Landau poles in grand unification

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Grand Unified Theory (GUT)

Simplicity, minimality with respect to the Standard Model (SM)

- in SM 3 gauge couplings $g_{1,2,3}$, $\rightarrow$ in GUT only 1: g_{GUT}
- in SM 5 representations Q, L, u^c, d^c, e^c $\rightarrow$ in GUT ≤ 2

$$(Q, u^c, e^c) = 10\ ; \quad (d^c, L) = \bar{5} \qquad \text{in} \ \ SU(5)$$

$$(Q, u^c, e^c, d^c, L, \nu^c) = 16 \qquad \text{in} \ \ SO(10)$$

$$(Q, u^c, e^c, d^c, L, \nu^c, d', L'^c, d'^c, L', s) = 27 \qquad \text{in} \ \ E_6$$

- in SM 4 Yukawas $Y_{U,D,E,N}$ $\rightarrow$ in GUT typically only 2 $\rightarrow$ predictions

- the GUT gauge structure explains electric charge quantization

  $\bar{5} = (d_1^c, d_2^c, d_3^c, \nu, e) \to 3q_{d^c} + q_\nu + q_e = 0 \to q_{d^c} = -q_e/3$

  $\to$ existence of magnetic monopoles predicted

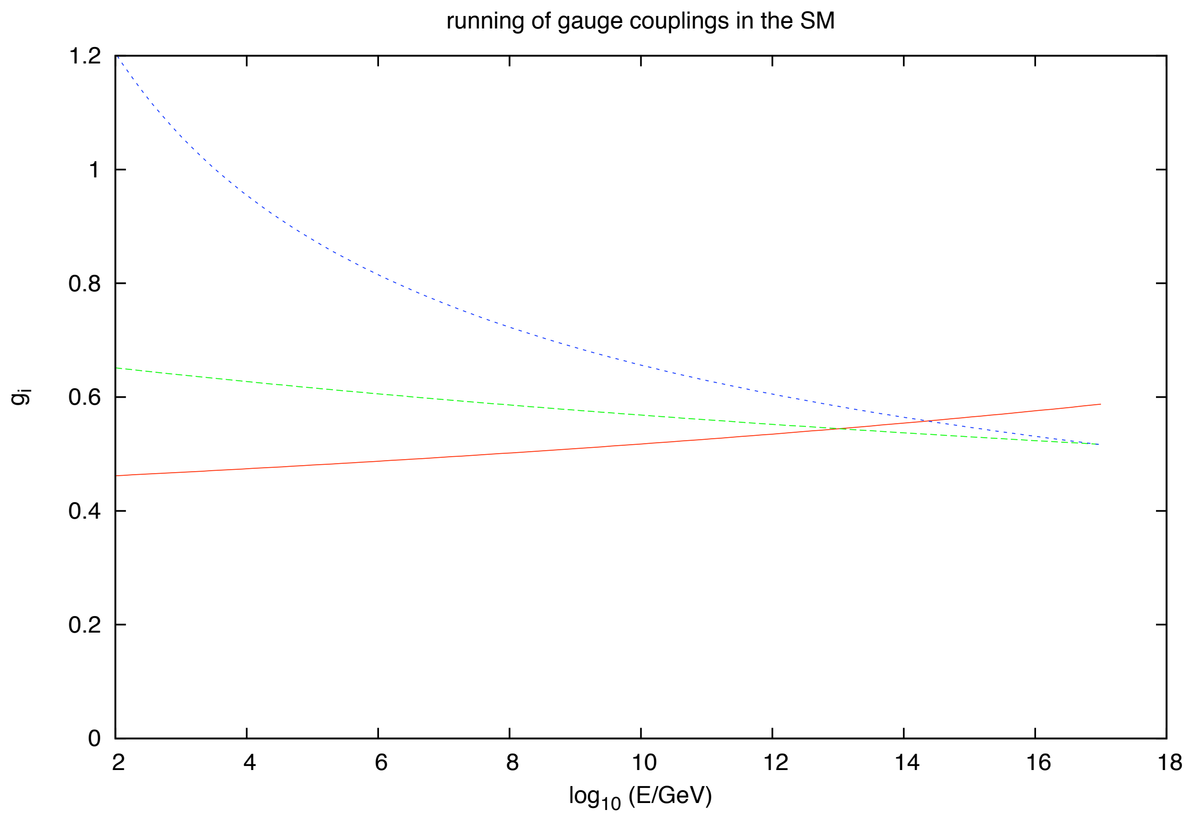
- GUTs are theories of proton decay:

$$\mathcal{L} = \frac{c_{ijkl}}{\Lambda^2} q_i q_j q_k l_l$$

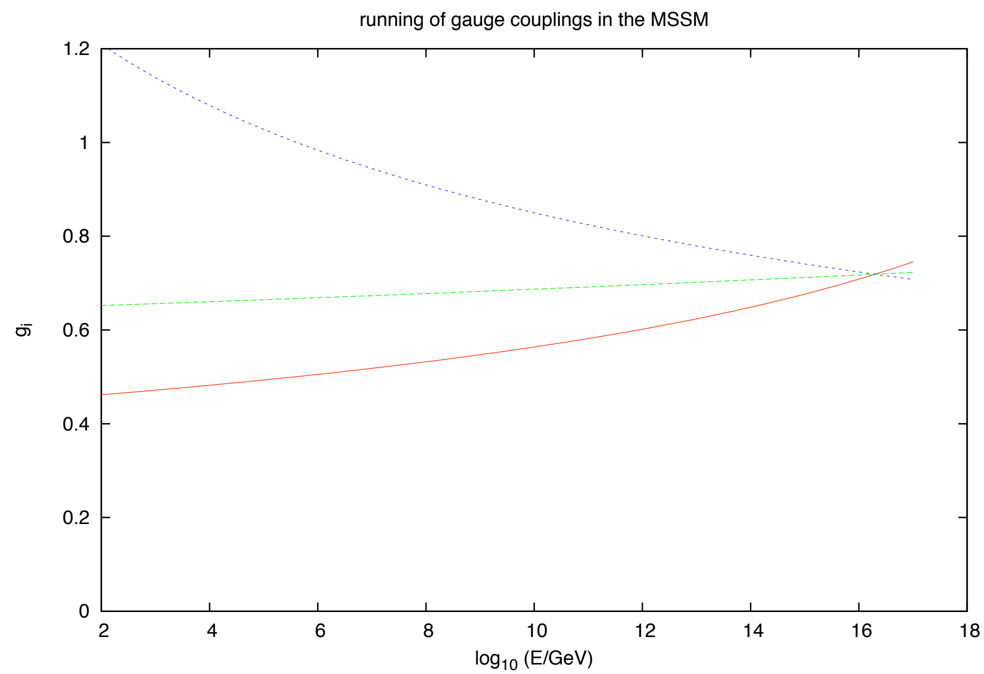
  in SM c_{ijkl}, Λ arbitrary

  in GUTs $\Lambda = M_{GUT}$ and c_{ijkl} predicted (model dependent)

But what does it mean $g_{1,2,3} \to g_5$ $(g_1 \neq g_2 \neq g_3)$? What if we run g_i, do they unify at some scale? Not completely:



New states needed. If you put MSSM at ≈ 1 TeV: unification at $M_{GUT} \approx 10^{16}$ GeV



Not unique solution, but enough to motivate supersymmetry

The simplest theory: SU(5)

matter: $3 \times (\bar{5}_F + 10_F)$

Higgs: $24_H + 5_H + \bar{5}_H$

$$
\begin{aligned}
W_Y &= 10_F \bar{5}_F \bar{5}_H + 10_F 10_F 5_H \\
W_H &= 24_H^3 + 24_H^2 + \bar{5}_H 5_H + 24_H \bar{5}_H 5_H \\
W_{RPV} &= 10_F \bar{5}_F \bar{5}_F + \bar{5}_F 5_H + 24_H \bar{5}_F 5_H
\end{aligned}
$$

Several good features (mentioned above) but it suffers from:

- neutrino massless (as in SM)
- why at least some RPV couplings very small?

R-parity $\phi_F \to -\phi_F, \phi_H \to \phi_H$ must be imposed

More promising theories: SO(10), E_6

Contain right-handed neutrino → see-saw mechanism automatic

R-parity:

$$R = (-1)^{3(B-L)+2S}$$

SO(10):

$$\begin{aligned} R(\langle 16 \rangle \equiv \tilde{\nu}_R) &= 1 \\ R(\langle 126 \rangle) &= 2 \end{aligned}$$

This means that if no 16 gets a vev → R-parity exact

In E_6 all you need is to give vevs only to components that in the SO(10) decomposition do not belong to 16, 144, etc

But 126 (and e.g. $351' \supset 126$ in E_6) is a large representation:

$$T(126) = 35$$

so that the corresponding beta function is

$$\beta_{SO(10)} = 3 \times \underbrace{C(G)}_{=8} - \sum_R T(R) < 0$$

becomes negative (and large)

In minimal renormalizable supersymmetric models

$$\begin{aligned} \beta_{SO(10)} &= -109 \\ \beta_{E_6} &= -159 \end{aligned}$$

$$\frac{d}{d\log\left(\mu/\Lambda\right)}\left(\frac{1}{\alpha}\right)=\frac{1}{4\pi}\beta=\frac{1}{4\pi}\left(b^{(1)}+\ldots\right)$$

At 1-loop the solution is

$$\frac{1}{\alpha(\mu)}=\frac{b^{(1)}}{4\pi}\log\mu/\Lambda$$

- $\alpha(\Lambda)=\infty$
- $\alpha(\mu)$ defined for $\mu<\Lambda$ because $b^{(1)}<0$
- such UV singularity at Λ is called the Landau pole
- in our case the Landau pole is already 1 order of magnitude (or less) above the GUT scale

Can we resolve this singularity with a non-trivial UV fixed point?

Add the 2-loop contribution

$$\beta = -N b^{(1)} + N^2 b^{(2)} \frac{\alpha}{4\pi} = 0$$

(N =number of d.o.f. in the loop $= \mathcal{O}(10^2 - 10^3)$) at

$$N \frac{\alpha}{4\pi} = \frac{b^{(1)}}{b^{(2)}} \lesssim 1$$

then the theory could be perturbative, i.e. the 3-loop contribution

$$N^3 b^{(3)} \left(\frac{\alpha}{4\pi}\right)^2$$

and higher terms could be neglected. This possible only if $b^{(1)}$ strangely (parametrically) small (Banks-Zaks).

This is not our case here. There is no parametric cancellation in $b^{(1)}$.

So even if our theory has a meaning and develops a UV asymptotically free fixed point, it will not be controlled by perturbation.

At this point interesting to mention that only recently an explicit non-trivial example of an asymptotically non-free theory with perturbatively controlled UV fixed point has been found.

It is based on the interplay of all 3 different couplings (gauge, Yukawa and Higgs).

Such a situation has been proved not to be possible in a similar simple way in susy.

On the other side problem of the Landau pole is essentially supersymmetric.

In non-susy β function typically much smaller

$$\begin{aligned} f &\to 2/3 \\ b &\to 1/3 \;\; or \;\; 1/6 \\ \text{less representations}, 126+\overline{126} &\to 126 \;\; only \end{aligned}$$

Typically in non-susy theories the Landau pole, if it exists, it is above M_{Planck}

Analytic perturbation theory

Hystorically there were some attempts to attack the Landau problem (Redmond,. . . , Bogolyubov, . . . , Shirkov, . . .)

Easier to start actually with QCD:

$$\alpha(Q^2) = \frac{1}{\beta \log\left(Q^2/\Lambda^2\right)}$$

Landau pole at $Q^2 = \Lambda^2$: non-analytic $\to$ violates causality

Need to force α to be analytic

$$\begin{aligned}\alpha_{an}(Q^2) &= \frac{1}{\pi}\int_0^\infty dz \frac{Im[\alpha(-z-i\epsilon')]}{z+Q^2-i\epsilon} \\ &= \frac{1}{\beta}\left(\frac{1}{\log\left(Q^2/\Lambda^2\right)}+\frac{\Lambda^2}{\Lambda^2-Q^2}\right)\end{aligned}$$

- No poles, only a branch cut for *real negative* Q^2.
- Corrections non-perturbative:

$$\frac{\Lambda^2}{Q^2} = e^{-\frac{1}{\alpha\beta}}$$

- For large $Q^2 \to \infty$ perturbative result recovered.

For $\beta < 0$ this not applicable $\rightarrow$ subtracted dispersion relation

Assuming that the β function is constant for all energies from zero to infinity:

$$\begin{aligned}
\alpha_{an}(Q^2) - \alpha_{an}(\mu^2) &= -\int_0^\infty \frac{((Q^2-\mu^2)/\beta)\, dz}{(z+Q^2)(z+\mu^2)\left[(\log(z/\Lambda^2))^2+\pi^2\right]} \\
&= \frac{1}{\beta}\left(\frac{1}{\log(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2-Q^2}\right) \\
&- \frac{1}{\beta}\left(\frac{1}{\log(\mu^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2-\mu^2}\right)
\end{aligned}$$

If $\beta > 0$ we can put $\mu \rightarrow \infty$, where $\alpha(\infty) = 0$ and recover previous unsubtracted result.

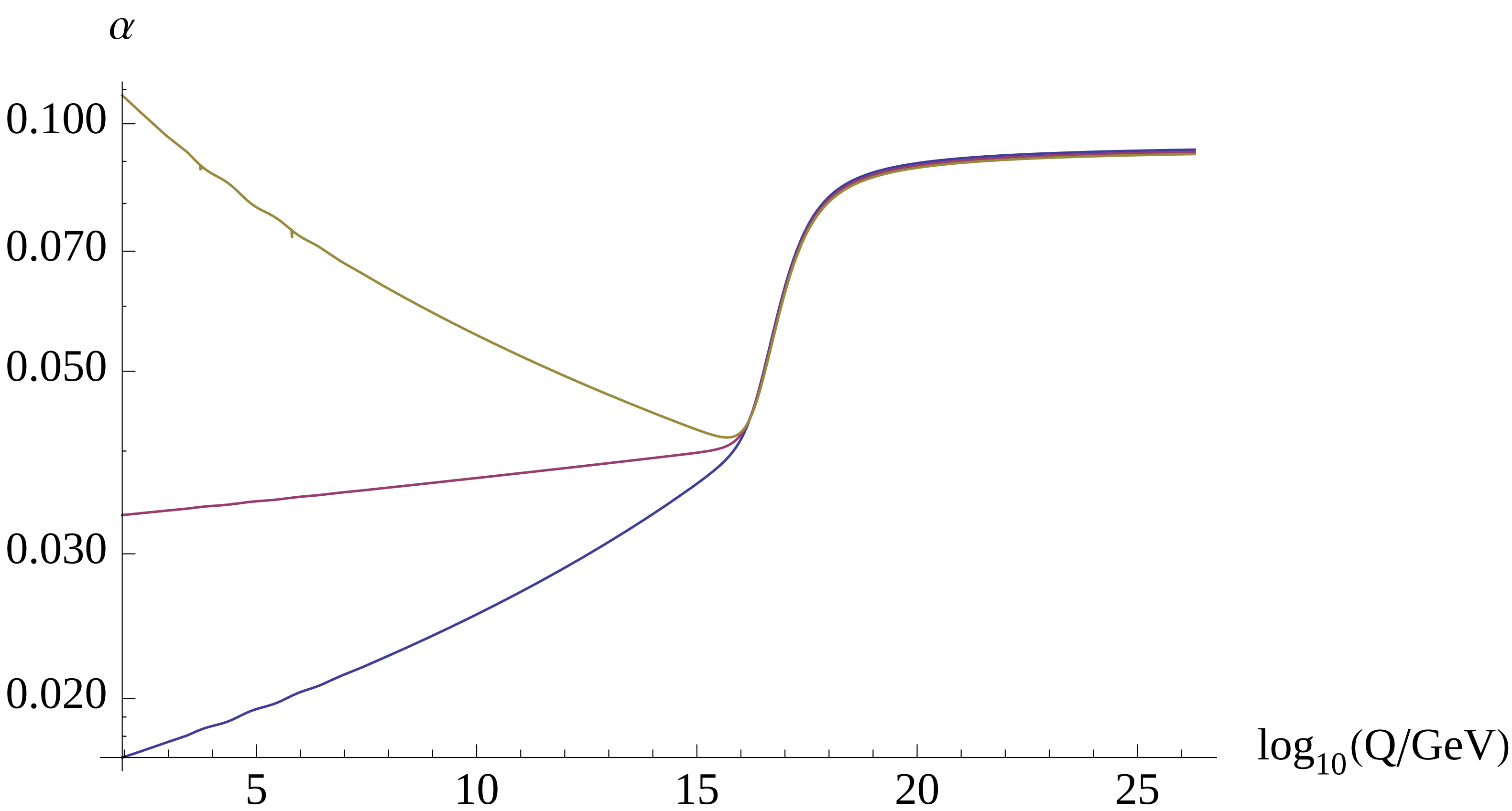
Can this help in our problem? We have here some thresholds, so Λ and β depend on the energy.

$z < M_{GUT}^2$:

$$\begin{aligned} \Lambda_i &= M_Z \exp\left(-\frac{1}{2\beta_i \alpha_i(M_Z)}\right) \\ \beta_i &= (-33/5, -1, 3) \\ \alpha_i(M_Z) &= (0.01696, 0.03344, 0.10830) \end{aligned}$$

$z > M_{GUT}^2$:

$$\begin{aligned} \Lambda_{GUT} &= M_{GUT} \exp\left(-\frac{1}{2\beta_{GUT} \alpha_i(M_{GUT})}\right) \\ \beta_{GUT} &= -159 \\ \alpha_i(M_{GUT}) &= 1/25 \end{aligned}$$



α
0.100
0.070
0.050
0.030
0.020
5
10
15
20
25
log10(Q/GeV)

So the theory stays perturbative.

But it is hard to say if this has anything to do with reality. Hard to check if the proposal is reliable.

To mention however that some check have been done.

Example, comparing large N expansion and analytic perturbation theory in $d = 2$ O(N) Gross-Neveu model: surprising agreement

Dualities

Remember that in QCD we have a similar problem, although in the IR.

Here the electric theory (QCD) becomes strong at low $\Lambda_{QCD} \lesssim 1$ TeV and is defined perturbatively only for $E \gg \Lambda_{QCD}$.

In the IR we have a weakly interacting (non-gauge) theory of mesons (magnetic theory): χPT

The two theories are believed to be equivalent, but they describe physics in the opposite regimes.

Can we have something of this kind in our GUT examples?

This is very difficult to do in ordinary theories, but in supersymmetric we have holomorphy of the superpotentiual W which helps us.

Two theories will be dual if

1) they have the same moduli space. This is described by all $\langle \phi_i \rangle$ which satisfy the e.o.m. This is typically a continuous family of solutions. This means also coincidence of all special (singular) points with enhanced symmetry

2) same number of massless d.o.f. at all points in moduli space. Heavy guys do not count, we should integrate them out. Duality is here only in deep IR.

3) Start typically at $W = 0$ or with just few terms. This brings a large amount of global symmetries which help. At the end we can add δW as small perturbations.

4) 't Hooft anomaly matching for unbroken global symmetries

Seiberg dualities

Simplest example: SQCD

Electric theory:

SU(N$_c$)

N_f quarks Q

N_f antiquarks $\bar{Q}$

The beta function is

$$\beta_{SU(N_c)} = 3 \times N_c - 2N_f \times \frac{1}{2}$$

To have a UV free theory $\rightarrow N_f < 3N_c$

Symmetries of the electric ($W = 0$):

$$[SU(N_c)]_{local} \times \left[SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1) \times U(1)_R\right]_{global}$$

$$\begin{aligned} Q &\sim (N_c, N_f, 1)\left(1, \frac{N_f - N_c}{N_f}\right) \\ \bar{Q} &\sim (\bar{N}_c, 1, \bar{N}_f)\left(-1, \frac{N_f - N_c}{N_f}\right) \end{aligned}$$

Mesons:

$M_{ij} = \bar{Q}_i Q_j \qquad i, j = 1, \ldots N_f$

Baryons:

$B_{j_1,\ldots,j_{N_c}} = \epsilon_{i_1\ldots i_{N_c}} Q^{i_1}_{j_1} \ldots Q^{i_{N_c}}_{j_{N_c}} \qquad i_1, \ldots i_{N_c} = 1, \ldots N_c$

$\bar{B}_{j_1,\ldots,j_{N_c}} = \epsilon_{i_1\ldots i_{N_c}} \bar{Q}^{i_1}_{j_1} \ldots \bar{Q}^{i_{N_c}}_{j_{N_c}} \qquad j_1, \ldots j_{N_c} = 1, \ldots N_f$

Magnetic theory:

$SU(N_f - N_c)$

N_f quarks q

N_f antiquarks $\bar{q}$

N_f^2 singlets T

The beta function is

$$\beta_{SU(N_f-N_c)} = 3 \times (N_f - N_c) - 2 \times N_f \times \frac{1}{2}$$

To have a UV free theory $\rightarrow N_f > 3N_c/2$

Symmetries of the magnetic ($W \sim T\bar{q}q$):

$$[SU(N_f - N_c)]_{local} \times [SU(N_f)_q \times SU(N_f)_{\bar{q}} \times U(1) \times U(1)_R]_{global}$$

$$\begin{aligned}
q &\sim \left(N_f - N_c, \bar{N}_f, 1\right)\left(\frac{N_c}{N_f - N_c}, \frac{N_c}{N_f}\right) \\
\bar{q} &\sim \left(\overline{N_f - N_c}, 1, N_f\right)\left(-\frac{N_c}{N_f - N_c}, \frac{N_c}{N_f}\right) \\
T &\sim \left(1, N_f, \bar{N}_f\right)\left(0, 2\frac{N_f - N_c}{N_f}\right)
\end{aligned}$$

Mesons:

$$m_{ij} = \bar{q}_i q_j \qquad i, j = 1, \ldots N_f$$

Baryons:

$$b_{j_1,\ldots,j_{N_c}} = \epsilon_{i_1 \ldots i_{N_c}} Q^{i_1}_{j_1} \ldots Q^{i_{N_c}}_{j_{N_c}} \qquad i_1, \ldots i_{N_c} = 1, \ldots N_c$$

$$\bar{b}_{j_1,\ldots,j_{N_c}} = \epsilon_{i_1 \ldots i_{N_c}} \bar{Q}^{i_1}_{j_1} \ldots \bar{Q}^{i_{N_c}}_{j_{N_c}} \qquad j_1, \ldots j_{N_c} = 1, \ldots N_f$$

Why are these two theories believed to be dual?

- same composites
- 't Hooft anomaly matching (global)^3
- the identity survive under different deformations (moving along the flat directions, adding mass terms)

A SO(10) example

Imagine we have

N_{10} copies of 10's

3 copies of 16's

The beta function is

$$\beta_{SO(10)} = 3 \times 8 - N_{10} \times 1 - 3 \times 2 = 18 - N_{10}$$

For $N_{10} > 18$ the theory has a Landau pole in the UV. This is the magnetic theory, well behaved in the IR. We look for a dual theory, well defined (asymptotically free) in the UV.

The magnetic theory has the following symmetries:

$$[SO(10)]_{local} \times [SU(N_{10}) \times SU(3) \times U(1) \times U(1)_R]_{global}$$

$$\begin{aligned} 10 = Q &\sim (10, N_{10}, 1)(-6, \frac{N_{10}-2}{N_{10}+6}) \\ 16 = \Psi &\sim (16, 1, 3)(N_{10}, \frac{N_{10}-2}{N_{10}+6}) \end{aligned}$$

Parton DOF $= 10N_{10} + 48$

Gauge invariants (composites):

$$
\begin{aligned}
16^4 &\sim (1,1,\bar{6})(4N_{10}, 4\frac{N_{10}-2}{N_{10}+6}) \\
10^2 &\sim (1, \frac{N_{10}(N_{10}+1)}{2}, 1)(-12, 2\frac{N_{10}-2}{N_{10}+6}) \\
16^2 10 &\sim (1, N_{10}, 6)(2N_{10}-6, 3\frac{N_{10}-2}{N_{10}+6}) \\
16^2 10^3 &\sim (1, \begin{pmatrix} N_{10} \\ 3 \end{pmatrix}, \bar{3})(2N_{10}-18, 5\frac{N_{10}-2}{N_{10}+6}) \\
16^4 10^4 &\sim (1, \begin{pmatrix} N_{10} \\ 4 \end{pmatrix}, \bar{6})(4N_{10}-24, 8\frac{N_{10}-2}{N_{10}+6}) \\
16^2 10^5 &\sim (1, \begin{pmatrix} N_{10} \\ 5 \end{pmatrix}, 6)(2N_{10}-30, 7\frac{N_{10}-2}{N_{10}+6})
\end{aligned}
$$

Hadron DOF $= \frac{1}{20}\left(N_{10}^5 - 5N_{10}^4 + 15N_{10}^3 - 15N_{10}^2 + 144N_{10} + 120\right)$

The DOF must match:

$$\text{Parton DOF} = \text{Goldstones} + \text{Hadron DOF} - \text{constraints}$$

Example for $N_{10} = 20$:

$$248 = 45 + 125850 - 125647$$

Constraints come from e.o.m.:

$$\frac{\partial W}{\partial \phi_i} = 0$$

$$W = W(16^4, 10^2, 16^2 10, 16^2 10^3, 16^4 10^4, 16^2 10^5)$$

The electric dual:

$$[SU(N_{10}-1)\times Sp(4)]_{local}\times[SU(N_{10})\times SU(2)\times U(1)\times U(1)_R]_{global}$$

Parton DOF:

$$\begin{aligned}
q &\sim (N_{10}-1,1,\bar{N}_{10},1)\\
q' &\sim (N_{10}-1,4,1,2)\\
\bar{q} &\sim (\overline{N_{10}-1},1,1,5)\\
s &\sim (\overline{\frac{(N_{10}-1)N_{10}}{2}},1,1,1)\\
t &\sim (1,4,1,4)\\
m &\sim (1,1,\frac{N_{10}(N_{10}+1)}{2},1)\\
n &\sim (1,1,N_{10},5)
\end{aligned}$$

Checks:

- in both theories all local or mixed local-global anomalies vanish
- global anomalies satisfy 't Hooft anomaly matching
- reproduction of known results in various limits
- mass deformations
- ...

We need also to use an adjoint to eventually break SO(10). If we add a 45 it is tough to find the dual

$\rightarrow$ deconfinement mechanism

Add extra gauge Sp(6) and $Z = (10, 6)$ under SO(10)$\times$Sp(6)

$$Z^\alpha_\mu J_{\alpha\beta} Z^\beta_\nu = (10 \times 10)_{AS} = 45_{\mu\nu} \quad , \quad J_{\alpha\beta} = -J_{\beta\alpha}$$

Any Sp($2N$) with $2N + 4$ fundamentals confine at the IR scale Λ_{Sp}.

Then Z becomes composite 45 of SO(10).

Adding a nontrivial superpotential for Z (perturbation) does the rest:

$$SO(10) \xrightarrow{\langle 45 \rangle} SM \times U(1)$$

Superconformal index

Progress done recently in checking whether two theories dual. One can compare the superconformal index. This is a (calculable finite) generalization of Witten's index. It automatically includes 't Hooft anomaly matching, but more than just it.

It uses infinite products like

$$
\begin{aligned}
(x;p)_\infty &= \prod_{j=0}^{\infty}(1-xp^j) \\
\Gamma(z;p,q) &= \prod_{j,k=0}^{\infty}\frac{1-z^{-1}p^{j+1}q^{k+1}}{1-zp^jq^k} \\
\Gamma(z_1,z_2;p,q) &= \Gamma(z_1;p,q)\Gamma(z_2;p,q)
\end{aligned}
$$

- We have global symmetries $SU(N_f)\times SU(N_f)$ both in electric and magnetic theory. So there are charges associated with generators of the Cartan subalgebra. Introduce the corresponding chemical potentials s_i and t_i.
- Introduce regulators p and q for the ∞ number of ground states (flat directions).
- Index depends on q,p,s_i,t_i

For the SQCD Seiberg duality we have

$$
\begin{aligned}
I_E &= \frac{(p;p)_\infty^{N_c-1}(q;q)_\infty^{N_c-1}}{N_c!} \\
&\times \oint_{|z_j|=1} \frac{\prod_{i=1}^{N_f}\prod_{j=1}^{N_c}\Gamma(s_i z_j, t_i^{-1}z_j^{-1};p,q)}{\prod_{1\le i<j\le N_c}\Gamma(z_i z_j^{-1}, z_i^{-1}z_j;p,q)} \prod_{j=1}^{N_c-1}\frac{dz_j}{2\pi i z_j} \\
I_M &= \frac{(p;p)_\infty^{\tilde N_c-1}(q;q)_\infty^{\tilde N_c-1}}{\tilde N_c!} \prod_{1\le i,j\le N_f}\Gamma(s_i t_j^{-1};p,q) \\
&\times \oint_{|z_j|=1} \frac{\prod_{i=1}^{N_f}\prod_{j=1}^{\tilde N_c}\Gamma(S^{\frac{1}{\tilde N_c}} s_i^{-1} z_j, T^{-\frac{1}{\tilde N_c}} t_i z_j^{-1};p,q)}{\prod_{1\le i<j\le \tilde N_c}\Gamma(z_i z_j^{-1}, z_i^{-1}z_j;p,q)} \prod_{j=1}^{\tilde N_c-1}\frac{dz_j}{2\pi i z_j}
\end{aligned}
$$

where

$\tilde N_c = N_f - N_c\ ,\ S = \prod_{i=1}^{N_f} s_i\ ,\ T = \prod_{i=1}^{N_f} t_i\ ,\ ST^{-1} = (pq)^{-\tilde N_c}$

Duality means that $I_E = I_M$ for any choice of s_i, t_i, p, q.

This has indeed been proved.

Not only for Seiberg SQCD, but for many other conjectured nontrivial dualities.

Limitations

But even if we find the electric dual of a realistic GUT, we still have the obvious problem, present already in QCD:

we have a magnetic theory defined in the IR and an electric theory defined in the UV, but we do not have any region in between in which both theories are perturbative.

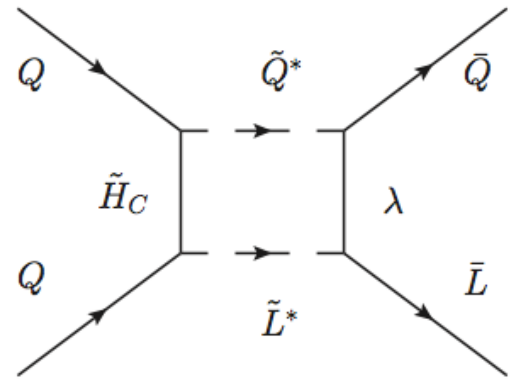
This sort of obvious: two different gauge theories cannot be equivalent and perturbative.

So there is no simple way to match the measured parameters of the magnetic (IR) theory with the non-measured parameters of the electric (UV) theory. This should be eventually done non-perturbatively (lattice).

Conclusions

Backup slides

The rate for nucleon decay in low-energy susy models is typically dominated by $d = 5$ operators (schematically)



$$\tau^{-1} \approx \left| \left(\frac{Y^2}{M_C} \right) \left(\frac{\alpha}{4\pi} \frac{m_\lambda}{m_{\tilde{f}}^2} \right) \right|^2 m_p^5$$

$Y^2 \ldots$ products of Yukawas

$M_C \ldots$ color triplet mass

$(\alpha \ldots)$ MSSM loop factor (λ gaugino or higgsino, $\tilde{f}$ sfermion)

$m_p^5 \ldots$ from strong QCD dynamics (lattice)

The lifetime depends a lot on the model considered

But for $m_{susy} \sim 1$ TeV this typically a problem.

Renormalizable sugra SU(5)

$(3 \times (10_F + \overline{5}_f) + 5_H + \overline{5}_H + 24_H + 24_V)$:

- $Y^2 = Y_U Y_D (= Y_U Y_E)$
- $M_C \approx 10^{14-15}$ GeV from RGE constraints
- sfermion mixing $\propto$ fermion mixing

+ LHC-friendly values for soft parameters

- $m_\lambda \approx m_{\tilde{f}} \lesssim \mathcal{O}(\text{TeV})$

$$\tau \approx 10^{29} \text{ years}$$

Too fast: $\tau_{exp}(p \to K^+ \bar{\nu}) \gtrsim 10^{33}$ yrs

Many different ways of solving this problems (larger m_{susy}, non-renormalizable operators, particular flavor structure, etc)

But large threshold corrections (many fields in the loop) suggest another solution

$$Z_H = \mathcal{O}(10^{-3})$$

$$Y_{SO(10)} = \underbrace{Z_H^{1/2}}_{\text{small}} \times \underbrace{Y_{SM}}_{\text{measured}} \rightarrow \text{small}$$

$$\tau_{d=5} \propto \frac{M_C^2 m_{susy}^2}{Y_{SO(10)}^2 m_p^5} \rightarrow \text{large enough}$$

This however tells us that perturbation may be lost