COLOR SUPERCONDUCTORS

Massimo Mannarelli
INFN-LNGS
massimo@lngs.infn.it

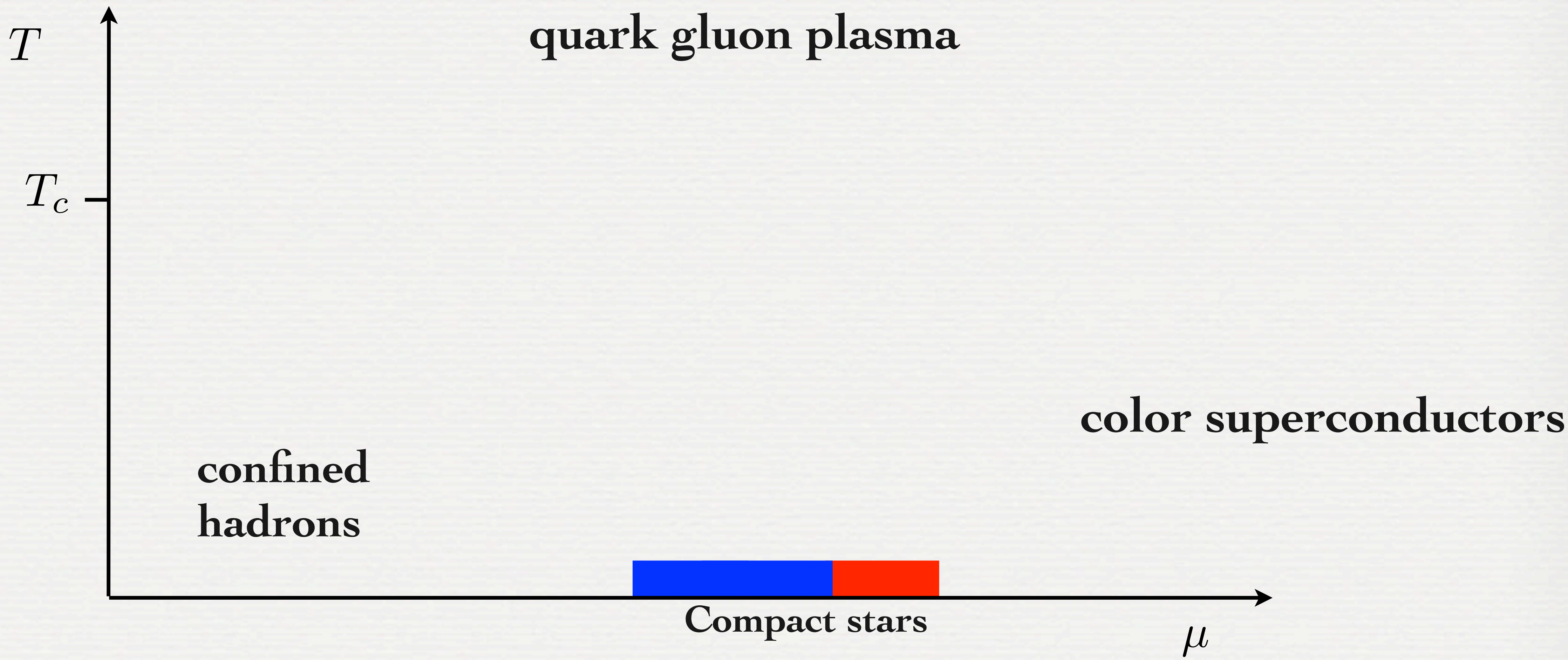
LNGS Sept 2015

OUTLINE

- ***General motivations***
- ***Superfluids and Superconductors***
- ***Color Superconductors***
- ***Crystalline Color Superconductors***

GENERAL MOTIVATIONS

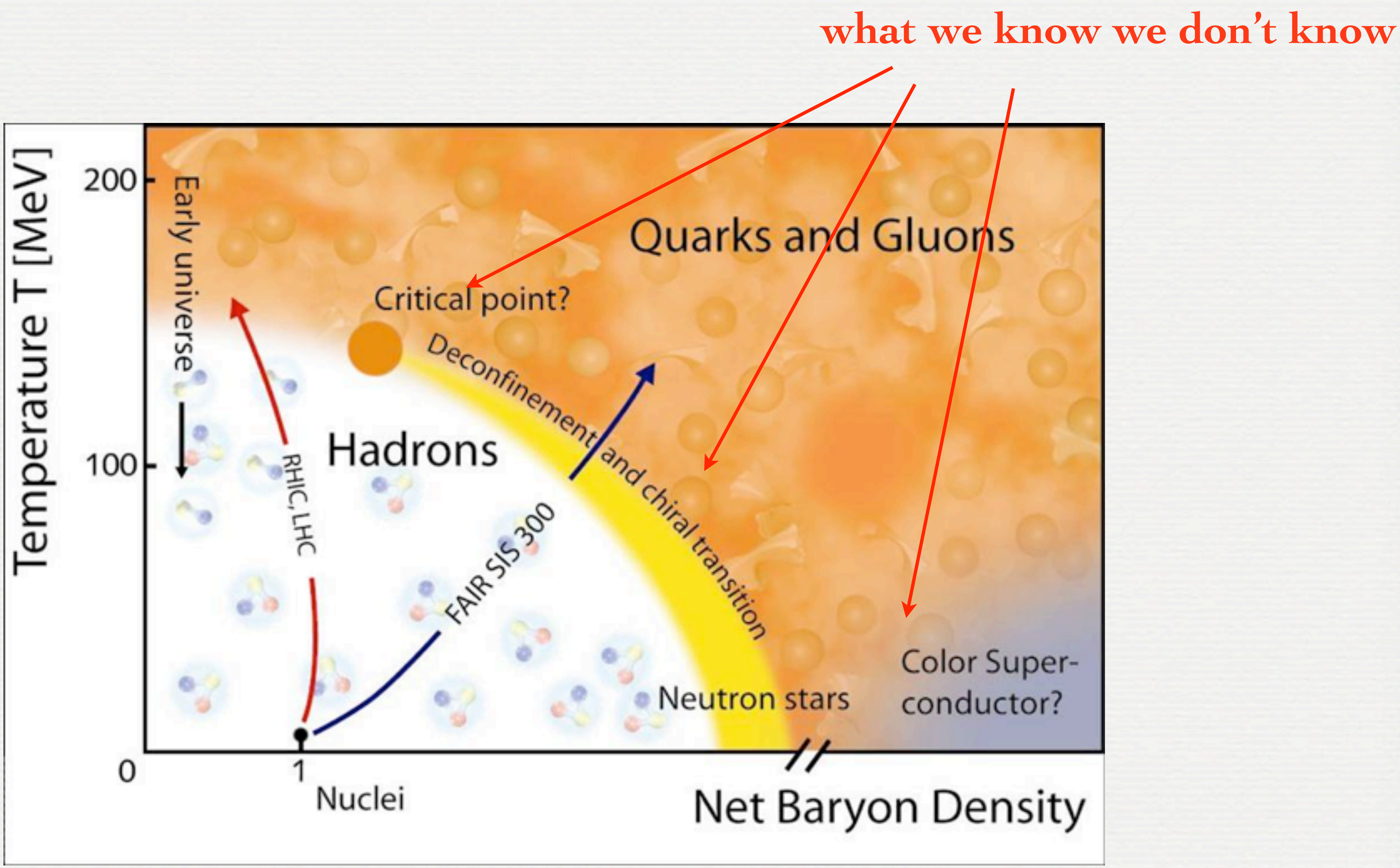
The QCD phase diagram



Few pieces of information about the phase diagram

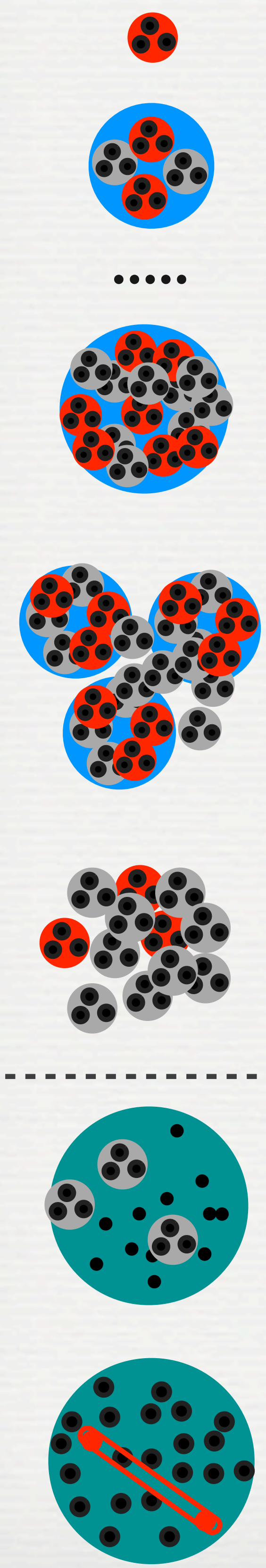
HOT MATTER	ENERGY-SCAN	LATTICE QCD	EMULATION
RHIC LHC	RHIC NA61/SHINE@CERN-SPS CBM@FAIR/GSI MPD@NICA/JINR	Different approaches. Sign problem.	Ultracold fermi atoms

How we think it should be



http://homepages.uni-regensburg.de/~sow28704/ftd_lqcd_ss2012/ftd_lqcd_ss2012.html

Increasing baryonic density



	Density	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H			
He	low		light nuclei
.....	......		.....
Ni		confining	heavy nuclei
neutron drip			
neutron proton soup	high		neutrons and protons
quark drip	very large	strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
quark soup	extreme	weak coupling	Cooper pairs of quarks NGBs

ρ

The physics at high (nonextreme) baryonic density is difficult

1. **QCD is nonperturbative**
2. **Lattice simulations do not work** **(Barbour et al. 1986 Nucl.Phys. B275 296)**
3. **No experimental facility (so far) can reproduce the correct conditions**

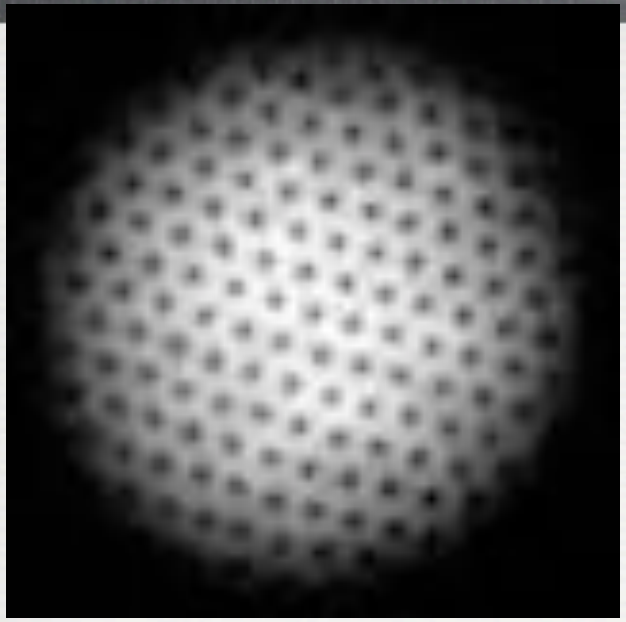
Let me rephrase it:

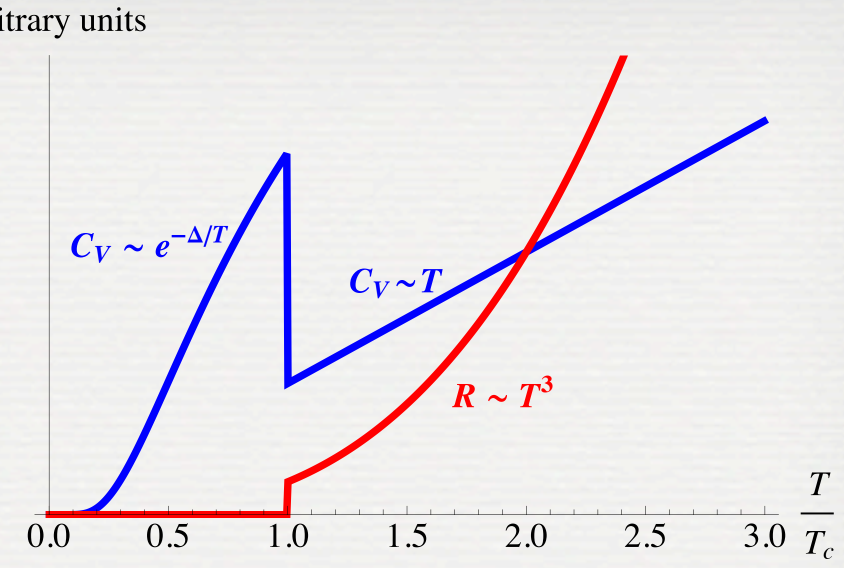
1. **We do not know how to do reliable computations**
2. **We do not have numerical methods for doing tests**
3. **We have no terrestrial lab for validating the theoretical results**

The way out:

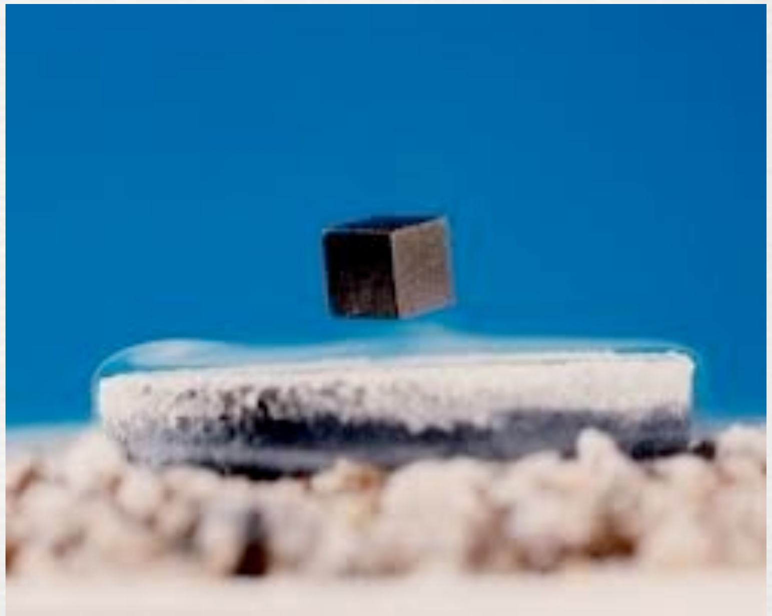
We can use symmetries and analogies for obtaining qualitative and semiquantitative results

We can use compact stars as the "lab"





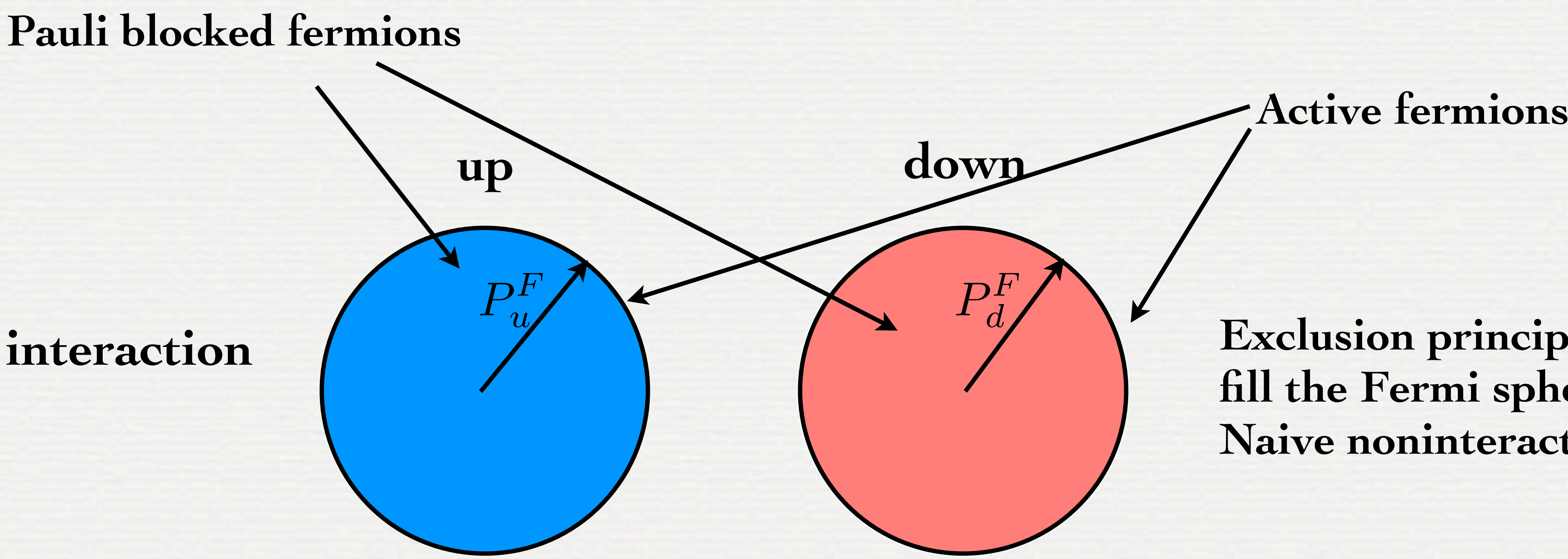
SUPERFLUIDS
AND
SUPERCONDUCTORS



BCS qualitative description

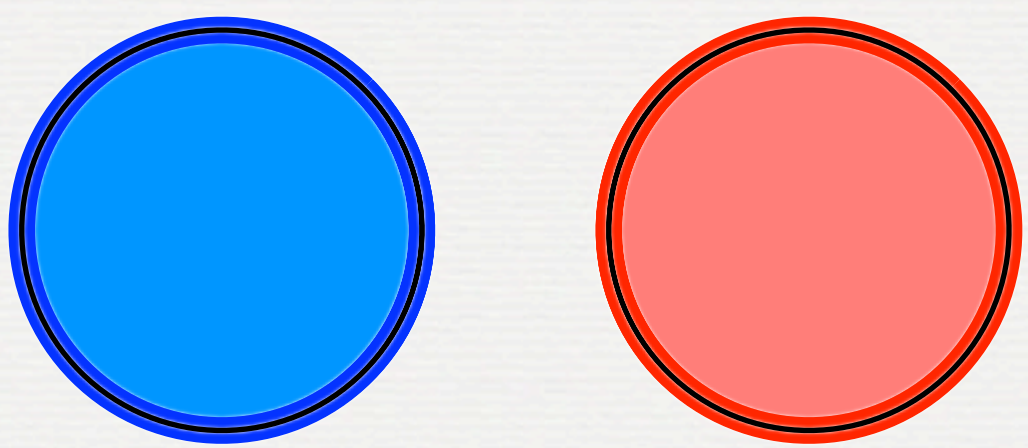
Degenerate system of spin up and spin down fermions

No interaction



Exclusion principle rules: fermions fill the Fermi sphere.
Naive noninteracting ground state.

Attractive interaction



Active fermions form correlated pairs.
BCS state

The naive Fermi sphere ground state is unstable: the BCS is the energy minimum.
We can integrate out the Pauli blocked fermions

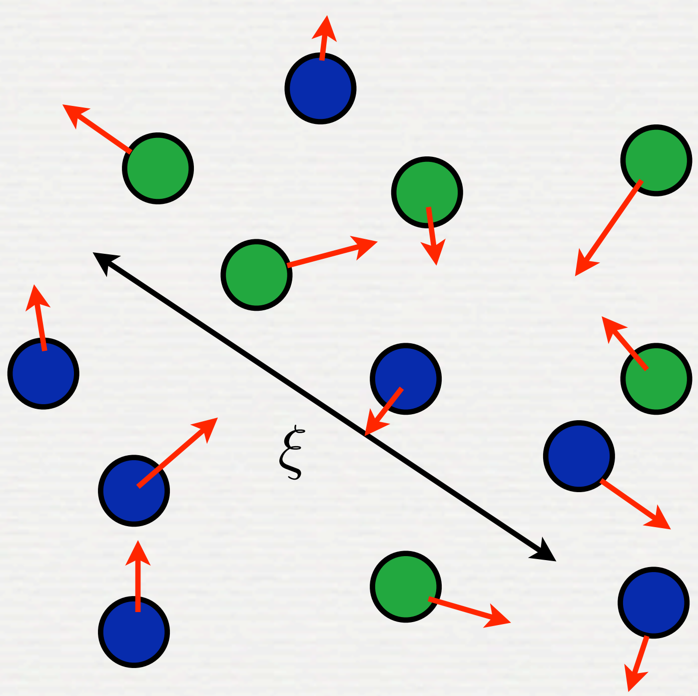
BCS semiquantitative description

fermions

spin up

spin down

momentum



Cooper pairs: "difermions" with total spin 0 and total momentum 0

Free energy gain $\sim \Delta$

$$\xi \sim \frac{v_F}{\Delta}$$

BCS: loosely bound pairs $\xi \gtrsim n^{-1/3}$

BEC: tightly bound pairs $\xi \lesssim n^{-1/3}$

Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with $v = \nabla\varphi \Rightarrow \nabla \times v = 0$ (irrotational or quantized vorticity)

Superconductor: "screening" of the magnetic field: Meissner effect (almost perfect diamagnet)

Superfluid

Broken global symmetry

Goldstone theorem → Transport of the quantum numbers of the broken group with almost no dissipation

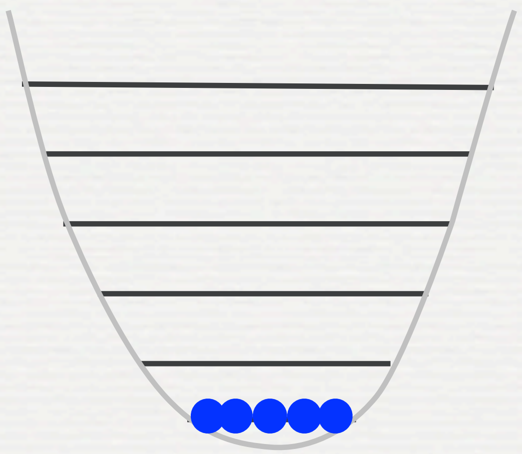
Superconductor

"Broken gauge symmetry"

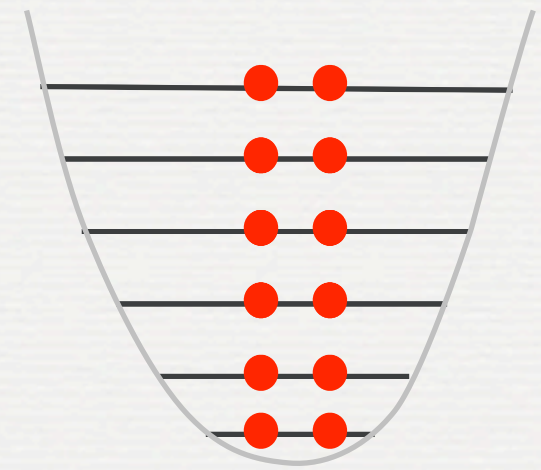
Higgs mechanism → Broken gauge fields with mass, M, penetrate for a length $\lambda \propto 1/M$

Fermionic and bosonic superfluids at T=0

^4He **BOSONS**



^3He **FERMIONS**



Bosons “like” to move together, no dissipation

^4He becomes superfluid at
$T_c \simeq 2.17$ K, Kapitsa et al (1938)

An arbitrary weak interaction leads to the formation of Cooper pairs

^3He becomes superfluid at
$T_c \simeq 0.0025$ K, Osheroff (1971)

BEC ← **BCS**

increasing the attractive interaction between fermions

Main concepts so far:

- Superfluid, spontaneous breaking of a global symmetry
- Superconductor, “spontaneous breaking” of a local symmetry
- Superfluids are weird systems: vanishing viscosity, quantized vorticity...
- Both fermions (like ^{3}He) and bosons (like ^{4}He) can become superfluid

Next:

What about quark matter? Can it be superfluid? Does it happen? Which are the consequences? ....

COLOR SUPERCONDUCTORS

A bit of history

- Quark matter inside compact stars, Ivanenko and Kurdgelaidze (1965), Paccini (1966) ...
- Quark Cooper pairing was proposed by Ivanenko and Kurdgelaidze (1969)
- With asymptotic freedom (1973) more robust results by Collins and Perry (1975), Baym and Chin (1976)
- Classification of some color superconducting phases: Bailin and Love (1984)

Interesting studies but predicted small energy gaps ~ 10 ÷100 keV
small/negligible phenomenological impact for compact stars

- A large gap with instanton models by Alford et al. (1998) and by Rapp et al. (1998)
- The color flavor locked (CFL) phase was proposed by Alford et al. (1999)

Do we have the ingredients?

Recipe for superconductivity

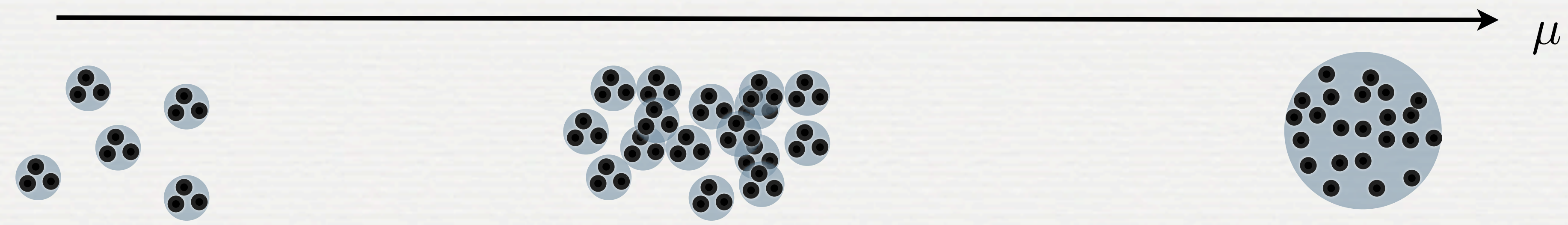
- Degenerate system of fermions
- Attractive interaction (in some channel)
- $T < T_c$

Color superconductivity

- At large μ, degenerate system of quarks
- Attractive interaction between quarks in the antitriplet color channel
- We expect $T_c \sim (10 - 100)$ MeV $>>$ $T_{star\text{-}core} \sim 10 \div 100$ keV

  THE SYSTEM IS EFFECTIVELY ULTRACOLD

Color superconductors

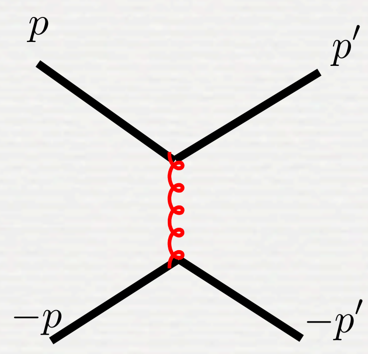


Degenerate system of quarks

Attractive interaction between quarks

$$3 \times 3 = \bar{3}_A + 6_S$$

attractive channel



$$p, p' \simeq p_f$$

Using quarks as building blocks, one has color, flavor as well as spin degrees of freedom: the game is more complicated

QCD, allows for a zoo of colored phases and one has to single out the one with the smallest free-energy

Example: "Standard BCS superconductors"

Consider a degenerate system of fermions with charge Q
Suppose there is an attractive S-wave interaction channel, say because of a phonon exchange

If the temperature is sufficiently low, there will be a condensate made by two fermions

$$\langle \psi\psi \rangle$$

The gauge transformation $\psi(x) \to e^{iQ\alpha(x)}\psi(x)$ **does not leave the condensate invariant**

Indeed the condensate has electric charge 2 Q and it "brakes" $U(1)_{\rm em}$

By the Higgs-Anderson mechanism, the photon gets mass: the Meissner mass and the magnetic component of the gauge field is screened.

Important points:
1) we have only used symmetries (no specific description of the interaction channel)
2) we have not used an external Higgs field, it has been the dynamics of the fermions to induce the symmetry breaking.

General expression of the gap parameter

Quite general color superconducting condensate:

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \sum_{I=1}^{3} \Delta_{\mathrm{I}} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

gap parameters

$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, 2, 3$ **flavor indices**

It has a flavor charge **It has a color charge** **It has a baryonic charge**

Δ_I **is the gap parameter between quarks whose flavor and color is not** ***I***

Δ_3 **is the gap parameter between** $\langle u_b d_r \rangle$ **and** $\langle u_r d_b \rangle$

Obs. the condensate has been written in a gauge variant way

2 Flavor color superconductor (2SC)

Suppose the strange quark mass is "large".
Strange quarks decouple and we effectively have only up and down quarks

$$\Delta_3 > 0\,,\; \Delta_2 = \Delta_1 = 0$$

The breaking pattern:

$$SU(3)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_S}_{\supset U(1)_Q} \to SU(2)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}} \times U(1)_S}_{\supset U(1)_{\tilde{Q}}}$$

- 5 gauge bosons acquire a mass: the system IS A COLOR SUPERCONDUCTOR
- No global symmetry is broken: the system IS NOT A SUPERFLUID
- No chiral symmetry breaking
- The photon is rotated (mixed with gauge and global symmetries).

The system is an "electrical" conductor

Color Flavor Locking (CFL)

Suppose that the strange quark mass is "small".
All quarks should be treated on an equal footing:

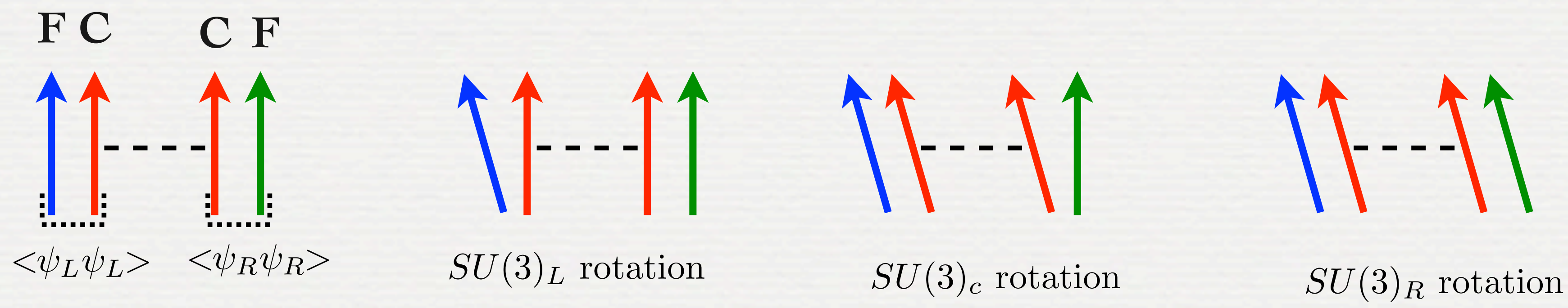
$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_{\rm CFL}$$

$$\langle \psi_{\alpha i} C\gamma_5 \psi_{\beta j}\rangle \propto \Delta_{\rm CFL} \sum_{I=1}^{3} \varepsilon^{\alpha\beta I}\epsilon_{ijI}$$

Alford, Rajagopal, Wilczek hep-ph/9804403

All quarks, contribute coherently to pairing. This phase maximizes the pairing.
It is very robust. It is expected to be the ground state of quark matter at very large densities

Locking color and flavor rotations:



This is "like" in the SU(2) x U(1) -> U(1) locking of the standard model

CFL breaking pattern

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

- 8 gluons acquire "magnetic mass" CFL IS A COLOR SUPERCONDUCTOR
- χSB: 8 (pseudo) Nambu-Goldstone bosons
- $U(1)_B$ breaking CFL IS A SUPERFLUID
- "Rotated" photon, mixing angle $\cos\theta = \dfrac{g}{\sqrt{g^2 + 4e^2/3}}$

The system is an electromagnetic transparent insulator

CRYSTALLINE COLOR SUPERCONDUCTORS

Additional ingredients

We have to consider the typical environment of compact stars

- It is "cold", with temperature of order tens of keV
- Matter is in weak equilibrium
- Matter is electrically neutral
- The strange quark mass might be comparable with the quark chemical potential

The first condition simplifies the treatment: we can take T=0.

The other conditions tend to disfavor CFL pairing

Fermi mismatch *(unpaired quark matter)*

Large quark number chemical potential
Quark number densities of "free" quarks

$$300 \text{ MeV} < \mu < 1 \text{ GeV}$$

$$n_i = C_i \frac{k_{F,i}^3}{3\pi^2} \qquad k_{F,i} = \sqrt{\mu_i^2 - m_i^2}$$

Only u,d,s quarks are relevant. Light quarks can be treated as massless.

weak equilibrium

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

→ $\mu_u = \mu_d - \mu_e$, $\quad \mu_d = \mu_s$

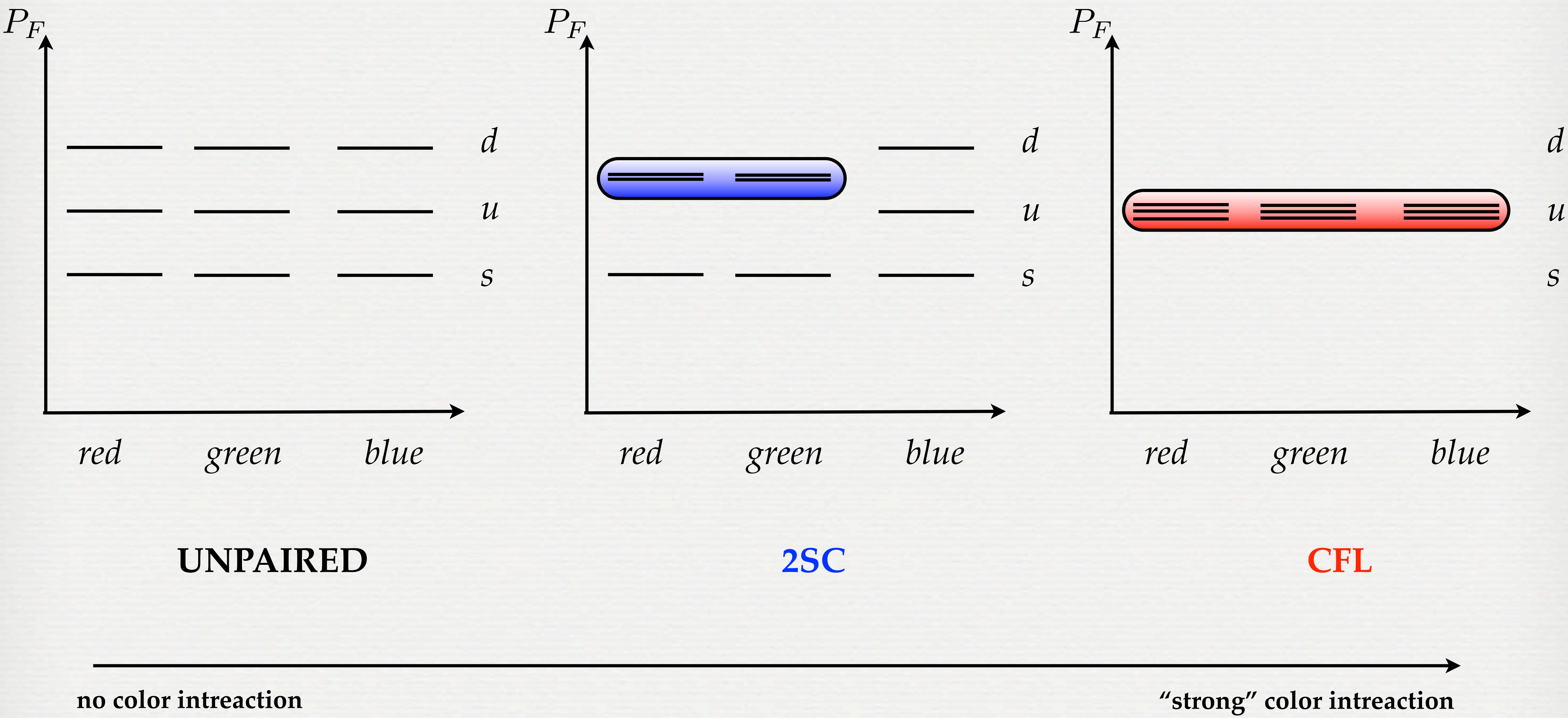
electric neutrality → $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$

⇒ $$\mu_e \simeq \frac{m_s^2}{4\mu}$$

$$\mu = \frac{\mu_u + \mu_d + \mu_s}{3}$$

$$p_d^F = \mu + \frac{1}{3}\mu_e \qquad p_u^F = \mu - \frac{2}{3}\mu_e \qquad p_s^F \simeq \mu - \frac{5}{3}\mu_e$$

Fixed mismatch, increasing coupling



Whenever there is BCS pairing, the Fermi surfaces have to match.

If the mismatch is too large, pairing cannot occur. The largest chemical potential mismatch which allows paring is named the **Chandrasekhar-Clogston limit** (derived for weakly coupled two level systems)

Chemical potential stress on the CFL phase

At a given interaction strength a large chemical potential difference tends to disrupt the CFL paring

Free energy gain $\propto \Delta_{\rm CFL}$

Free energy cost $\propto \delta\mu \sim \frac{M_s^2}{4\mu}$

CFL favored for

$$\Delta_{\rm CFL} > c\frac{M_s^2}{\mu}$$

Various possibile transitions:

- **2SC phase**
- **gapless CFL phase (unstable)**
- **CFL-K0**
- **Crystalline color superconductors**

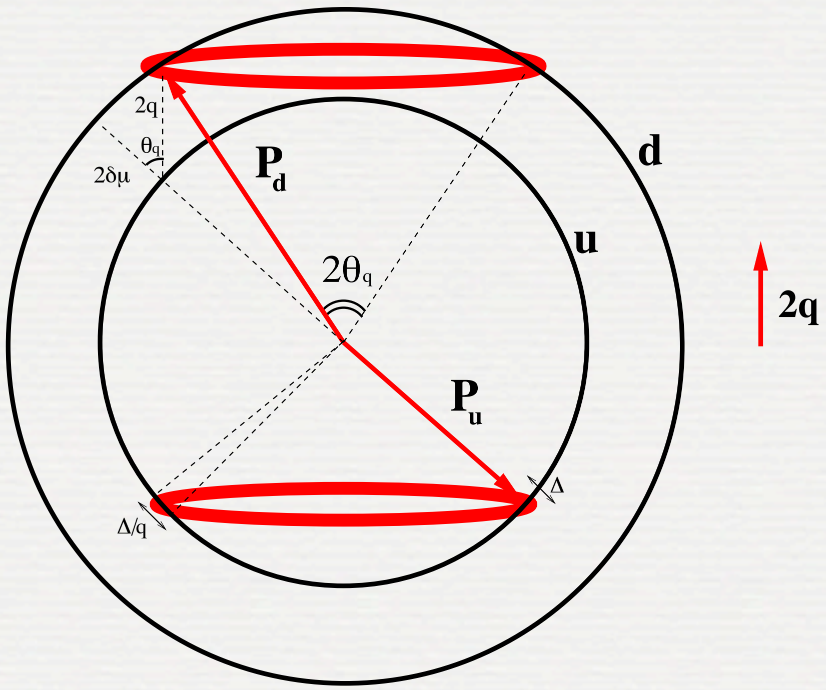
LOFF (or FFLO)-phase

LOFF: Larkin-Ovchinnikov and Fulde-Ferrel (1964)

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the superconducting phase named LOFF is favored with Cooper pairs of non-zero total momentum**

For two flavors

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$



- **In momentum space**

$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\,\delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

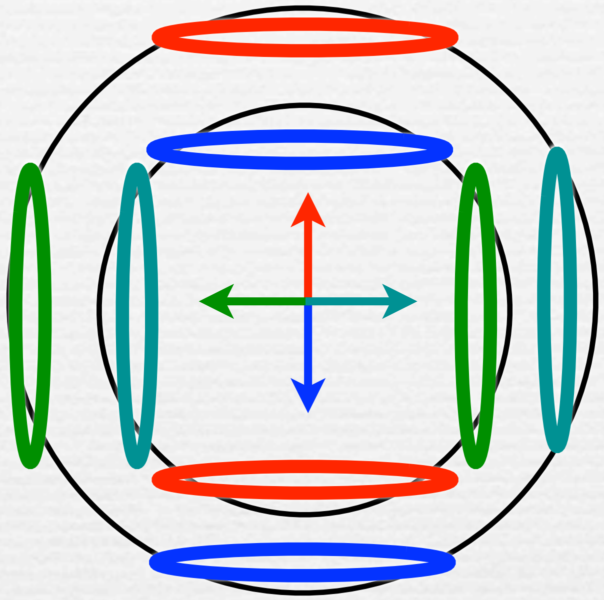
- **In coordinate space**

$$< \psi(\boldsymbol{x})\psi(\boldsymbol{x}) > \sim \Delta\, e^{i2\boldsymbol{q}\cdot\boldsymbol{x}}$$

Inhomogeneous superconductor, with a spatially modulated condensate in the spin 0 channel

The dispersion law of quasiparticles is gapless in some directions, but no instability

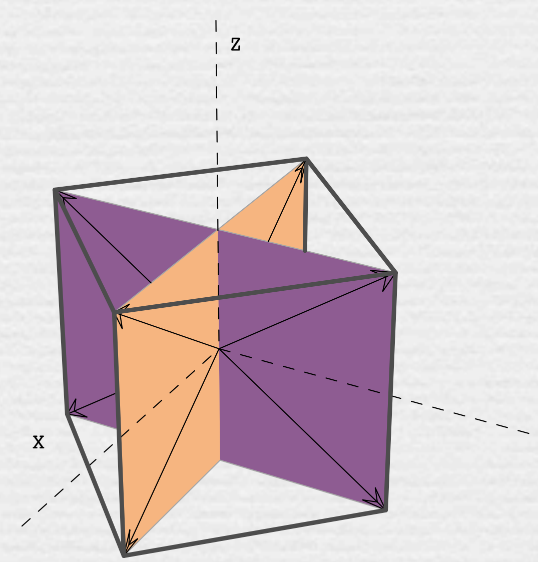
Crystalline structures: CCSC phase



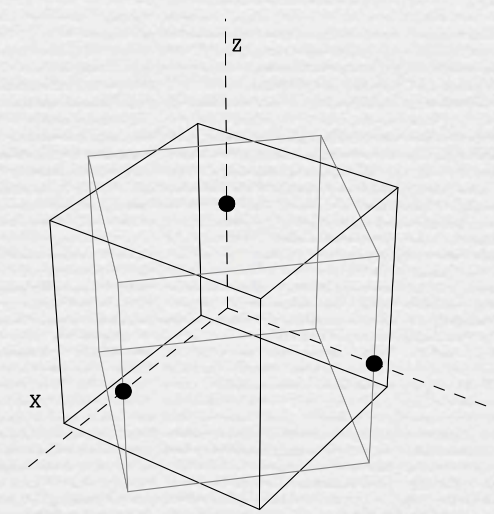
- Compliacted structures can be obtained combining more plane waves

- “no-overlap” condition between ribbons

CX



2cube45z



- Three flavors

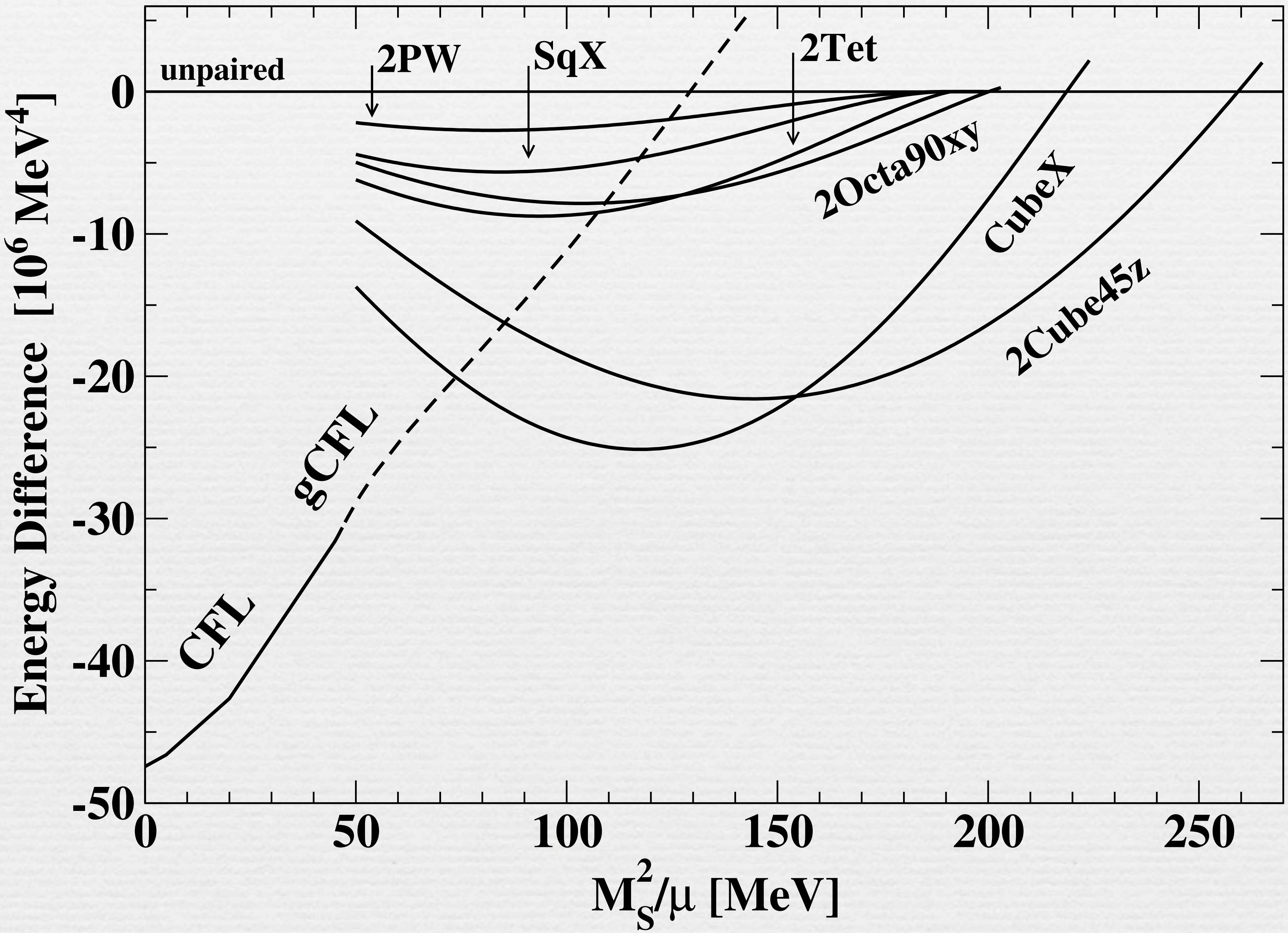
$$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I^m\}} e^{2i\mathbf{q}_I^m \cdot \mathbf{r}} \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

simplifications

$$\mathbf{q}_I^m = q\, \mathbf{n}_I^m$$

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Free energy evaluation



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Fermionic dispersion laws

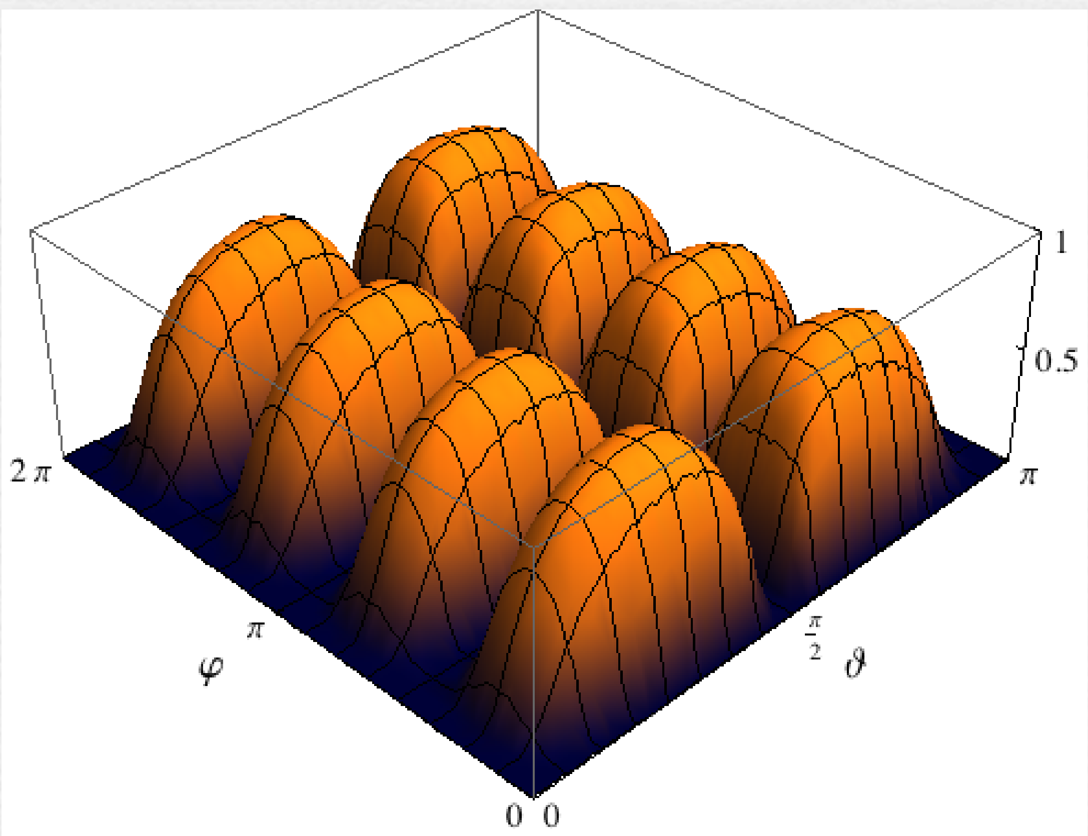
Fermions have an unisotropic gapless dispersion law:

direction dependent velocity

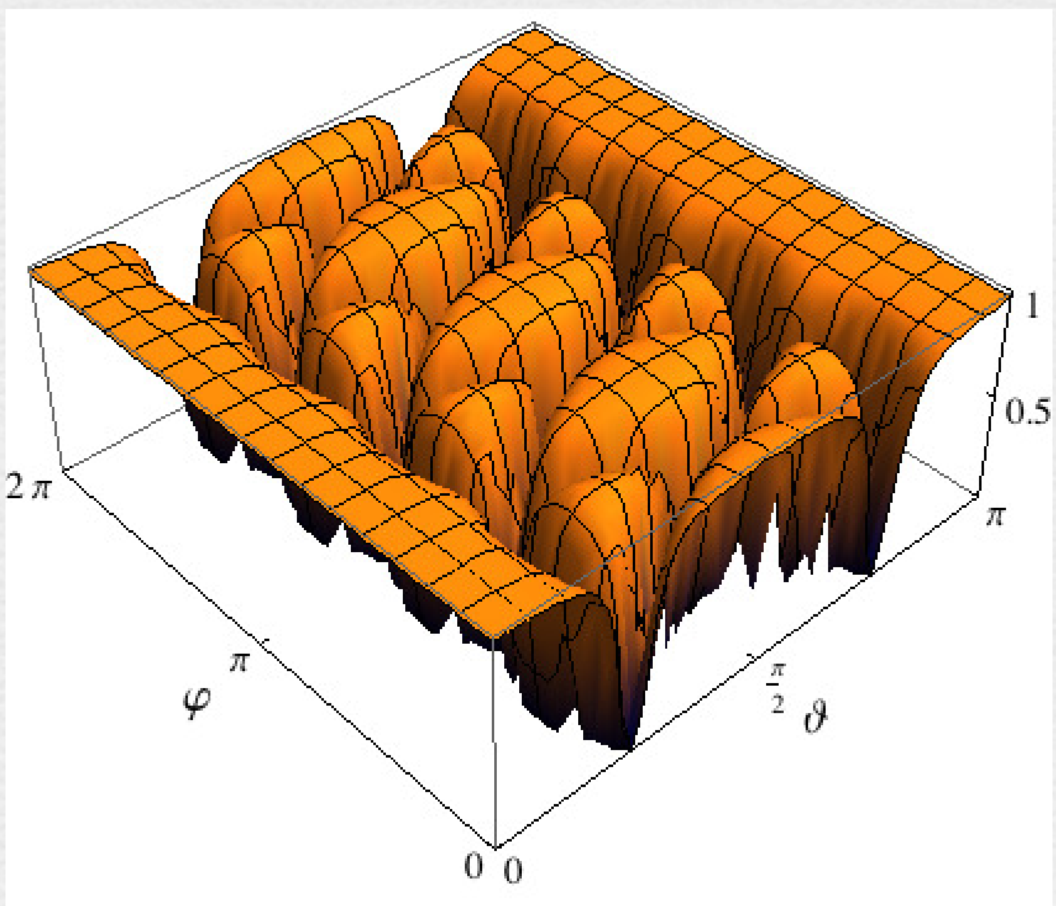
$$E = c(\theta, \phi)\, \xi$$

Velocity of fermions in two different structures

BCC

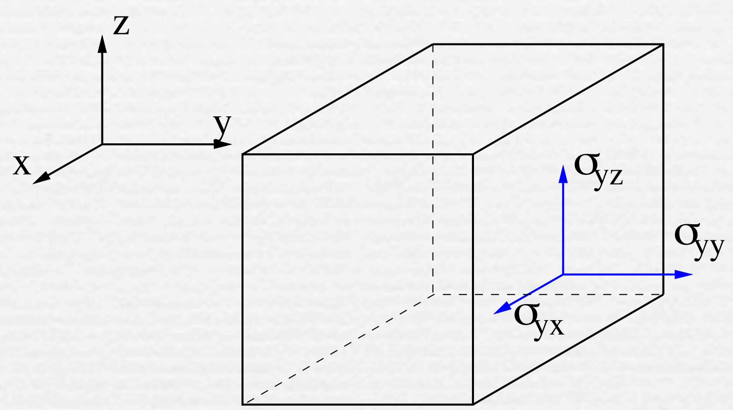


FCC



MM et al. 1302.4624

Shear modulus



The shear modulus describes the response of a crystal to a shear stress

$$\nu^{ij} = \frac{\sigma^{ij}}{2s^{ij}} \qquad \text{for} \quad i \neq j$$

σ^{ij} **stress tensor acting on the crystal**

s^{ij} **strain (deformation) matrix of the crystal**

- **Crystalline structure given by the spatial modulation of the gap parameter**
- **It is this pattern of modulation that is rigid (and oscillates)**

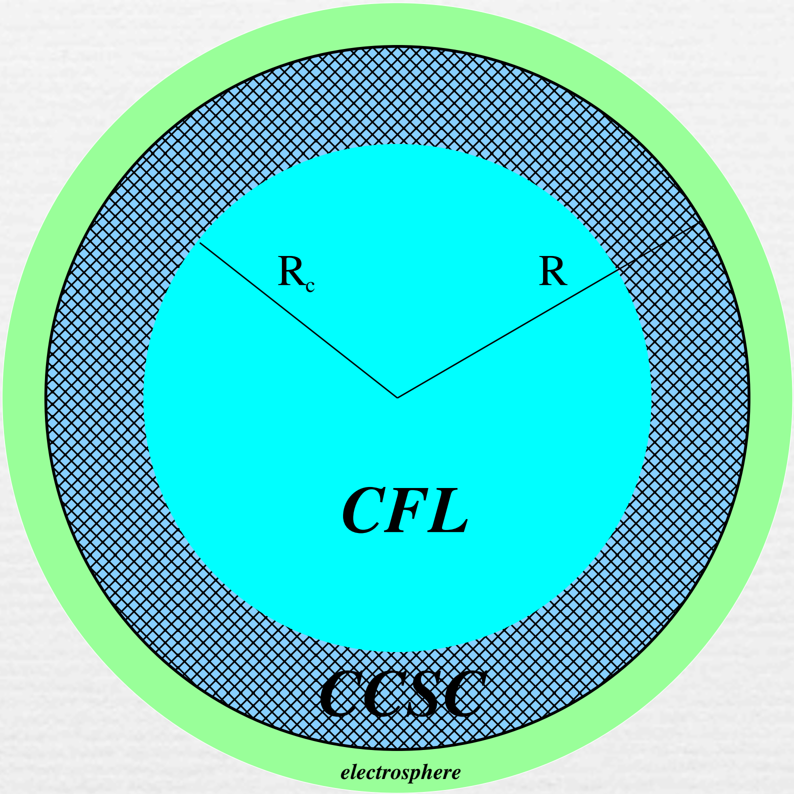
$$\nu = 2.47 \frac{\text{MeV}}{\text{fm}^3} \left(\frac{\Delta}{10\text{MeV}}\right)^2 \left(\frac{\mu}{400\text{MeV}}\right)^2$$

More rigid than diamond!!
20 to 1000 times more rigid than the crust of neutron stars

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

EM signals from bare strange stars

M.M., G.Pagliaroli, A. Parisi, L. Pilo, arXiv:1403.0128



Since the crust is rigid it can sustain large torsional oscillations

Since the star is bare, it has a large positive charge (compensated by the electrosphere)

Frequency of oscillation

$$\omega \propto \frac{1}{R - R_c}\sqrt{\frac{\nu}{\rho}}$$

about 1 GHz for a few centimeters thick crust

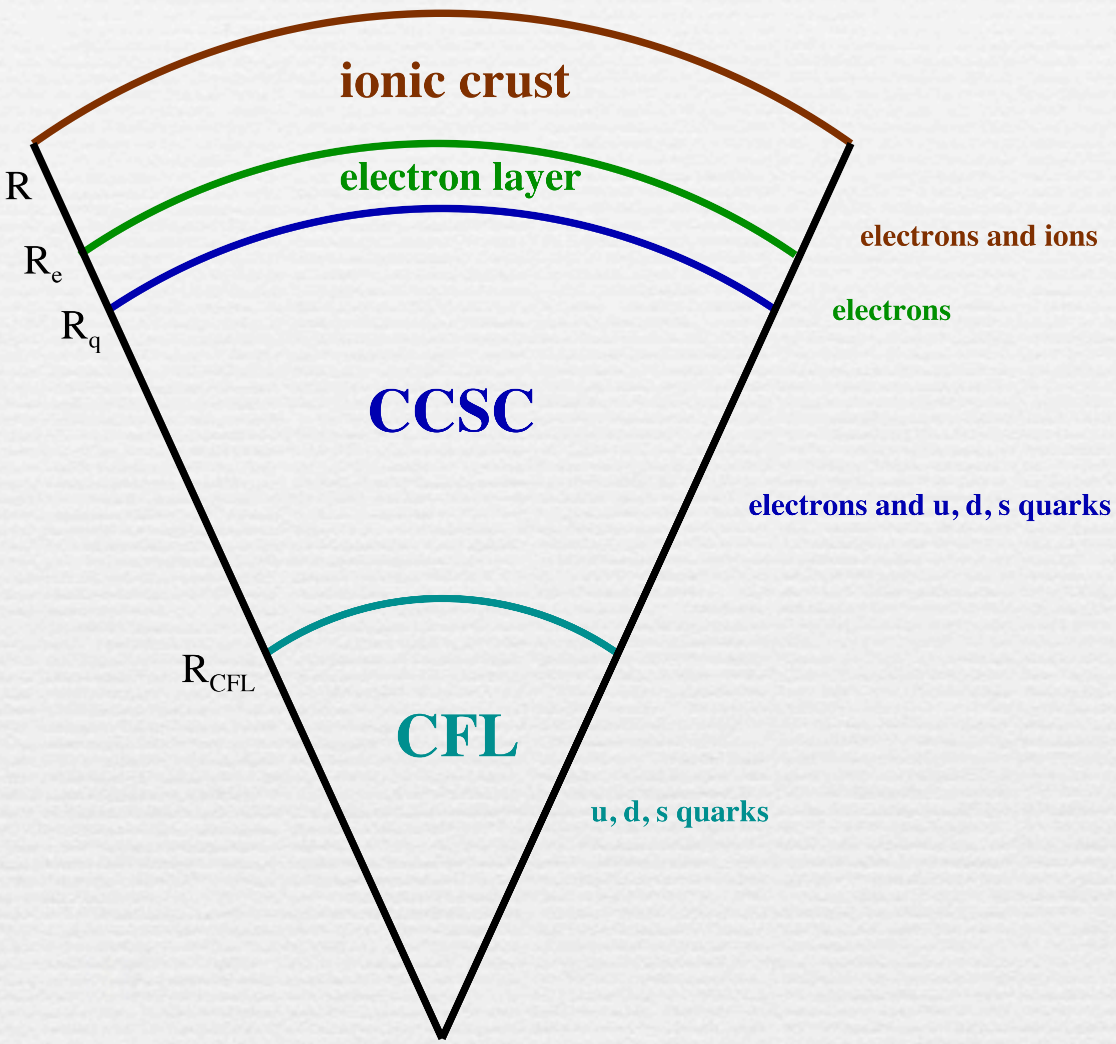
Estimated emitted power

$$P \propto 10^{50} \text{ erg/s}$$

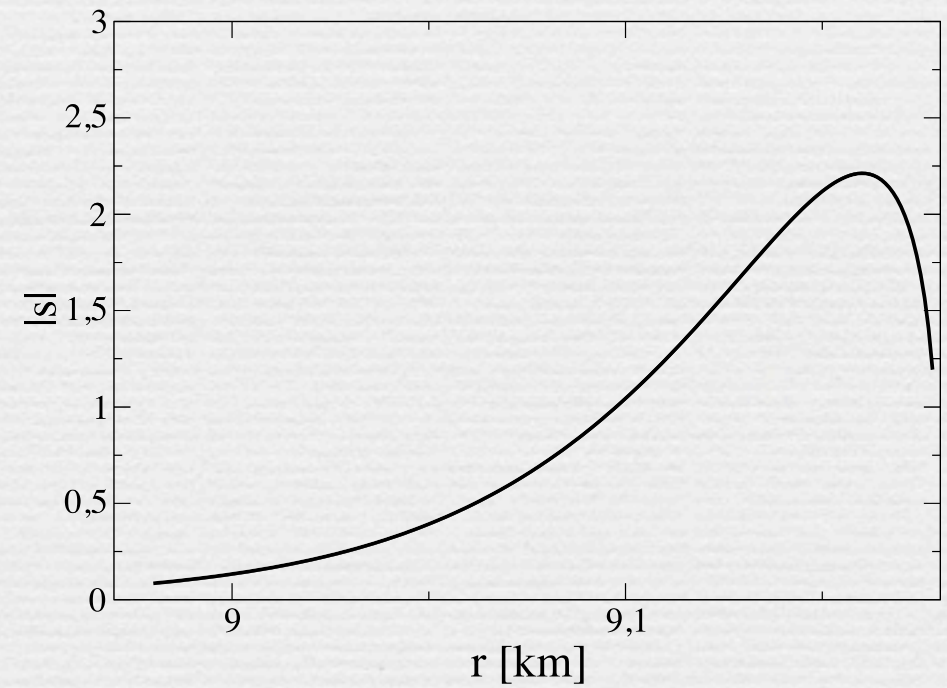
assuming a giant Vela-like glitch as the trigger

Torsional oscillations of nonbare strange stars

M.M., G.Pagliaroli, A. Parisi, L. Pilo, F. Tonelli arXiv:1504.07402



A compact star with two crusts!!
A ionic crust on the top of a CCSC crust



The shear strain has a maximum close to the star surface.
Thus, a torsional oscillation "easily" can break the ionic crust

Summary

- Motivated by compact stellar observations, the study of matter in extreme conditions allows to shed light on some properties of QCD
- Color superconductivity is a phase of matter predicted by QCD at extreme densities
- Crystalline color superconductors are extremely rigid, more rigid than any known material.
- The study of various observables related to the large rigidity are under way