Compact binary waveform recovery from the cross-correlated data of two detectors by matched filtering with spinning templates

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Method

Pattern recognition is a difficult problem in gravitational astrophysics. Here we attempt to recover a gravitational signal h_{inj} immersed in S5 LIGO-like noise by a matched filtering. Templates were generated with spinning waveforms [1]. The matched filtering follows the method described in Ref. [2]. Namely, we calculate the overlap between the noisy injection $h_{n,i}$ and spinning templates $h_{template}$:

$$O\left[h_{n,i},h_{template}\right]=\frac{\langle h_{n,i}|h_{template}\rangle}{\sqrt{\langle h_{n,i}|h_{n,i}\rangle\langle h_{template}|h_{template}\rangle}}\ ,$$

$$\langle h_1|h_2\rangle=4Re\int_{f_{\min}}^{f_{\max}}\frac{\tilde{h}_1\left(f\right)\tilde{h}_2^*\left(f\right)}{S_n\left(f\right)}df\ ,$$

where tilde and star denote Fourier transforms and complex conjugates, respectively, and $S_n(f)$ is the power spectral density of the noise. We choose f_{min}=50Hz and f_{max}=600Hz, lying in the best sensitivity band of the LIGO detectors, also the post-Newtonian prediction for the waveform is quite accurate there.

On theoretical grounds, infinite long data series would be required for exact determination of the power spectrum. In order to achive stability of the power spectral density of order of 1%, we would like to have at least 100 periods of the lowest frequency, therefore a minimal length of the templates of 100x(1/50Hz)=2 sec was imposed.

We compute:
- the above overlap both for the Hanford and Livingston detectors (O(H) and O(L), respectively)
- the correlated match: the overlap ($O_{corr}(H,L)$) using the correlated quantities of the Hanford and Livingstone signals for $h_{n,i}$, $h_{template}$ and noises. The correlation of h_1 and h_2 in Fourier space is

$$\widetilde{H}\left(f\right)=\widetilde{h}_1\left(f\right)\widetilde{h}_2^*\left(f\right)$$

- the following quantity for the two detectors, according to the method implemented in **SpinSpiral** [3]:

$$O\left(H,L\right)=\frac{\langle h_{n,i}|h_{template}\rangle_H+\langle h_{n,i}|h_{template}\rangle_L}{\sqrt{\left(\langle h_{n,i}|h_{n,i}\rangle_H+\langle h_{n,i}|h_{n,i}\rangle_L\right)\left(\langle h_{template}|h_{template}\rangle_H+\langle h_{template}|h_{template}\rangle_L\right)}}\ ,$$

The spinning waveforms are based on the **SpinTaylor** [1], based on [4]. Antenna functions were computed with the **XLALComputeDetAMResponse()** function in **DetResponse.c** under the **LAL package** [1] and based on [5].

[1] https://www.lsc-group.phys.uwm.edu/daswg/projects/lal.html
[2] B. Vaishnav, I. Hinder, F. Herrmann, D. Shoemaker, Phys. Rev. D 76 084020 (2007)
[3] V. Raymond, M. V. van der Sluys, I. Mandel, V. Kalogera, C. Roever, N. Christensen, arXiv:0912.3746 (2009)
[4] A. Buonanno, Y. Chen, M. Vallisneri, Phys. Rev. D 67 104025 (2003); Erratum-ibid. D 74 029904 (2006)
[5] W. G. Anderson, P. R. Brady, J. D. E. Creighton, É. É. Flanagan, Phys. Rev. D 63 042003 (2001)

The injected waveforms

Fig1: Spinning waveforms h_+ (left) and h_x (right)

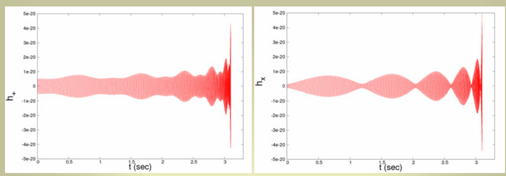


Table 1: The parameters of the injected signal (h_+, h_x): masses m_i, magnitudes of the dimensionless spins χ_i, initial direction of the spin vectors in the frame assigned by the orbital angular momentum (L_N) (Fig. 4 in [4]) given by $\cos\kappa_i$ and ψ_i, initial polar and azimuthal angles α and φ_L of L_N in the source frame assigned by the total angular momentum (J) (Fig. 1 in [4]), the angle Θ between the direction of the detector and J, and the distance d_L of the source.

name	$m_1(M_\odot)$	$m_2(M_\odot)$	χ_1	χ_2	$\cos\kappa_1$	ψ_1	$\cos\kappa_2$	ψ_2	α	φ_L	Θ	$d_L(M_\odot)$
injection	3.553	3.358	0.983	0.902	0.984	1.109	0.978	0.957	1.116	2.833	1.430	1.000

Fig 2: Spinning waveforms at Hanford (top left) and Livingston (top right), mixed in LIGO S5-like noise (bottom)

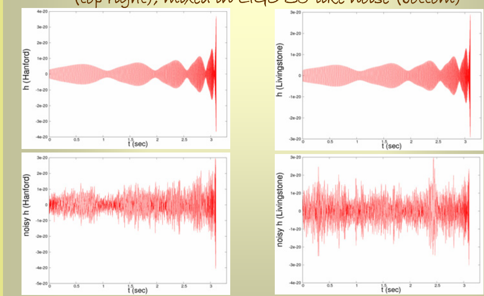
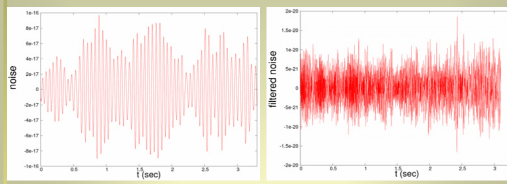


Table 2: The parameters of the antenna functions (F_+, F_x). Here θ, φ and ψ (polarization angle) give the relation between the detector and the source frame.

name	φ	θ	ψ
injection	3.657	0.278	0.000

Detector noise

Fig 3: The characteristic unfiltered (left) and filtered (right) noise



Assumptions in the signal search

- stellar mass black holes in the mass range of 3-10 $M_\odot$
- both with high spins, the parameters χ_1 and χ_2 in the range: **0.7-1**
- $\cos\kappa_i$ and ψ_i generated randomly
- distance (d_L), angles α, φ_L, Θ and θ, φ, ψ fixed to the values given in Table 1 and 2, respectively
- time when signal starts assumed known

Questions to answer

- Can rapidly spinning systems be reliably reconstructed with currently employed spinning waveforms?
- Can the spin parameters be estimated with currently employed spinning templates?

Results

The injection and the best templates from the correlation

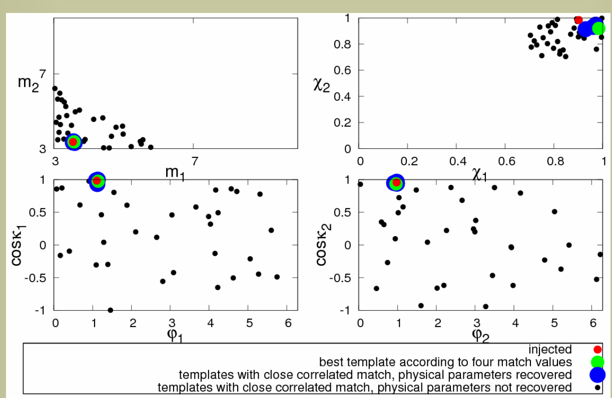


Fig 4:

- injected signal (red)
- best template from the correlated match, other three matches reasonable, and physical parameters recovered (green)
- templates with close correlated match, other three matches poor, and physical parameters recovered (blue)
- templates with close correlated match, other three matches poor, and physical parameters not recovered (black)

Table 3: Match values and parameters for the green and blue templates of Fig 4

name	$O_{corr}(H,L)$	$O(H)$	$O(L)$	$O(H,L)$	$m_1(M_\odot)$	$m_2(M_\odot)$	χ_1	χ_2	$\cos\kappa_1$	ψ_1	$\cos\kappa_2$	ψ_2
injection	0.937	0.912	0.874	0.897	3.553	3.358	0.983	0.902	0.984	1.109	0.978	0.957
green	0.923	0.702	0.632	0.674	4.805	3.336	0.985	0.919	0.974	1.143	0.935	0.963
blue 1	0.910	−0.007	0.052	0.017	3.598	3.304	0.972	0.948	0.933	1.132	0.948	0.982
blue 2	0.924	0.209	0.367	0.271	3.547	3.326	0.961	0.931	0.998	1.132	0.938	0.982
blue 3	0.928	0.219	0.188	0.207	3.574	3.341	0.973	0.931	0.997	1.131	0.968	0.981
blue 4	0.896	0.026	−0.032	0.003	3.561	3.392	0.930	0.913	0.990	1.090	0.942	0.986

Fig 5: Best template (green) of Fig 4 at Hanford (top) and Livingston (bottom)

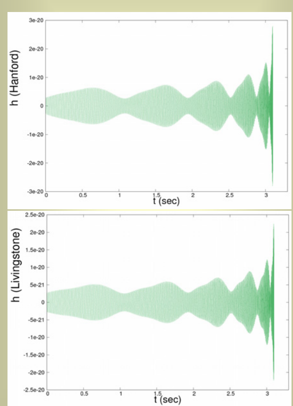
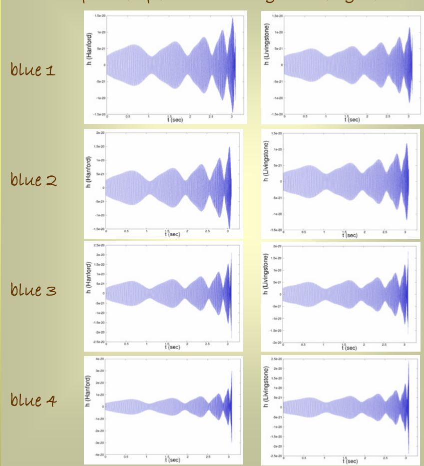


Fig 6: The blue templates of Fig 4 at Hanford (left) and Livingston (right)



Conclusion

1. The templates with high values of the correlated match contain a subset of templates with all varied parameters well recovered!!!
   → Improvement over 1-detector match!!!
2. One of these could be picked up on the basis of reasonably high values of the other three matches O(H), O(L), and O(H,L).
   → Selection criteria

Work supported by the Polányi Program of the Hungarian National Office for Research and Technology (NKTH) and the Hungarian Scientific Research Fund (OTKA) grant 69036.