The Lamb shift, `proton charge radius puzzle' etc.

Savely Karshenboim

Pulkovo Observatory (ГАО РАН) (St. Petersburg)
&
Max-Planck-Institut für Quantenoptik (Garching)

MAX-PLANCK-INSTITUTE
OF QUANTUM OPTICS
GARCHING

Outline

- Different methods to determine the proton charge radius
  - *spectroscopy of hydrogen (and deuterium)*
  - *the Lamb shift in muonic hydrogen*
  - *electron-proton scattering*
- The proton radius: the state of the art
  - *electric charge radius*
  - *magnetic radius*

Different methods to determine the proton charge radius

- *Spectroscopy of hydrogen (and deuterium)*

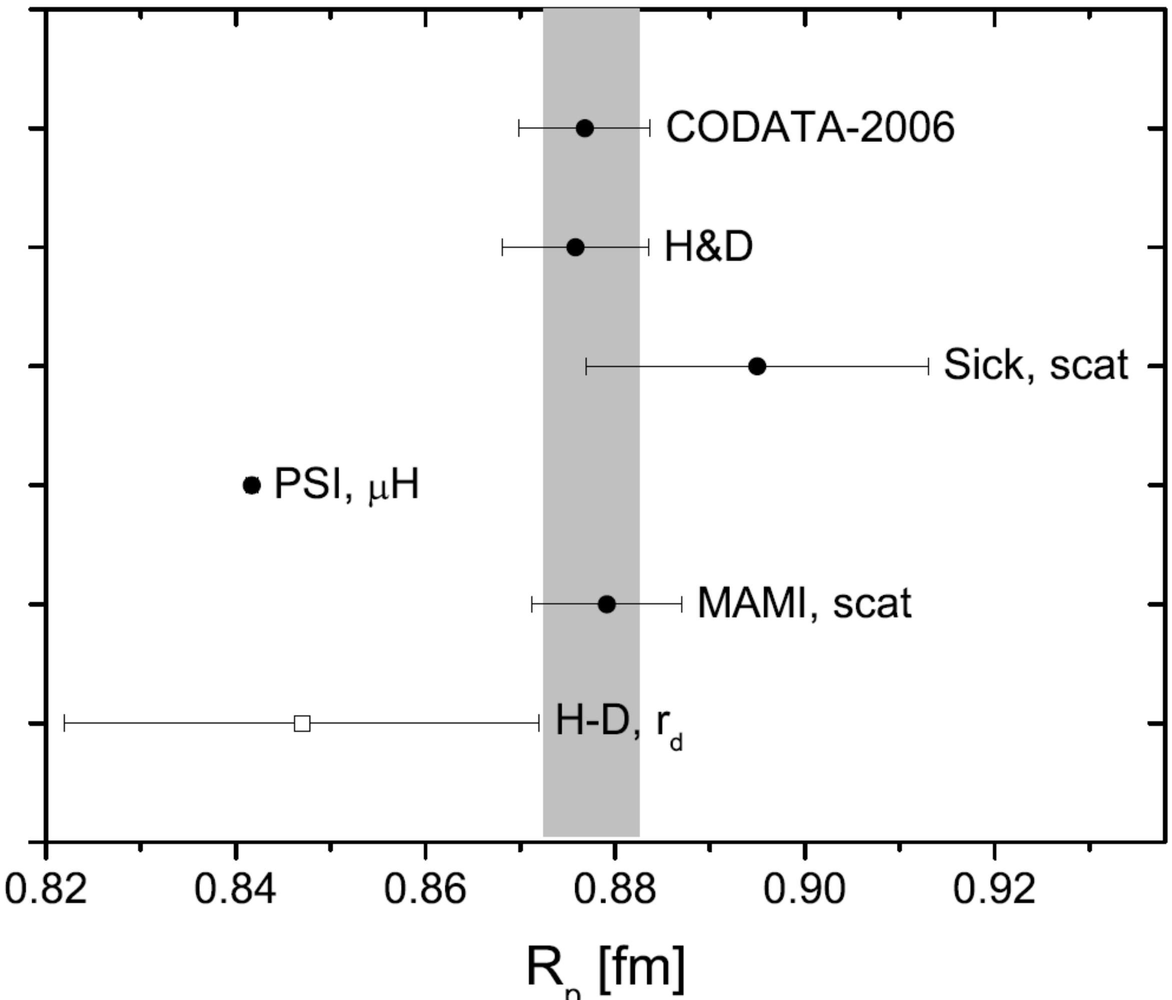
- *The Lamb shift in muonic hydrogen*

Spectroscopy produces a model-independent result, but involves a lot of theory and/or a bit of modeling.

- *Electron-proton scattering*

Studies of scattering need theory of radiative corrections, estimation of two-photon effects; the result is to depend on model applied to extrapolate to zero momentum transfer.

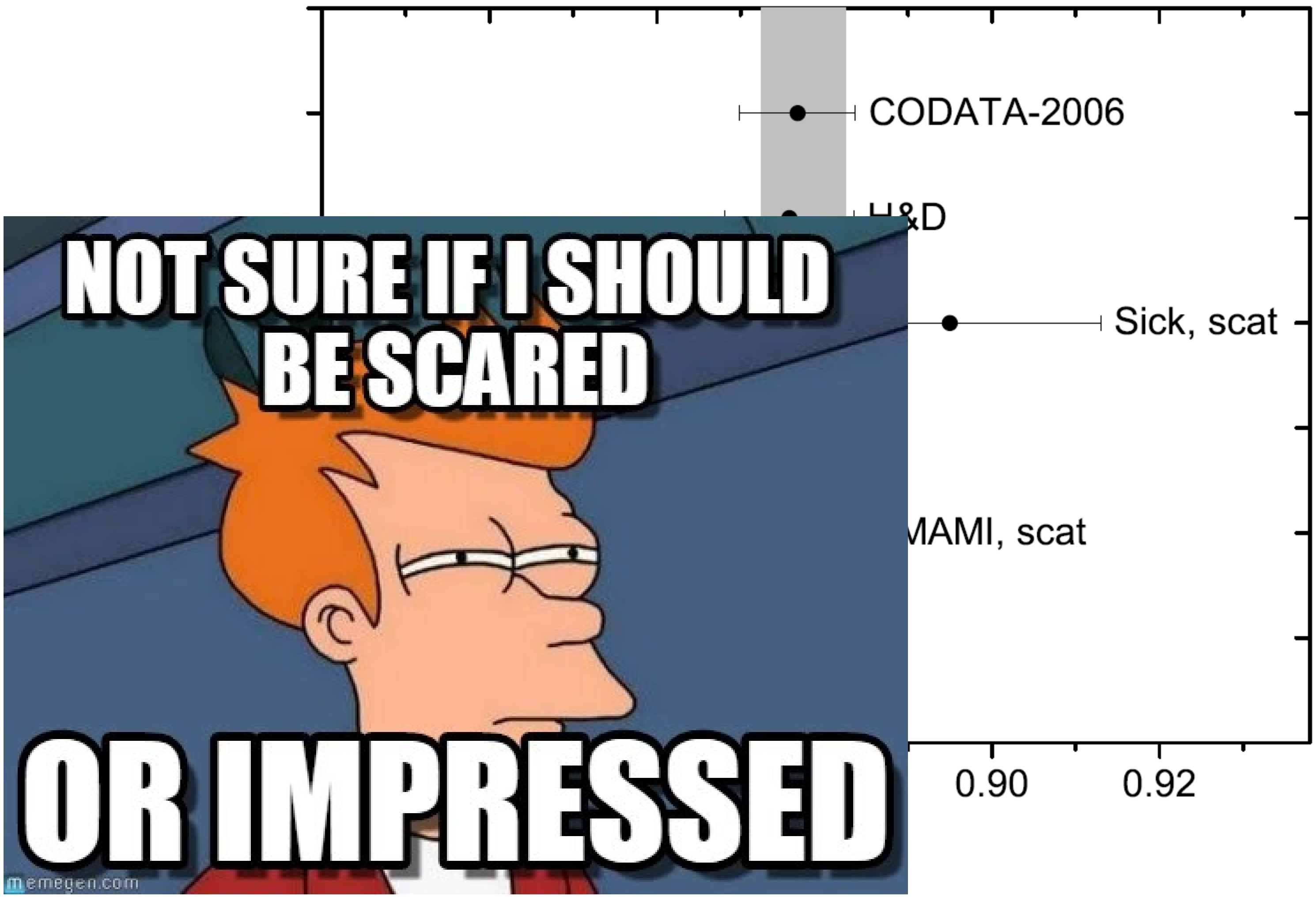
Different methods to determine the proton charge radius



roton

ring need
ative
stimation
effects;
depend
lied to
zero
ansfer.

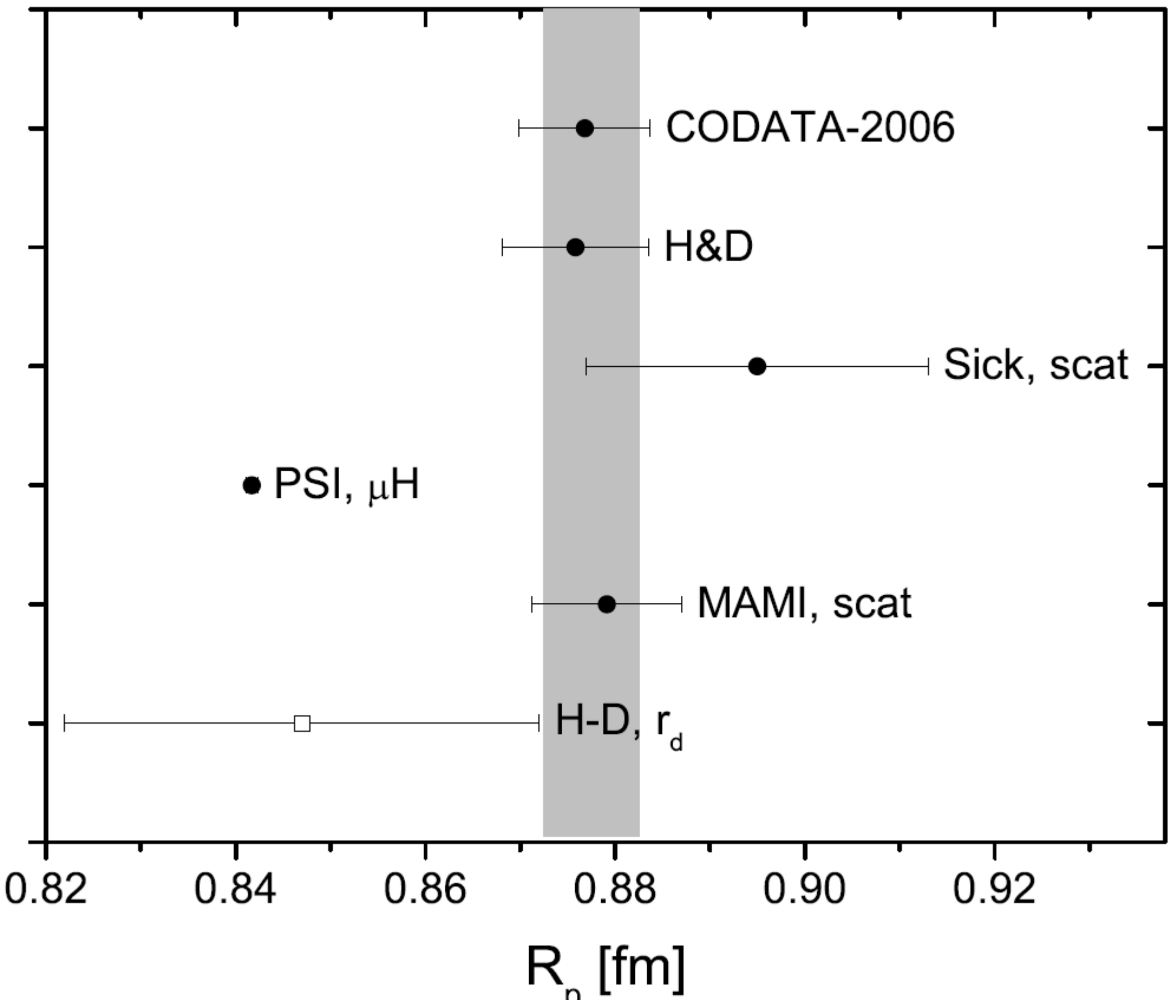
Different methods to determine the proton charge radius



roton

ring need
ative
stimation
effects;
depend
lied to
zero
ansfer.

The proton charge radius: spectroscopy vs. empiric fits

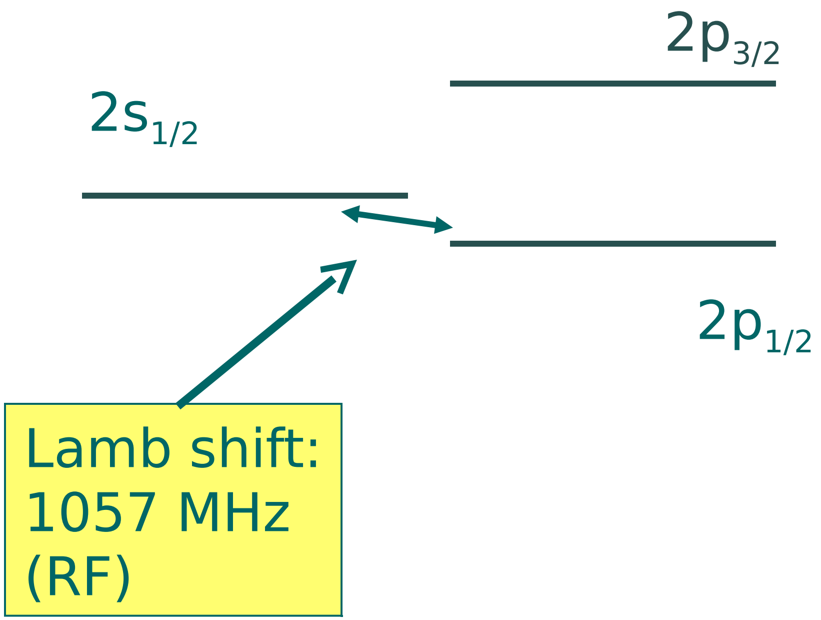


roton

ring need
ative
stimation
effects;
depend
lied to
zero
ansfer.

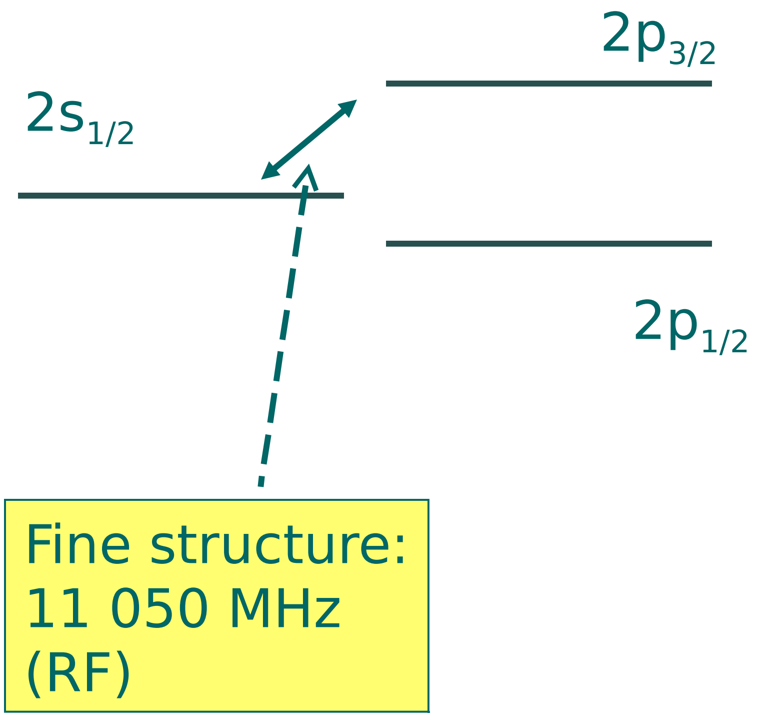
Lamb shift measurements in microwave

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz)



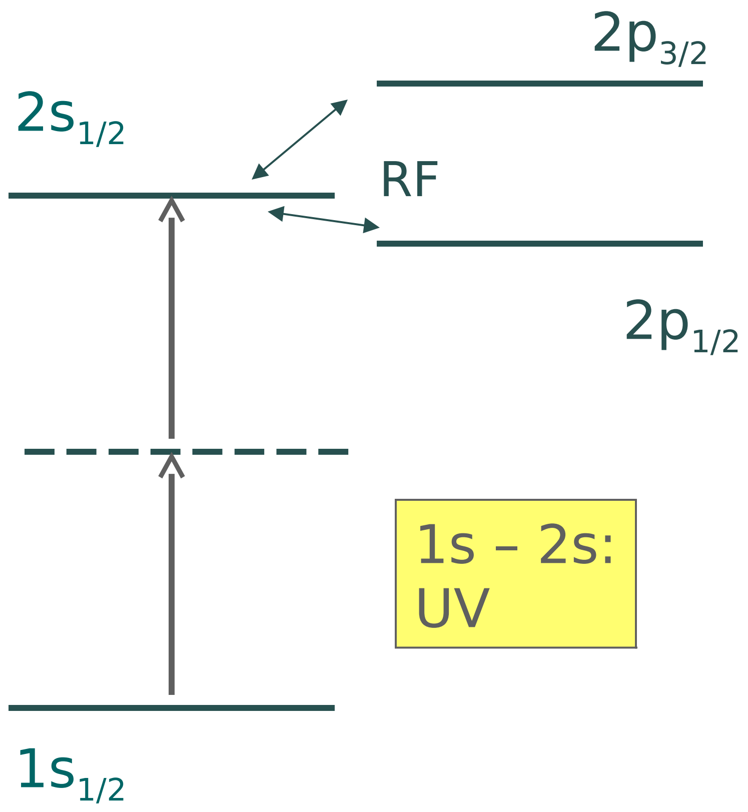
Lamb shift measurements in microwave

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz) or a big contribution into the fine splitting $2p_{3/2}$ – $2s_{1/2}$ 11 THz (fine structure).



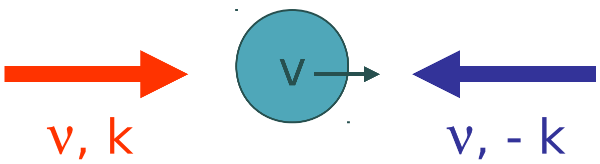
Lamb shift measurements in microwave & optics

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz) or a big contribution into the fine splitting $2p_{3/2}$ – $2s_{1/2}$ 11 THz (fine structure).
- *However, the best result for the Lamb shift has been obtained up to now from UV transitions (such as 1s – 2s).*



Two-photon Doppler-free spectroscopy of hydrogen atom

Two-photon spectroscopy



is free of linear Doppler effect.

That makes cooling relatively not too important problem.

All states but 2s are broad because of the E1 decay.

The widths decrease with increase of n.

However, higher levels are badly accessible.

Two-photon transitions double frequency and allow to go higher.

Spectroscopy of hydrogen (and deuterium)

Two-photon spectroscopy involves a number of levels strongly affected by QED.

In "old good time" we had to deal only with 2s Lamb shift.

Theory for p states is simple since their wave functions vanish at $r=0$.

Now we have more data and more unknown variables.

Spectroscopy of hydrogen (and deuterium)

Two-photon spectroscopy involves a number of levels strongly affected by QED.

In "old good time" we had to deal only with 2s Lamb shift.

Theory for p states is simple si… functions…

Now we hav…

The idea is based on theoretical study of

$$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

which we understand much better since any short distance effect vanishes for $\Delta(2)$.

Theory of p and d states is …

… the 1s Lamb shift L_{1s} & R_∞.

The Lamb shift in the hydrogen atom

S. G. Karshenboĭm

D.I. Mendeleyev Russian Metrology Research Institute, 198005 St. Petersburg, Russia

(Submitted 6 April 1994)

Zh. Eksp. Teor. Fiz. **106**, 414–424 (August 1994)

A theoretical expression is derived for the difference $\Delta E_L(1s_{1/2}) - 8\Delta E_L(2s_{1/2})$ in Lamb shifts

Z. Phys. D 39, 109–113 (1997)

ZEITSCHRIFT FÜR PHYSIK D

The Lamb shift of excited *S*-levels in hydrogen and deuterium atoms

Savely G. Karshenboim*

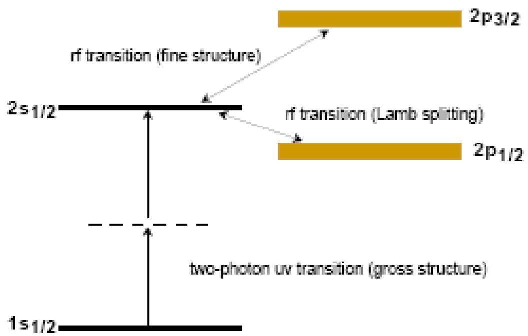
Spectroscopy of hydrogen (and deuterium)

- Two-photon spectroscopy involves a number of levels strongly affected by QED.
- In "old good time" we had to deal only with 2s Lamb shift.
- Theory for p states is simple since their wave functions vanish at r=0.
- *Now we have more data and more unknown variables.*
- *The idea is based on theoretical study of*

  $$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

  which we understand much better since any short distance effect vanishes for $\Delta(2)$.
- Theory of p and d states is also simple.
- That leaves only two variables to determine: the 1s Lamb shift L_{1s} & R_∞.

Spectroscopy of hydrogen (and deuterium)



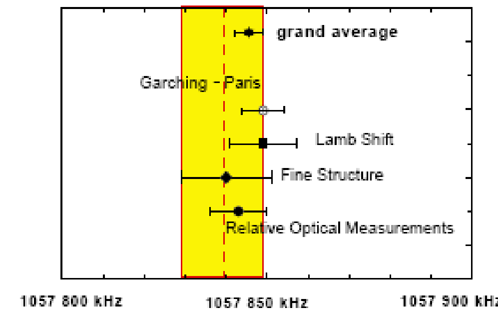


Fig. 9. Measurement of the Lamb shift in the hydrogen atom. The most accurate experimental result comes from a comparison of the $1s-2s$ interval measured at MPQ (Garching) [38] and the $2s-ns/d$ intervals at LKB (Paris) [39], where $n=8,10,12$ (see also [33] for detail). Three more results are shown for the average values extracted from direct *Lamb shift* measurements, measurements of the *fine structure* and a comparison of *two optical* transitions within a *single* experiment (i.e., a relative optical measurement). The filled part is for theory. Theory and evaluation of the experimental data are presented according to Ref. [36].

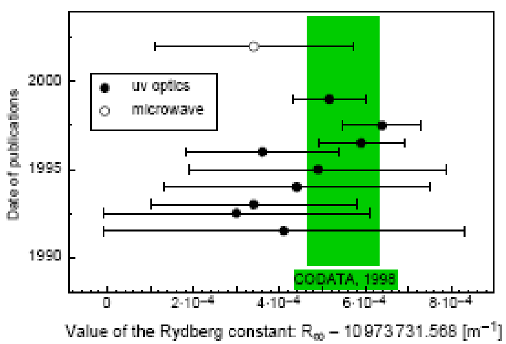


Fig. 8. Progress in determination of the Rydberg constant by means of two-photon Doppler-free spectroscopy of hydrogen and deuterium. The label *CODATA* stands for the recommended value of the Rydberg constant $R_\infty(1998)$ [21] from Eq. (12). The most recent original value is a preliminary result from MIT obtained by microwave means [37].

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT

Physics Reports 422 (2005) 1–63

PHYSICS REPORTS

www.elsevier.com/locate/physrep

Precision physics of simple atoms: QED tests, nuclear structure and fundamental constants

Savely G. Karshenboim[a, b, *]

[a] *D.I. Mendeleev Institute for Metrology, 190005 St. Petersburg, Russian Federation*
[b] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*

Lamb shift ($2s_{1/2} – 2p_{1/2}$) in the hydrogen atom

Uncertainties:

- Experiment: 2 ppm
- QED: < 1 ppm
- Proton size: 2 ppm

There are data on a number of transitions, but most of them are correlated.

H & D spectroscopy

- Complicated theory
- Some contributions are not cross checked
- More accurate than experiment
- No higher-order nuclear structure effects

$$
\begin{aligned}
\nu_H(1s-2s) = \frac{3}{4}\,c\,\mathrm{R}_\infty \Bigg\{ 1 \quad & + \quad \left[\frac{11}{48}(Z\alpha)^2 + \frac{43}{384}(Z\alpha)^4 + \frac{851}{12288}(Z\alpha)^6 + \dots\right] \\
& + \quad \frac{m_e}{m_p}\left[-1 - \frac{13}{24}(Z\alpha)^2 - \frac{17}{64}(Z\alpha)^4 + \dots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^2 \left[1 + \frac{41}{48}(Z\alpha)^2 + \dots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^3 \left[-1 + \dots\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\frac{m_e}{m_p}\left[-\frac{7}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{9}\ln k_0(2s) + \frac{64}{9}\ln k_0(1s) - \frac{112}{3}\ln 2 - \frac{805}{54}\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\left(\frac{m_e}{m_p}\right)^2 \left[\frac{7}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{8}{3}\ln k_0(2s) - \frac{64}{3}\ln k_0(1s) + 112\ln 2 + \frac{889}{18}\right] + \dots \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left[-\frac{28}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{4}{9}\log k_0(2s) + \frac{32}{9}\log k_0(1s) - \frac{266}{135}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{28}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{4}{3}\log k_0(2s) - \frac{32}{3}\log k_0(1s) + \frac{14}{5}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left(\frac{m_e}{m_p}\right)^2\left[-\frac{56}{3}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{3}\log k_0(2s) + \frac{64}{3}\log k_0(1s) - \frac{14}{15}\right] \\
& + \quad \alpha(Z\alpha)^3\left[\frac{14}{3}\log 2 - \frac{2989}{288}\right] \\
& + \quad \alpha(Z\alpha)^3\frac{m_e}{m_p}\left[-14\log 2 + \frac{2989}{96}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^4\left[\frac{7}{8}\ln^2\frac{1}{(Z\alpha)^2} + \left(-\frac{208}{9}\ln 2 + \frac{347}{90}\right)\ln\frac{1}{(Z\alpha)^2} + 71.626974\right] + \dots \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^2\left[-\frac{7}{2}\pi^2\ln 2 + \frac{70\pi^2}{81} + \frac{15253}{1944} + \frac{21}{4}\zeta(3)\right] \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{21}{2}\pi^2\ln 2 - \frac{70\pi^2}{27} - \frac{15253}{648} - \frac{63}{4}\zeta(3)\right] \\
& + \quad 50.2976\left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^3 \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^4\left[\frac{56}{81}\ln^3\frac{1}{(Z\alpha)^2} + \frac{1}{27}\ln^2\frac{1}{(Z\alpha)^2}\right. \\
& \qquad + \left(-\frac{246337}{32400} - \frac{385\pi^2}{81} + \frac{1126}{135}\ln 2 - \frac{7\pi^2}{4}\ln 2 - \frac{248}{27}\ln^2 2 - 34.845333\right)\ln\frac{1}{(Z\alpha)^2} \\
& \qquad \left. + 147(25)\right] + \dots \\
& + \quad \left(\frac{\alpha}{\pi}\right)^3(Z\alpha)^2\left[-\frac{248659831}{279936} + \frac{1765757\pi^2}{29160} - \frac{11137\pi^4}{9720} + \frac{7952}{27}\ln 2 - \frac{33509\pi^2}{324}\ln 2\right. \\
& \qquad \left. + \frac{1673\pi^2}{405}\log^2 2 + \frac{497}{81}\log^4 2 + \frac{588497}{6912}\zeta(3) + \frac{847\pi^2}{216}\zeta(3) - \frac{595}{72}\zeta(5)\right] + \dots \\
& + \quad \frac{\alpha(Z\alpha)^3}{\pi^2}\frac{m_e}{m_p}\left[\frac{3136}{81} - \frac{245\pi^2}{108} + \frac{14\pi^2}{3}\ln 2 - 14\zeta(3) - \frac{14}{9}\pi(Z\alpha)\ln^2\frac{1}{(Z\alpha)^2}\right] \\
& - \quad \frac{14}{9}(Z\alpha)^2\left(\frac{m_e c R_p}{\hbar}\right)^2 \\
& + \quad \dots \Bigg\}
\end{aligned}
$$

H & D spectroscopy

- Complicated theory
- Some contributions are not cross checked

$$
\begin{aligned}
\nu_H(1s-2s) = \frac{3}{4}\,c\,\mathrm{R}_\infty \Bigg\{ 1 \quad & + \quad \left[\frac{11}{48}(Z\alpha)^2 + \frac{43}{384}(Z\alpha)^4 + \frac{851}{12288}(Z\alpha)^6 + \ldots\right] \\
& + \quad \frac{m_e}{m_p}\left[-1 - \frac{13}{24}(Z\alpha)^2 - \frac{17}{64}(Z\alpha)^4 + \ldots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^2 \left[1 + \frac{41}{48}(Z\alpha)^2 + \ldots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^3 \left[-1 + \ldots\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\frac{m_e}{m_p}\left[-\frac{7}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{9}\ln k_0(2s) + \frac{64}{9}\ln k_0(1s) - \frac{112}{3}\ln 2 - \frac{805}{54}\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\left(\frac{m_e}{m_p}\right)^2\left[\frac{7}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{8}{3}\ln k_0(2s) - \frac{64}{3}\ln k_0(1s) + 112\ln 2 + \frac{889}{18}\right] + \ldots \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left[-\frac{28}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{4}{9}\log k_0(2s) + \frac{32}{9}\log k_0(1s) - \frac{266}{135}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{28}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{4}{3}\log k_0(2s) - \frac{32}{3}\log k_0(1s) + \frac{14}{5}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left(\frac{m_e}{m_p}\right)^2\left[-\frac{56}{3}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{3}\log k_0(2s) + \frac{64}{3}\log k_0(1s) - \frac{14}{15}\right]
\end{aligned}
$$

Fig. 1. It is a relation between the $1s$–$2s$ transition frequency $\nu_{\mathrm{H}}(1s\text{–}2s)$ and the Rydberg constant R_∞. A correction for the difference between the center of gravity of the $1s$ and $2s$ hyperfine multiplets and their triplet component is not included. This figure is an example of a complicated relationship, and is not intended to be read.

- No higher-order nuclear structure effects

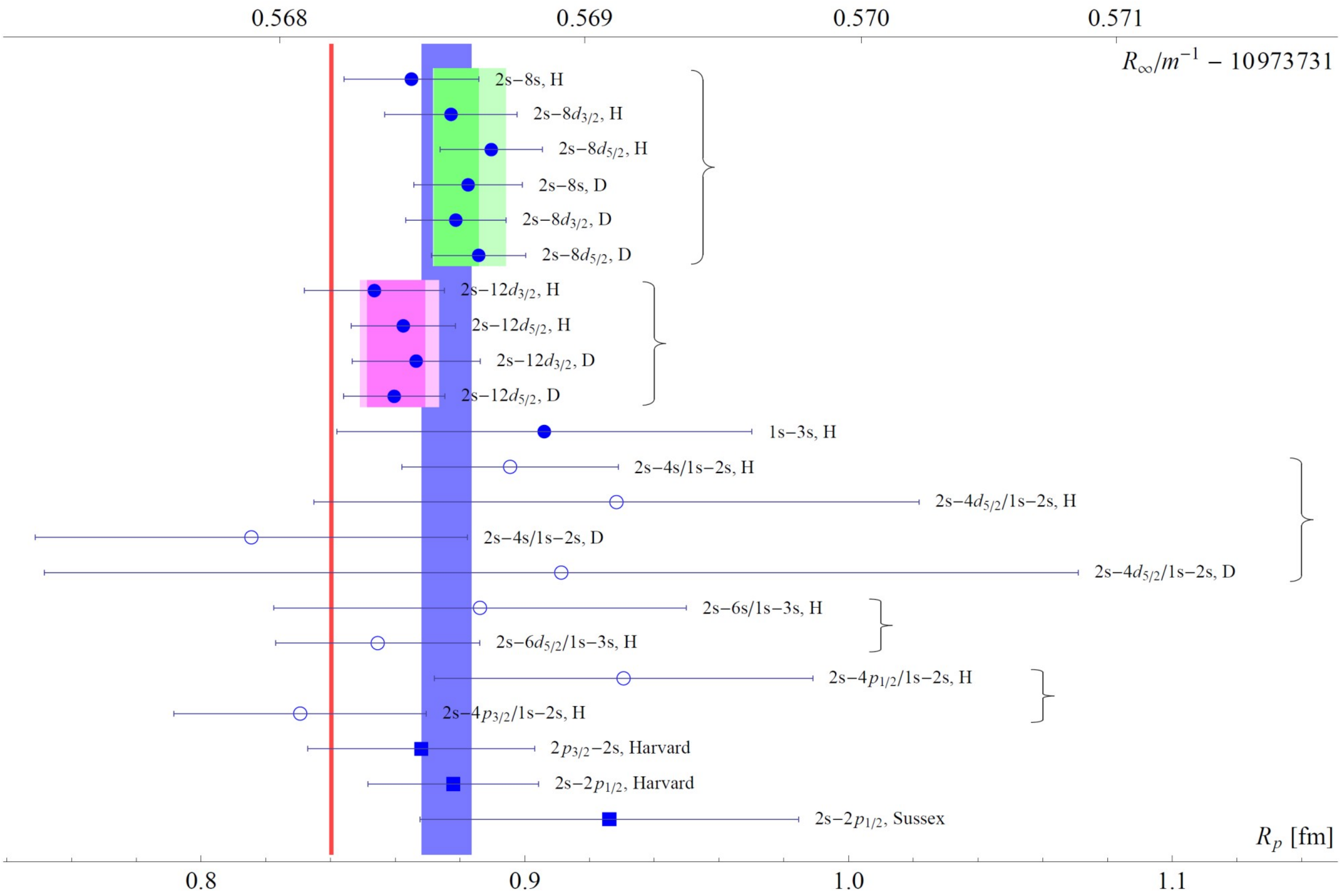
$$
\begin{aligned}
& + \quad 50.2976\left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^3 \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^4\left[\frac{56}{81}\ln^3\frac{1}{(Z\alpha)^2} + \frac{1}{27}\ln^2\frac{1}{(Z\alpha)^2}\right. \\
& \qquad + \left(-\frac{246337}{32400} - \frac{385\pi^2}{81} + \frac{1126}{135}\ln 2 - \frac{7\pi^2}{4}\ln 2 - \frac{248}{27}\ln^2 2 - 34.845333\right)\ln\frac{1}{(Z\alpha)^2} \\
& \qquad \left. + 147(25)\right] + \ldots \\
& + \quad \left(\frac{\alpha}{\pi}\right)^3(Z\alpha)^2\left[-\frac{248659831}{279936} + \frac{1765757\pi^2}{29160} - \frac{11137\pi^4}{9720} + \frac{7952}{27}\ln 2 - \frac{33509\pi^2}{324}\ln 2\right. \\
& \qquad \left. + \frac{1673\pi^2}{405}\log^2 2 + \frac{497}{81}\log^4 2 + \frac{588497}{6912}\zeta(3) + \frac{847\pi^2}{216}\zeta(3) - \frac{595}{72}\zeta(5)\right] + \ldots \\
& + \quad \frac{\alpha(Z\alpha)^3}{\pi^2}\frac{m_e}{m_p}\left[\frac{3136}{81} - \frac{245\pi^2}{108} + \frac{14\pi^2}{3}\ln 2 - 14\zeta(3) - \frac{14}{9}\pi(Z\alpha)\ln^2\frac{1}{(Z\alpha)^2}\right] \\
& - \quad \frac{14}{9}(Z\alpha)^2\left(\frac{m_e c R_p}{\hbar}\right)^2 \\
& + \quad \ldots \Bigg\}
\end{aligned}
$$

H & D spectroscopy

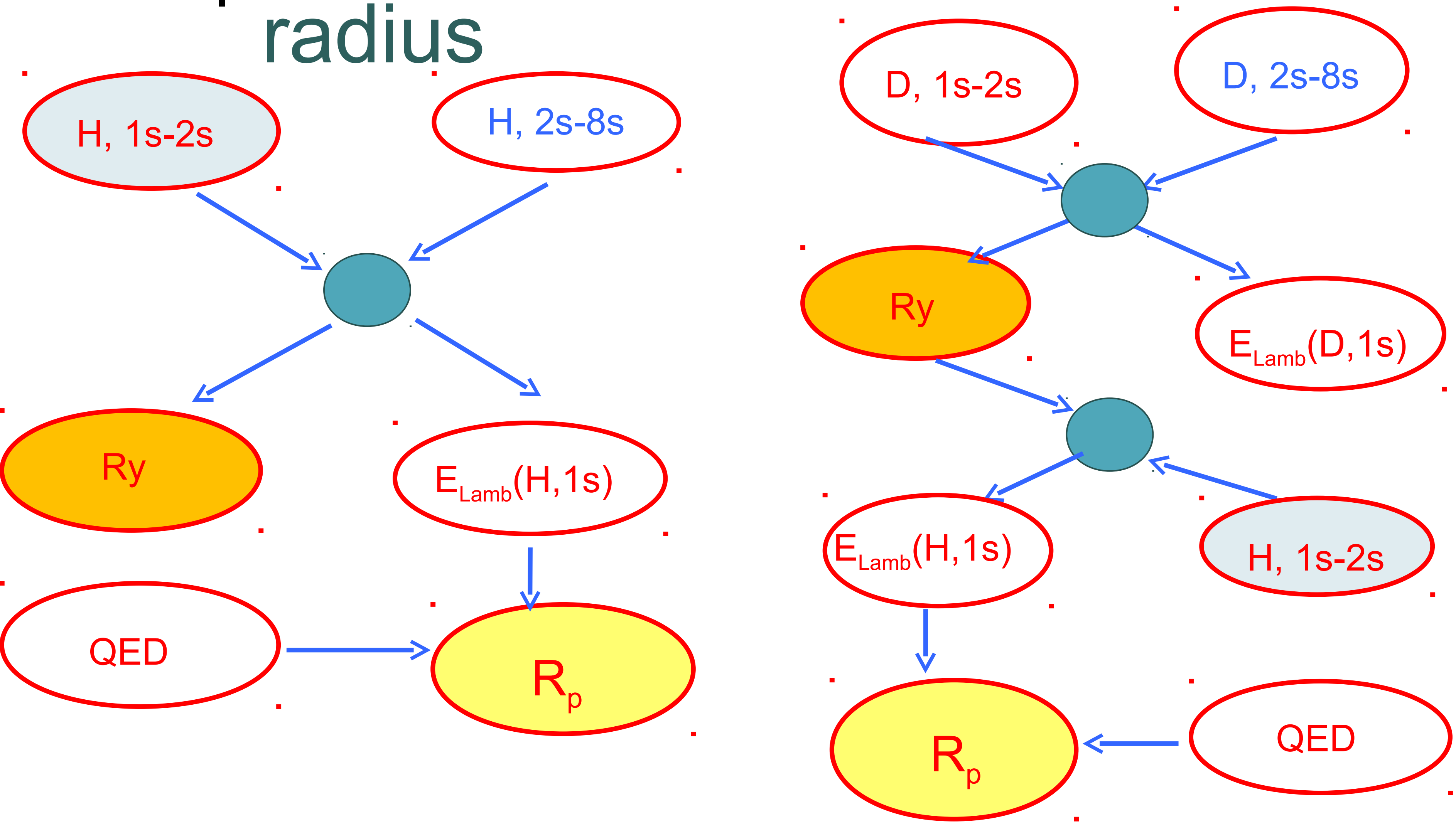
- Complicated theory
- Some contributions are not cross checked
- More accurate than experiment
- No higher-order nuclear structure effects

$$
\begin{aligned}
\nu_H(1s-2s) = \frac{3}{4}\, c\,\mathrm{R}_\infty \bigg\{ 1 \quad & + \quad \left[\frac{11}{48}(Z\alpha)^2 + \frac{43}{384}(Z\alpha)^4 + \frac{851}{12288}(Z\alpha)^6 + \dots\right] \\
& + \quad \frac{m_e}{m_p}\left[-1 - \frac{13}{24}(Z\alpha)^2 - \frac{17}{64}(Z\alpha)^4 + \dots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^2 \left[1 + \frac{41}{48}(Z\alpha)^2 + \dots\right] \\
& + \quad \left(\frac{m_e}{m_p}\right)^3 \left[-1 + \dots\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\frac{m_e}{m_p}\left[-\frac{7}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{9}\ln k_0(2s) + \frac{64}{9}\ln k_0(1s) - \frac{112}{3}\ln 2 - \frac{805}{54}\right] \\
& + \quad \frac{(Z\alpha)^3}{\pi}\left(\frac{m_e}{m_p}\right)^2 \left[\frac{7}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{8}{3}\ln k_0(2s) - \frac{64}{3}\ln k_0(1s) + 112\ln 2 + \frac{889}{18}\right] + \dots \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left[-\frac{28}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{4}{9}\log k_0(2s) + \frac{32}{9}\log k_0(1s) - \frac{266}{135}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{28}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{4}{3}\log k_0(2s) - \frac{32}{3}\log k_0(1s) + \frac{14}{5}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^2\left(\frac{m_e}{m_p}\right)^2\left[-\frac{56}{3}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{3}\log k_0(2s) + \frac{64}{3}\log k_0(1s) - \frac{14}{15}\right] \\
& + \quad \alpha(Z\alpha)^3\left[\frac{14}{3}\log 2 - \frac{2989}{288}\right] \\
& + \quad \alpha(Z\alpha)^3\frac{m_e}{m_p}\left[-14\log 2 + \frac{2989}{96}\right] \\
& + \quad \frac{\alpha}{\pi}(Z\alpha)^4\left[\frac{7}{8}\ln^2\frac{1}{(Z\alpha)^2} + \left(-\frac{208}{9}\ln 2 + \frac{347}{90}\right)\ln\frac{1}{(Z\alpha)^2} + 71.626974\right] + \dots \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^2\left[-\frac{7}{2}\pi^2\ln 2 + \frac{70\pi^2}{81} + \frac{15253}{1944} + \frac{21}{4}\zeta(3)\right] \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^2\frac{m_e}{m_p}\left[\frac{21}{2}\pi^2\ln 2 - \frac{70\pi^2}{27} - \frac{15253}{648} - \frac{63}{4}\zeta(3)\right] \\
& + \quad 50.2976\left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^3 \\
& + \quad \left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^4\bigg[\frac{56}{81}\ln^3\frac{1}{(Z\alpha)^2} + \frac{1}{27}\ln^2\frac{1}{(Z\alpha)^2} \\
& \qquad + \left(-\frac{246337}{32400} - \frac{385\pi^2}{81} + \frac{1126}{135}\ln 2 - \frac{7\pi^2}{4}\ln 2 - \frac{248}{27}\ln^2 2 - 34.845333\right)\ln\frac{1}{(Z\alpha)^2} \\
& \qquad + 147(25)\bigg] + \dots \\
& + \quad \left(\frac{\alpha}{\pi}\right)^3 (Z\alpha)^2\bigg[-\frac{248659831}{279936} + \frac{1765757\pi^2}{29160} - \frac{11137\pi^4}{9720} + \frac{7952}{27}\ln 2 - \frac{33509\pi^2}{324}\ln 2 \\
& \qquad + \frac{1673\pi^2}{405}\log^2 2 + \frac{497}{81}\log^4 2 + \frac{588497}{6912}\zeta(3) + \frac{847\pi^2}{216}\zeta(3) - \frac{595}{72}\zeta(5)\bigg] + \dots \\
& + \quad \frac{\alpha(Z\alpha)^3}{\pi^2}\frac{m_e}{m_p}\left[\frac{3136}{81} - \frac{245\pi^2}{108} + \frac{14\pi^2}{3}\ln 2 - 14\zeta(3) - \frac{14}{9}\pi(Z\alpha)\ln^2\frac{1}{(Z\alpha)^2}\right] \\
& - \quad \frac{14}{9}(Z\alpha)^2\left(\frac{m_e c R_p}{\hbar}\right)^2 \\
& + \quad \dots \bigg\}
\end{aligned}
$$

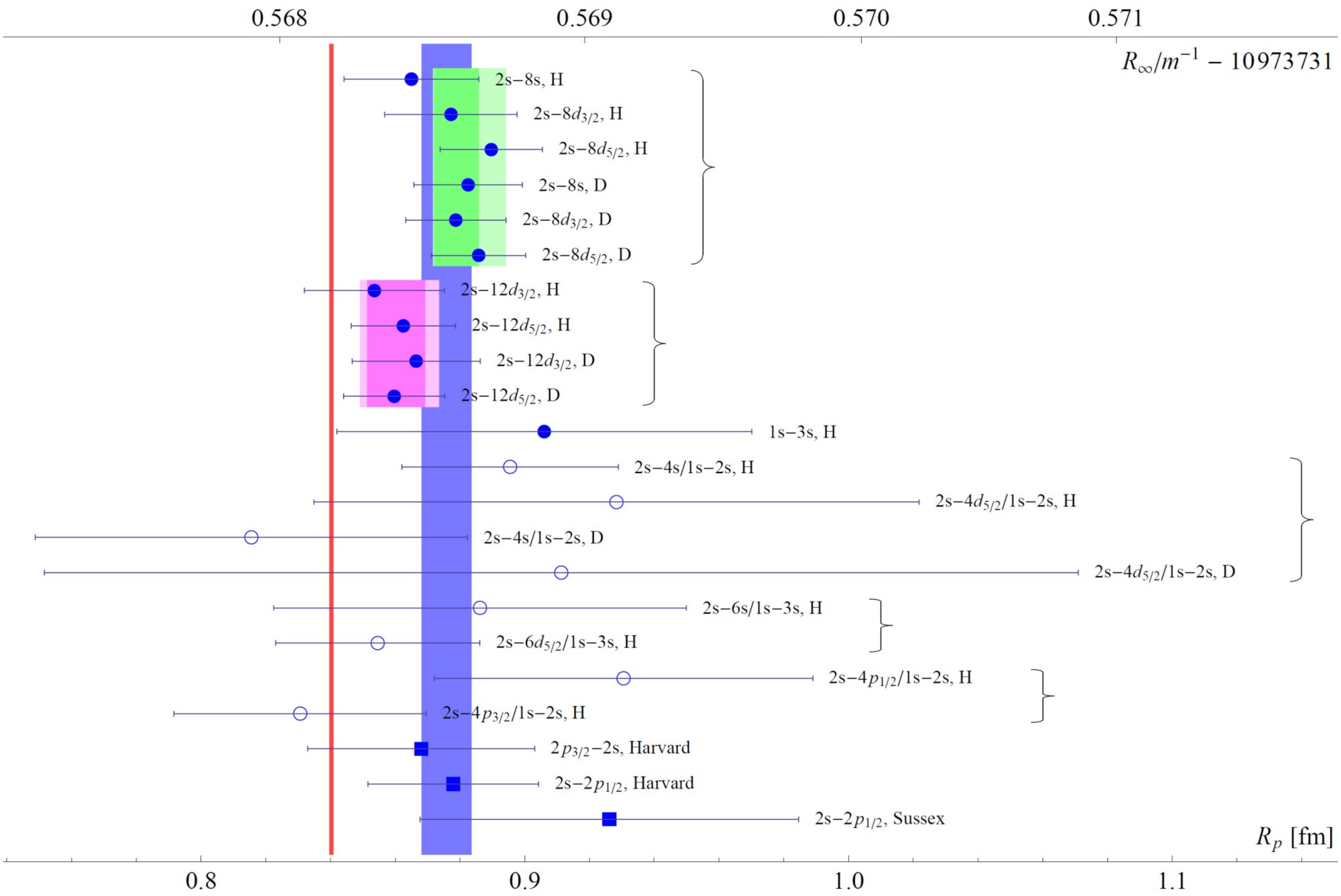
Proton radius from hydrogen



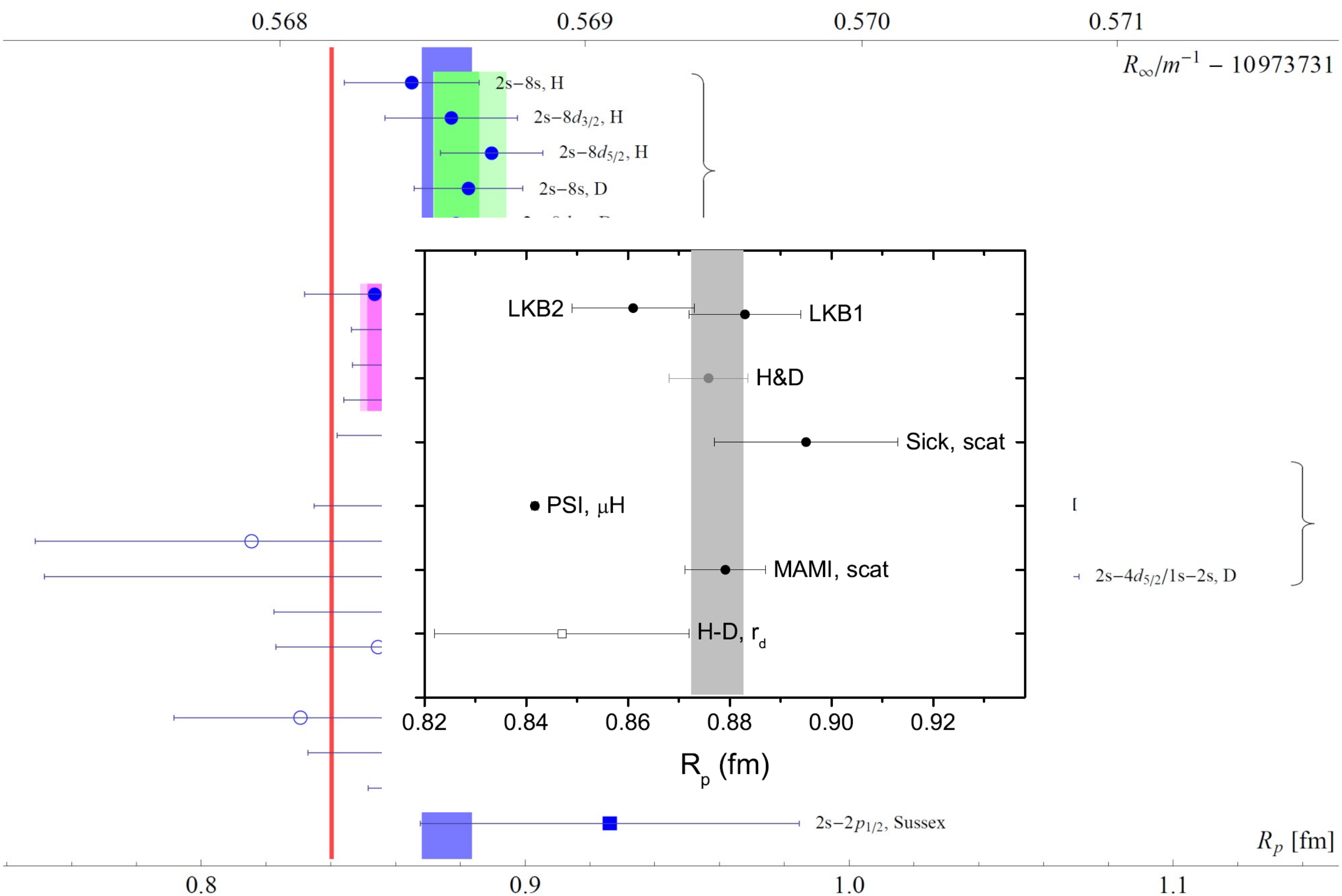
Optical determination of Rydberg constant and proton radius



Proton radius from hydrogen



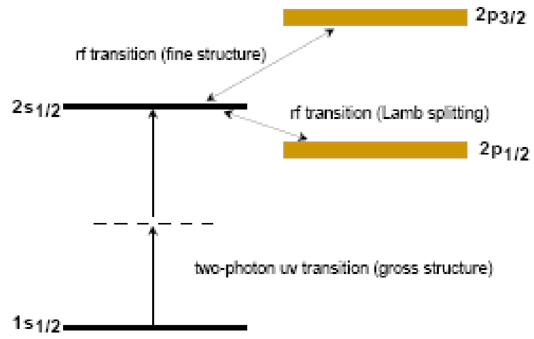
Proton radius from hydrogen



The Lamb shift in muonic hydrogen

- Used to believe: *since a muon is heavier than an electron, muonic atoms are more sensitive to the nuclear structure.*
- Not quite true. **What is important**: scaling of various contributions with *m*.

- Scaling of contributions
  - **nuclear finite size effects:** ~ m^3;
  - standard Lamb-shift QED and its uncertainties: ~ m;
  - width of the 2p state: ~ m;
  - nuclear finite size effects for HFS: ~ m^3



The Lamb shift in muonic hydrogen: experiment

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

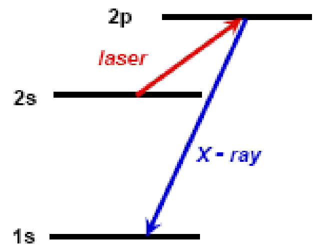


Fig. 16. Level scheme of the PSI experiment on the Lamb shift in a muonic hydrogen [88] (not to scale). The hyperfine structure is not shown.

The Lamb shift in muonic hydrogen: experiment

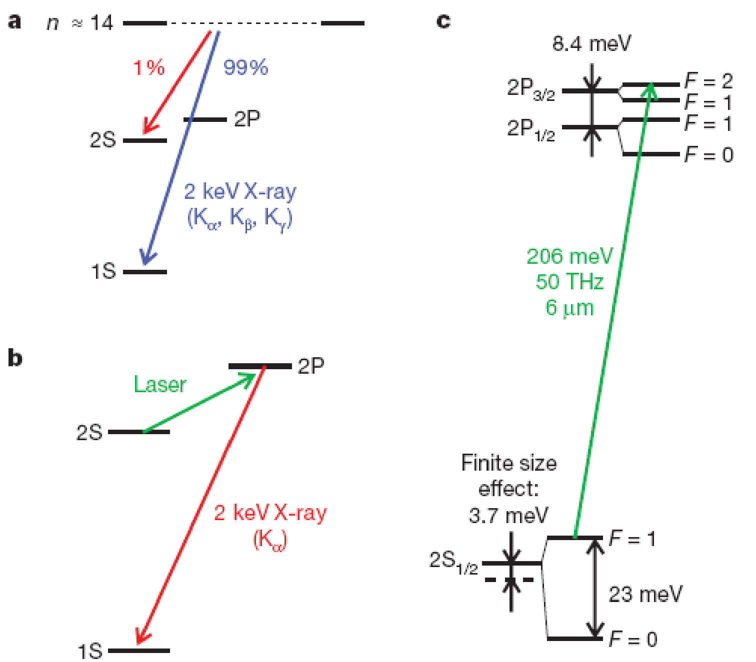


Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. **a**, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6$ μm.

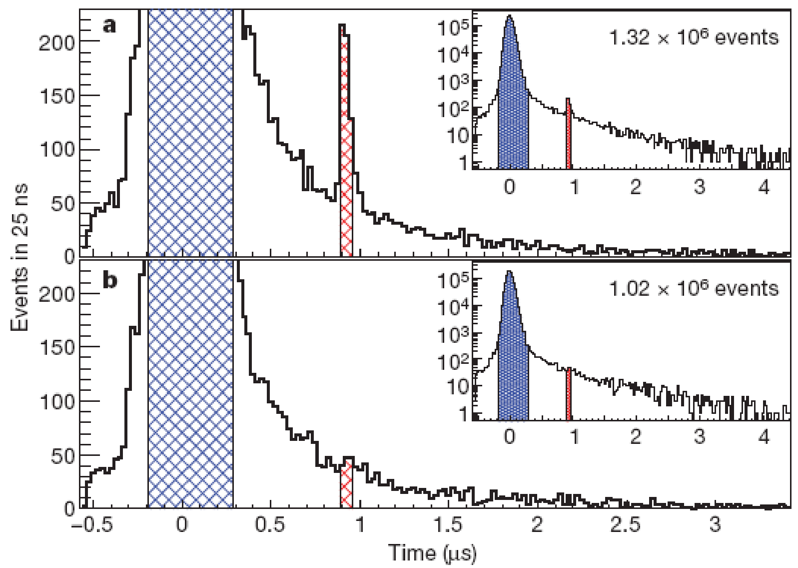


Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

The Lamb shift in muonic hydrogen: experiment

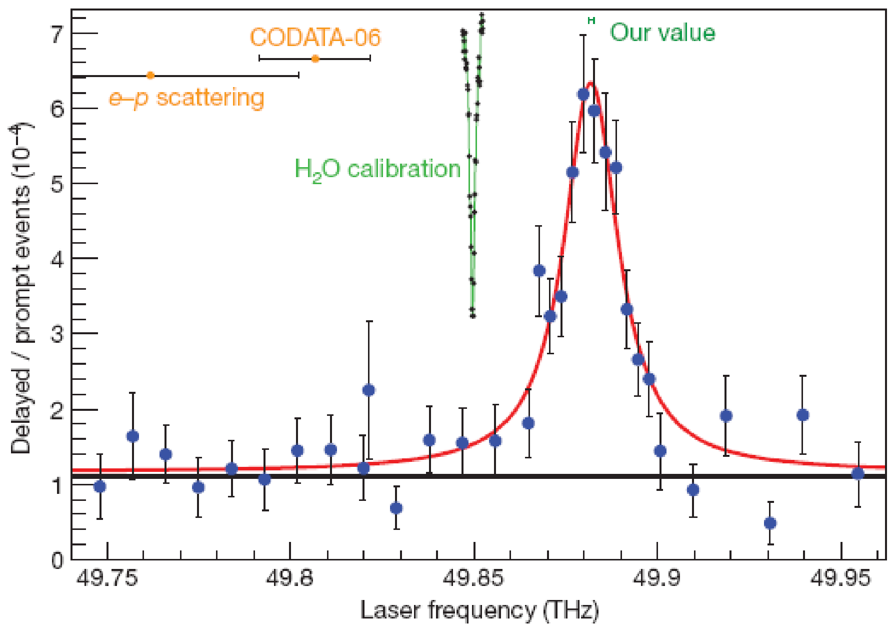


Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[3] or electron scattering[1,2] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the ±1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

Theoretical summary

#	ΔE [meV]	Ref.
Unperturbed quantum mechanics		
0	$\mathbf{-0.050\,88}$	Table I
Specific QED		
1	$\mathbf{205.026\,12}$	Table II
2	$\mathbf{1.658\,85}$	Table II
3	$0.007\,52$	Table II
4	$-0.000\,89(2)$	Table II
5	$-0.002\,54$	Table II
6	$-0.001\,52$	Table II
Re-scaled QED		
7	$\mathbf{-\,0.667\,69}$	Table IV
8	$\mathbf{-\,0.044\,97}$	Table IV
Proton-line QED		
9	$-0.010\,41$	Eq. (12)
Proton-finite-size		
10	$\mathbf{-5.1974}\ r_p^2$	Table V
12	$-0.0282\ r_p^2$	Table V
13	$0.0006\ r_p^2$	Table V
14	$0.063\,54\ r_p^2 - 0.0259(35)$	Table VI
Proton polarizability		
15	$0.0088(21)$	Eq. (31)
Hadronic VP		
16	$0.010\,6(10)$	Eq. (35)
Total	$205.9067(42) - 5.1620\ r_p^2$	

Theory of Lamb Shift in Muonic Hydrogen

Savely G. Karshenboim[a)]
Max-Planck-Institut für Quantenoptik, Garching 85748, Germany and Pulkovo Observatory, St. Petersburg 196140, Russia

Evgeny Yu. Korzinin and Valery A. Shelyuto
D. I. Mendeleev Institute for Metrology, St. Petersburg 190005, Russia

Vladimir G. Ivanov
Pulkovo Observatory, St. Petersburg 196140, Russia

The Lamb shift in muonic hydrogen: theory

#	Contribution	Ref.	Our selection Value	Our selection Unc.	Pachucki[1–3] Value	Pachucki[1–3] Unc.	Borie[5] Value	Borie[5] Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

$$\Delta E_{LS} = 206.0573(45) - 5.2262\, r_p^2 + 0.0347\, r_p^3 \text{ meV} \quad (1)$$

- Discrepancy ~ 0.300 meV.
- Only few contributions are important at this level.
- **They are reliable.**

Theory of H and μH:

- Rigorous
- Ab initio
- Complicated
- Very accurate
- Partly not cross checked
- Needs no higher-order proton structure

- Rigorous
- Ab initio
- Transparent
- Very accurate
- Cross checked
- Needs higher-order proton structure (much below the discrepancy)

Theory of H and μH:

- Rigorous
- Ab initio
- Complicated
- Very accurate
- Partly not cross checked
- Needs no higher-order proton structure

- Rigorous
- Ab initio
- Transparent
- Very accurate
- Cross checked
- Needs higher-order proton structure (much below the discrepancy)

The *th* uncertainty is much below the level of the discrepancy

Spectroscopy of H and μH:

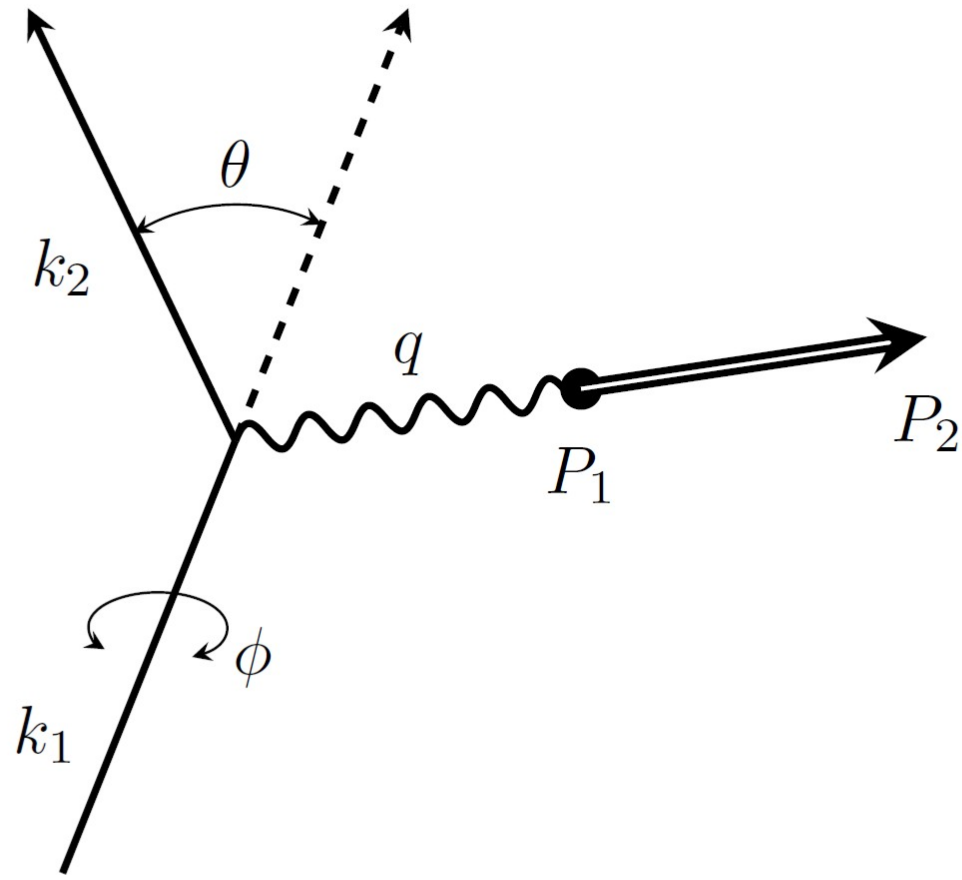
- Many transitions in different labs.
- One dominates.
- Correlated.
- Metrology involved.
- The discrepancy is much below the line width.
- Sensitive to various systematic effects.

- One experiment
- A correlated measurement on μD
- No real metrology
- Discrepancy is of few line widths.
- Not sensitive to many perturbations.

H vs μH:

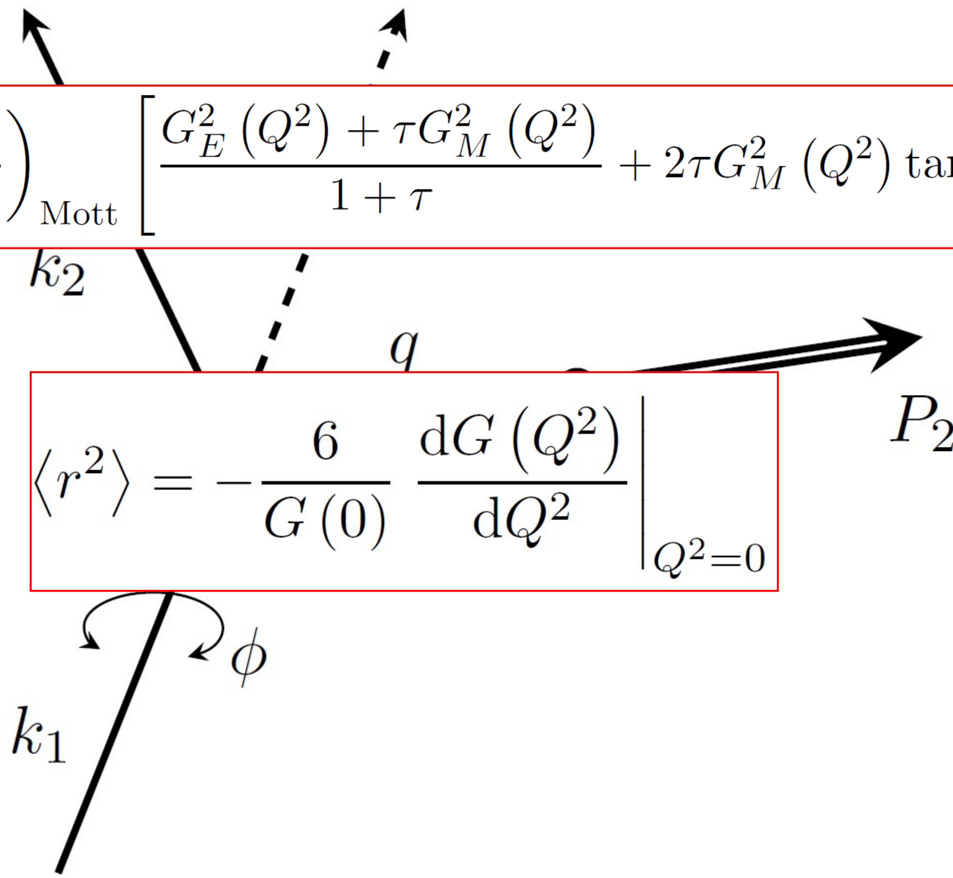
- μH: much more sensitive to the R_p term:
  - less accuracy in theory and experiment is required;
  - easier for estimation of systematic effects etc.
- H experiment: easy to see a signal, hard to interpret.
- μH experiment: hard to see a signal, easy to interpret.

Elastic electron-proton scattering



Elastic electron-proton scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)}{1+\tau} + 2\tau G_M^2\left(Q^2\right)\tan^2\frac{\theta}{2}\right]$$



$$\left\langle r^2 \right\rangle = -\frac{6}{G(0)} \left.\frac{dG\left(Q^2\right)}{dQ^2}\right|_{Q^2=0}$$

Electron-proton scattering: new Mainz experiment

High-precision determination of the electric and magnetic form factors of the proton

J. C. Bernauer,[1,*] P. Achenbach,[1] C. Ayerbe Gayoso,[1] R. Böhm,[1] D. Bosnar,[2] L. Debenjak,[3]
M. O. Distler,[1,†] L. Doria,[1] A. Esser,[1] H. Fonvieille,[4] J. M. Friedrich,[5] J. Friedrich,[1] M. Gómez Rodríguez
de la Paz,[1] M. Makek,[2] H. Merkel,[1] D. G. Middleton,[1] U. Müller,[1] L. Nungesser,[1] J. Pochodzalla,[1]
M. Potokar,[3] S. Sánchez Majos,[1] B. S. Schlimme,[1] S. Širca,[6,3] Th. Walcher,[1] and M. Weinriefer[1]

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\mathrm{stat.}}(4)_{\mathrm{syst.}}(2)_{\mathrm{model}}(4)_{\mathrm{group}}\,\mathrm{fm},$$

$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.777(13)_{\mathrm{stat.}}(9)_{\mathrm{syst.}}(5)_{\mathrm{model}}(2)_{\mathrm{group}}\,\mathrm{fm}.$$

Electron-proton scattering: evaluations of `the World data'

- Mainz:

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}(4)_{\text{group}}\,\text{fm},$$
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.777(13)_{\text{stat.}}(9)_{\text{syst.}}(5)_{\text{model}}(2)_{\text{group}}\,\text{fm}.$$

- JLab (similar results also from Ingo Sick)

$$\langle r_E^2 \rangle^{1/2} = 0.875 \pm 0.008_{exp} \pm 0.006_{fit}\ \text{fm} \quad (3)$$
$$\langle r_M^2 \rangle^{1/2} = 0.867 \pm 0.009_{exp} \pm 0.018_{fit}\ \text{fm}, \quad (4)$$

- Charge radius:

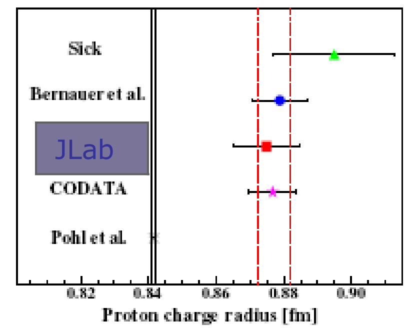


FIG. 3: (Color online) The proton RMS charge radius from previous *ep* scattering analysis (Sick [40]), Mainz low Q^2 measurement (Bernauer *et al.* [37]) and this work compared to the CODATA [41] and muonic hydrogen spectroscopy (Pohl *et al.* [42]). The red dashed lines show the combined results from CODATA, Bernauer *et al.* and this work, while the black dotted lines show the Pohl *et al.* uncertainty.

High Precision Measurement of the Proton Elastic Form Factor Ratio $\mu_p G_E/G_M$ at Low Q^2

X. Zhan,[1,2] K. Allada,[3] D. S. Armstrong,[4] J. Arrington,[2] W. Bertozzi,[1] W. Boeglin,[5] J.-P. Chen,[6] K. Chirapatpimol,[7] S. Choi,[8] E. Chudakov,[6] E. Cisbani,[9,10] P. Decowski,[11] C. Dutta,[12] S. Frullani,[9] E. Fuchey,[13] F. Garibaldi,[9] S. Gilad,[1] R. Gilman,[6,14] J. Glister,[15,16] K. Hafidi,[2] B. Hahn,[4] J.-O. Hansen,[6] D. W. Higinbotham,[6] T. Holmstrom,[17] R. J. Holt,[2] J. Huang,[1] G. M. Huber,[18] F. Itard,[13] C. W. de Jager,[6] X. Jiang,[14] J. Johnson,[19] J. Katich,[4] R. de Leo,[20] J. J. LeRose,[6] R. Lindgren,[7] E. Long,[21] D. J. Margaziotis,[22] S. May-Tal Beck,[23] D. Meekins,[6] R. Michaels,[6] B. Moffit,[1,6] B. E. Norum,[7] M. Olson,[24] E. Piasetzky,[25] I. Pomerantz,[25] D. Protopopescu,[26] X. Qian,[27] Y. Qiang,[27,6] A. Rakhman,[28] R. D. Ransome,[14] P. E. Reimer,[2] J. Reinhold,[29] S. Riordan,[7] G. Ron,[25,30] A. Saha,[6] A. J. Sarty,[31] B. Sawatzky,[6,32] E. C. Schulte,[14] M. Shabestari,[7] A. Shahinyan,[33] S.Širca,[34,35] P. Solvignon,[2,6] N. F. Sparveris,[1,32] S. Strauch,[36] R. Subedi,[7] V. Sulkosky,[1,6] I. Vilardi,[20] Y. Wang,[37] B. Wojtsekhowski,[6] Z. Ye,[38] and Y. Zhang[39]
(Jefferson Lab Hall A Collaboration)

Electron-proton scattering: evaluations of `the World data'

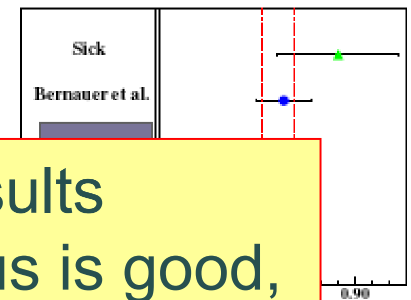
- Mainz:

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}(4)_{\text{group}}\,\text{fm},$$
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.777(13)_{\text{stat.}}(9)_{\text{syst.}}(5)_{\text{model}}(2)_{\text{group}}\,\text{fm}.$$

- JLa
resu
Ingo

$$\langle r_E^2 \rangle^{1/2} = 0.875$$
$$\langle r_M^2 \rangle^{1/2} = 0.867$$

- Charge radius:



radius from pre-
ow Q^2 measure-
ared to the CO-
Pohl *et al.* [42]).
from CODATA,
otted lines show

The consistency of the results on the proton charge radius is good, but not sufficient.

The agreement is required between the form factors, which is an open question for the moment.

Ratio $\mu_p G_E/G_M$ at Low Q^2

in,[5] J.-P. Chen,[6] K. Chirapatpimol,[7]
. Fuchey,[13] F. Garibaldi,[9] S. Gilad,[1]
R. Gilman,[6,14] J. Glister,[15,16] K. Hafidi,[2] B. Hahn,[4] J.-O. Hansen,[6] D. W. Higinbotham,[6] T. Holmstrom,[17] R. J. Holt,[2]
J. Huang,[1] G. M. Huber,[18] F. Itard,[13] C. W. de Jager,[6] X. Jiang,[14] J. Johnson,[19] J. Katich,[4] R. de Leo,[20]
J. J. LeRose,[6] R. Lindgren,[7] E. Long,[21] D. J. Margaziotis,[22] S. May-Tal Beck,[23] D. Meekins,[6] R. Michaels,[6]
B. Moffit,[1,6] B. E. Norum,[7] M. Olson,[24] E. Piasetzky,[25] I. Pomerantz,[25] D. Protopopescu,[26] X. Qian,[27] Y. Qiang,[27,6]
A. Rakhman,[28] R. D. Ransome,[14] P. E. Reimer,[2] J. Reinhold,[29] S. Riordan,[7] G. Ron,[25,30] A. Saha,[6] A. J. Sarty,[31]
B. Sawatzky,[6,32] E. C. Schulte,[14] M. Shabestari,[7] A. Shahinyan,[33] S. Širca,[34,35] P. Solvignon,[2,6] N. F. Sparveris,[1,32]
S. Strauch,[36] R. Subedi,[7] V. Sulkosky,[1,6] I. Vilardi,[20] Y. Wang,[37] B. Wojtsekhowski,[6] Z. Ye,[38] and Y. Zhang[39]
(Jefferson Lab Hall A Collaboration)

Electron-proton scattering: evaluations of `the World data'

- Mainz:

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}(4)_{\text{group}}\,\text{fm},$$
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.777(13)_{\text{stat.}}(9)_{\text{syst.}}(5)_{\text{model}}(2)_{\text{group}}\,\text{fm}.$$

- JLab (similar results also from Ingo Sick)

$$\langle r_E^2 \rangle^{1/2} = 0.875 \pm 0.008_{\text{exp}} \pm 0.006_{\text{fit}}\,\text{fm} \quad (3)$$
$$\langle r_M^2 \rangle^{1/2} = 0.867 \pm 0.009_{\text{exp}} \pm 0.018_{\text{fit}}\,\text{fm}, \quad (4)$$

Magnetic radius does not agree!

- Charge radius:

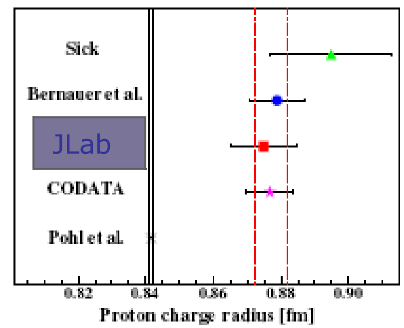


FIG. 3: (Color online) The proton RMS charge radius from previous ep scattering analysis (Sick [40]), Mainz low Q^2 measurement (Bernauer *et al.* [37]) and this work compared to the CODATA [41] and muonic hydrogen spectroscopy (Pohl *et al.* [42]). The red dashed lines show the combined results from CODATA, Bernauer *et al.* and this work, while the black dotted lines show the Pohl *et al.* uncertainty.

High Precision Measurement of the Proton Elastic Form Factor Ratio $\mu_p G_E/G_M$ at Low Q^2

X. Zhan,[1,2] K. Allada,[3] D. S. Armstrong,[4] J. Arrington,[2] W. Bertozzi,[1] W. Boeglin,[5] J.-P. Chen,[6] K. Chirapatpimol,[7] S. Choi,[8] E. Chudakov,[6] E. Cisbani,[9,10] P. Decowski,[11] C. Dutta,[12] S. Frullani,[9] E. Fuchey,[13] F. Garibaldi,[9] S. Gilad,[1] R. Gilman,[6,14] J. Glister,[15,16] K. Hafidi,[2] B. Hahn,[4] J.-O. Hansen,[6] D. W. Higinbotham,[6] T. Holmstrom,[17] R. J. Holt,[2] J. Huang,[1] G. M. Huber,[18] F. Itard,[13] C. W. de Jager,[6] X. Jiang,[14] J. Johnson,[19] J. Katich,[4] R. de Leo,[20] J. J. LeRose,[6] R. Lindgren,[7] E. Long,[21] D. J. Margaziotis,[22] S. May-Tal Beck,[23] D. Meekins,[6] R. Michaels,[6] B. Moffit,[1,6] B. E. Norum,[7] M. Olson,[24] E. Piasetzky,[25] I. Pomerantz,[25] D. Protopopescu,[26] X. Qian,[27] Y. Qiang,[27,6] A. Rakhman,[28] R. D. Ransome,[14] P. E. Reimer,[2] J. Reinhold,[29] S. Riordan,[7] G. Ron,[25,30] A. Saha,[6] A. J. Sarty,[31] B. Sawatzky,[6,32] E. C. Schulte,[14] M. Shabestari,[7] A. Shahinyan,[33] S. Širca,[34,35] P. Solvignon,[2,6] N. F. Sparveris,[1,32] S. Strauch,[36] R. Subedi,[7] V. Sulkosky,[1,6] I. Vilardi,[20] Y. Wang,[37] B. Wojtsekhowski,[6] Z. Ye,[38] and Y. Zhang[39]
(Jefferson Lab Hall A Collaboration)

Certainty of the derivatives

- $\Delta f/\Delta t$ – we can find it in a model independent way if we have accurate data and an estimation of the higher-order Taylor terms.
- Data are roughly with 1%.
- R_p is wanted within 1%.
- We need fits!
- We can use fits only if we know the exact shape.

Certainty of the derivatives

- Δf/Δt – we can find it in a model independent way if we have

- Data are roughly with 1%.
- R_p is wanted within 1%.

We need fits!

We can use fits only if we know the exact shape.

Narrowing the area increases the uncertainty: e.g.:
Hill and Paz, 2010
Kraus et al., 2014

Certainty of the derivatives

Fifty years:

•data improved (quality, quantity);

•accuracy of radius stays the same;

•systematic effects:

increasing complicity of the fit.

ughly

d

estimation of the higher-order Taylor terms.

We need fits!

- We can use fits only if we know the exact shape.

Analytic properties: is that important?

- The form factors are measured in a finite space-like region.
- The fits present the form factors for all the momenta.
- The form factors fits in time-like region are wrong.
- Their analytic properties are inappropriate.
- Their behavior at larger momenta is unreasonable.

Analytic properties:

`Dispersion fits' always produce smaller values of the radius, but usually they have bad χ^2.

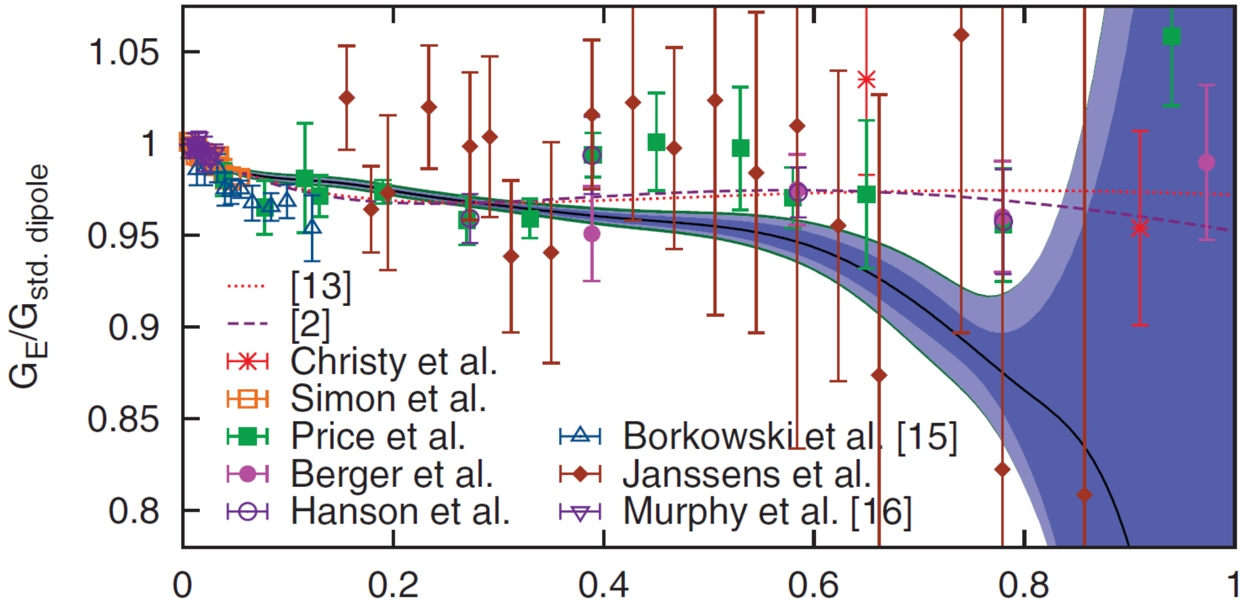
Mergell et al., 1995; Belushkin et all, 2007, Lorenz et al., 2012; Adamuscin et al., 2012

Is it possible to produce a fit with a good value of χ^2 and consistent with our knowledge about the imaginary part ?

Asymptotic behavior: is that important?

- The form factors are measured in a finite space-like region.
- The fits present the form factors for all the momenta.
- The form factors fits in time-like region are wrong.
- Their analytic properties are inappropriate.
- Their behavior at high momenta is unreasonable.

Asymptotic behavior: is that important?



s

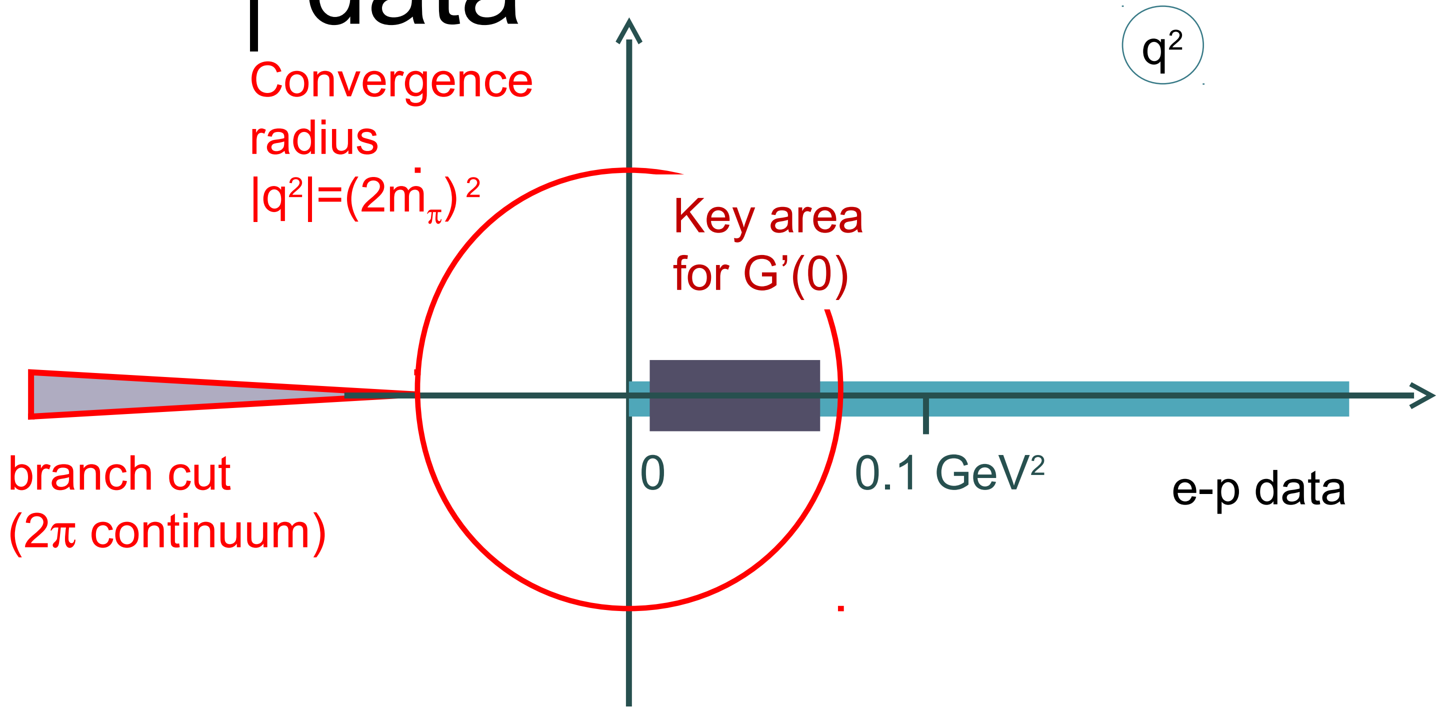
on

the form factors for all the momenta.

inappropriate.

- Their behavior at high momenta is unreasonable.

Analytic properties and the data



Different methods to determine the proton charge radius

- *spectroscopy of hydrogen (and deuterium)*
- *the Lamb shift in muonic hydrogen*
- *electron-proton scattering*

- Comparison:

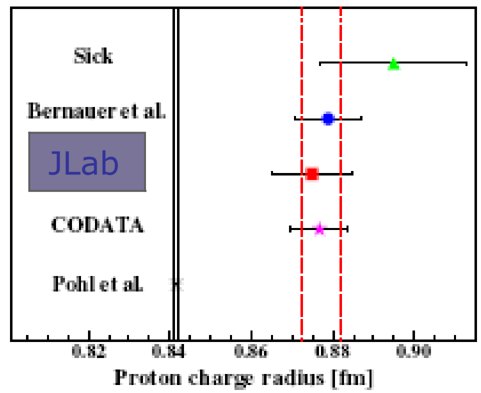


FIG. 3: (Color online) The proton RMS charge radius from previous *ep* scattering analysis (Sick [40]), Mainz low Q^2 measurement (Bernauer *et al.* [37]) and this work compared to the CODATA [41] and muonic hydrogen spectroscopy (Pohl *et al.* [42]). The red dashed lines show the combined results from CODATA, Bernauer *et al.* and this work, while the black dotted lines show the Pohl *et al.* uncertainty.

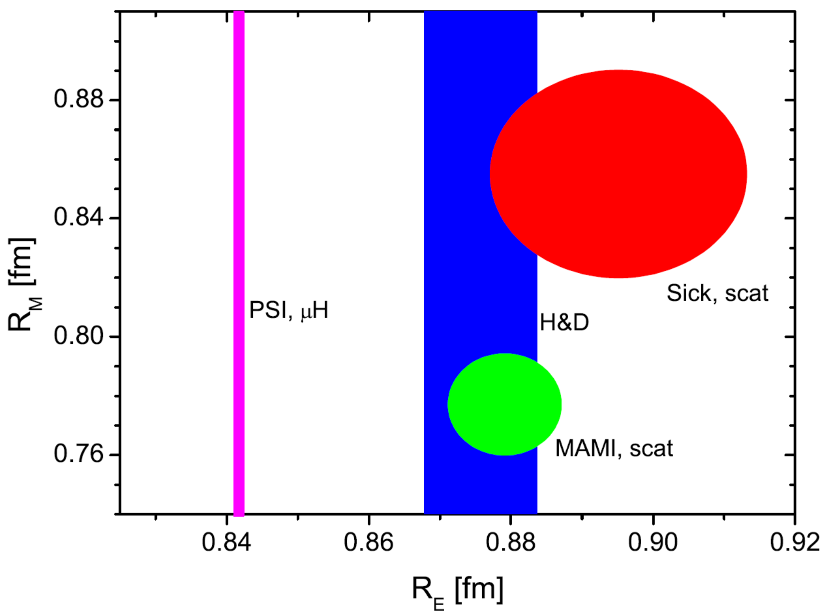
Present status of proton radius: three convincing results

charge radius and the Rydberg constant: a strong discrepancy.

- If I would bet:
  - *systematic effects in hydrogen and deuterium spectroscopy*
  - *error or underestimation of uncalculated terms in 1s Lamb shift theory*
- Uncertainty and model-independence of scattering results.

magnetic radius:

a strong discrepancy between different evaluation of the data and maybe between the data



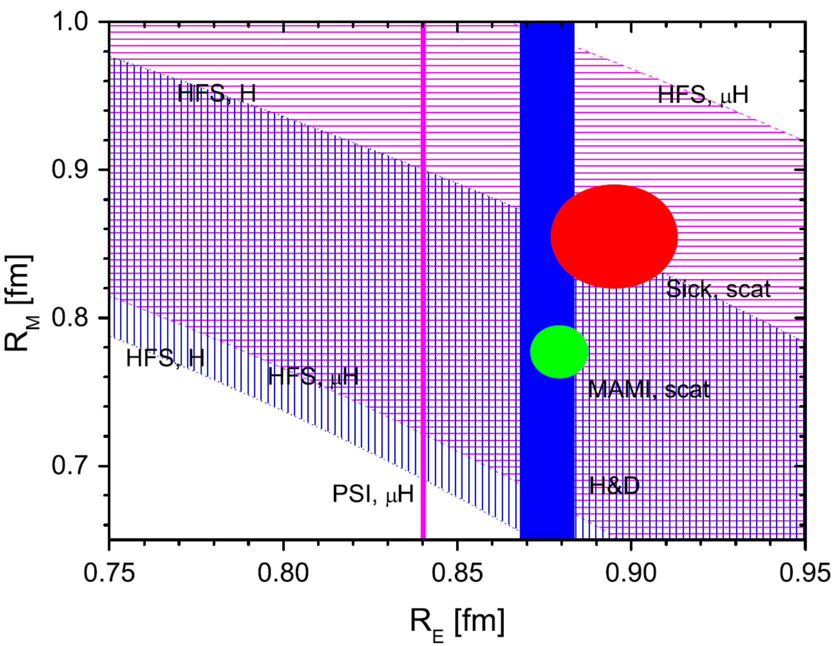
Present status of proton radius: three convincing results

charge radius and the Rydberg constant: a strong discrepancy.

- If I would bet:
  - *systematic effects in hydrogen and deuterium spectroscopy*
  - *error or underestimation of uncalculated terms in 1s Lamb shift theory*
- Uncertainty and model-independence of scattering results.

magnetic radius:

a strong discrepancy between different evaluation of the data and maybe between the data

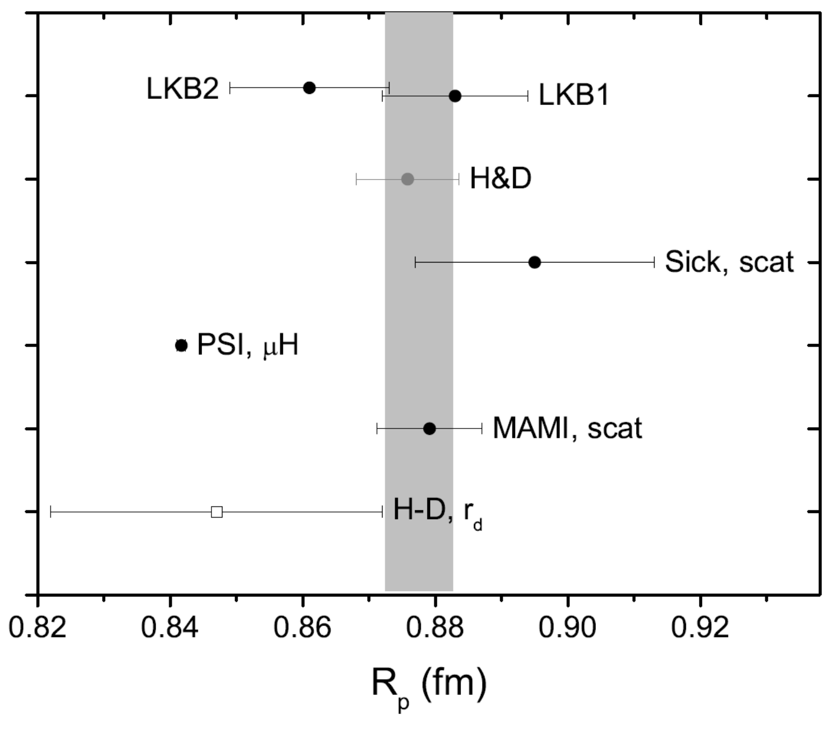


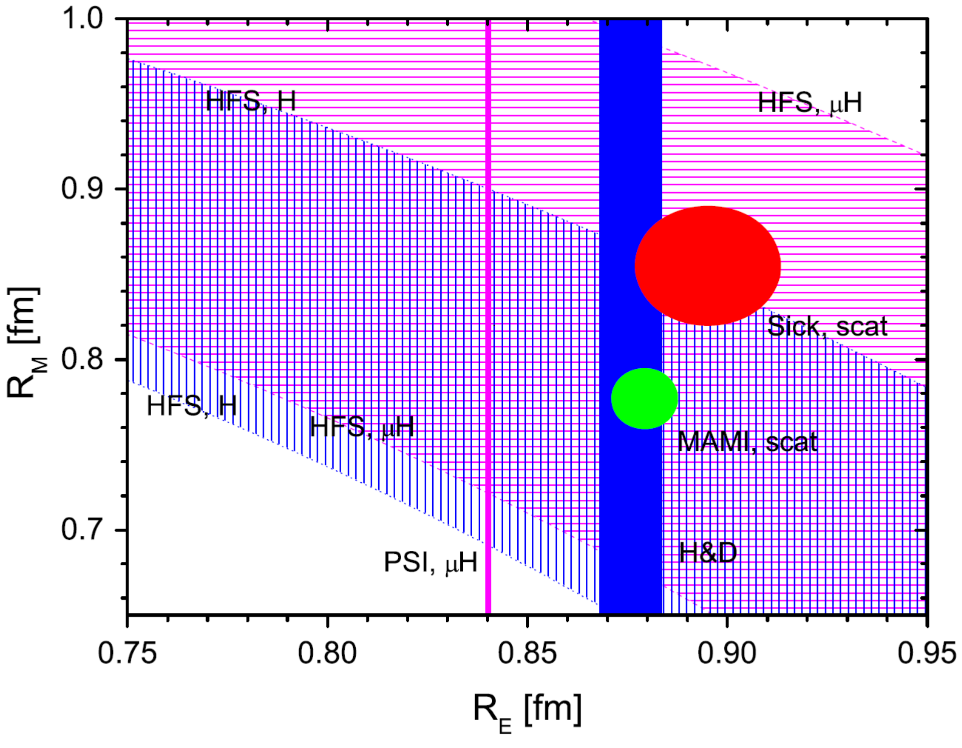
What is next?

- *new evaluations of scattering data (old and new)*
- *new spectroscopic experiments on hydrogen and deuterium*
- *evaluation of data on the Lamb shift in muonic deuterium (from PSI) and new value of the Rydberg constant*

- *systematic check on muonic hydrogen and deuterium theory*

Where we are


LKB2
LKB1
H&D
Sick, scat
PSI, μH
MAMI, scat
H-D, r_d
0.82
0.84
0.86
0.88
0.90
0.92
R_p (fm)

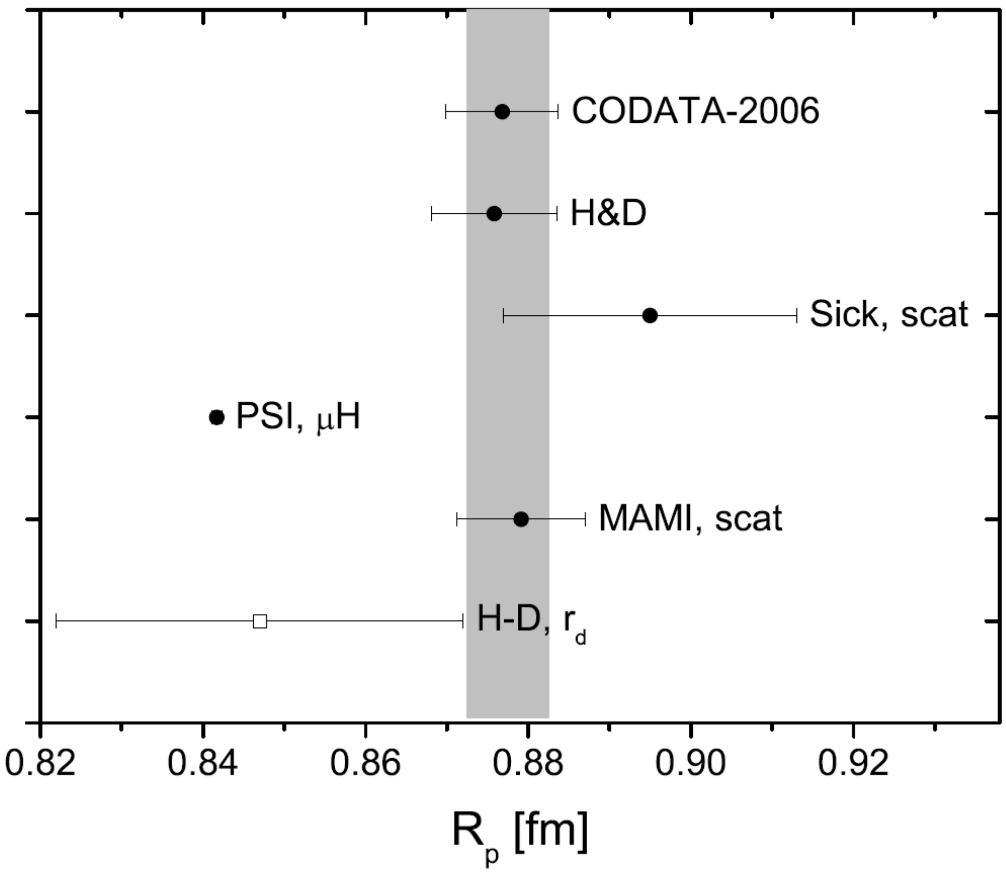

1.0
0.9
0.8
0.7
R_M [fm]
HFS, H
HFS, μH
Sick, scat
HFS, H
HFS, μH
MAMI, scat
PSI, μH
H&D
0.75
0.80
0.85
0.90
0.95
R_E [fm]

What's new?

- Hydrogen:
  - A preliminary MPQ result on 2s-4p is consistent with μH.
- Muonic atoms:
  - μD is consistent with μH (PSI) + isotopic H-D 1s-2s (MPQ).
- Scattering data:
  - Jlab's people and I. Sick state that world data and MAMI's are not quite consistent.
  - More studies of different shapes (conformal mapping etc.)

What's new?

- Hydrogen:



- Scattering data:
  - Jlab's people and I. Sick state that world data and MAMI's are not quite consistent.
  - More studies of different shapes (conformal mapping etc.)

Conferences

Precision physics and fundamental constants (FFK)

Oct., 12-16, 2015

Budapest

Precision physics of simple atoms (PSAS)

May, 22-26, 2016

Jerusalem