UNIVERSITY OF
BIRMINGHAM

EGO

# Thermal noise issues of the Advanced Virgo core optics design

**Stefan Hild**
*University of Birmingham*

ET/ADV thermal noise workshop, Rome, Febuary 2009

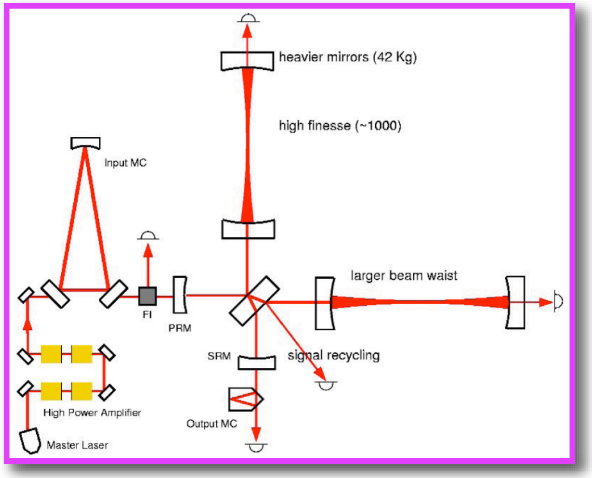
# Overview

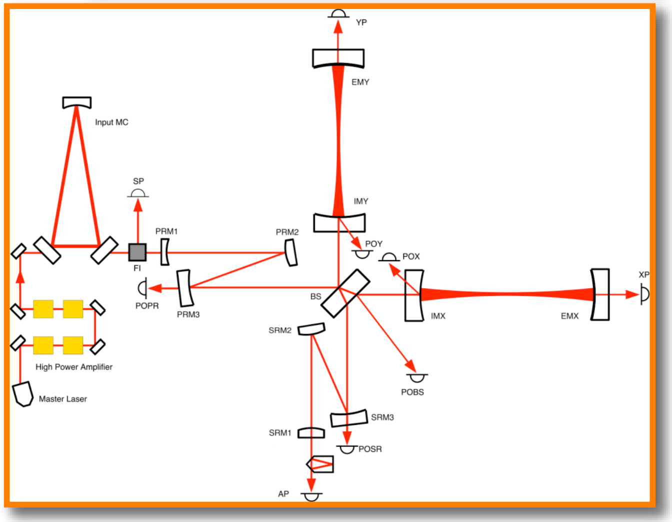
- **Motivation of accurate thermal noise models for Advanced Virgo:**
  - Required as input for important design decisions
  - Required as input for sensitivity optimization
- **Current status of the Advanced Virgo GWINC model:**
  - Documentation, availability, history…
  - Coating Brownian
    - Current implementation
    - Beam sizes and asymmetric ROCs
    - Test mass shape?
  - Substrate Brownian
  - Thermo-optic
- **Examples of recent investigations**
  - Thermal noise issues in case of non-degenerate recycling cavities.
  - Sensitivity improvement from application of LG modes
- **My personal wish list for the future**

# Motivation: Design decisions (I)

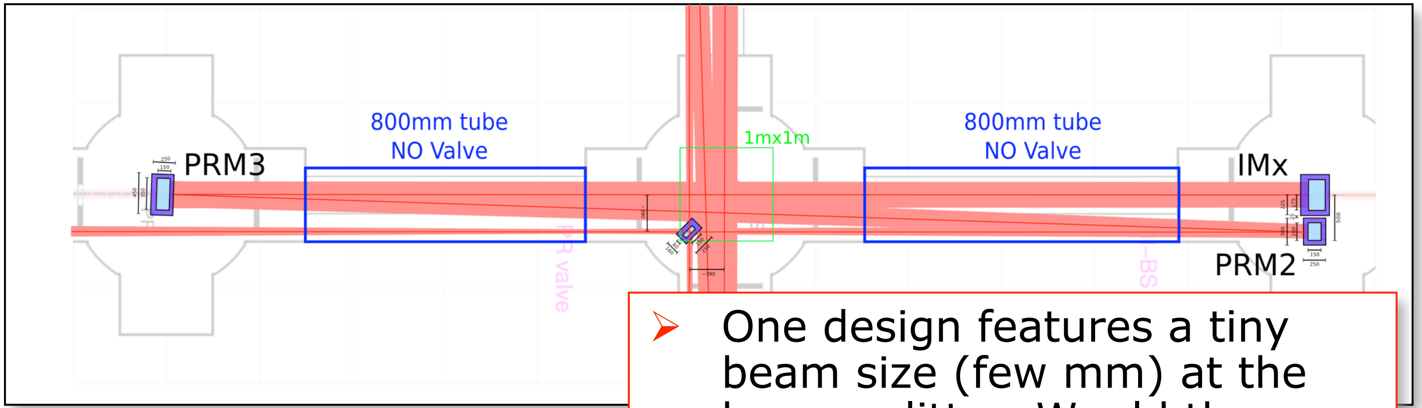
- Currently two different choices for the recycling cavity design are discussed:
  - Marginally stable (like initial Virgo)
  - Non-degenerate (like Advanced LIGO)

- Before decision can be taken a myriad of questions need to be answered:
  - Some of these are related to thermal noise!



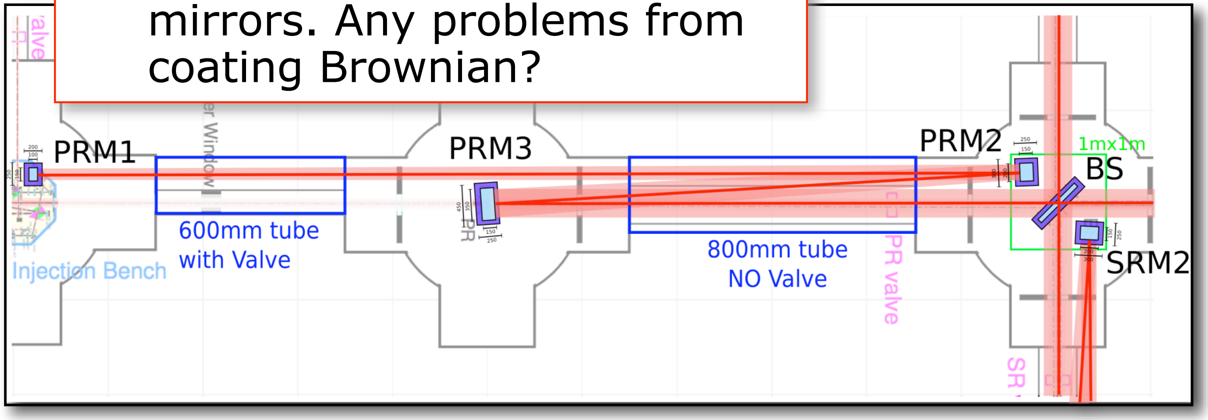


# Motivation: Design decisions (II)



- One design features a tiny beam size (few mm) at the beam splitter. Would thermo-refractive noise be problem?

- Other designs consider 1mm beams on the recycling mirrors. Any problems from coating Brownian?



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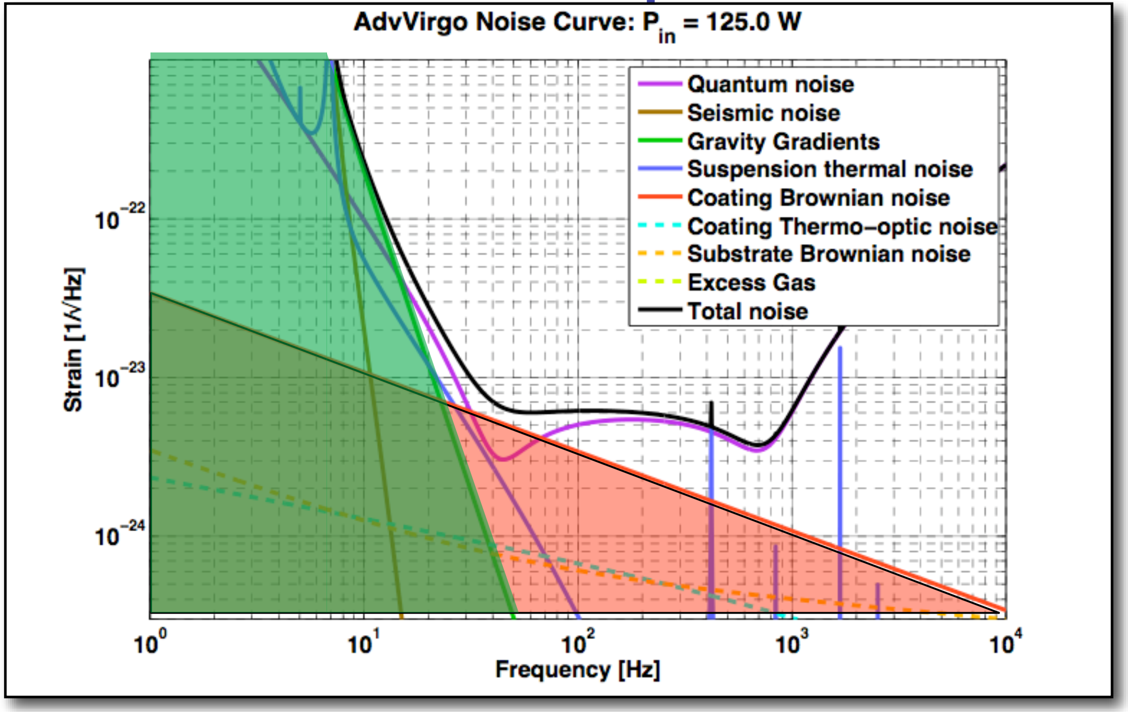
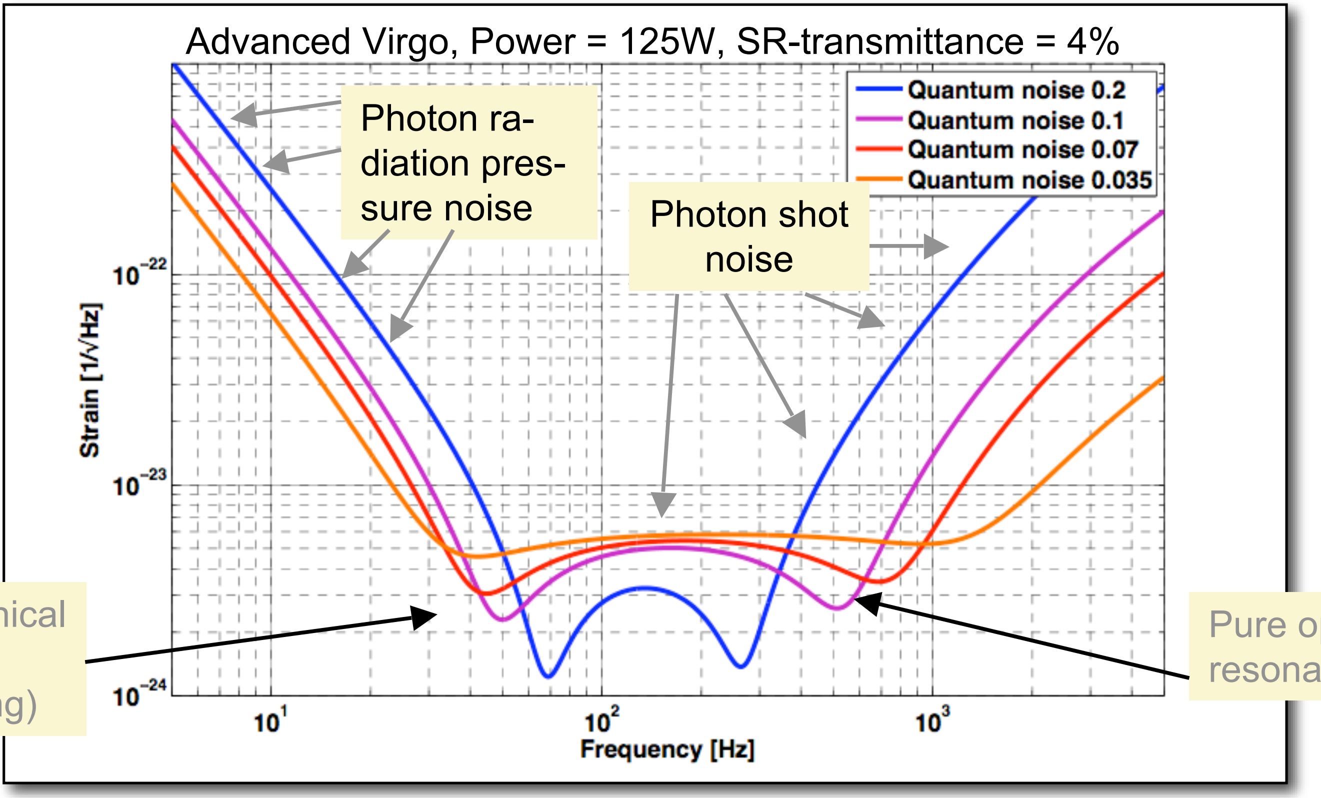
# Limits of the optimization



- Our optimisation is limited by Coating thermal noise and Gravity Gradient noise … what's about suspension thermal noise (see talks by Michele, Paola)?
- **Quantum noise to be optimised!**
- We have three knobs available for this optimisation: 1) Optical power, 2) Signal recycling tuning, 3) Signal Recycling trans-mittance

# Optimization Parameter 1: Signal-Recycling (de)tuning



Photon radiation pressure noise

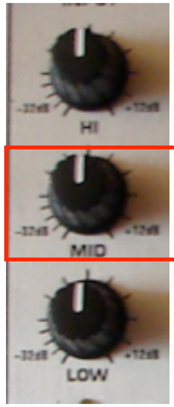
Photon shot noise

Opto-mechanical Resonance (Optical spring)

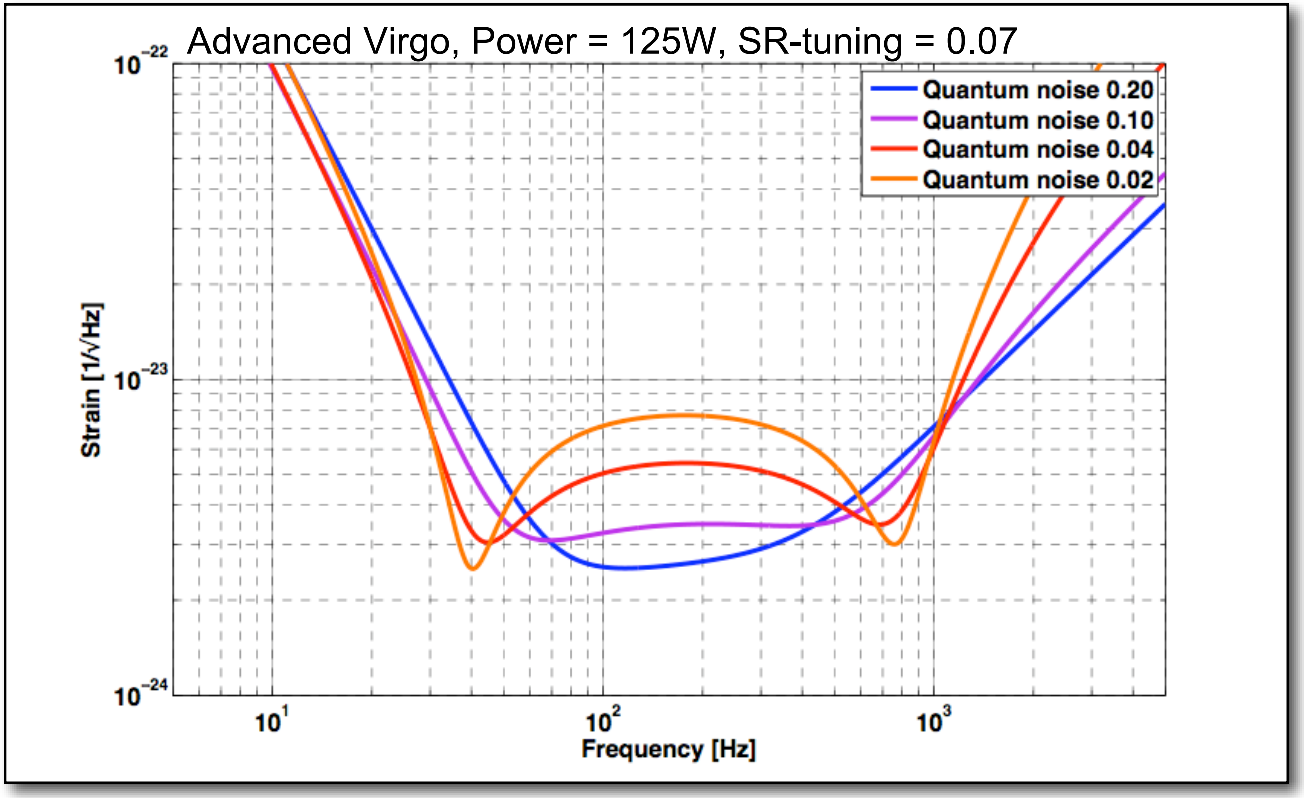
Pure optical resonance

- ➢ Frequency of pure optical resonance goes down with SR-tuning.
- ➢ Frequency of opto-mechanical resonance goes up with SR-tuning

# Optimization Parameter 2:
## Signal-Recycling mirror transmittance



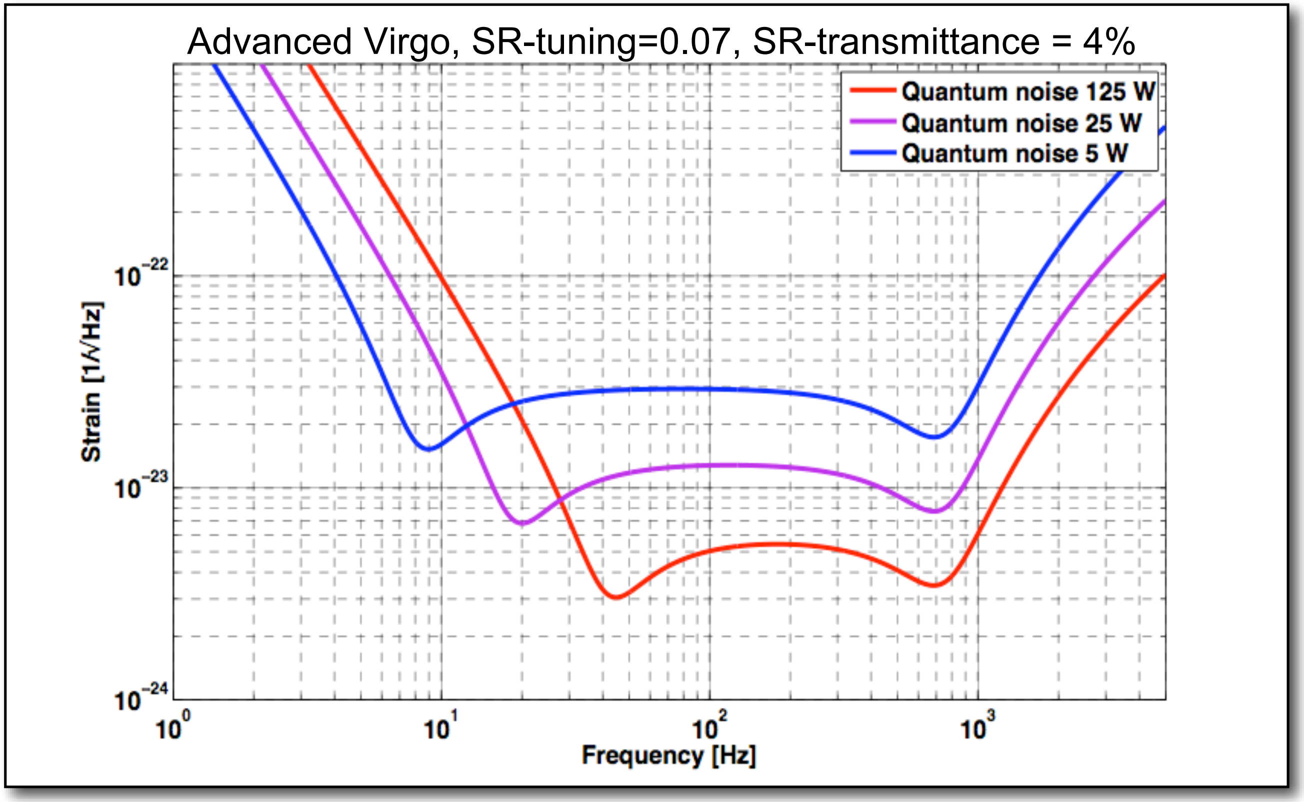
knob 2



- ➢ Resonances are less developed for larger SR transmittance.

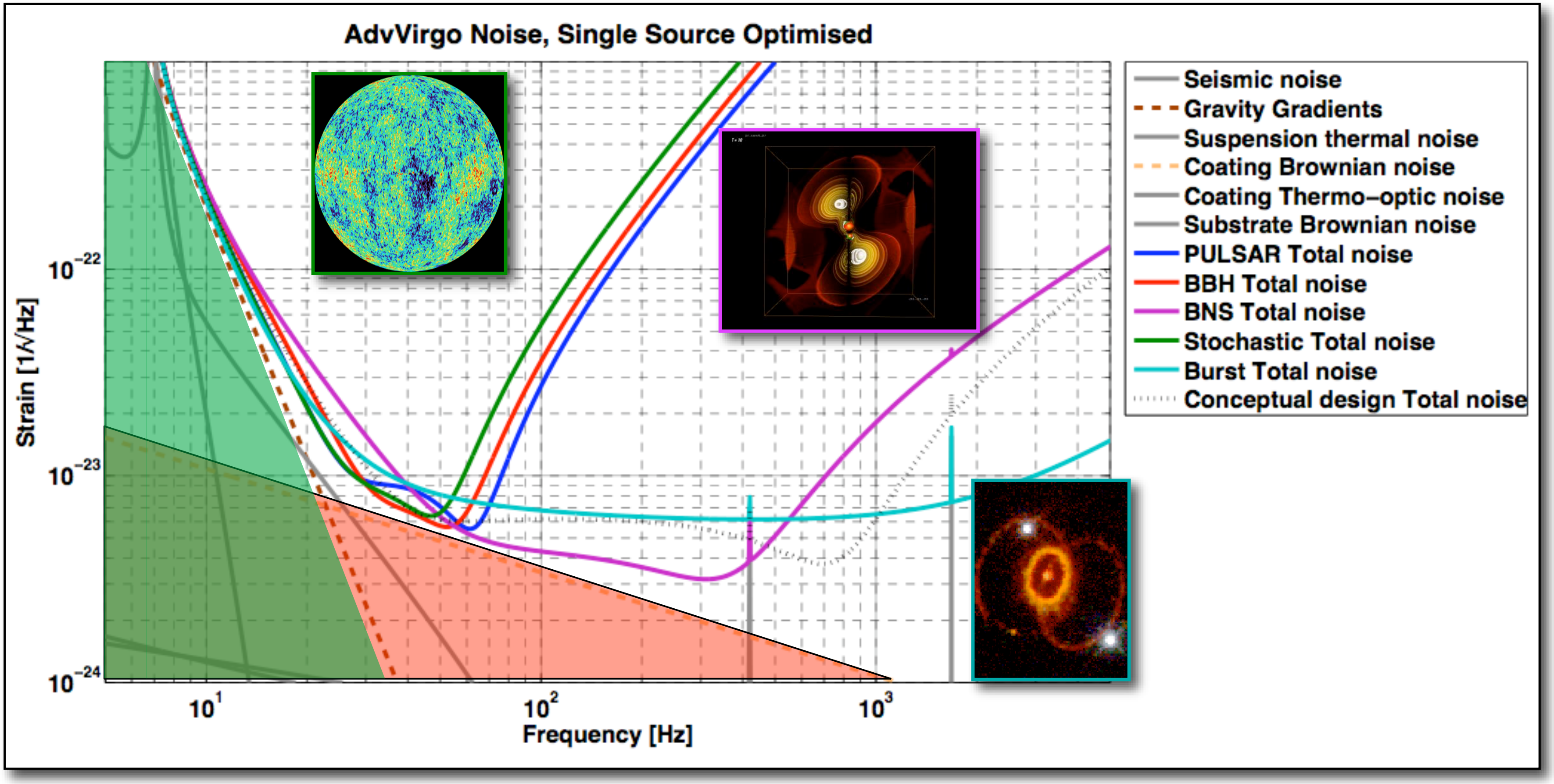
# Optimization Parameter 3: Laser-Input-Power

knob 3



- ➢ High frequency sensitivity improves with higher power (Shotnoise)
- ➢ Low frequency sensitivity decreases with higher power (Radiation pressure noise)

# Optimal configurations



**Curves show the optimal sensitivity for a single source type.**

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# SVN repository for AdV GWINC

- GWINC (previously BENCH) is set of Matlab tools for simulating and manipulating the fundamental noises of GWD. It was developed within the LIGO Scientific collaboration.

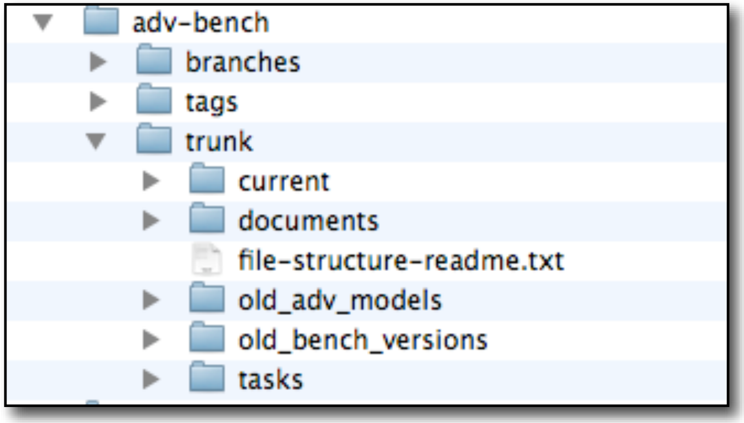
- There is a dedicated SVN for Advanced Virgo GWINC work

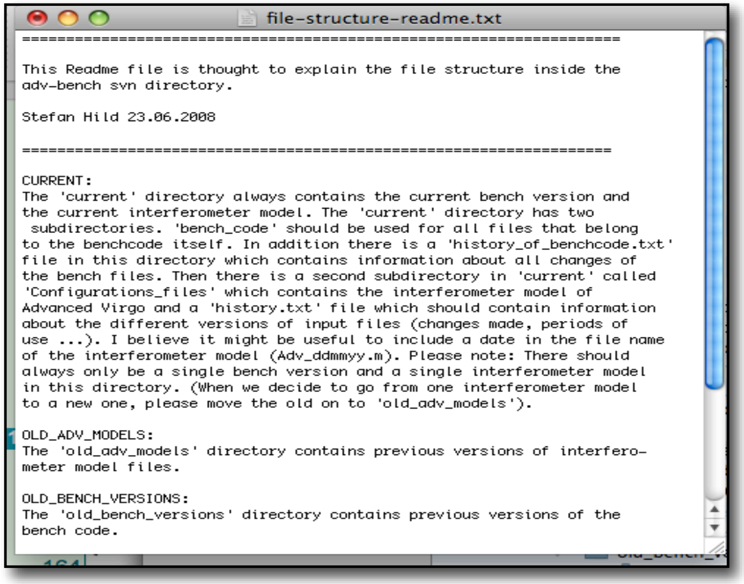
- All GWINC related input files and codes are stored in a subversion repository including backup and version control.

- This svn can be read by the public (no username or password required):
  - Server: svn://lnx0.sr.bham.ac.uk
  - Repository: adv-bench

# SVN repository for AdV Bench

- Place of current (up-to-date) Advanced Virgo GWNIC model.
- Also storage of all outdated codes and files (allows comparison, reproducibility and crosschecking with previous analyses, such for instance the conceptual design)
- File-structure-readme.txt: detailed description of how to use the file structure and where to find what

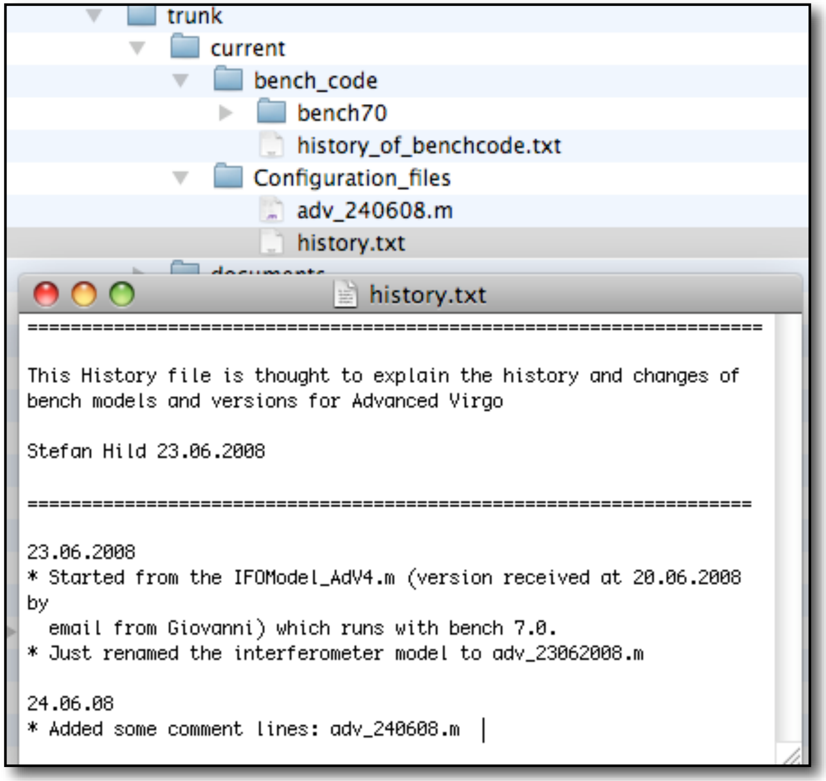




```
======================================================================

This Readme file is thought to explain the file structure inside the
adv-bench svn directory.

Stefan Hild 23.06.2008

=====================================================================

CURRENT:
The 'current' directory always contains the current bench version and
the current interferometer model. The 'current' directory has two
 subdirectories. 'bench_code' should be used for all files that belong
to the benchcode itself. In addition there is a 'history_of_benchcode.txt'
file in this directory which contains information about all changes of
the bench files. Then there is a second subdirectory in 'current' called
'Configurations_files' which contains the interferometer model of
Advanced Virgo and a 'history.txt' file which should contain information
about the different versions of input files (changes made, periods of
use ...). I believe it might be useful to include a date in the file name
of the interferometer model (Adv_ddmmyy.m). Please note: There should
always only be a single bench version and a single interferometer model
in this directory. (When we decide to go from one interferometer model
to a new one, please move the old on to 'old_adv_models').

OLD_ADV_MODELS:
The 'old_adv_models' directory contains previous versions of interfero-
meter model files.

OLD_BENCH_VERSIONS:
The 'old_bench_versions' directory contains previous versions of the
bench code.
```

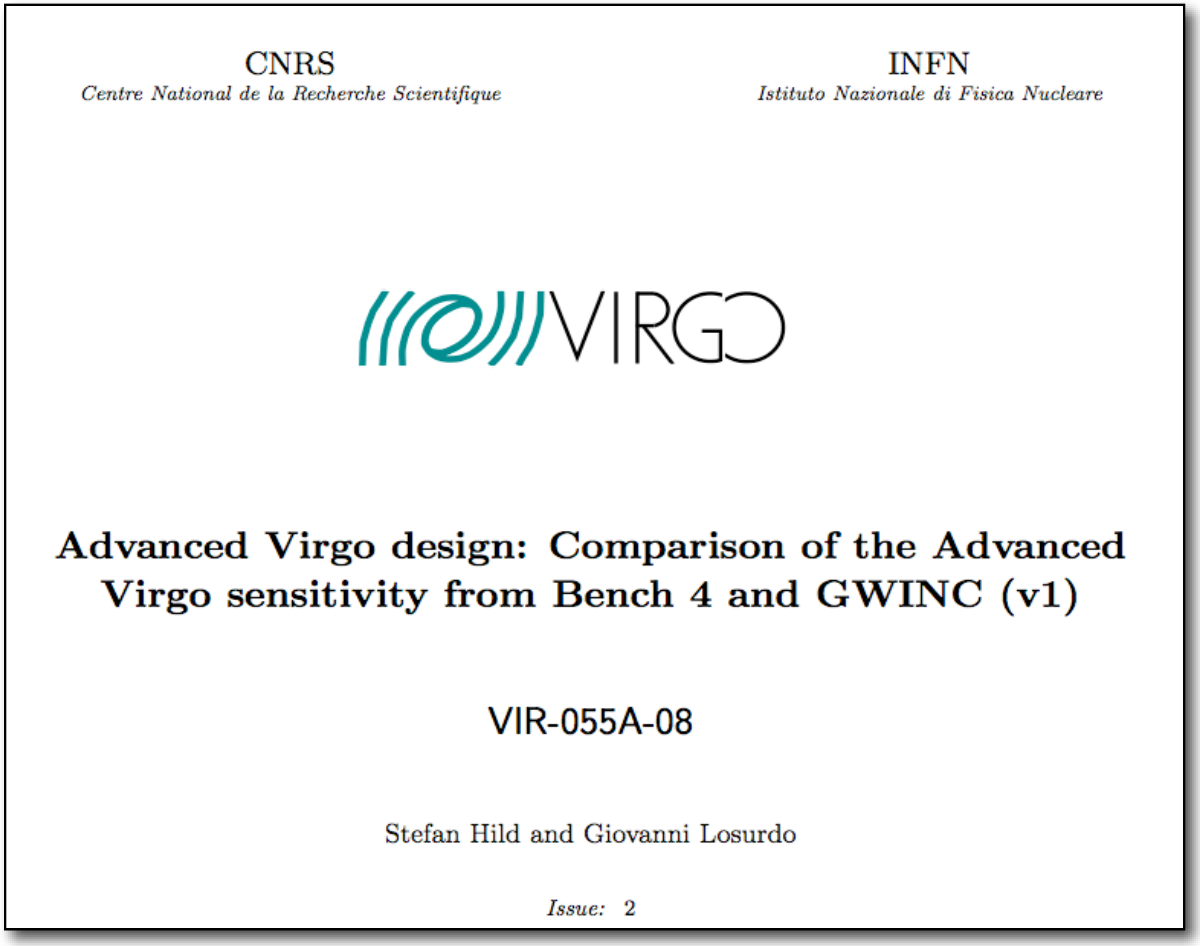
# History and integrity of GWINC files

- First layer: inherent version control of the SVN system.
- Second layer: In addition we keep track by manually maintained history files (for input files as well as for the code itself).



```
====================================================================

This History file is thought to explain the history and changes of
bench models and versions for Advanced Virgo

Stefan Hild 23.06.2008

===================================================================

23.06.2008
* Started from the IFOModel_AdV4.m (version received at 20.06.2008
by
  email from Giovanni) which runs with bench 7.0.
* Just renamed the interferometer model to adv_23062008.m

24.06.08
* Added some comment lines: adv_240608.m
```

# More information can be found



CNRS
*Centre National de la Recherche Scientifique*

INFN
*Istituto Nazionale di Fisica Nucleare*

VIRGO

**Advanced Virgo design: Comparison of the Advanced Virgo sensitivity from Bench 4 and GWINC (v1)**

VIR-055A-08

Stefan Hild and Giovanni Losurdo

*Issue:* 2

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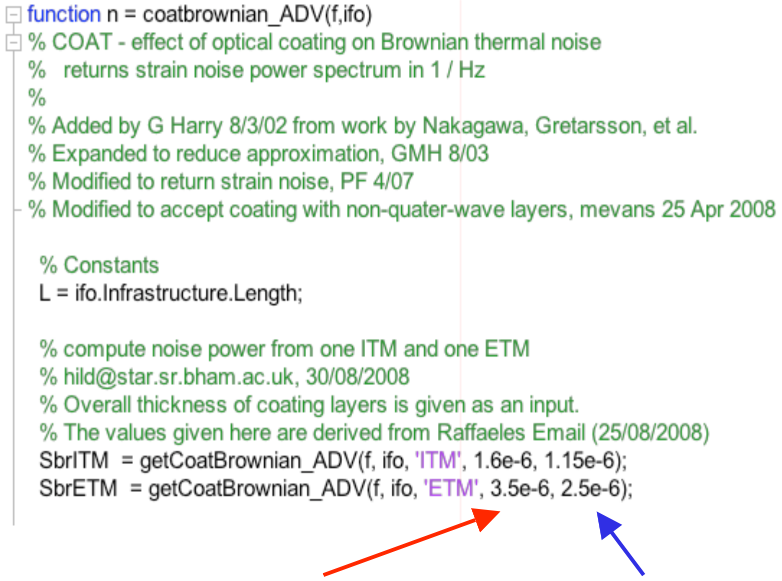
# Coating Brownian in GWINC: current implementation (I)

- Makes use of G.Harry et al: ‘Titania-doped tantala/silica coatings for gravitational-wave detection Classical and Quantum Gravity’, 2007, 24, 405-415.
- Several Matlab subroutines involved.
- Originally GWINC takes a mirror reflectivities and calculates from that the number of coating layers. In ADV GWINC we changed to directly insert coating thicknesses given by LMA.

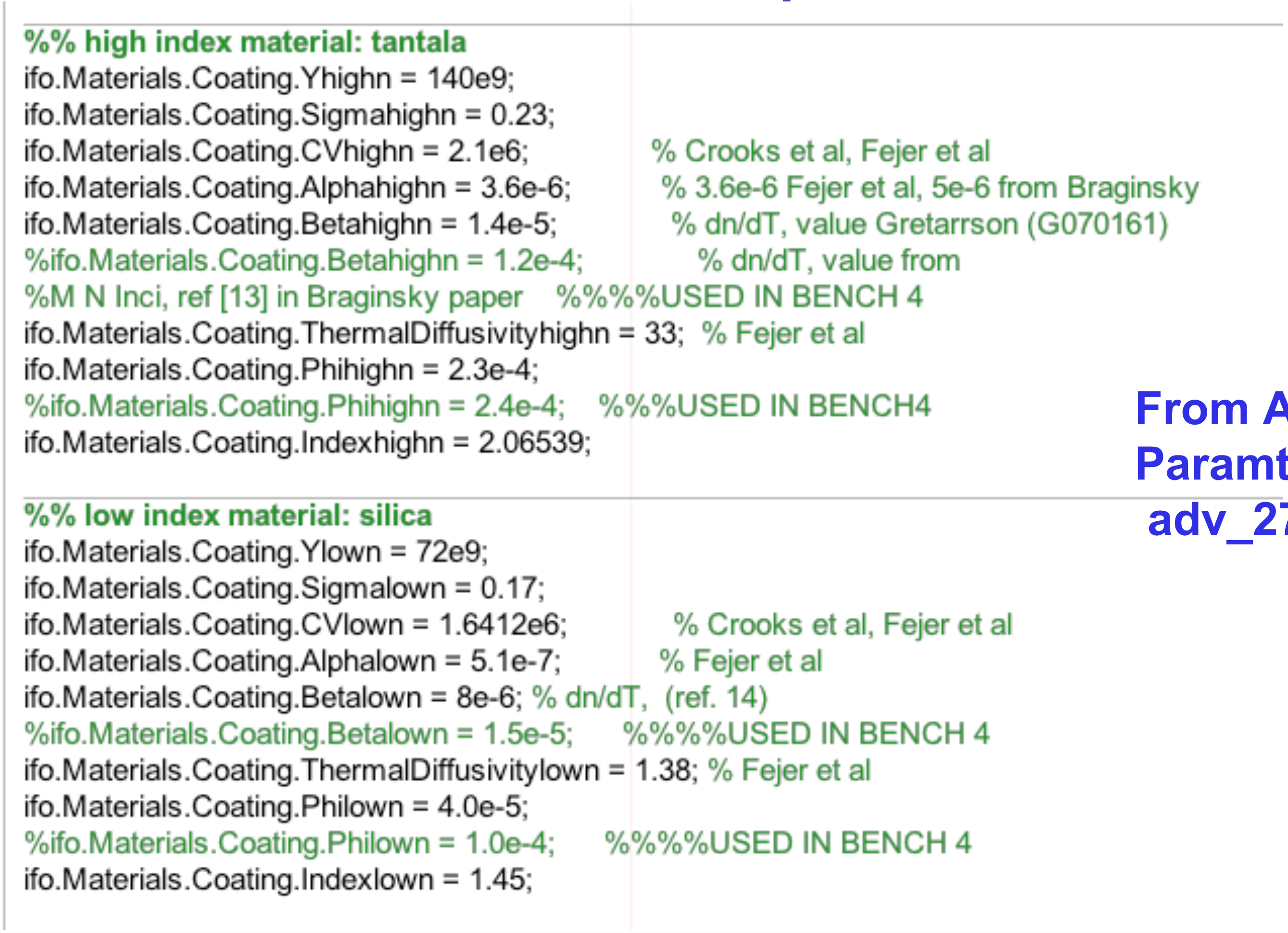


```
function n = coatbrownian_ADV(f,ifo)
% COAT - effect of optical coating on Brownian thermal noise
%   returns strain noise power spectrum in 1 / Hz
%
% Added by G Harry 8/3/02 from work by Nakagawa, Gretarsson, et al.
% Expanded to reduce approximation, GMH 8/03
% Modified to return strain noise, PF 4/07
% Modified to accept coating with non-quater-wave layers, mevans 25 Apr 2008

  % Constants
  L = ifo.Infrastructure.Length;

  % compute noise power from one ITM and one ETM
  % hild@star.sr.bham.ac.uk, 30/08/2008
  % Overall thickness of coating layers is given as an input.
  % The values given here are derived from Raffaeles Email (25/08/2008)
  SbrITM  = getCoatBrownian_ADV(f, ifo, 'ITM', 1.6e-6, 1.15e-6);
  SbrETM  = getCoatBrownian_ADV(f, ifo, 'ETM', 3.5e-6, 2.5e-6);
```

low-n material thickness

high-n material thickness

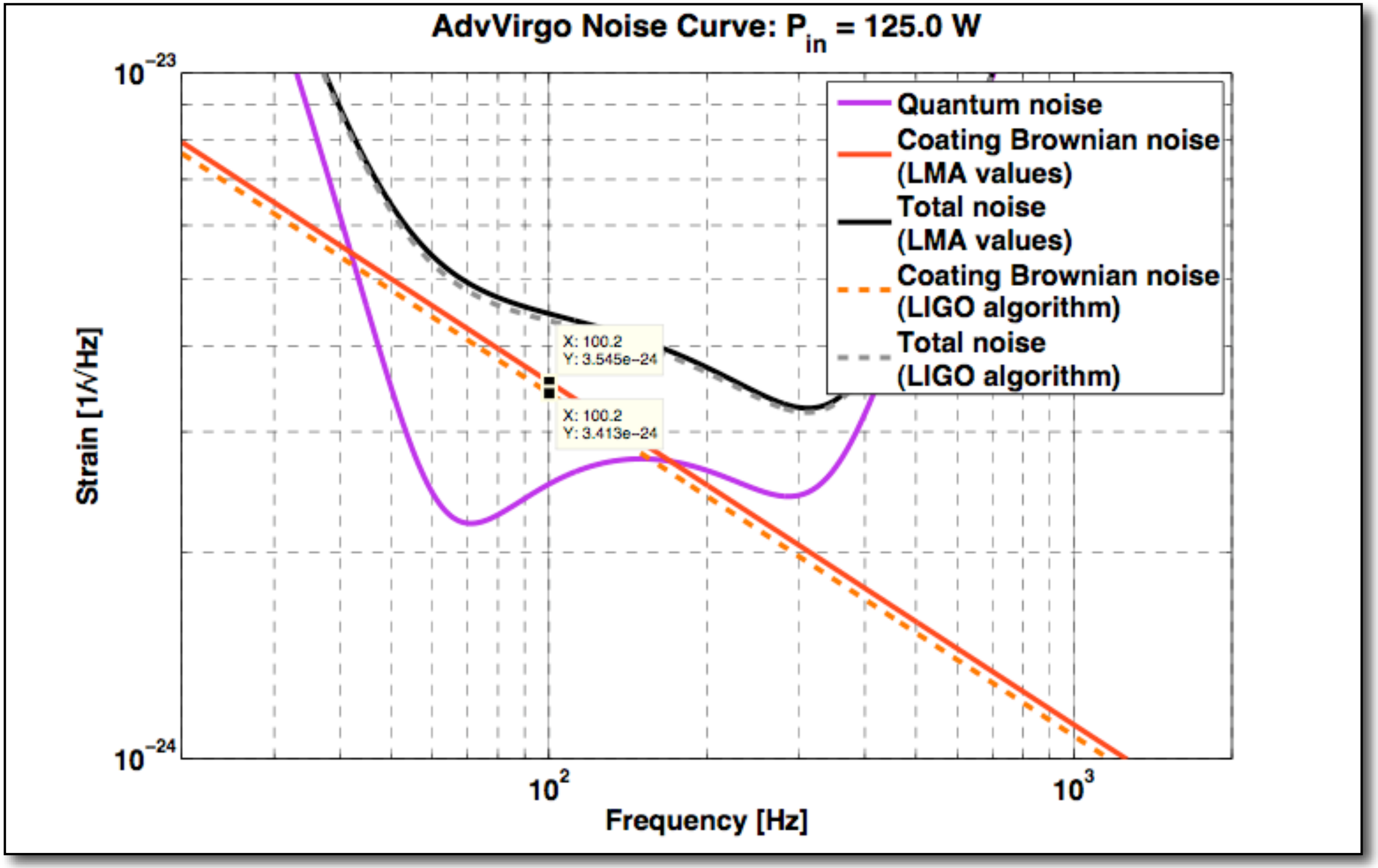
# Coating Brownian in GWINC: current implementation (II)



```
%% high index material: tantala
ifo.Materials.Coating.Yhighn = 140e9;
ifo.Materials.Coating.Sigmahighn = 0.23;
ifo.Materials.Coating.CVhighn = 2.1e6;           % Crooks et al, Fejer et al
ifo.Materials.Coating.Alphahighn = 3.6e-6;        % 3.6e-6 Fejer et al, 5e-6 from Braginsky
ifo.Materials.Coating.Betahighn = 1.4e-5;          % dn/dT, value Gretarrson (G070161)
%ifo.Materials.Coating.Betahighn = 1.2e-4;           % dn/dT, value from
%M N Inci, ref [13] in Braginsky paper    %%%%USED IN BENCH 4
ifo.Materials.Coating.ThermalDiffusivityhighn = 33;  % Fejer et al
ifo.Materials.Coating.Phihighn = 2.3e-4;
%ifo.Materials.Coating.Phihighn = 2.4e-4;    %%%USED IN BENCH4
ifo.Materials.Coating.Indexhighn = 2.06539;

%% low index material: silica
ifo.Materials.Coating.Ylown = 72e9;
ifo.Materials.Coating.Sigmalown = 0.17;
ifo.Materials.Coating.CVlown = 1.6412e6;           % Crooks et al, Fejer et al
ifo.Materials.Coating.Alphalown = 5.1e-7;           % Fejer et al
ifo.Materials.Coating.Betalown = 8e-6; % dn/dT,  (ref. 14)
%ifo.Materials.Coating.Betalown = 1.5e-5;     %%%%USED IN BENCH 4
ifo.Materials.Coating.ThermalDiffusivitylown = 1.38; % Fejer et al
ifo.Materials.Coating.Philown = 4.0e-5;
%ifo.Materials.Coating.Philown = 1.0e-4;      %%%%USED IN BENCH 4
ifo.Materials.Coating.Indexlown = 1.45;
```

**From Advanced Virgo Paramter input file: adv_270608.m**

# Coating Brownian in GWINC: current implementation (III)



# Beam Geometry

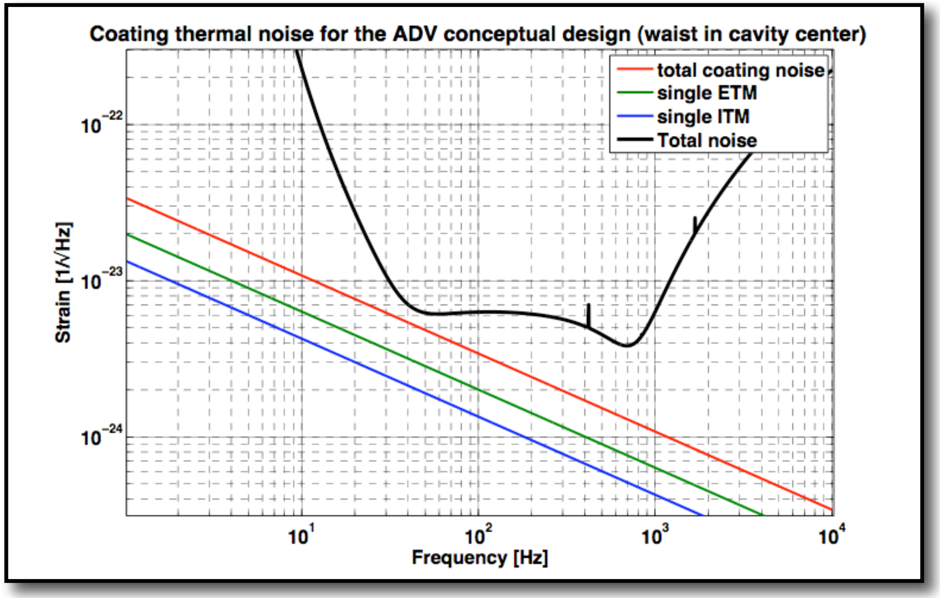
- Intuitively one would think the lowest coating noise is achieved when beam waist is at the center of the cavity (=> equal beam size at ITM and ETM),

  ***BUT:***

- Coating noise for ITM and ETM are different, due to their different number of coating layer:

$$\bar{v} = C(S_T + \gamma^{-1} S_S),$$

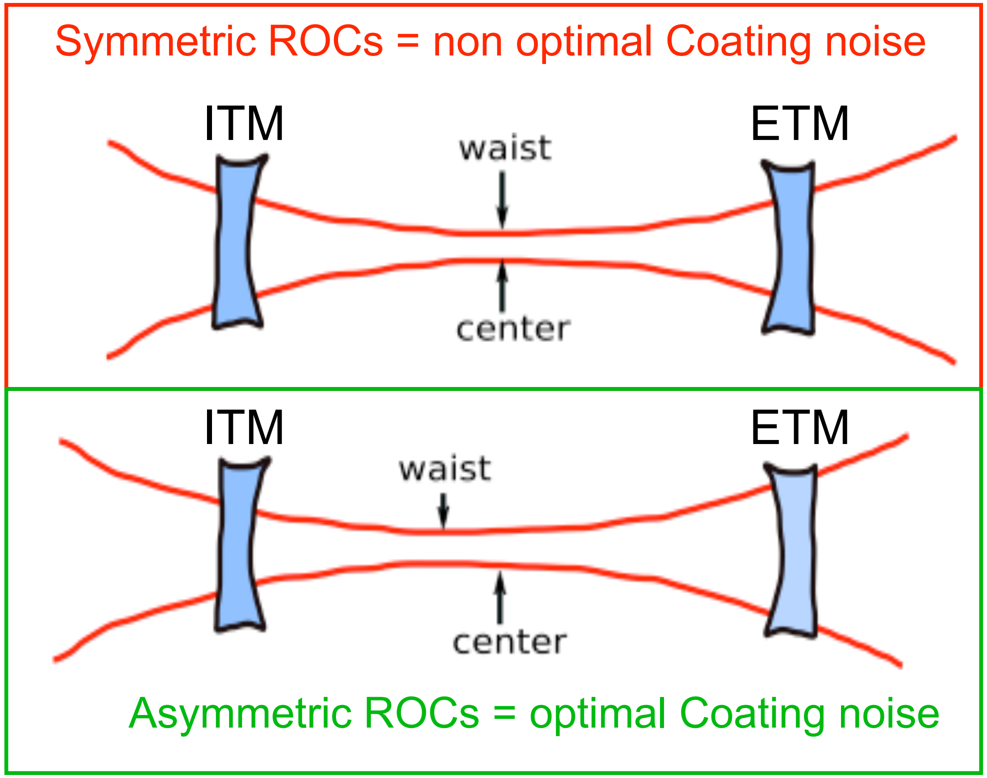
*J. Agresti et al (LIGO-P060027-00-Z)*



- For equal beam size ETM has higher noise.

# Optimal Waist Position

- In order to minimize the thermal noise we have to make the beam larger on ETM and smaller on ITM.
- Equivalent to moving the waist closer to ITM.
- Nice side effect, the beam in the central central area would be slightly smaller.

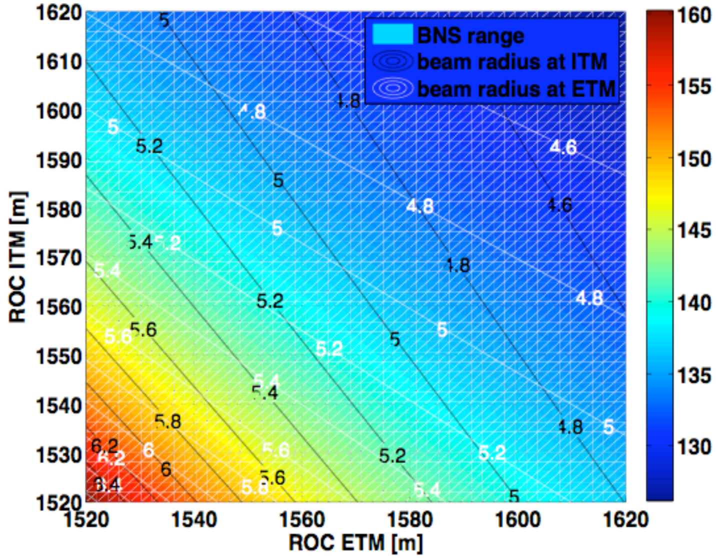


Symmetric ROCs = non optimal Coating noise

Asymmetric ROCs = optimal Coating noise

# Beam Size

- Principle Rule:
  - The larger the beam the better the detector sensitivity
  - Larger beams make nearly everything else more complicated / more expensive.

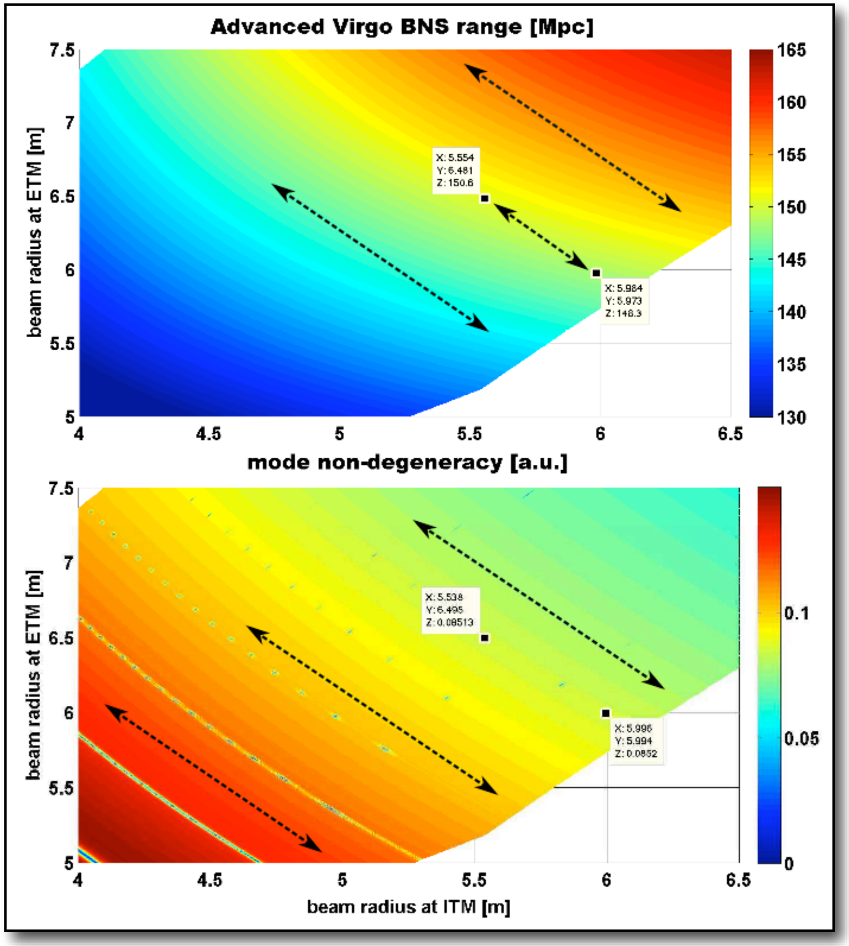


- Advantages of large beams:
  - Reduced thermal noise of test masses (especially coating Brownian)
  - Slightly reduced contribution from residual gas pressure

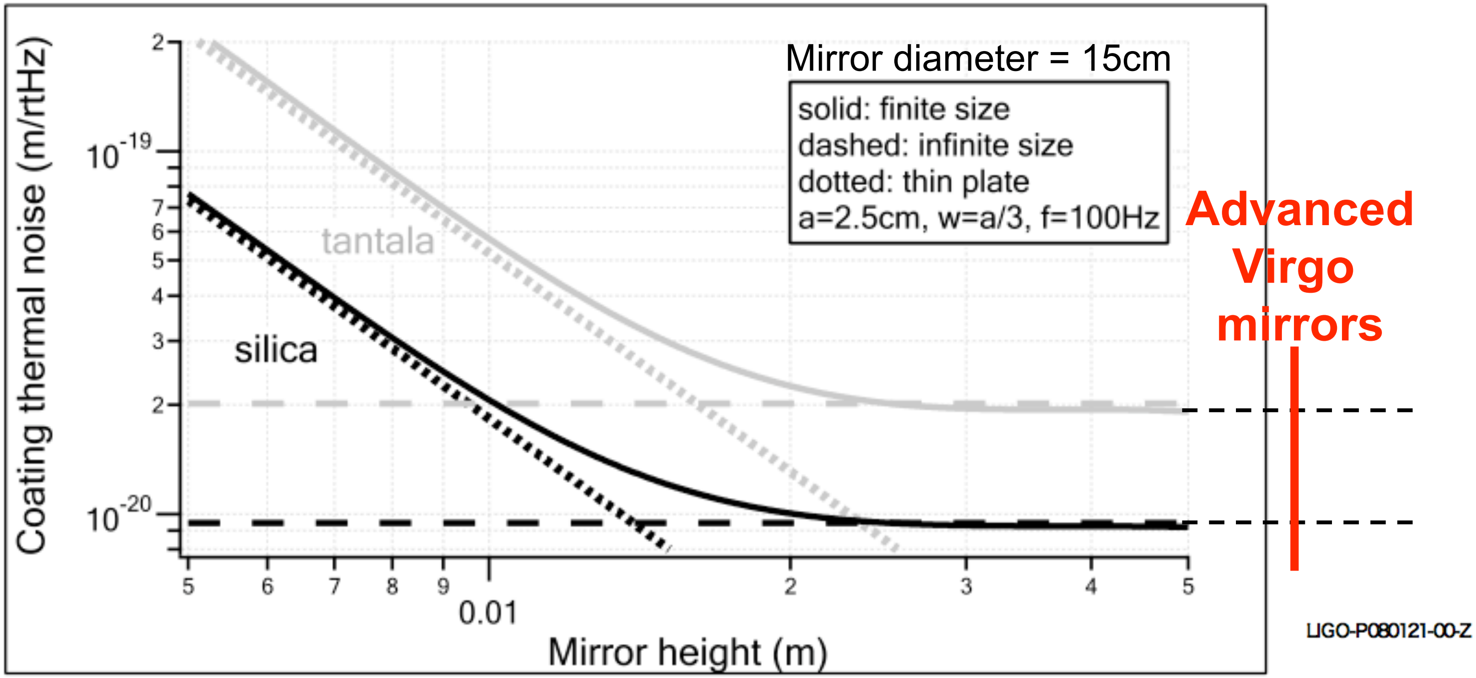
- Disadvantages of large beams:
  - Higher clipping losses
  - Larger test masses (especially BS, because of 45deg angle)
  - Larger apertures are required (vacuum system, actuators, etc)
  - Large telescopes (input, output, pick-off beams)
  - More sensitive to ROC deviations

# Choice of ROCs/beam size: Sensitivity vs Mode-non-degeneracy

- In general mode-non-degeneracy and sensitivity go opposite.
- Asymmetric ROCs are beneficial:
  - For identical mode-non-degeneracy (parallel to arrows in lower plot) we can increase sensitivity (parallel to arrow in upper plot) by going towards the upper left corner.
  - This means making beam larger on ETM and smaller on ITM.



# Test mass aspect ratio and Coating Brownian noise



- ➢ No significant change for ADV

**Coating thermal noise of a finite-size cylindrical mirror**

Kentaro Somiya[a] and Kazuhiro Yamamoto[b]

[a] *Theoretical Astrophysics, California Institute of Technology, Pasadena, California, 91125*

[b] *Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Callinstr. 38, 30167 Hannover, Germany*

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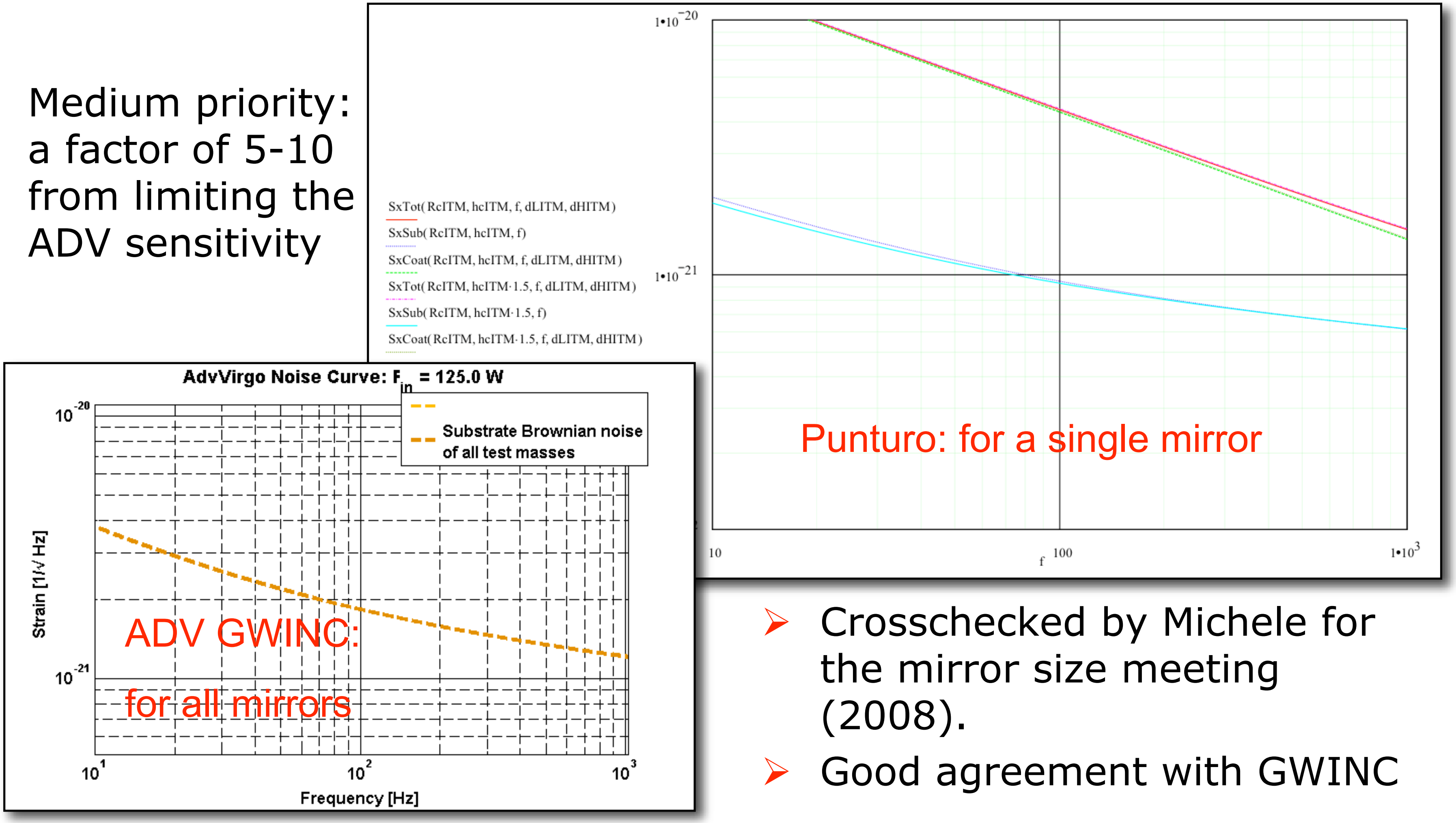
# Substrate Brownian noise

- Medium priority: a factor of 5-10 from limiting the ADV sensitivity



Punturo: for a single mirror

ADV GWINC: for all mirrors

- Crosschecked by Michele for the mirror size meeting (2008).
- Good agreement with GWINC

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# Coating Thermo-optic

- GWINC includes the findings from Matt Evan et al presented at ELBA 2008.
- Otherwise …

? ? ?

- ….SO FAR NO CHECKS PERFORMED

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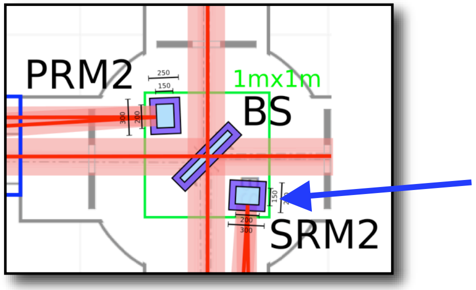
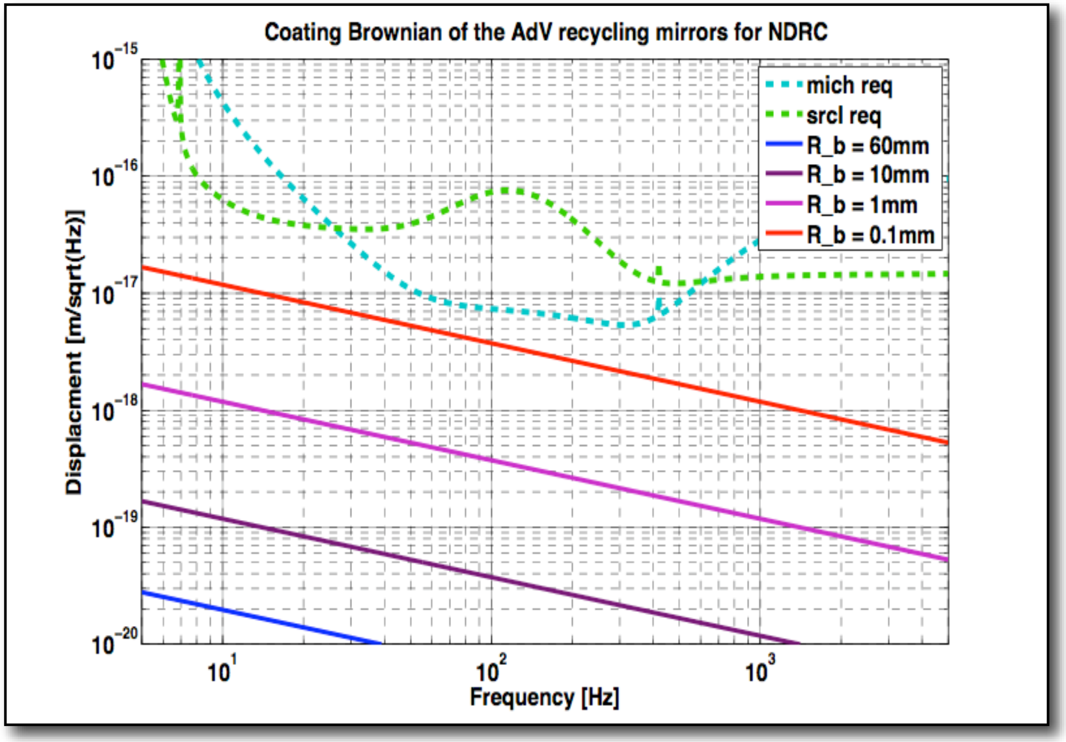
# Coating Brownian of the Signal-Recycling mirrors in NDRC

- How small can we make the beam on the SRM before we ran into coating noise?



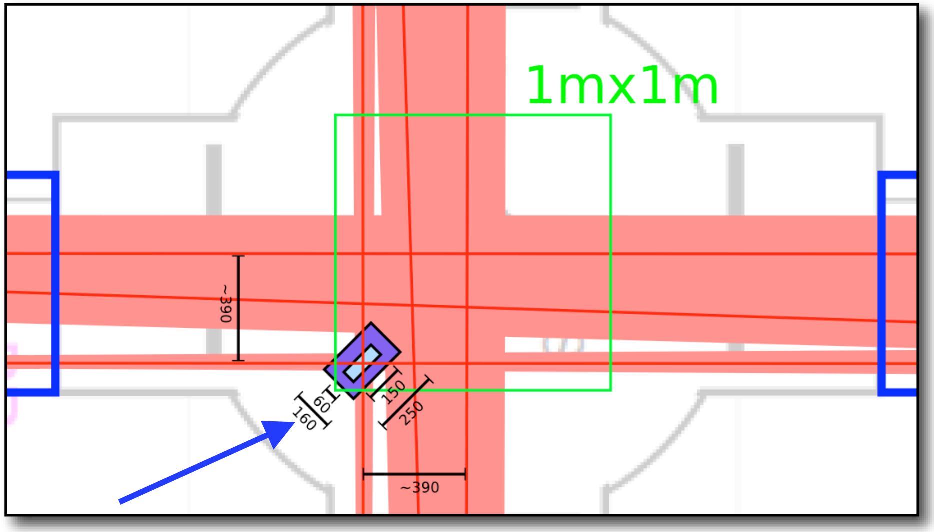
- Used the GWINC routine for the calculation.
- Result: Everything larger 0.5mm is fine. => No problem!!

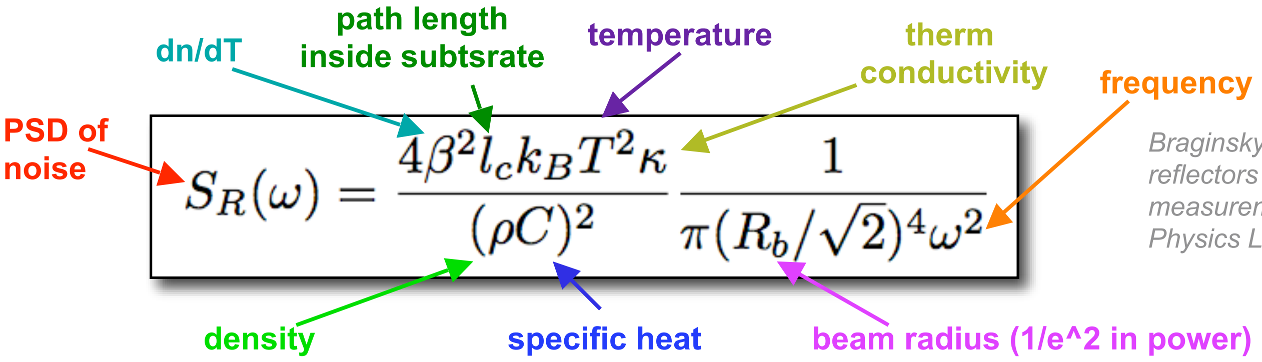


*S.Hild, M. Barsuglia and A .Freise: 'Thermal Noise Constraints for the Advanced Virgo Non-Degenerate Recycling Cavity Design', Virgo-note in preparation*

# Thermo-refractive noise of the beam splitter (II)

- One NDRC design features a tiny BS.
- How small can we make the beam before thermo-refractive noise becomes a problem?





dn/dT

path length inside subtsrate

temperature

therm conductivity

frequency

PSD of noise

$$S_R(\omega) = \frac{4\beta^2 l_c k_B T^2 \kappa}{(\rho C)^2} \frac{1}{\pi (R_b/\sqrt{2})^4 \omega^2}$$

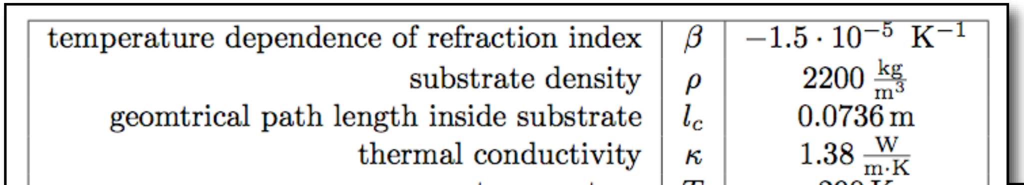
density

specific heat

beam radius (1/e^2 in power)

*Braginsky and Vyatchanin: 'Corner reflectors and quantum-non-demolition measurements in GW antennae Physics Letters A, 2004, 324, 345*

# Thermo-refractive noise of the beam splitter (II)

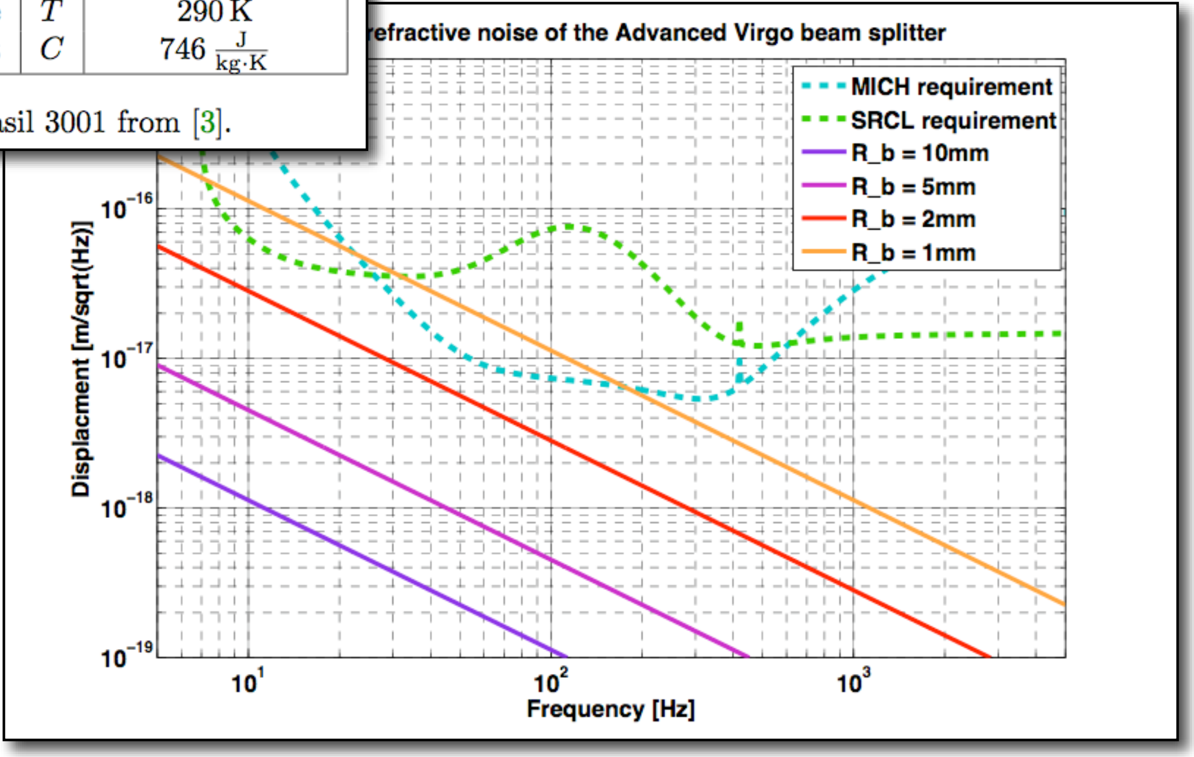


| | | |
|---|---|---|
| temperature dependence of refraction index | $\beta$ | $-1.5 \cdot 10^{-5}\ \mathrm{K}^{-1}$ |
| substrate density | $\rho$ | $2200\ \frac{\mathrm{kg}}{\mathrm{m}^3}$ |
| geomtrical path length inside substrate | $l_c$ | $0.0736\,\mathrm{m}$ |
| thermal conductivity | $\kappa$ | $1.38\ \frac{\mathrm{W}}{\mathrm{m \cdot K}}$ |
| temperature | $T$ | $290\,\mathrm{K}$ |
| specific heat | $C$ | $746\ \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$ |

**Table 1:** Material properties of Suprasil 3001 from [3].

- Result: As long as the beam radius is larger than 2mm no problems from thermo-refractive noise.

*S.Hild, M. Barsuglia and A .Freise: 'Thermal Noise Constraints for the Advanced Virgo Non-Degenerate Recycling Cavity Design', Virgo-note in preparation*



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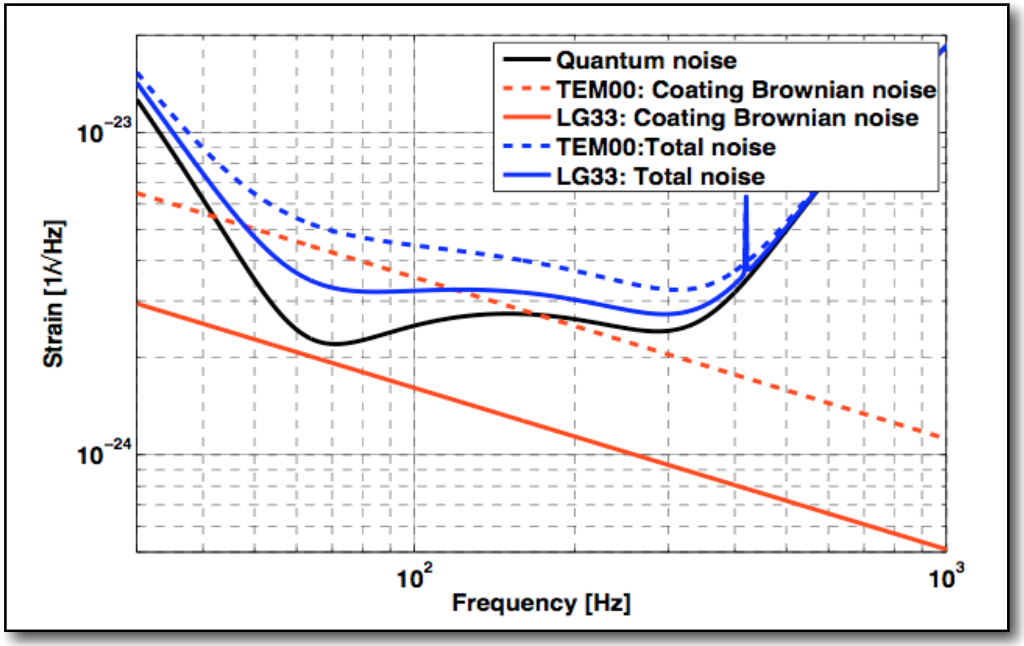
# Application of LG modes for Advanced Virgo upgrades?

- Switch beam geometry from $TEM_{00}$ to $LG_{33}$
- Requires mirror replacement (different ROC)
- Reduces coating Brownian by a factor 2.2.
  *Vinet: personal communication*
- Reduces substrate Brownian by a factor 2.7
  *Mours, B.; Tournefier, E. & Vinet, J. Thermal noise reduction in interferometric GW antennas: using high order TEM modes, CQG, 2006, 23, 5777*
- Increases thermo-elastic by a factor 1.7
  *Vinet: personal communication*



**BNS Inspiral range increases from 148 Mpc to 195 Mpc => increase of event rate by a factor 2.3**

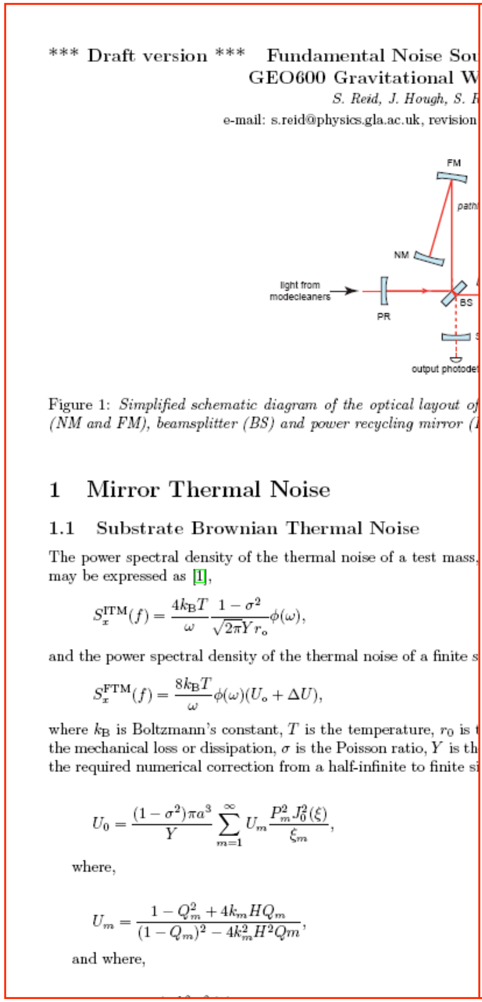
*S. Chelkowski, S.Hild and A .Freise: 'Prospects of higher-order Laguerre Gauss modes in future GW detectors', arXiv:0901.4931v1 [gr-qc]*

# Overview

- **Motivation of accurate thermal noise models for Advanced Virgo:**
  - Required as input for important design decisions
  - Required as input for sensitivity optimization
- **Current status of the Advanced Virgo GWINC model:**
  - Documentation, availability, history…
  - Coating Brownian
    - Current implementation
    - Beam sizes and asymmetric ROCs
    - Test mass shape?
  - Substrate Brownian
  - Thermo-optic
- **Examples of recent investigations**
  - Thermal noise issues in case of non-degenerate recycling cavities.
  - Sensitivity improvement from application of LG modes
- **My personal wish list for the future**

# My personal wish list for the future

- We should compile ONE list of consistent material parameters which available for the public and continuously updated.
- Each of the important thermal noises in GWINC should be checked by an expert from Virgo.
  - High priority: coating Brownian
  - Medium priority: thermo-optic
- Please use Advanced Virgo GWINC: more people = more checking etc
- In parallel to (and independent of) the complex GWINC code it would be nice to have a short document (5 pages) containing equations, references and parameters for all relevant thermal noise sources.



*** Draft version *** Fundamental Noise Sou
GEO600 Gravitational W
*S. Reid, J. Hough, S. R*
e-mail: s.reid@physics.gla.ac.uk, revision

Figure 1: *Simplified schematic diagram of the optical layout of*
*(NM and FM), beamsplitter (BS) and power recycling mirror (I*

## 1 Mirror Thermal Noise

### 1.1 Substrate Brownian Thermal Noise

The power spectral density of the thermal noise of a test mass,
may be expressed as [1],

$$S_x^{\mathrm{ITM}}(f) = \frac{4k_{\mathrm{B}}T}{\omega}\frac{1-\sigma^2}{\sqrt{2\pi}Yr_{\mathrm{o}}}\phi(\omega),$$

and the power spectral density of the thermal noise of a finite s

$$S_x^{\mathrm{FTM}}(f) = \frac{8k_{\mathrm{B}}T}{\omega}\phi(\omega)(U_{\mathrm{o}}+\Delta U),$$

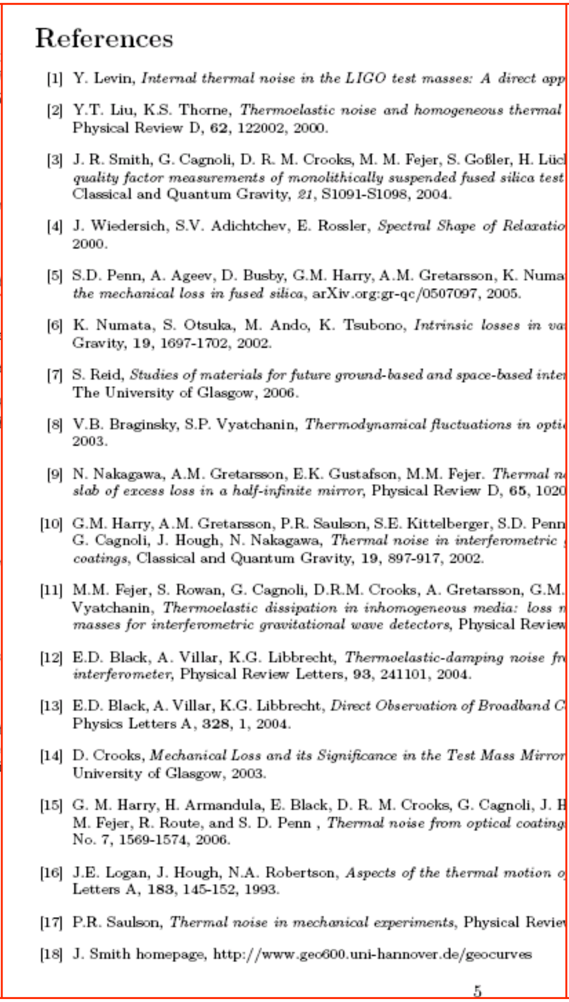
where $k_{\mathrm{B}}$ is Boltzmann's constant, $T$ is the temperature, $r_0$ is t
the mechanical loss or dissipation, $\sigma$ is the Poisson ratio, $Y$ is th
the required numerical correction from a half-infinite to finite si

$$U_0 = \frac{(1-\sigma^2)\pi a^3}{Y}\sum_{m=1}^{\infty}U_m\frac{P_m^2 J_0^2(\xi)}{\xi_m},$$

where,

$$U_m = \frac{1-Q_m^2+4k_m HQ_m}{(1-Q_m)^2-4k_m^2H^2Qm},$$

and where,



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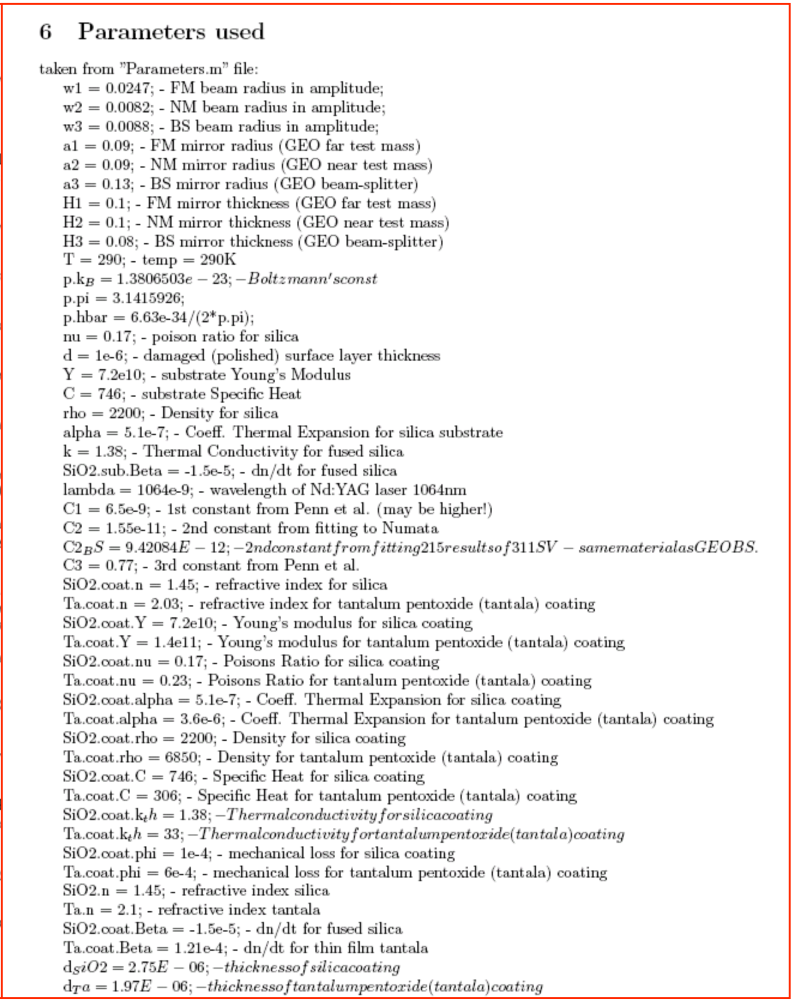


## 6 Parameters used

taken from "Parameters.m" file:
w1 = 0.0247; - FM beam radius in amplitude;
w2 = 0.0082; - NM beam radius in amplitude;
w3 = 0.0088; - BS beam radius in amplitude;
a1 = 0.09; - FM mirror radius (GEO far test mass)
a2 = 0.09; - NM mirror radius (GEO near test mass)
a3 = 0.13; - BS mirror radius (GEO beam-splitter)
H1 = 0.1; - FM mirror thickness (GEO far test mass)
H2 = 0.1; - NM mirror thickness (GEO near test mass)
H3 = 0.08; - BS mirror thickness (GEO beam-splitter)
T = 290; - temp = 290K
p.k$_B$ = $1.3806503e - 23; -Boltzmann'sconst$
p.pi = 3.1415926;
p.hbar = 6.63e-34/(2*p.pi);
nu = 0.17; - poison ratio for silica
d = 1e-6; - damaged (polished) surface layer thickness
Y = 7.2e10; - substrate Young's Modulus
C = 746; - substrate Specific Heat
rho = 2200; - Density for silica
alpha = 5.1e-7; - Coeff. Thermal Expansion for silica substrate
k = 1.38; - Thermal Conductivity for fused silica
SiO2.sub.Beta = -1.5e-5; - dn/dt for fused silica
lambda = 1064e-9; - wavelength of Nd:YAG laser 1064nm
C1 = 6.5e-9; - 1st constant from Penn et al. (may be higher!)
C2 = 1.55e-11; - 2nd constant from fitting to Numata
C2$_BS = 9.42084E - 12; -2ndconstantfromfitting215resultsof311SV - samematerialasGEOBS.$
C3 = 0.77; - 3rd constant from Penn et al.
SiO2.coat.n = 1.45; - refractive index for silica
Ta.coat.n = 2.03; - refractive index for tantalum pentoxide (tantala) coating
SiO2.coat.Y = 7.2e10; - Young's modulus for silica coating
Ta.coat.Y = 1.4e11; - Young's modulus for tantalum pentoxide (tantala) coating
SiO2.coat.nu = 0.17; - Poisons Ratio for silica coating
Ta.coat.nu = 0.23; - Poisons Ratio for tantalum pentoxide (tantala) coating
SiO2.coat.alpha = 5.1e-7; - Coeff. Thermal Expansion for silica coating
Ta.coat.alpha = 3.6e-6; - Coeff. Thermal Expansion for tantalum pentoxide (tantala) coating
SiO2.coat.rho = 2200; - Density for silica coating
Ta.coat.rho = 6850; - Density for tantalum pentoxide (tantala) coating
SiO2.coat.C = 746; - Specific Heat for silica coating
Ta.coat.C = 306; - Specific Heat for tantalum pentoxide (tantala) coating
SiO2.coat.k$_t h = 1.38; -Thermalconductivityforsilicacoating$
Ta.coat.k$_t h = 33; -Thermalconductivityfortantalumpentoxide(tantala)coating$
SiO2.coat.phi = 1e-4; - mechanical loss for silica coating
Ta.coat.phi = 6e-4; - mechanical loss for tantalum pentoxide (tantala) coating
SiO2.n = 1.45; - refractive index silica
Ta.n = 2.1; - refractive index tantala
SiO2.coat.Beta = -1.5e-5; - dn/dt for fused silica
Ta.coat.Beta = 1.21e-4; - dn/dt for thin film tantala
d$_S iO2 = 2.75E - 06; -thicknessofsilicacoating$
d$_T a = 1.97E - 06; -thicknessoftantalumpentoxide(tantala)coating$

# END

# Cavity Stability and Choice of ROCs

- Definition of mode-non-degeneracy:
  - Gouy-phase shift of mode of order l+m:

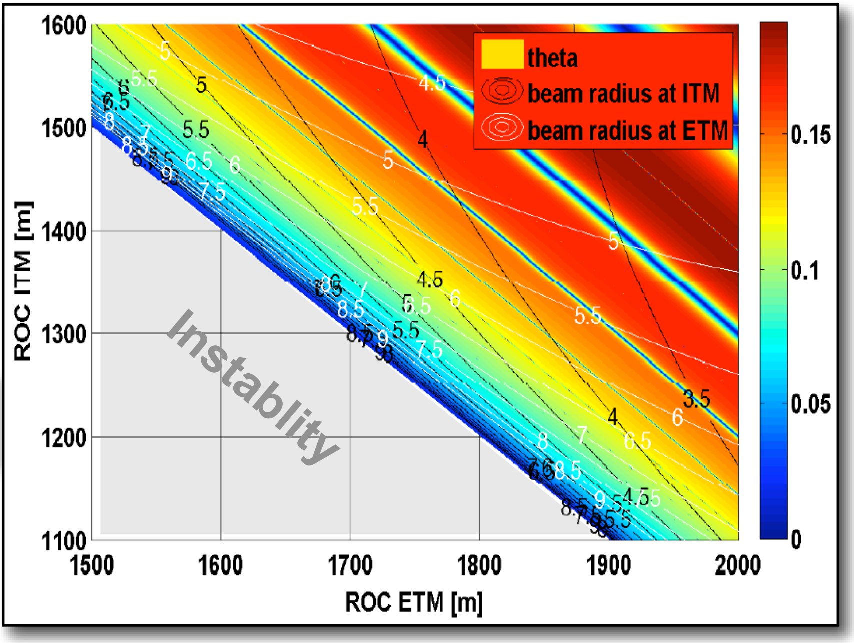
$$\phi_{l+m} = (l+m)\frac{1}{\pi}\arccos\sqrt{(1-\frac{L}{R_{c,i}})(1-\frac{L}{R_{c,e}})}.$$

  - Mode-non-degeneracy for a single mode is:

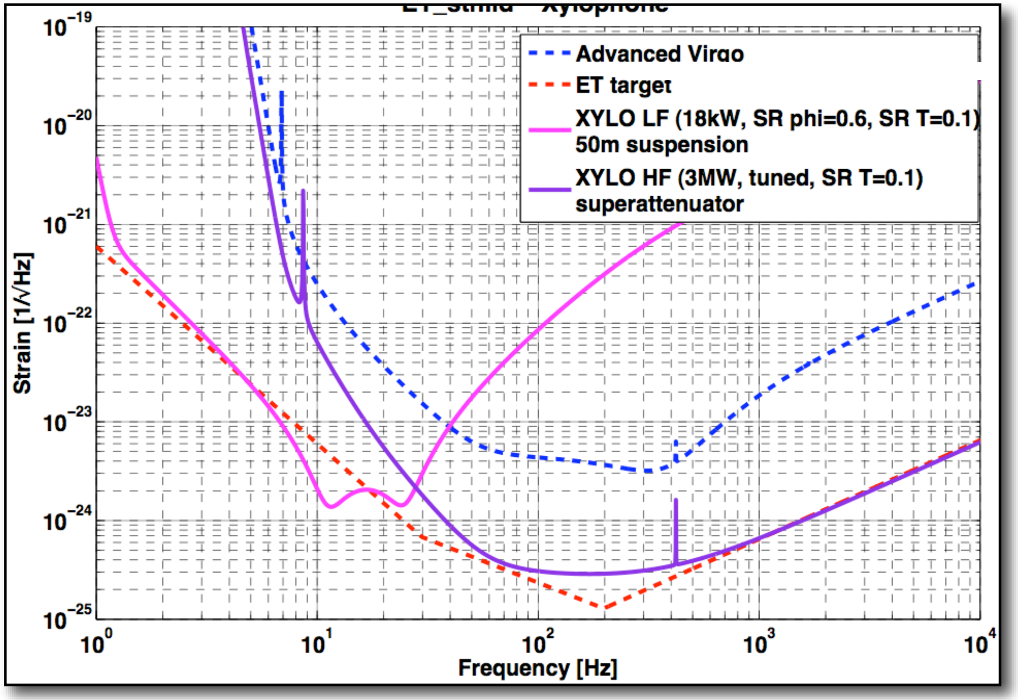
$$\Psi_{l+m}(L, R_{c,i}, R_{c,e}) = |\phi_{l+m} - \mathrm{round}(\phi_{l+m})|.$$

  - Figure of merit for combining all modes up to the order N:

$$\Theta_N(L, R_{c,i}, R_{c,e}) = \frac{1}{\sqrt{\sum_{k=1}^{N} \frac{1}{\Psi_k^2} \frac{1}{k!}}}$$



# Xyolophon: More than one detector to cover the full bandwidth



**Low Frequency IFO:** low optical power, cryogenic test masses, sophisticated low frequency suspension, underground, heavy test masses.

**High Frquency IFO:** high optical power, room temperature, surface location, squeezed light