



# Compact binary waveform recovery from the cross-correlated data of two detectors by matched filtering with spinning templates

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## Method

Pattern recognition is a difficult problem in gravitational astrophysics. Here we attempt to recover a gravitational signal  $h_{inj}$  immersed in SS LIGO-like noise by a matched filtering. Templates were generated with spinning waveforms [1]. The matched filtering follows the method described in ref. [2]. Namely, we calculate the overlap between the noisy injection  $h_{n,i}$  and spinning templates  $h_{template}$ :

$$O(h_{n,i}, h_{template}) = \frac{(h_{n,i}|h_{template})}{\sqrt{(h_{n,i}|h_{n,i})(h_{template}|h_{template})}}$$
$$(h_1|h_2) = 4\text{Re} \int_{f_{min}}^{f_{max}} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$$

where tilde and star denote Fourier transforms and complex conjugates, respectively, and  $S_n(f)$  is the power spectral density of the noise. We choose  $f_{min}=50$ Hz and  $f_{max}=600$ Hz, lying in the best sensitivity band of the LIGO detectors, also the post-Newtonian prediction for the waveform is quite accurate there.

On theoretical grounds, infinite long data series would be required for exact determination of the power spectrum. In order to achieve stability of the power spectral density of order of 1%, we would like to have at least 100 periods of the lowest frequency, therefore a minimal length of the templates of  $100 \times (1/50\text{Hz}) = 2$  sec was imposed.

We compute:

- > the above overlap both for the Hanford and Livingston detectors ( $O(H)$  and  $O(L)$ , respectively)
- > the correlated match: the overlap ( $O_{corr}(H,L)$ ) using the correlated quantities of the Hanford and Livingston signals for  $h_{n,i}$ ,  $h_{template}$  and noises. The correlation of  $h_1$  and  $h_2$  in Fourier space is

$$\tilde{h}(f) = \tilde{h}_1(f)\tilde{h}_2^*(f)$$

- > the following quantity for the two detectors, according to the method implemented in Spinspiral [3]:

$$O(H,L) = \frac{(h_{n,i}|h_{template})_H + (h_{n,i}|h_{template})_L}{\sqrt{((h_{n,i}|h_{n,i})_H + (h_{n,i}|h_{n,i})_L)((h_{template}|h_{template})_H + (h_{template}|h_{template})_L)}}$$

The spinning waveforms are based on the spinTaylor [4], based on [4]. Antenna functions were computed with the XLALComputeDetAMResponse() function in DetResponse.c under the LAL package [1] and based on [5].

[1] <https://www.lsc-group.phys.uwm.edu/daswg/projects/lal.html>  
[2] B. Vaishnav, I. Hinder, F. Herrmann, D. Shoemaker, Phys. Rev. D 76 084020 (2007)  
[3] V. Raymond, M. V. van der Sluis, I. Mandel, V. Kalogera, C. Roever, N. Christensen, arXiv:0912.3746 (2009)  
[4] A. Buonanno, Y. Chen, M. Vallisneri, Phys. Rev. D 67 104025 (2003); Erratum-ibid. D 74 029904 (2006)  
[5] W. G. Anderson, P. R. Brady, J. D. E. Creighton, E. Flanagan, Phys. Rev. D 63 042003 (2001)

## The injected waveforms

Fig 1: Spinning waveforms  $h_+$  (left) and  $h_x$  (right)

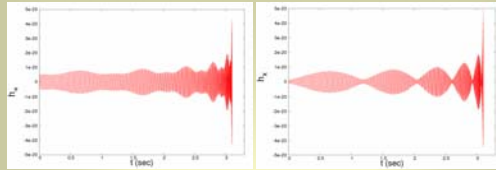


Table 1: The parameters of the injected signal ( $h_+$ ,  $h_x$ ): masses  $m_i$ , magnitudes of the dimensionless spins  $\chi_i$ , initial direction of the spin vectors given by  $\cos\kappa_i$  and  $\psi_i$  in the frame with the line of sight on the z-axis, and the initial orbital angular momentum ( $L_x$ ) in the x-z plane, spanning the angle  $\Theta$ , and the distance  $d_L$  of the source. The initial phase is 0; the initial time being also fixed to 0.

name	$m_1(M_\odot)$	$m_2(M_\odot)$	$\chi_1$	$\chi_2$	$\cos\kappa_1$	$\psi_1$	$\cos\kappa_2$	$\psi_2$	$\Theta$	$d_L(M_{pc})$
injection	3.553	3.358	0.983	0.902	0.984	1.109	0.978	0.957	1.430	1.000

Fig 2: Spinning waveforms at Hanford (top left) and Livingston (top right), mixed in LIGO SS-like noise (bottom)

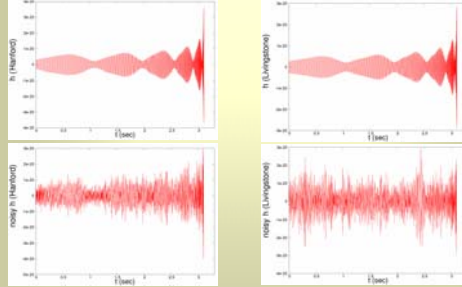
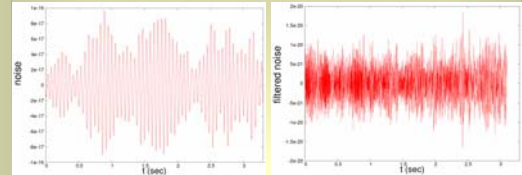


Table 2: The parameters of the antenna functions ( $F_+$ ,  $F_x$ ). Here  $\beta$ ,  $\varphi$  and  $\psi$  (polarization angle) give the relation between the detector and the source frame.

name	$\beta$	$\varphi$	$\psi$
injection	3.657	0.278	0.000

## Detector noise

Fig 3: The characteristic unfiltered (left) and filtered (right) noise



## Assumptions in the signal search

- > stellar mass black holes in the mass range of 3-10  $M_\odot$
- > both with high spins, the parameters  $\chi_1$  and  $\chi_2$  in the range: 0.7-1
- >  $\cos\kappa_i$  and  $\psi_i$  generated randomly
- > distance ( $d_L$ ), angles  $\alpha$ ,  $\varphi_1$ ,  $\Theta$  and  $\beta$ ,  $\varphi$ ,  $\psi$  fixed to the values given in Table 1 and 2, respectively
- > time when signal starts assumed known

## Questions to answer

- > Can rapidly spinning systems be reliably reconstructed with currently employed spinning waveforms?
- > Can the spin parameters be estimated with currently employed spinning templates?

## Results

### The injection and the best templates from the correlation

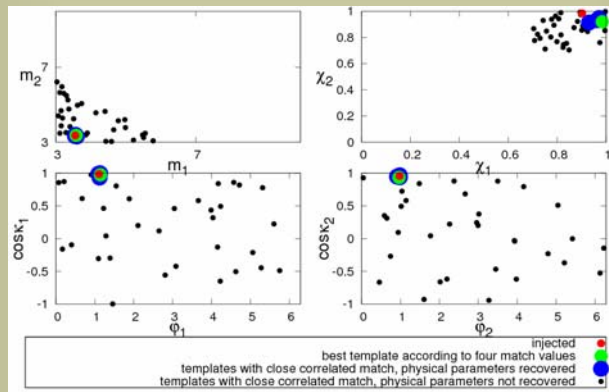


Fig 4:

- injected signal (red)
- best template from the correlated match, other three matches reasonable, and physical parameters recovered (green)
- templates with close correlated match, other three matches poor, and physical parameters recovered (blue)
- templates with close correlated match, other three matches poor, and physical parameters not recovered (black)

Fig 5: Best template (green) of Fig 4 at Hanford (top) and Livingston (bottom)

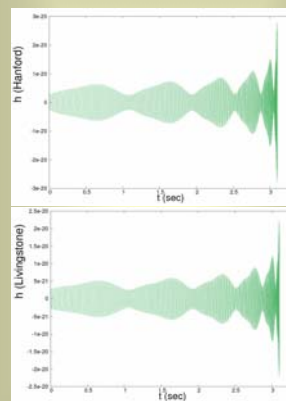
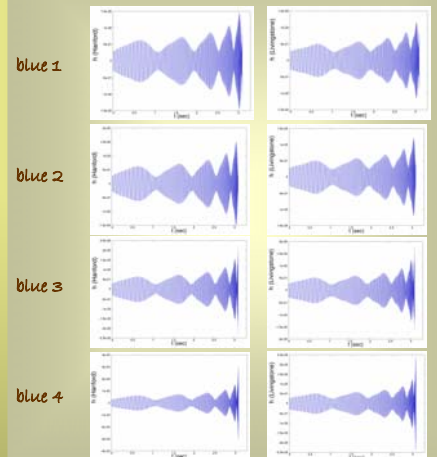


Fig 6: The blue templates of Fig 4 at Hanford (left) and Livingston (right)



## Conclusion

1. The templates with high values of the correlated match contain a subset of templates with all varied parameters well recovered!!!  
→ Improvement over 1-detector match!!!
2. One of these could be picked up on the basis of reasonably high values of the other three matches  $O(H)$ ,  $O(L)$ , and  $O(H,L)$ .  
→ Selection criteria

Table 3: Match values and parameters for the green and blue templates of Fig 4

name	$O_{corr}(H,L)$	$O(H)$	$O(L)$	$O(H,L)$	$m_1(M_\odot)$	$m_2(M_\odot)$	$\chi_1$	$\chi_2$	$\cos\kappa_1$	$\psi_1$	$\cos\kappa_2$	$\psi_2$
injection	0.937	0.912	0.874	0.897	3.553	3.358	0.983	0.902	0.984	1.109	0.978	0.957
green	0.923	0.702	0.632	0.674	4.805	3.336	0.985	0.919	0.974	1.143	0.935	0.963
blue 1	0.910	-0.007	0.052	0.017	3.598	3.304	0.972	0.948	0.933	1.132	0.948	0.982
blue 2	0.924	0.209	0.367	0.271	3.547	3.326	0.961	0.931	0.998	1.132	0.938	0.982
blue 3	0.928	0.219	0.188	0.207	3.574	3.341	0.973	0.931	0.997	1.131	0.968	0.981
blue 4	0.896	0.026	-0.032	0.003	3.561	3.392	0.930	0.913	0.990	1.090	0.942	0.986