Heavy flavour decay and fragmentation with the Analityc Coupling model

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In collaboration with:
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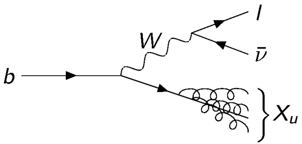
Outline

Inclusive B decays: $B \to X_q + \langle \text{non QCD partons} \rangle, \quad q \equiv u, s$



Semileptonic charmless decay:

$$B \to X_u + l + \bar{\nu}$$

- Energy scales: $m_b \geq E_X \geq m_X, \quad Q = E_X + \sqrt{E_X^2 - m_X^2}$
- To avoid the ~ 50 times larger $B \to X_c l \nu$ background kinematic cuts are necessary: $m_X < m_D \simeq (1.8 GeV), \quad E_l > (m_B^2 - m_D^2)/2m_B, \; q^2 > (m_B - m_D)^2$
- This means $m_X \sim \sqrt{E_X \Lambda_{QCD}} \ll E_X$, $(E_X \sim O(m_B), \quad m_X \gg \Lambda_{QCD})$.

Threshold region $\Rightarrow$ pQCD resummation and modelling Fermi motion.

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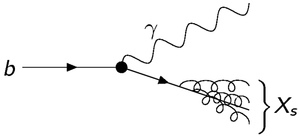

Radiative decay:
$b \to X_s + \gamma$

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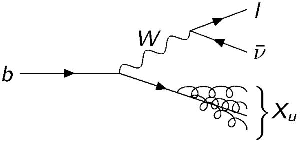

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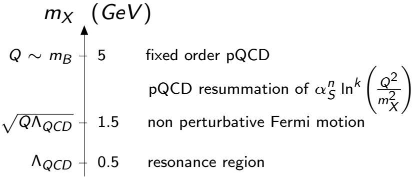

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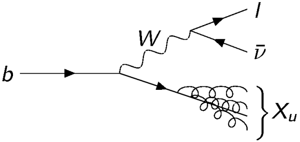

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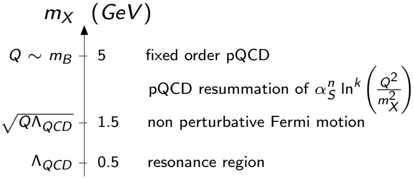

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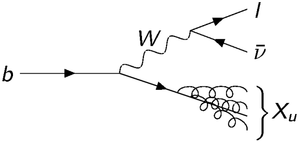

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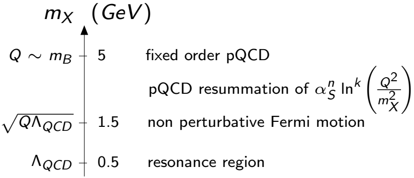

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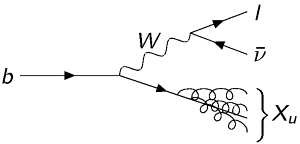

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m_X (GeV)		
$Q \sim m_B$	5	fixed order pQCD
		pQCD resummation of $\alpha_S^n \ln^k \left(\frac{Q^2}{m_X^2} \right)$
$\sqrt{Q\Lambda_{QCD}}$	1.5	non perturbative Fermi motion
Λ_{QCD}	0.5	resonance region

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Threshold region ⇒ pQCD resummation and modelling Fermi motion.

Inclusive B decays: factorization

- Resummation formula for radiative decays:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r[\alpha_S(Q)]\,\sigma[t;\alpha_S(Q)] + d_r[t;\,\alpha_S(Q)]\ ,$$

where $t \equiv m_X^2/m_b^2$ and $Q = 2\,E_X$.
$Q = m_b(1+m_X^2/m_b^2) \simeq m_b, \quad \alpha_S(Q) \simeq \alpha_S(m_b) \simeq 0.22$.

- Resummation formula for semileptonic charmless decays [Aglietti ('01)]:

$$\frac{1}{\Gamma_s}\frac{d^3\Gamma_s}{dxdudw} = C_s\,[x,w;\alpha_S(Q)]\ \sigma\,[u;\alpha_S(Q)] + d_s\,[x,u,w;\alpha_S(Q)]$$

where $x \equiv \frac{2E_l}{m_b}$, $w \equiv \frac{2E_X}{m_b}$, $u \equiv \frac{1-\sqrt{1-4y}}{1+\sqrt{1-4y}}$, $y \equiv \frac{m_X^2}{Q^2} = \frac{m_X^2}{4E_X^2}$.
$Q = m_b(1+m_X^2/m_b^2 - q^2/m_b^2)$; q^2 is the dilepton invariant mass.

We can not put $\alpha_S(Q) \simeq \alpha_S(m_b)$ in the form factor σ: $\alpha_S(Q) = \alpha_S(wm_b)$.

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Inclusive B decays: Threshold resummation

In the Mellin space the threshold resummed form factor reads [Sterman ('87), Catani & Trentadue ('89)]:

$$\ln \sigma_N = \int_0^1 dy \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2y)] + D[\alpha_S(Q^2y^2)] \right\}$$

$$\text{where} \quad A(\alpha_S) = \sum_{n=1}^{\infty} A_n\,\alpha_S^n, \quad B(\alpha_S) = \sum_{n=1}^{\infty} B_n\,\alpha_S^n, \quad D(\alpha_S) = \sum_{n=1}^{\infty} D_n\,\alpha_S^n \ .$$

When $y = \frac{m_X^2}{Q^2} \to 0$ this formula involves $\alpha_S(k_\perp^2)$ evaluated at the Landau pole: it is necessary a prescription (outside pQCD), f.i. the Minimal Prescription [Catani, Mangano, Nason & Trentadue ('96)].

Idea: introduce non-perturbative effects (Fermi motion) in the resummation formula using an effective coupling without the Landau pole [Aglietti, Ricciardi ('04),Aglietti,G.F., Ricciardi ('06)].

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Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda^2_{QCD}$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} \ .$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

$$\bar{\alpha}_S(Q^2) = \frac{1}{2\pi i}\int_0^\infty \frac{ds}{s+Q^2}\, Disc_s\, \alpha_S(-s), \quad \text{space-like.}$$

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- Semi-inclusive B decays are time-like processes:

$$\tilde{\alpha}_S(k_\perp^2) = \frac{i}{2\pi}\int_0^{k_\perp^2} ds\, Disc_s\, \frac{\bar{\alpha}_S(-s)}{s}, \quad \text{time-like.}$$

- At leading order we have:

$$\tilde{\alpha}_S^{lo}(k_\perp^2) = \frac{1}{\beta_0}\left(\frac{1}{2} - \frac{1}{\pi}\arctan\frac{\ln\frac{k_\perp^2}{\Lambda_{QCD}^2}}{\pi}\right),$$

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- The well defined quantity

$$\alpha_0 = \frac{1}{\mu_I}\int_0^{\mu_I}\tilde{\alpha}_S(k_\perp^2)\, dk_\perp \simeq 0.44, \quad \text{with} \quad \mu_I = 2\, GeV$$

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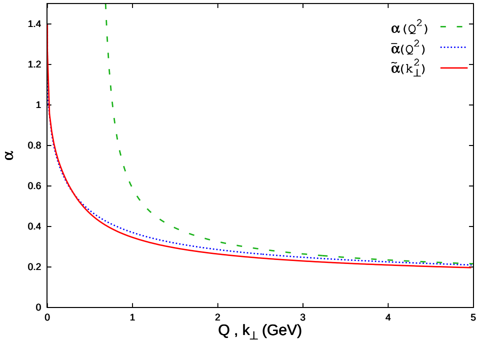
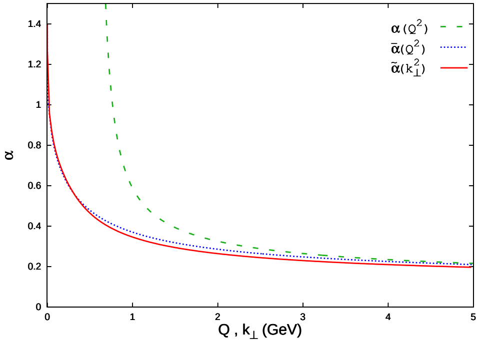


Figure 1: Time-like and space-like analytic couplings compared with the standard one.

We have an improved threshold resummation formula.

$$\ln \sigma_N = \int_0^1 dy \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2 y^2}^{Q^2 y} \frac{dk_\perp^2}{k_\perp^2} \tilde{A}\left[\tilde{\alpha}_S(k_\perp^2)\right] + \tilde{B}\left[\tilde{\alpha}_S(Q^2 y)\right] + \tilde{D}\left[\tilde{\alpha}_S(Q^2 y^2)\right] \right\}$$



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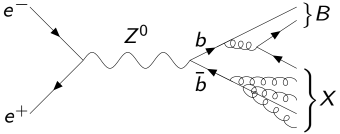
b-quark fragmentation

$$e^+e^- \to Z^0 \to B + X$$



- Energy scales: $m_Z \geq E_b \geq m_b$, $Q = m_Z \sim 90\ GeV$
- Threshold region: $x_b \equiv \frac{2E_b}{m_Z} \to 1$
- The b quark energy distribution factorizes as [Mele,Nason ('91), Cacciari,Catani ('01)]:

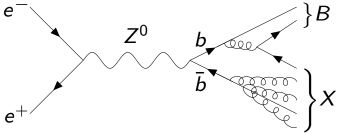
$$\frac{1}{\sigma}\frac{d\sigma}{dx_b}(x_b;m_Z,m_b) = C(x_b;m_Z,\mu_F) \otimes E(x_b;\mu_F,\mu_{0F}) \otimes D^{ini}(x_b;\mu_{0F},m_b)$$

- The soft effects contained in D^{ini} have the same resummed expression as in b decays [Gardi ('05)].

$$\ln D_N^{ini} \sim \int_0^1 dy\, \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2y^2}^{\mu_{0F}^2} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + D[\alpha_S(Q^2y^2)] \right\}$$

b-quark fragmentation

$$e^+e^- \to Z^0 \to B + X$$



- Energy scales: $m_Z \geq E_b \geq m_b$, $Q = m_Z \sim 90\ GeV$
- Threshold region: $x_b \equiv \frac{2E_b}{m_Z} \to 1$
- The b quark energy distribution factorizes as [Mele,Nason ('91), Cacciari,Catani ('01)]:

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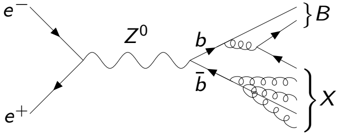

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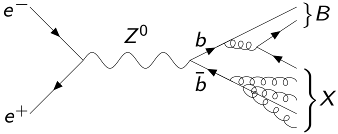

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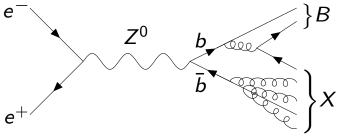

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Phenomenological Analysis

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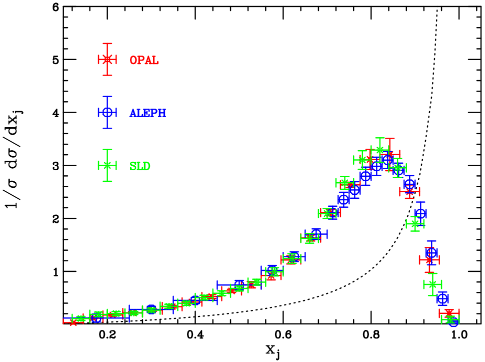

Figure 2: B-hadron spectrum: fixed-order prediction compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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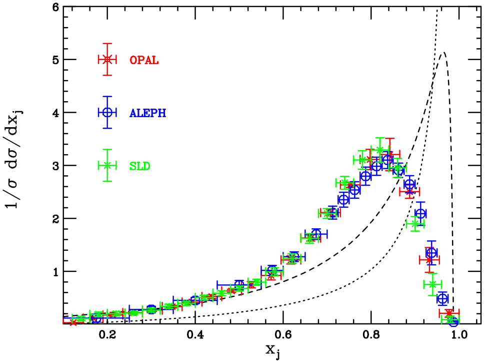

Figure 2: B-hadron spectrum: prediction of the NLL resummation with the MP compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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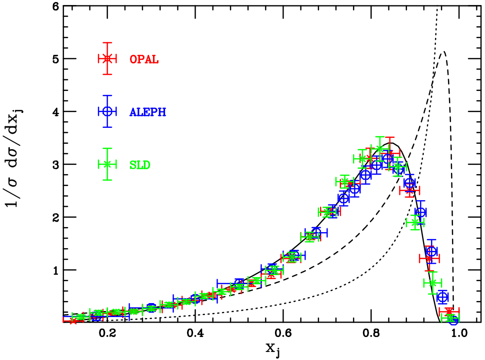


Figure 2: B-hadron spectrum: prediction of the NNLL analytic coupling model [Aglietti,Corcella,G.F. ('06)] compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

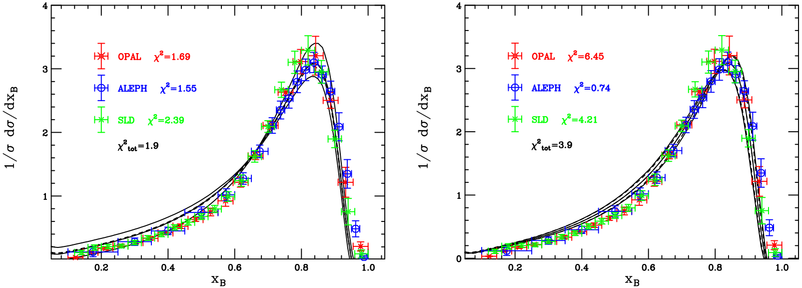


Figure 3: Model dependence on the factorizations scales (left): $\mu_{0F} = m_b/2$, m_b, $2m_b$; $\mu_F = m_Z/2$, m_Z, $2m_Z$ and on $\alpha_S(m_Z)$ and on m_b (right): $\alpha_S(m_Z) = 0.117,\ 0.119,\ 0.121$; $m_b = 4.7,\ 5.0,\ 5.3$ GeV.

Radiative decay: hadron mass and photon energy distribition

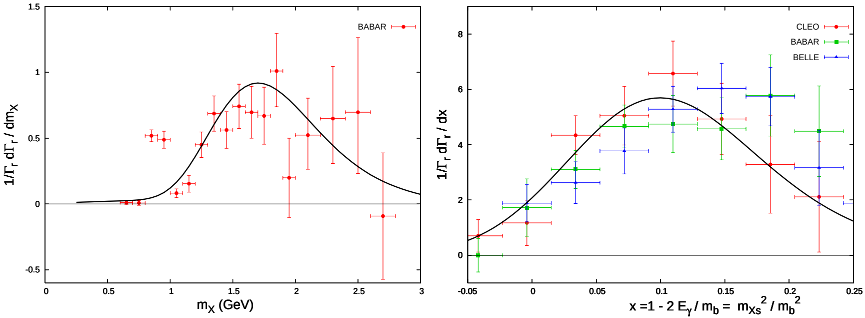
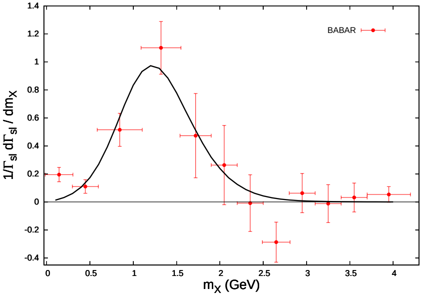


Figure 4: Invariant hadron mass distribution in the radiative decay: prediction of the model compared with the experimental data [BaBar ('05)].The K^* peak cannot clearly be accounted for in a perturbative QCD framework.
Photon energy spectrum in the radiative decay: prediction of the model compared with data [Cleo ('01), BaBar ('05), Belle ('05)]

Semileptonic decay: hadron mass distribution



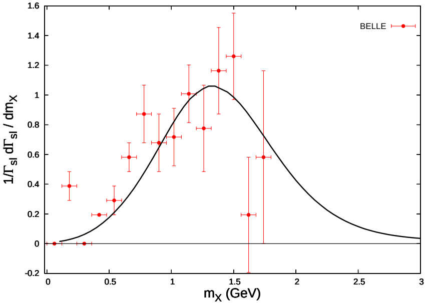


Figure 5: Invariant hadron mass distribution in the semileptonic decay: prediction of the model compared with the experimental [Belle ('04), BaBar ('05)]. Note the π and the ρ peaks at small hadron masses.

Semileptonic decay: electron energy distribution

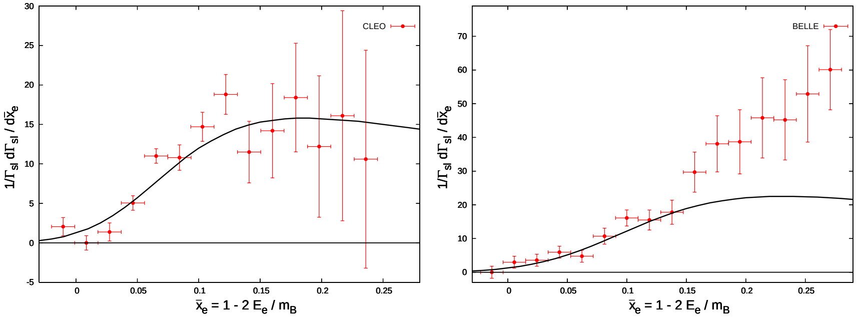


Figure 6: Inclusive charmless electron spectrum in the semileptonic decay: prediction of the model compared with data [Cleo ('01) and Belle ('04)]. To include the Doppler effect, we have convoluted our spectrum with a Gaussian distribution with $\sigma \sim 100$ MeV. Our model predicts a maximum around the energy $E_e = 2$ GeV, below which data are not available.

Semileptonic decay: electron energy distribution

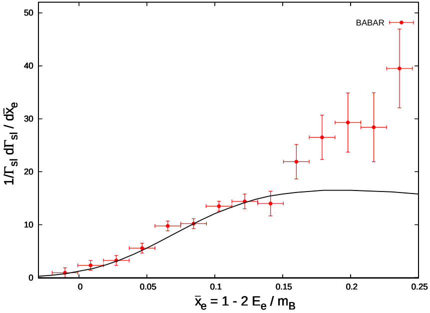
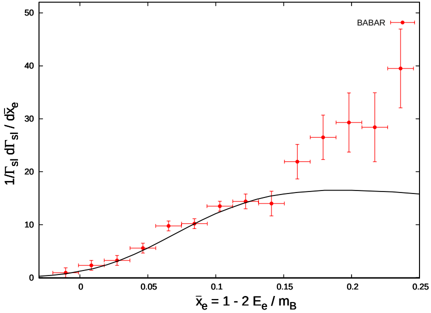
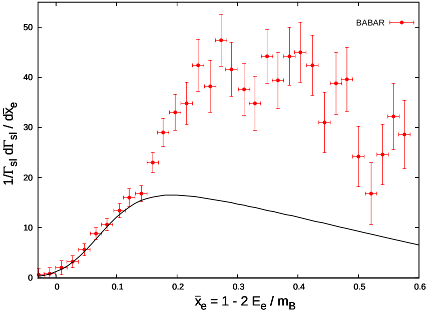


Figure 7: Inclusive charmless electron spectrum in the semileptonic decay: prediction of the model compared with data [Babar ('05)]. To include the Doppler effect, we have convoluted our spectrum with a Gaussian distribution with $\sigma \sim 100$ MeV. We do not known whether the discrepancy is related to a deficiency of our model or to an under-estimate charm background.

Semileptonic decay: electron energy distribution





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Hadron energy distribution in the semileptonic decay

It is the only single differential distribution in the semileptonic decay which (for $E_X > m_b/2$) permits the direct extraction of the QCD form factor.

$$\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dw} = C_{w_1}(\alpha_S)\left\{1 - C_{w_2}(\alpha_S)\Sigma[w-1;\alpha_S(m_b)] + H(w;\alpha_S)\right\} \quad (w>1)$$

where $\Sigma[u;\alpha_S] = \int_0^u du'\sigma(u';\alpha_S)$.

$$\int_1^2 \frac{1}{\Gamma_{sl}}\frac{d\Gamma_{sl}}{dw}dw = 0.2$$

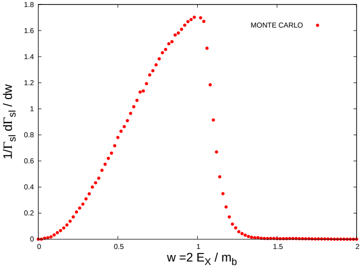

Figure 8: Hadron energy distribution in the semileptonic decay generated by a Monte Carlo based on the model: see the "Sudakov shoulder" [Catani & Webber ('07)].

Extraction of $\alpha_S(m_Z)$

$$b \to u \; l \; \nu$$

$$\begin{aligned}
\alpha_S(m_Z) &= 0.119 \pm 0.003 \qquad (m_{X_u} : \; BABAR) \\
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\end{aligned}$$

$$\alpha_S(m_Z) = 0.1176 \pm 0.0020 \qquad (PDG08)$$

Extraction of $|V_{ub}|$: [Aglietti,Di Ludovico,G.F.,Ricciardi ('08)]

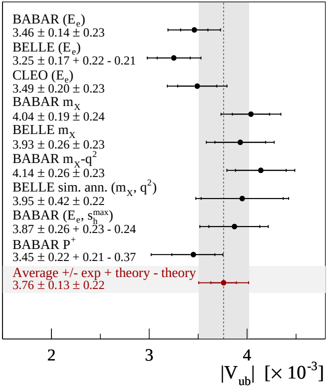

Figure 9: $|V_{ub}|$ values for the uncorrelated analyses and their average

$$|V_{ub}| = 3.76 \pm 0.13_{exp} \pm 0.22_{th} \ .$$

Semi-leptonic charmed decay and V_{cb}

- To describe the process $B \to X_c \; l \; \nu$ we need a new formalism to take in account the non-vanishing charm mass m_c: [Aglietti, Di Giustino, G.F. & Trentadue ('06)]

$$\sigma_N(Q^2, m^2) = \sigma_N(Q^2)\; \delta_N(Q^2, m^2), \qquad r \equiv \frac{m^2}{Q^2} \simeq 0.1$$

$$\ln\delta_N = \int_0^1 dy \frac{(1-y)^{r(N-1)}-1}{y}\left\{-\int_{m^2y^2}^{m^2y}\frac{dk_\perp^2}{k_\perp^2}A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2y)] + D[\alpha_S(m^2y^2)]\right\}$$

- This formula have been checked against explicit α_S order computations: the radiative decay $b \to s\gamma$ with $m_s \neq 0$ and DIS $\nu_\mu + s \to c + \mu$ with $m_c \neq 0$, finding complete agreement.
- Using this formula, the full $O(\alpha_S)$ triple differential distribution [Trott ('04), Aquila, Gambino, Ridolfi & Uraltsev ('05)] and the model previously described we are confident that we can provide quantitative description of the data.

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Conclusions

- Heavy flavour physics in the threshold region is plagued by large logarithmic perturbative corrections and non perturbative effects : all order resummation and a model for non perturbative physics is needed.
- Through the analytic QCD coupling and NNLL threshold resummation we have developed a model that describes with good accuracy the measured spectra.
- The extracted values for $\alpha_S(m_Z)$ are in agreement with the PDG avarage and the extraction of $|V_{ub}|$ is in complete agreement with the SM fit.
- We propose a thresold resummation formula for processes involving jets initiated by massive quarks. It can be used to apply the model in the semileptonic charmed decays.

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Backup Slides

Logarithmic corrections

- The probability for a light quark to evolve into a jet with an invariant mass smaller than m_X is written in leading order as:

$$J(m_X) = 1 + \alpha_S \frac{C_F}{\pi} \int_0^1 \frac{d\omega}{\omega} \int_0^1 \frac{d\theta^2}{\theta^2} \left[\Theta\left(\frac{m_X^2}{Q^2} - \omega\,\theta^2 \right) - 1 \right]$$

$$= 1 - \alpha_S \frac{C_F}{2\,\pi} \log^2\left(\frac{Q^2}{m_X^2} \right).$$

- Both integrals diverge for $\omega = 0$ (soft singularity) and for $\theta = 0$ (collinear singularity), but their sum is finite.

- "Complete" real-virtual cancellation occurs only for $m_X = Q$, i.e. in the completely inclusive evolution of the quark line, while for $m_X < Q$ there is a left-over double logarithm.

- $\alpha_S(m_B) \simeq 0.2$, $\log^2\left(\frac{Q^2}{m_X^2}\right) \sim 6$ if $m_X \sim \sqrt{m_B \Lambda_{QCD}}$

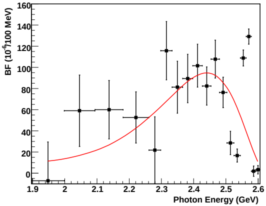
Fermi motion

$$B \to X + \langle \text{non QCD partons} \rangle \qquad p_B = p_X + q \quad (p_b = p_{\hat{X}} + q)$$

$$p_b = p_B + k' \quad \text{with} \quad k' \sim O(\Lambda_{QCD})$$

$$m_{\hat{X}}^2 = (p_b - q)^2 = (p_X + k')^2 = m_X^2 + 2p_X \cdot k' + k'^2 \simeq m_X^2 + 2E_X k'_+$$

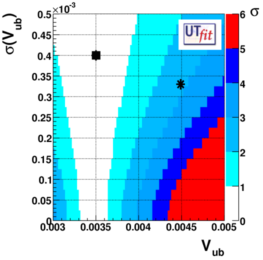
- Fermi motion can be described by shape function $f(k'_+)$ which represents the distribution of the effective mass $m_B + k'_+$ of the heavy quark [Bigi et al.('93)].
- Non-perturbative (f.i. lattice QCD) calculation of $f(k'_+)$ does not exist: models with free parameter to be extracted from the $B \to X_s\gamma$ data have been proposed [Neubert et al.('05); Gambino et al.('07)].
- Experimental data do not permit an accurate extraction of the shape function.



We propose a general model to describes semi-inclusive B decays, based on NNLL threshold resummation and on an effective QCD coupling which we have tested with precise LEP and SLD data.

$$|V_{ub}|^{excl.} = (35.0 \pm 4.0) \times 10^{-4}$$
$$|V_{ub}|^{incl.} = (44.9 \pm 3.3) \times 10^{-4}$$
$$|V_{ub}|^{SM} = (33.4 \pm 1.6) \times 10^{-4}$$

[UTfit Coll. ('06)]



Radiative decay $B \to X_s\, \gamma$

- Invariant mass distribution:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r[\alpha_S(Q)]\,\sigma[t;\alpha_S(Q)] + d_r[t;\alpha_S(Q)]\ ,$$

where $t \equiv m_X^2/m_b^2$ and $Q = 2\,E_X$.
$Q = m_b(1+m_X^2/m_b^2) \simeq m_b, \quad \alpha_S(Q) \simeq \alpha_S(m_b) \simeq 0.22$.

- $C_r(\alpha_S) = C_r^{(0)} + \alpha_S\, C_r^{(1)} + \cdots$
short-distance (process dependent) hard factor.
- $\Sigma(t;\alpha_S) = \int_0^t \sigma(t';\alpha_S)dt' = \sum_{n=0}^{\infty}\sum_{k=0}^{2n} \Sigma_{n,k}\, \alpha_S^n\, \ln^k(1/t)$
long-distance dominated (universal) QCD form factor.
- $d_r(t;\alpha_S) = d_r^{(0)}(t) + \alpha_S\, d_r^{(1)}(t) + \cdots$
short-distance (process dependent) remainder function, to have good approximation also in the region $m_X \leq E_X$: $\lim_{t\to 0}\int_0^t d_r(t';\alpha_S)dt' = 0$.

Semileptonic charmless decay $B \to X_u + l + \nu$

- Triple differential distribution reads [Aglietti ('01)]:

$$\frac{1}{\Gamma_s}\frac{d^3\Gamma_s}{dxdudw} = C_s\left[x, w; \alpha_S(Q)\right] \sigma\left[u; \alpha_S(Q)\right] + d_s\left[x, u, w; \alpha_S(Q)\right]$$

where $x \equiv \frac{2E_l}{m_b}$, $w \equiv \frac{2E_X}{m_b}$, $u \equiv \frac{1-\sqrt{1-4y}}{1+\sqrt{1-4y}}$, $y \equiv \frac{m_X^2}{Q^2} = \frac{m_X^2}{4E_X^2}$.
$Q = m_b(1 + m_X^2/m_b^2 - q^2/m_b^2)$; q^2 is the dilepton invariant mass.

We can not put $\alpha_S(Q) \simeq \alpha_S(m_b)$ in the form factor σ: $\alpha_S(Q) = \alpha_S(wm_b)$.

- $C_s(x, w;\, \alpha_S) = C_s^{(0)}(x, w) + \alpha_S\, C_s^{(1)}(x, w) + \cdots$
short-distance (process dependent) hard factor.
- $d_s(x, u, w; \alpha_S) = d_s^{(0)}(x, u, w) + \alpha_S\, d_s^{(1)}(x, u, w) + \cdots$
short-distance (process dependent) remainder function.

Non universality effects

- Universality of long-distance effects studied by comparing the logarithmic structure of different spectra.
- Spectra not involving integration over hadron energy: same infrared structure of the hadron invariant mass distribution of the radiative decay i.e pure short-distance relation

$$\Sigma(u;\alpha_S) = \int_0^u \sigma(u';\alpha_S)du' = 1 + \sum_{n=1}^{\infty}\sum_{k=1}^{2n} \Sigma_{n,k}\,\alpha_S^n \ln^k \frac{1}{u}$$

- Spectra involving integration over hadron energy: different infrared structure from each other and from the hadron invariant mass distribution of the radiative decay i.e not pure short-distance relation [Aglietti, Ricciardi & G.F. ('05)].

$$\Sigma_U(u;\alpha_S) = \frac{\int_0^1\int_0^w C(x,w;\alpha_S)dx\Sigma(u;\alpha_S(wm_b))dw}{\int_0^1\int_0^w C(x,w;\alpha_S)\,dx\,dw} = 1 + \sum_{n=1}^{\infty}\sum_{k=1}^{2n}\Sigma_{U\,n,k}\alpha_S^n \ln^k \frac{1}{u}$$

Threshold resummation with analytic coupling

- The threshold resummation formula with the analytic coupling reads

$$\ln \sigma_N = \int_0^1 dy \, \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} \tilde{A}[\tilde{\alpha}_S(k_\perp^2)] + \tilde{B}[\tilde{\alpha}_S(Q^2y)] + \tilde{D}[\tilde{\alpha}_S(Q^2y^2)] \right\}$$

- The coefficients for the time-like coupling are obtained by imposing the equality:

$$A(\alpha_S) = \tilde{A}(\tilde{\alpha}_S), \quad B(\alpha_S) = \tilde{B}(\tilde{\alpha}_S), \quad D(\alpha_S) = \tilde{D}(\tilde{\alpha}_S),$$

- Expressing the time-like coupling in terms of the standard one, we obtain:

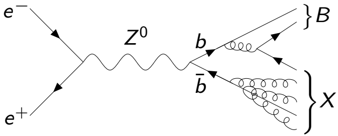
$$\tilde{A}_1 = A_1; \quad \tilde{A}_2 = A_2; \quad \tilde{A}_3 = A_3 + \frac{(\pi\beta_0)^2}{3} A_1 \simeq 0.31 + 0.72 \simeq 1;$$

analogous relations hold for $\tilde{B}_i$ and $\tilde{D}_i$.

- As part of our model the Mellin integration and the inversion to x-space are performed exactly in numerical way

b-quark fragmentation

$$e^+e^- \to Z^0 \to B + X$$



- Energy scales: $m_Z \geq E_b \geq m_b$, $Q = m_Z \sim 90\ GeV$
- Threshold region: $x_b \equiv \frac{2\,E_b}{m_Z} \to 1$
- At α_S order $x_b + x_{\bar{b}} + \omega = 2$, we obtain $1 - x_b = \frac{1}{2} x_{\bar{b}}\, \omega\, (1 - \cos\theta_{g\bar{b}})$
- We have resummed NNLL large logarithmic contributions that affect the spectrum in the threshold region ($x_b \to 1$). [Aglietti, Corcella & G.F. ('06)].

- The energy distribution of the b quark factorizes as

$$\frac{1}{\sigma}\frac{d\sigma}{dx_b}(x_b;m_Z,m_b)=C(x_b;m_Z,\mu_F)\otimes E(x_b;\mu_F,\mu_{0F})\otimes D^{ini}(x_b;\mu_{0F},m_b)$$

  - $C(x_b;\ m_Z,\mu_F)$ is a coefficient function, describing the emission off a light parton.
  - $E(x_b;\mu_F,\mu_{0F})$ is an evolution operator from the scale $\mu_F \sim m_Z$ down to $\mu_{0F} \sim m_b$. It resums mass logarithms $\ln^k \frac{m_Z^2}{m_b^2}$;
  - $D^{ini}(x_b;\ \mu_{0F},m_b)$ is the initial condition of the perturbative fragmentation function at the scale $\mu_{0F} \simeq m_b$.
- The soft effects are contained in D^{ini} has the same resummed expression as the (perturbative) shape function in b decays

$$\ln D_N^{ini} \sim \int_0^1 dy\, \frac{(1-y)^{N-1}-1}{y}\left\{\int_{Q^2y^2}^{\mu_{0F}^2}\frac{dk_\perp^2}{k_\perp^2}\, A[\alpha_S(k_\perp^2)] + D[\alpha_S(Q^2y^2)]\right\}$$

Phenomenological Analysis

b-quark fragmentation: $e^+e^- \to Z^0 \to B + X, \qquad x_b = \frac{2E_b}{m_Z}$

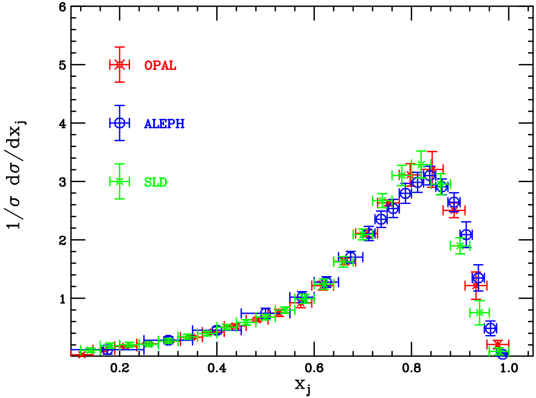


Figure 2: *B*-hadron spectrum: experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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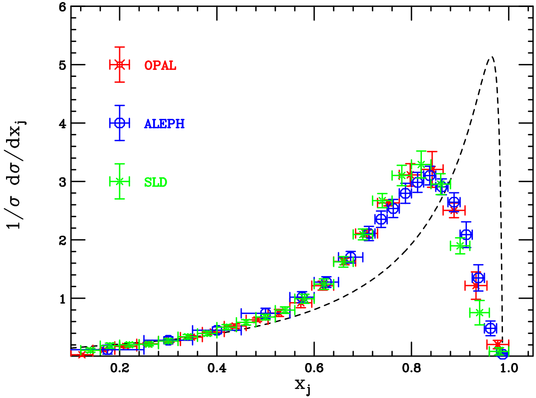

Figure 2: B-hadron spectrum: prediction of the NLL resummation with the MP compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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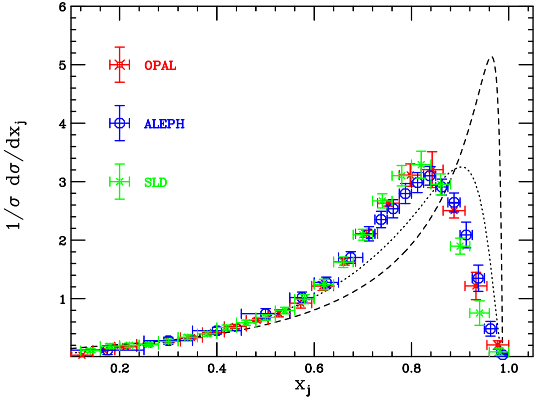

Figure 2: B-hadron spectrum: prediction of the NLL analytic coupling model compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

Phenomenological Analysis

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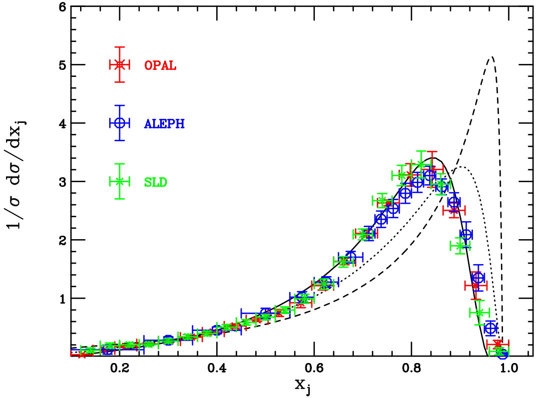

Figure 2: B-hadron spectrum: prediction of the NNLL analytic coupling model compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

A possible new measure

Hadronic energy distribution in the semileptonic decay

- The QCD form factor can be experimentally measured from the m_X or the E_γ distribution of the radiative decay:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r(\alpha_S)\,\sigma[t;\alpha_S(m_b)] + d_r(t;\,\alpha_S)\ ,$$

- The only single differential distribution in the semileptonic decay which permits the direct extraction of the QCD form factor is the hadronic energy distribution for $w \equiv 2E_X/m_b > 1$:

$$\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dw} = C_{w_1}(\alpha_S)\Big\{1 - C_{w_2}(\alpha_S)\Sigma[w-1;\alpha_S(m_b)] + H(w;\alpha_S)\Big\} \quad (\text{w}>1)$$

where $\Sigma[u;\alpha_S] = \int_0^u du'\sigma(u';\alpha_S)$.

$$\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dw} = L^{(0)}(w) + \alpha_S L^{(1)}(w) + O(\alpha_S^2) \quad (\text{w}<1)$$

Semileptonic decay: hadronic energy distribution

$$\int_1^2 \frac{1}{\Gamma_{sl}} \frac{d\Gamma_{sl}}{dw} dw = 0.2$$

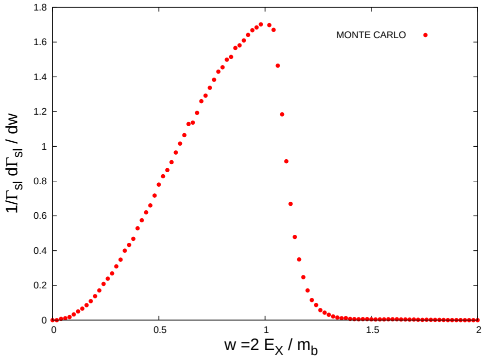

Figure 8: Hadronic energy distribution in the semileptonic decay generated by a Monte Carlo based on the model: see the "Sudakov shoulder".