THEORY OF THE LAMB SHIFT IN MUONIC HYDROGEN

Savely Karshenboim

Max-Planck-Institut für Quantenoptik (Garching)
&
Pulkovo Observatory ( ) (St. Petersburg)

MPQ
MAX-PLANCK-INSTITUTE
OF QUANTUM OPTICS
GARCHING

OUTLINE

- Level structure
- QED
  - Unperturbed energy levels
  - Specific QED
  - Re-scaled QED
  - Proton-line QED
  - Hadronic vacuum polarization
- Proton structure
  - Leading term
  - External field
  - Two-photon exchange
  - Recoil proton-size
  - Proton polarizability
- Comparison of theory and experiment
  - Proton radius

OUTLINE

- Level structure

- QED
  - Specific QED
  - Re-scaled QED

- L
  - Re
  - Pr... polarizability
    - Co

PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

Krzysztof Pachucki*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straβe 1, 85748 Garching, Germany
(Received 28 August 1995)

Michael I. Eides Howard Grotch Valery A. Shelyuto

Theory of
Light Hydrogenic
Bound States

Annals of Physics 326 (2011) 500–515
Contents lists available at ScienceDirect
ELSEVIER
Annals of Physic
journal homepage: www.elsevier.c

Lamb shift in muonic hydrogen—I. Veri
of theoretical predictions

U.D. Jentschura *
Department of Physics, Missouri University of Science and Technology, Rolla, Missou
National Institute of Standards and Technology, Gaithersburg, Maryland, MD 20899,

Annals of Physics 327 (2012) 733–763
Contents lists available at SciVerse ScienceDirect
ELSEVIER
Annals of Physics
journal homepage: www.elsevier.com/locate/aop

Lamb shift in light muonic atoms — Revisited

E. Borie *
Karlsruhe Institute of Technology, Institut für Hochleistungsimpuls and Mikrowellentechnik (IHM),
D-76344 Eggenstein-Leopoldshafen, Germany

Annals of Physics 331 (2013) 127–145
Contents lists available at SciVerse ScienceDirect
ELSEVIER
Annals of Physics
journal homepage: www.elsevier.com/locate/aop

Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

Aldo Antognini [a,*], Franz Kottmann [a], François Biraben [b], Paul Indelicato [b], François Nez [b], Randolf Pohl [c]
[a] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland*
[b] *Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France*
[c] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*

UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

- Effective-Dirac-equation approach

$$E = m_r(f_D - 1) - \frac{m_r^2}{2(M+m)}(f_D - 1)^2$$

- Breit-Hamiltonian approach

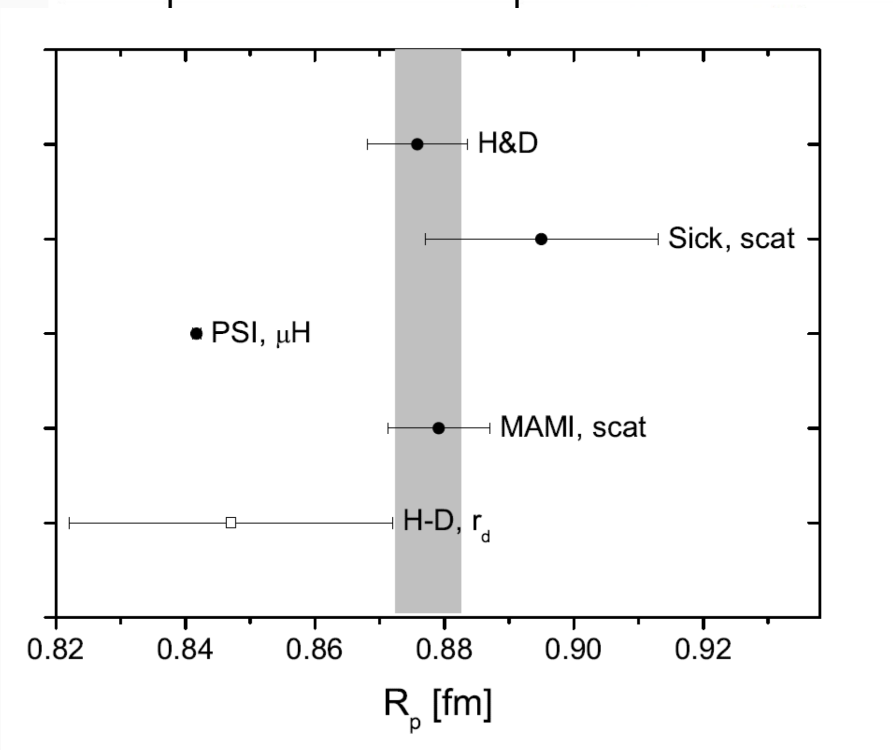
$$\Delta E_{\rm BG}(nl) = \frac{(Z\alpha)^4 m_r^3}{2n^3 M^2}\left(\frac{1}{j+1/2} - \frac{2}{3}\right)(1 - \delta_{l0})$$

UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

#	Designation	Order	Ref.	ΔE [meV]
0.1	Rel	$(Z\alpha)^{4+}m$		0
0.2	Rel-Rec*	$(Z\alpha)^4 m^2/M$		0
0.3	BG*	$(Z\alpha)^4(m/M)^2 m$	[31]	**0.057 47**
0.4	BP*†	$(Z\alpha)^4(m/M)^2 m$	Table XII	**−0.108 35**

UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

#	Designation	Order	Ref.	ΔE [meV]
				0
				0
		m	[31]	**0.057 47**
		m	Table XII	**−0.108 35**

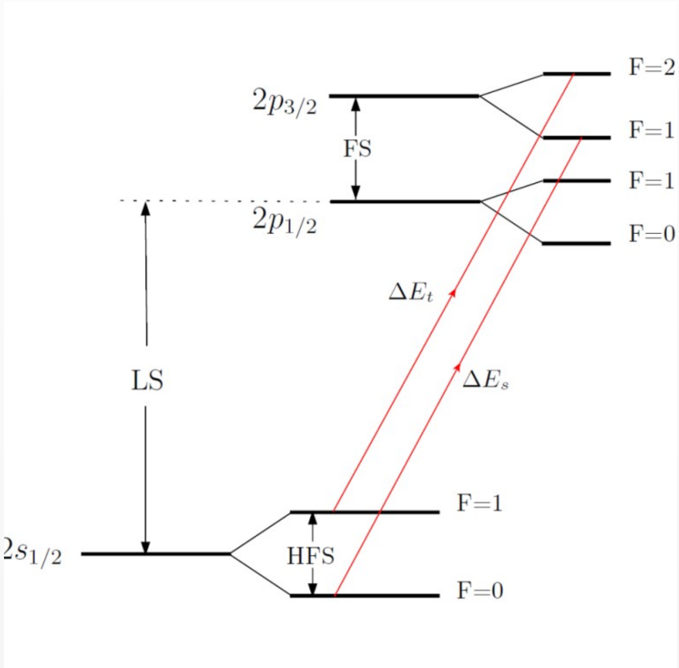


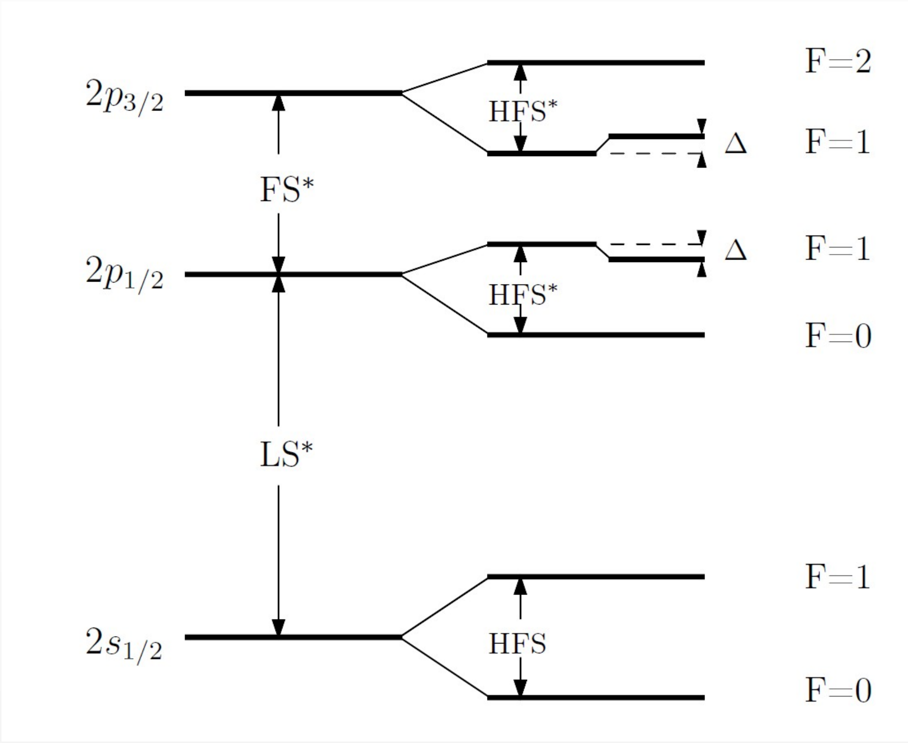
UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

#	Designation	Order	Ref.	ΔE [meV]
0.1	Rel	$(Z\alpha)^{4+}m$		0
0.2	Rel-Rec*	$(Z\alpha)^4 m^2/M$		0
0.3	BG*	$(Z\alpha)^4 (m/M)^2 m$	[31]	**0.057 47**
0.4	BP*†	$(Z\alpha)^4 (m/M)^2 m$	Table XII	**−0.108 35**

LEVELS STRUCTURE

- Experiment





Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen

Aldo Antognini,[1,2*] François Nez,[3] Karsten Schuhmann,[2,4] Fernando D. Amaro,[5] François Biraben,[3] João M. R. Cardoso,[5] Daniel S. Covita,[5,6] Andreas Dax,[7] Satish Dhawar Marc Diepold,[1] Luis M. P. Fernandes,[5] Adolf Giesen,[4,8] Andrea L. Gouvea,[5] Thomas Graf,[8] Theodor W. Hänsch,[1,9] Paul Indelicato,[3] Lucile Julien,[3] Cheng-Yang Kao,[10] Paul Knowles, Franz Kottmann,[2] Eric-Olivier Le Bigot,[3] Yi-Wei Liu,[10] José A. M. Lopes,[5] Livia Ludhova,[11] Cristina M. B. Monteiro,[5] Françoise Mulhauser,[11] Tobias Nebel,[1] Paul Rabinowitz,[12] Joaquim M. F. dos Santos,[5] Lukas A. Schaller,[11] Catherine Schwob,[3] David Taqqu,[13] João F. C. A. Veloso,[6] Jan Vogelsang,[1] Randolf Pohl[1]

PHYSICAL REVIEW VOLUME 163, NUMBER 1 5 NOVEMBER 1967

Precise Theory of the Zeeman Spectrum for Atomic Hydrogen and Deuterium and the Lamb Shift*

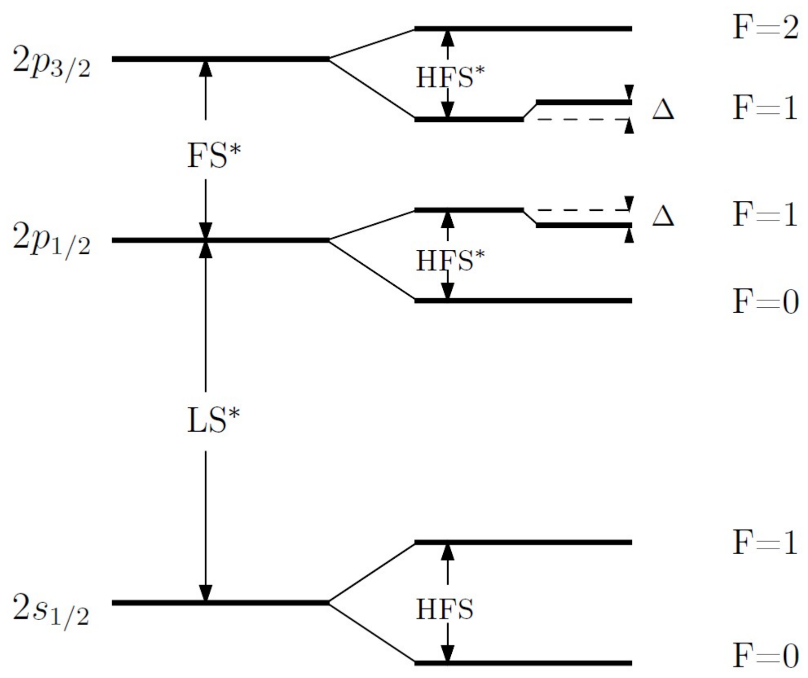
Stanley J. Brodsky and Ronald G. Parsons†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

UNPERTURBED ENERGY LEVELS: BRODSKY-PARSONS TERM

- HFS mixing

$$\langle 2p_{1/2}(F=1)|H_{\mathrm{HFS}}|2p_{3/2}(F=1)\rangle \neq 0$$

$$\Delta \simeq 0.145 \text{ meV}$$



PHYSICAL REVIEW VOLUME 163, NUMBER 1 5 NOVEMBER 1967

Precise Theory of the Zeeman Spectrum for Atomic Hydrogen and Deuterium and the Lamb Shift*

STANLEY J. BRODSKY AND RONALD G. PARSONS†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

Krzysztof Pachucki*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany

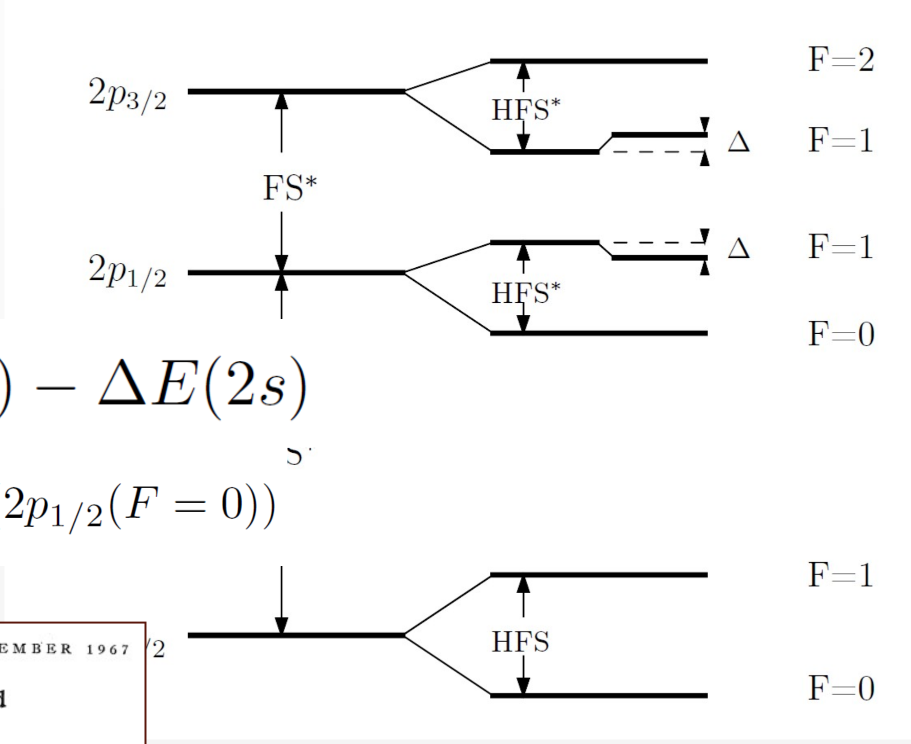
(Received 28 August 1995)

UNPERTURBED ENERGY LEVELS: BRODSKY-PARSONS TERM

- HFS mixing

$$\langle 2p_{1/2}(F=1)|H_{\mathrm{HFS}}|2p_{3/2}(F=1)\rangle \neq 0$$

- We consider



$$\Delta E_{\mathrm{L}}(2p_{1/2}-2s) \;\equiv\; \Delta E(2p_{1/2})-\Delta E(2s)$$

$$\Delta E(2p_{1/2}) \;\equiv\; \frac{3}{4}\Delta E(2p_{1/2}(F=1))+\frac{1}{4}\Delta E(2p_{1/2}(F=0))$$

PHYSICAL REVIEW VOLUME 163, NUMBER 1 5 NOVEMBER 1967

Precise Theory of the Zeeman Spectrum for Atomic Hydrogen and Deuterium and the Lamb Shift*

STANLEY J. BRODSKY AND RONALD G. PARSONS†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

ISSN 1063-7788, Physics of Atomic Nuclei, 2008, Vol. 71, No. 1, pp. 125–135. © Pleiades Publishing, Ltd., 2008.
Original Russian Text © A.P. Martynenko, 2008, published in Yadernaya Fizika, 2008, Vol. 71, No. 1, pp. 126–136.

ELEMENTARY PARTICLES AND FIELDS
Theory

Fine and Hyperfine Structure of P-Wave Levels in Muonic Hydrogen

A. P. Martynenko*
Samara State University, ul. Akademika Pavlova 1, Samara, 443011 Russia

PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

Krzysztof Pachucki*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
(Received 28 August 1995)

UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

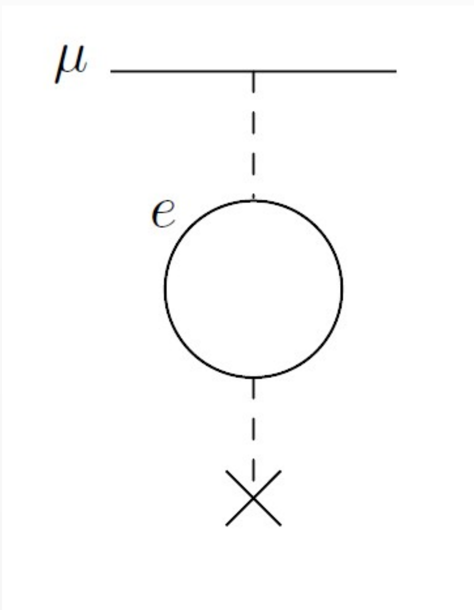
#	Designation	Order	Ref.	ΔE [meV]
0.1	Rel	$(Z\alpha)^{4+}m$		0
0.2	Rel-Rec*	$(Z\alpha)^4 m^2/M$		0
0.3	BG*	$(Z\alpha)^4(m/M)^2 m$	[31]	**0.057 47**
0.4	BP*†	$(Z\alpha)^4(m/M)^2 m$	Table XII	**−0.108 35**

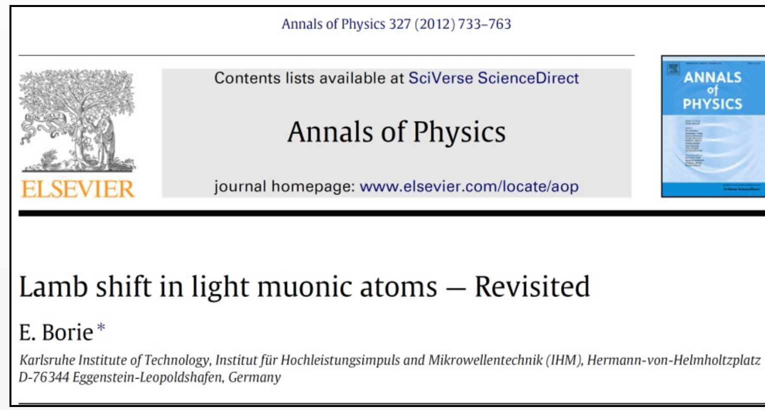
SPECIFIC QED FOR MUONIC HYDROGEN

#	Designation	Order	Ref.	ΔE [meV]
1.1	eVP1 (NR)*	$\alpha(Z\alpha)^2 m$		**205.007 36**
1.2	eVP1 (Rel)	$\alpha(Z\alpha)^4 m$		0.020 84
1.3	eVP1 (Rel-Rec)*	$\alpha(Z\alpha)^4 \frac{m^2}{M}$	[25, 34]	−0.002 08
2	eVP2 (NR)*	$\alpha^2(Z\alpha)^2 m$	[21, 50]	**1.658 85**
3	eVP3 (NR)*	$\alpha^3(Z\alpha)^2 m$	[51, 52]	0.007 52
4	LbL*†	$\alpha^5 m$	Table III	−0.000 89(2)
5	eVP+SE	$\alpha^2(Z\alpha)^4 m$	[53]	−0.002 54
6	SE[eVP]	$\alpha^2(Z\alpha)^4 m$	[26, 54]	−0.001 52

SPECIFIC QED FOR MUONIC HYDROGEN

- The leading term





Annals of Physics 327 (2012) 733–763

Contents lists available at SciVerse ScienceDirect

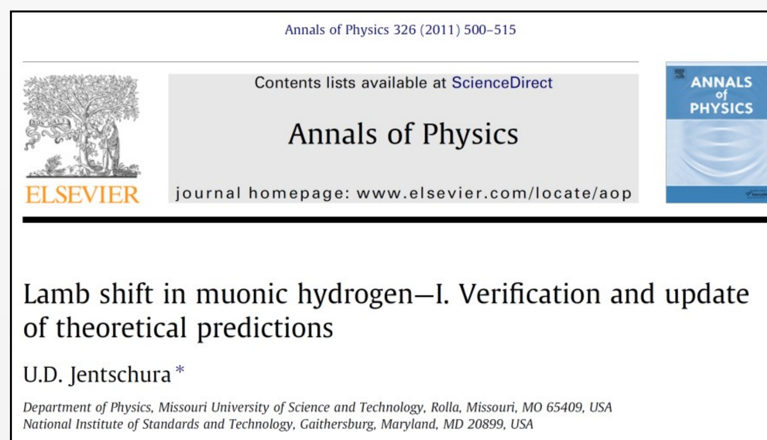
Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Lamb shift in light muonic atoms — Revisited

E. Borie *

Karlsruhe Institute of Technology, Institut für Hochleistungsimpuls and Mikrowellentechnik (IHM), Hermann-von-Helmholtzplatz 1, D-76344 Eggenstein-Leopoldshafen, Germany



Annals of Physics 326 (2011) 500–515

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

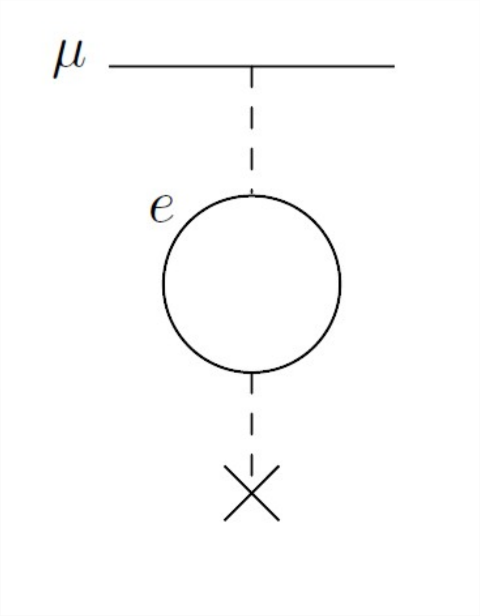
Lamb shift in muonic hydrogen—I. Verification and update of theoretical predictions

U.D. Jentschura *

Department of Physics, Missouri University of Science and Technology, Rolla, Missouri, MO 65409, USA
National Institute of Standards and Technology, Gaithersburg, Maryland, MD 20899, USA

SPECIFIC QED FOR MUONIC HYDROGEN

- The leading term



Annals of Physics 327 (2012) 733–763

Contents lists available at SciVerse ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Lamb shift in light muonic atoms — Revisited

PHYSICAL REVIEW A **85**, 032509 (2012)

Relativistic recoil corrections to the electron-vacuum-polarization contribution in light muonic atoms

Savely G. Karshenboim*
Max-Planck-Institut für Quantenoptik, Garching, D-85748, Germany and Pulkovo Observatory, St. Petersburg, RU-196140, Russia

Vladimir G. Ivanov
Pulkovo Observatory, St. Petersburg, RU-196140, Russia

Evgeny Yu. Korzinin
D. I. Mendeleev Institute for Metrology, St. Petersburg, RU-190005, Russia

Annals of Physics

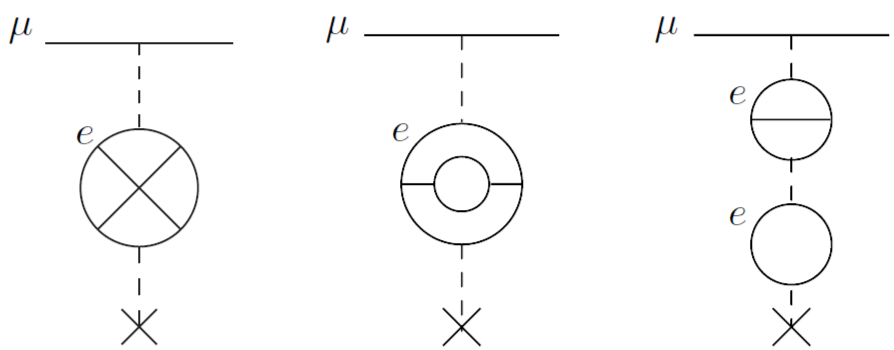
journal homepage: www.elsevier.com/locate/aop

Lamb shift in muonic hydrogen—I. Verification and update of theoretical predictions

U.D. Jentschura*

Department of Physics, Missouri University of Science and Technology, Rolla, Missouri, MO 65409, USA
National Institute of Standards and Technology, Gaithersburg, Maryland, MD 20899, USA

SPECIFIC QED FOR MUONIC HYDROGEN: EVP (3)



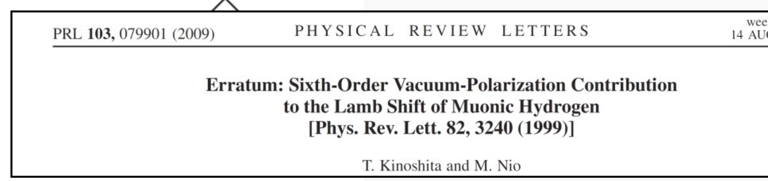

RAPID COMMUNICATIONS
PHYSICAL REVIEW A **81**, 060501(R) (2010)

Nonrelativistic contributions of order $\alpha^5 m_\mu c^2$ to the Lamb shift in muonic hydrogen and deuterium, and in the muonic helium ion

S. G. Karshenboim*
D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia and Max-Planck-Institut für Quantenoptik, Garching D-85748, Germany

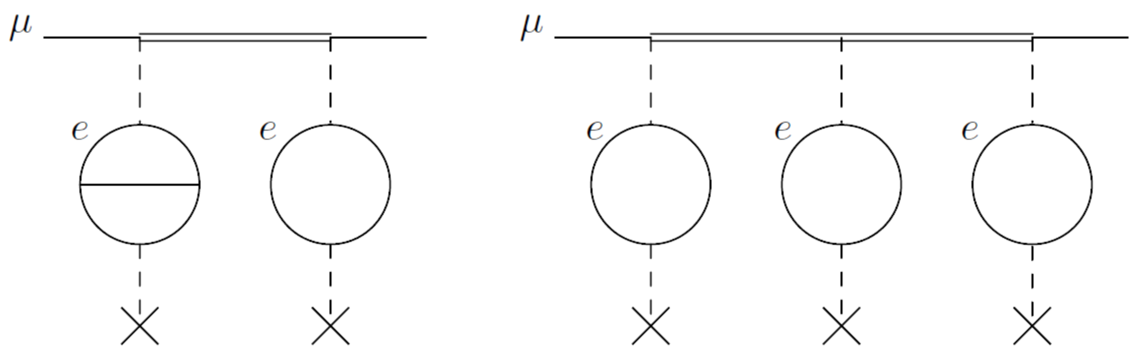
V. G. Ivanov
Pulkovo Observatory, St. Petersburg RU-196140, Russia and D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia

E. Yu. Korzinin and V. A. Shelyuto
D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia

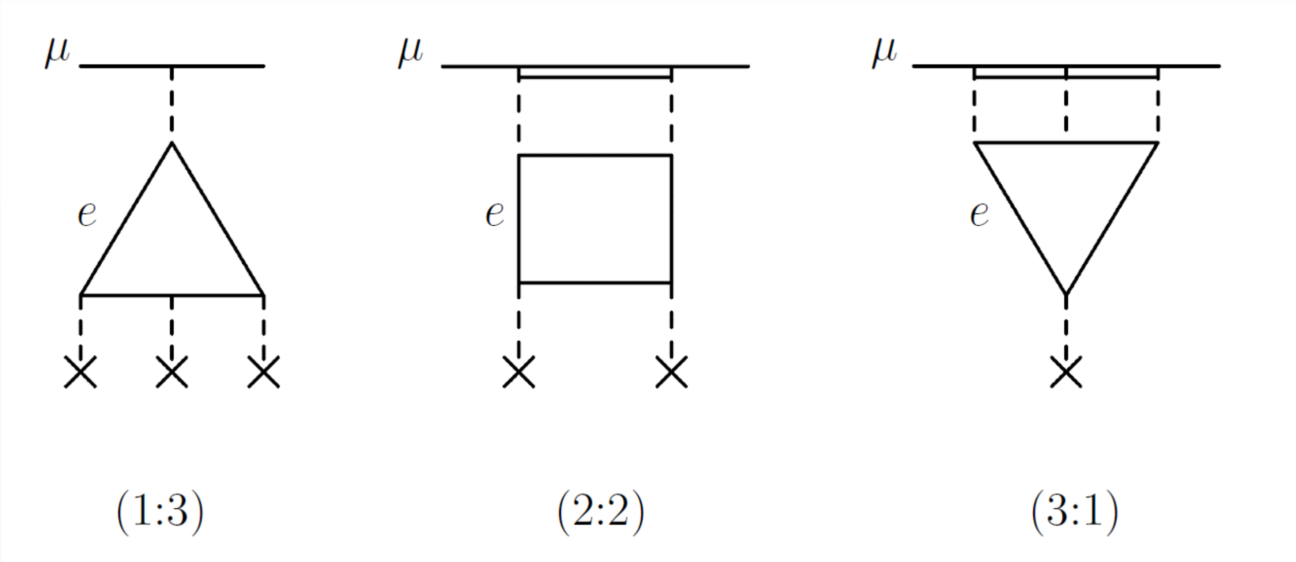

PRL **103**, 079901 (2009) PHYSICAL REVIEW LETTERS week e
14 AUGU

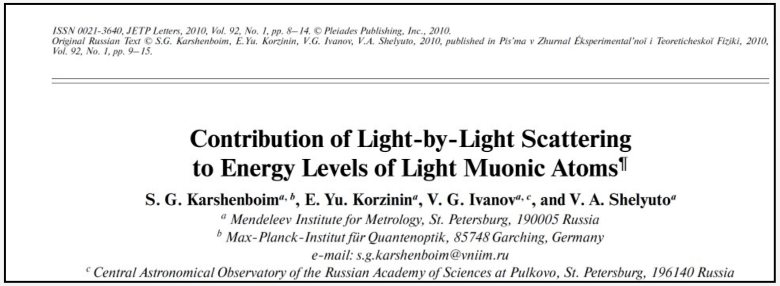
Erratum: Sixth-Order Vacuum-Polarization Contribution to the Lamb Shift of Muonic Hydrogen [Phys. Rev. Lett. 82, 3240 (1999)]

T. Kinoshita and M. Nio



SPECIFIC QED FOR MUONIC HYDROGEN: LIGHT-BY-LIGHT




ISSN 0021-3640, JETP Letters, 2010, Vol. 92, No. 1, pp. 8–14. © Pleiades Publishing, Inc., 2010.
Original Russian Text © S.G. Karshenboim, E.Yu. Korzinin, V.G. Ivanov, V.A. Shelyuto, 2010, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2010, Vol. 92, No. 1, pp. 9–15.

Contribution of Light-by-Light Scattering to Energy Levels of Light Muonic Atoms¶

S. G. Karshenboim[a, b], E. Yu. Korzinin[a], V. G. Ivanov[a, c], and V. A. Shelyuto[a]
[a] Mendeleev Institute for Metrology, St. Petersburg, 190005 Russia
[b] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany
e-mail: s.g.karshenboim@vniim.ru
[c] Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo, St. Petersburg, 196140 Russia

SPECIFIC QED FOR MUONIC HYDROGEN

#	Designation	Order	Ref.	ΔE [meV]
1.1	eVP1 (NR)*	$\alpha(Z\alpha)^2 m$		**205.007 36**
1.2	eVP1 (Rel)	$\alpha(Z\alpha)^4 m$		0.020 84
1.3	eVP1 (Rel-Rec)*	$\alpha(Z\alpha)^4 \frac{m^2}{M}$	[25, 34]	−0.002 08
2	eVP2 (NR)*	$\alpha^2(Z\alpha)^2 m$	[21, 50]	**1.658 85**
3	eVP3 (NR)*	$\alpha^3(Z\alpha)^2 m$	[51, 52]	0.007 52
4	LbL*†	$\alpha^5 m$	Table III	−0.000 89(2)
5	eVP+SE	$\alpha^2(Z\alpha)^4 m$	[53]	−0.002 54
6	SE[eVP]	$\alpha^2(Z\alpha)^4 m$	[26, 54]	−0.001 52

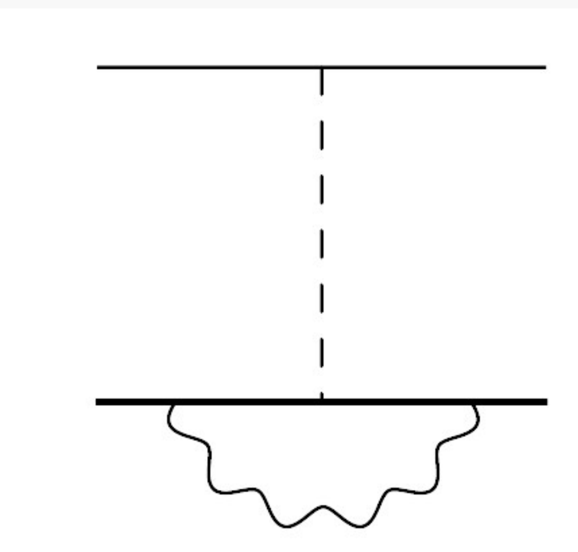
RE-SCALED HYDROGENIC THEORY

#	Designation	Order	ΔE [meV]
7.1	QED (Rad)*	$\alpha(Z\alpha)^4 m$	$-\mathbf{0.663\,45}$
7.2	QED (Rad)	$\alpha(Z\alpha)^5 m$	$-0.004\,43$
7.3.	QED (Rad-Rec)	$\alpha(Z\alpha)^5 \frac{m^2}{M}$	$0.000\,19$
8	QED (Rec)*	$(Z\alpha)^5 m^2/M$	$-\mathbf{0.044\,97}$

PROTON-LINE QED

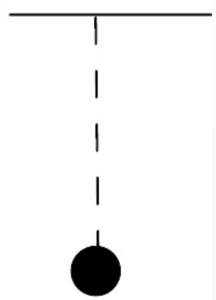
- Infrared divergence in the radius

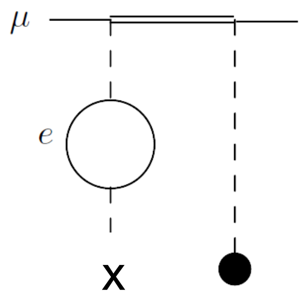
$$\Delta R_p^2 = \frac{\alpha}{\pi}\frac{1}{m_p^2}\ln\left(\frac{m_p^2}{\lambda^2}\right)$$

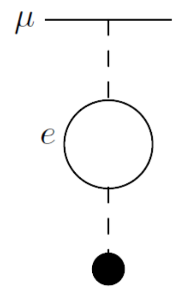


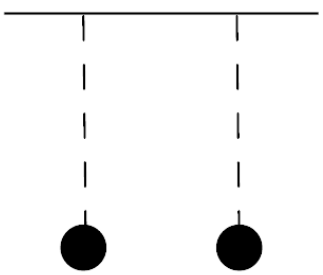
$$\Delta E_{\mathrm{pQED}}(nl) = \frac{4(Z^2\alpha)(Z\alpha)^4}{\pi n^3}\frac{m_r^3}{M^2} \times\left\{\left[\frac{1}{3}\ln\frac{M}{(Z\alpha)^2 m_r}+\frac{11}{72}\right]\delta_{l0} - \frac{1}{3}\ln k_0(nl)\right\},$$

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTIONS





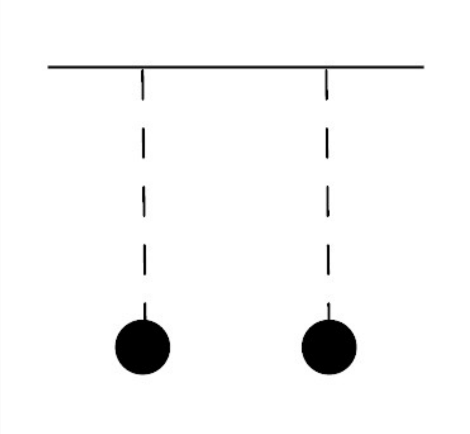




#	Designation	Order	Ref.	ΔE [meV]	
				Value	Estimation
10	PS (NR)	$(Z\alpha)^4 m$		$\mathbf{-5.1974}\ r_p^2$	$-\mathit{3.7}$
11	PS (Rel)	$(Z\alpha)^6 m$	[65–67]	$-0.0016\ r_p^2 - 0.000\,04(r_p^2)^2$	$-\mathit{0.0011}$
12	PS (eVP)	$\alpha(Z\alpha)^4 m$	Eq. (15)	$-0.0282\ r_p^2$	$-\mathit{0.020}$
13	PS (SE)	$\alpha(Z\alpha)^4 m$	[6]	$0.0006\ r_p^2$	*0.0005*
14.1	PS (Fri) term	$(Z\alpha)^5 m$	[69]	$-0.0251(35) + 0.062\,44\ r_p^2$	*0.019*

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: PROBLEM

- Friar term



$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5 m_r^4}{\pi} I_{\rm Fr} ,$$
$$I_{\rm Fr} = \frac{\pi}{48} \int d^3r\, d^3r' \rho_E(\mathbf{r}) \rho_E(\mathbf{r}') |\mathbf{r} - \mathbf{r}'|^3$$
$$= \frac{\pi}{48} \langle r^3 \rangle_2 ,$$

$$I_{\rm Fr} = \int_0^\infty \frac{dq}{q^4} \left[\left(G_E(q^2) \right)^2 - 1 - 2G'_E(0)\, q^2 \right]$$

Volume 80B, number 3 PHYSICS LETTERS

FINITE-SIZE CORRECTIONS TO THE ENERGY LEVELS OF LIGHT MUONIC ATOMS☆

J.L. FRIAR
Theoretical Division, Los Alamos Scientific Laboratory, Los Alamos, NM 87545, USA

PHYSICAL REVIEW A VOLUME 53, NUMBER 4

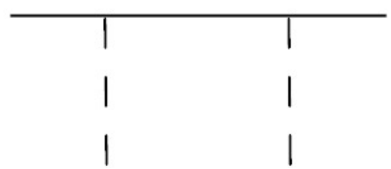
Theory of the Lamb shift in muonic hydrogen

Krzysztof Pachucki*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
(Received 28 August 1995)

$$G'_E(0) = -\frac{1}{6} R_p^2$$

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: PROBLEM

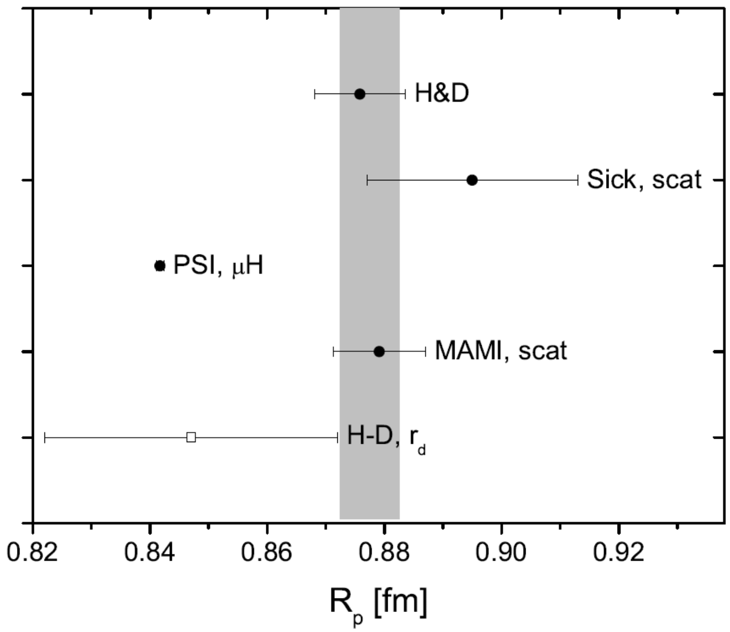
- Friar term



$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5 m_r^4}{\pi} I_{\rm Fr} ,$$

$$I_{\rm Fr} = \frac{\pi}{48} \int d^3r\, d^3r' \rho_E(\mathbf{r})\rho_E(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^3$$

$$= \frac{\pi}{48} \langle r^3 \rangle_2 ,$$



$${}_{\rm Fr} = \int_0^\infty \frac{dq}{q^4} \left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\, q^2 \right]$$

$$G'_E(0) = -\frac{1}{6} R_p^2$$

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: PROBLEM

- Friar term

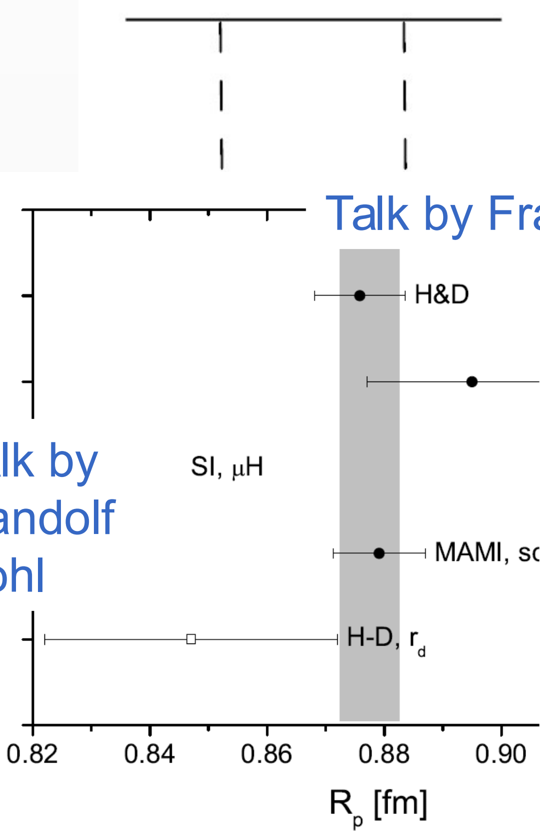
$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5\, m_r^4}{\pi}\, I_{\rm Fr}\,,$$

$$\pi \int d^3r\, d^3r'\, \rho_E(\mathbf{r})\rho_E(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^3$$

$$2\,,$$

$$_E(q^2)\big)^2 - 1 - 2G'_E(0)\, q^2\Big]$$

$$_E(0) = -\frac{1}{6} R_p^2$$



Talk by François Nez

Talk by Randolf Pohl

Talk by John Arrington

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: METHOD

$$I = \int_0^\infty dq \ldots \equiv I_< + I_> \equiv \int_0^{q_0} dq \ldots + \int_{q_0}^\infty dq \ldots$$

N·H
ELSEVIER
27 January 1997
PHYSICS LET
Physics Letters A 225 (1997) 97–106

Nuclear structure-dependent radiative corrections to the hydrogen hyperfine splitting

Savely G. Karshenboim[1]
D.I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 198005, Russian Federation

PHYSICAL REVIEW D **90,** 053012 (2014)

Self-consistent value of the electric radius of the proton from the Lamb shift in muonic hydrogen

Savely G. Karshenboim*
Max-Planck-Institut für Quantenoptik, Garching 85748, Germany and Pulkovo Observatory, St. Petersburg 196140, Russia

The recoil correction to the proton-finite-size contribution to the Lamb shift in muonic hydrogen

Savely G. Karshenboim*
Max-Planck-Institut für Quantenoptik, Garching, 85748, Germany and Pulkovo Observatory, St.Petersburg, 196140, Russia

Evgeny Yu. Korzinin and Valery A. Shelyuto
D. I. Mendeleev Institute for Metrology, St.Petersburg, 190005, Russia

Vladimir G. Ivanov
Pulkovo Observatory, St.Petersburg, 196140, Russia

Poster by Evgeny Korzinin

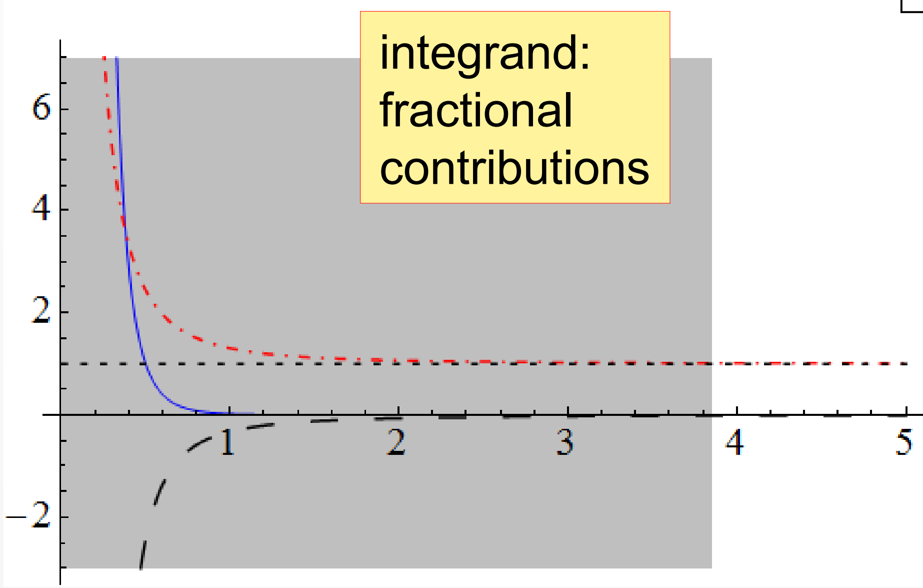
EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: RESULTS

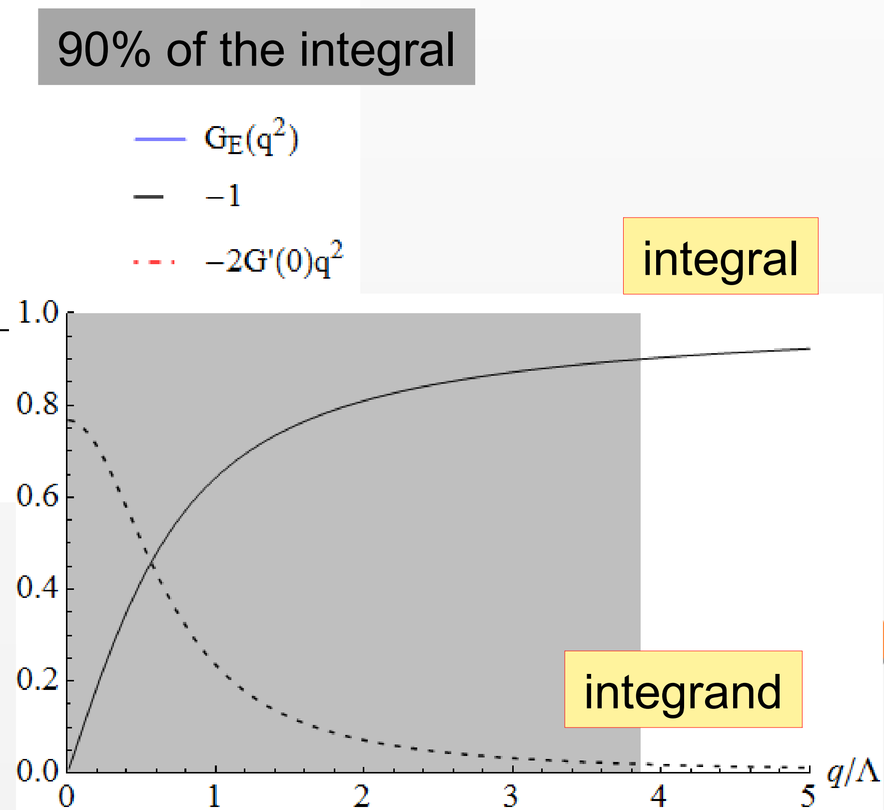
#	Designation	Order	Ref.	ΔE [meV]	
				Value	Estimation
10	PS (NR)	$(Z\alpha)^4 m$		$\mathbf{-5.1974}\ r_p^2$	$\boldsymbol{-3.7}$
11	PS (Rel)	$(Z\alpha)^6 m$	[65–67]	$-0.0016\ r_p^2 - 0.000\,04 (r_p^2)^2$	-0.0011
12	PS (eVP)	$\alpha(Z\alpha)^4 m$	Eq. (15)	$-0.0282\ r_p^2$	-0.020
13	PS (SE)	$\alpha(Z\alpha)^4 m$	[6]	$0.0006\ r_p^2$	0.0005
14.1	PS (Fri) term	$(Z\alpha)^5 m$	[69]	$-0.0251(35) + 0.062\,44\ r_p^2$	0.019

THE LAMB SHIFT IN MUONIC HYDROGEN: CONSISTENCY PROBLEM

- The integrand includes
  Three terms:

$$I_3^E \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$

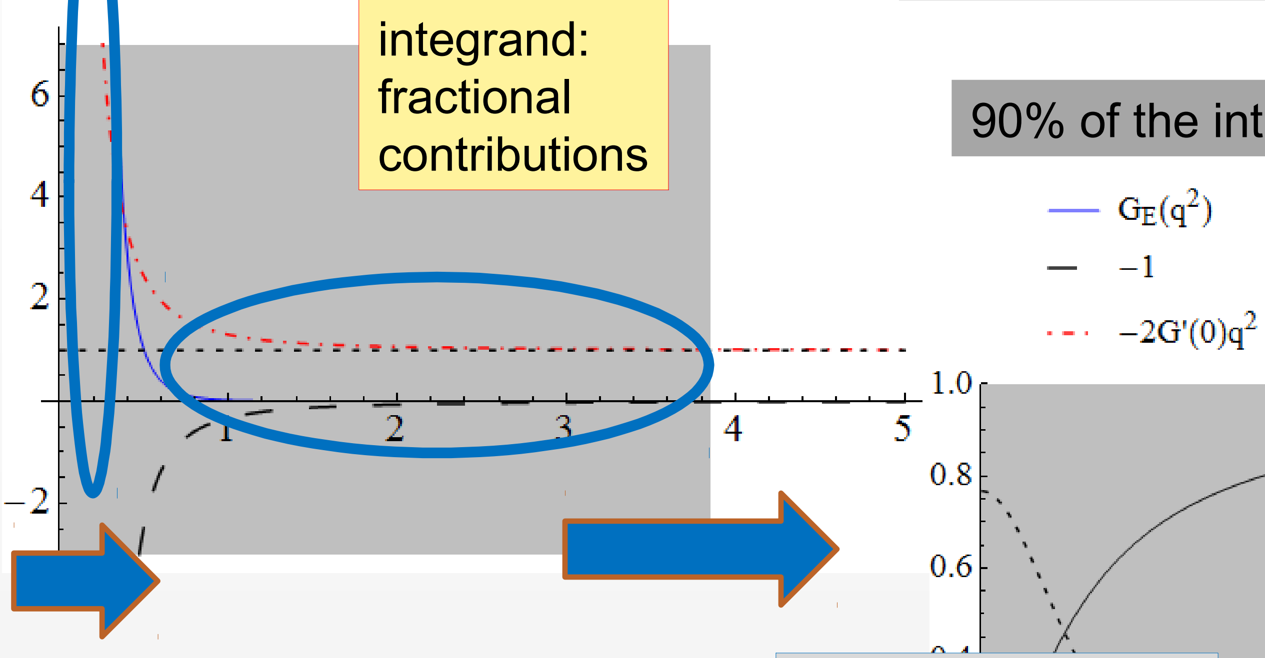


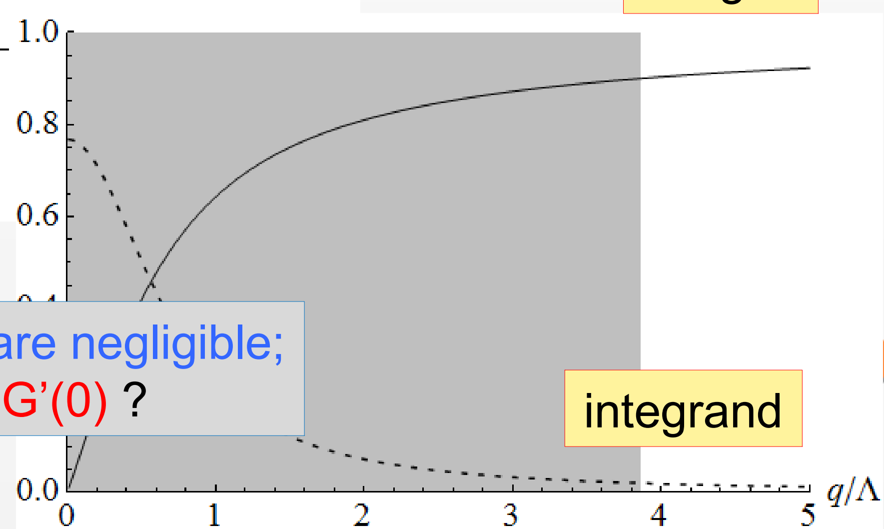


THE LAMB SHIFT IN MUONIC HYDROGEN: CONSISTENCY PROBLEM

- The integrand includes
  Three terms:

$$I_3^E \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\,q^2\right]$$





STRATEGY OF THE EVALUATION

- Split the integral

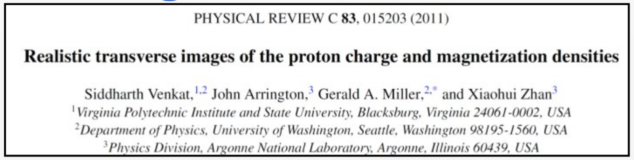
$$I = \int_0^\infty dq \ldots \equiv I_< + I_> \equiv \int_0^{q_0} dq \ldots + \int_{q_0}^\infty dq \ldots$$

- Low momentum

$$\left(G_E(q^2)\right)^2 \quad \simeq \quad 1 - \frac{R_E^2}{3} q^2 + C^{\text{dip}} (1 \pm 1)\, q^4$$

$$I_{3>}^{\text{E}} = \int_{q_0}^\infty \frac{dq}{q^4} \left(G_E(q^2)\right)^2 - \frac{1}{3q_0^3} + \frac{1}{3} \frac{R_E^2}{q_0}$$

- High momentum



PHYSICAL REVIEW C **83**, 015203 (2011)

Realistic transverse images of the proton charge and magnetization densities

Siddharth Venkat,[1,2] John Arrington,[3] Gerald A. Miller,[2,*] and Xiaohui Zhan[3]

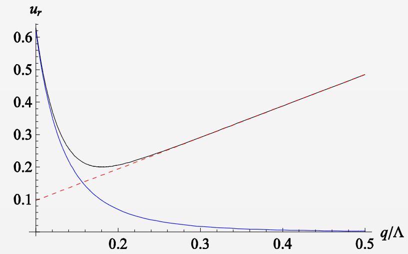
[1]*Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0002, USA*

[2]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*

[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

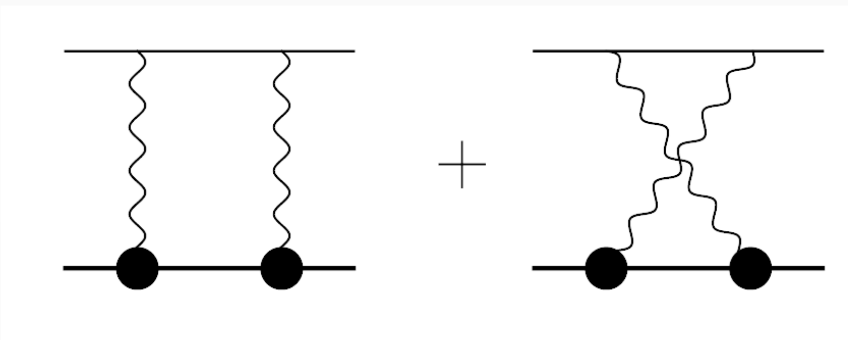
$$\delta I_{3>}^{\text{E}} = \frac{1}{3q_0^3} \frac{2\delta G_E(q_0^2)}{G_E(q_0^2)} \left(G_{\text{dip}}(q_0^2)\right)^2$$

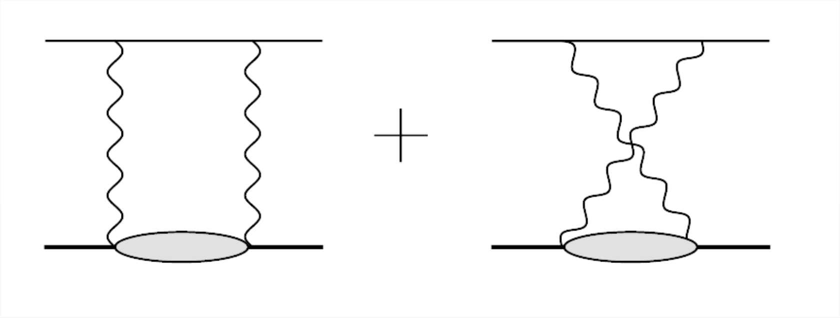
- Minimization of the uncertainty



TWO-PHOTON EXCHANGE

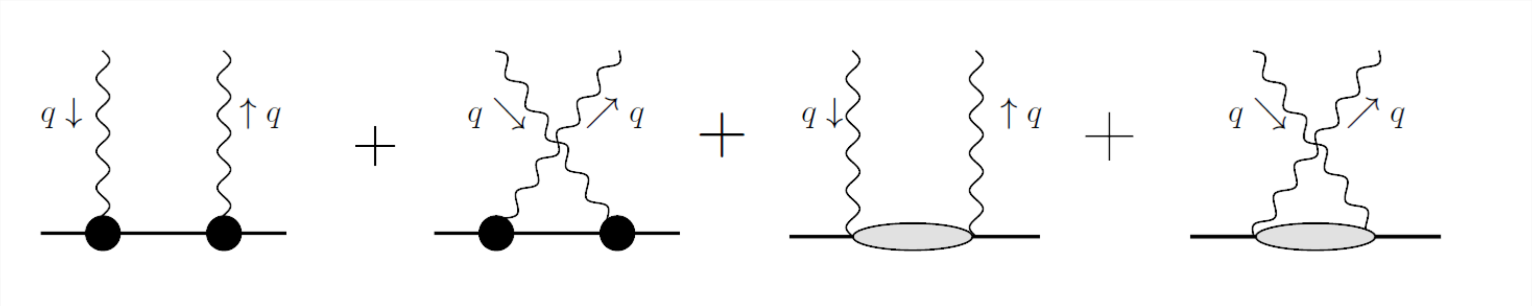
- Elastic





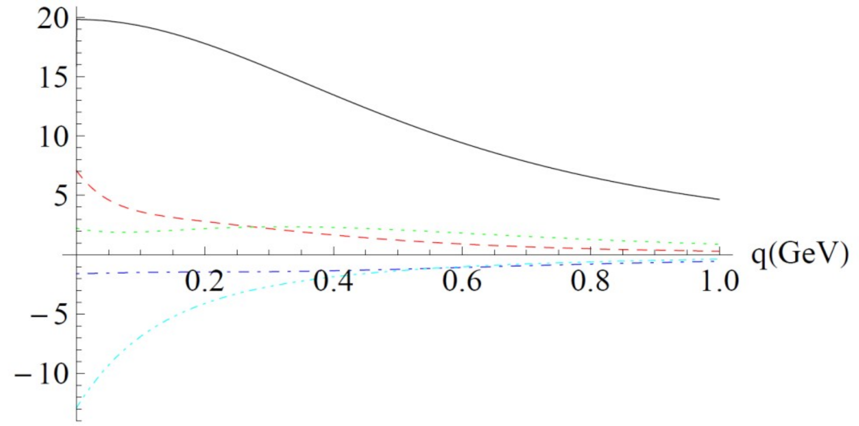
- Virtual Compton amplitude

Talk by Mike Birse



ELASTIC TPE: RECOIL & FINITE SIZE

$$
\begin{aligned}
I_{\rm rec} &= I_\kappa + I_{\rm EF} + I_{\rm M1} + I_{\rm M2} \;, \\
I_\kappa &= \kappa \int_0^\infty \frac{dq}{q^4}\left\{(2+\kappa) f_{M1} + f_{M2}\right\} \;, \\
I_{\rm EF} &= \int_0^\infty \frac{dq}{q^4} f_{EF}(m,M;q^2)\left[\left(G_E(q^2)\right)^2 - 1\right] \;, \\
I_{\rm M1} &= \int_0^\infty \frac{dq}{q^4} f_{M1}\left[\left(G_M(q^2)\right)^2 - (1+\kappa)^2\right] \;, \\
I_{\rm M2} &= \int_0^\infty \frac{dq}{q^4} f_{M2}\left[G_M(q^2)G_E(q^2) - (1+\kappa)\right] \;, (21)
\end{aligned}
$$



PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

Krzysztof Pachucki*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
(Received 28 August 1995)

Proton Polarizability and Lamb Shift in the Muonic Hydrogen Atom

A. P. Martynenko and R. N. Faustov[1)]
Samara State University, ul. Akad. Pavlova 1, Samara, 443011 Russia

PHYSICAL REVIEW A **84**, 020102(R) (2011)

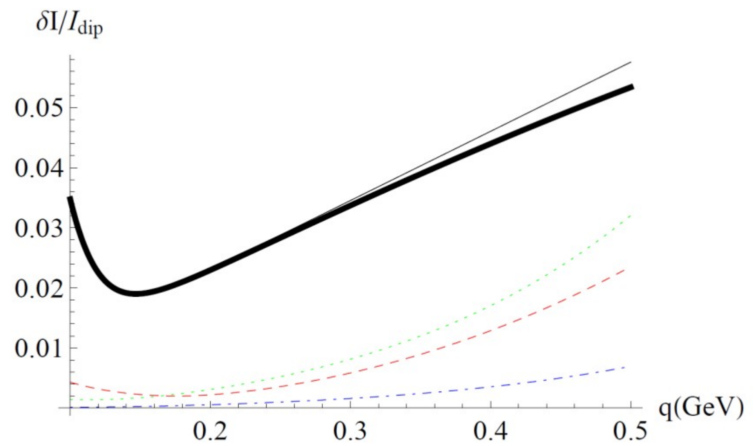
Higher-order proton structure corrections to the Lamb shift in muonic hydrogen

Carl E. Carlson[1,2] and Marc Vanderhaeghen[3]
[1]*Helmholtz Institut Mainz, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*
[2]*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*
[3]*Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*

Eur. Ph
DOI 10.
Regul

Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory

M.C. Birse[a] and J.A. McGovern
Theoretical Physics Division, School of Physics and Astronomy, The University of Manchester, Manchester, M13 9PL, UK



ELASTIC TPE: RECOIL & FINITE SIZE

#	Designation	ΔE [meV]	
		Value	Estimation
14.1	eTPE:Fri	$0.062\, r_p^2 - 0.025(4)$	*0.019*
14.2	eTPE:κ^*	$-0.003\,05$	*−0.003*
14.3	eTPE:EF*	$0.001\,07\, r_p^2 + 0.001\,36(4)$	*0.002*
14.4	eTPE:M1*	$0.001\,88(3)$	*0.002*
14.5	eTPE:M2*	$-0.000\,016\, r_p^2 - 0.000\,90$	*−0.0009*
14	eTPE	$0.064\, r_p^2 - 0.026(4)$	*0.019*

PROTON POLARIZABILITY: SUBTRACTION AND DISPERSION RELATIONS

$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

$$\Delta E_{\rm inel}(nl) = -\frac{2\alpha^2}{mM}|\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2} \int_{\nu_{\rm th}}^\infty d\nu \left[\frac{\widetilde{\gamma}_1(\nu, q^2) F_1(\nu, q^2)}{\nu} + \frac{\widetilde{\gamma}_2(\nu, q^2) F_2(\nu, q^2)}{q^2/M}\right],$$

$$\Delta E_{\rm inel}(2p_{1/2} - 2s_{1/2}) = 13.0(0.6)\ \mu{\rm eV}$$

PHYSICAL REVIEW A **87**, 052501 (2013)

Muonic-hydrogen Lamb shift: Dispersing the nucleon-excitation uncertainty with a finite-energy sum rule

Mikhail Gorchtein,[1,*] Felipe J. Llanes-Estrada,[2] and Adam P. Szczepaniak[3]
[1]*Institut für Kernphysik, Universität Mainz, 55128 Mainz, Germany*
[2]*Departamento de Fisica Teorica I, Universidad Complutense de Madrid, Madrid 28040, Spain*
[3]*Department of Physics and Center for Exploration of Energy and Matter, Indiana University, Bloomington, Indiana 47403, USA*

PROTON POLARIZABILITY: SUBTRACTION AND DISPERSION RELATIONS

$$\Delta E_{\mathrm{pol}}(nl) = \Delta E_{\mathrm{inel}}(nl) + \Delta E_{\mathrm{sub}}(nl)$$

$$\Delta E_{\mathrm{sub}}(nl) = \frac{4\pi\alpha^2}{m} |\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2} \frac{\gamma_1(\tau_\mu)}{\sqrt{\tau_\ell}} \overline{T}_1(0, q^2)$$

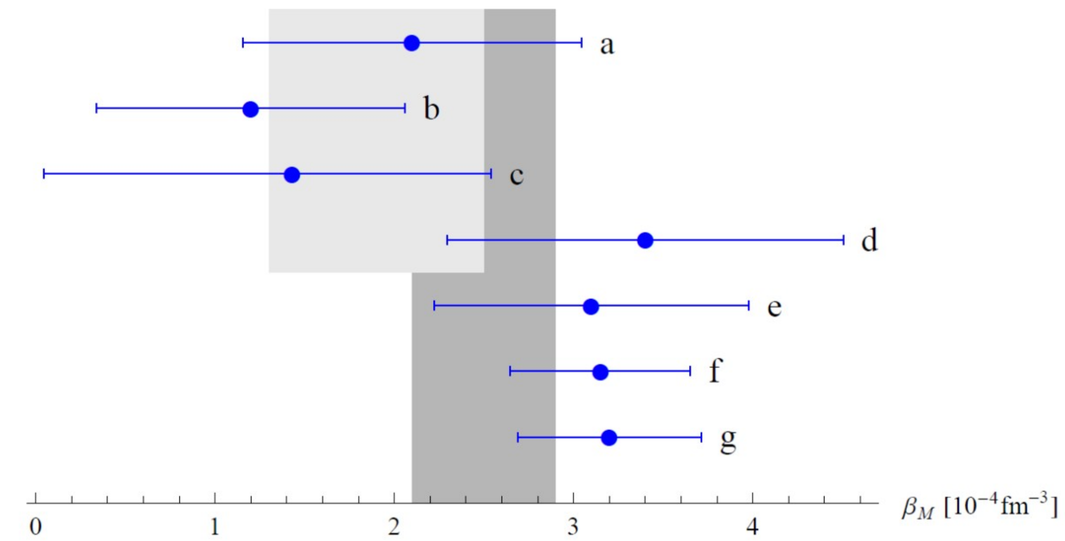
$$\overline{T}_1(0,0) = \beta_M$$

PROTON POLARIZABILITY: SUBTRACTION AND DISPERSION RELATIONS

$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

$$\Delta E_{\rm sub}(nl) = \frac{4\pi\alpha^2}{m}|\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2}\frac{\gamma_1(\tau_\mu)}{\sqrt{\tau_\ell}}\overline{T}_1(0,q^2)$$

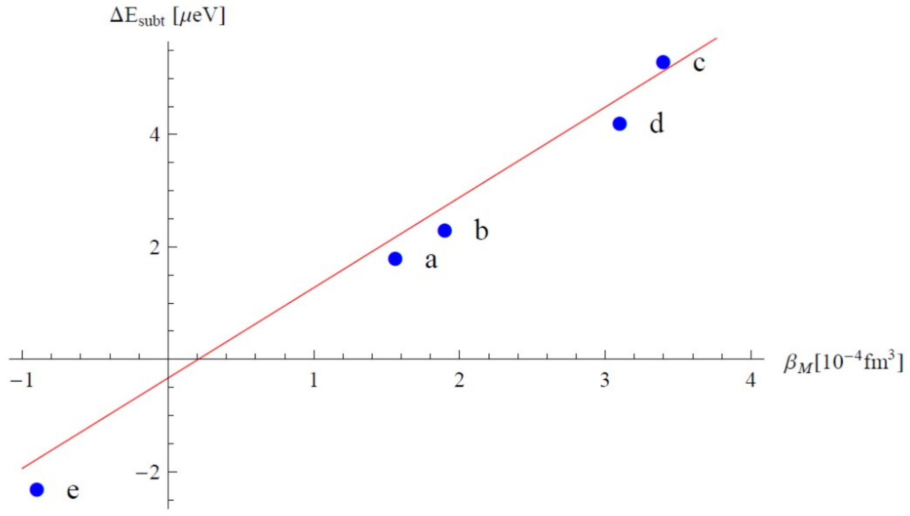
$$\overline{T}_1(0,0) = \beta_M$$

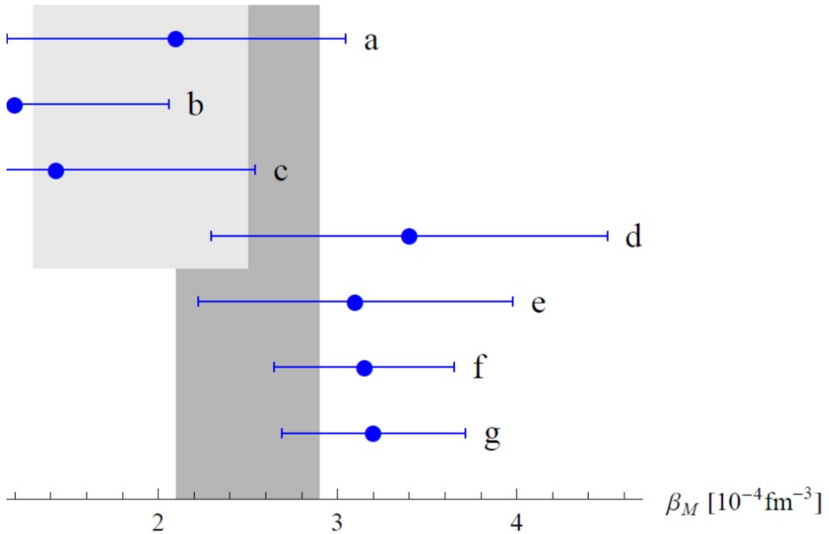


PROTON POLARIZABILITY: SUBTRACTION AND DISPERSION RELATIONS

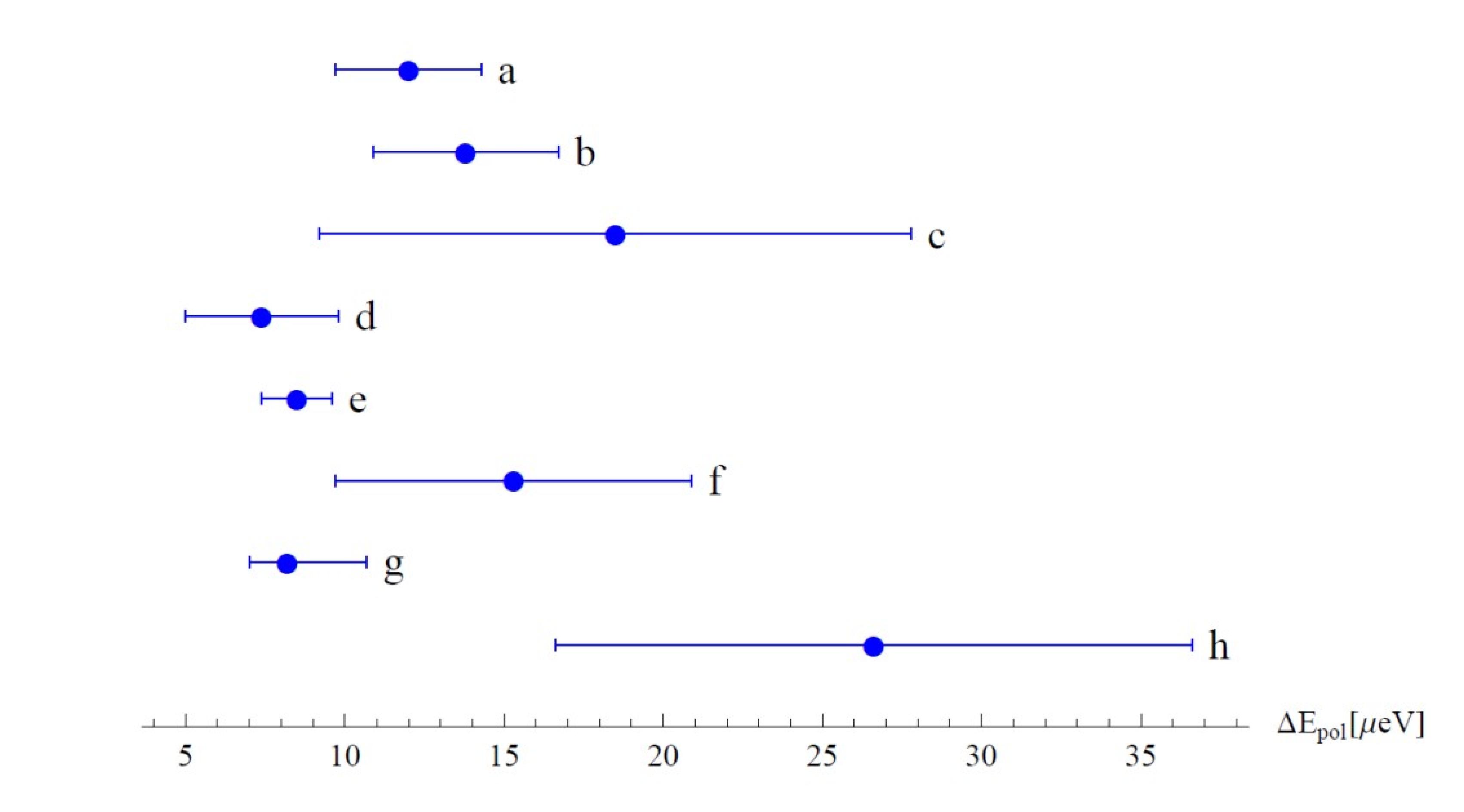
$$\Delta E_{\text{pol}}(nl) = \Delta E_{\text{inel}}(nl) + \Delta E_{\text{sub}}(nl)$$

$$\Delta E_{\text{sub}}(nl) = \frac{4\pi\alpha^2}{m} |\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2} \frac{\gamma_1(\tau_\mu)}{\sqrt{\tau_\ell}} \overline{T}_1(0, q^2)$$



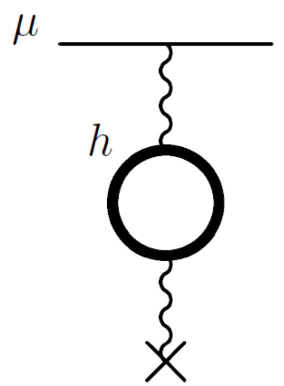


PROTON POLARIZABILITY: OTHER CALCULATIONS



Talk by Mike Birse and Poster by Franziska Hagelstein

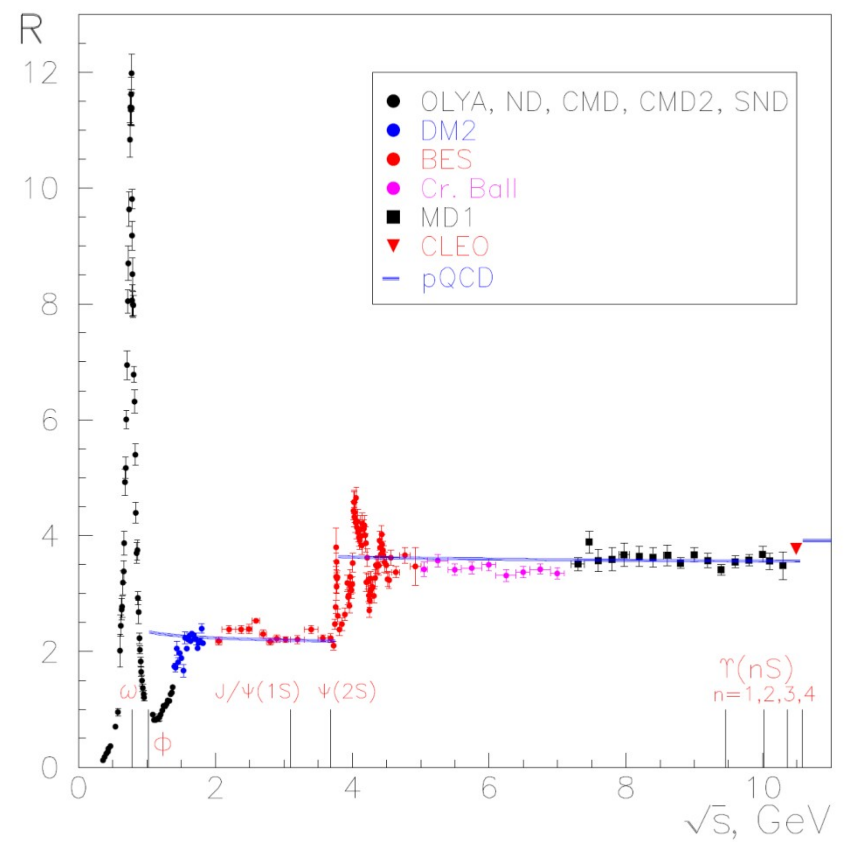
HADRONIC VACUUM POLARIZATION



$$\Delta E_{\rm hVP}(nl) = -4\pi(Z\alpha)\Pi'_h(0)|\psi_{nl}(0)|^2$$

$$\Pi_h(q^2) = \frac{\alpha}{\pi}\, q^2 \int \frac{ds\, \rho_h(s)}{q^2+s}\,.$$

$$\rho_h(s) = \frac{R(s)}{3s}$$



Talk by Ivan Logashenko

THEORETICAL SUMMARY

#	ΔE [meV]	Ref.
Unperturbed quantum mechanics		
0	$-\mathbf{0.050\,88}$	Table I
Specific QED		
1	$\mathbf{205.026\,12}$	Table II
2	$\mathbf{1.658\,85}$	Table II
3	$0.007\,52$	Table II
4	$-0.000\,89(2)$	Table II
5	$-0.002\,54$	Table II
6	$-0.001\,52$	Table II
Re-scaled QED		
7	$-\mathbf{0.667\,69}$	Table IV
8	$-\mathbf{0.044\,97}$	Table IV

Proton-line QED		
9	$-0.010\,41$	Eq. (12)
Proton-finite-size		
10	$-\mathbf{5.1974}\ r_p^2$	Table V
12	$-0.0282\ r_p^2$	Table V
13	$0.0006\ r_p^2$	Table V
14	$0.063\,54\ r_p^2 - 0.0259(35)$	Table VI
Proton polarizability		
15	$0.0088(21)$	Eq. (31)
Hadronic VP		
16	$0.010\,6(10)$	Eq. (35)
Total	$205.9067(42) - 5.1620\ r_p^2$	

THEORY VS. EXPERIMENT: PROTON CHARGE RADIUS

$$\Delta E(2p_{1/2} - 2s_{1/2}) = \left[205.9067(42) - 5.1620\, r_p^2\right] \ \text{meV}$$

$$\Delta E_L \equiv \Delta E(2p_{1/2} - 2s) = 202.2622(23)\, \text{meV}$$

$$R_p = 0.840\,25(55) \ \text{fm}$$

THEORY VS. EXPERIMENT: PROTON CHARGE RADIUS

$$\Delta E(2p_{1/2} - 2s_{1/2}) = \left[205.9067(42) - 5.1620\, r_p^2\right] \text{ meV}$$

$$\Delta E_L \equiv \Delta E(2p_{1/2} - 2s) = 202.2622(23)\,\text{meV}$$

$$R_p = 0.840\,25(55)\ \text{fm}$$

Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen

Aldo Antognini,[1,2]* François Nez,[3] Karsten Schuhmann,[2,4] Fernando D. Amaro,[5] François Biraben,[3] João M. R. Cardoso,[5] Daniel S. Covita,[5,6] Andreas Dax,[7] Satish Dhawar Marc Diepold,[1] Luis M. P. Fernandes,[5] Adolf Giesen,[4,8] Andrea L. Gouvea,[5] Thomas Graf,[8] Theodor W. Hänsch,[1,9] Paul Indelicato,[3] Lucile Julien,[3] Cheng-Yang Kao,[10] Paul Knowles,[1 Franz Kottmann,[2] Eric-Olivier Le Bigot,[3] Yi-Wei Liu,[10] José A. M. Lopes,[5] Livia Ludhova,[11 Cristina M. B. Monteiro,[5] Françoise Mulhauser,[11] Tobias Nebel,[1] Paul Rabinowitz,[12] Joaquim M. F. dos Santos,[5] Lukas A. Schaller,[11] Catherine Schwob,[3] David Taqqu,[13] João F. C. A. Veloso,[6] Jan Vogelsang,[1] Randolf Pohl[1]

nd the magnetic radius, r_M = 0.87(6)
adius, r_E = 0.84087(39) femtometer,
10-CODATA value and at 7σ variance

THEORY VS. EXPERIMENT: PROTON CHARGE RADIUS

$$\Delta E(2p_{1/2} - 2s_{1/2}) = \left[205.9067(42) - 5.1620\, r_p^2\right] \ \text{meV}$$

$$\Delta E_L \equiv \Delta E(2p_{1/2} - 2s) = 202.2622(23)\,\text{meV}$$

$$R_p = 0.840\,25(55)\ \text{fm}$$

PHYSICAL REVIEW D **90,** 053012 (2014)

Self-consistent value of the electric radius of the proton from the Lamb shift in muonic hydrogen

Savely G. Karshenboim*

Max-Planck-Institut für Quantenoptik, Garching 85748, Germany and Pulkovo Observatory, St. Petersburg 196140, Russia

value of the electric radius o

be $R_E = 0.840\,22(56)$ fm.

The results are obtained in cooperation with

- Vladimir Ivanov (Pulkovo Obs)
- Evgeny Korzinin (VNIIM)
- Valery Shelyuto (VNIIM)

I AM GRATEFUL FOR FRUITFUL AND STIMULATING DISCUSSIONS

Jose Manuel Alarcon, Igor Anikin, Aldo Antognini, Andrej Arbuzov, John Arrington, Jan Bernauer, Michael Birse, Edith Borie, Vladimir Braun, Carl Carlson, Victor Chernyak, Michael Distler, Dieter Drechsel, Simon Eidelman, Misha Eides, Ron Gilman, Misha Gorshteyn, Richard Hill, Franz Kottmann, Vadim Lensky, Ina Lorenz, Judith McGovern, Ulf Meissner, Makiko Nio, Krzysztof Pachucki, Vladimir Pascalutsa, Gil Paz, Randolf Pohl, Guy Ron, Akaki Rusetsky, Ingo Sick, Marc Vanderhaeghen