Recent progress in determination of fundamental constants (CODATA 2010-2014)

Savely G Karshenboim

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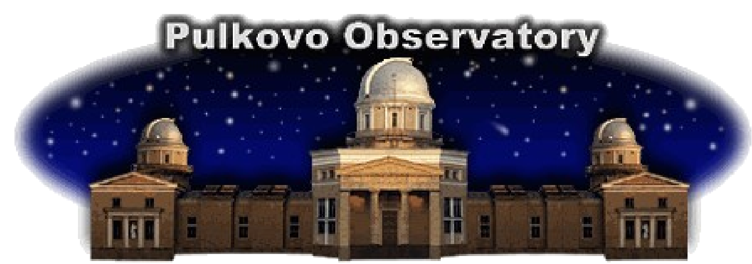
and Max-Planck-Institut für Quantenoptik (Garching)

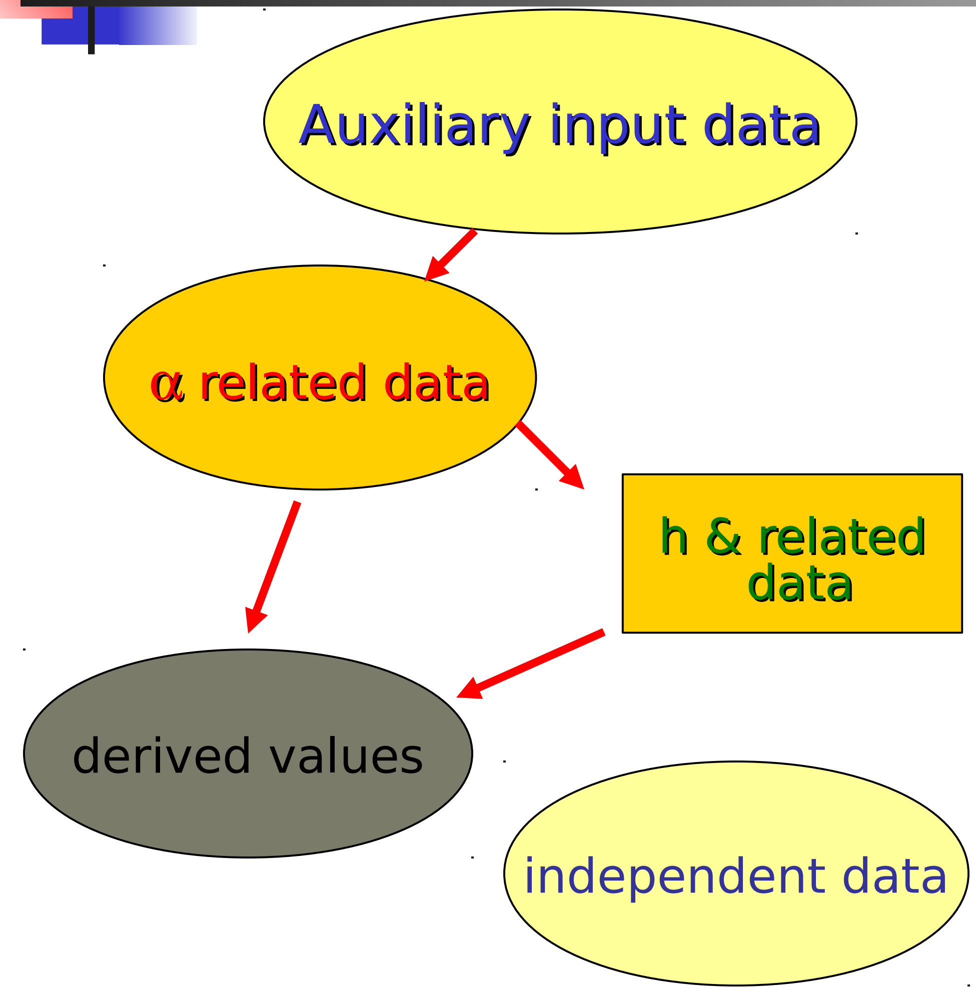
MAX-PLANCK-INSTITUTE OF QUANTUM OPTICS GARCHING
MPQ



Outline

- *structure of input and output*
- *auxiliary data*
  - Rydberg and R_p
  - m_e/m_p
- α
- *h*
- *mass of a particle*
- *independent constants*
  - *G*
  - *k*
  - $(g-2)_\mu$
- *progress: 2006 vs. 2010*
- *problems*

Structure of the input data and output values



- Auxiliary data = **exact** + the most accurate data which are to be evaluated <u>prior</u> the adjustment: R_∞, m_e/m_p, atomic masses.
- α related data: h/m, hN_A ...
- h related data: e, e/h, ...
- The lines (→) are equations: e.g., theoretical expressions for h/M, the Lamb shift, ...
- Some data are measured, a lot are derived: m_p [kg], m_e [Mev/c^2], ...
- G is uncorrelated,...

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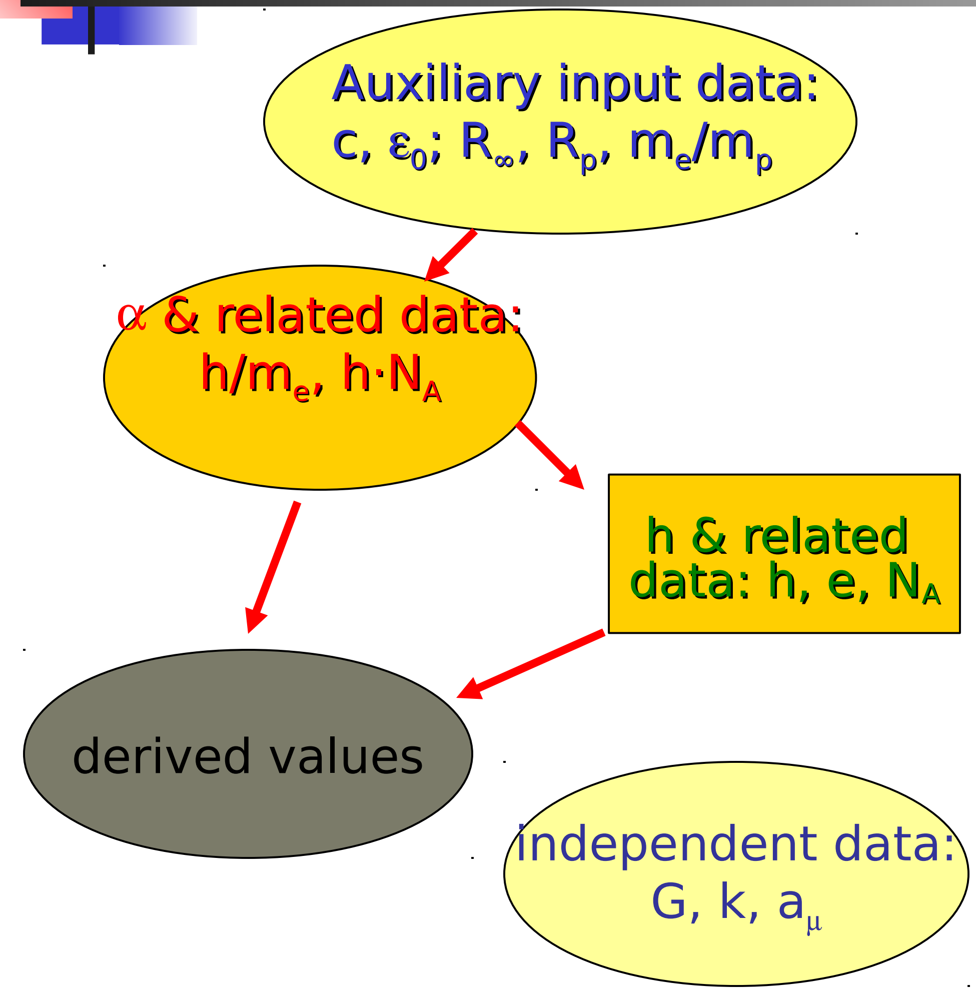

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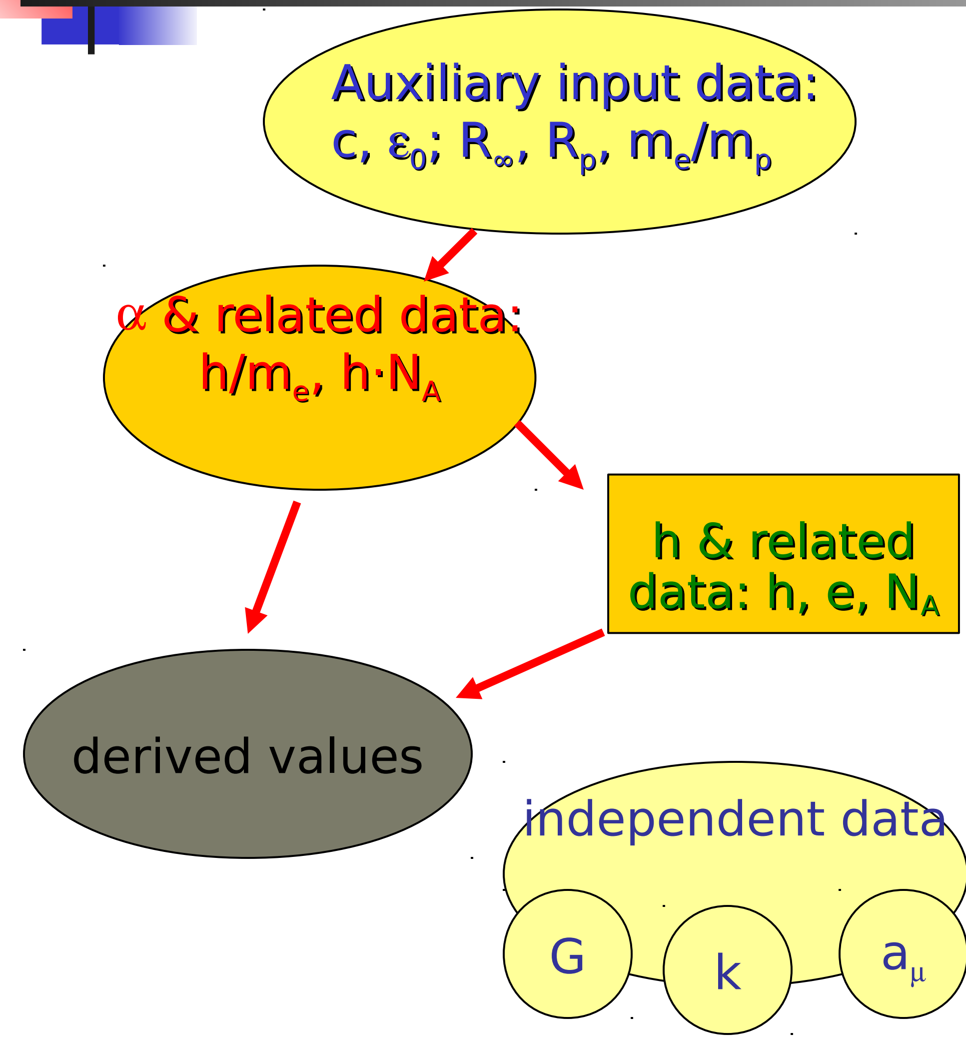

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Auxiliary data

- exact:
- the most accurate

Quantity	Symbol	Value	u_r
speed of light in vacuum	c	$299\,792\,458\ \mathrm{m\,s^{-1}}$	exact
magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}}$	exact
electric constant	$\epsilon_0 = 1/(c^2\mu_0)$	$8.854\,187\,817\ldots \times 10^{-12}\ \mathrm{F\,m^{-1}}$	exact
atomic mass of ^{12}C	$m(^{12}\mathrm{C})$	$12\ \mathrm{u}$	exact
Rydberg constant	R_∞	$10\,973\,731.568\,539(55)\ \mathrm{m^{-1}}$	$[5.0 \times 10^{-12}]$
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,45(75)$	$[4.1 \times 10^{-10}]$
electron mass	m_e	$5.485\,799\,0946(22) \times 10^{-4}\ \mathrm{u}$	$[4.0 \times 10^{-10}]$
proton rms charge radius	R_p	$0.8775(51) \times 10^{-15}\ \mathrm{m}$	$[5.9 \times 10^{-3}]$

Example: multiplicative vs. additive: R_∞ vs. α

- equations:

$$\alpha^2 = R_\infty \frac{h}{m_e c}$$

$$c_1 R_\infty c + c_2 \alpha^2 R_\infty c = \nu$$

- uncertainty:
  - $R_\infty \sim 10^{-11}$
  - $\alpha \sim 10^{-9} – 10^{-10}$
  - $\alpha^2 \sim 10^{-4} \times 10^{-9}$

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`almost´

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Atomic & nuclear masses

Quantity	Symbol	Value	u_r
atomic mass of ^{16}O	$m(^{16}\mathrm{O})$	15.994 914 619 57(18) u	$[1.1 \times 10^{-11}]$
atomic mass of ^{28}Si	$m(^{28}\mathrm{Si})$	27.976 926 534 96(62) u	$[2.2 \times 10^{-11}]$
atomic mass of ^{87}Rb	$m(^{87}\mathrm{Rb})$	86.909 180 535(10) u	$[1.2 \times 10^{-10}]$

Quantity	Symbol	Value	u_r
proton mass	m_p	1.007 276 466 812(90) u	$[8.9 \times 10^{-11}]$
deuteron mass	m_d	2.013 553 212 712(77) u	$[3.8 \times 10^{-11}]$
triton mass	m_t	3.015 500 7134(25) u	$[8.2 \times 10^{-10}]$
helion mass	m_h	3.014 932 2468(25) u	$[8.3 \times 10^{-10}]$
alpha particle mass	m_α	4.001 506 179 125(62) u	$[1.5 \times 10^{-11}]$

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

- LKP (Paris), MPQ (Garching),...

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen
- MAMI = Mainzer Mikrotron
- old world data

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen
- CREMA collaboration @ PSI

Spectroscopy of hydrogen (and deuterium)

- Two-photon spectroscopy involves a number of levels strongly affected by QED.
- In “old good time” we had to deal only with 2s Lamb shift.
- Theory for p states is simple since their wave functions v
- *Now we have*

- *The idea is based on theoretical study of*

$$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

which we understand much better since any short distance effect vanishes for $\Delta(2)$.

- Theory of p and d states is

R_∞.

The Lamb shift in the hydrogen atom

S. G. Karshenboĭm

D.I. Mendeleyev Russian Metrology Research Institute, 198005 St. Petersburg, Russia
(Submitted 6 April 1994)
Zh. Eksp. Teor. Fiz. **106**, 414–424 (August 1994)

A theoretical expression is derived for the difference $\Delta E_L(1s_{1/2}) - 8\Delta E_L(2s_{1/2})$ in Lamb shifts

Z. Phys. D 39, 109–113 (1997)

FÜR PHYSIK D
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The Lamb shift of excited *S*-levels in hydrogen and deuterium atoms

Savely G. Karshenboim*

The Lamb shift in muonic hydrogen: experiment

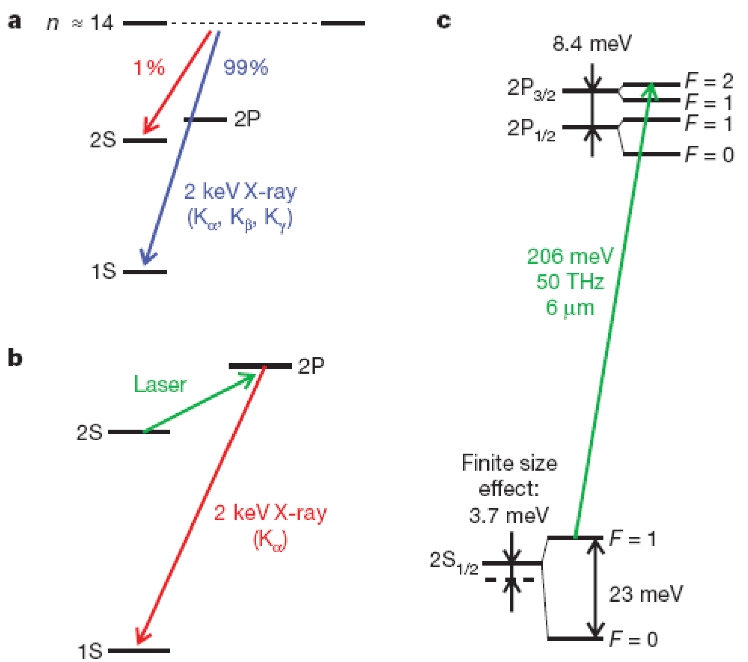


Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. a, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6\ \mu m$.

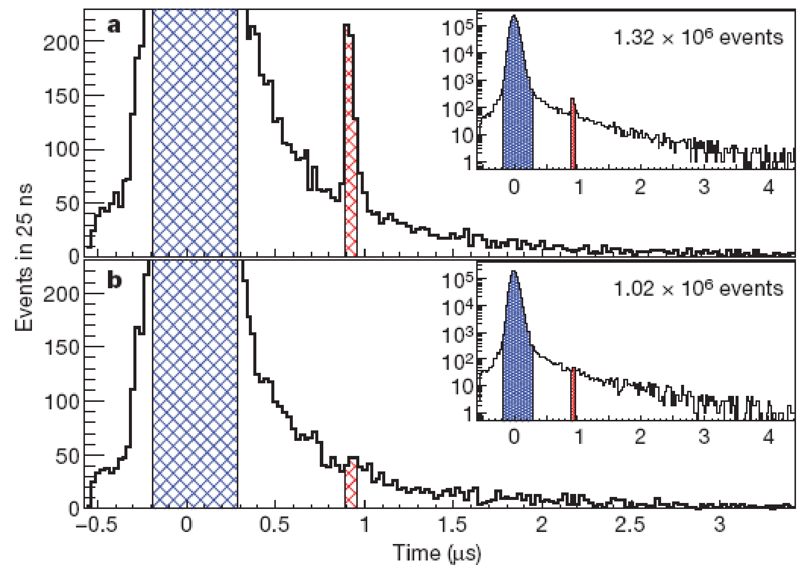


Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

The Lamb shift in muonic hydrogen: experiment

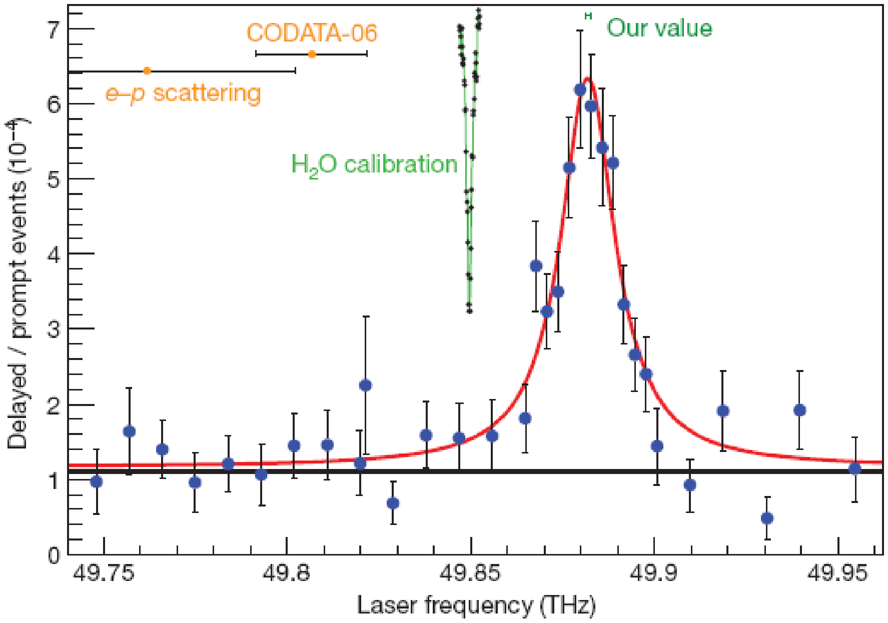
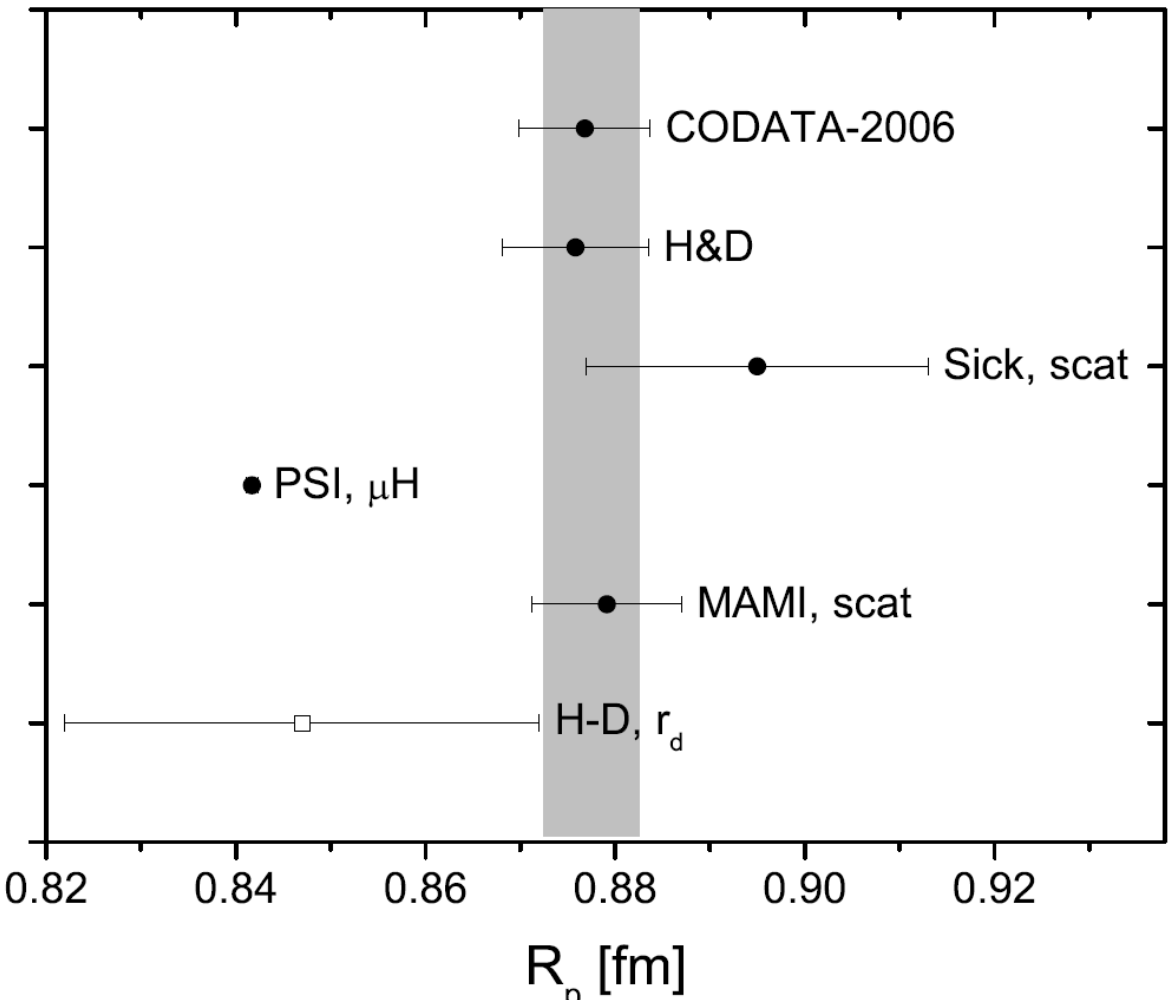


Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[3] or electron scattering[1,2] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the ±1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

Proton radius puzzle



4e) Proton Radius from e – p scattering (all numbers in fm): TGFC meeting

Arrington and Sick

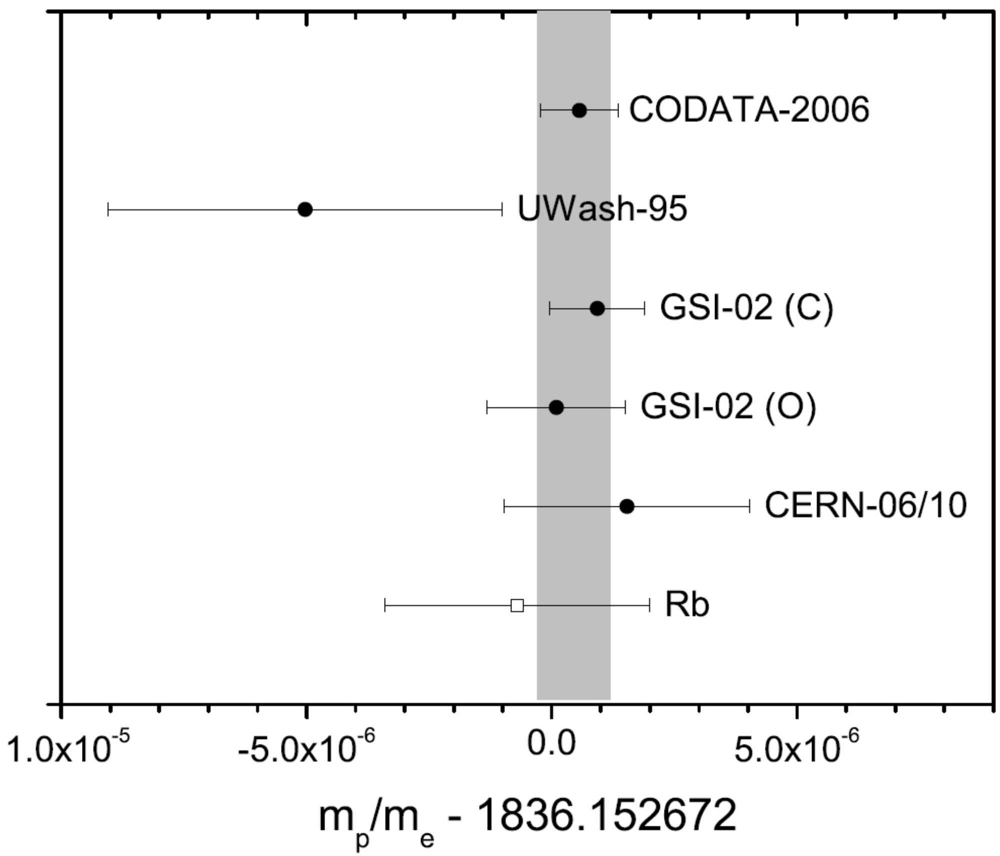
- World w/o Mainz: 0.887(8)
- Jlab: 0.875(10)
  - Simple average: 0.881(11)
- Mod. Mainz: 0.875(15)
  - **Weighted mean: 0.879(11)**

Bernauer and Distler

- Mainz spline: 0.8750(67)
- Mainz poly: 0.8830(77)
- World w/o Mainz: 0.887(8)
- Jlab: 0.875(10)
  - **Weighted mean: 0.880(11)**

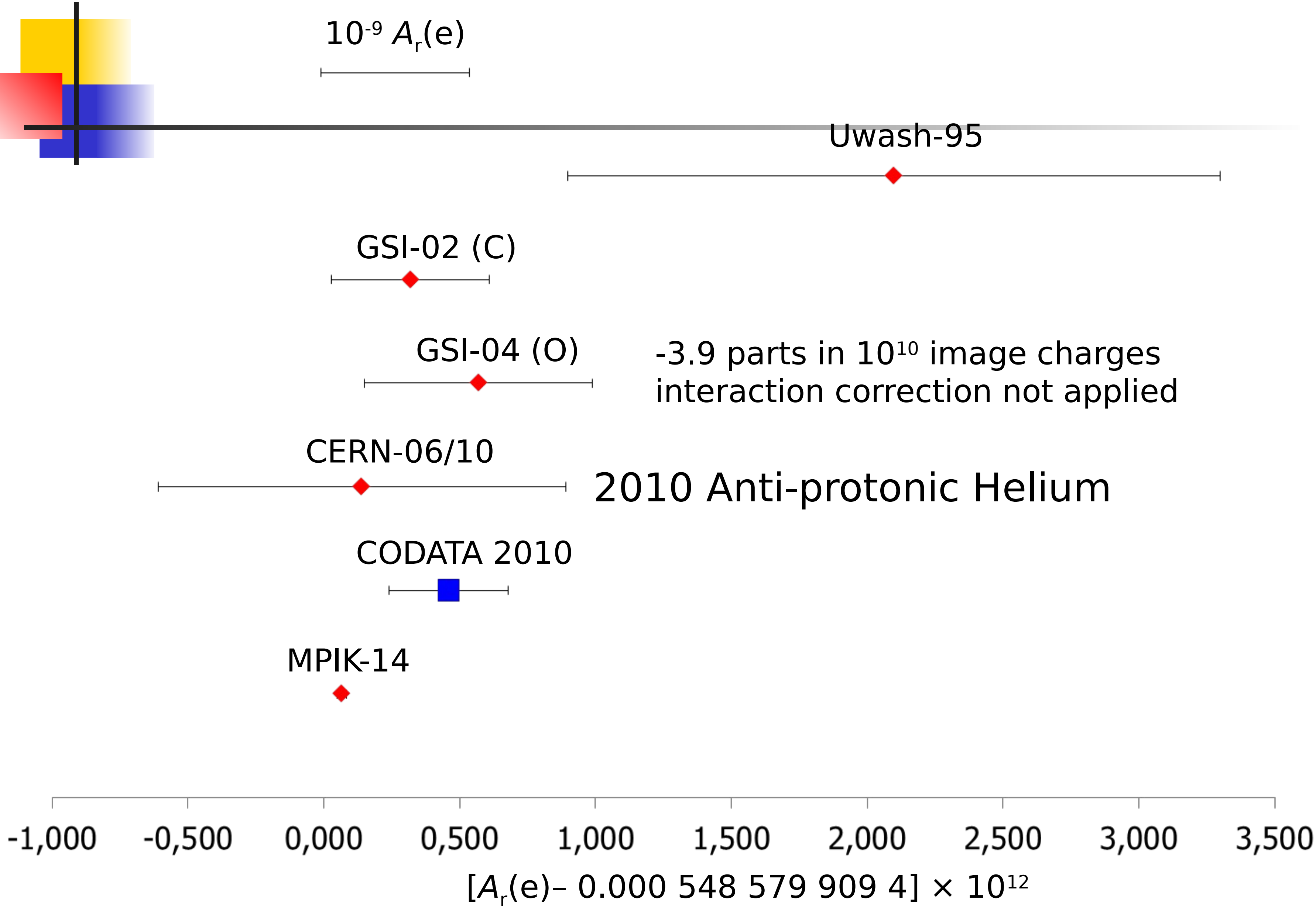
Average value and uncertainty: 0.879(11)

electron-to-proton mass ratio



- cyclotron frequencies of e & p (UWash)
- g factor of a bound e in H-like ion (magnetic moment precession vs. ion cyclotron frequency) @ Mainz
- antiprotonic He spectroscopy (ASACUSA @ CERN)

Electron mass: TGFC meeting



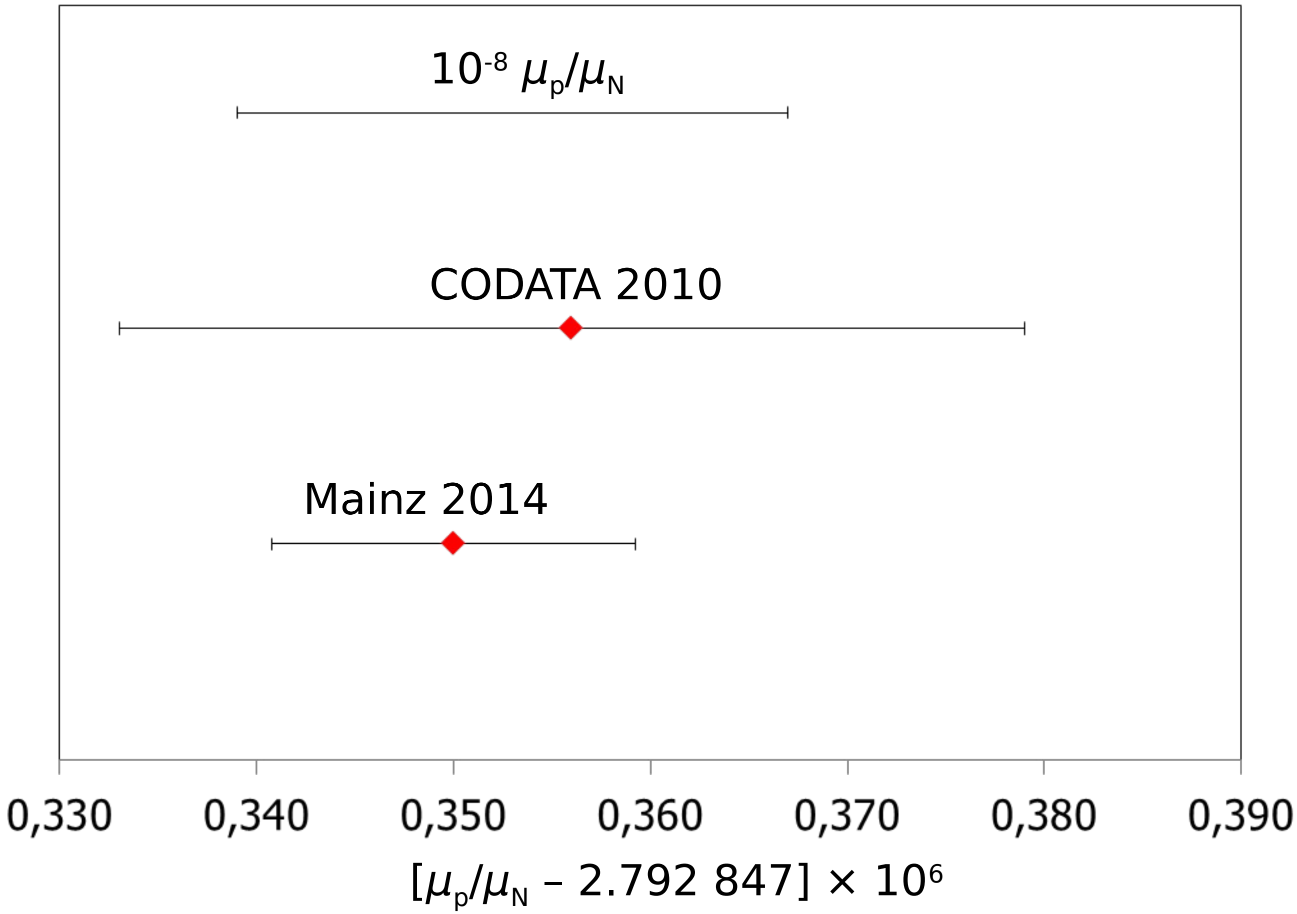
LETTER

doi:10.1038/nature13388

Direct high-precision measurement of the magnetic moment of the proton

A. Mooser[1,2]†, S. Ulmer[3], K. Blaum[4], K. Franke[3,4], H. Kracke[1,2], C. Leiteritz[1], W. Quint[5,6], C. C. Rodegheri[1,4], C. Smorra[3] & J. Walz[1,2]

Factor 2.5 more accurate



α block

- equations:

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- input data
  - α
  - h/m_e

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- equations:

$$\alpha^2 m_e c$$

$$m(^{12}\mathrm{C})/12 \cdot N_A = 1\ \mathrm{g\,mol}^{-1}$$

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- m_p in u
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- input data
  - α
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  - h/m_p
  - h/m_{at}
- output
  - $h \cdot N_A$

$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times \left(m(^{12}\mathrm{C})/12 \cdot N_A\right)$$

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- input data
  - α
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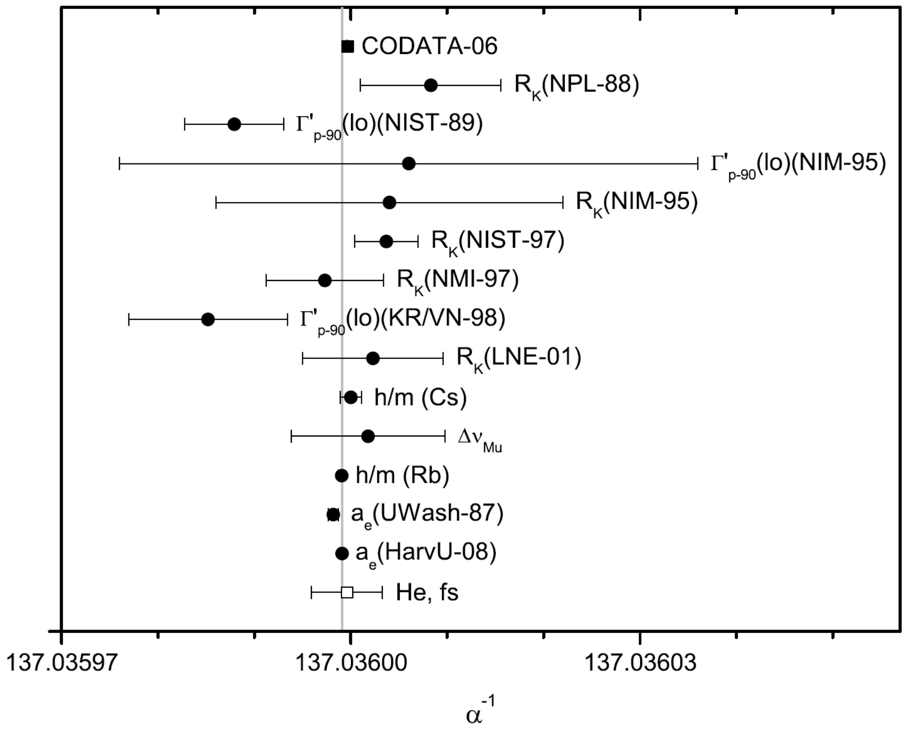
  - $h \cdot N_A$

$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times (m(^{12}\mathrm{C})/12 \cdot N_A)$$

α block

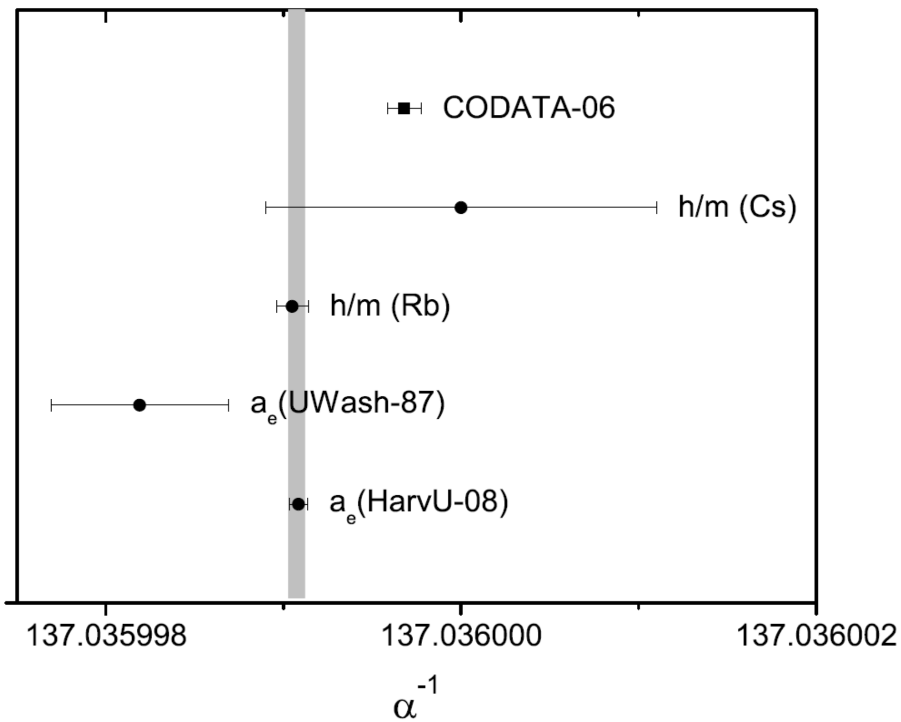
Quantity	Symbol	Value	u_r
inverse fine structure constant	α^{-1}	137.035 999 074(44)	$[3.2 \times 10^{-10}]$
molar Planck constant	$h \cdot N_A$	$3.990\,312\,7176(28) \times 10^{-10}$ J s mol^{-1}	$[7.0 \times 10^{-10}]$
quantum of circulation	$h/(2m_e)$	$3.636\,947\,5520(24) \times 10^{-4}$ m^2 s^{-1}	$[6.5 \times 10^{-10}]$
Compton wavelength	$\lambda_C = h/(m_e c)$	$2.426\,310\,2389(16) \times 10^{-12}$ m	$[6.5 \times 10^{-10}]$
von Klitzing constant	$R_K = h/e^2$	25 812.807 4434(84) Ω	$[3.2 \times 10^{-10}]$
muon-electron mass ratio	m_μ/m_e	206.768 2843(52)	$[2.5 \times 10^{-8}]$

α block



- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quatum Hall standard vs calculable capacitor: R_K

α block



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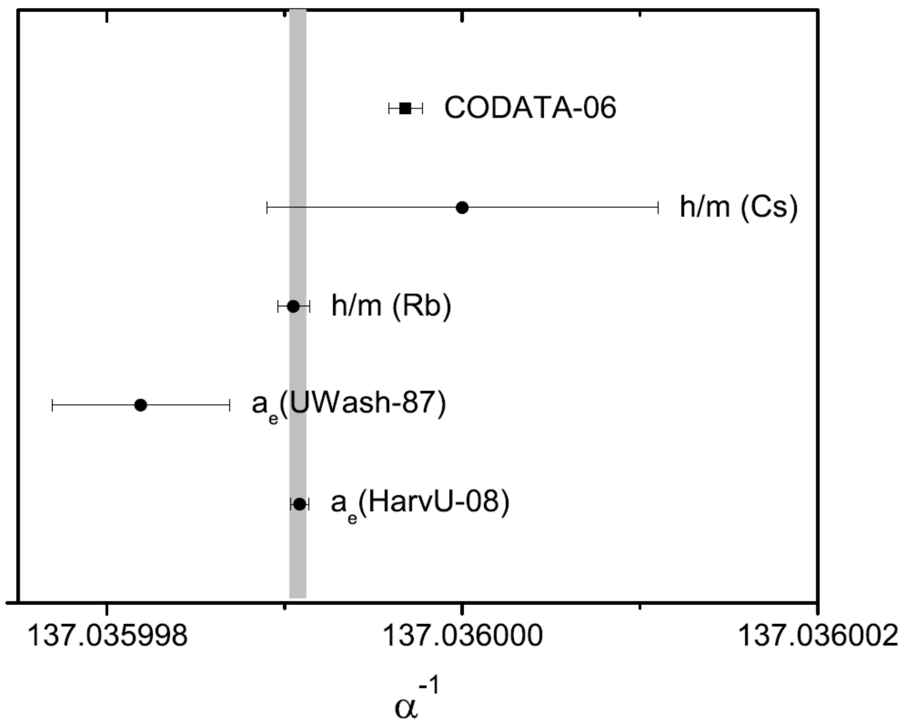
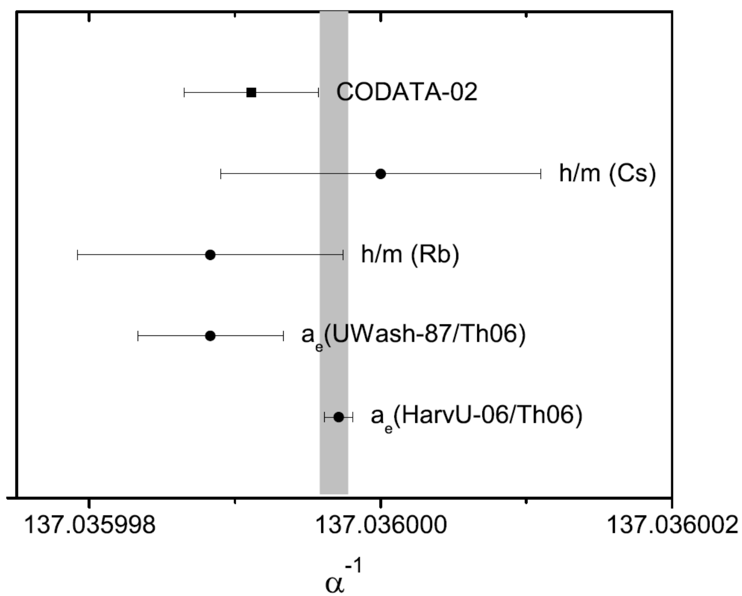
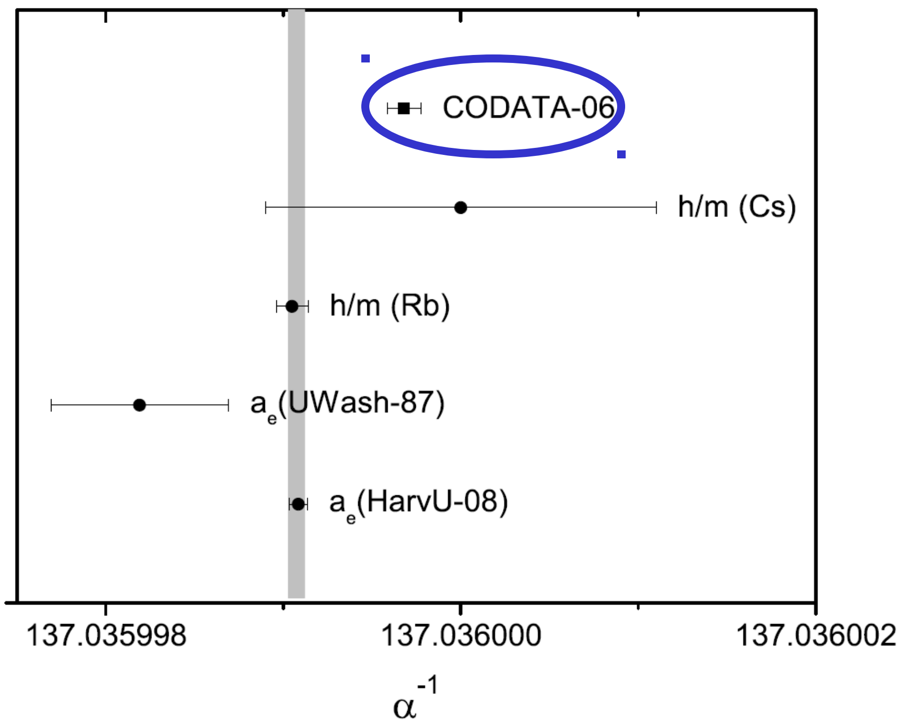

CODATA-06
h/m (Cs)
h/m (Rb)
a_e(UWash-87)
a_e(HarvU-08)
137.035998
137.036000
137.036002
α^-1

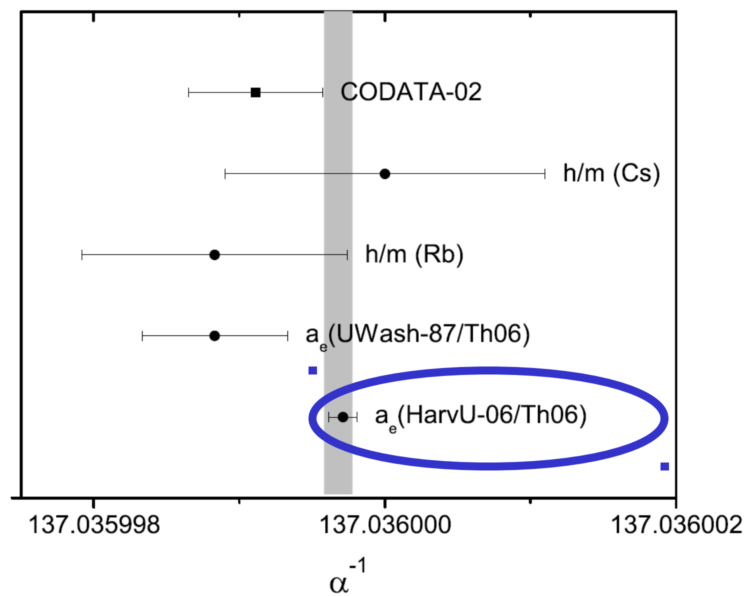
- 2006:


CODATA-02
h/m (Cs)
h/m (Rb)
a_e(UWash-87/Th06)
a_e(HarvU-06/Th06)
137.035998
137.036000
137.036002
α^-1

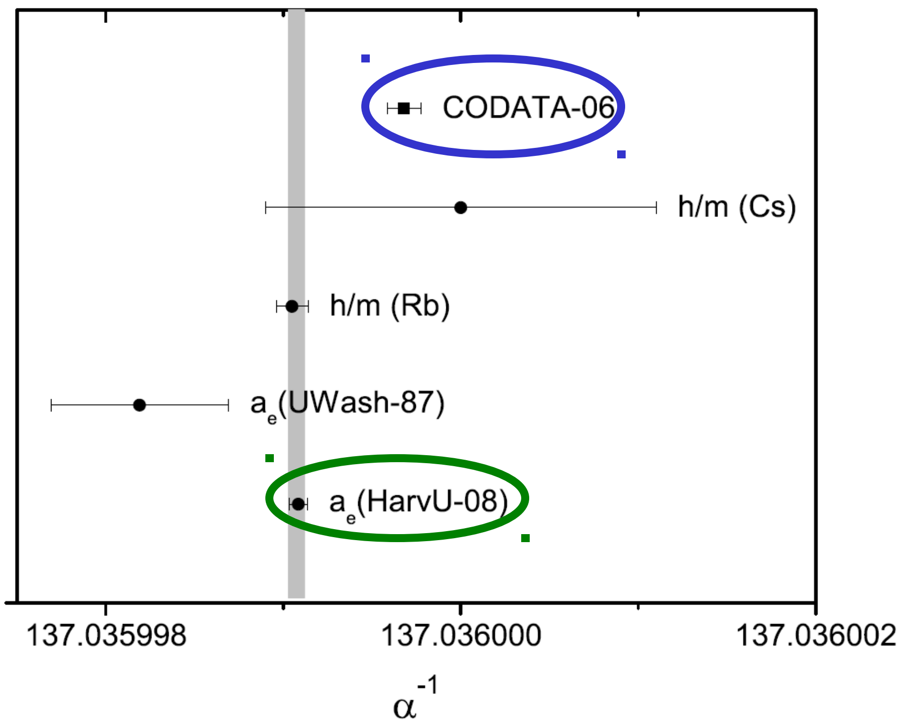
α block



- 2006:
  - old CODATA – old a_e

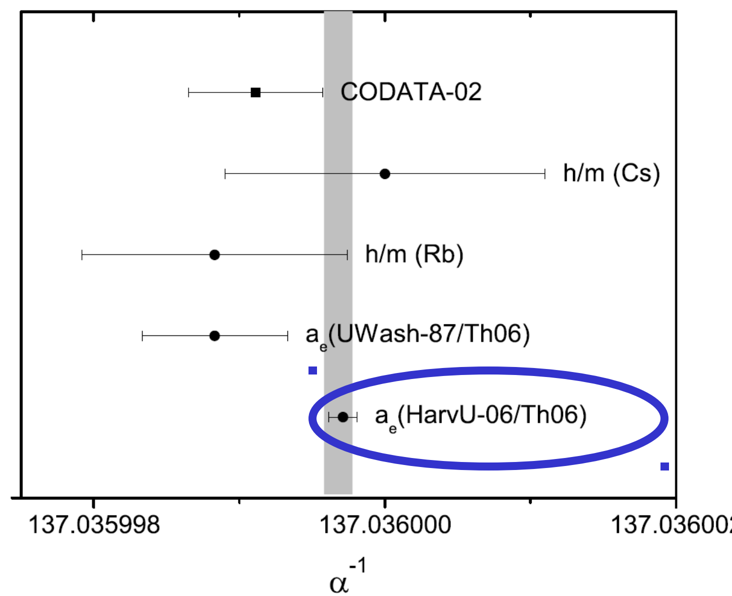


α block



- a_e theory jump

- 2006:



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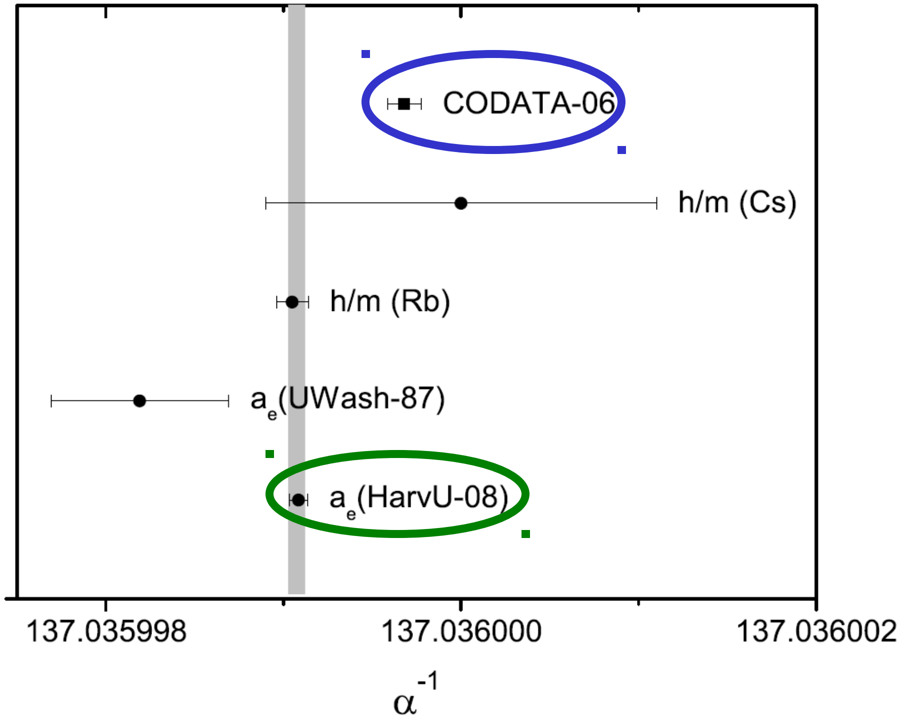

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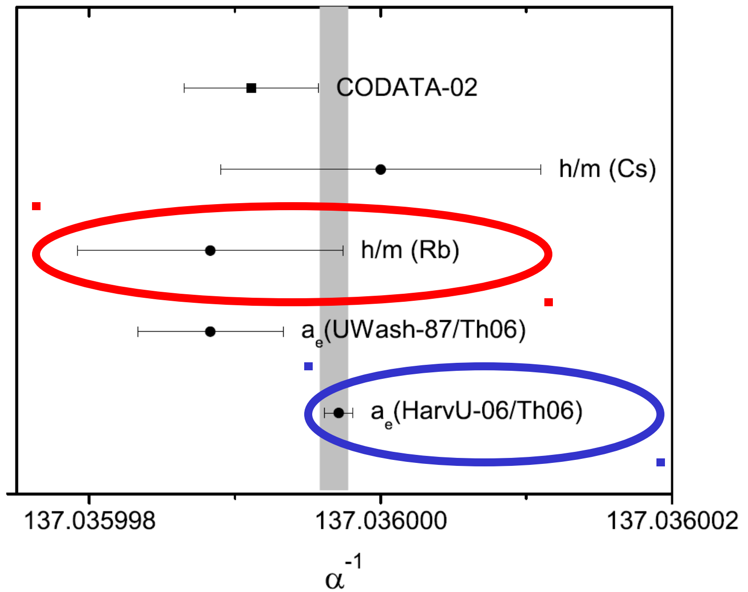
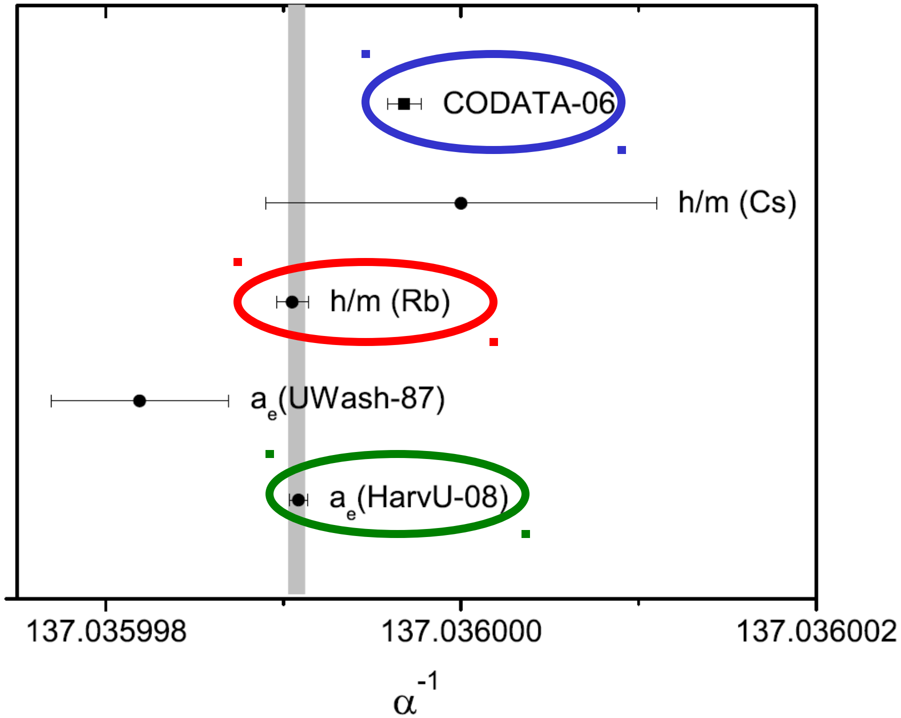


- not sensitive

α block



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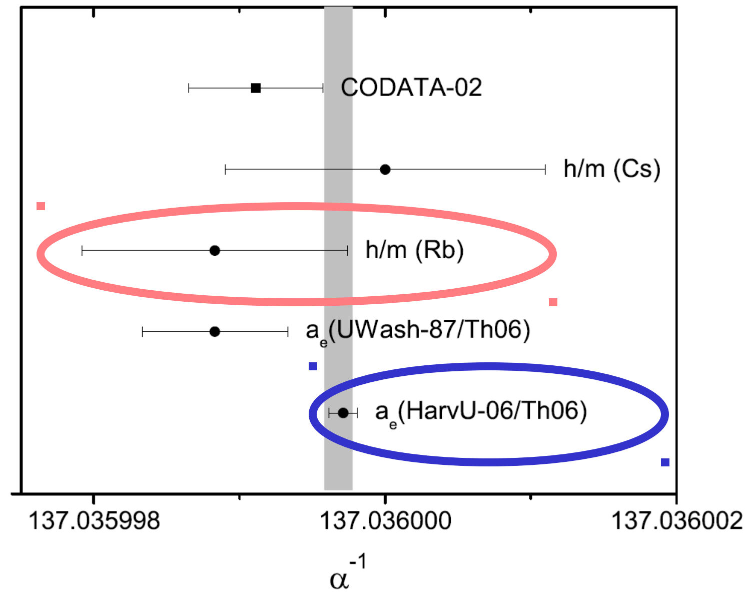
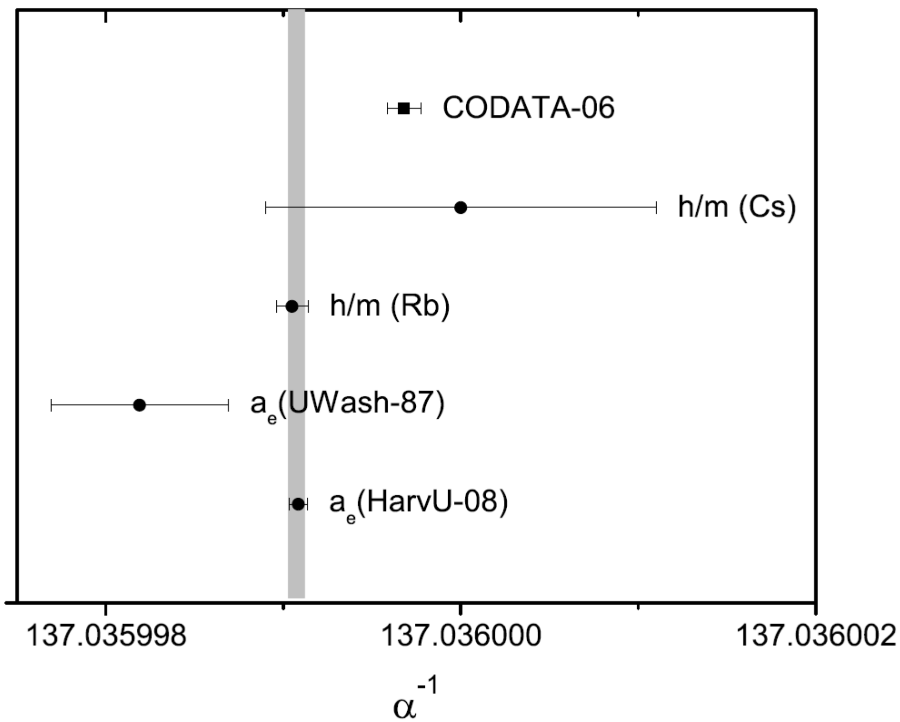


- sensitive

α block

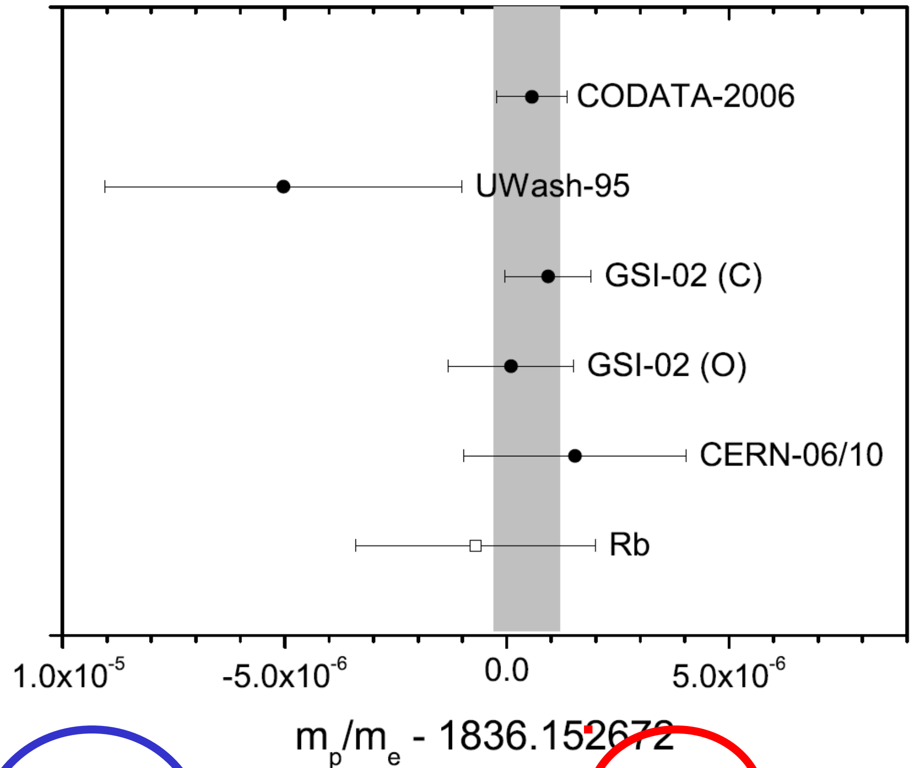


- QED vs Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- 5-loop corrections to $(g-2)_e$

Tenth-Order QED Contribution to the Electron $g-2$ and an Improved Value of the Fine Structure Constant

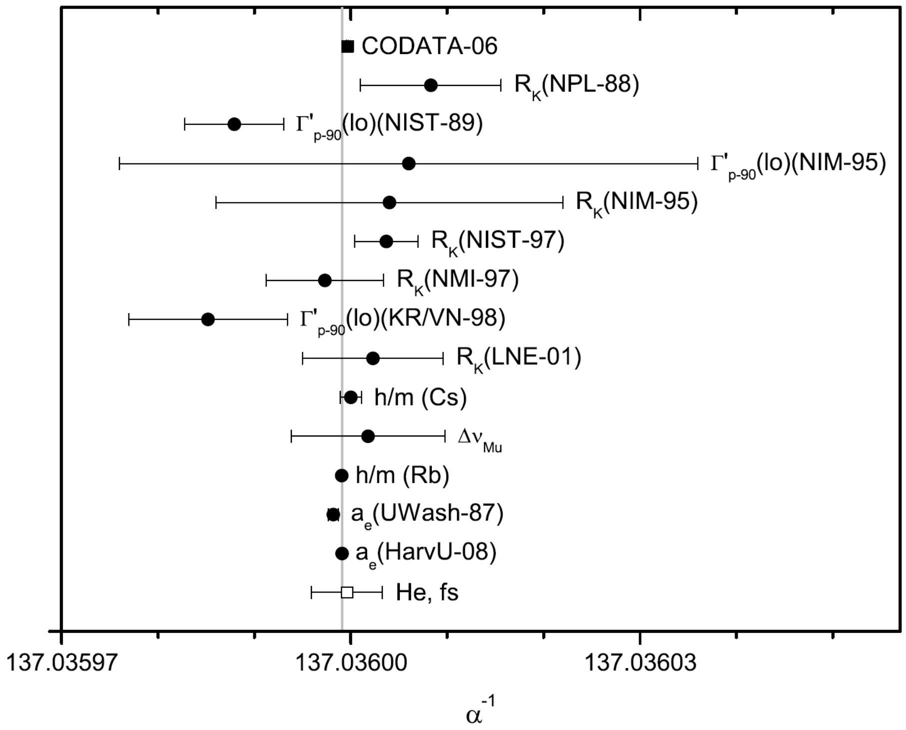
Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

m_e/m_p vs α: accuracy is close!



$$\frac{m_p}{m_e}[\mathrm{Rb}] = \frac{m_p[\mathrm{u}]}{m_{\mathrm{Rb}}[\mathrm{u}]}\,\frac{1}{h/m_{\mathrm{Rb}}}\,\frac{\alpha^2[a_e]\,c^2}{2R_\infty} = 1836.152\,6713(27)\,, \quad [u_r = 1.5\times 10^{-9}]$$

α block

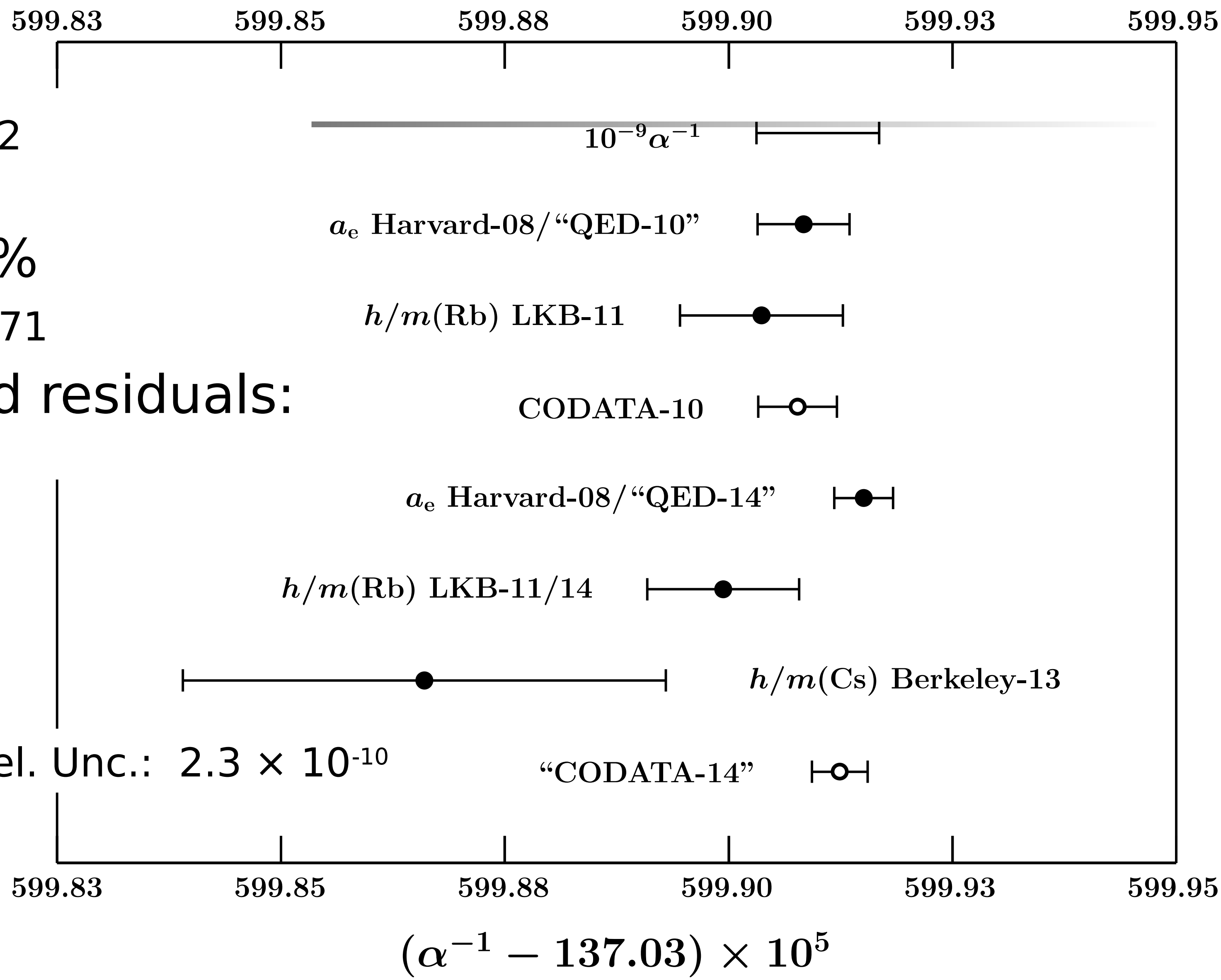


- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quatum Hall standard vs calculable capacitor: R_K

2014 Input data related to the Fine-structure constant: TGFC meeting

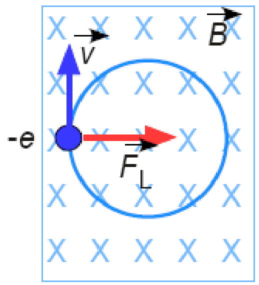
χ^2: 5.82
DOF: 2
Prob. χ^2: 5.8%
R_B: 1.71
Max. reduced residuals:
1.51, 1.70

'CODATA-14' Rel. Unc.: 2.3×10^{-10}



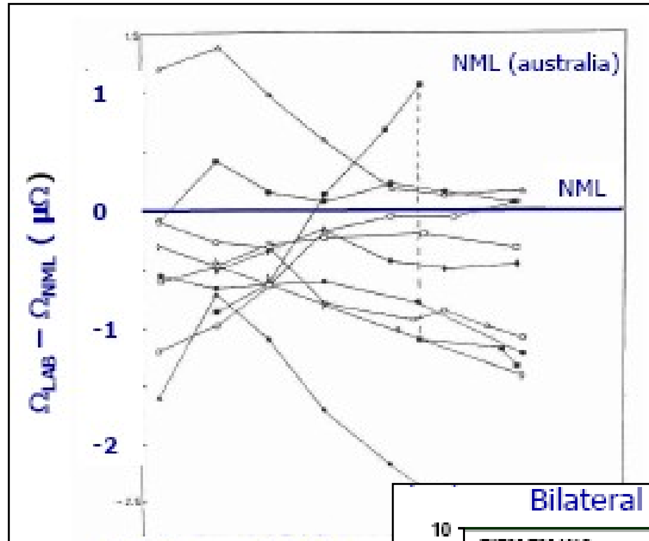
Quantum Hall effect and a standard of resistance

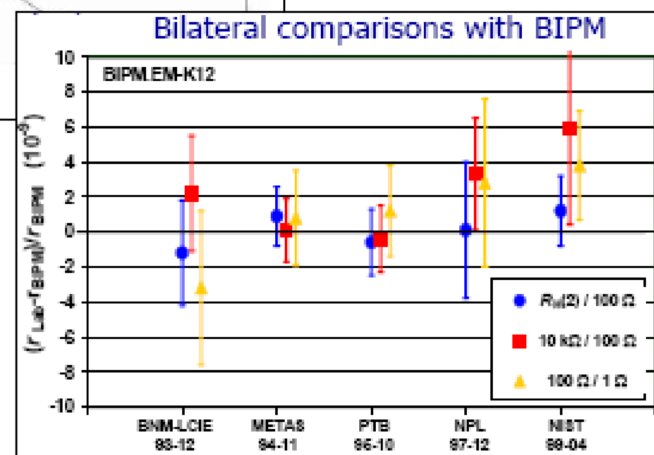
Classical Hall effect (1879)

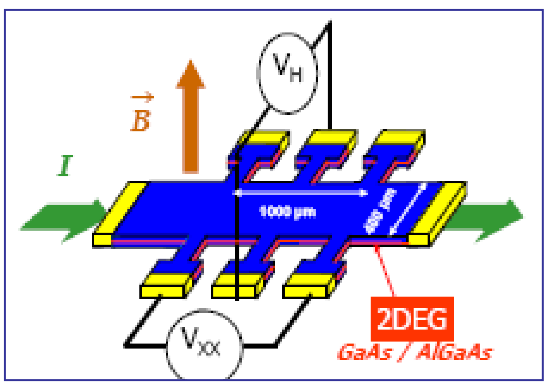


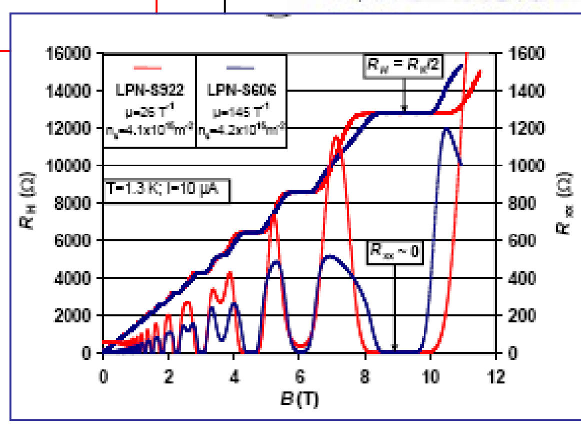
Lorentz Force

$$\vec{F}_L = q(\vec{v} \times \vec{B})$$



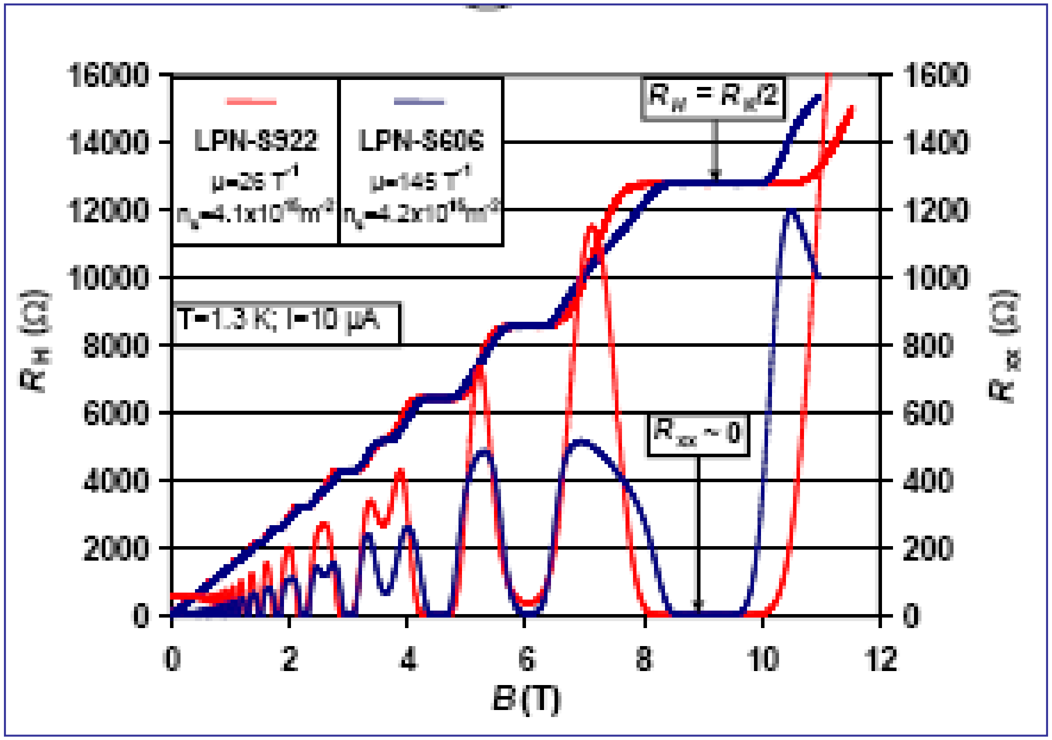






W. Poirier, Les Houches, 2007

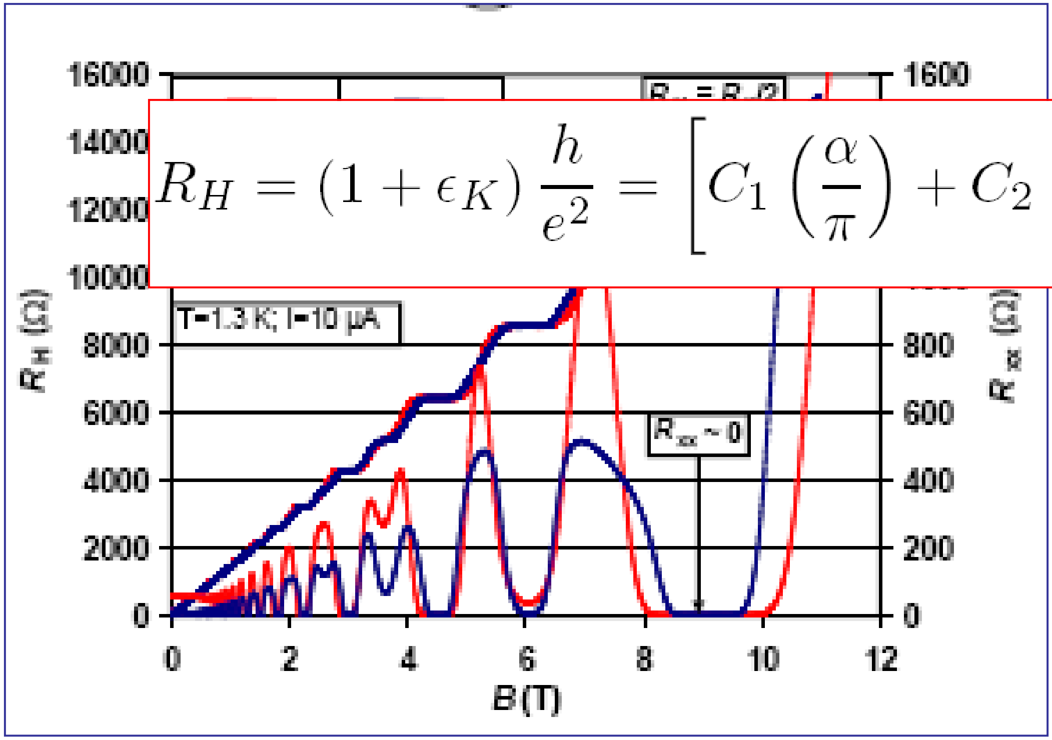
Needs for a `theory´ for QHE



- steps R_n
- rationa$R_n = R_1/n$
- universal$R_1 = R_H$
- relation to α

$$R_H = R_K \equiv h/e^2$$

Needs for a `theory´ for QHE



- steps R_n

$$R_H = (1 + \epsilon_K) \frac{h}{e^2} = \left[C_1 \left(\frac{\alpha}{\pi} \right) + C_2 \left(\frac{\alpha}{\pi} \right)^2 + C_3 \left(\frac{\alpha}{\pi} \right)^3 + \dots \right] \frac{h}{e^2}$$

$1/n$

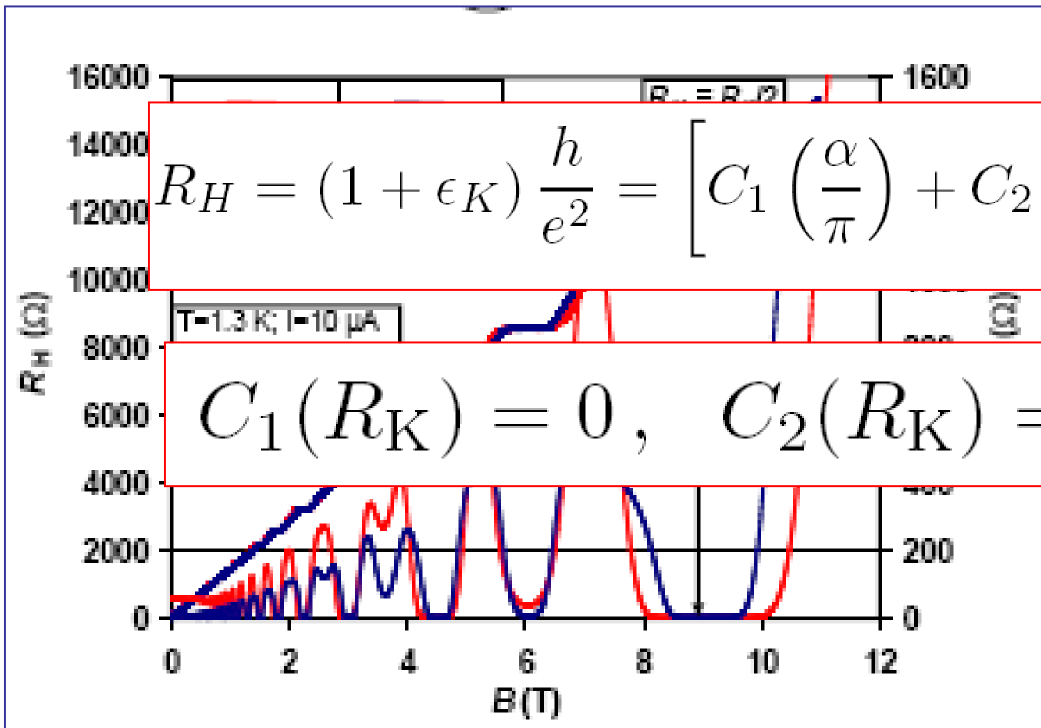
- universal $R_1 = R_H$

- relation to α

$$R_H = R_K \equiv h/e^2$$

Needs for a `theory´ for QHE

- steps R_n



$$R_H = (1+\epsilon_K)\,\frac{h}{e^2} = \left[C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + \dots\right]\frac{h}{e^2}$$

$1/n$

$$C_1(R_{\mathrm{K}}) = 0\,, \quad C_2(R_{\mathrm{K}}) = 0\,, \quad C_3(R_{\mathrm{K}}) \simeq 2(2)$$

R_H

- relation to α

$$R_H = R_K \equiv h/e^2$$

h block

known from α block

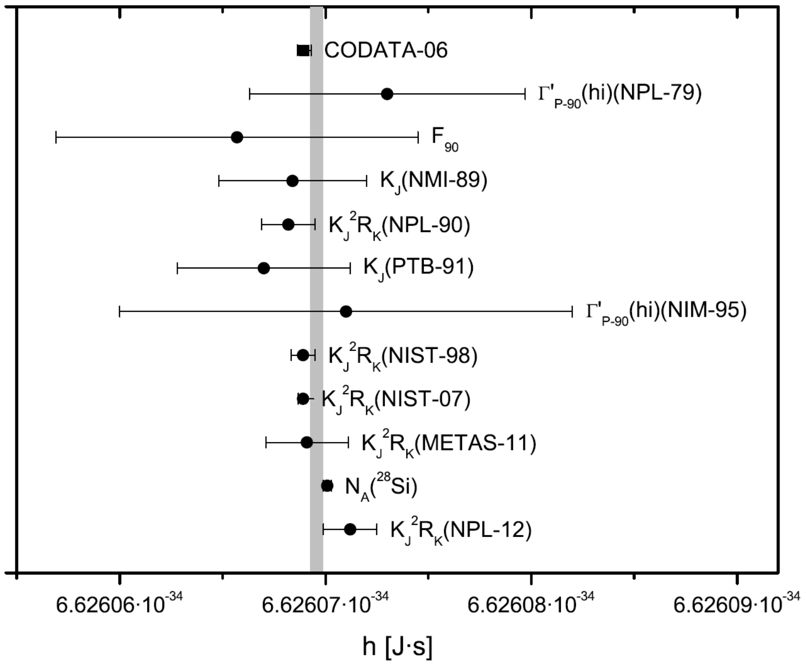
- $$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$
- $h \cdot N_A$
- h/m_e

- input:
  - h
  - e
  - N_A
- output
  - m_e
  - μ_B

h block

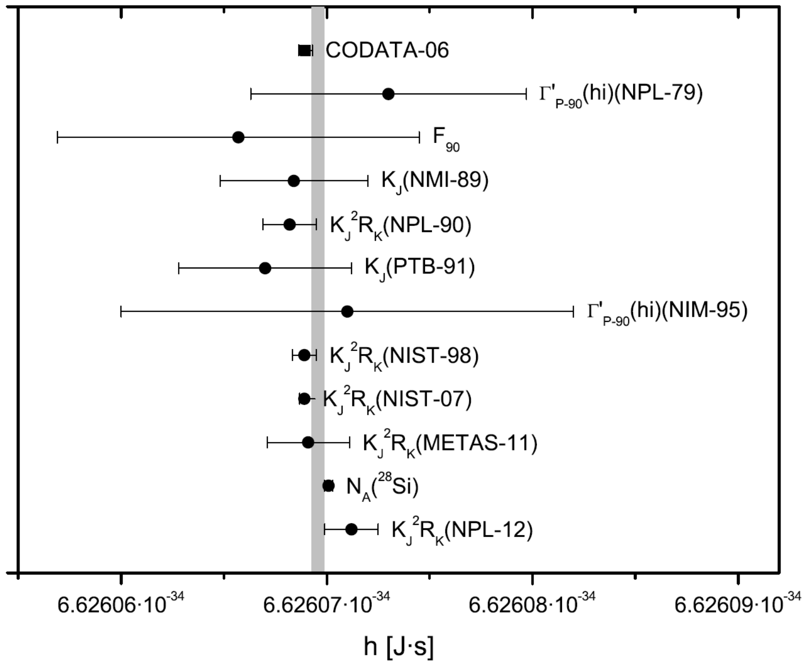
Quantity	Symbol	Value	u_r
Planck constant	h	$6.626\,069\,57(29) \times 10^{-34}$ J s	$[4.4 \times 10^{-8}]$
elementary charge	e	$1.602\,176\,565(35) \times 10^{-19}$ C	$[2.2 \times 10^{-8}]$
Avogadro constant	N_A	$6.022\,141\,29(27) \times 10^{23}$ mol^{-1}	$[4.4 \times 10^{-8}]$
Faraday constant	$F = e \cdot N_A$	$96\,485.3365(21)$ C mol^{-1}	$[2.2 \times 10^{-8}]$
electron charge to mass quotient	e/m_e	$1.758\,820\,088(39) \times 10^{11}$ C kg^{-1}	$[2.2 \times 10^{-8}]$
electron gyromagnetic ratio	$\gamma_e = 2\mu_e/\hbar$	$1.760\,859\,708(39) \times 10^{11}$ s^{-1} T^{-1}	$[2.2 \times 10^{-8}]$
electron mass	m_e	$9.109\,382\,91(40) \times 10^{-31}$ kg	$[4.4 \times 10^{-8}]$
		$0.510\,998\,928(11)$ MeV$/c^2$	$[2.2 \times 10^{-8}]$
proton mass	m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$
		$938.272\,046(21)$ MeV$/c^2$	$[2.2 \times 10^{-8}]$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$927.400\,968(20) \times 10^{-26}$ J T^{-1}	$[2.2 \times 10^{-8}]$
nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,53(11) \times 10^{-27}$ J T^{-1}	$[2.2 \times 10^{-8}]$
Josephson constant	$K_J = 2e/h$	$483\,597.870(11) \times 10^{9}$ Hz V^{-1}	$[2.2 \times 10^{-8}]$

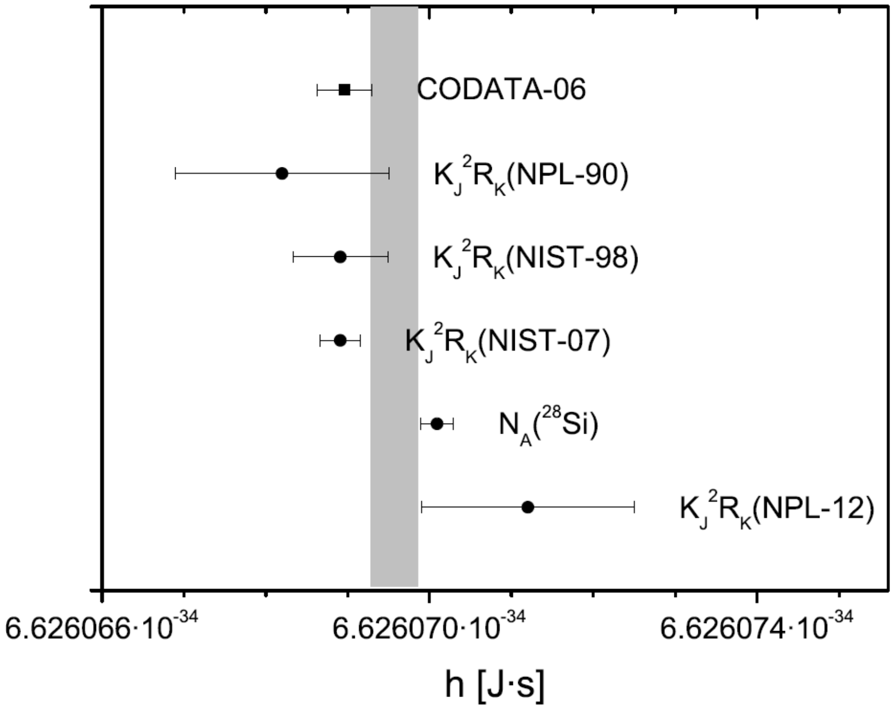
h block



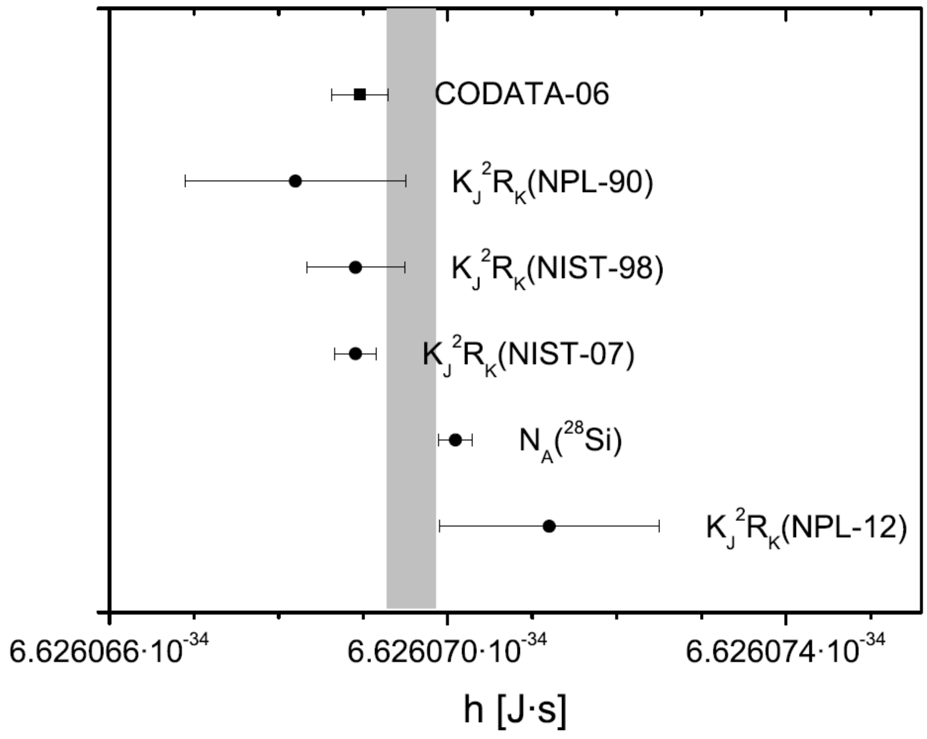
CODATA-06
Γ'P-90(hi)(NPL-79)
F90
KJ(NMI-89)
KJ2RK(NPL-90)
KJ(PTB-91)
Γ'P-90(hi)(NIM-95)
KJ2RK(NIST-98)
KJ2RK(NIST-07)
KJ2RK(METAS-11)
NA(28Si)
KJ2RK(NPL-12)
6.62606·10-34
6.62607·10-34
6.62608·10-34
6.62609·10-34
h [J·s]

h block: the most important data





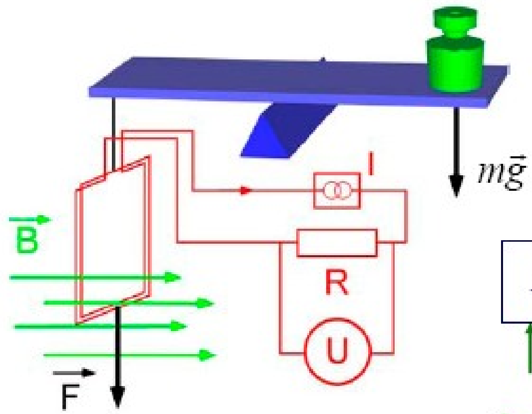
h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si

watt-ballance

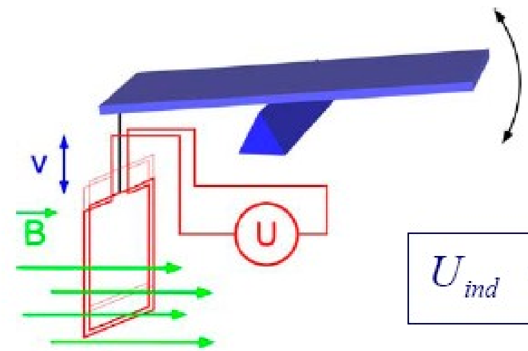
WB Principle (1): static phase / weighing mode



$$I \cdot \int d\ell \times B = mg$$

$$I = \frac{U_j}{R}$$ (QHE & JVS)

WB Principle (2): dynamic phase / velocity mode



$$U_{ind} = v \cdot \int d\ell \times B$$

WB Principle (3): combination of modes

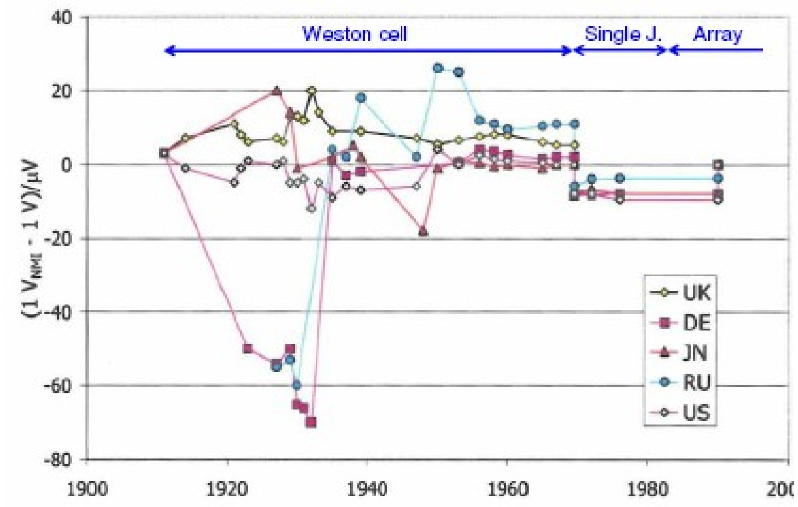
Only if $G_m = G_e$

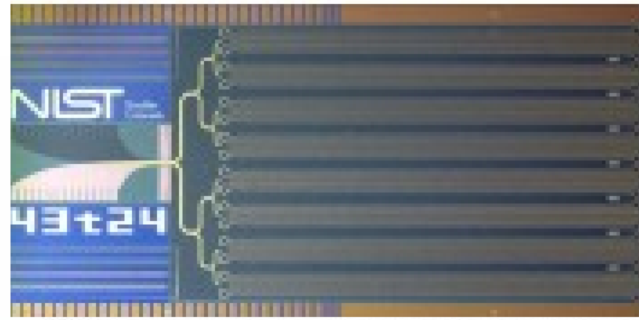
$$G(B,\ell) = \underbrace{\frac{mg}{I}}_{\text{static}} = \underbrace{\frac{U}{v}}_{\text{dynamic}} \Rightarrow \boxed{UI = m\,g\,v}$$

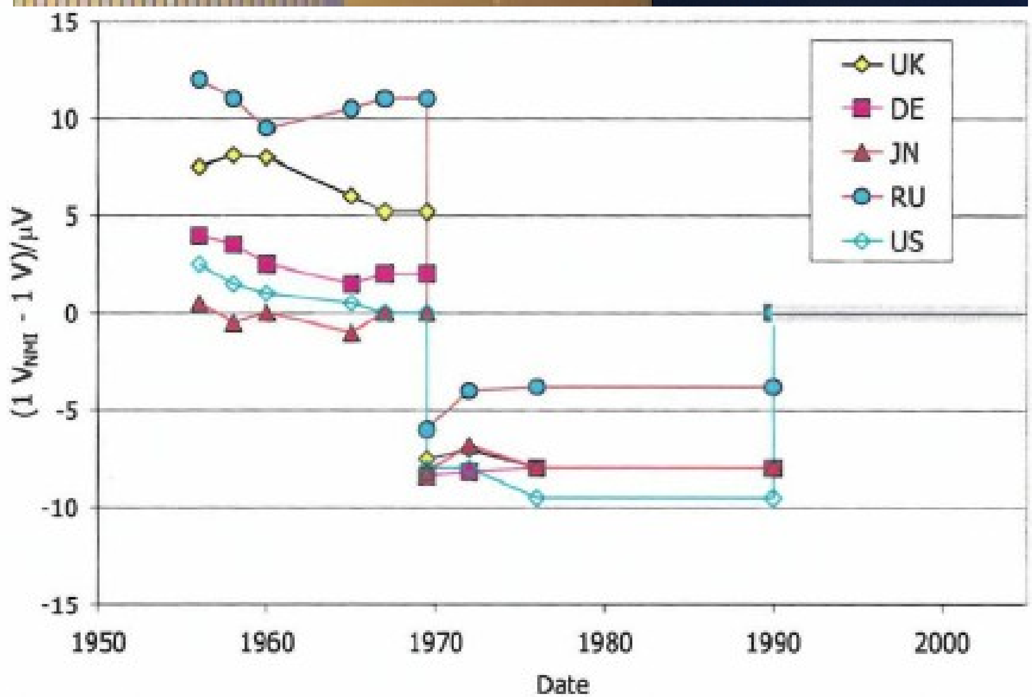
electrical power — mechanical power

B. Jeanneret, Les Houches, 200

Josephson effect and quantum volt stardar







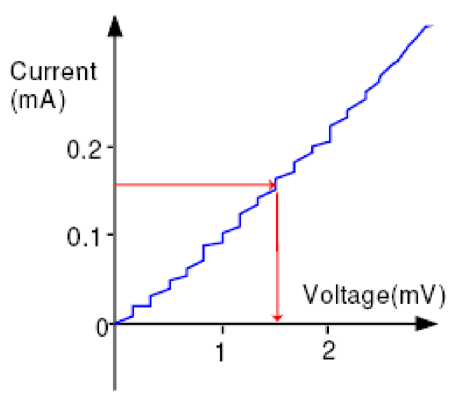
Irradiation with microwave:

- Cooper pairs synchronize with radiation
- Voltage steps appear

$$V_n = n\frac{h}{2e}f$$

Shapiro step, 1963

V_1 ~ 145 µV @ 70 GHz



B. Jeanneret, Les Houches, 200

watt-ballance

WB Principle (1): static phase / weighing mode

$$K_J = \frac{2e}{h}$$

$$R_K = \frac{h}{e^2}$$

phase / velocity mode

WB Pri ination of

$$_{ind} = v \cdot \int d\ell \times B$$

Only if $G_m = G_e$

mg U

$$\Rightarrow UI = mgv$$

$$\frac{1}{R_K K_J^2} = \frac{h}{4}$$

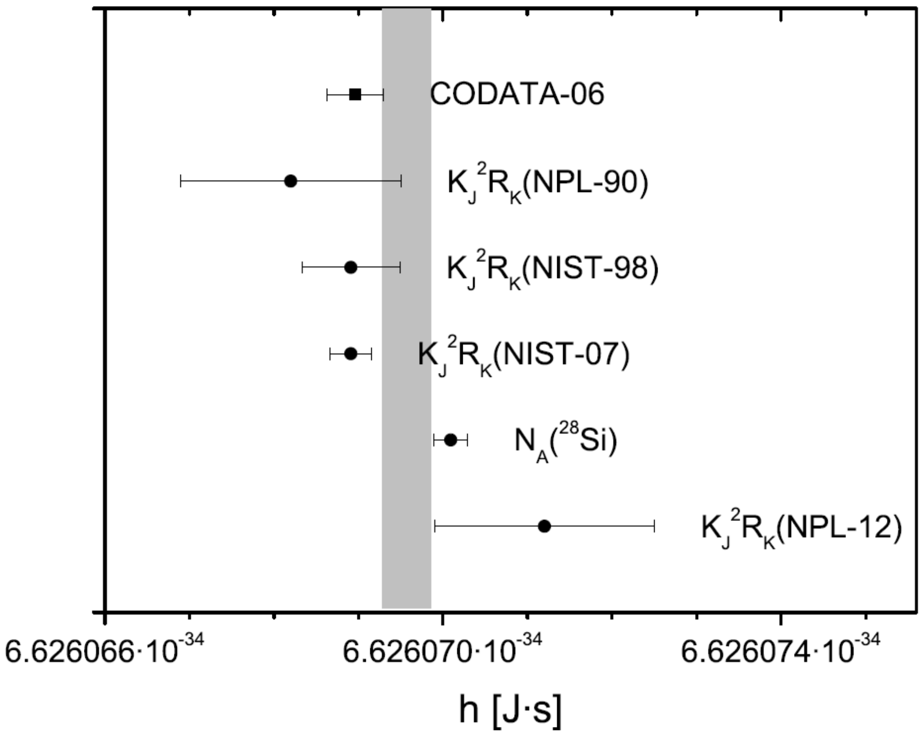
amic

electrical power

mechanical power

B. Jeanneret, Les Houches, 200

h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si

Monocrystale of ^{28}Si

monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: 92%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

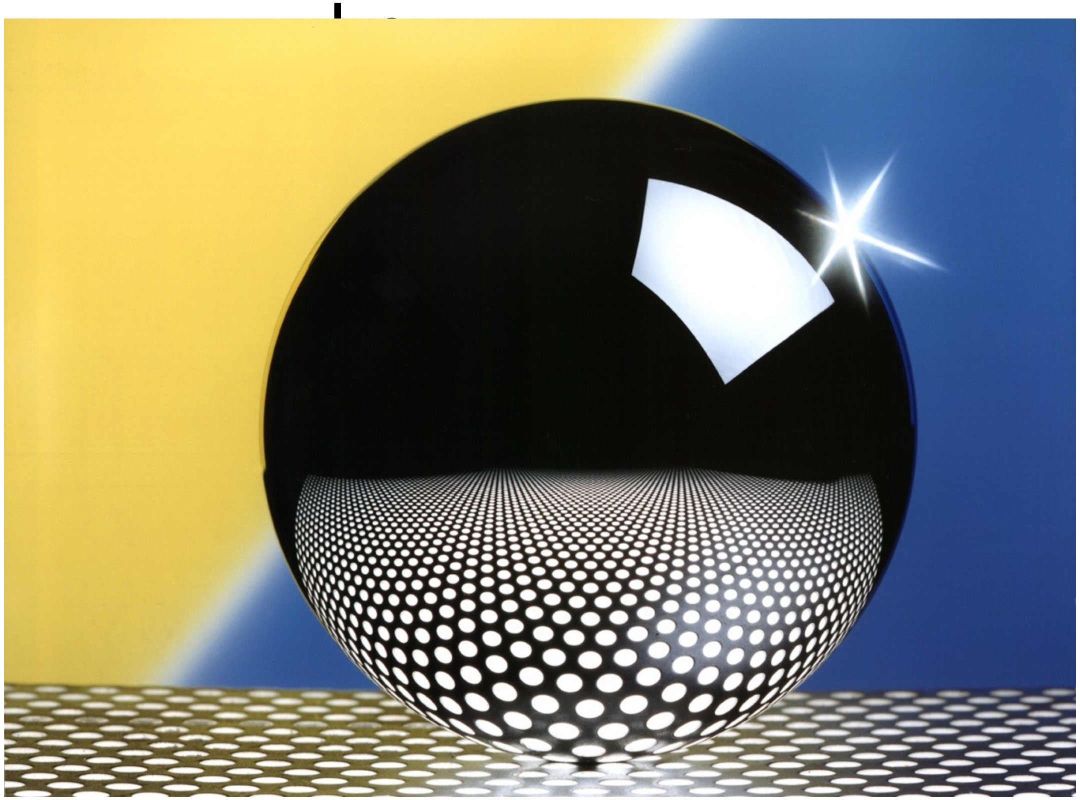
monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

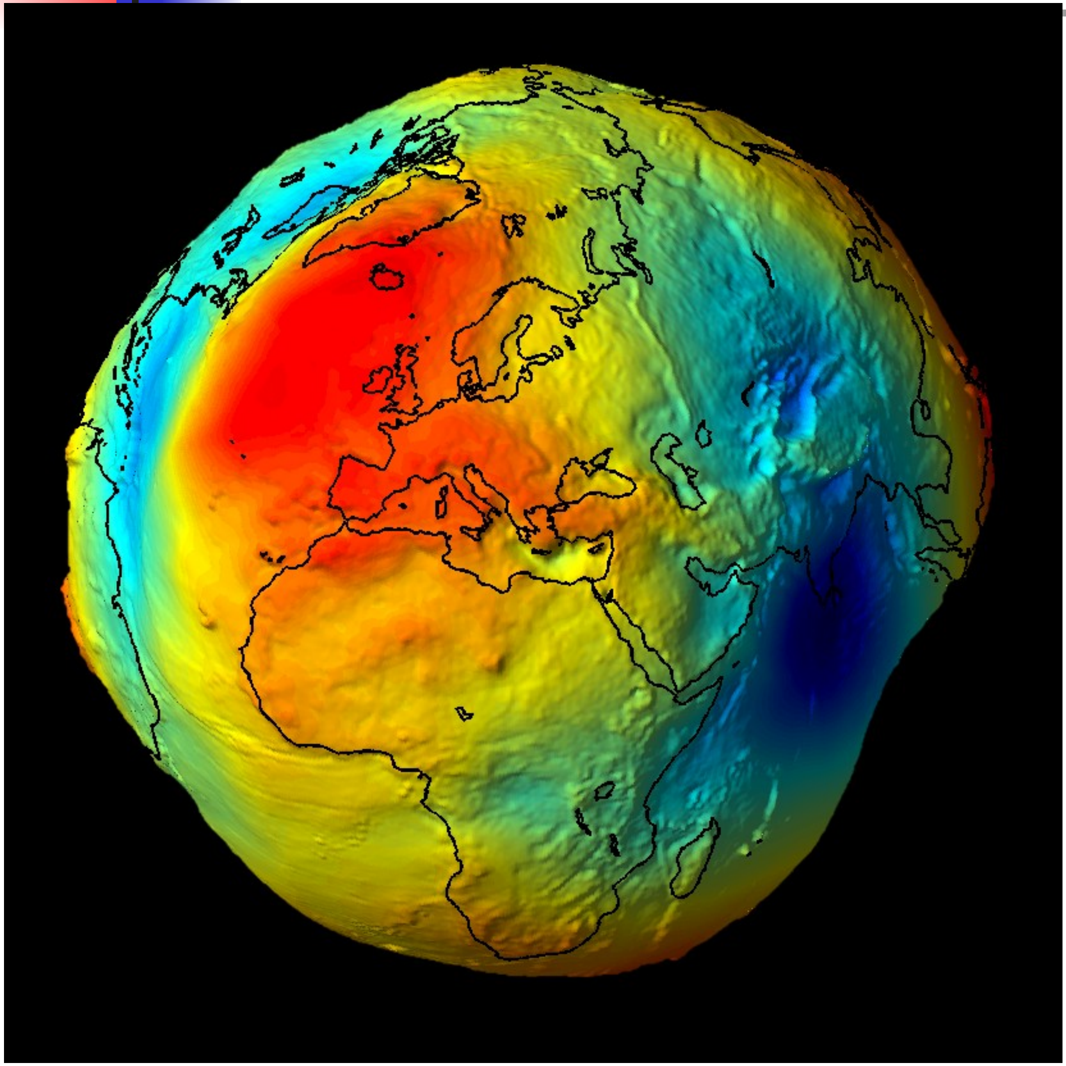
monocrystale ~ 1



isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

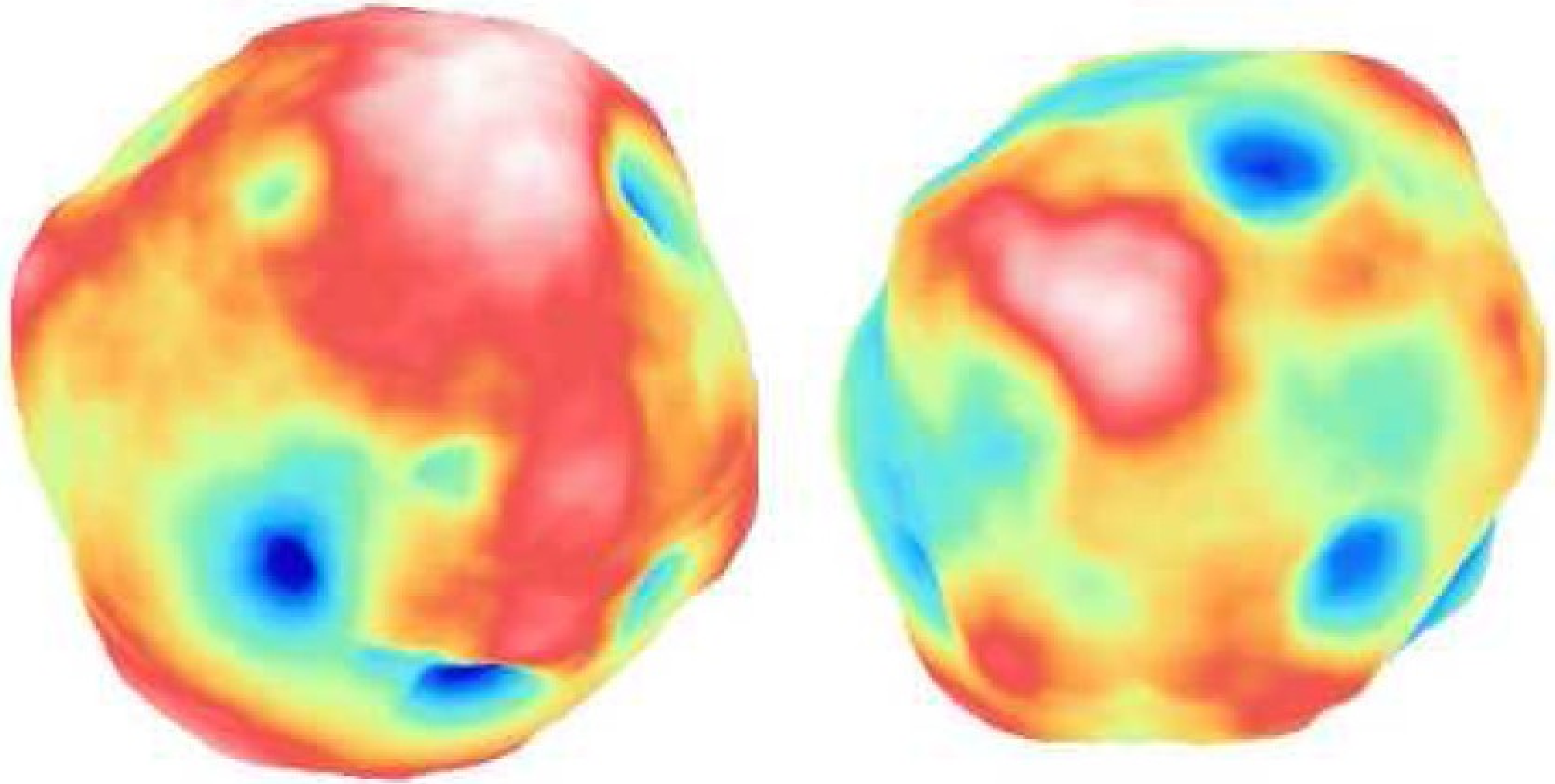


isotopic composition

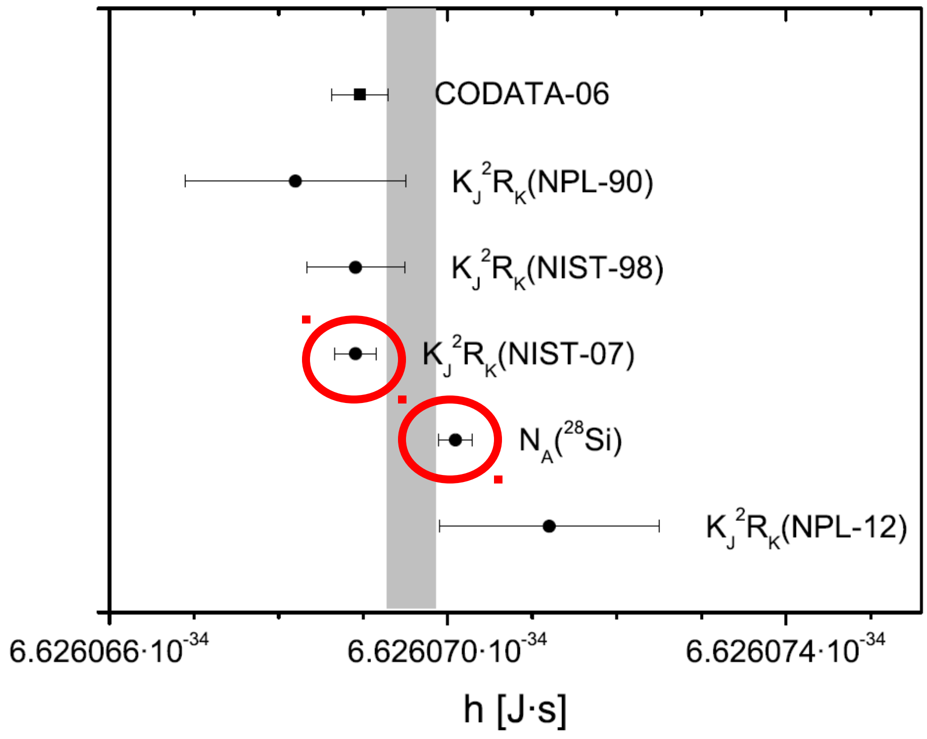
- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

monocrystale ~ 1   isotopic

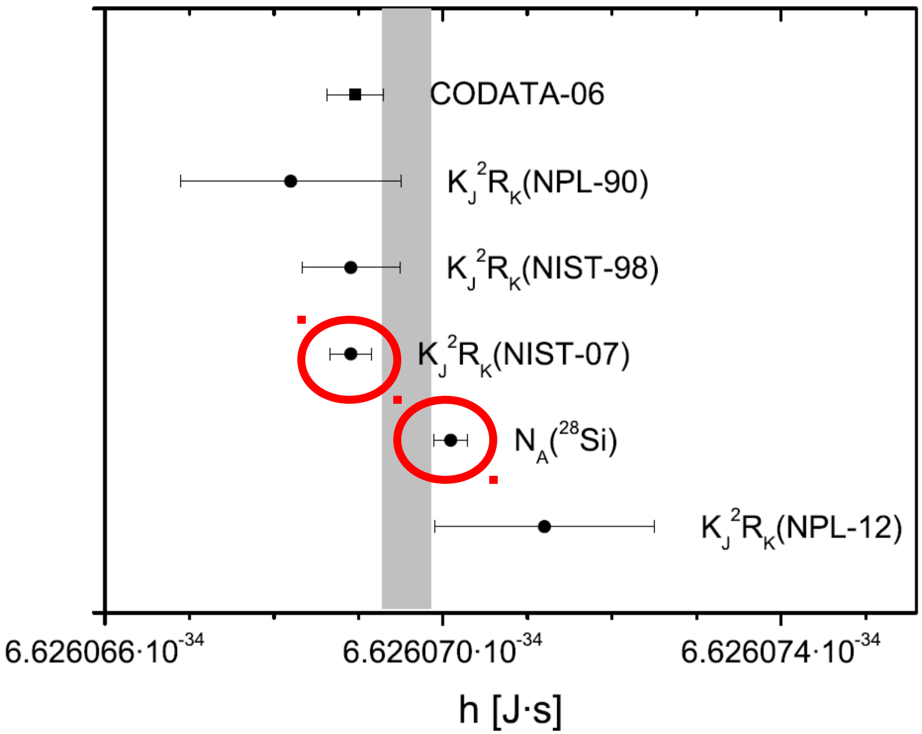


h block: the most important data

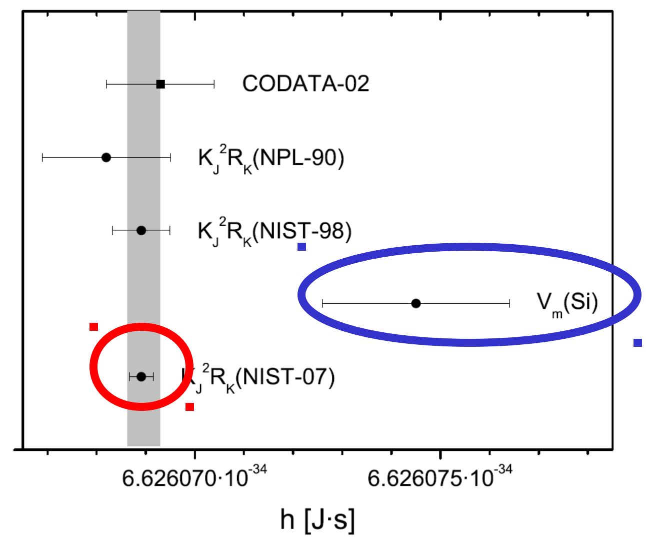


- watt ballance
- Avogadro constant from ehrhiched Si
- problem remains

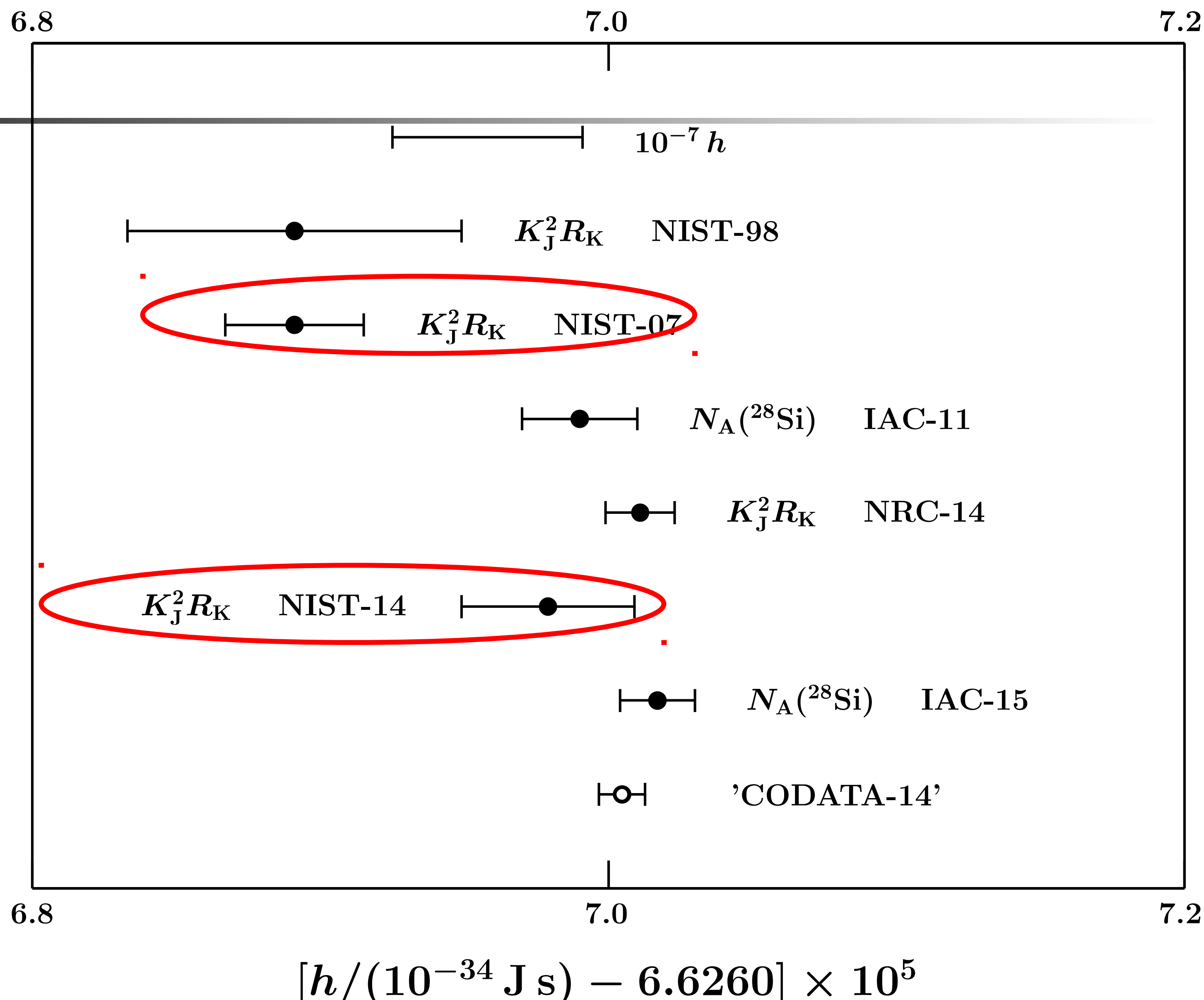
h block: the most important data



- watt ballance
- Avogadro constant from



2014 Input data related to the Planck constant: TGFC meeting



$h = 6.626\ 070\ 038(81) \times 10^{-34}$ J sec

$[1.2 \times 10^{-8}]$

χ^2: 8.49

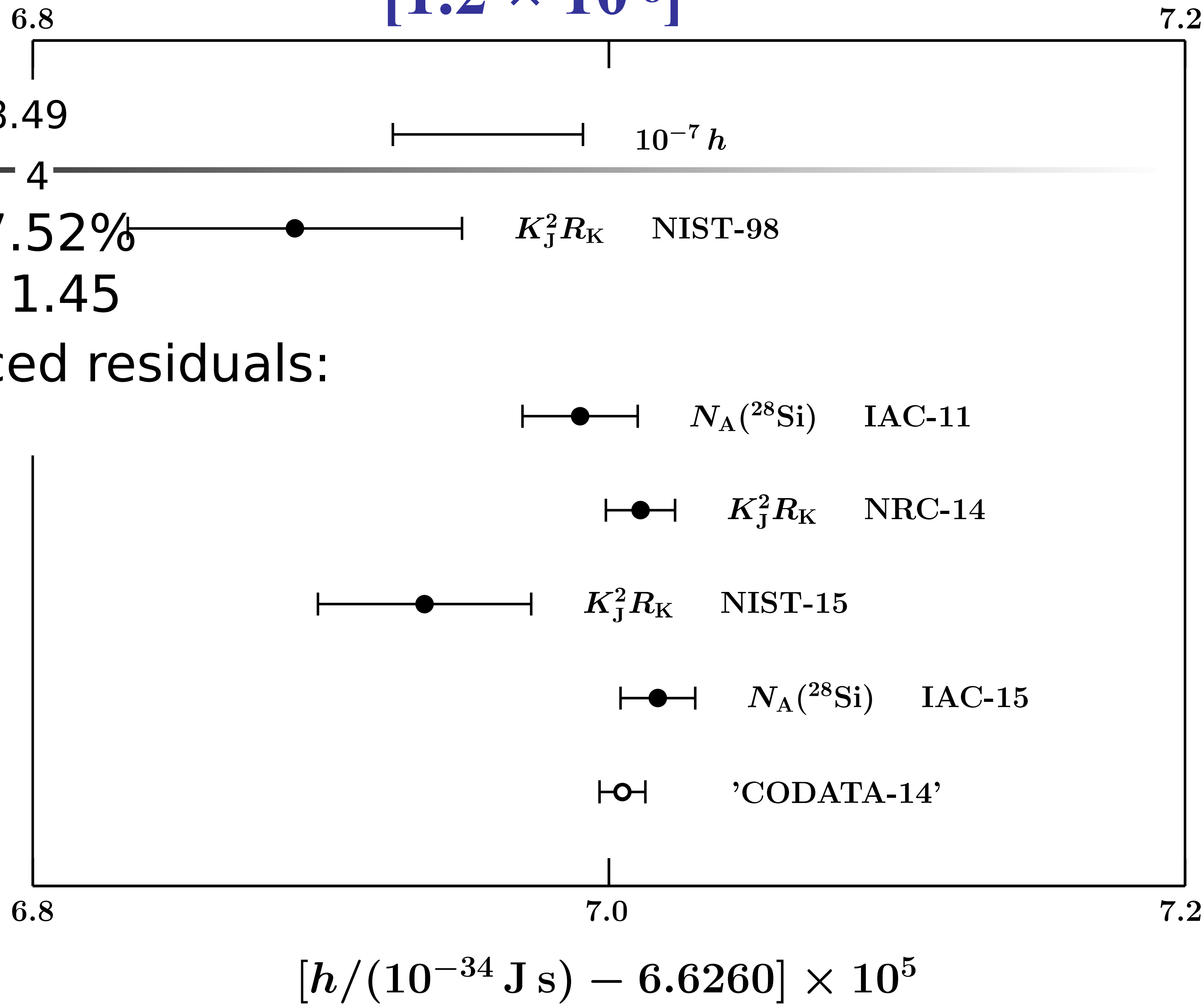
DOF: 4

Prob. χ^2: 7.52%

R_B: 1.45

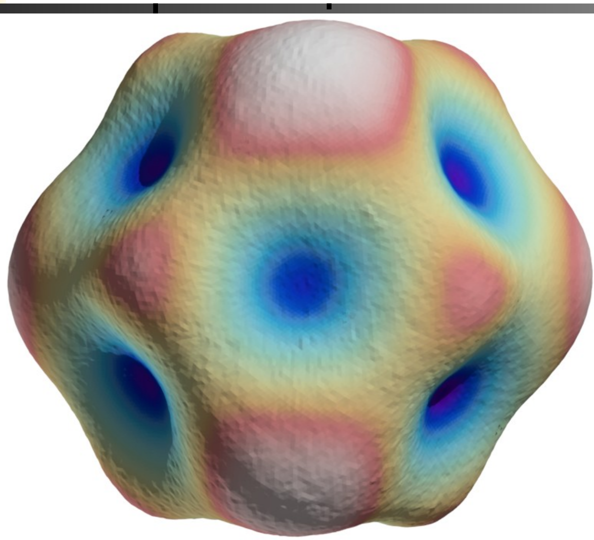
Max. reduced residuals:

1.96, 1.84

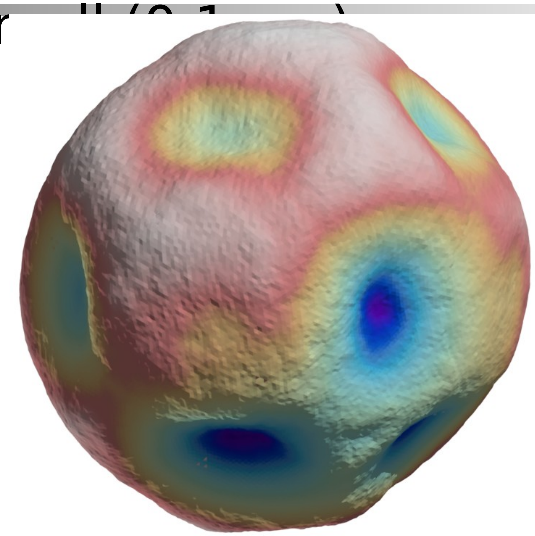


Sphere repolishing

- The AVO28-S5 and AVO28-S8 spheres have been repolished at PTB.
- Surface metallic layers (Ni and Cu) were near-completely removed.
- The … dentedly sr …



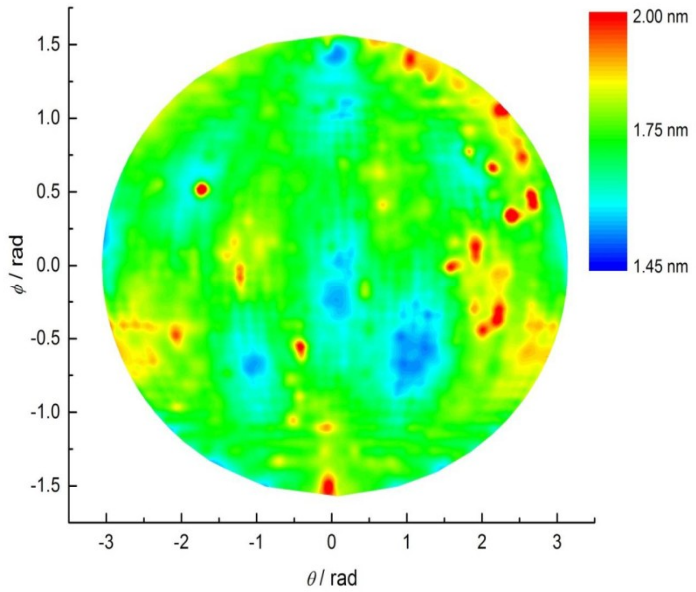
AVO28-S5c, (p-v)$_{diameter}$ = 69 nm



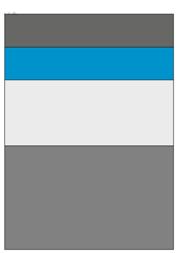
AVO28-S8c, (p-v)$_{diameter}$ = 38 nm

Sphere surface

The spheres were repolished: the metal contamination was removed, but the surface layer has been changed.

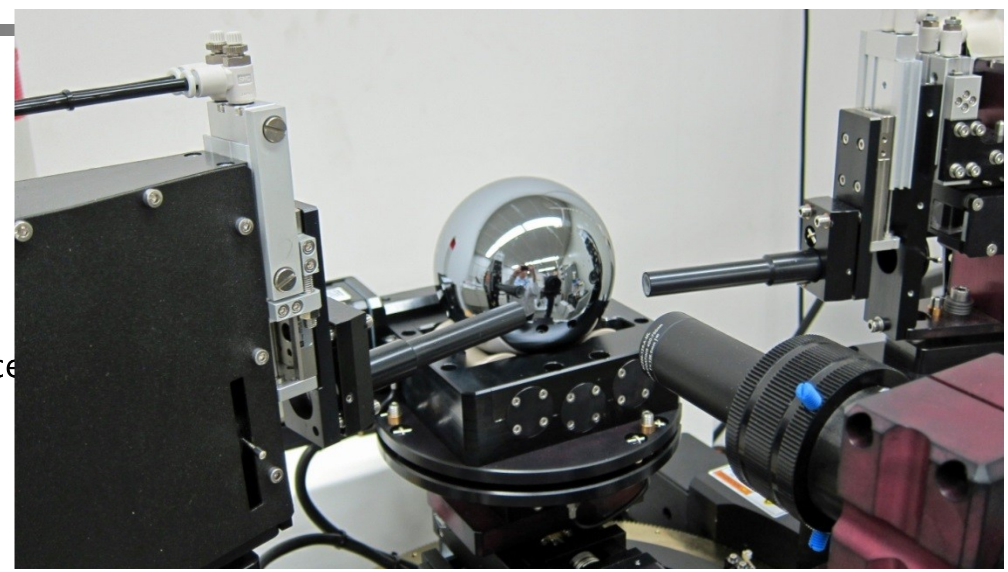


Topography of the SiO_2 thickness



x-ray fluorescence
gravimetry
x-ray reflectometry, x-ray fluorescence
spectral ellipsometry

Model of the surface layers and measurement techniques

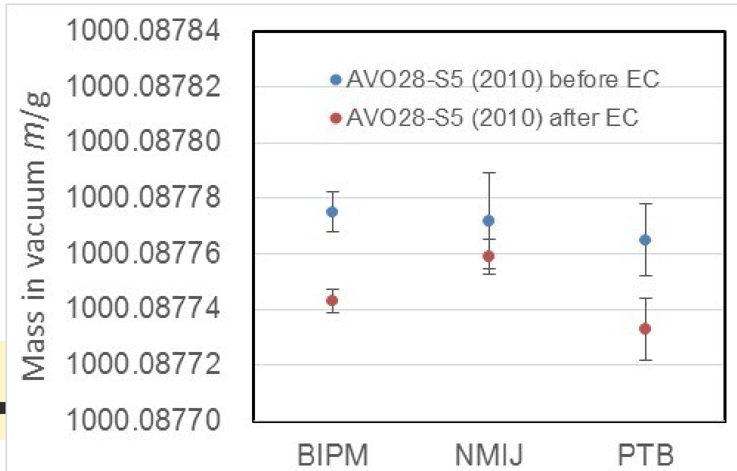


Spectral ellipsometer at the NMIJ

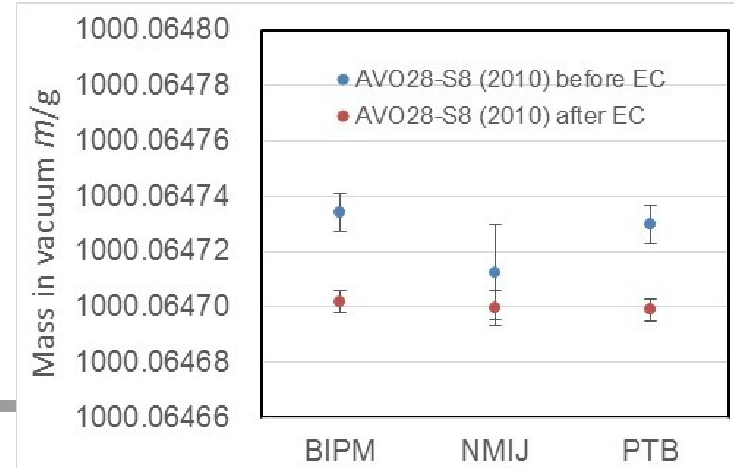
Extraordinary calibration and sphere mass

2010

S5

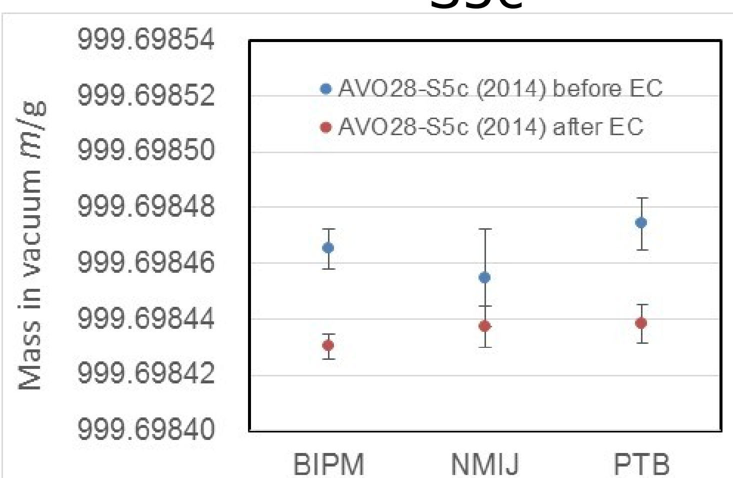


S8

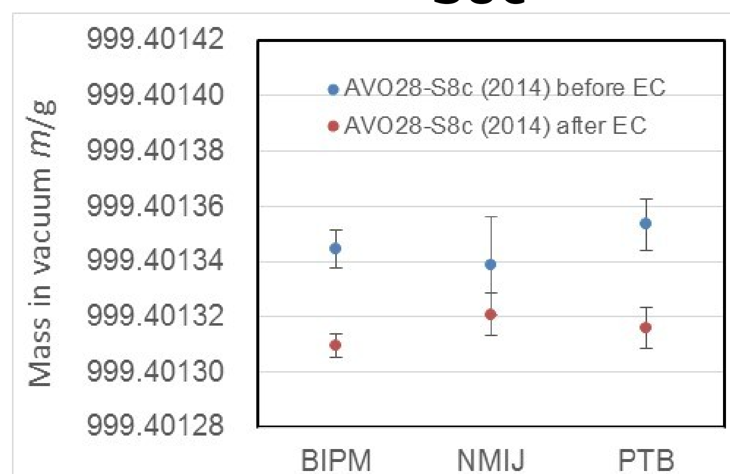


2014

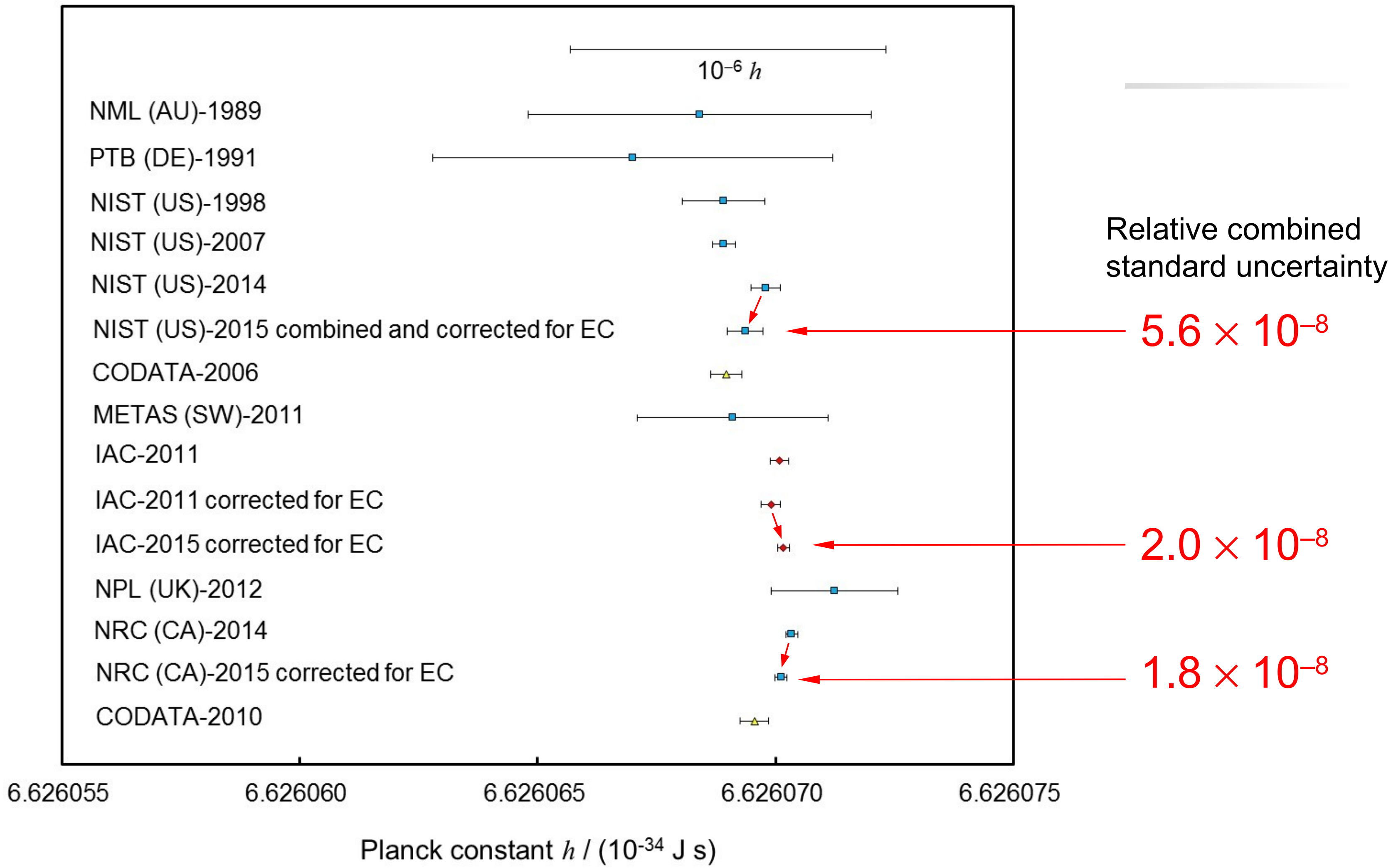
S5c



S8c



Planck constant (2015)



Mass of a proton in different units

Symbol	Value	u_r
m_p	$1.007\,276\,466\,812(90)$ u	$[8.9 \times 10^{-11}]$
m_p	$1836.152\,672\,45(75)\ m_e$	$[4.1 \times 10^{-10}]$
m_pc^2/h	$2.268\,731\,8139(16) \times 10^{23}$ Hz	$[7.1 \times 10^{-10}]$
m_pc^2	$938.272\,046(21)$ MeV	$[2.2 \times 10^{-8}]$
m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$

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- auxiliary data

Mass of a proton in different units

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- α block

Mass of a proton in different units

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m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$

- h block

Independent constants

Quantity	Symbol	Value	u_r
Newtonian constant of gravitation	G	$6.673\,84(80) \times 10^{-11}\ \mathrm{m^3\,s^{-2}\,kg^{-1}}$	$[1.2 \times 10^{-4}]$
Planck mass	$m_P = \sqrt{\hbar c/G}$	$2.176\,51(13) \times 10^{-8}\ \mathrm{kg}$	$[6.0 \times 10^{-5}]$
Boltzmann constant	k	$1.380\,6488(13) \times 10^{-23}\ \mathrm{J\,K^{-1}}$	$[9.1 \times 10^{-7}]$
molar gas constant	$R = k\,N_A$	$8.3144621(75)\ \mathrm{J\,K^{-1}\,mol^{-1}}$	$[9.1 \times 10^{-7}]$
Stefan-Boltzmann constant	$\sigma = (\pi^2/60)(k^4/\hbar^3 c^2)$	$5.670\,373(21) \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	$[3.6 \times 10^{-6}]$
anomalous magnetic moment of muon	a_μ	$1.165\,920\,91(63) \times 10^{-3}$	$[5.4 \times 10^{-7}]$

Independent constants: G

$$GM_\odot = 1.327\,124\,4210(1) \times 10^{20}\ \mathrm{m^3 s^{-2}}$$

$$GM_\oplus = 3.986\,004\,418(8) \times 10^{14}\ \mathrm{m^3 s^{-2}}$$

IESR, 2010

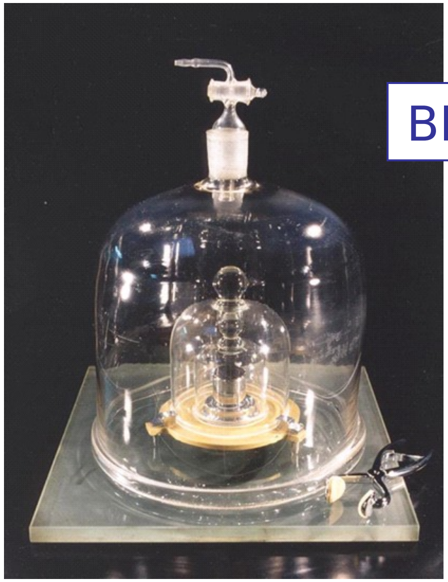
- $\delta G/G \sim 10^{-4}$

$$M_\odot = 1.988\,55(24) \times 10^{30}\ \mathrm{kg}$$
$$M_\oplus = 5.972\,58(72) \times 10^{24}\ \mathrm{kg}$$

PSR J0737-3039/A/B

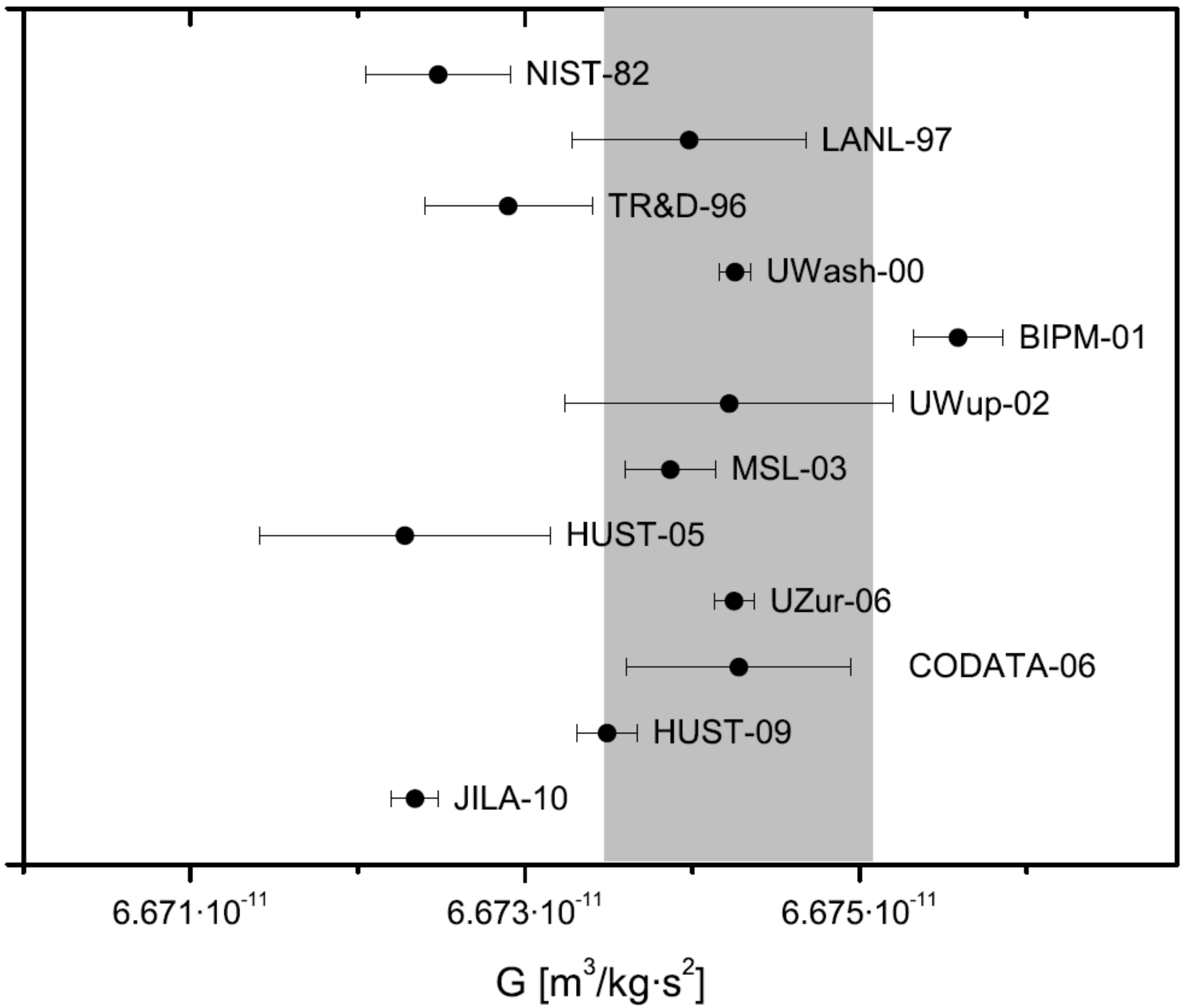
$$M_m = 1.3381(7)\ M_\odot = 2.6609(14) \times 10^{30}\ \mathrm{kg}$$
$$M_p = 1.2489(7)\ M_\odot = 2.4835(14) \times 10^{30}\ \mathrm{kg}$$

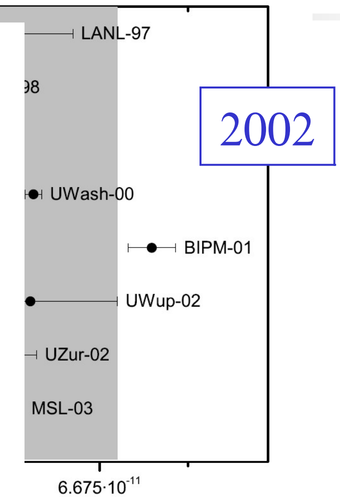
Kramer et al., 2006

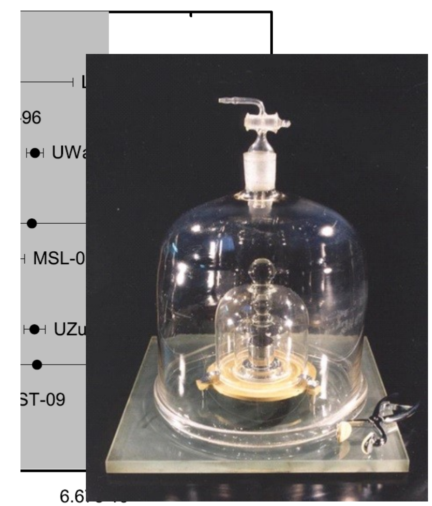


BIPM 1889

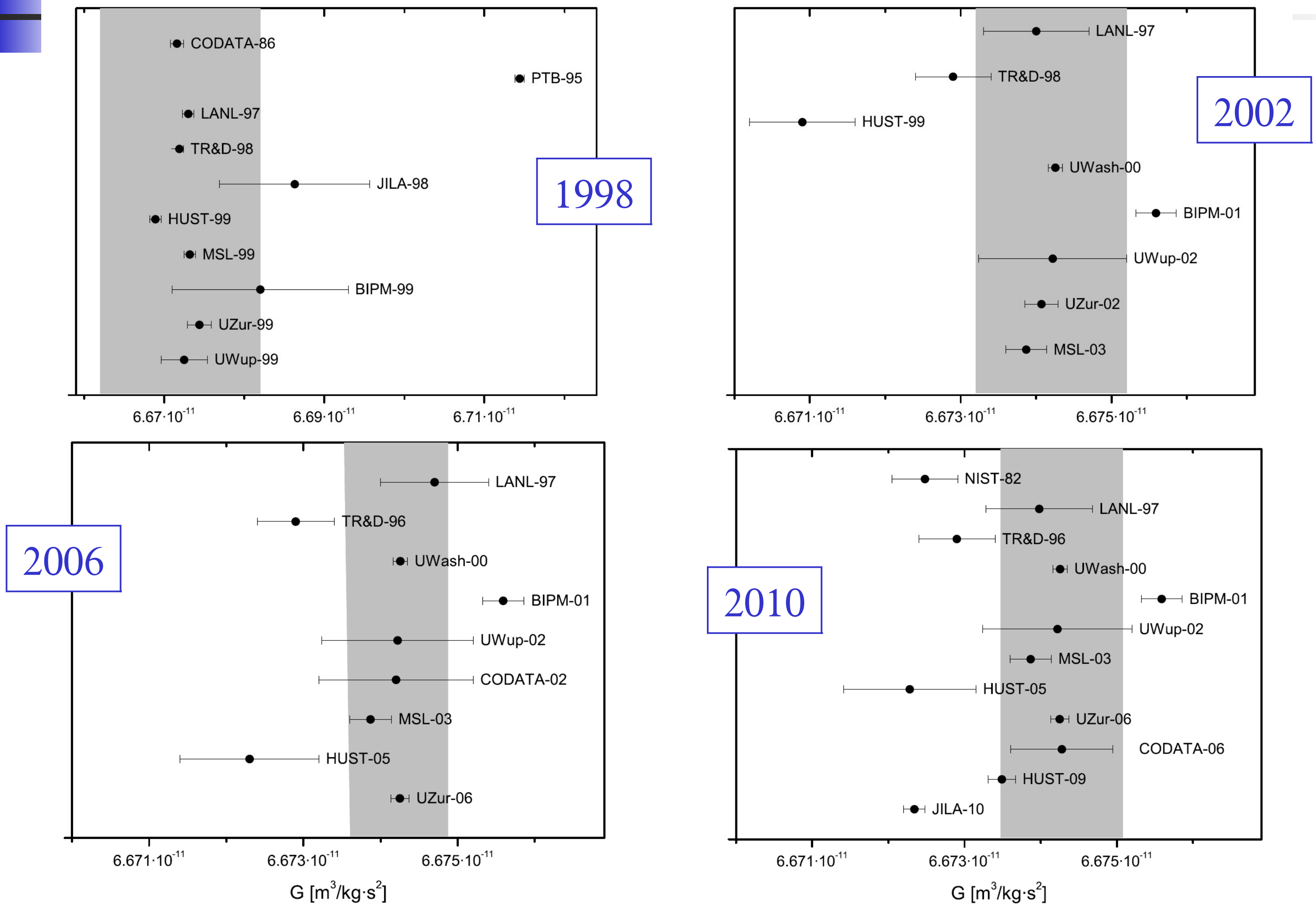
Independent constants: G



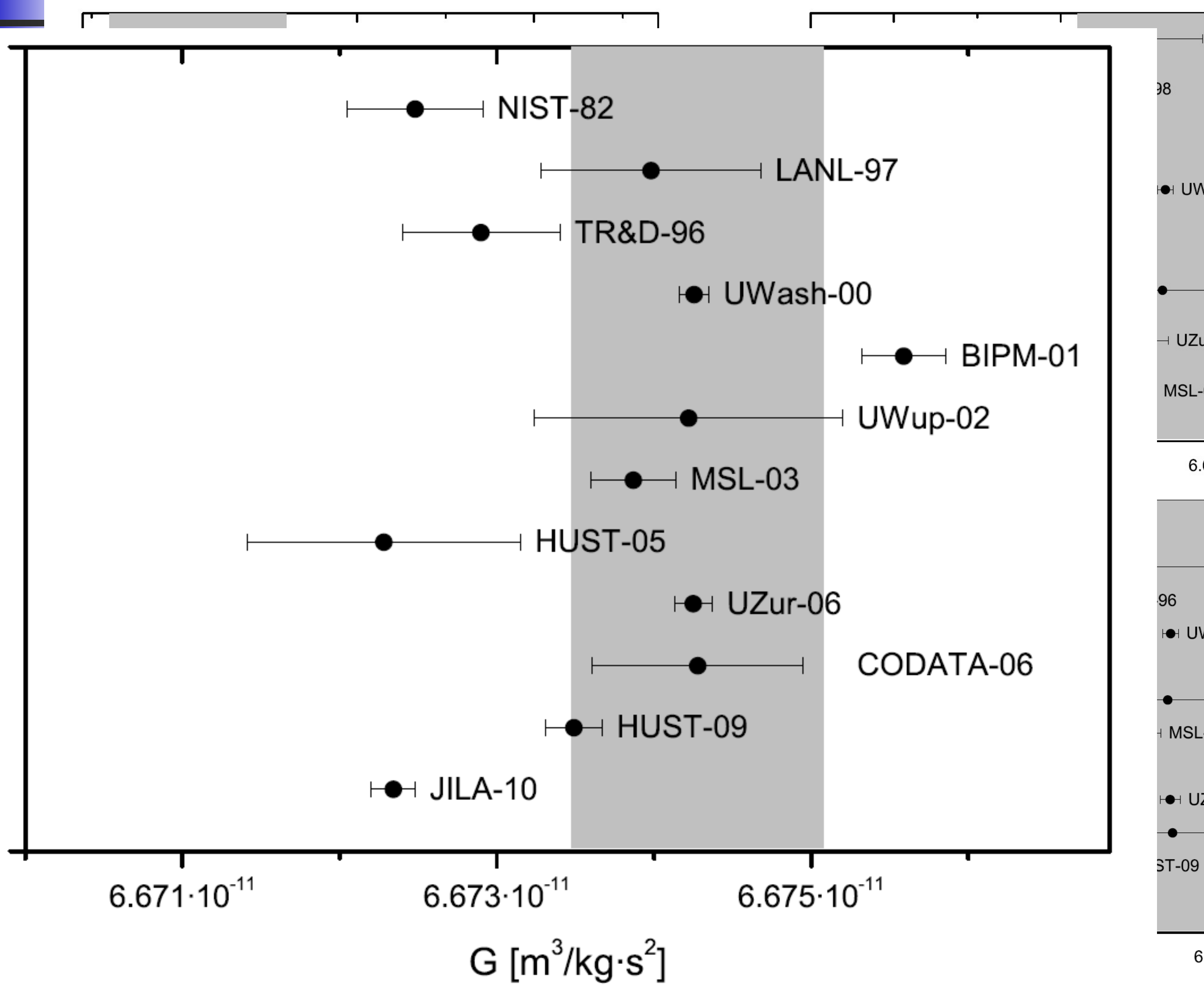


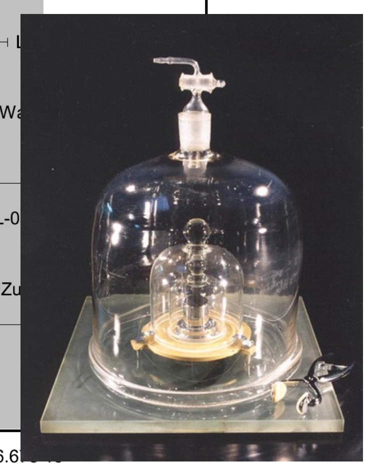


Independent constants: G



Independent constants: G





$G = 6.67408(31)\times10^{-11}$ m^{3}kg^{-1}s^{-2}

[4.7×10^{-5}]: TGFC meeting

With expansion factor of 6.3

χ^2: 8.05

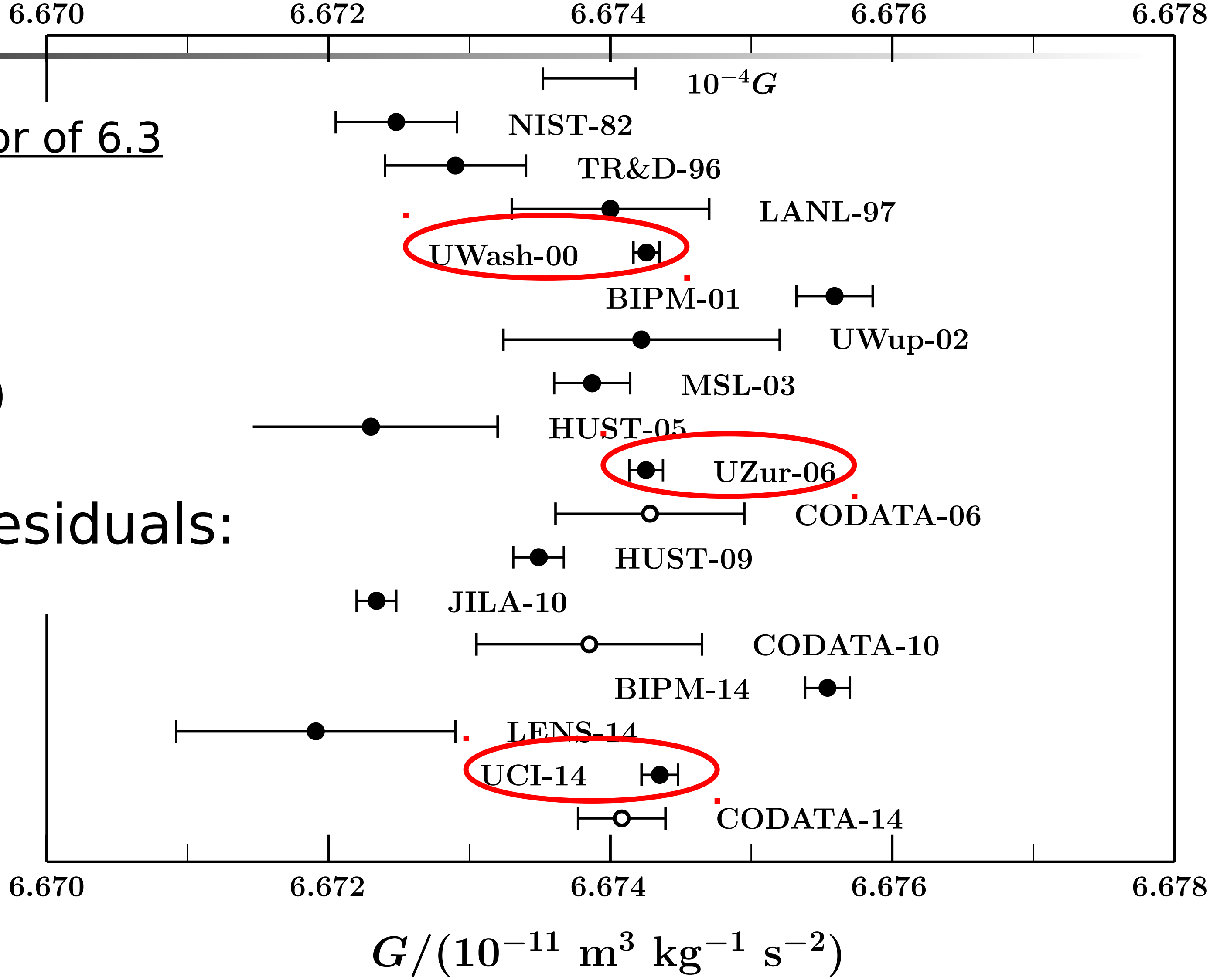
DOF: 13

Prob. χ^2: 84%

R_B: 0.79

Max. reduced residuals:

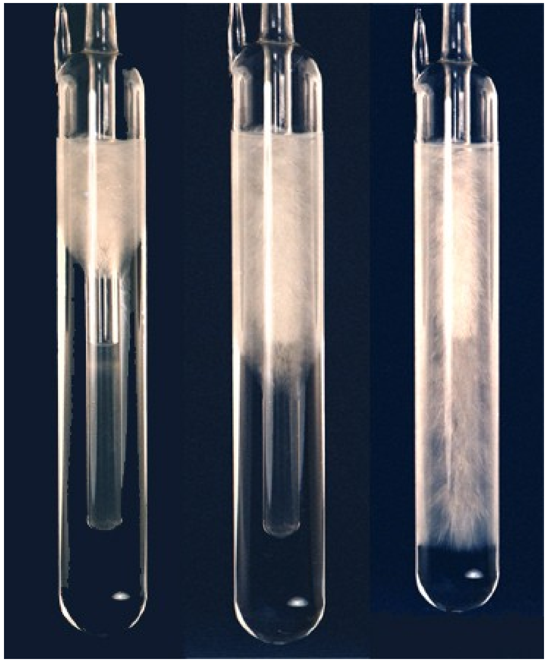
-1.98, 1.44



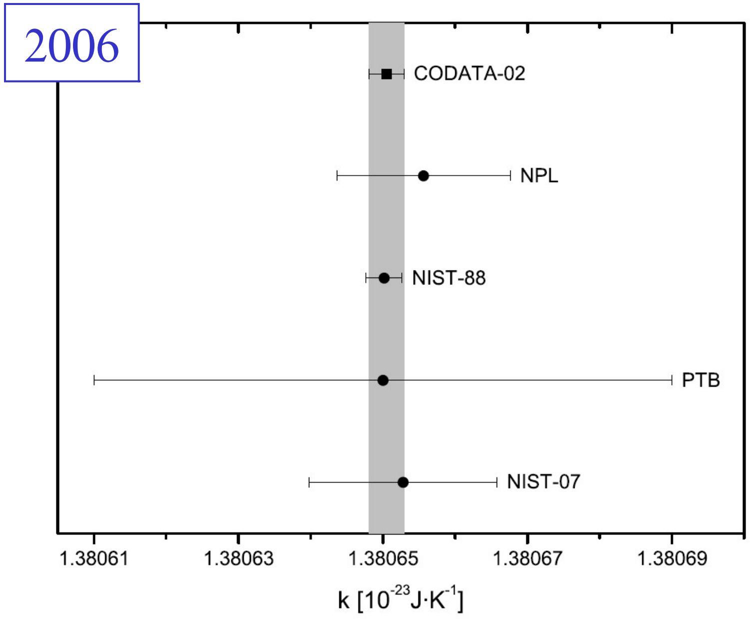
Independent constants: k

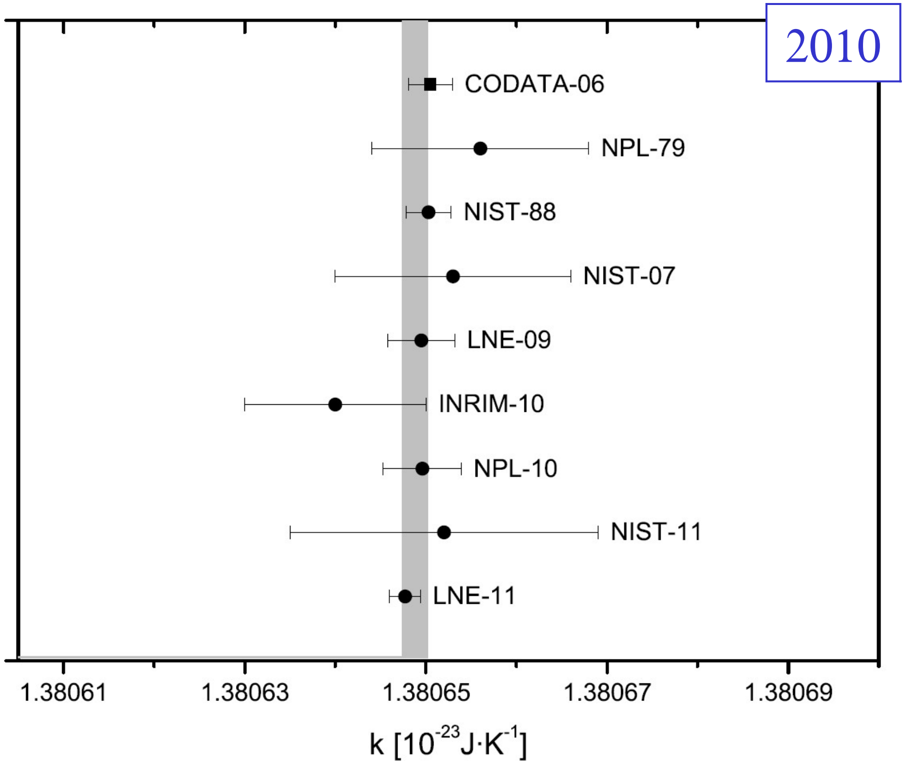
$$T_{\mathrm{CMB}} = 2.725\,48(57)\ K$$

Fixsen, 2009: COBE



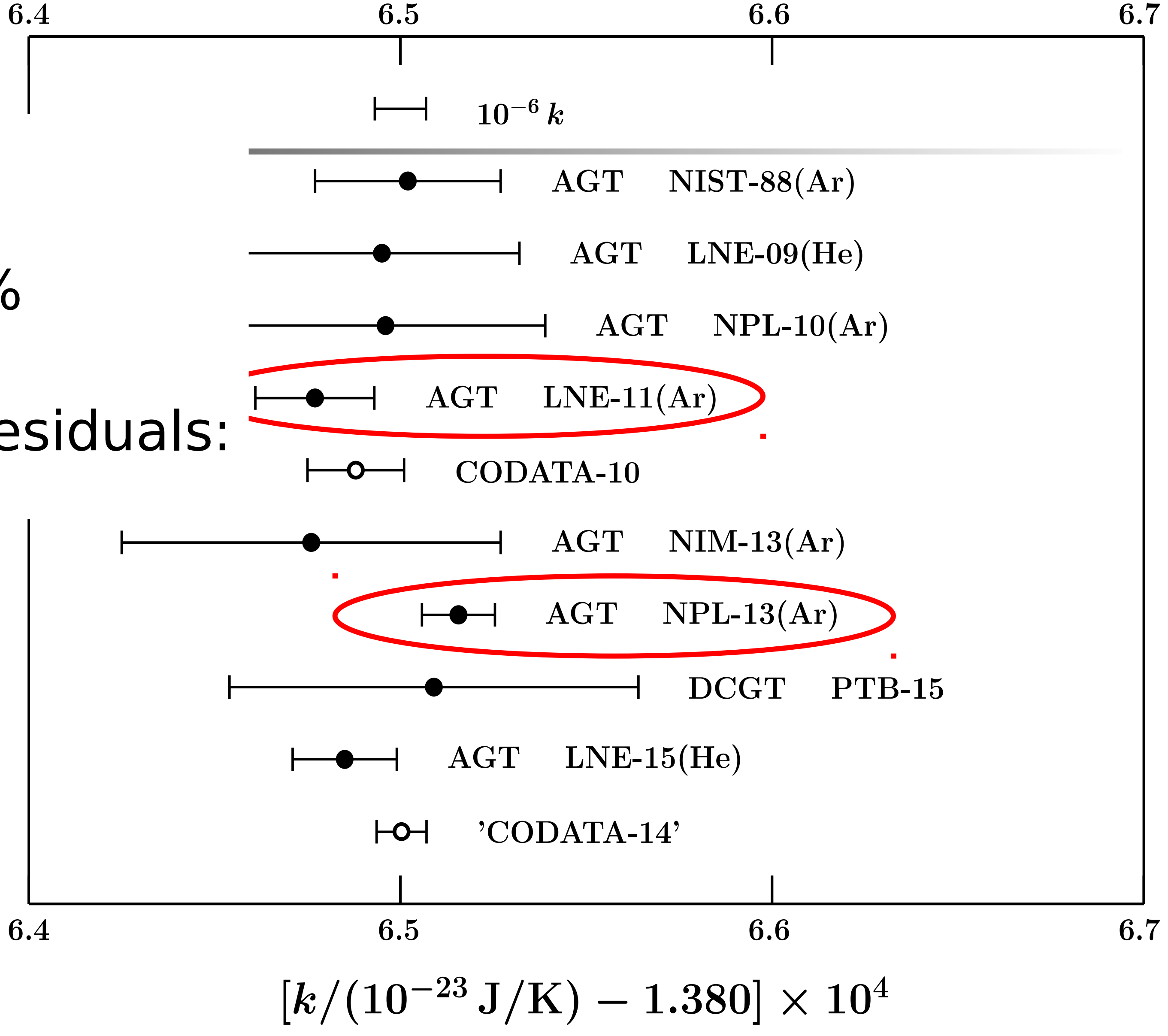
Independent constants: k



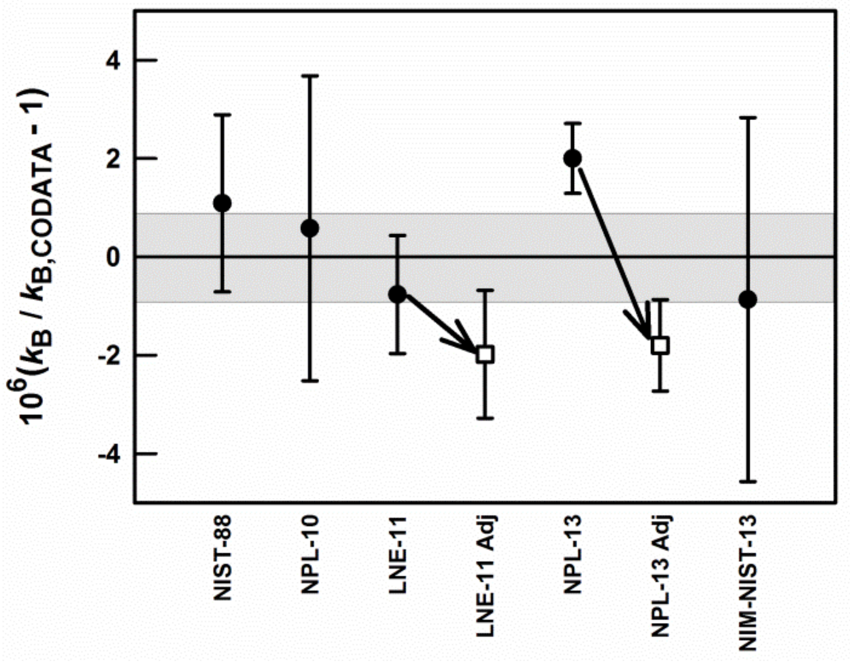


4b) 2014 Input data related to the Boltzmann constant: TGFC meeting

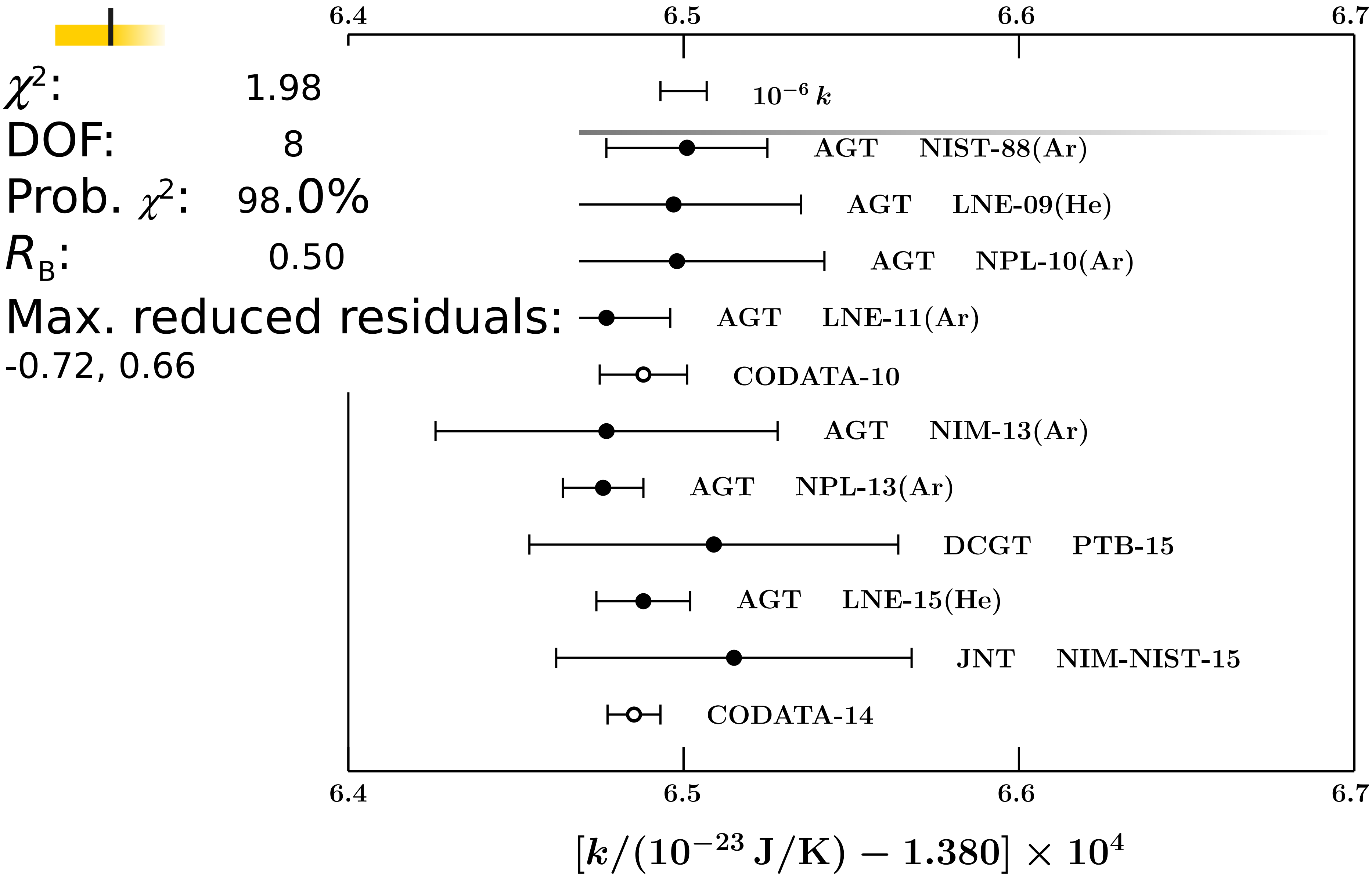
χ^2: 5.50
DOF: 7
Prob. χ^2: 60.0%
R_B: 0.89
Max. reduced residuals:
-1.28, 1.55



“Improving acoustic determinations of the Boltzmann constant with mass spectrometer measurements of the molar mass of argon,”
I. Yang, L. Pitre, M. R. Moldover, J. Zhang, X. Feng, and J. S. Kim1, Metrologia, available online November 2015

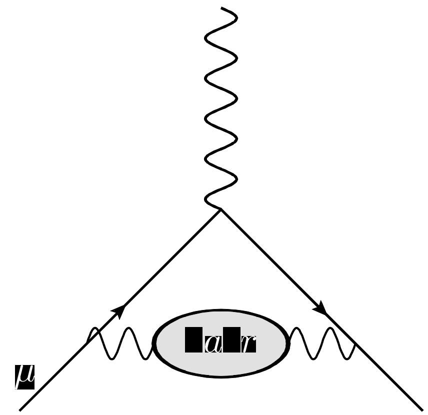


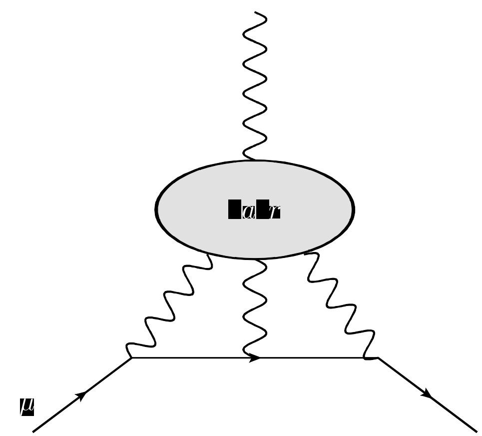
$k = 1.380\ 648\ 52(79) \times 10^{-23}$ J K^{-1}
$[5.7 \times 10^{-7}]$



Independent constants: a_μ

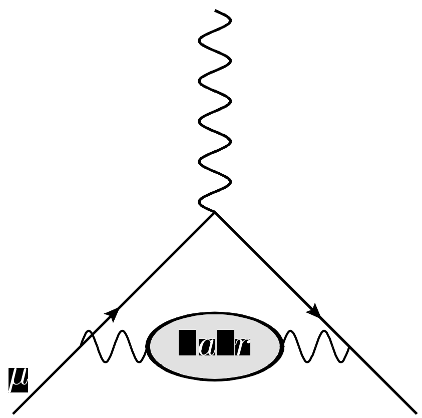
- QED is not a problem
- hadronic effects

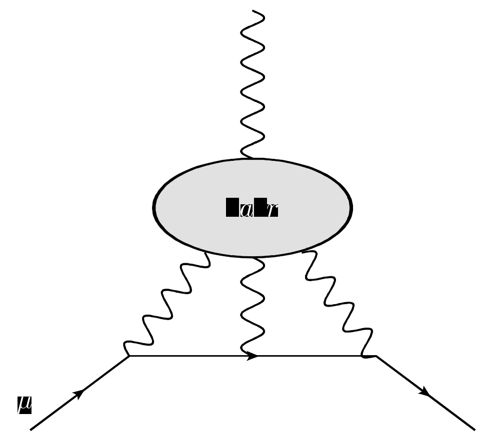




Independent constants: a_μ

- data
- models





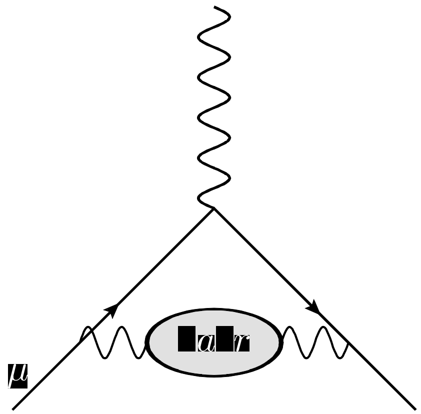
$$a_\mu(\mathrm{LO-hVP}) = 695.5(4.1) \times 10^{-10}$$

$$a_\mu(\mathrm{hLbL}) = 10.5(2.6) \times 10^{-10}$$

expt. uncertainty 6.3×10^{-10}

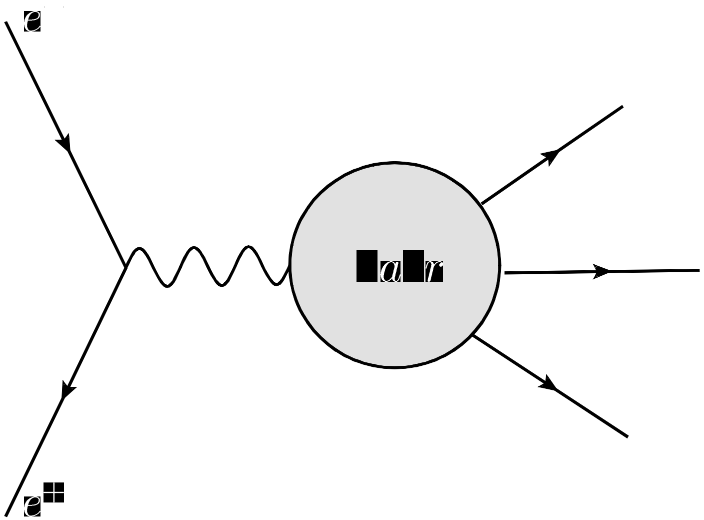
Independent constants: a_μ

- e^+e^-
- τ decay

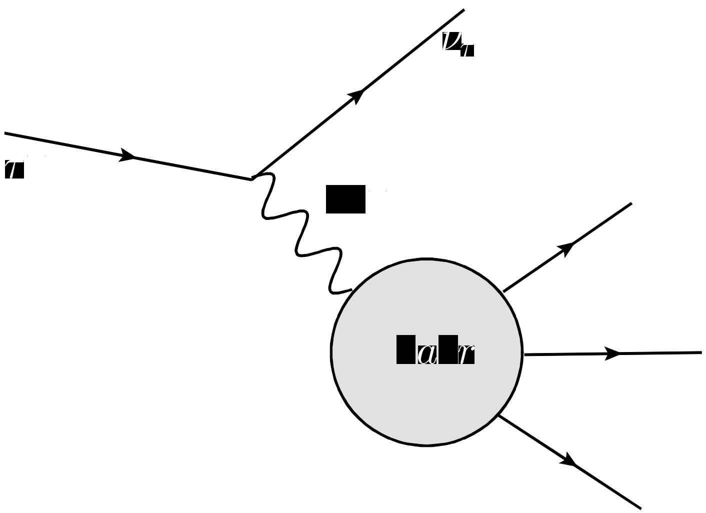


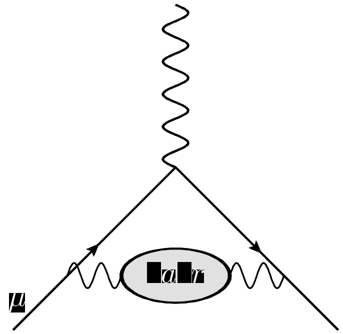
Independent constants: a_μ

- e^+e^-

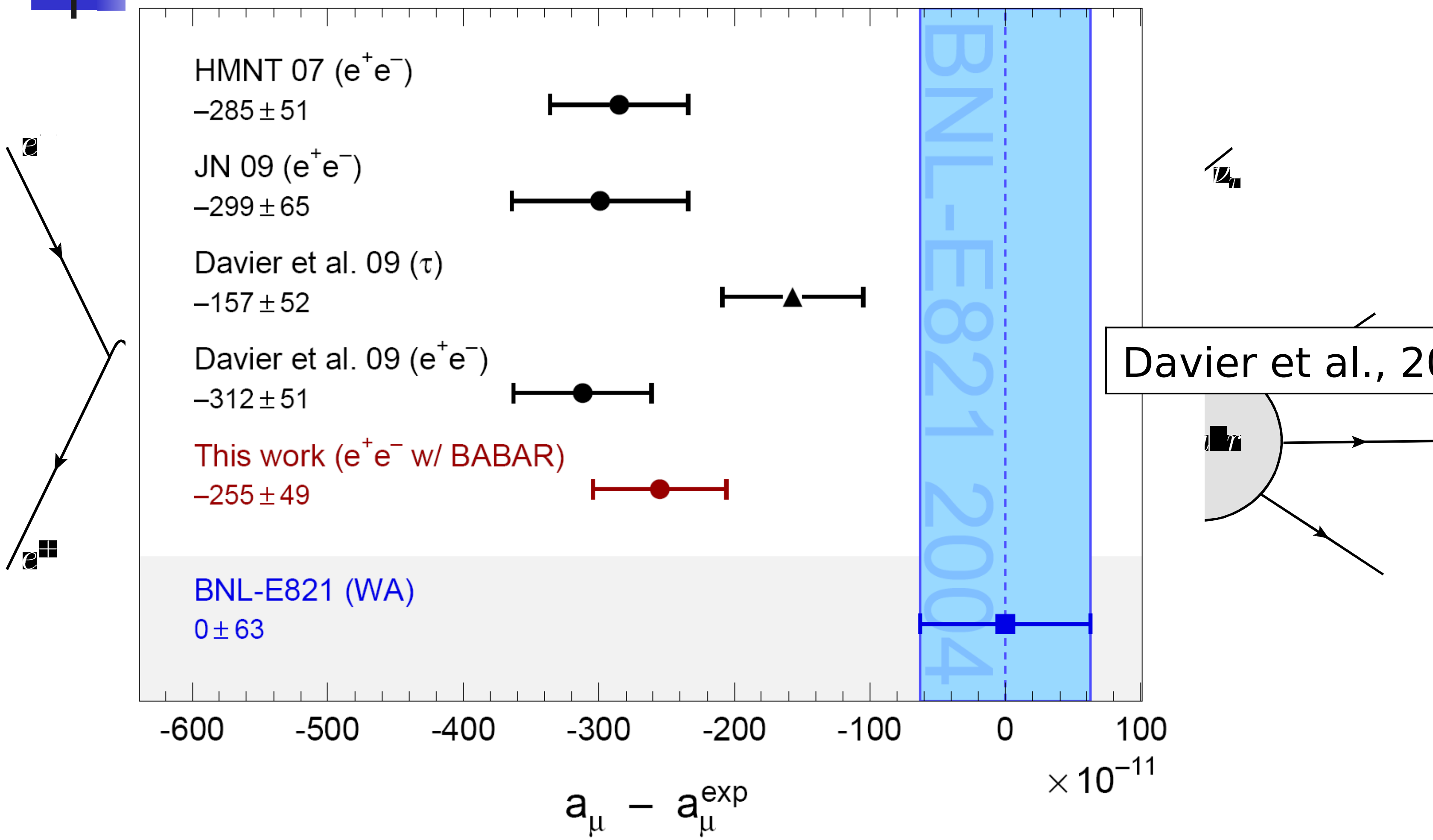


- τ decay



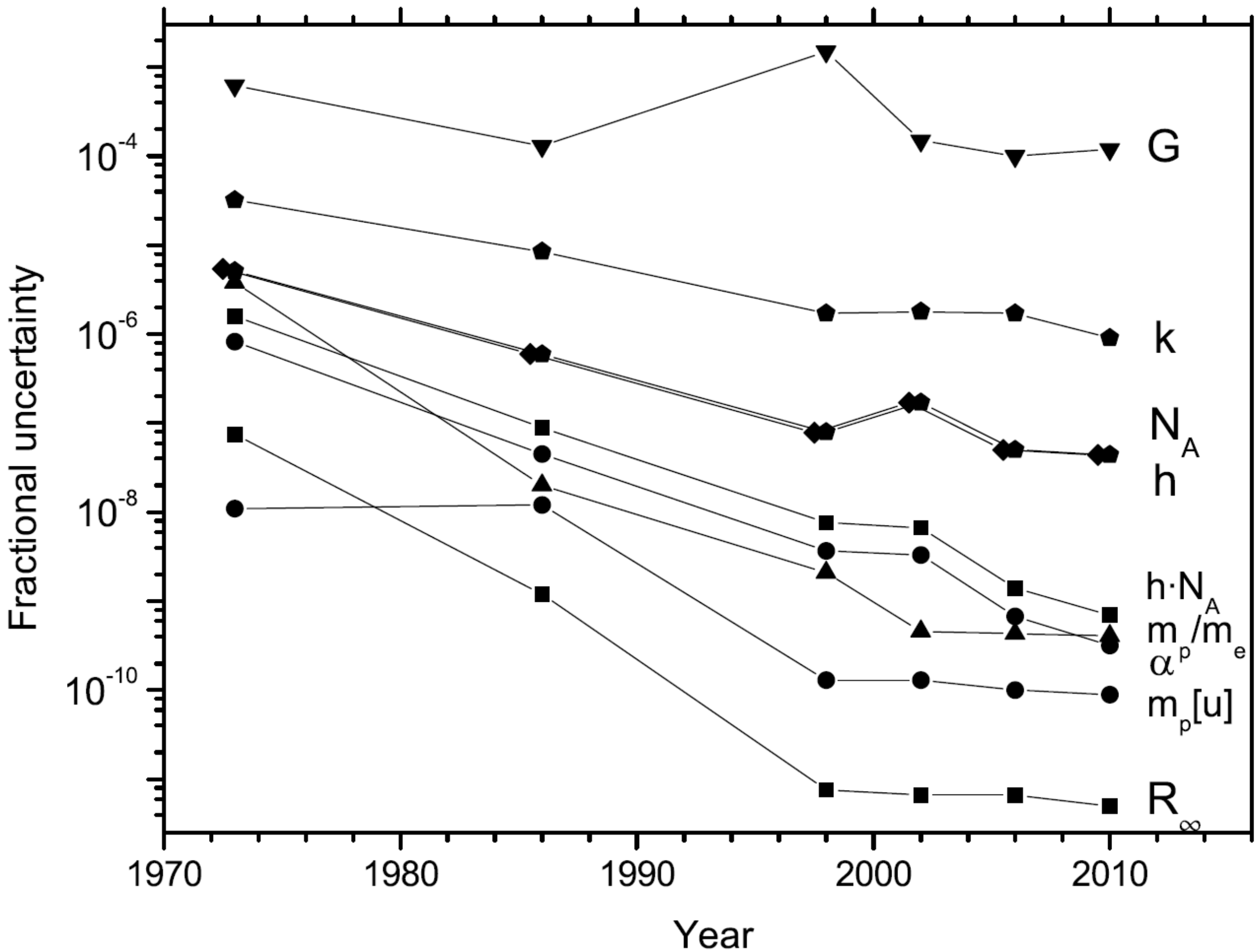


Independent constants: a_μ



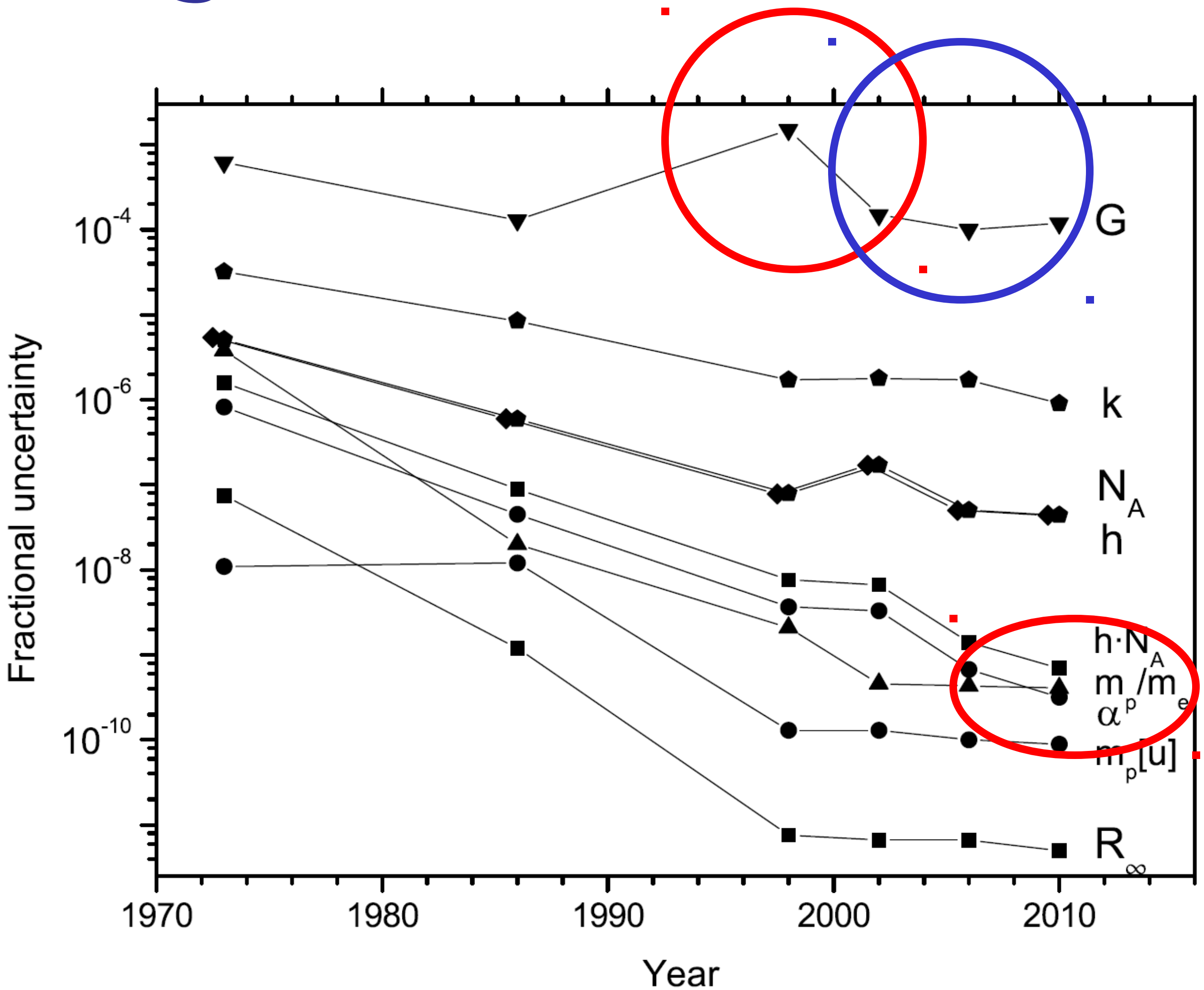
Davier et al., 2010

Progress



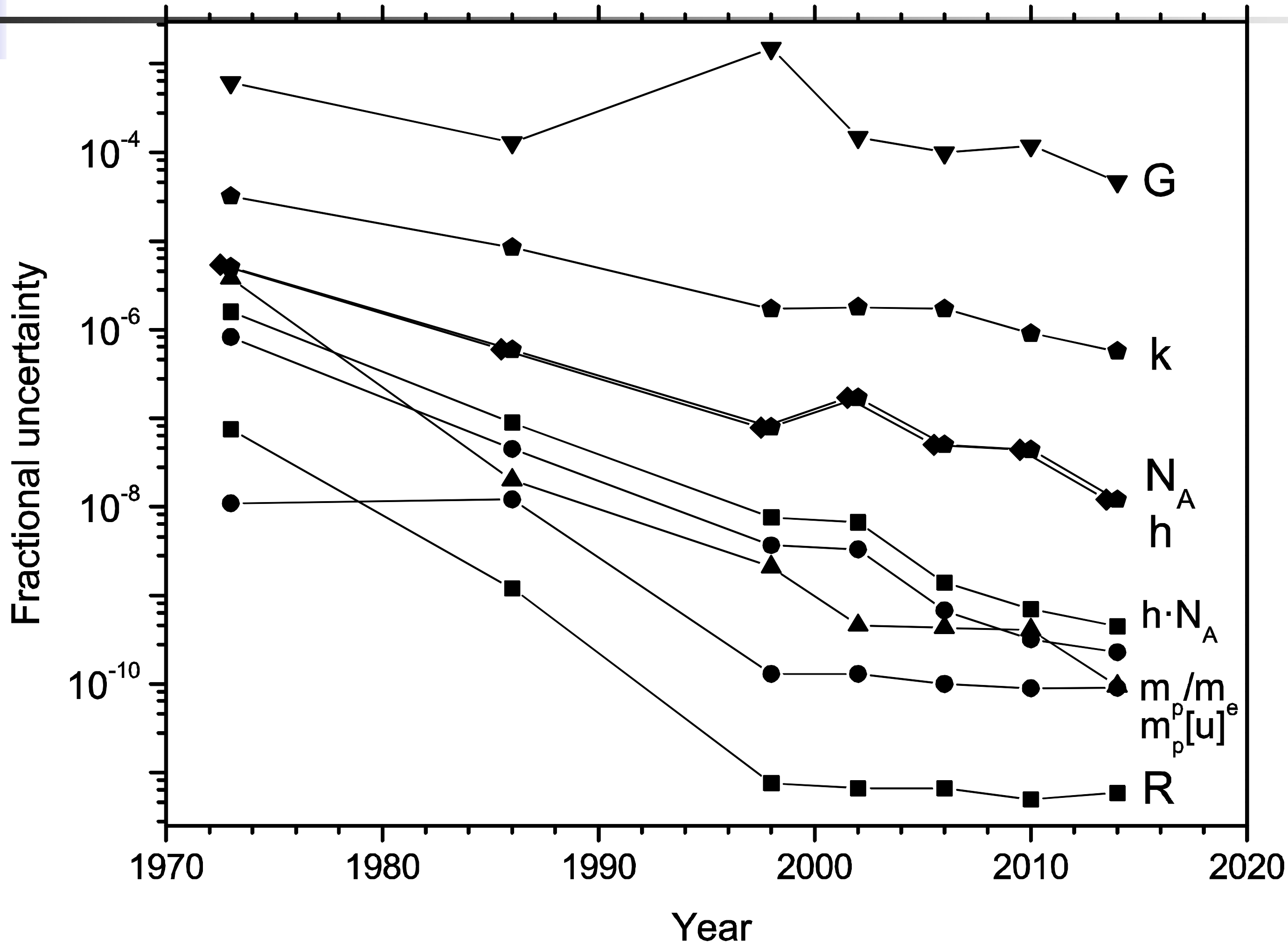
Fractional uncertainty
10^-4
10^-6
10^-8
10^-10
1970
1980
1990
2000
2010
Year
G
k
N_A
h
h·N_A
m_p/m_e
α
m_p[u]
R_∞

Progress



G
k
N_A
h
$h \cdot N_A$
m_p/m_e
α
$m_p[u]$
R_∞
10^{-4}
10^{-6}
10^{-8}
10^{-10}
Fractional uncertainty
1970
1980
1990
2000
2010
Year

Progress



Fractional uncertainty
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1970
1980
1990
2000
2010
2020
Year
G
k
N_A
h
$h \cdot N_A$
m_p/m_e
$m_p[u]$
R

Progress

Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
α	6.8×10^{-10}	44.2×10^{-10}	6.50	3.2×10^{-10}	0.47
h	5.0×10^{-8}	9.2×10^{-8}	1.84	4.4×10^{-8}	0.88
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Progress

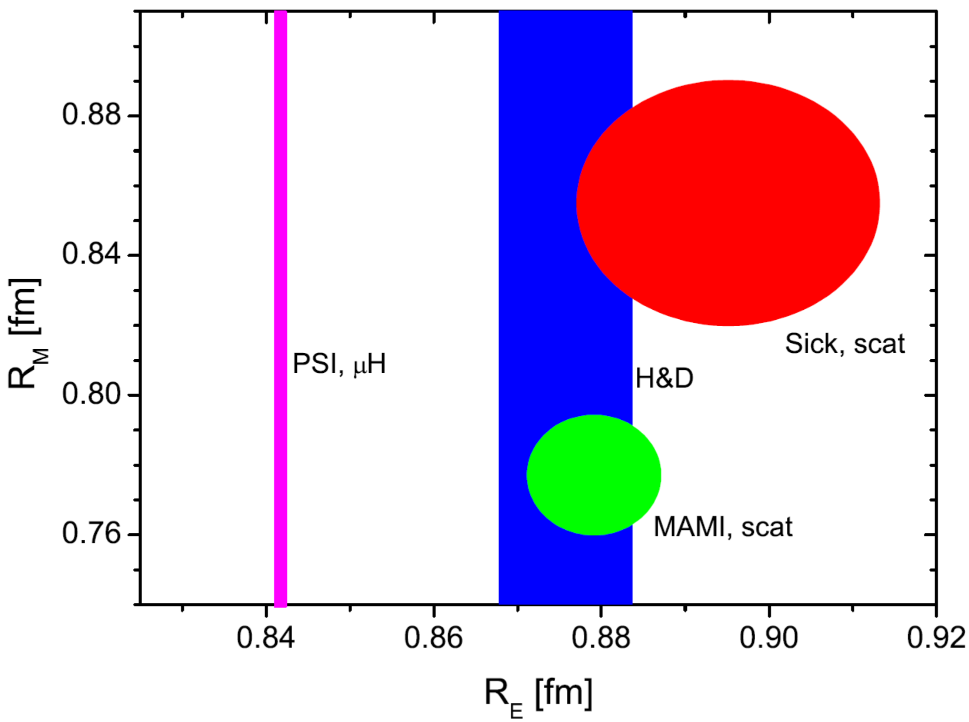
Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
α	6.8×10^{-10}	44.2×10^{-10}	6.50	3.2×10^{-10}	0.47
h	5.0×10^{-8}	9.2×10^{-8}	1.84	4.4×10^{-8}	0.88
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

Problems

- R_∞ & R_p
- m /m



+ better accuracy in scattering

+ new method for R_p

- discrepancy in data

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ slow progress in two methods

+ no discrepancies

overlap with α data

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ better accuracy
+ two methods
+ sensitivity to 5 loops

– 6-sigma jump

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

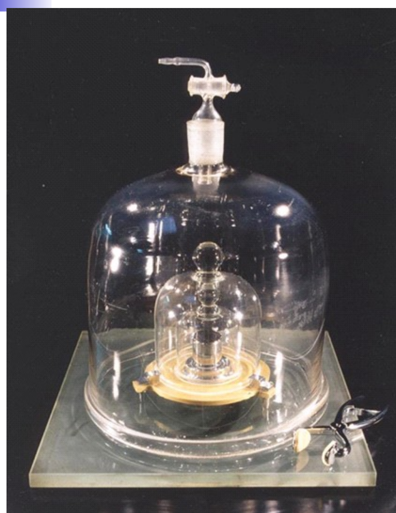
+ natural-silicon discrepacy resolved

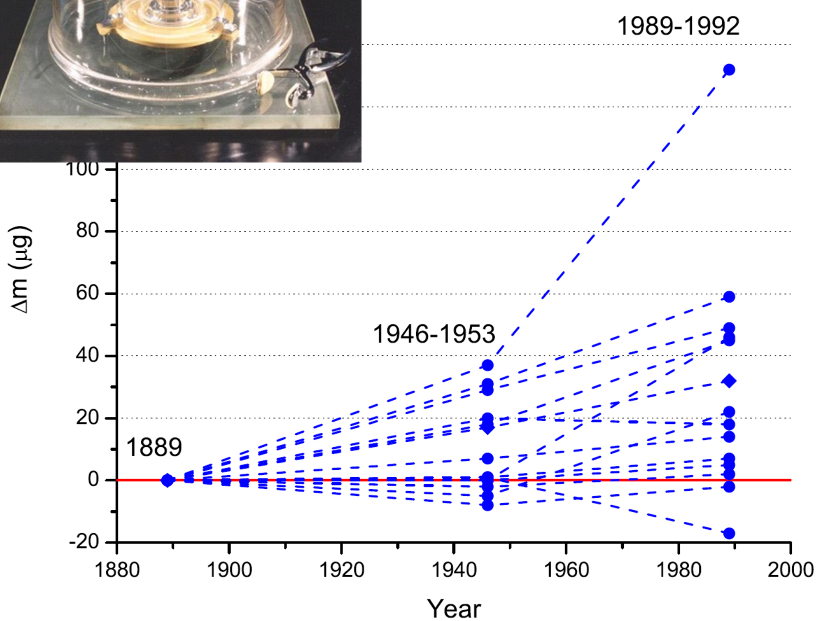
+ better accuracy for Avodagro

- new discrepancy

NPL $\rightarrow$ NRC

Problems





+ natural-silicon discrepacy resolved

+ better accuracy for Avodagro

- new discrepancy

NPL $\rightarrow$ NRC

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ more accurate results

– bigger scatter

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ more accurate results

+ more methods

+ efforts for atomic/molecular spectroscopy

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

  - still a discrepancy between e^+e^- & τ
  - still a disagreement of theory and experiment

CODATA Recommended Values of the Fundamental Physical Constants: 2010*

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National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

*This report was prepared by the authors under the auspices of the CODATA Task Group on Fundamental Constants. The members of the task group are:

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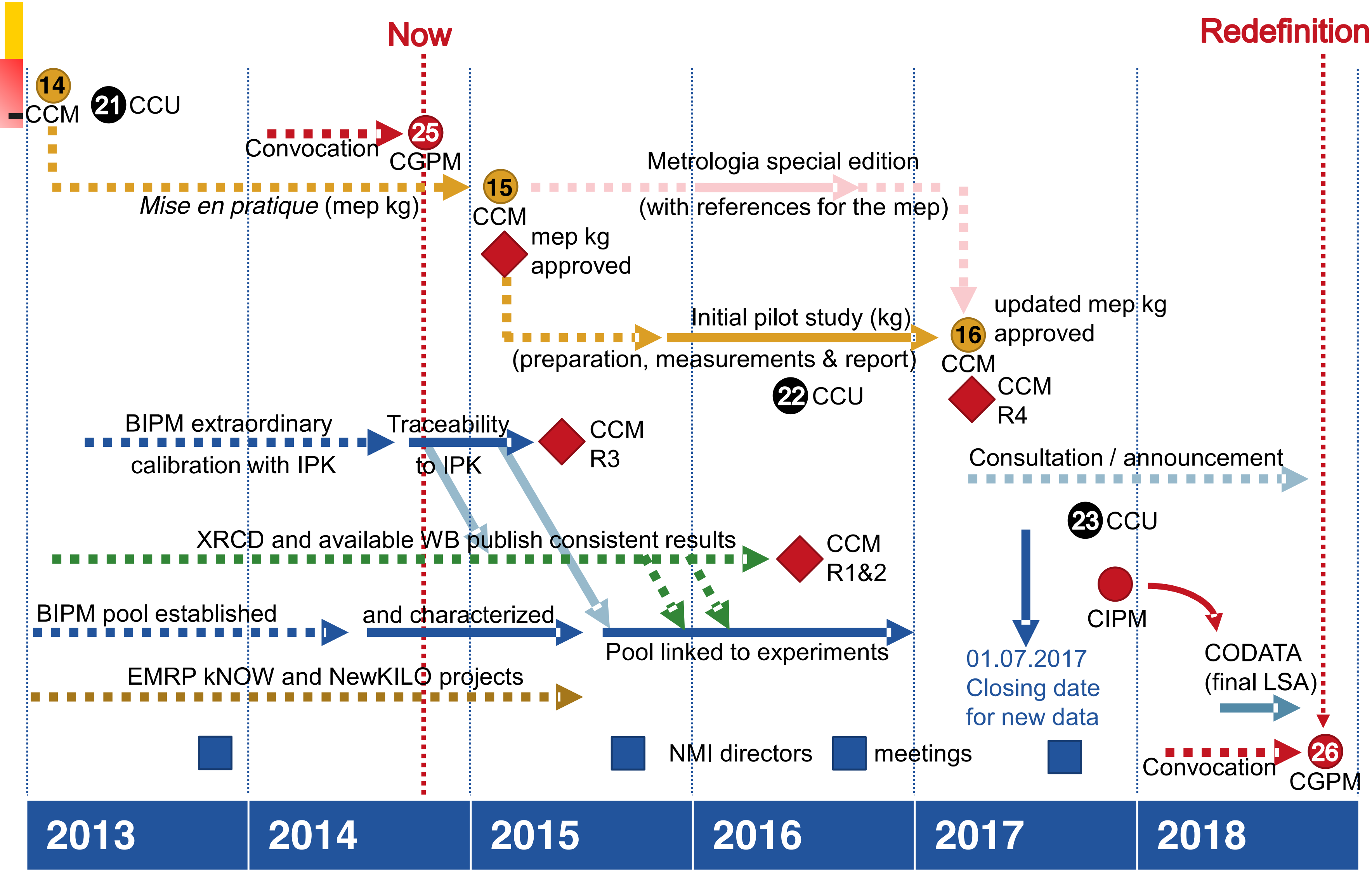
Z. Zhang, National Institute of Metrology, China (People's Republic of)

Towards a quantum SI system

- the ampere to be defined via a fixed value of *e*
- the kilogram to be defined via a fixed value of *h*
- the kelvin it to be defined via a fixed value of *k*
- to be reproduced from ohm (quantum Hall effect) and volt (Josephson effect)
- to be reproduced with watt balances and Avogadro spheres

Timeline - on the road to redefinition: TGFC meeting

Joint CCM and CCU roadmap for the new SI



Conditions from CCM Recommendation G1 (2013)