Black holes (BH) – old and new questions

Outline:

1) Some basic properties of classical BH (Introduction)
2) BH evaporation
3) Transplanckian frequensities (whether they are significant)
4) Quantum properties of horizon
5) Paradoxes
6) BH and quantum gravity

Schwarzschild BH

$$ds^2 = -\left(1 - \frac{2mG}{rc^2}\right)dt^2 + \left(1 - \frac{2mG}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad ; \quad r_g = \frac{2mG}{c^2}$$

↓ charged

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Omega^2$$

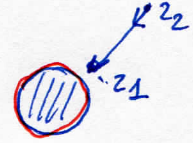
extremal

$$f(r) = 1 - \frac{r_g}{r} + \frac{Q^2 G}{r^2} = \left(1 - \frac{r_{g+}}{r}\right)\left(1 - \frac{r_{g-}}{r}\right) \rightarrow \left(1 - \frac{r_g}{r}\right)^2$$

↓

$$f(r) = 1 - \frac{r_g}{r} + \frac{Q^2 G}{r^2} \pm \frac{r^2}{R^2}$$ (BH in dS or AdS)

Radial motion of massless particles



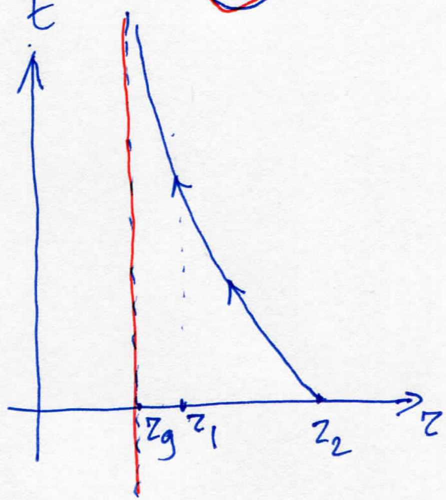
$$ds^2 = 0 = -f dt^2 + f^{-1} dr^2 \rightarrow dt = \pm \frac{dr}{f} = \frac{r\,dr}{r - r_g} \rightarrow$$

$$\xrightarrow{\text{time to reach } r_2 \to r_1} t_{12} = r_{12} + r_g \ln \frac{r_2 - r_g}{r_1 - r_g}$$

For $r_1 \to r_g$ $\quad t_{12} \simeq r_g \ln \frac{r_g}{r_1 - r_g} \to \infty$

Time to reach the Planck dist. $l_p = m_p^{-1}$ from. hor.

$$\hat{t} \simeq r_g \ln \frac{r_g}{l_p}$$



Nonsingular coordinates (on horizon)

$$ds^2 = -\left(1 - \frac{r_g}{r}\right)dt^2 + \left(1 - \frac{r_g}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

$$dr^* = \frac{dr}{1 - r_g/r} \rightarrow r^* = r + r_g \ln\left((r - r_g)/r_g\right)$$

$$v = t + r^* \; ; \; u = t - r^*$$

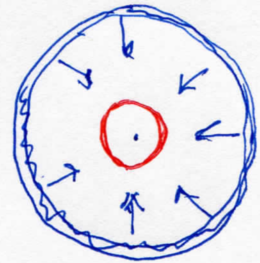
$$(t, r) \rightarrow (v, r)$$

$$ds^2 = -\left(1 - \frac{r_g}{r}\right)dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

Collaps of light-like shell (Vaidya's metric)

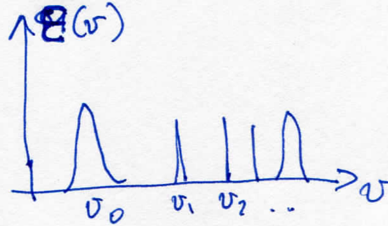
$$ds^2 = -\left(1 - \frac{2GM(v)}{r}\right)dv^2 + 2\,dv\,dr + r^2 d\Omega^2 =$$

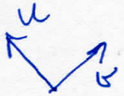
$$= -\left(1 - \frac{2GM(v)}{r}\right)du\,dv + r^2 d\Omega^2$$

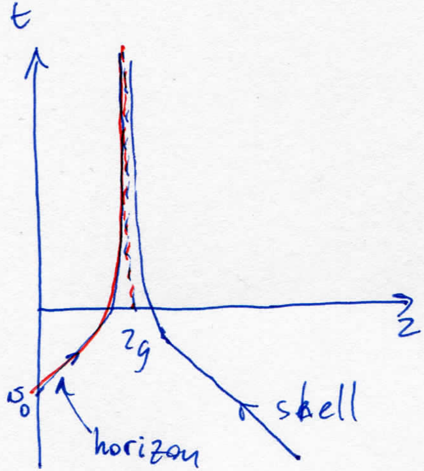


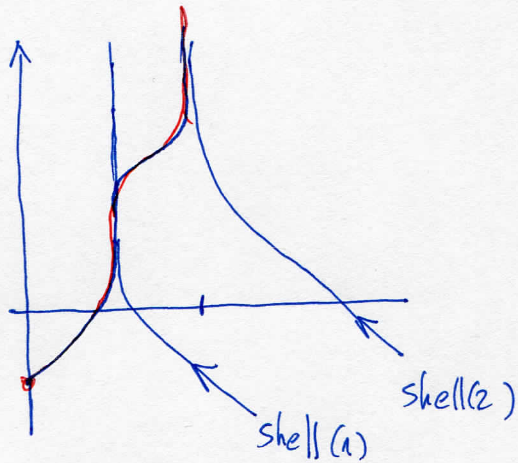
$$M(v) = \int_{-\infty}^{v} dv_1\, \varepsilon(v_1) \Longrightarrow \sum_i{}' m_i\, \theta(v - v_i)$$

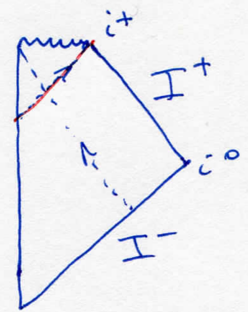
masses of δ-shells



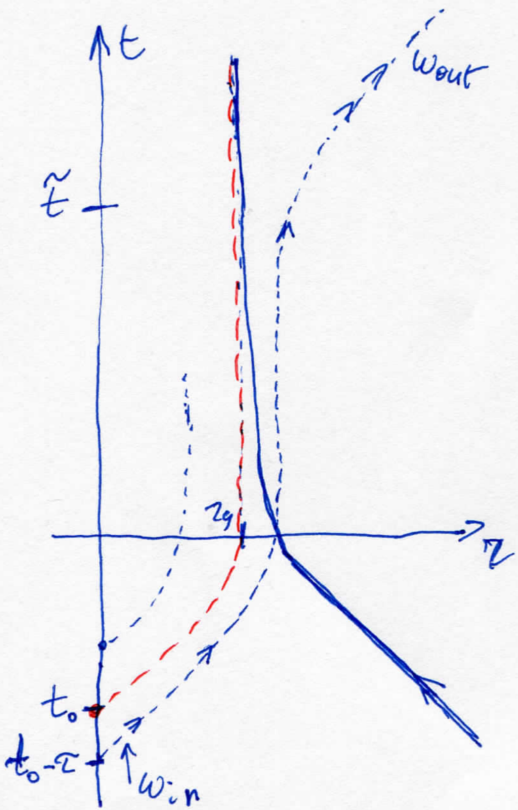








Trajectories of rays in the case of collapsing shell



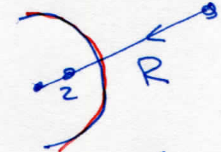
$$\tau(\tilde{t}) \sim r_g e^{-\tilde{t}/r_g}$$

$$\omega_{in} \sim \omega_{out} \cdot \exp\left(t/r_g\right) \longrightarrow$$

$$\xrightarrow{\omega_{out} \sim 1/r_g} r_g^{-1} \exp\left(t/r_g\right)$$

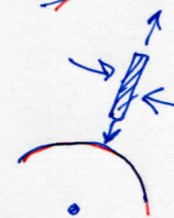
Some essential properties ~~of~~ BH

1) Eigentime to cross the horizon for massive body (observer)



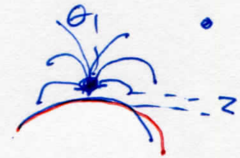
$$\tau(z) = \left(R/z_g\right)^{1/2} \int_z^R \frac{dz_1 \sqrt{z_1}}{\sqrt{R-z_1}} \sim R\left(R/z_g\right)^{1/2}$$

2) Deformation of body

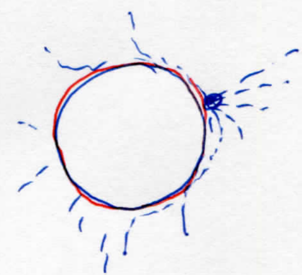


$$\ddot{\delta \xi}^{\mu} \sim R^{\mu}_{\ \lambda\sigma\rho}\, \delta^{\sigma} u^{\lambda} u^{\rho} \ ; \quad |R^{\cdot}_{\ \cdot\cdot\cdot}| \sim z_g/z^3$$

3) Trajectories of particles near BH horizon



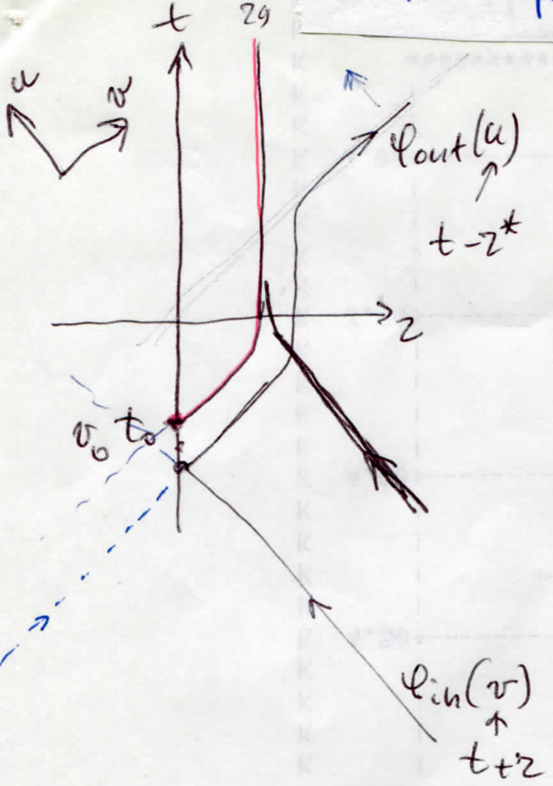
$$\theta^2_{max} \sim \frac{z-z_g}{z_g}$$



4) Charge and fields near BH

5) Entropy of BH $\quad S_{BH} \sim z_g^2 \rightarrow T \sim 1/z_g$

BH evaporation (Hawking) (S-wave)



$\varphi_{in}(v) \to \varphi_{out}(u)$ | $e^{-i\omega v} \to e^{-i\omega' u'}$

$u(v) = v - 2z_g \ln \frac{v_0 - v}{2z_g}$ | $\varphi_{in}(v) = \varphi_{out}(u(v))$

$$e^{-i\omega u(v)} = e^{-i\omega v}\left[\frac{2z_g}{v_0 - v}\right]^{2iz_g\omega} \theta(v_0 - v)$$

$$\varphi_{in} \sim \sum_\omega \left(a_\omega e^{-i\omega v} + a^+_\omega e^{i\omega v}\right)$$

$$\varphi_{out} \sim \sum_\omega \left(b_\omega e^{-i\omega u} + b^+_\omega e^{i\omega u}\right)$$

$$b_\omega = \sum_\varepsilon \left(\alpha_{\varepsilon\omega} a_\varepsilon + \beta_{\varepsilon\omega} a^+_\varepsilon\right)$$

$\rho_b \sim \exp\left[-\int_{\omega_1\omega_2} b^+_{\omega_1} b^+_{\omega_2} \gamma_{\omega_1\omega_2}\right]$; $\gamma_{\omega_1\omega_2} \sim \int_\varepsilon \alpha^{-1}_{\omega_1\varepsilon} \beta_{\varepsilon\omega_2}$

$$\beta_{\omega,\varepsilon}, \alpha_{\omega,\varepsilon} \sim \sqrt{\frac{\omega}{\varepsilon}}\, e^{i(\varepsilon \pm \omega)}\, e^{\pm\pi\omega z_g/2}\, \Gamma(1 \pm 2i\omega z_g)$$

Euclidian approach to BH entropy and evaporation

Near the horizon: $r = r_g + y \; ; \; y \ll r_g$: $\;\; \Big| \; y = \rho^2/2r_g$

$$ds^2 \simeq -\frac{y}{r_g}dt^2 + \frac{r_g}{y}dy^2 + r_g^2 d\Omega^2 \rightarrow -\frac{\rho^2}{4r_g^2}dt^2 + d\rho^2 + dx_\perp^2$$

$$t \to i\tau: \; ds_E^2 = \frac{\rho^2}{4r_g^2}d\tau^2 + d\rho^2 + dx_\perp^2 \Longrightarrow \underbrace{\rho^2 d\alpha^2 + d\rho^2}_{} + dx_\perp^2 \qquad \Big| \; \frac{\tau}{2r_g} = \alpha$$

flat 2 space in polar (ρ, α) coord

$0 \le \alpha \le 2\pi$

All fields must be periodic in $\underline{\underline{\alpha}}$

$$\int \mathcal{D}\varphi_i \exp\left(-I(\phi_i, g)\right)$$

$$T_\omega = 1/2\pi \longrightarrow T_\tau = 1/4\pi r_g \longrightarrow T(r) \simeq \frac{1}{r_g}\sqrt{\frac{r}{r - r_g}}$$

Quantization of the horizons of BH

$$S = A/4\ell_p^2 \;;\; \ell_p^2 = \frac{\hbar G}{c^3} \;;\; T = \frac{\partial E}{\partial S} \sim 1/r_g$$

Where is localized entropy of BH ?

Heavy string $\to$ BH $\begin{cases} S_{string} \sim M \cdot \ell_p \\ S_{BH} \sim M^2 \ell_p^2 \end{cases}$

In Loop Gravity the area is quantized

$$A = \sum_k{}' a_k(j_k) \;;\; a(j) = 8\pi \gamma \ell_p^{-2} \sqrt{j(j+1)}$$

Spectra of BH evaporation: $\begin{cases} \text{1) Termal} \\ \text{2) discrete} \\ \text{3) discrete; LG quanta.} \end{cases}$

1) Transplankian frequencies in the BH evaporation

2) Unitary evaporation of BH

3) Complementarity of BH.....

4) Entanglement; Firewall; paradoxes

5) Quantum fluctuation of horizon $\delta z_g \sim l_p \longleftrightarrow l_p \sqrt{\frac{z_g}{l_p}}$

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