Measuring the black hole mass function using LISA capture sources

Jonathan Gair, Christopher Tang (IoA, Cambridge) and Marta Volonteri (Michigan)

Introduction

LISA should detect between a few tens and a few hundreds of inspirals of compact objects into supermassive black holes in the centres of galaxies [1], so called extreme-mass-ratio inspirals (EMRIs). LISA will be able to measure the parameters of each system to very high precision [2], but, from an astrophysical point of view, the statistics of these black holes are more important than precise measurements for individual systems. Here we discuss how the set of LISA EMRI events can be used to place constraints on the mass function of black holes.

EMRI detection

EMRIs begin in the Universe at a rate $n(\mathbf{x})\mathcal{R}(\mathbf{x})$ in systems with parameters **x**, where $n(\mathbf{x})$ is the number density of these systems and $\mathcal{R}(\mathbf{x})$ is the intrinsic rate of inspirals in such systems. Only events that are loud enough can be detected, which can be modelled by saying all events with SNR $\rho > \rho_{\rm th} \approx 30$ can be seen but no others. This defines a range of times (the "observable lifetime", $\tau(\mathbf{x})$) during the inspiral at which LISA could start taking data and the source be detected (see Figure 1, right). $\tau(\mathbf{x})$ was calculated for circular and equatorial inspirals in [1].

The start time of an EMRI is random, so the number of events that LISA will detect is drawn from a Poisson process with mean $r(\mathbf{x}) = \tau(\mathbf{x})n(\mathbf{x})\mathcal{R}(\mathbf{x})$.

Our analysis could include plunge time as a parameter in **x**. This eliminates $\tau(\mathbf{x})$, but it is replaced by the completeness of the LISA observation. The observable lifetime prescription assumes 100% completeness for $\rho > \rho_{\rm th}$, and 0% for $\rho < \rho_{\rm th}$. This is a reasonable model if we assume we use an SNR cut for source selection.

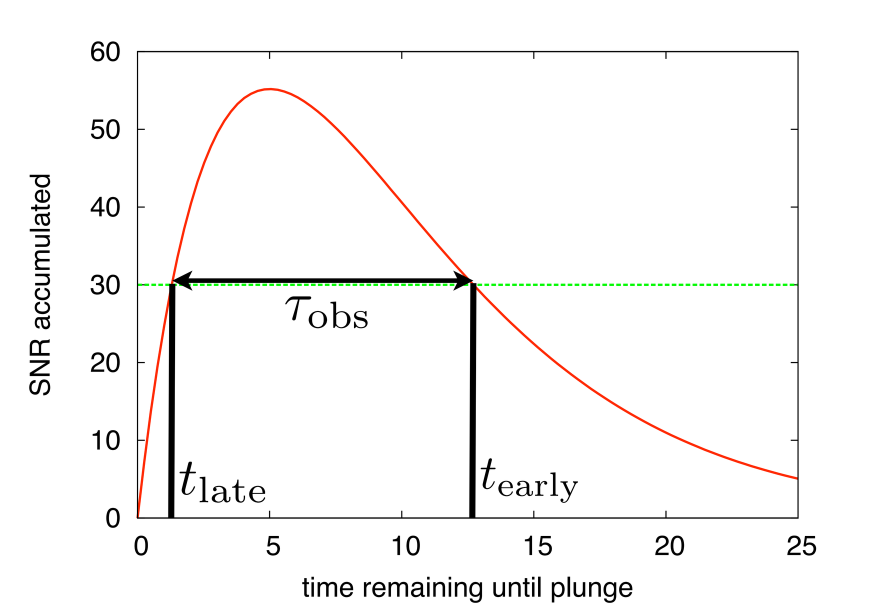

Figure 1. LISA can only detect sources if enough SNR is accumulated over the mission lifetime. If LISA turns on too early in the inspiral, the source is too quiet, while if LISA turns on too late there is insufficient time before plunge to accumulate the needed SNR. The difference between these times defines the "observable lifetime" of a source.

Methodology

For a first analysis, we assumed $\mathcal{R}(\mathbf{x})$ was known [3], that all EMRIs were circular, equatorial inspirals into non-spinning black holes and looked at the distribution of events in black hole mass, M, and redshift, z, only. We binned the observations into bins of M and z so that parameter measurement errors could be ignored. The data, D, is then the number of LISA EMRI events, n_i, in each bin, i; the model prescribes the number density of black holes $n(M, z; \mathbf{y})$ and is dependent on some parameters **y**; and the likelihood is given by a product of Poisson probabilities for each bin

$$p(D|\mathbf{y}, H) = \prod_{i=1}^{K} \frac{(r(M_i, z_i; \mathbf{y})^{n_i} {\rm e}^{-r(M_i,z_i;\mathbf{y})}}{n_i!}$$

Bayes Theorem then gives us the posterior on the model parameters **y**

$$P(\mathbf{y}|D, H) = \frac{p(D|\mathbf{y}, H)p(\mathbf{y}|H)}{p(D|H)}$$

We can explore this posterior using Markov Chain Monte Carlo techniques. This analysis could also be done in a continuum limit, including posterior pdfs for the parameters of each individual event, but the results will be broadly the same.

Current constraints on the black hole mass function

The black hole mass function can be inferred from the observed galaxy luminosity function and the $L - \sigma$ and $M_{\rm bh} - \sigma$ relations [4]. It is well fit by

$$\frac{{\rm d}n}{{\rm d}\log M} = \frac{A(M/M_\odot)^\alpha}{B + (M/M_\odot)^\beta}$$

The data is poorly constrained in the LISA range, $M_{\rm bh} < 10^7 M_\odot$, but a single power-law is likely a good approximation.

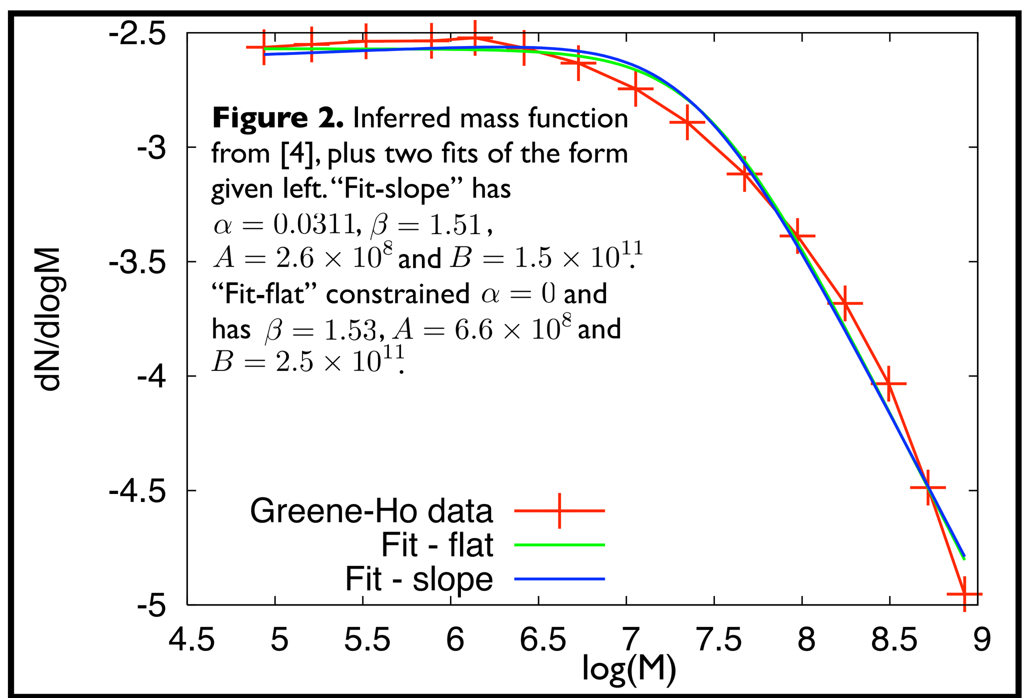

Figure 2. Inferred mass function from [4], plus two fits of the form given left. "Fit-slope" has $\alpha = 0.0311$, $\beta = 1.51$, $A = 2.6 \times 10^8$ and $B = 1.5 \times 10^{11}$. "Fit-flat" constrained $\alpha = 0$ and has $\beta = 1.53$, $A = 6.6 \times 10^8$ and $B = 2.5 \times 10^{11}$.

References

[1] Gair, J R, Class. Quantum Grav. **26** 094034 (2009), arxiv:0811.0188
[2] Barack, L, & Cutler, C, Phys. Rev. D**69** 082005 (2004), arxiv:gr-qc/0310125
[3] Hopman, C, Class. Quantum Grav. **26** 094028 (2009), arxiv:0901.1667
[4] Greene, J E, & Ho, L C, Astrophys. J. **698** 198 (2009), arxiv:0705.0020

Results - non-evolving mass function

We took the black hole mass function to have a simple power law form in black hole mass and to be independent of redshift. Such a model has two parameters - an amplitude and a slope

$$\frac{{\rm d}n}{{\rm d}\log M} = A_0 \left(\frac{M}{3 \times 10^6 M_\odot}\right)^{\alpha_0}$$

Typical posterior distributions for these parameters, obtained by MCMC, are shown below. The posterior pdfs for $\ln(A_0)$ and α_0 are both well fit by Gaussians.

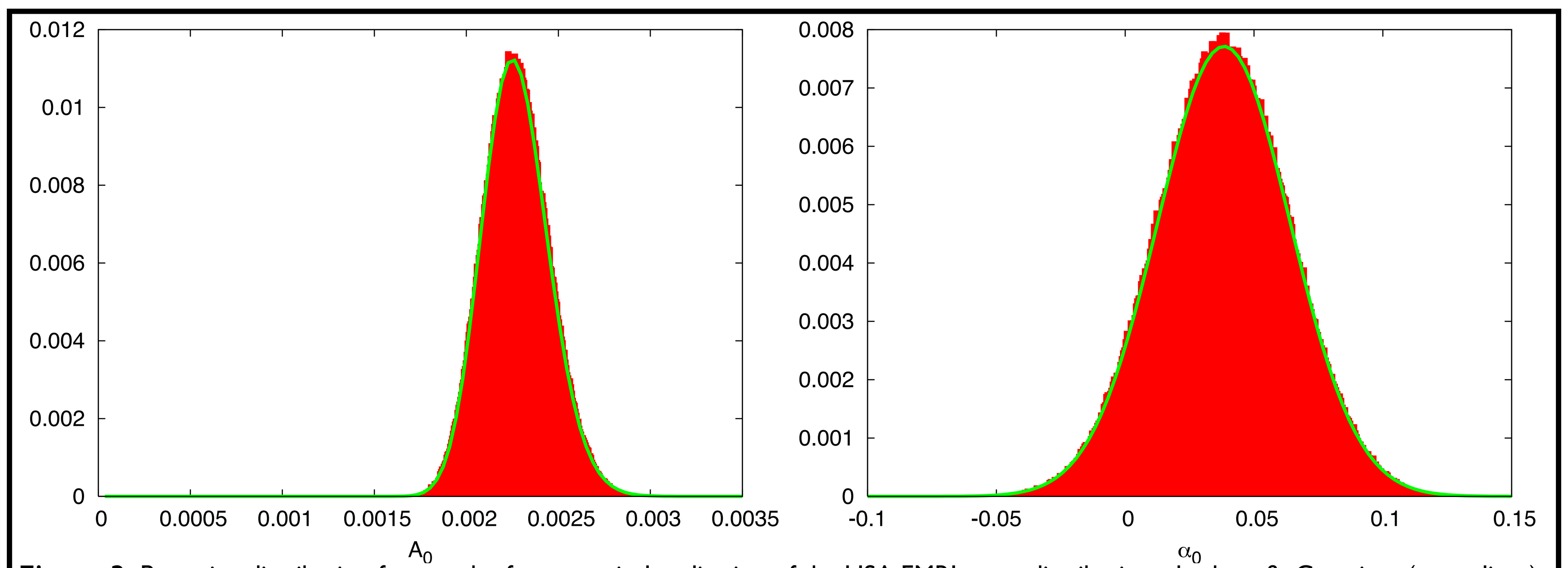

Figure 3. Posterior distribution for A_0 and α_0 from a typical realisation of the LISA EMRI event distribution, plus best-fit Gaussians (green lines). The observed set of EMRI events was generated from the distribution with $A_0 = 0.002{\rm Mpc}^{-3}$ and $\alpha_0 = 0$.

The width of the Gaussian characterizes the error we would expect to make in measuring the parameters using LISA EMRI events. The plots below show how these errors vary with the true values of A_0 and α_0 of the Universe.

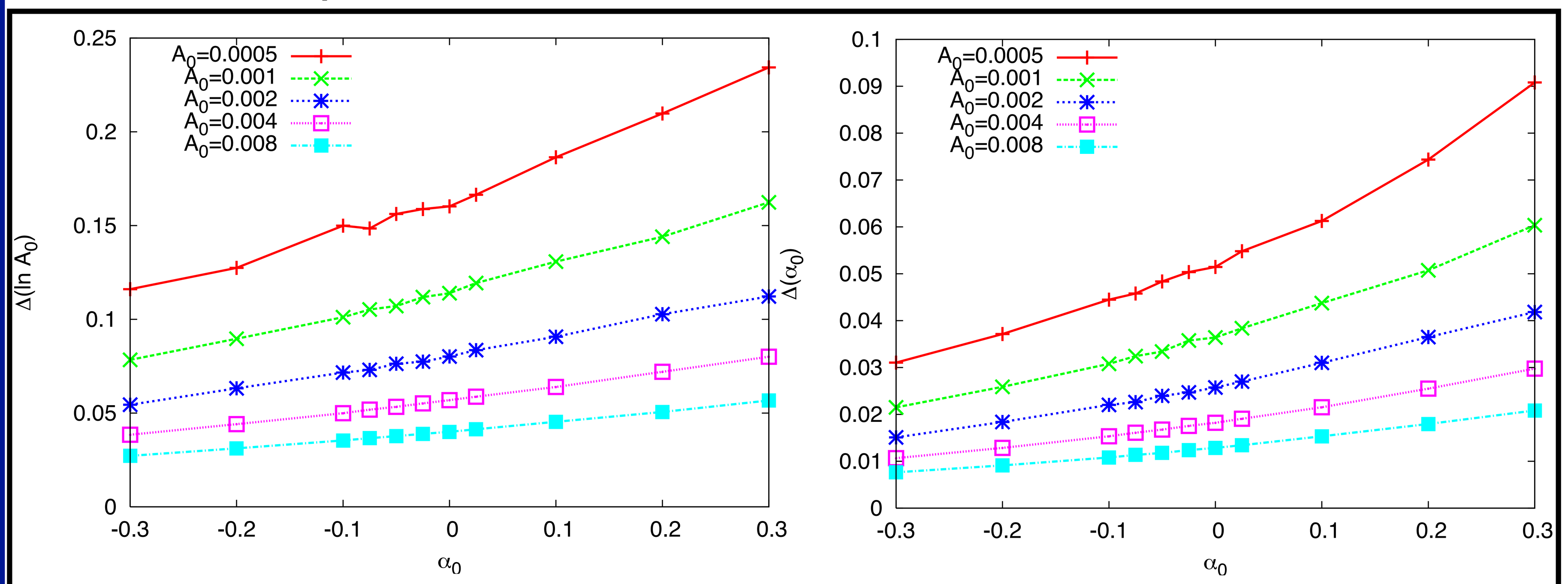

Figure 4. Dependence of the typical error in a measurement of A_0 (left) and α_0 (right), on the true values of α_0 (x-axis) and A_0 (different lines).

LISA EMRI events should be able to measure the amplitude of the mass function to a precision $\Delta(\ln A_0) \sim 0.025 - 0.25$ and the slope to $\Delta(\alpha_0) \sim 0.01 - 0.1$.

Results - redshift-dependent mass function

We can include redshift evolution in the mass-function with the ansatz

$$\frac{{\rm d}n}{{\rm d}\log M} = A_0(1+z)^{A_1} \left(\frac{M}{3 \times 10^6 M_\odot}\right)^{\alpha_0 - \alpha_1 z}$$

The posterior pdfs for the four parameters are shown in Figure 5 for a realisation with $A_0 = 0.002{\rm Mpc}^{-3}$ and $\alpha_0 = A_1 = \alpha_1 = 0$.

The width of the pdfs for A_1 and α_1 indicate that the black hole mass-function would have to change in amplitude by a factor of ~ 2 or in slope by $\Delta\alpha \sim 0.3$ out to redshift $z \approx 1$ for LISA EMRIs to detect that there is a redshift dependence.

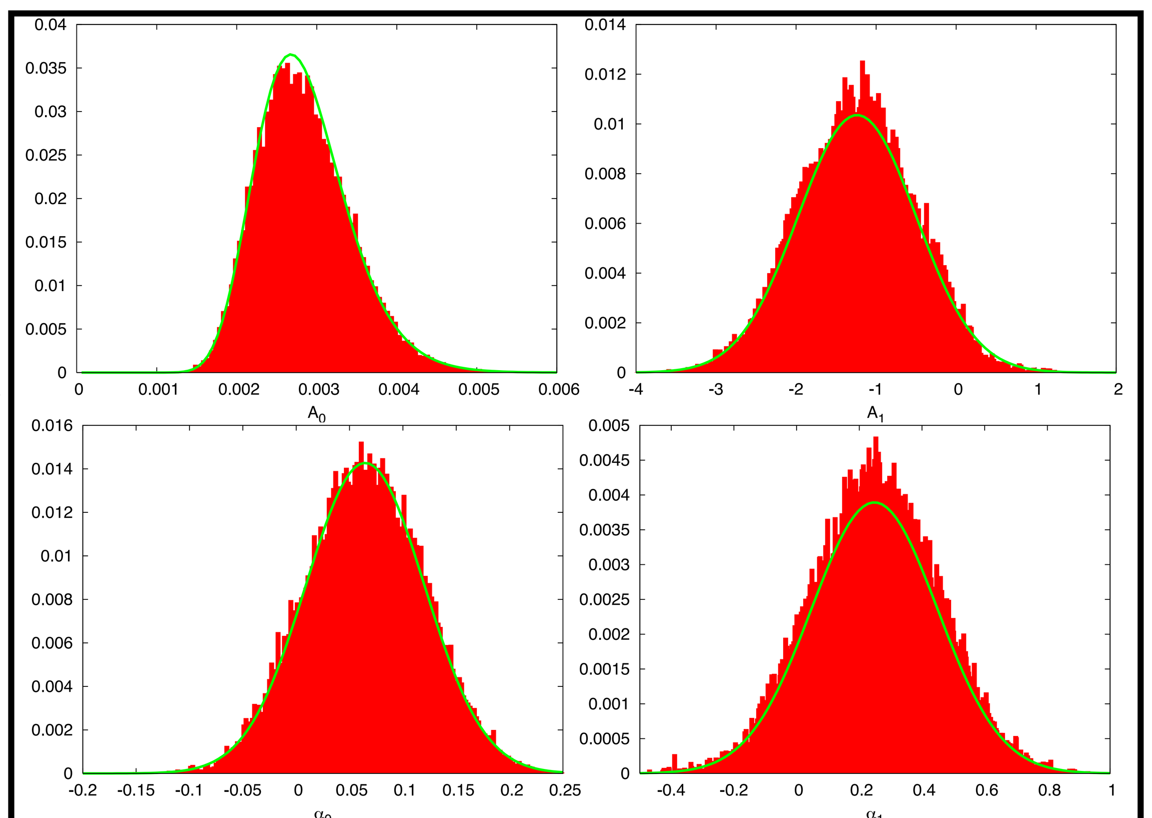

Figure 5. Typical posterior pdfs and best-fit Gaussians for the four parameters of the redshift dependent model.

Summary and Future Work

These results indicate that LISA EMRI events can be used to measure the slope of the black-hole mass function at low redshift to a precision $\Delta(\alpha_0) \lesssim 0.1$, and the normalisation to a precision of $\sim 10\%$, but we are unlikely to be able to detect an evolution in the mass function. Future problems to address include:

- What can we learn from combined observations, e.g., using LISA supermassive black hole mergers or data from electromagnetic telescopes?
- What can LISA events tell us about black hole spins and processes operating in stellar clusters, e.g., mass segregation and EMRI capture mechanisms?
- Can LISA observations determine both $n(\mathbf{x})$ and $\mathcal{R}(\mathbf{x})$ or just $n(\mathbf{x})\mathcal{R}(\mathbf{x})$?
- Extend these results to EMRIs on generic, i.e., eccentric and inclined, orbits.