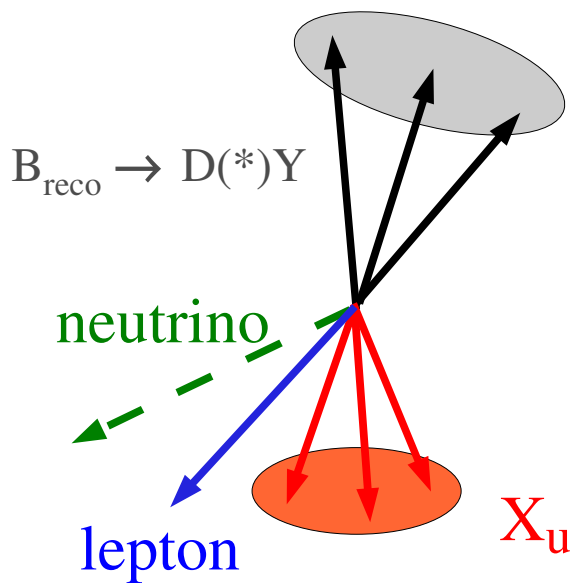
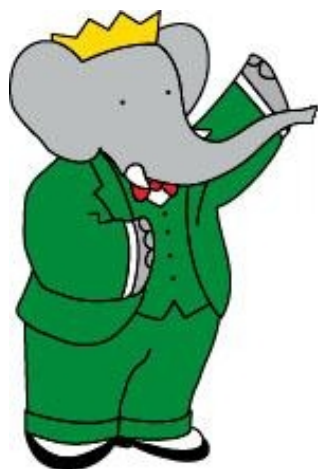


# Inclusive semileptonic B decays: $|V_{ub}|$



*Nicola Gagliardi (Università di Padova)*<sub>1</sub>

# Outline

- **Motivation:**

- ✓ Semileptonic decays and Inclusive  $B \rightarrow X_u l \nu$  theory;

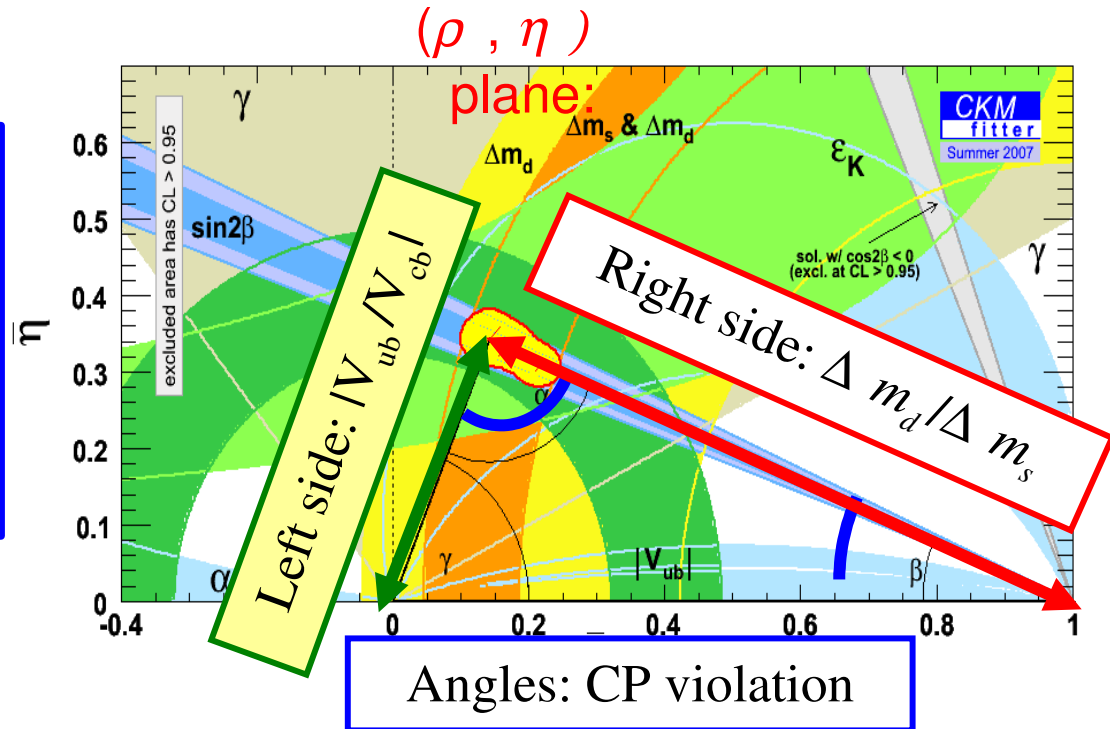
- **Inclusive  $B \rightarrow X_u l \nu$ :**

- ✓ BaBar Measurement:  $|V_{ub}|$  with endpoint method;
- ✓ BaBar Measurement:  $|V_{ub}|$  with hadronic tag;
- ✓ World Average and HQE parameters;
- ✓ Belle multivariate analysis (CKM 2008);
- ✓  $X_u$  hadronic moments;
- ✓ Weak Annihilation in  $B \rightarrow X_u l \nu$  decays;

- **Conclusions.**

# Motivation

Precision of  $\sin 2\beta$  (3%)  
 outstripped the other  
 measurements:  
 must improve the others to  
 make more stringent test



Left side of the triangle:

- $|V_{cb}|$  known with a precision of  $\sim 2\%$
- $|V_{ub}|$  known with a precision of  $\sim 8\%$

Error on the length of the side opposite to b dominated by errors on  $|V_{ub}|$



**Improved precision on  $|V_{ub}|$  needed**

# Inclusive $B \rightarrow X_u l \nu$

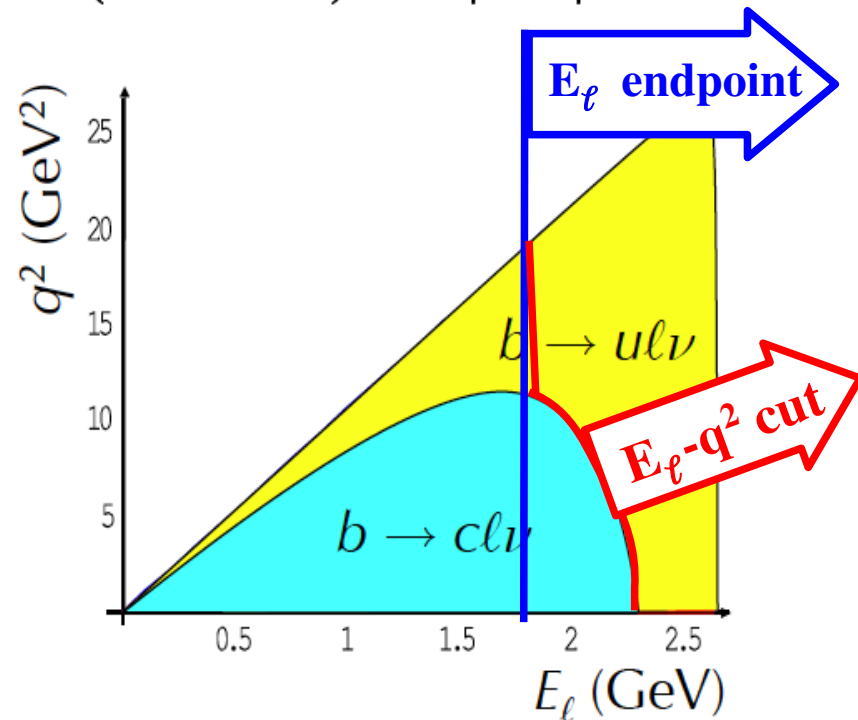
$$\Gamma(\bar{B} \rightarrow X_u l \bar{\nu}) = \underbrace{\frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3}}_{\text{free quark decay}} \left[ 1 + \underbrace{O(\alpha_s)}_{\text{perturbative correction}} + \underbrace{O(1/m_b^2)}_{\text{non perturbative correction}} + H.C. \right] \quad \text{OPE} \sim 5\% \text{ uncertainty}$$

$$\frac{\Gamma(b \rightarrow ul\nu)}{\Gamma(b \rightarrow cl\nu)} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$

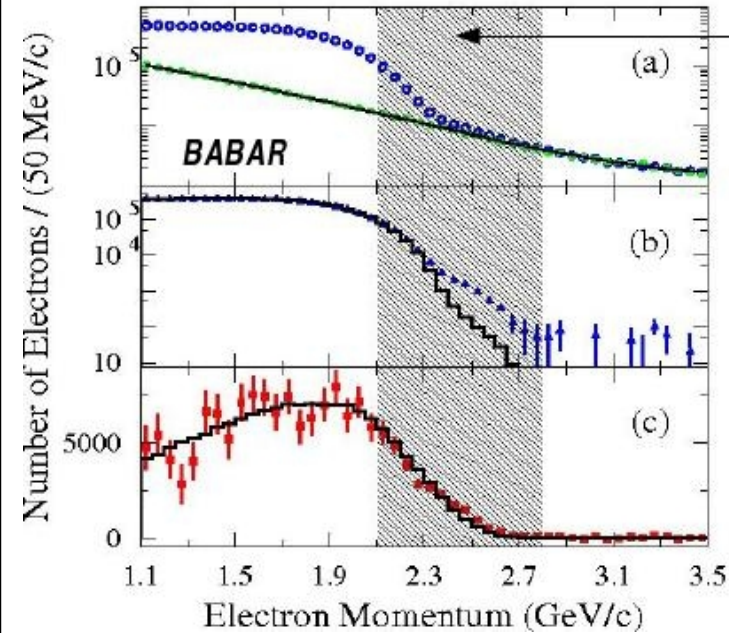
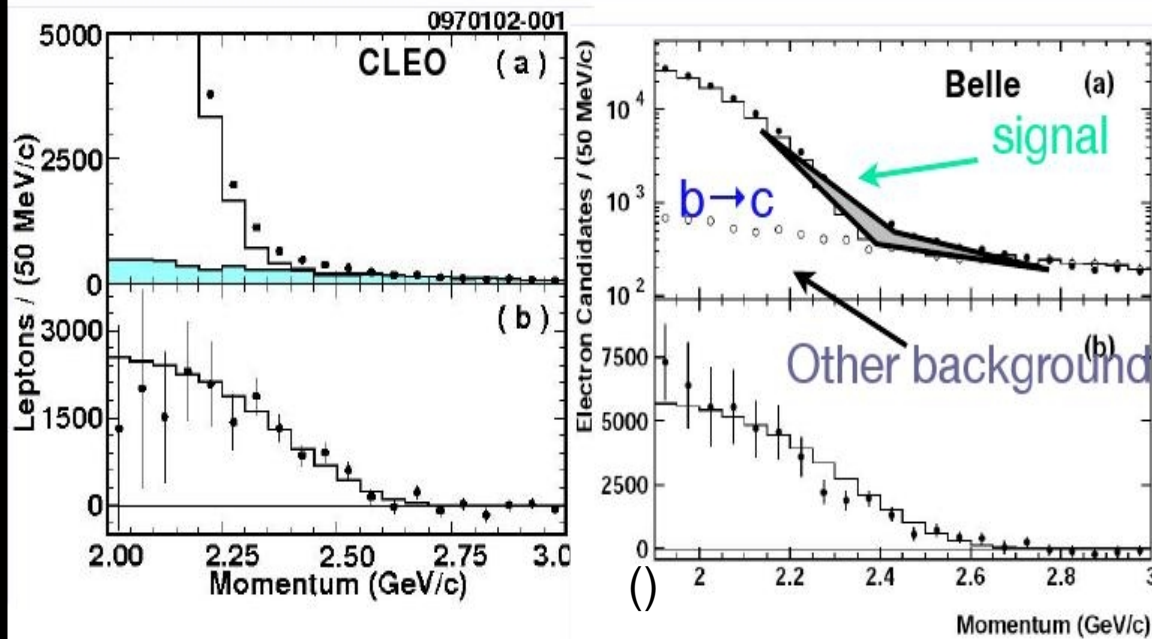
- $m_u \ll m_c$  different kinematics
- measure  $\Delta B(B \rightarrow X_u l \nu)$  in a region where the S/N is good and the  $\Delta\Gamma_u$  is reliably calculable (exclude  $b \rightarrow cl\nu$  decays)
- OPE convergence is compromised ( $O(1/m_b)$ )

$$\Delta B(B \rightarrow X_u l \nu) = \tau_B |V_{ub}|^2 \zeta_c$$

- theoretical acceptances are sensitive to b quark motion (Fermi motion) parametrized by **Shape Function**. Detailed shape not known, in particular the tail but mean and r.m.s constrained ( $B \rightarrow X_c l \nu$  and  $B \rightarrow X_s \gamma$  moments).



# Endpoint method



Subtract offpeak data scaled to onpeak luminosity bin-by-bin;  
Fit MC to data in low energy region to constrain  $B \rightarrow X_c l \nu$  from data

$B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu + B \rightarrow D^* l \nu$  (ratio fixed)

$B \rightarrow D^{**} l \nu$ ,  $B \rightarrow D^{(*)} \pi l \nu$

(Goiti & Roberts)

$B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu + B \rightarrow D^* l \nu$  ( $D/D^*$  fixed)

$B \rightarrow D^{**} l \nu$   $D^{**}/D+D^*$  fitted

Simultaneous fit for non-BB,  
 $B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu$ ,  $B \rightarrow D^* l \nu$ ,  $B \rightarrow D^{**} l \nu$ ,  
 $B \rightarrow D^{(*)} \pi l \nu$ ,  
other background

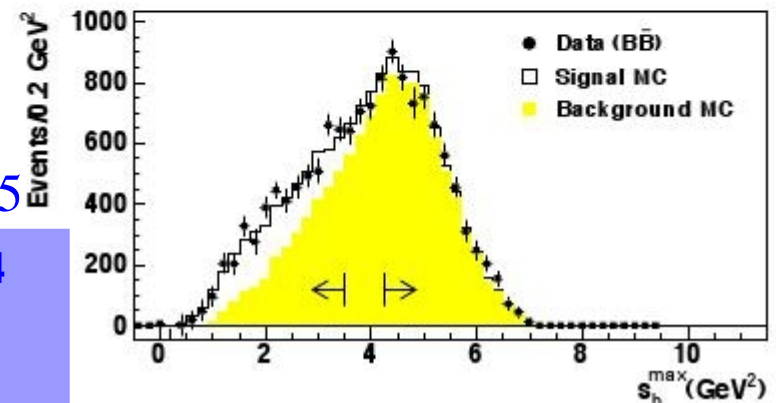
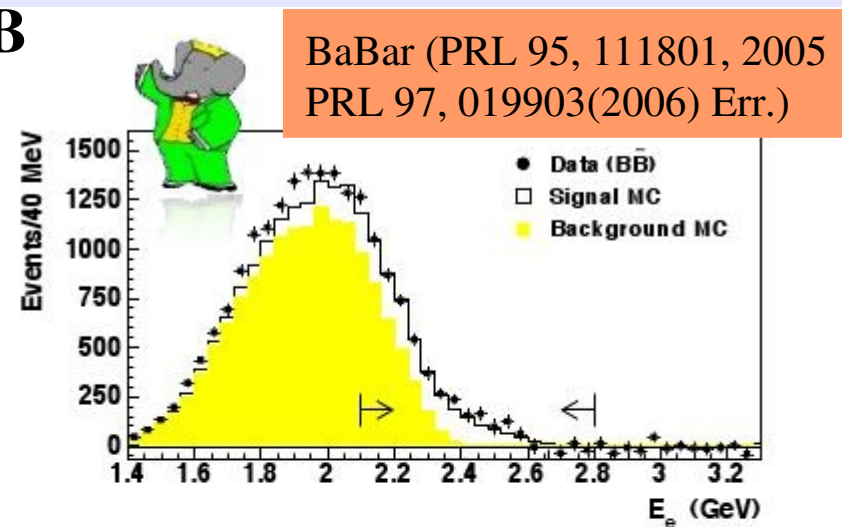
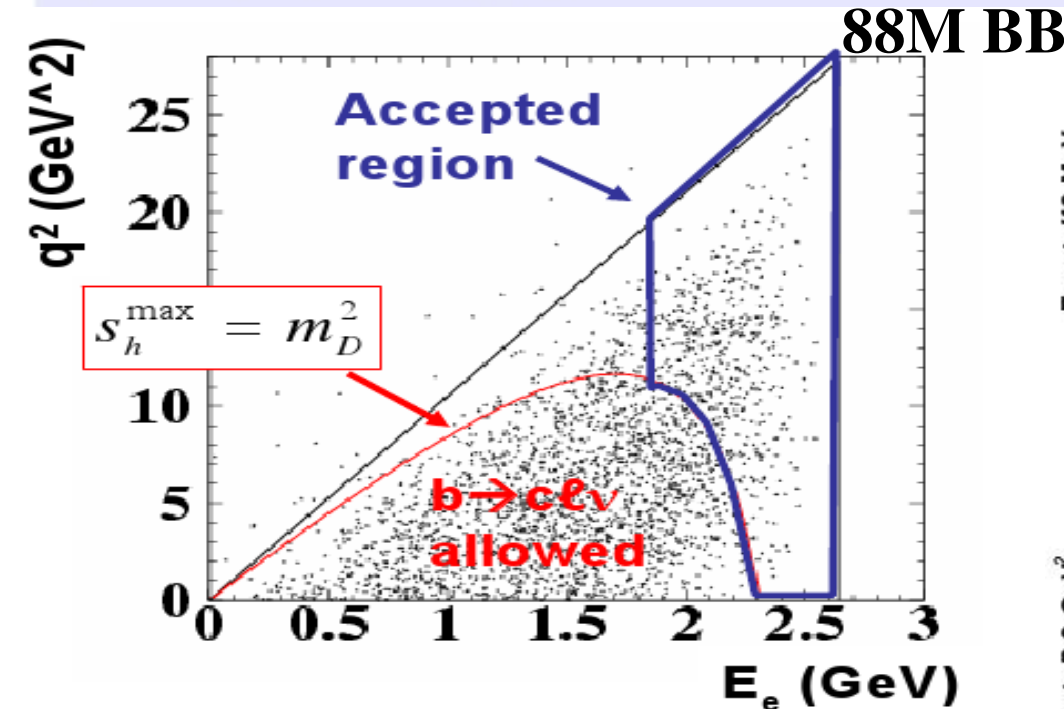
		$E_{cut}$	$\Delta BR \times 10^{-4}$	$ V_{ub} _{BLNP} [x10^{-3}]$
PRL 88, 231803, 2002	Belle	1,9	$8,47 \pm 0,37_{stat} \pm 1,53_{sys}$	$4,74 \pm 0,44_{exp} \pm 0,35_{the}$
PLB621, 28, 2005	BaBar	2,0	$5,72 \pm 0,41_{stat} \pm 0,65_{sys}$	$4,29 \pm 0,24_{exp} \pm 0,35_{the}$
PRD 73, 012006, 2006	CLEO	2,0	$4,22 \pm 0,33_{stat} \pm 1,78_{sys}$	$3,94 \pm 0,46_{exp} \pm 0,37_{the}$

# Improved Endpoint method: the neutrino reconstruction

Maximally allowed hadronic mass for given  $E_l$  &  $q^2$  can be determinate:

$$s_h^{max} = m_B^2 + q^2 - 2m_B(E_l \eta_- + q^2 \eta_+ / 4 E_e), \quad \pm 2 E_e > \pm \sqrt{q^2} \eta_{\pm} \quad \eta_{\pm} = \sqrt{(1 \pm \beta)/(1 \mp \beta)}$$

$$s_h^{max} = m_B^2 + q^2 - 2m_B \sqrt{q^2}, \quad \text{otherwise} \quad \beta \approx 0.06$$



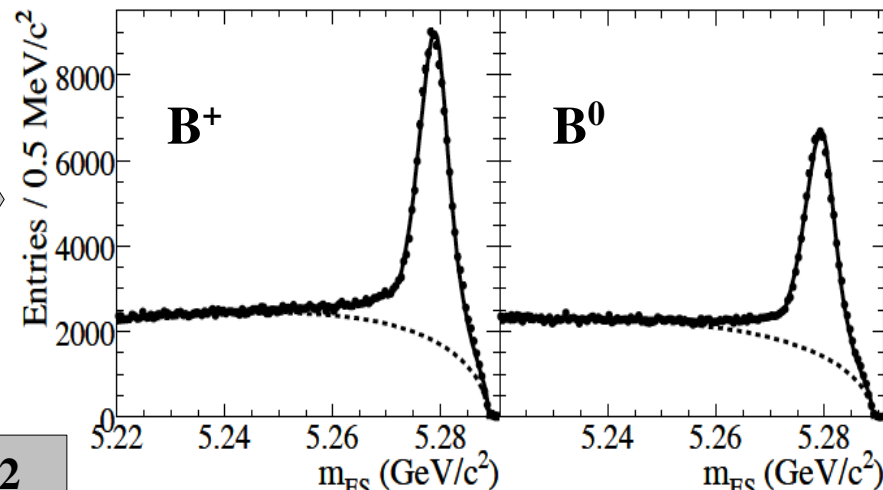
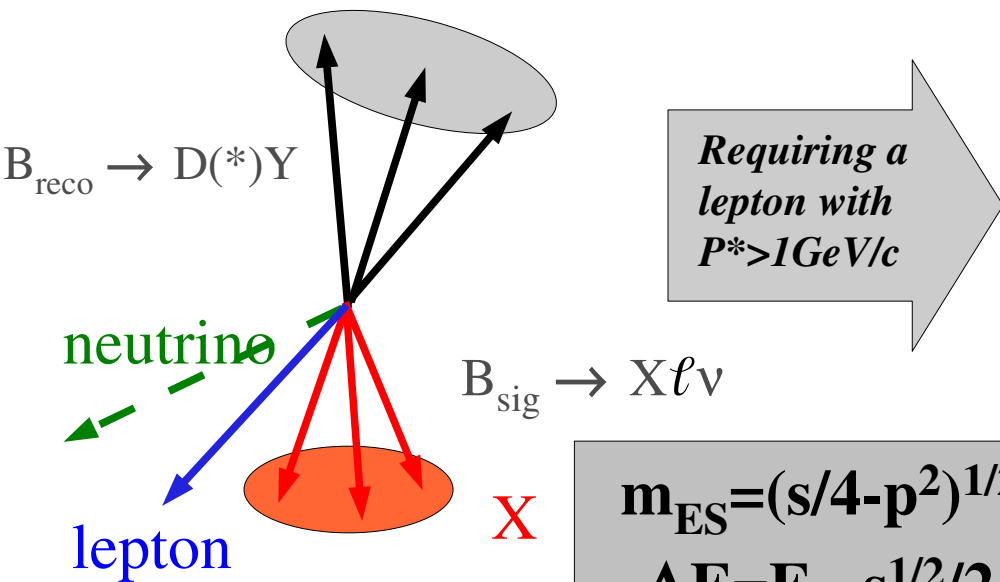
The neutrino reconstruction reduces S/B to  $\sim 0.5$

$$\Delta B(2.0-2.6) = (4.41 \pm 0.42 \pm 0.42) \times 10^{-4}$$

$$|V_{ub}|_{BLNP} = (4.41 \pm 0.30_{\text{exp}} \pm 0.42_{\text{the}}) \times 10^{-3}$$

# Inclusive $|V_{ub}|$ with hadronic tag

Fully reconstruct one B in hadronic decays:  $B \rightarrow D(*)Y$   $Y = n\pi + m\pi^0 + pK_S + qK$



- $\mathbf{P}_{\text{miss}} = \mathbf{P}_Y \text{ (4S)} - \mathbf{P}_{\text{reco}} - \mathbf{P}_X - \mathbf{P}_{\text{lepton}}$
- $\mathbf{m}_X$ : all remaining particles

$$m_{\text{ES}} = (s/4 - p^2)^{1/2}$$

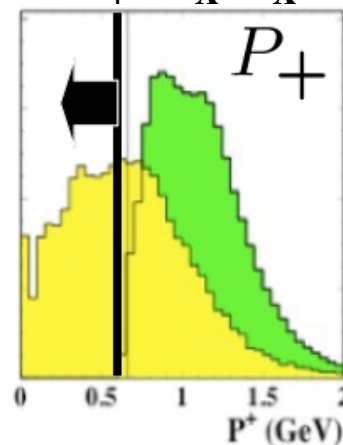
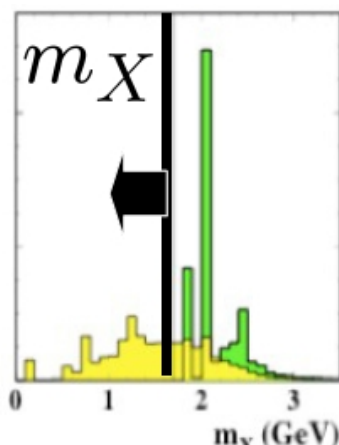
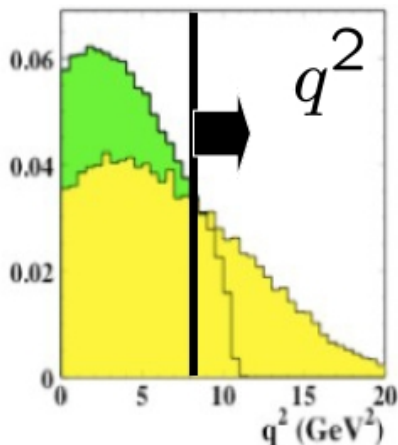
$$\Delta E = E_B - s^{1/2}/2$$

$$P_+ = m_X - |p_X|$$

$m_X$  and  $P_+$  require a sample of  $B_{\text{reco}}$

Experimental Resolution leads to irreducible  $b \rightarrow c\ell\nu$  contamination

Not to scale



$b \rightarrow u/\nu$

$b \rightarrow c/\nu$

# Inclusive $|V_{ub}|$ with hadronic tag: results

Measure the partial branching ratio relative to the number of total semileptonic events

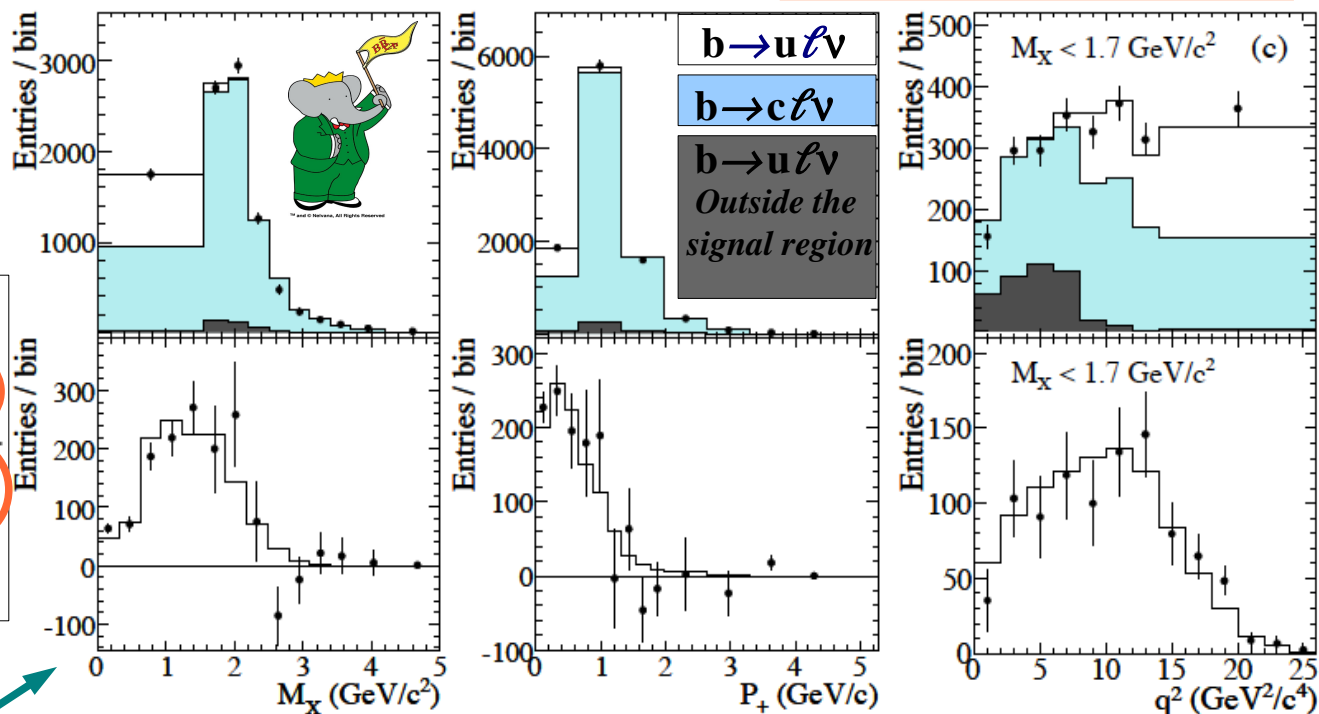
Unfolding factor

$$\frac{\Delta B(X_u \ell \nu)}{B(X \ell \nu)} = \frac{N_{b \rightarrow u}}{N_{X \ell \nu}} \cdot F \cdot \epsilon_{sel}$$

Signal efficiency

## 383M BB

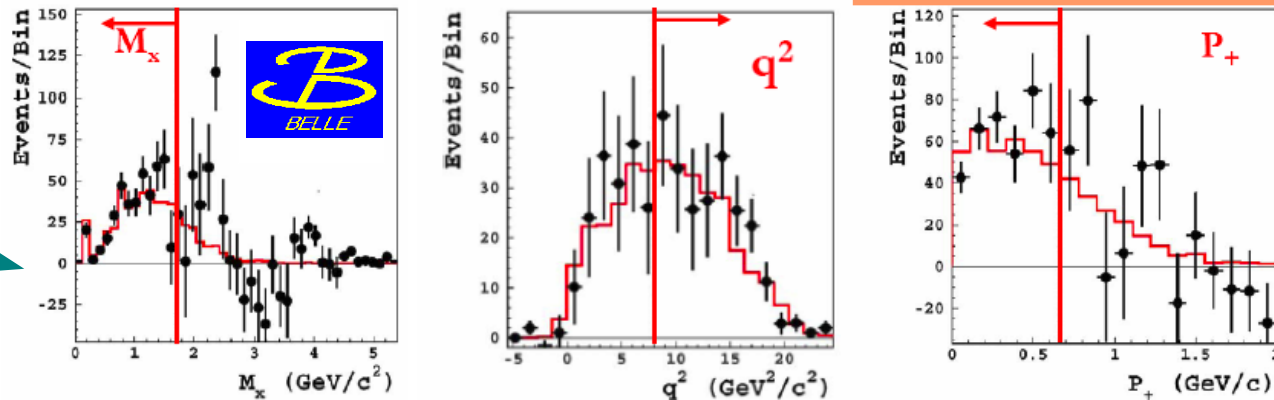
PRL 100, 171802 (2008)



## 375M BB

PRL95,241801(2005)

Background subtracted distributions: not efficiency corrected





# Inclusive $|V_{ub}|$ with hadronic tag: results

Kinematic Region	$N_u$	$ V_{ub}  \times 10^{-3}$	stat.	syst.	theory	Model
$M_X < 1.55 \text{ GeV}/c^2$	$803 \pm 60$	4,27	$\pm 0,16$	$\pm 0,13$	$\pm 0,30$	BLNP
		4,56	$\pm 0,17$	$\pm 0,14$	$\pm 0,32$	DGE
$P_+ < 0.66 \text{ GeV}/c^2$	$633 \pm 63$	3,88	$\pm 0,19$	$\pm 0,16$	$\pm 0,28$	BLNP
		3,99	$\pm 0,20$	$\pm 0,16$	$\pm 0,24$	DGE
$M_X < 1.55 \text{ GeV}/c^2$ & $q^2 > 8 \text{ GeV}^2/c^2$	$562 \pm 55$	4,57	$\pm 0,22$	$\pm 0,19$	$\pm 0,30$	BLNP
		4,64	$\pm 0,23$	$\pm 0,19$	$\pm 0,25$	DGE
		4,93	$\pm 0,24$	$\pm 0,20$	$\pm 0,36$	BLL

Bosh,Lange,Neuber,Paz  
PRL93,221801(2004)  
PRD72,073006(2005)

Andersen,Gardi  
JHEP0601,097(2006)

Bauer,Ligeti,Luke  
PRD64,113004(2001)

**Single analysis  
with 9% tot. uncertainty**

*Compatibility taking into account the large correlations (stat. & syst.) between the different cuts:*

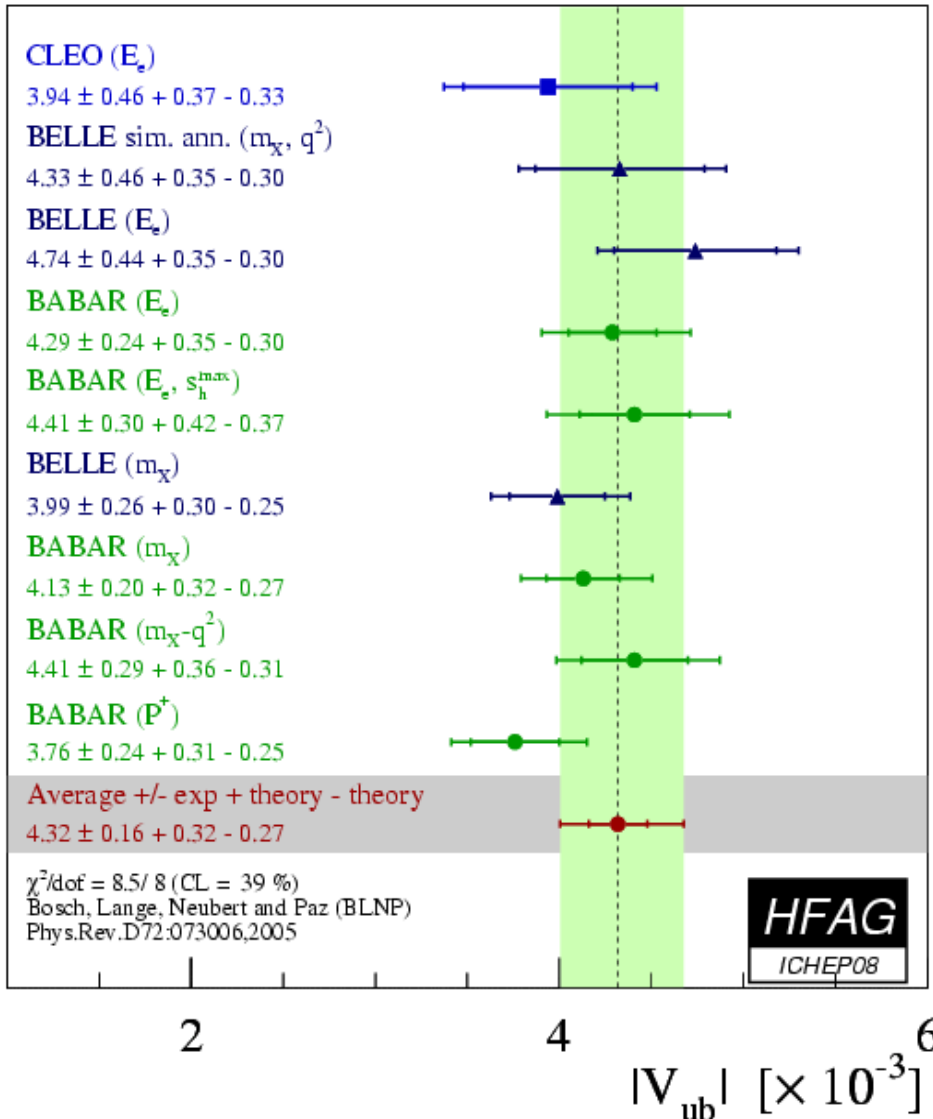
- $m_x$  and  $(m_x, q^2)$  agree at  $1\sigma$
- $P_+$  differs by  $2.5\sigma$

Experimental systematics on  $\Delta B$  ( $B \rightarrow Xu\ell\nu$ ) expressed in %

Method	Detector	Shape function	$\mathcal{B}(\bar{B} \rightarrow Xu\ell\bar{\nu})$ $X_u = \pi, \rho, \dots$	Gluon splitting	$\mathcal{B}(\bar{B} \rightarrow X_c\ell\bar{\nu})$	$B \rightarrow D^*\ell\bar{\nu}$ form factors	$\mathcal{B}(D)$	$m_{ES}$ fit	Monte Carlo statistics	Total
$M_X$	1.92	0.90	2.08	1.62	0.87	0.21	0.44	3.71	3.22	6.07
$P_+$	3.88	1.31	2.22	1.47	2.80	0.39	0.73	3.98	4.62	8.38
$M_X, q^2$	3.83	2.43	2.71	1.02	1.17	0.55	0.79	5.17	4.29	8.81

# $|V_{ub}|$ results (HFAG average, BLNP)

- Many different theoretical approach  
 $\Rightarrow$  many  $|V_{ub}|$  values

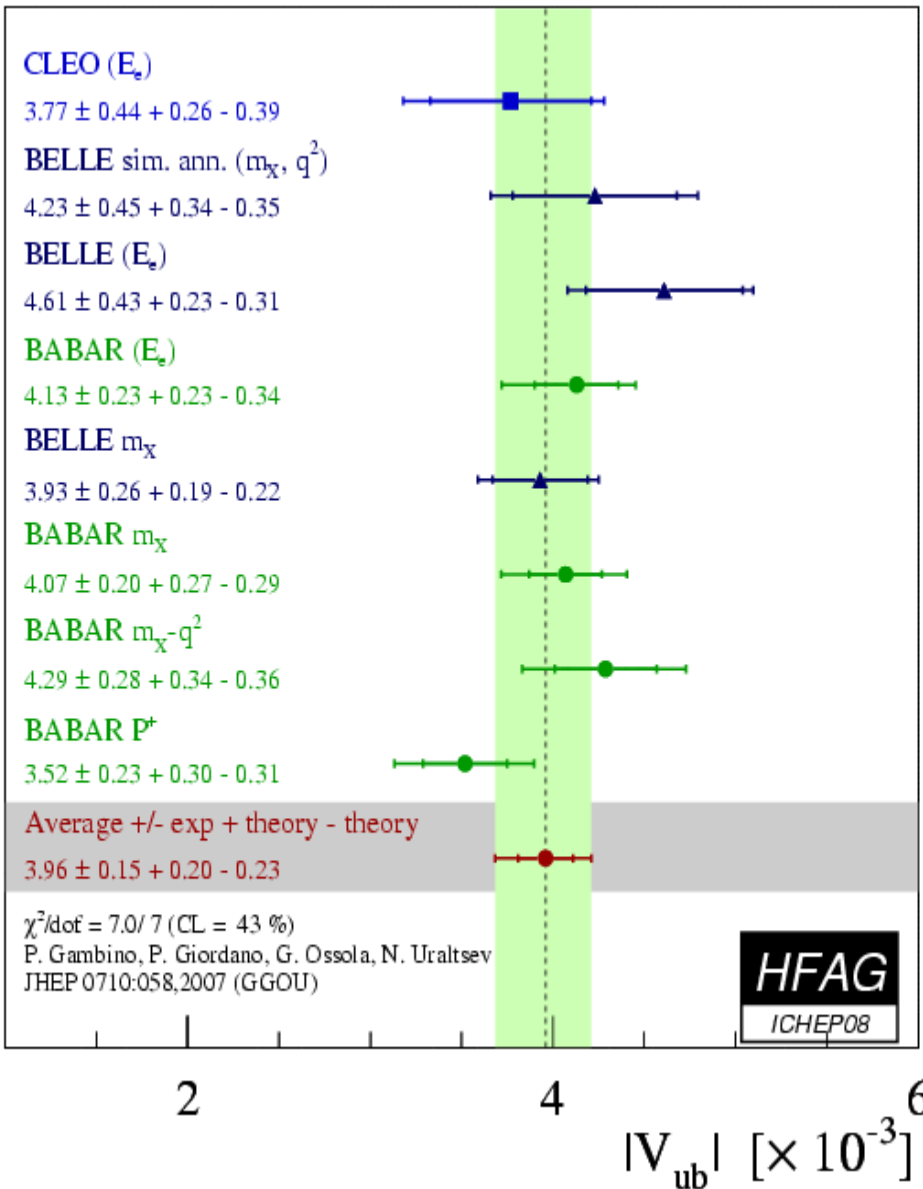


- Here only BLNP, with  $m_b$  from  $B \rightarrow X_c \ell \nu + B \rightarrow X_s \gamma$  Global Fit (Kinetic Scheme), including also uncertainty on the  $KS \Leftrightarrow SF$  Scheme translation

$$|V_{ub}| = (4.32 \pm 0.16 \pm 0.32) \times 10^{-3}$$

$\delta V_{ub} $	+8.2% -7.2%
Statistical	2.1%
Exp.systematics	2.3%
$b \rightarrow c \ell \nu$ model	1.3%
$b \rightarrow u \ell \nu$ model	1.4%
HQ parameters	5.7%
SF + Sub. SF	2.4%
matching	3.7%
Weak Annihilation	1.6%

# $|V_{ub}|$ results (HFAG average, GGOU)



Gambino, Giordano, Ossola, Uraltsev  
JHEP0710:058(2007)

$$|V_{ub}| = (3.96 \pm 0.15 \pm 0.23) \times 10^{-3}$$

$\delta V_{ub} $	+6.3% -7.0%
<b>Statistical</b>	<b>2.3%</b>
<b>Exp.systematics</b>	<b>2.2%</b>
<b><math>b \rightarrow c \ell \nu</math> model</b>	<b>1.5%</b>
<b><math>b \rightarrow u \ell \nu</math> model</b>	<b>1.5%</b>
<b>Non pert.-</b>	<b>3.9%</b>
<b>Higher order par.</b>	<b>1.8%</b>
<b><math>q^2</math> tail model</b>	<b>2.6%</b>
<b>Weak Annihilation</b>	<b>-3.1%</b>

# Determination of HQE parameters

The global fits to  $B \rightarrow X_c \ell \nu$  and  $B \rightarrow X_s \gamma$  moments provide input for the HQE parameters needed in calculating  $B \rightarrow X_u \ell \nu$  partial rates. These HQE parameters are also used to constraint the first and second moments of the shape function.

- HQE connect the inclusive  $b \rightarrow c \ell \nu$  decay width to  $|V_{cb}|$

$$\Gamma_{SL} \sim |V_{cb}|^2 m_b^5 [z_0(r) + 0/m_b + z_2(r, \mu_\pi^2/m_b^2, \mu_G^2/m_b^2) + z_3(r, \rho_D^3/m_b^3, \rho_{LS^3}/m_b^3) + ..]$$

- Similar expressions for moments of various inclusive distributions:

⇒ Hadron mass moments  $\langle M_X^n \rangle_{E > E_{cut}}$

⇒ Lepton energy moments  $\langle E_{l e p^n} \rangle_{E > E_{cut}}$

⇒ Photon energy moments in  $b \rightarrow s \gamma$  :  $\langle E_\gamma^n \rangle_{E > E_{cut}}$



**Extract  $|V_{cb}|$ ,  $BR_{c \ell \nu}$**

**and the HQE parameters**

$m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS^3}$

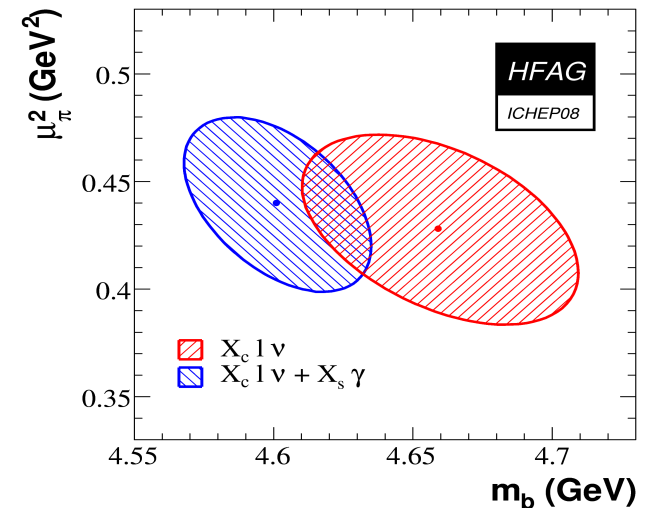
- Global fit with all available results:

✓ Babar, Belle, CLEO, CDF, Delphi

$$m_B(\text{KS}) = 4.601 \pm 0.034 \text{ GeV}$$

$$\mu_\pi^2(\text{KS}) = 0.440 \pm 0.040 \text{ GeV}^2$$

**The  $B \rightarrow X_c \ell \nu$  decays are crucial to reduce the uncertainty for the  $|V_{ub}|$**



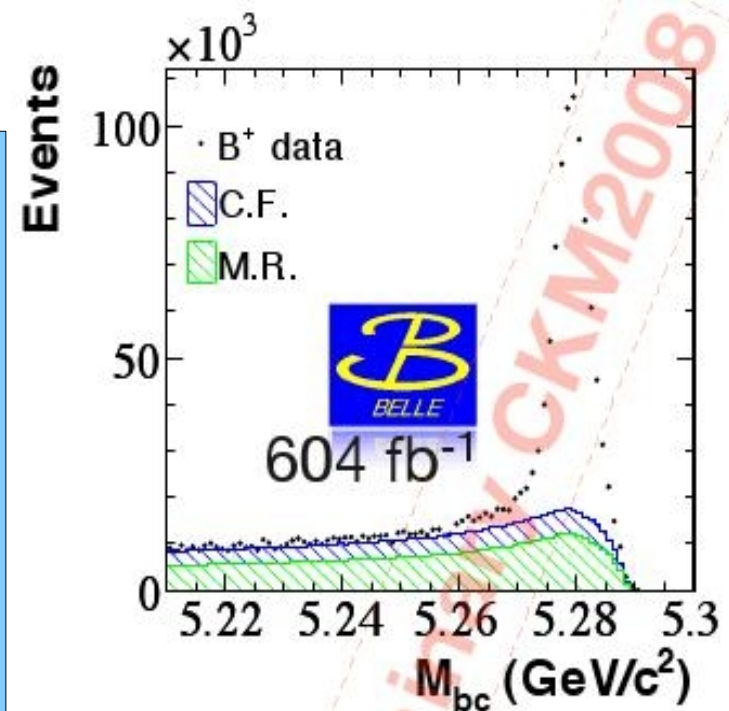
# Belle Multivariate analysis (CKM 2008)

The irreducible uncertainties in the measurements to date are related to limited phase space:

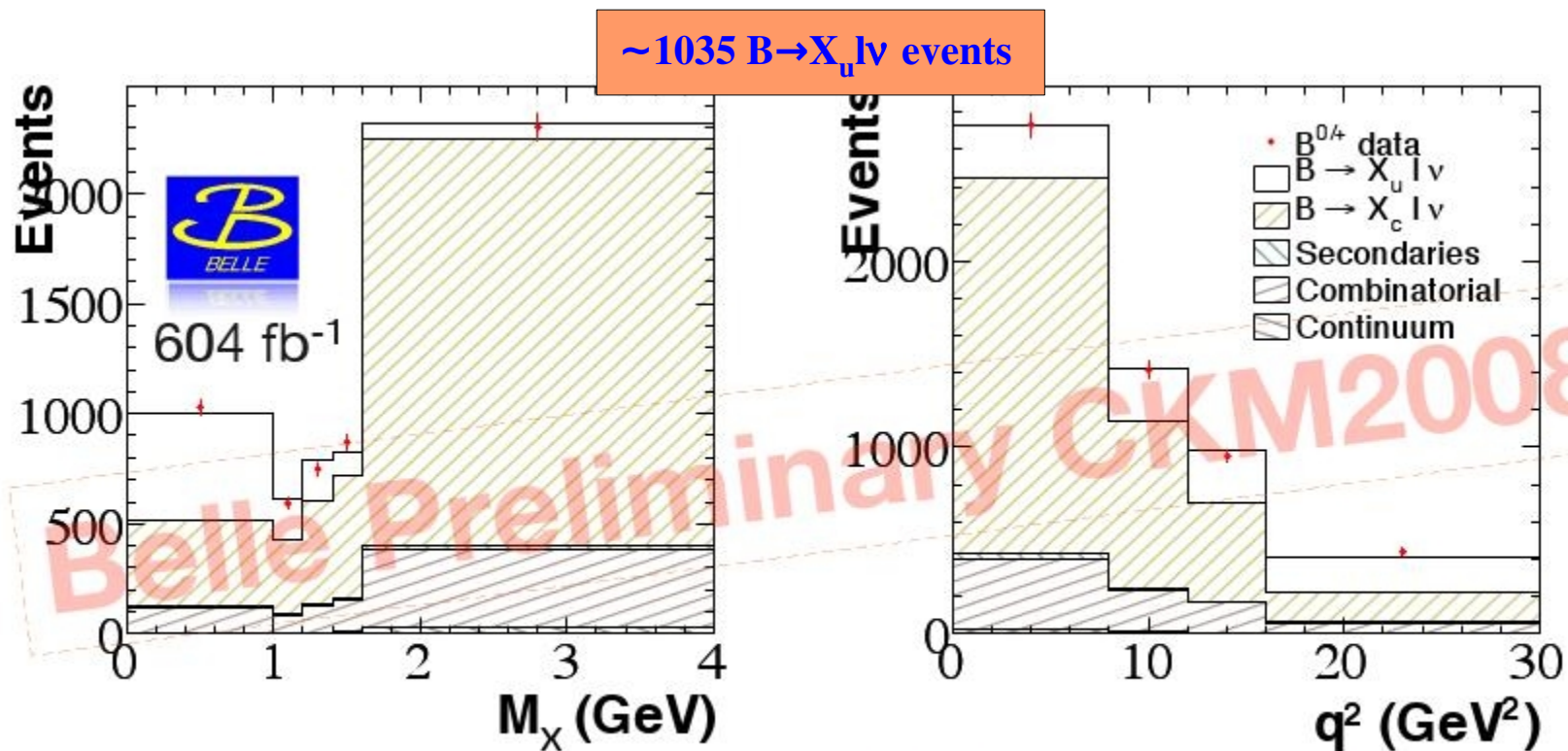
- exploit the many non-linear correlation between kinematic and event variables available in B-beam sample that separate  $b \rightarrow u$  and  $b \rightarrow c$ .
- **Boosted decision tree** based selection, use  $\sim 20$  event parameters from the full reconstruction sample

*No need to place stringent, hard cuts that result in zero efficiency!*

- ✓ Signal side: reconstruct high momentum lepton ( $p_{\text{cms}} > 1 \text{ GeV}/c$ );
- ✓ Event Level:  $Q(B^+_{\text{reco}}) \times Q(\text{lepton}) = -1$ ;
- ✓ BDT cut with many input parameters:  $M^2_{\text{miss}}, Q_{\text{total}}, Q_{\text{lepton}}, N_{\text{lepton}}, Q(B), D^*$  partial reconstruction etc... ;
- ✓ 2D fit to  $M_X, q^2$  with background and signal floated to determine background yield;
- ✓ Measure absolute rate.



# Belle Multivariate analysis: results



2D fit in  $q^2$   $m_X$ . (projections shown)

Kinematic Region	$B(B \rightarrow X_u l \nu) \times 10^{-3}$		$m_b$	$ V_{ub} $ ( $10^{-3}$ ) % error	Theory
	$\Delta$ (stat.)	sys.)			
$P_{\text{lepton}} > 1.0$ GeV	1.96 x (1 ± 0.088 ± 0.076)		(kinetic) 4.613 GeV, $m_{\text{up}} = 0.440 \text{ GeV}^2$	4.42 (± 3.1 ± 5.1)	GGOU (thanks P. Giordano)
			(MSbar) 4.243 GeV	4.47 (± 6.7)	DGE (thanks E. Gardi)

~90% total phase space, thus theory error less correlated to other  $V_{ub}$  determinations

# $|V_{ub}|$ using SF independent analyses

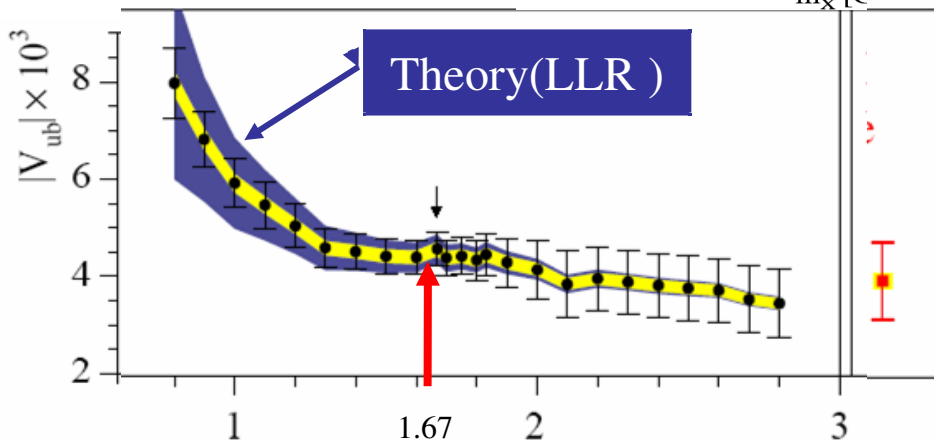
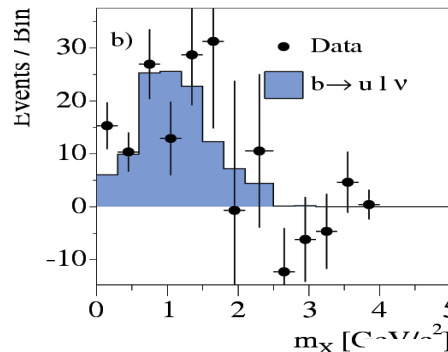
- QCD interaction affecting  $b \rightarrow s\gamma$  and  $b \rightarrow u\ell\nu$  are the same Lange,Neuber,Paz JHEP0510:084 (2005)
- Take ratio of weightheted rates

$$\int_0^{m_{max}} \frac{d\Gamma(b \rightarrow u)}{dm_X} dm_X \longleftrightarrow \int_{E_{min}}^{m_B/2} \frac{d\Gamma(b \rightarrow s\gamma)}{dE_\gamma} W(E_\gamma, E_{min}) dE_\gamma$$

Leibovich,Low,Rothstein  
PRD61,053006(2000)  
PLB513,83 (2001)

PRL96,221801(2006)

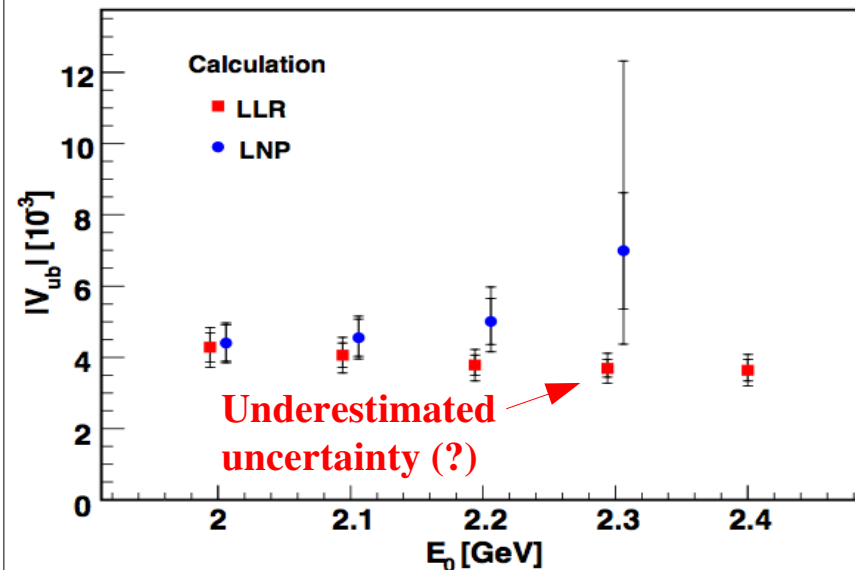
- Measuring  $m_X$  using the  $B_{reco}$  sample
- Statistics limited (89 M BB)



$$|V_{ub}| = (4.43 \pm 0.30_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.29_{\text{theo.}}) \times 10^{-3}$$

Golubev,Skovpen,Luth  
PRD76,114003(2007) using the BaBar data

- Using the  $E_\ell$  endpoint spectrum (BaBar data PRD73,012006(2006)
- consistent results (except at high  $E_\ell$ )



# $|V_{ub}|$ results (different calculation)

HFAG Ave. (BLNP)

$$4.32 \pm 0.16 + 0.32 - 0.27$$

HFAG Ave. (DGE)

$$4.26 \pm 0.14 + 0.19 - 0.13$$

HFAG Ave. (GGOU)

$$3.96 \pm 0.15 + 0.20 - 0.23$$

HFAG Ave. (ADFR)

$$3.76 \pm 0.13 \pm 0.22$$

HFAG Ave. (BLL)

$$4.87 \pm 0.24 \pm 0.38$$

BABAR (LLR)

$$4.92 \pm 0.32 \pm 0.36$$

BABAR endpoint (LLR)

$$4.28 \pm 0.29 \pm 0.48$$

BABAR endpoint (LNP)

$$4.40 \pm 0.30 \pm 0.47$$

Excl. HPQCD

$q^2 > 16 \text{ GeV}^2$

Ufit LP2007



$$|V_{ub}|_{\text{Ufit}} = (3.44 \pm 0.16) \times 10^{-3}$$

$|V_{ub}| [\times 10^{-3}]$

approach  
OPE

*Different theoretical calculation*

*Results vary from  $3.76 \times 10^{-3}$  (AC) to  $4.92 \times 10^{-3}$  (LLR)*

SF free

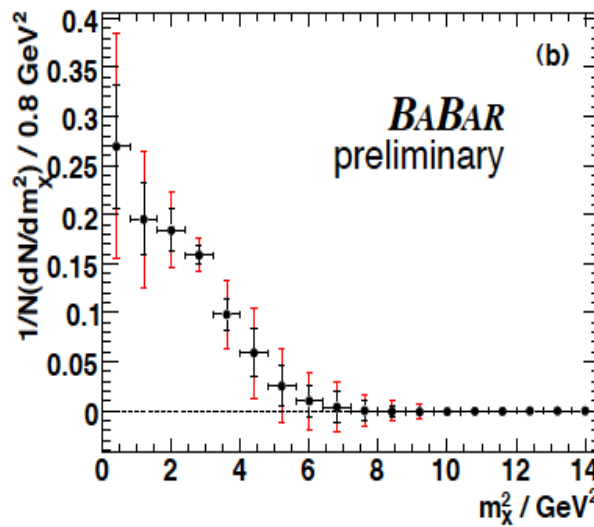
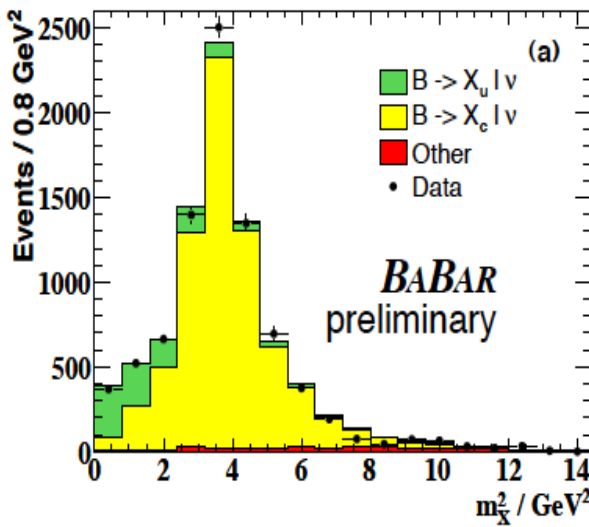
Comparison with Excl. (HPQCD)

$$|V_{ub}|_{\text{excl.}} = (3.40 \pm 0.20^{+0.59}_{-0.39 \text{ FF}}) \times 10^{-3}$$



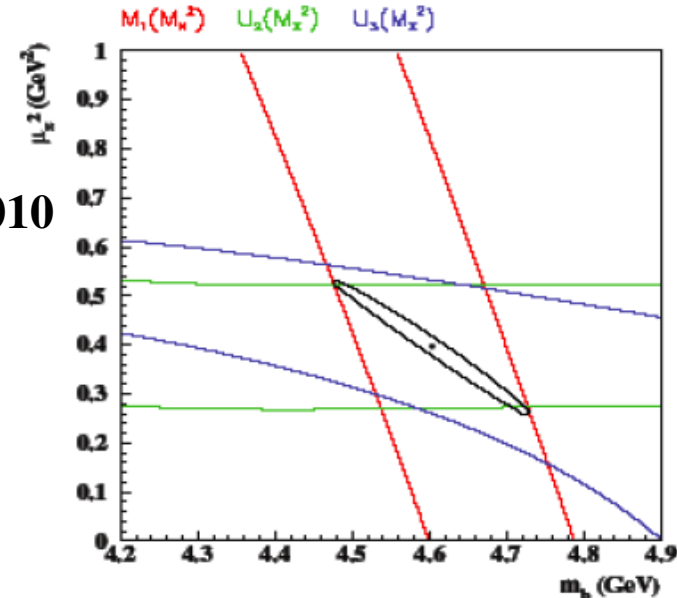
# Hadronic Moments in $B \rightarrow X_u \ell \nu$ decays

- Measure hadronic mass spectrum over full  $m_X$  range
- Mass moments related to  $m_b$ : extract moments with upper cut  $m_X^2 < 6.4 \text{ GeV}^2$



ArXiv: 0801.2985[hep-ex]

Mom	Stat.	Syst.	
<b>M1</b>	$1.96 \pm 0.34$	$\pm 0.53$	$\text{GeV}^2$
<b>U2</b>	$1.92 \pm 0.59$	$\pm 0.87$	$\text{GeV}^4$
<b>U3</b>	$1.79 \pm 0.62$	$\pm 0.78$	$\text{GeV}^6$



Calculations of Gambino, Ossola, Uraltsev JHEP09(2005)010

*First measurement of  $m_b$  in  $B \rightarrow X_u \ell \nu$  decays  
(in the Kinetic scheme)*

$$m_b = 4.604 \pm 0.250 \text{ GeV}$$

*compatible with Global Fit*

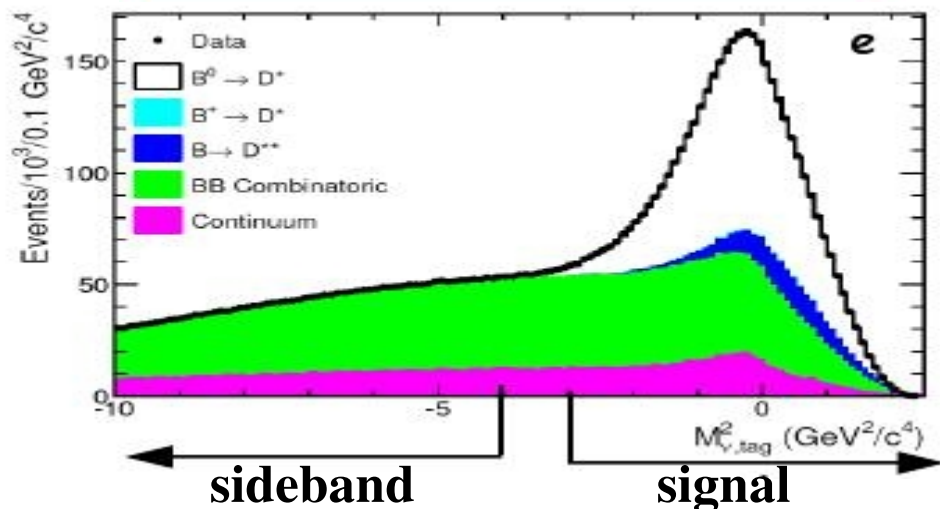
# Weak Annihilation in $B \rightarrow X_u \ell \nu$

- Small contribution to  $B \rightarrow X_u \ell \nu$  decays:

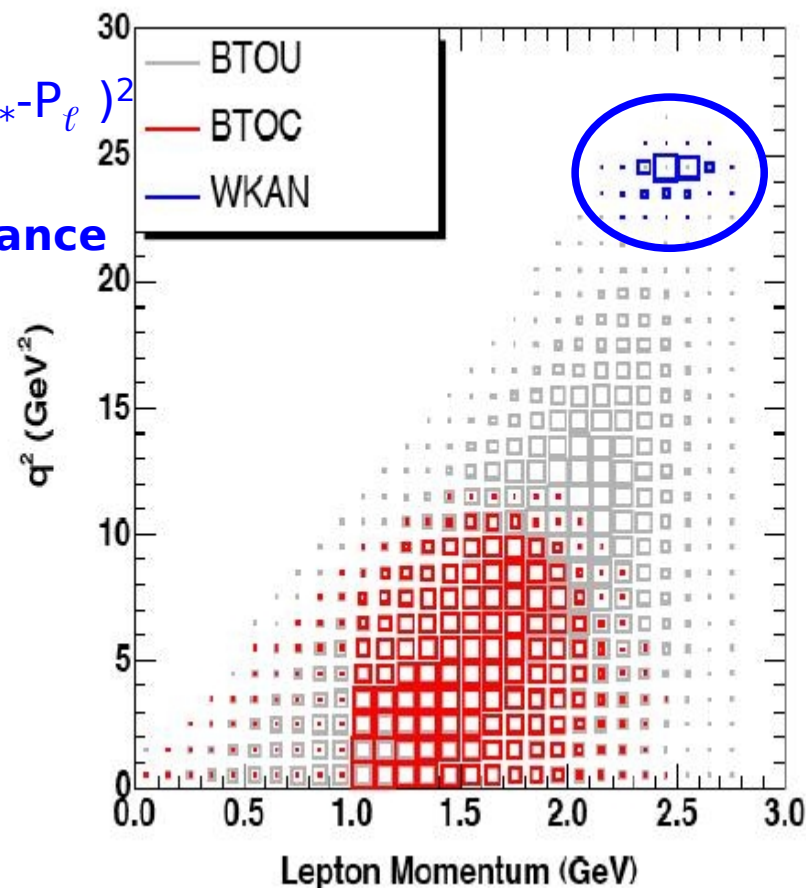
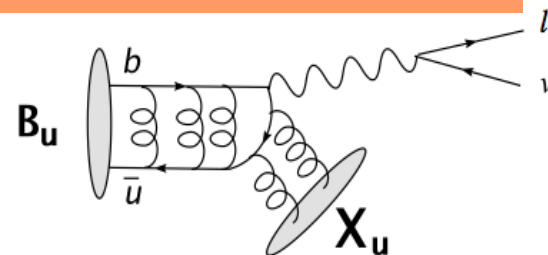
$$\frac{|\Gamma_{WA}|}{\Gamma_u} < 7.4\% \text{ at } 90\% \text{ C.L.} \quad \text{CLEO, studying the } q^2 \text{ spectra} \\ \text{PRL96,121801 (2006)}$$

- Introduce difference between  $B^0$  and  $B^+$  decays
- Tag with partial reconstructed  $B^0 \rightarrow D^{*+} \ell \nu$
- Neutrino mass from kinematics:  $m_\nu^2 = (P_B - P_{D^*} - P_\ell)^2$
- Compare  $B^0$  partial rate to charge averaged  $B \rightarrow X_u \ell \nu$  rate in the large  $p_\ell$  region (to enhance the WA contribution) PRD73,012006(2006)**

$$N_{B^0} = (3606.4 \pm 9.2_{\text{stat}} \pm 47.1_{\text{sist}}) \times 10^3$$

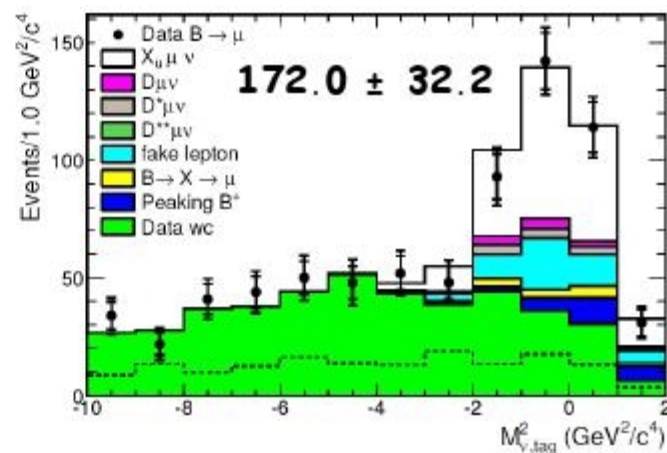
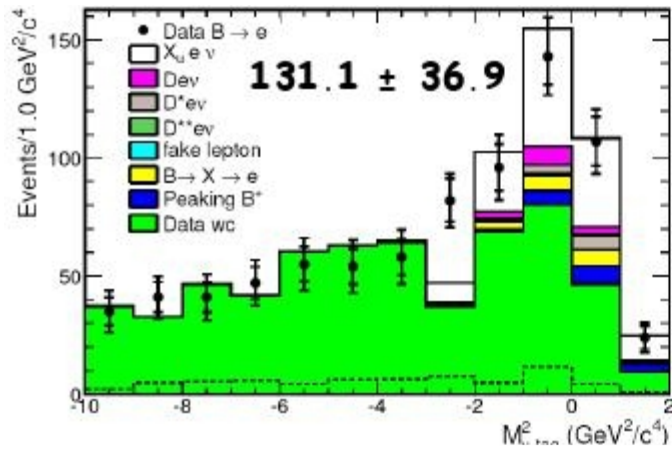


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# Weak Annihilation in $B \rightarrow X_u l \nu$

for  $\Delta p = 2.3-2.6 \text{ GeV}/c \sim 300 B \rightarrow X_u l \nu$  events (  $e + \mu$  )



**Charge asymmetry:**  $A^{+/-0} = \frac{\Delta\Gamma^+ - \Delta\Gamma^0}{\Delta\Gamma^+ + \Delta\Gamma^0}$

$\Delta p$	$\Delta\mathcal{B}(B) \cdot 10^4 [8]$	$\Delta\mathcal{B}(B^0) \cdot 10^4$	$A^{+/-0}$
2.2 – 2.6 GeV/c	$2.31 \pm 0.10 \pm 0.18$	$2.62 \pm 0.33 \pm 0.16$	$-0.17 \pm 0.15 \pm 0.11$
2.3 – 2.6 GeV/c	$1.46 \pm 0.06 \pm 0.10$	$1.30 \pm 0.21 \pm 0.07$	$0.08 \pm 0.15 \pm 0.08$
2.4 – 2.6 GeV/c	$0.75 \pm 0.04 \pm 0.06$	$0.76 \pm 0.15 \pm 0.05$	$-0.05 \pm 0.20 \pm 0.10$

$$\frac{|\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3 - 2.6)} \text{ at } 90\% C.L.$$

-  $f_{WA}(\Delta p)$  is the fraction of WA in a  $\Delta p$  interval

# Conclusion

- **Determination of  $|V_{ub}|$  is crucial to over-constraint UT**
- **Significant improvements of understanding semileptonic decays in the last years; thanks to the continuous theory and experiment interaction;**
- **Inclusive  $B \rightarrow X_u \ell \nu$  achieved  $< 9\%$  error on  $|V_{ub}|$  (crucial the role of  $m_b$ )**
- **Waiting for a complete analysis of the full BaBar dataset (full phase space analysis)**