Quantum gravity (Q.G)

Space is flat at our distances (~ 1 m.) and on the scale of 'usual particles physics' $\sim 10^{-13} \div 10^{-15}$ cm.

But when we go to much smaller distances ($\sim 10^{-33}$ cm.) the geometry of our space exhibit very large quantum fluctuations

Quantum G. effects on large ($\gg \ell_p$) scales are weak, and are described by tenzor gravitons, corresponding to small fluctuation of metric tenzor $g_{\mu\nu}(x)$)

$$8\pi G = m_p^{-2}$$

$$m_p \simeq 2 \cdot 10^{19}\ \text{GeV} \sim 10^{-5}\ \text{g}$$

$$\ell_p \sim \hbar / m_p c \sim 10^{-33}\ \text{cm}$$

$$t_p \sim \ell_p / c \sim 10^{-43}\ \text{sec}$$

TeV scale gravity $[10^{-16} \div 10^{-17}\ \text{cm}]$?

Plan of lectures

1. Introduction : main goals and problems in QG
- 2. The perturbative gravity — simplest structures
- 3 The high-energy behaviour in the perturbative Q.G.
- 4. What can be measured. Experimental implications of QG.
- 5. Non perturbative Q.G. An elementary introduction and results.

1. Canonical QG
2. From metrics to connection and holonomies (loops)
3. Gravitational spin nets
4. Quantum geometry
5. Some applications (Black holes, Cosmology)

1. $\int\sqrt{-g}\,\mathcal{L}$; $\mathcal{L} = m_p^2 R + a_2 m_p^4 R^2 + \dots$; $x^\mu \to y^\mu(x)$

2. $O_i(x) = \{R(x), R_{\mu\nu}R^{\mu\nu}, \dots\}$

3. $F_n(i) = \langle O_1(x_1) O_2(x_2) \dots O_n(x_n) \rangle_0$

4. Theory is discrete for invariant $\langle\cdot\rangle_0$

expectation values $F_n(i)$ over invariant vacuum don't depend on x_i^μ

5. Ground state (and space of states based on it) is noninvariant $\longrightarrow \langle g_{\mu\nu}(x)\rangle_0 = \eta_{\mu\nu} \neq 0$

6. In this case we have only effective theory valid for small quantum fluctuations $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu} m_p^{-1}$
$|h_{\mu\nu}| \ll |\eta_{\mu\nu}| m_p$

7. Virtual momenta must be cut on a scale $\sim m_p$, because for smaler distances $\langle g_{\mu\nu}\rangle_0$ is unessential and all Green functions freeze on $F_n(i)$

8 All infinities are gauge artifacts

$$\{|\vec{k}_1, \vec{k}_2, \dots \vec{k}_n\rangle\} \Longrightarrow \left(\frac{\{g_{ik}(\vec{x})\}}{G_{diff.}} \,,\; \frac{\{e_i^a(\vec{x})\}}{G_{diff}\cdot G_L} \right)$$

n-graviton states

$$\frac{\text{Space of all metrix}}{\text{Sym. group}}$$

$$\{C_n\} \Longleftarrow \left(\frac{\{\omega_i^{ab}(\vec{x})\}}{G_{diff}\cdot G_L} \,,\; \frac{\{A_i^a(\vec{x})\}}{G_{diff}\cdot G_L} \right)$$

Space of all spin-nets =
= Space of all collections of quantized space cells

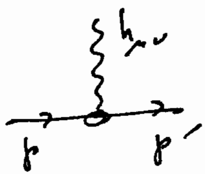
$$\frac{\text{Space of all connections}}{\text{Sym. Group}}$$

$$\frac{O(3)\ \text{Gauge fields}}{\text{Sym. Gr.}}$$

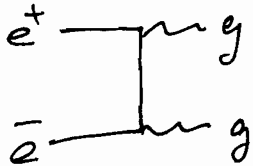
Perturbative gravitons

$$g_{\mu\nu}(x) \simeq \eta_{\mu\nu} + h_{\mu\nu}(x)/m_p \quad ; \quad \langle g_{\mu\nu}(x)\rangle_0 \overset{?}{=} \eta_{\mu\nu}$$

$$\mathcal{L} = m_p^2 R + \mathcal{L}(\varphi \ldots) \simeq h_{\mu\nu} \Box h_{\mu\nu} + \ldots + m_p^{-1} h_{\mu\nu} T_{\mu\nu}$$



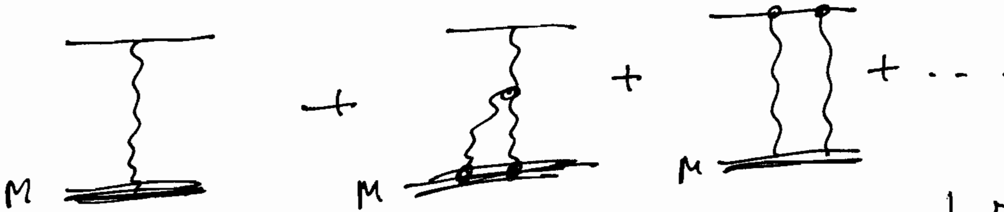
$$\longleftrightarrow \quad m_p^{-1} h_{\mu\nu} \langle p' | T_{\mu\nu} | p \rangle \sim m_p^{-1} h_{\mu\nu} (p_\mu p_\nu' + p_\mu' p_\nu)$$



$$\longrightarrow \sigma \sim \frac{1}{m_e^2}\left(\frac{s^2}{m_p^4}\right) = \frac{1}{m_e^2}\left(\frac{1\,\mathrm{GeV}}{m_p}\right)^4 \left(\frac{s}{1\,\mathrm{GeV}^2}\right)^2$$

$$\underset{10^{-76}}{\uparrow} \qquad \longleftrightarrow 10^{-12}$$

Strong and weak gravitational fields

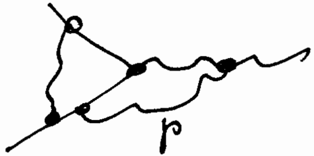


$$\varphi = \frac{MG}{r} + \dots = \left(\frac{r_M}{r}\right)\left(\frac{c^2}{2}\right)\left(1 + \frac{r_m}{r} + \dots\right)$$

$r_M = \dfrac{2GM}{c^2}$ – Gravitational radius of particle (body)

$$g_{00} \simeq 1 + \varphi \frac{2}{c^2}$$

Higher orders



1. Every loop adds the divergent factor $\int^{\Lambda} d^4p / m_p^2 p^2 \sim \Lambda^2 / m_p^2$
2. If we somehow cut such integrals on $\Lambda \sim m_p$ then all loop corrections are of the same order

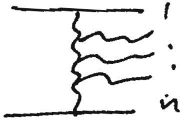
$\updownarrow$

3. Nonrenormalizability
4. $\langle g_{\mu\nu}(x) \rangle = \eta_{\mu\nu}$ is unstable
5. Possible cancelations in $N=8$ supergravity ?
6. Other approaches to QG (strings, composite gravitons...)

High energy behaviour in perturbative QG.

$$\sigma \sim \frac{1}{m_p^2}\left(\frac{s}{m_p k_{\perp min}}\right)^2$$

No Froissart bound on QG – because gravitons are massless

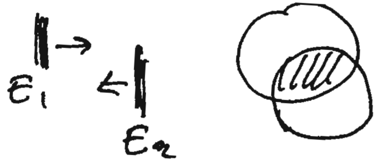


$$\sigma_n \sim \frac{\Delta^n}{m_p^2}\left(\frac{s}{m_p^2}\right)^2 \ln^n \frac{s}{m_p^2}$$

$\sum_n$  $\Rightarrow$  ← QG analog of Pomeron with $\alpha_G(0) \simeq 3+\Delta$

Unitarisation of ↑ → picture of black disks collision

$$R(E_i) \sim E/m_p^2 \rightarrow \sigma \sim E_1 E_2 / m_p^4$$



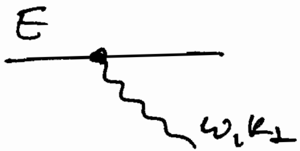
Black holes production as a result of collision at $s > m_p^2$

Graviton-parton bremsstrahlung spectra

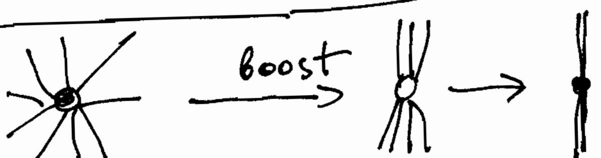
in QED:

$$dn \sim \alpha_{QED} \cdot \frac{d\omega}{\omega} \frac{dk_\perp^2}{k_\perp^2}$$

in QG:



$$dn \sim \left(\frac{E}{m_p}\right)^2 \frac{d\omega}{\omega} \, dk_\perp^2$$



Coulomb field at rest

$$\text{QED: } F_{-\perp} \sim \alpha\, \delta(x^-) \frac{x_\perp}{x_\perp^2}$$

$$\text{Grav: } R_{-\perp-\perp} \sim \frac{E}{m_p^2} \delta(x^-) \frac{x_\perp x_\perp}{x_\perp^4}$$

Do we at all need QG. What can be measured

1. Non conservation of all global quantum numbers ?
2. Induced operators. Grav. proton decay $\tau \sim m_h^{-1}\left(\frac{m_P}{m_h}\right)^4 \sim 10^{46}\,y$
3. CP and CPT breaking
4. $E^2 = m^2 + \vec{p}^{\,2} + \frac{c}{m_P^2}\vec{p}^{\,4} + \dots$; $c \sim \pm 1$; $\sim 1/m_P^2 R^2$?
5. Decoherence on Planck scale
6. Quant. corrections to BH. evaporations $\left\{ \begin{array}{l} \text{relict B.H} \\ m_{BH} \gtrsim 10^{10} g \\ R \sim 10^{-19} cm \end{array} \right.$ $T_H \lesssim 10^4\,GeV$
7. Relict gravitons and gravitino
8. Tracks in CMB

TeV scale QG.

Non perturbative QG

Canonical
Loop QG
Quantum geometry

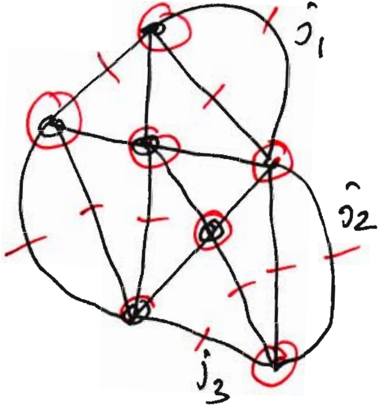
$\boxed{\text{No new degrees of freedom are introduced. Quantization of standard general relativity}}$

1. Diff. invariance $\leftrightarrow$ Background independence

2. $g_{\mu\nu} \to \Gamma^{\lambda}_{\mu\nu}$ – Now the main variables are special connections $A^a_i = \omega^a_i + \gamma K^a_i$, and not the metrics $g_{\mu\nu}$

Then the conjugated variables $\sim -i\frac{\partial}{\delta A^a_i}$ are E^i_a – space repers

3) Then one can find that states in QG are described by the spin-nets: samples of connected quantized space sells.



4) Areas and volumes of such sells are quantized

$$S \sim \ell_p^2 \gamma \sum_{\text{edges}} \sqrt{j(j+1)}$$

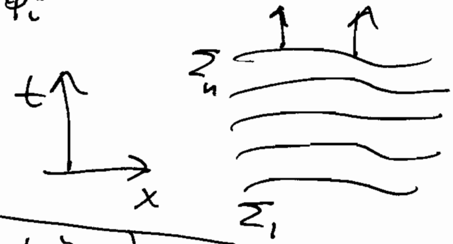
$$j = 1, 2, \dots$$

$$V \sim \ell_p^3 \gamma^{3/2} \sum_{\text{nodes}} f(j_1, j_2, \dots)$$

Quantum gravity (nonperturbative)

$$I = m_p^2 \int d^4x \sqrt{-g}\cdot R + I_\phi[\phi_i, g_{\mu\nu}] \rightarrow \frac{\delta I}{\delta g_{\mu\nu}} = 0; \quad \frac{\delta I}{\delta \phi_i} = 0$$

$$\mathcal{L}_g = m_p^2 R + c\, m_p^{-2} R^2 + c'\, m_p^{-4} R^4 + \ldots$$



1. Functional integral approach (Misner... Hawking)

$$\langle (g_{\mu\nu}, \phi)_{out} | (g_{\mu\nu}, \phi)_{in} \rangle \sim \int_{in}^{out} \mathcal{D}\phi_i \mathcal{D}g_{\mu\nu} \exp\left(i\, I[g, \phi_i] \right)$$

2. Canonical quantization (Dirac, ...)

Dynamical variables $(g_{\mu\nu},\ P_{\mu\nu} = \delta\mathcal{L}/\delta\partial_t g_{\mu\nu}) \rightarrow H = \dot{g}_{\mu\nu} P_{\mu\nu} - \mathcal{L}$

One must define:

a) Commutation relations $[g_{..}(x), P_{..}(y)]_- \sim i\delta^{\cdot}_{\cdot}\delta^{\cdot}_{\cdot}\delta^3(x-y)$

b) Space of states $\rightarrow$ (all 3-metrics), ...

c) Action of H and other operators in this space

Due to (diff.) simmetry of I

$$x_\mu \to x'_\mu(x_\nu)$$

variables $p_{\mu 0}$ are not well defined (constrained)
It is convenient to choose coordinates and $g_{\mu\nu}$ in such a way that constraints thies simple form $p_{\mu,0} = 0$

$$(\text{ADM}):\ ds^2 = -(N\,dt)^2 + g_{ik}(dx^i + N^i dt)(dx^k + N^k dt)$$

$$I = \int dt \int d^3x \left[N\sqrt{g_3}\,(K_{ij}K^{ij} - K^2 + R_3)\right) + \partial_i(\quad)_i$$

$$K_{ij} = (\nabla_i N_j + \nabla_j N_i - \dot{g}_{ij})/2N = \nabla_i^{(4)} n_j$$

$$p^0 = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0 \quad , \quad p^i = \frac{\partial \mathcal{L}}{\partial \dot{N}_i} = 0$$

$$p^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{g}_{ij}} = -\sqrt{g_3}\,(K^{ij} - g^{ij} K)$$

Hamiltonian: $H = \int d^3x \, [p^0 \dot{N} - p^i \dot{N}_i + N\mathcal{H} + N_i \mathcal{H}^i]$.

where

$$\mathcal{H} = \sqrt{g_3}\,(K_{ij}K^{ij} - K^2 - R_3^2)\ ; \quad \mathcal{H}^i = -2\,p^{ij}{}_{,j}$$

$$\frac{\partial}{\partial t} p^\mu = 0 \rightarrow [p^\mu, H] = 0 \rightarrow \begin{cases} \mathcal{H}^i = 0 \\ \mathcal{H} = 0 \end{cases}$$

Quantization:

$\Psi[g_{ij}(\vec{x})]$ — space of all functions of $g_{ij}(\vec{x})$

with operators $p^{ij} = -i\dfrac{\delta}{\delta g_{ij}}$, $p^i = -i\dfrac{\delta}{\delta N^i}$, $p = -i\dfrac{\delta}{\delta N}$; $[g_{ij}(x), p^{kl}(y)] = i\delta^3(x,y)\,\delta^{kl}_{ij}$

$$\mathcal{H}^i = 0 \rightarrow \nabla_j\left(\frac{\delta\Psi}{\delta g_{ij}}\right) = 0 \quad ; \quad \mathcal{H}\Psi = 0 \longleftrightarrow \frac{\partial}{\partial t}\Psi = 0$$

space diff. constraint

Hamilton constraint

$\mathcal{F} = \{ g_{ik}(\vec{x}) \}$ – Space of all 3-metrics

Λ = Space of all local diffeomorphisms

The space of states for QG can be defined as factorspace $\left(\frac{\mathcal{F}}{\Lambda}\right) = \Phi$ with additional Hamiltonian constraint

$$\mathcal{H}\Phi = 0$$

Is it possible to find some simple system of base vectors in this space $[\Phi, \mathcal{H}\Phi = 0]$

Possible strategy:

1). Find a simple base in Φ

2) Apply the constraint $\mathcal{H}\Phi = 0$

Metrics $\to$ Vierbein (repers)

$$g_{\mu\nu}(x) = e_\mu^a(x)\, e_\nu^a(x)\, \eta^{\mu\nu}$$

$$D_\mu e_\nu^a = \nabla_\mu e_\nu^a - \omega_\mu^{ab} e_{\nu b}$$

$$\nabla_\mu e_\nu^i = \partial_\mu e_\nu^i - \Gamma_{\mu\nu}^\alpha e_\alpha^i$$

$$\omega_{\mu ab} = e_a^\nu \nabla_\mu e_{b\nu} = \frac{1}{2}\left[\Omega_{cab} - \Omega_{abc} + \Omega_{bca}\right] e_\mu^c$$

coefficients of anholonomy

Symmetry becomes larger:

1. Diff invariance
2. Local $O(3,1)$ invariance

$$\mathcal{L} = \sqrt{-g}\, R \to e \cdot e^{\mu a} [\nabla_\mu, \nabla_\nu] e_a^\nu = e\left(\omega_{\mu a}^{\ a} \cdot \omega^{\mu a}_{\ \ a} - \omega_{\mu ab}\, \omega^{\mu ab}\right) + \partial_\mu(\ )_\mu$$

$$p_a^\mu = \frac{\delta \mathcal{L}}{\delta \dot{e}_\mu^a} = \frac{1}{2} e\, e_b^\mu (K_{ab} - \delta_{ab} K)$$

$$K_{ab} = e_a^\mu e_b^\nu \nabla_\mu e_{\mu \underline{\underline{0}}} = \Omega_{\underline{\underline{0}} ab}$$

extrinsic curvature

Hamiltonian

$$H = p^a \dot{e}^\mu_a - \mathcal{L} \Rightarrow N\mathcal{H} + N^i \mathcal{H}_i$$

Constraints:

1. Lorentz constraint: $L_{ab} \equiv e_{\mu[a} P^\mu_{b]} \approx 0$

2. Diff. constr: $\mathcal{H}_a \equiv D_m P^m_a \approx 0$

3. Hamilt. constr. $\mathcal{H} = \frac{e}{4}\left(K_{ab}K^{ab} - K^2\right) - eR^{(3)} \approx 0$

Quantisation

$$[e_{ma}(x), p^n_b(y)]_- = i\delta_{ab}\delta^n_m \delta^3(\vec{x},\vec{y}) \; ; \; p^n_b = -i\frac{\delta}{\delta e_n{}^a(x)}$$

$$L_{ab}\Psi(e) = 0 \; ; \; \mathcal{H}_a \Psi(e) = 0 \; ; \; \mathcal{H}\Psi(e) = 0$$

New (Ashtekar–Sen) variables

$$\omega_i^{ab} \to \omega_i^c = \omega_i^{ab} \epsilon^{abc}$$

all indices in 3space Σ

New variables are connections

$$A_i^a = \omega_i^a + \gamma K_i^a$$

; γ – Barbero–Immirzi parameter

γ – similar to θ param in QCD

$I = \int R_{\dots} g^{\cdot\cdot} g^{\cdot\cdot} \sqrt{-g} + $
$+ \gamma^{-1} \int R_{\dots} \epsilon^{\dots}$

Conjugate momenta:

$$E_a^i \equiv \frac{\partial \mathcal{L}}{\partial A_i^a} \Leftrightarrow -i \frac{\delta}{\delta A_i^a}$$

$$E_a^i = \frac{1}{\gamma} e_a^i \cdot e$$

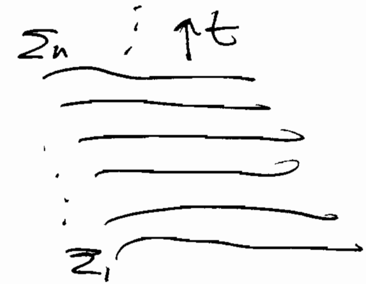
inverse dreibein

$$A_i^a \to F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a - \epsilon^{abc} A_i^b A_j^c$$

A_i^a – $SU(2) \leftrightarrow O(3)$ gauge field

$$A_i^{ab} \leftrightarrow A_i^{ab} \epsilon^{abc}$$

Kinematical space of states



$$\mathcal{C} = \frac{[\text{Space of connections } A_i^a(\vec{x}) \text{ on } \Sigma]}{G(\text{diff. on } \Sigma)\,\mathcal{G}(\text{local 3 rotations})}$$

Constraints:

① $\mathcal{D}_i E^i = 0$ [Gauss (Lorentz) constr.]

② $\mathrm{Tr}(F_{ij} E^i) = 0$ [Diff. constr.]

} Kinem. constr.

E, A, F — SU(2) or SO(3) matrices

③ Hamilt. constr.

$$\mathrm{Tr}\left(E^i E^j \left(F_{ij} + \tfrac{1}{2}(\gamma^2+1) K_{[i} K_{j]} + \Lambda \epsilon_{ijl} E^l + \dots\right)\right) = 0$$

} Dynam. constr.

Simple case $\gamma = i \Leftrightarrow$ complex connection ?

Speciale case $\gamma = \pm i$

A_m^a – multiplication operator

$$E_a^m = -i \frac{\delta}{\delta A_m^a}$$

Hamilt. constr.:

$$\epsilon_{abc} \frac{\delta^2}{\delta A_m^a \delta_n^b} \left(F_{mnc} - i \Lambda \epsilon_{mnk} \frac{\delta}{\delta A_k^c} \right) \Psi[A] = 0$$

Chern-Simons Lagrangian.

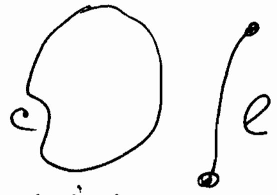
$$\mathcal{L}_{cs} = \epsilon^{mnk} \left(A_m^a \partial_n A_k^a + i A_m^a A_n^b A_k^c \epsilon^{abc} \right)$$

Kodama state $\Psi_K \sim \exp\left(\frac{i}{\Lambda} \int d^3x \, \mathcal{L}_{cs} \right)$

Quasi Class: $F_{mnc} = \Lambda \epsilon_{mni} E_c^i$

$A_{ia} = \Lambda^{1/2} f \delta_{ia}$; $E_{ia} \sim f^2 \delta_{ia}$ ← De-Sitter space

From Connections A_i^a to holonomies

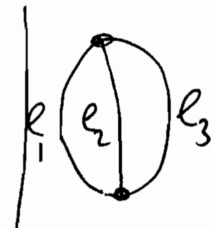


$$A_i^{ab} \longrightarrow h_c[A] = \mathbb{P}\exp\left(\int_c A_i dx^i\right) = \lim\,(1+dx_1^i A_i)(1+dx_2^i A_i)\ldots\ldots(\ldots)$$

$$\left\{\begin{array}{l} W_c[A] = \mathrm{Tr}\, h_c[A] \\ h^{m_1}_{n_1}(\ell_1)\, h^{m_2}_{n_2}(\ell_2)\, h^{m_3}_{n_3}(\ell_3)\, \epsilon_{m_1 m_2 m_3}\, \epsilon^{n_1 n_2 n_3} \\ - - - - . \\ f(h(\ell_1), \ldots\ldots h(\ell_n)) \end{array}\right\}$$

invariant under

$G(\text{dif.}\,\Sigma) \circ \mathcal{G}(\text{loc.rot.})$



$$\Psi[A] \longleftrightarrow \widetilde{\Psi}[C] = \int \mathcal{D}A\, \Psi[A]\, W_c[A]$$

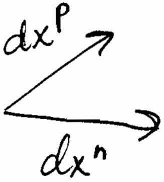
$\longleftarrow$ Loop 'Fourier' transformation

Generalization: $h(e) \to h_j(e) = \mathbb{P}\exp\left(\int_e \overset{(j)}{A}_m dx^m\right)$

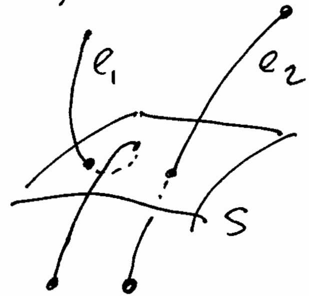
Now $A^{(j)}$ is 'j' representation of $O(3)$

$$[h(e)]^n_m \to [h_j(e)]^{n_1 \ldots\ldots n_j}_{m_1 \ldots\ldots m_j}$$

Variables conjugate to holonomies



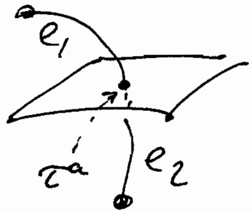
$$dE_a = E_a^m dx^n dx^p \epsilon_{mnp}$$ ← flux of E_a^m through area $dx \wedge dx$



$$E^a[S] = \int_S dE^a$$ ← flux of E through surface S

Poisson brackets (commutator):

$$[h_e, E^a[S]]_- = i\gamma I(e,S)\,(h_{e_1}\tau^a h_{e_2})$$



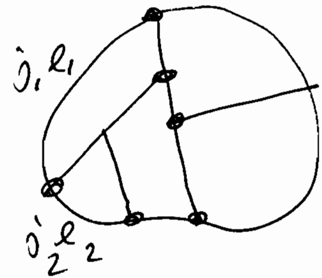
Excurse in QCD: [lattice 3+1]

$$\left|\begin{matrix}\text{Space of physical}\\ \text{states}\end{matrix}\right\rangle = \frac{\text{space of all } \vec{A}(x)}{\text{local } SU(3)}$$

$$\begin{cases} W_c = \mathrm{Tr}\left(\mathbb{P}\exp\left(i\oint_c \vec{A}\,d\vec{x}\right)\right) \\ \Psi = \sum_{c_i} a(c_1\ldots c_n)\,(W_{c_1}\cdot W_{c_2}\cdots W_{c_n})|0\rangle \end{cases}$$

$$W_e^{j} = \left[\mathbb{P}\exp\left(i\int \vec{A}^{(j)}\, d\vec{x}\right)\right]^{a_1\ldots a_j}_{b_1\ldots b_j}$$

$$\Psi = \sum_{c,j} a(\quad)\underbrace{(W^{(j_1)}W^{(j_2)}\cdots W^{(j_n)})|0\rangle}_{\text{spin-net state}}$$



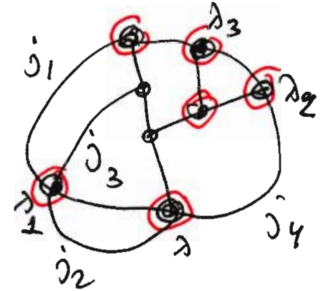
Spin net states in gravity (LQG)

1. Use j-holonomies $h_j(e) = \mathbb{P}\exp\left(\int A_m^{(j)} dx^m\right)$

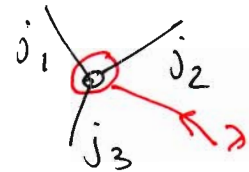
2. Build various spin net states

$$\underbrace{\Psi_{j_1 \dots j_n}}_{\text{lines}}\underbrace{(\lambda_1, \dots \lambda_n)}_{\text{vertexes}} \sim \left[h_{j_1}(e_1)\, h_{j_2}(e_2) \dots h_{j_n}(e_n)\right]|0\rangle$$

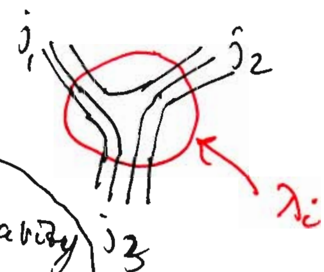
λ_i – interviners – generalizing Clebsch-Gordan coefficients



3. Net states $\Psi_{j_1 \dots j_n}(\lambda_i)$ fulfill Gauss and diff. constraint



4. $\Psi_{\dots}$ — Don't depend on the shape of contours l_i

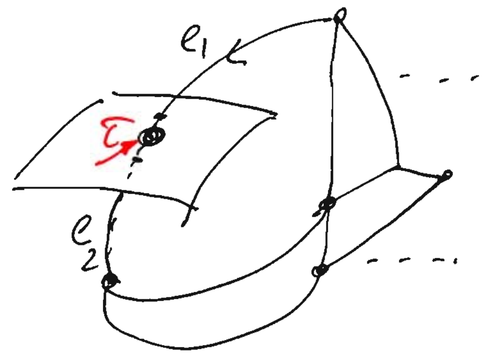


5. $$\Psi = \sum_{G, j, \lambda} a(j; G, \lambda)\, \Psi^{(G)}_{j \dots}(\lambda)$$ ← Kinematical space of state in gravity

6. A_i, F_{ij}, h_j act on Ψ as multiplication operators

The action of Flux operator on a spin net state

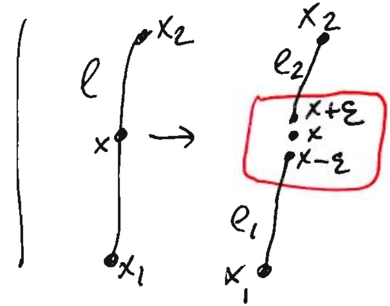
$$E^a(S) = \int_S E^{am}\,\epsilon_{mne}\,dx^m dx^e$$



$$E^a(S)\,|h(e)\rangle = i\gamma\, I(e,S)\,|h(e_1)\tau^a h(e_2)\rangle$$

$$[A_a^i(x), E_j^b(y)] = i\delta_a^b\,\delta_j^i\,\delta^3(x,y)$$

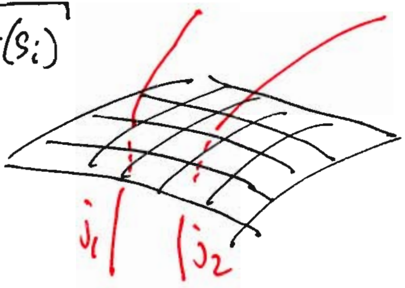
$$h(e) = \underbrace{\mathbb{P}\exp\left(\int_{x_1}^{x-\varepsilon} dx^m A_m\right)}_{h(e_1)}\left(1+\varepsilon\, dx^m A_m^a(x)\,\tau^a\right)\cdot \underbrace{\mathbb{P}\exp\left(\int_{x+\varepsilon}^{x_2} dx^m A_m\right)}_{h(e_2)}$$



The area operator and its eigenvalues on spin net states

$$A_S = \int_S d^2x \sqrt{g_2} = \gamma \int \sqrt{dE_i^a \, dE_j^a \, g^{ij}} = \gamma \sum_{S_i} \sqrt{E^a(S_i)\, E^a(S_i)}$$

det of the induced 2-metrics on S



$$E^a(S)\, E^a(S) \left| \overset{j}{h}(e) \right\rangle = (8\pi)\, \ell_p^2 \gamma \left| \overset{j}{h}(e_1)\, \tau^a \tau^a\, h(e_2) \right\rangle =$$

$$= 8\pi \ell_p^2 \gamma \, [j(j+1)] \left| \overset{j}{h}(e) \right\rangle$$

On line with momentum j

$$\hat{A}_S \Psi = \left(8\pi \ell_p^2 \gamma \sum_k \sqrt{j_k (j_k+1)} \right) \Psi$$

The Volume operator

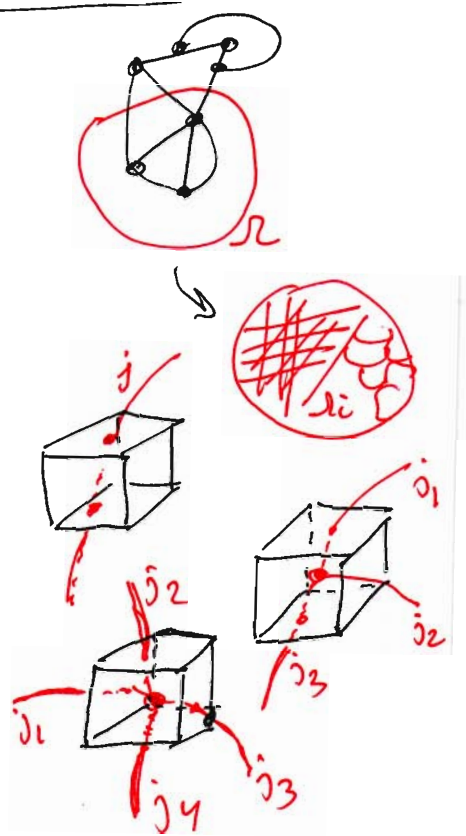
$$V_\Omega = \frac{1}{3!}\int_\Omega d^3x\left[E^a_m E^b_n E^c_k \epsilon^{mnk}\epsilon_{abc}\right]^{1/2} =$$

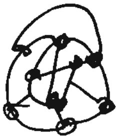
$$= \frac{1}{3!}\int_\Omega \sqrt{dE^a\, dE^b\, dE^c\, \epsilon_{abc}} = \frac{1}{3!}\sum_n \sqrt{E^a(\Omega_n)E^b E^c \epsilon_{abc}}$$

$$\epsilon_{abc}E^a E^b E^c \left| h^{j_1}(e_1)\, h^{j_2}(e_2)\dots\right\rangle \Rightarrow$$

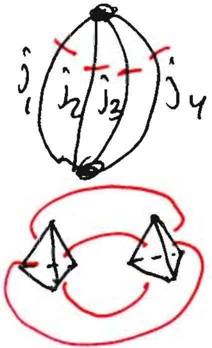
$$\Rightarrow \ell_p^3 \gamma^{3/2} \sum_{j_1 j_2 j_3} \tau^a|j_1\rangle\, \tau^b|j_2\rangle\, \tau^c|j_3\rangle \otimes \left| h_{j_1}(e_1)\dots\right\rangle$$

Volumes are concentrated only on nodes with 'valence' ≥ 4, and take discret values



$\left(\widehat{\text{Volume of}}\right)$ | Spin Net state  $\rangle = \left(\sum_{\text{nodes}} v_i\right)$ |  $\rangle$

$$V(j_1 \dots j_n) = \ell_p^3 \gamma^{3/2} \varphi(j_1 \dots j_n)$$



Large and 'smooth$^?$' spin-nets $\approx$

$\approx$ continuous 3 space

Like a big piece of solid (liquide) state containing many atoms

Hamiltonian constraint (time \`evolution?' operator)

$$0 = \hat{H} = \int_\Sigma d^3x \, N(x) \frac{E^i_a E^j_b}{\sqrt{\det E}} \left(\epsilon^{abc} F_{ijc} - \tfrac{1}{2}(1+\gamma^2) K^a_{[i} K^b_{j]} + \dots \right)$$

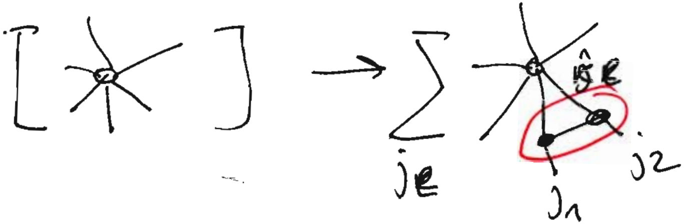
Regularization (Thiemann (1998))

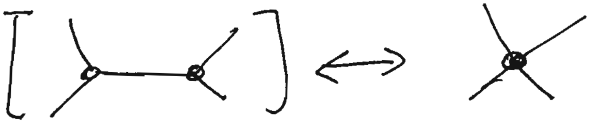
$$\epsilon^{abc} \epsilon_{ije} \frac{E^i_a E^j_b}{\sqrt{\det E}} = \frac{1}{4\gamma} \left[A^c_e(x), \int d^3x \sqrt{\det E} \right]_-$$

$$K^i_a \equiv \gamma^{-1}(A^i_a - \Gamma^i_a) \sim \left[A^i_a, \left[\int d^3x \, F^l_{ab} \frac{E^a_j E^b_k \epsilon^{ljk}}{\sqrt{\det E}}, \int d^3x \sqrt{\det E} \right]_- \right]_-$$

V

Action of $\hat{H}$ on spin-net states | $H = H_1 + H_K + H_M$

$\hat{H}_1$ 

$\hat{H}_K$ 

?

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Spliting and joining of space cells

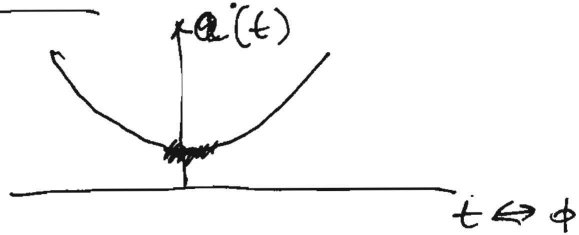
Spin Foam models of QG

Inclusion of matter fields [Scalars, fermions... on nodes.
Gauge fields are situated on links

Quantization of cross-sections (all) $\sim n \ell_p^2$

- Spin nets with boundary (external!)
- Boundary with fixed area $S = \sum \sigma_i$
  has $\neq 0$ entropy $\sim \gamma \cdot S / \ell_p^2$

Cosmology
Inflation



$\Psi(a, \phi)$