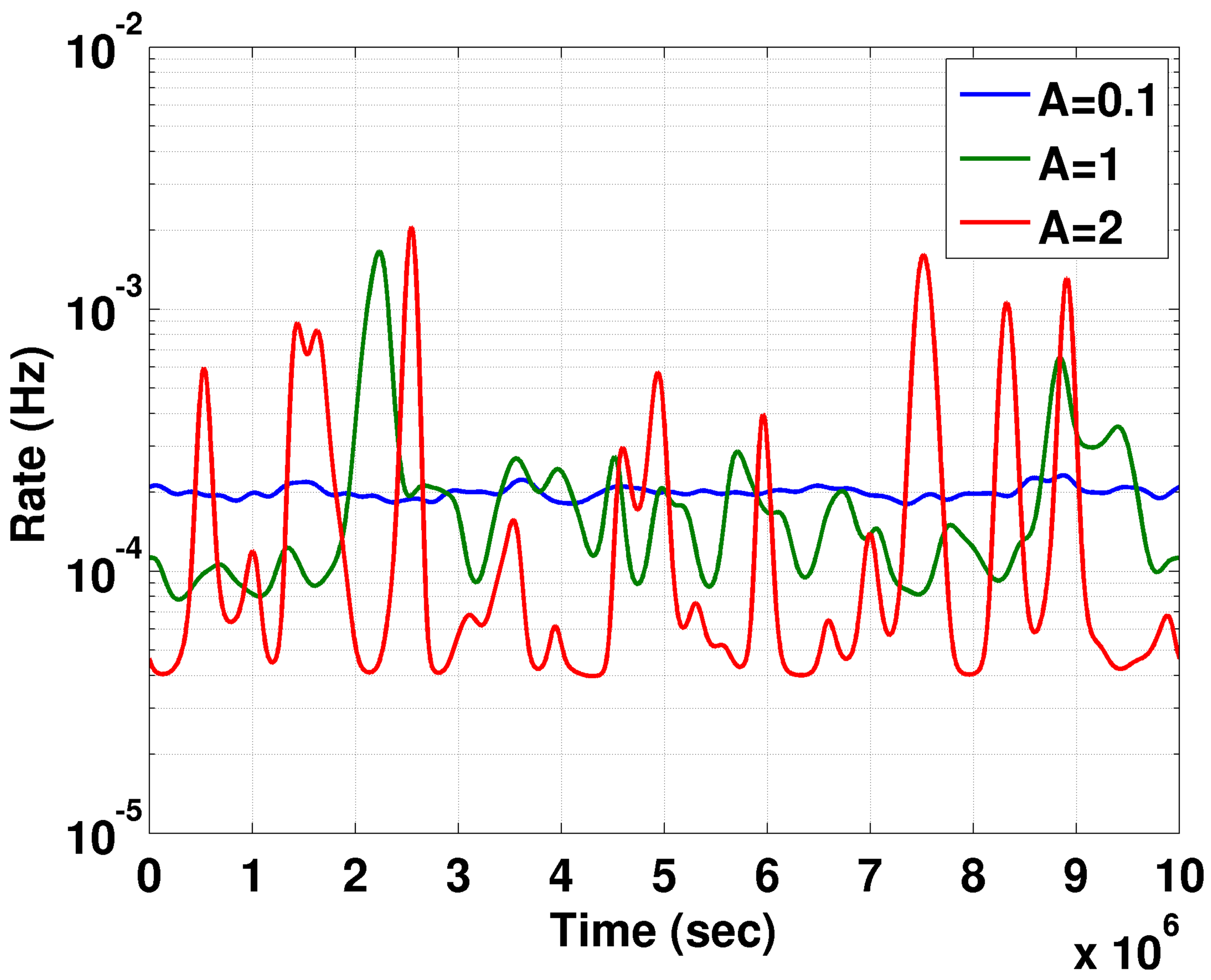
Relative error introduced by not taking into account rate non uniformity

$$\frac{\text{Std}(FA_{\text{Err}})}{FA_1 FA_2 \Delta t} = \sqrt{\frac{1}{P}\left(1+\frac{1}{R}\right) a_1^2(P) a_2^2(P)} \qquad FA_{\text{Err}} = FA(0) - \frac{1}{R}\sum FA(\mathcal{T}_k)$$

Need to be maximized over the possible non stationarity time scales (maximize over P)

A model of non stationarities

In order to test the error prediction, an ad-hoc non stationary rate generator is used. It yields a random rate fluctuations with time scales longer than T_{NS} and a non stationarity amplitude parametrized by A.



Examples of generated rate fluctuations for $T_{NS} = 10^5$ s and mean rate of 2×10^{-4} Hz

[1] Michal Wąs et al.
On the background estimation by time slides in a network of gravitational wave detectors.
Classical and Quantum Gravity, 27(1):015005, 2009.

Background estimation limited by Poisson statistics

- Time slides - an estimator of zero lag background rate

$$\widehat{FA} = \frac{1}{R}\sum_{k=1}^{R} \widetilde{FA}(\mathcal{T}_k) \quad \text{estimator of} \quad \widetilde{FA}(0)$$

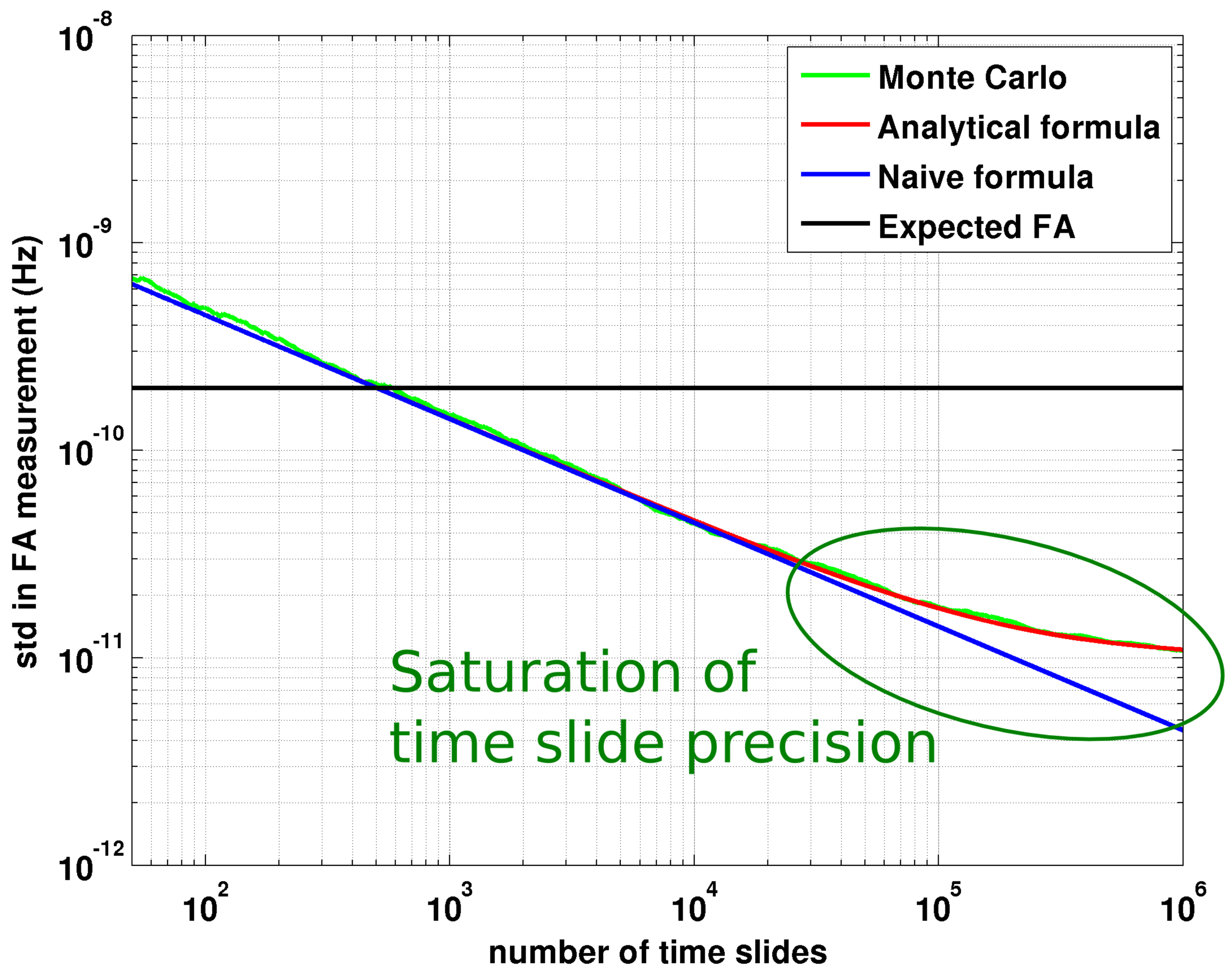
- Estimator properties for coincidence Poisson processes [1]

$$\text{Mean}\, \widehat{FA} = FA_1 FA_2 \Delta t$$

$$\text{Var}\, \widehat{FA} = FA_1 FA_2 \frac{\Delta t}{T}\left[\frac{1}{R} + FA_1 \Delta t + FA_2 \Delta t\right]$$

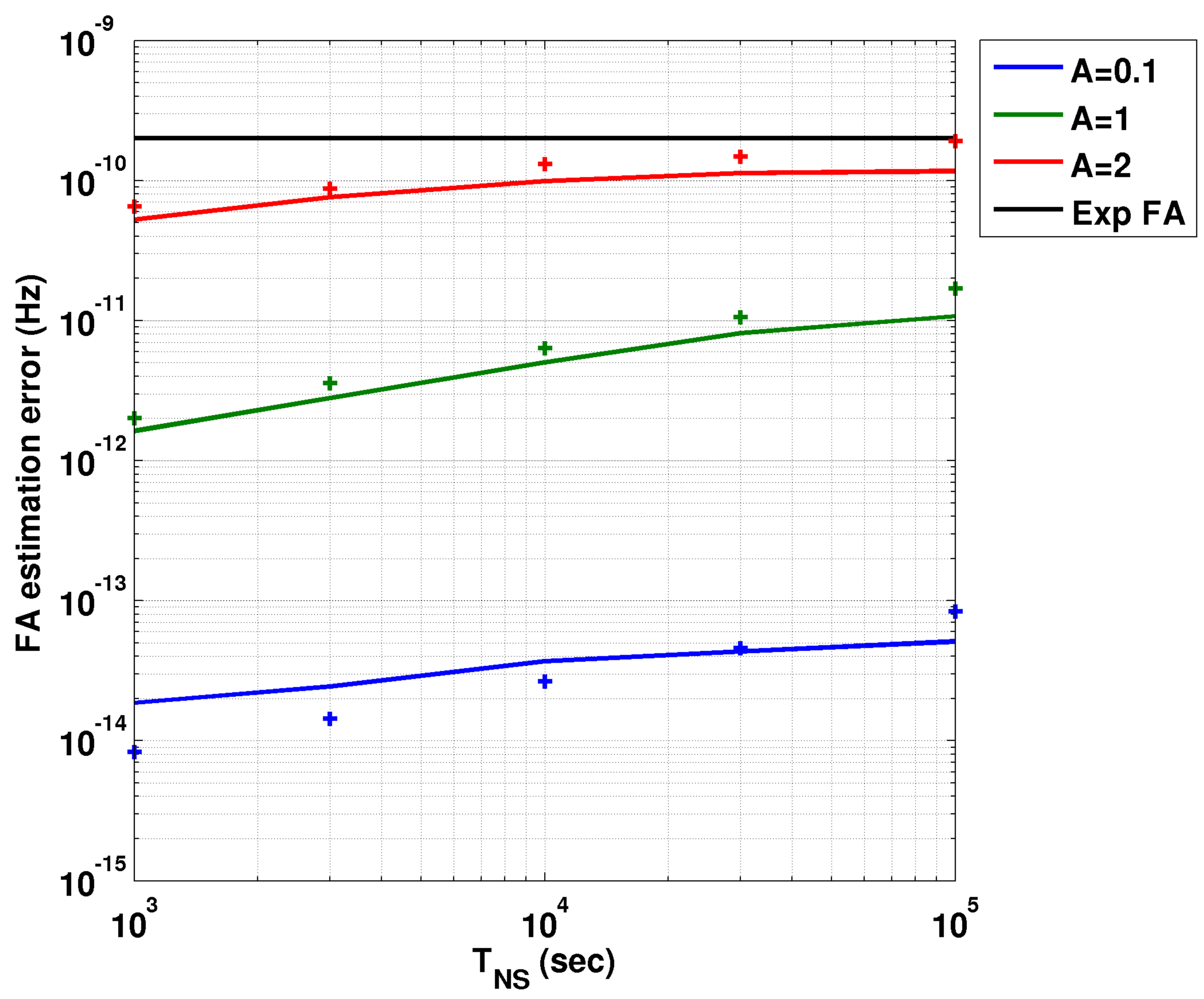
- Extra term comes from single detector rate measurement error

$$\widetilde{FA}_1 = FA_1 \pm \sqrt{\frac{FA_1}{T}} = \frac{N}{T} \pm \frac{\sqrt{N}}{T}$$



All Monte Carlos have been performed with $FA_1 = 2\times10^{-4}$ Hz, $FA_2 = 5\times10^{-5}$ Hz, $\Delta t = 20$ ms,
500 trials and a $T = 10^7$ s long run length. This parameters choice correspond to a typical detection scenario, the background coincidence probability is 10^{-3} per data run.
⇒ At least a few 1000s time slides are required in a 3 sigma detection scenario

Measure of error introduced by random time slides



Error introduced by making large time slides. Prediction (solid line) and Monte Carlo (pluses). Black solid line: expected background coincidence rate.
⇒ For $A \lesssim 2$ (amplitude of rate fluctuations lower than ~ 100), random time slides can be used.

Conclusions

- Background estimation by time slide is central to most transient GW searches, it raises the question of its precision.
  - The precision saturates with the number of time slides.
  - Time slides longer or shorter than non stationarities time scales?
  - Non stationarities can cause large errors in the background estimation
- Limitation for parameters typical of a GW data analysis
  - A few 1000s time slides are needed for background estimation at the 3 sigma level
  - For non stationarities on 20 minutes time scale only a few 100s of time slides of two detectors are possible
  - ⇒ Time slides longer than non stationarity time scale are mandatory
  - If non stationarity amplitude is too large background estimation by time slides is questionable