Parton picture of the gravitational interaction at transplanckian energies and black hole (BH) creation

(hep-ph/0208021) → (Phys. Atom. Nucl. 66, 2128 (03))

Outline:

1 Introduction

2. Gravitational field of fast particle and classical picture of particles collision.

3. Parton-graviton structure of fast particle

4. Unitarisation ↔ black parton-graviton disk.

5. Amplitudes and inclusive cross-sections

6. Interaction in final state ↔ instability of produced particles configurations and BH creation

Some main References

Collision in classical GR

P.D. D'Eath, Phys Rev D18, (1977) 990

P.D. D'Eath and P.N. Payne, Phys. Rev D46 (1992) 658

G. 't Hooft, Phys. Lett, B198 (1987) 61

Graviton ladder, ...

L.N. Lipatov, Phys. Lett, 116B (1982) 411
Nucl. Phys. B365 (1991) 614

D. Amati, M. Ciafaloni, G. Veneziano,
Phys. Lett B197 (1987) 81
Nucl. Phys. B347 (1990) 550

Some new papers (BH creation in collision)

T. Banks and W. Fischler – hep-th/9906038

S.B. Giddings and S. Thomas – hep-ph/0106219

M.B. Voloshin – hep-ph/0107119, 0111099

D.M. Eardley and S.B. Giddings, gr-qc/0201034

M. Cavaglia – hep-ph/0210296 (Review)

The main reasons why the transplanckian collisions can be interesting:

1. Gravitational interaction at $\sqrt{s} \gg m_p$ becomes strong

2. Black hole production with big cross-sections

3. Planck scale can 'move' to TeV region

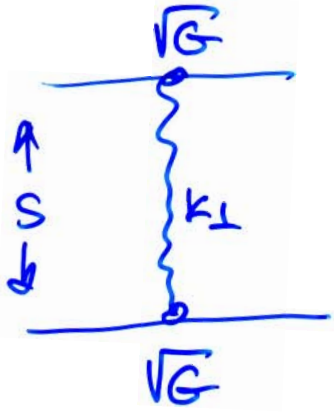
Specific:

Graviton is massless $\Rightarrow$

$\Rightarrow$ violation of Froissart bound $\Rightarrow$

$\Rightarrow$ different (from usual strong interaction) dynamics.

Gravitational interaction in perturbation theory:



$$A \sim G s^2 \frac{1}{k_\perp^2}$$

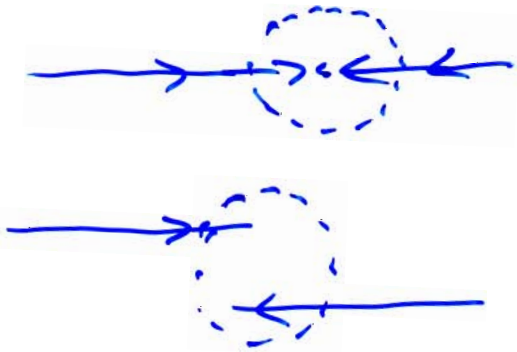
$$\sigma_{el} \sim \int |A|^2 d k_\perp^2 \Big/ s^2 \sim$$

$$\sim G^2 s^2 k_{\perp min}^{-2} \sim B_0^2 \left(\frac{s}{m_p^2}\right)^2$$

$$B_0 \sim k_{min}^{-1}$$

$$s^2 \int d^2 b \sim s^2 B_0^2$$

Classical picture of collision with $E \gg m_p \sim 10^{19}$ GeV and 'creation' of BH (horizon)

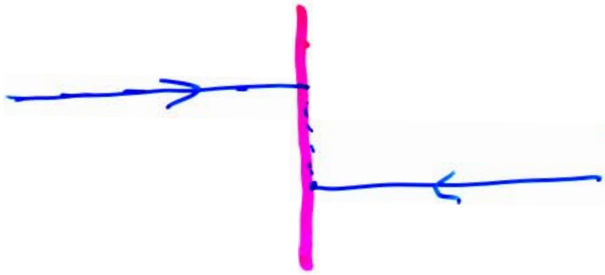


$$R_g = 2\sqrt{s} \cdot G$$

$$\sigma_{in} \simeq \pi R_g^2(s) \sim \frac{4\pi}{m_p^2}\left(\frac{s}{m_p^2}\right)$$

Planck scale 'can'? move from 10^{19} GeV $\longrightarrow$ to TeV region

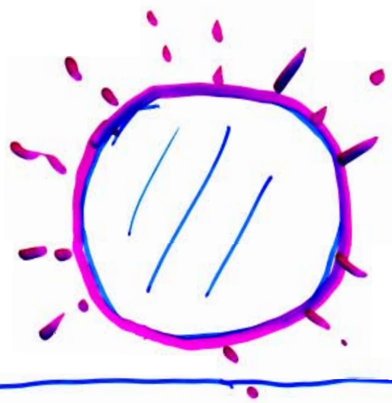
Classical picture of BH production



$$x_\perp < \frac{\sqrt{s}}{m_p^2}$$

Formation of horizon-trapping surface —

Classical stabilisation of BH shape



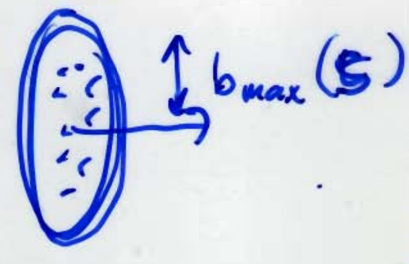
Hawking evaporation of BH

$$\tau \sim \frac{1}{m_p}\left(\frac{\sqrt{s}}{m_p}\right)^3 \quad \longleftrightarrow \quad t \sim \frac{1}{m_p}\left(\frac{\sqrt{s}}{m_p}\right)$$

evaporation time — reaction time

$$\sigma_{in}^{(BH)} \sim \frac{1}{m_p^2}\left(\frac{s}{m_p^2}\right) \sim \begin{cases} \sim 10^{-66}\,\text{cm}^2 & \text{— for } m_p \sim 10^{19}\,\text{GeV} \\ \sim 10^{-34}\,\text{cm}^2 & \text{— for } m_p \sim 1\,\text{TeV} \end{cases}$$

Froissart limit

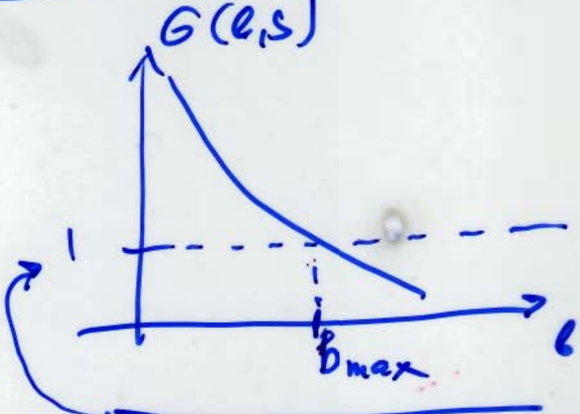


1) All masses > 0

local parton density

$$\sigma(b,s) \sim \left[s^n \cdot e^{-\mu b} \right]$$

local (in b) cross-section



$$\sigma_{in}(s) \sim \int \sigma(b,s)\, d^2 b \sim \pi b^2_{max}(s) \sim$$
$$\sim \left(\frac{\pi n^2}{\mu^2}\right) \ln^2 s$$

$\sigma(b,s) \leq 1$ ← Unitarity

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2) Some masses $= 0$

→ parton density $\sim 1/b^{\nu}$

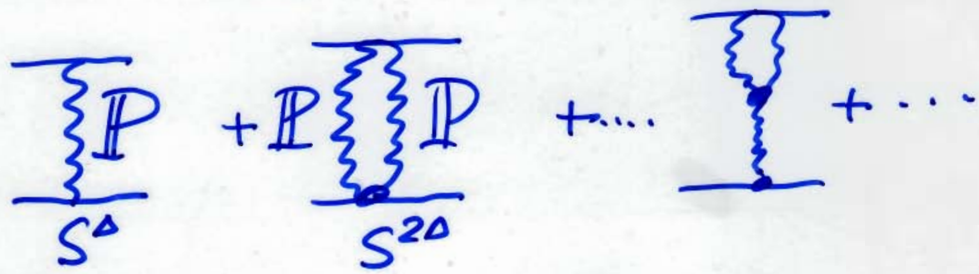
$$\sigma(b,s) \sim \left[s^n \cdot b^{-\nu} \right] \to b_{max} \sim s^{n/\nu}$$

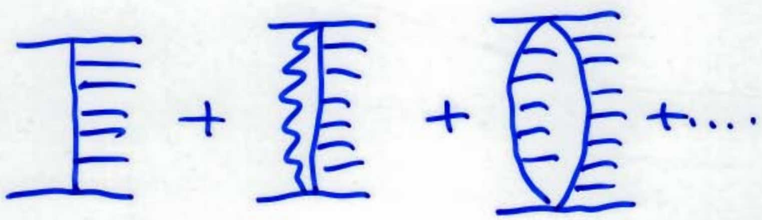
$$\downarrow \sigma_{in} = \int_b \sigma(b,s) \lesssim s^{(2n/\nu)}$$

$$e^{-\mu b} \leftrightarrow e^{-b^2/2\alpha' \ln s}$$

High energy hadronic interactions

In Pomeron terms:

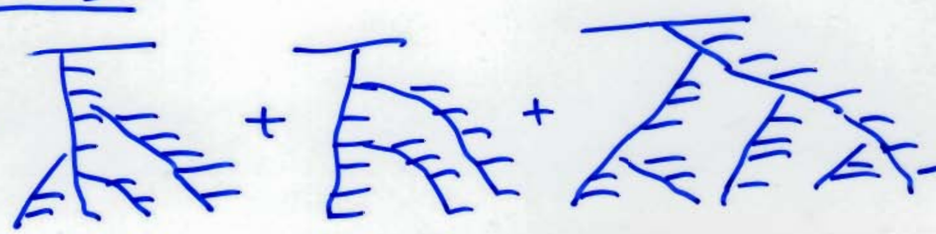
$A =$ 

$\sigma_{in} \Rightarrow$ 

↓ Unitarization
Froissart_like asymptotic.
-collision of black disks

$\begin{cases} \sigma_{in} \sim \bar{\mu}^2 \ln^2 E \\ N(E) \sim \ln^3 E \end{cases}$

In parton terms:

$\left| \text{Fock state of fast hadron} \right\rangle \simeq$ 

Parton cascading with mean multiplicty
of low energy partons $\sim E^{\Delta}$
Pomeron cut's correspond to simultaneus interaction
of far chains with target
↓saturation ⟷ parton gluing

$\left|\text{saturated black parton disk} \right\rangle$
with radius $\sim \mu^{-1} \ln E$

Collision of two 'Froissart' black disks

$\sigma_{in} = \pi (R(E_1) + R(E_2))^2 \sim \mu^2 \ln^2 (E_1 \cdot E_2)$

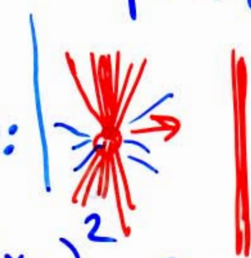
Gravitational field of fast particle ($E \gg m$):

Boosted Newton (Schwarzschild) metric:

$$\gamma = E/m$$

$$g_{\mu\nu} - g^{(0)}_{\mu\nu} \simeq \frac{2GP_\mu P_\nu}{m} \frac{1}{\sqrt{x_\perp^2 + \gamma^2(z-\beta t)^2}} \; ; \quad \beta = p/E$$

$\downarrow$ Aihelburg-Sexel metric ($E = \text{const}$, $m \to 0$):



$$ds^2 = dx^+ dx^- + 4GE \cdot \ln x_\perp^2 \cdot \delta(x^-)(dx^-)^2 - (dx_\perp)^2 \rightarrow \delta(x^-)$$

$$x^\pm = t \pm z$$

Generalization:

$$ds^2 = dx^+ dx^- + f(x^-, x_\perp)(dx^-)^2 - (dx_\perp)^2$$

$\downarrow$ Einstein equations reduce to a simple Poisson form:

$$R_{--} = \partial_\perp^2 f(x^-, x_\perp) = G\,T_{--}(x^-, x_\perp)$$

$$T_{--} = \sum_n \varepsilon_n \delta^2(\vec{x}_\perp - \vec{x}_{n\perp})\,\delta(x^- - x_n^-)$$

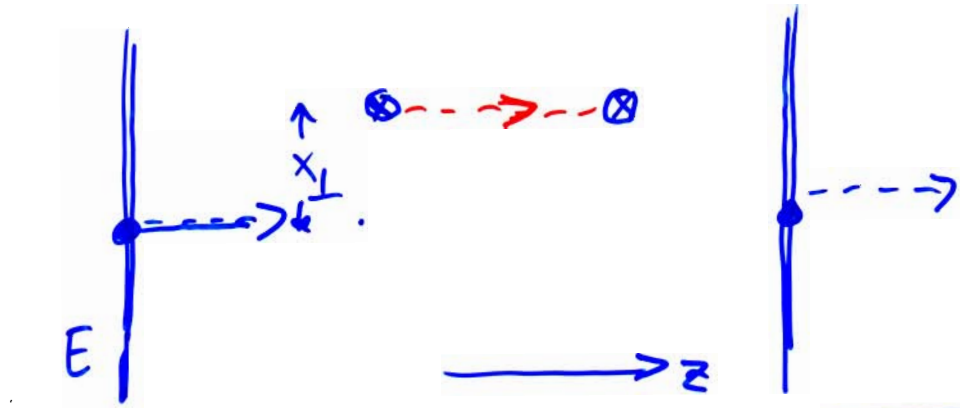
$\downarrow$

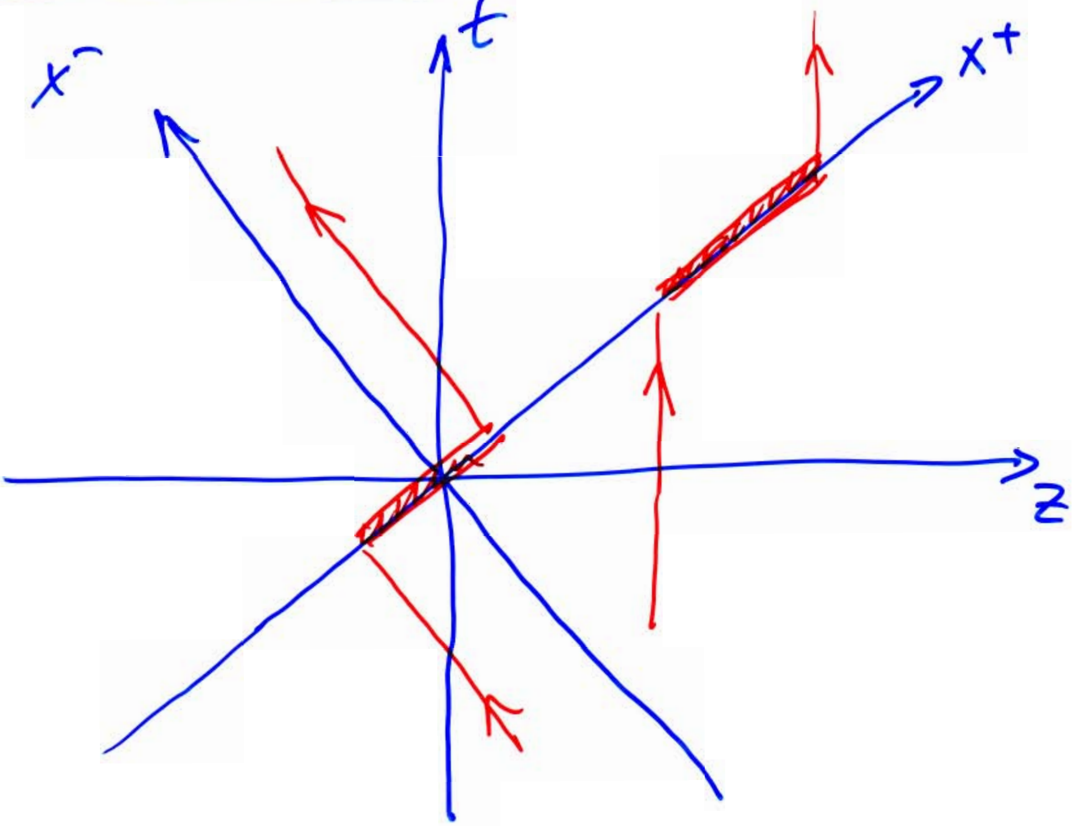
$$f = 4GE \sum_n \left(\frac{\varepsilon_n}{E}\right) \ln\left(\bar{x}_\perp - \bar{x}_{n\perp}\right)^2 \cdot \delta(x^- - x_n^-)$$

$$R_{-\perp-\perp} = \partial_\perp \partial_\perp f \sim GE\,\delta(x^-)\frac{x_\perp x_\perp}{x_\perp^4}$$

$\uparrow$ for one point-like particle

AS shock-front collision with a test particle





e^{-imt}

trapping (capture) time:

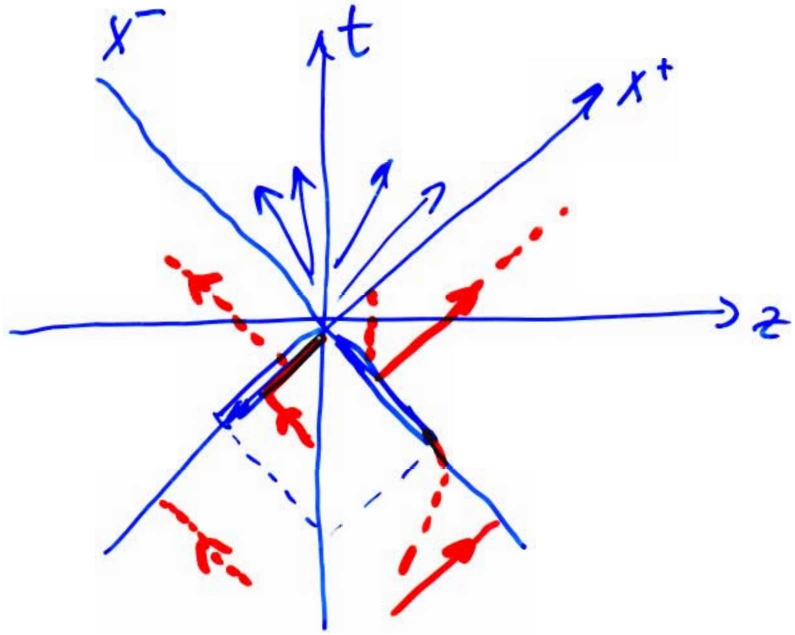
$$\tilde{t} = 8GE\ln\frac{L}{x_\perp}$$

Phase change for the wave function of 'test' particle with $\vec{p}=0$

$$\delta = m\tilde{t} = 8mEG\ln L/x_\perp$$

$$S(E,x_\perp) = e^{i\delta(E,x_\perp)}$$ ← 't Hooft's S-matrix

Two AS disks interaction

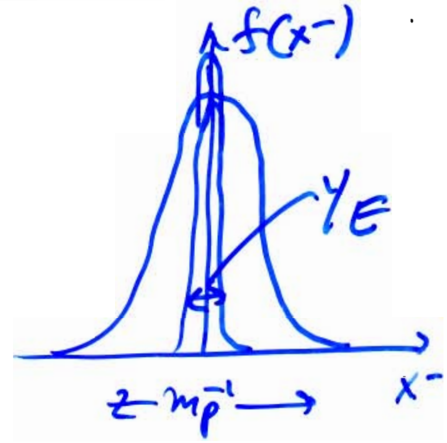


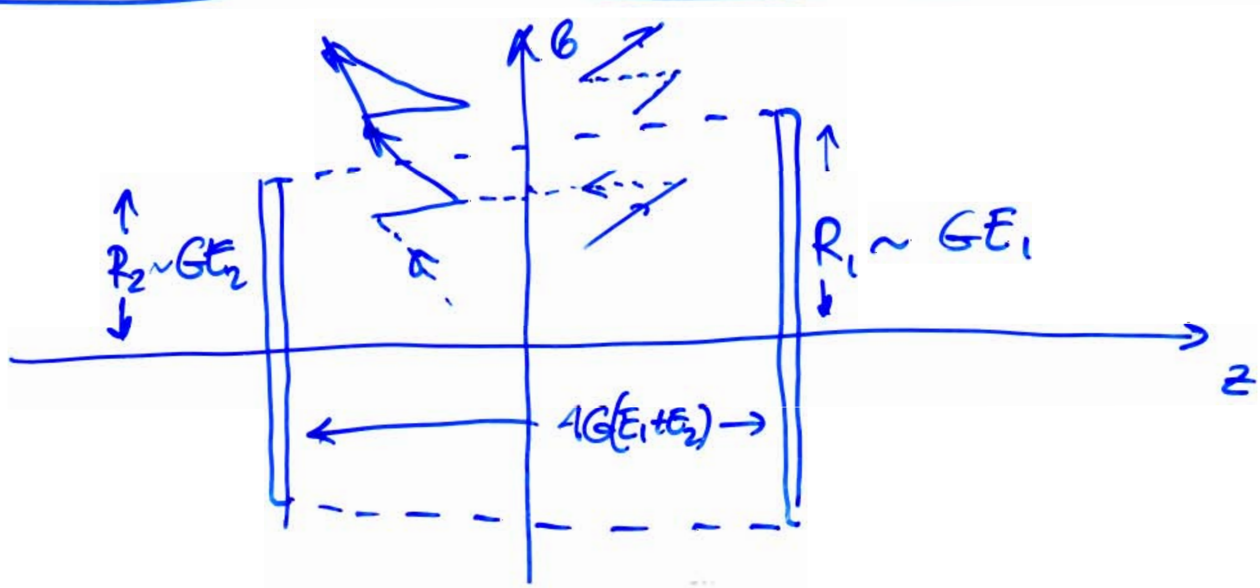
$$ds^2 = dx^+ dx^- + f_{E_1}(x^-, x_\perp)(dx^-)^2 + f_{E_2}(x^+, x_\perp)(dx^+)^2 + (d\vec{x}_\perp)^2$$

$$\partial_\perp^2 f(x^-, x_\perp) = G T_{--} \sim G E \delta^2(x_\perp)\delta(x^-) \longrightarrow f \sim 4 G E \delta(x^-) \ln x_\perp^2$$

$$|R_{\perp - \perp -}| \gg m_p^2$$

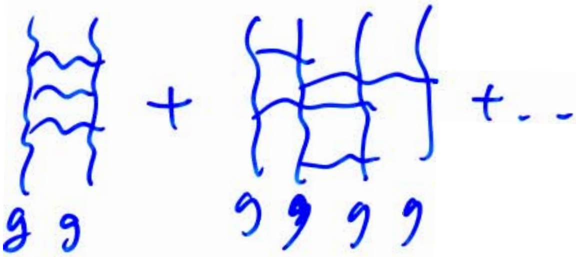
$$\partial_\perp^2 f + \left[C m_p^{-2} (\partial_\perp^2 f)^2 + \dots \right] = G T_{--}$$





if Supposed:

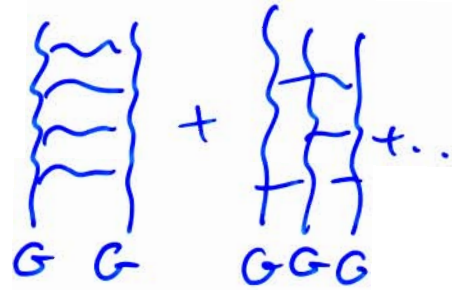
Cuttof of graviton virtualities at Planck $\leftrightarrow$ string scale

QCD: $\mathbb{P}$ $\simeq$ 

BFRL

$m_{min} > 0$

$\alpha_{\mathbb{P}}(0) = 1 + \Delta$

Gravity: $\mathcal{G} \simeq$ 

$m_{min} = 0$

$\downarrow$

No Froissart bound

$\alpha_{\mathcal{G}}(0) = 3 + \Delta$

$\downarrow$

$$\sigma^{\mathcal{G}}_{in}(s,b) \sim \left(\frac{s}{m_P}\right)^{2+\Delta} \frac{1}{(b\, m_P)^4}$$

$\downarrow$

Unitarization

$$\sigma_{in}(s,b) \sim 1 - \exp\left[-2\sigma^{\mathcal{G}}_{in}(s,b)\right] \to \int \sigma_{in}\, d^2b \sim m_P^{-4}\, s$$

Corresponds to collision of black disk with radius $R(E) \sim \sqrt{s}$ with target

WW spectra for gravitons

$$\sum_\lambda a^\lambda(k)\,\epsilon^{(\lambda)}_{\mu\nu} = \omega\int d^3x\, e^{ikx}\left(g_{\mu\nu} - g^{(0)}_{\mu\nu}\right)$$

AS metric

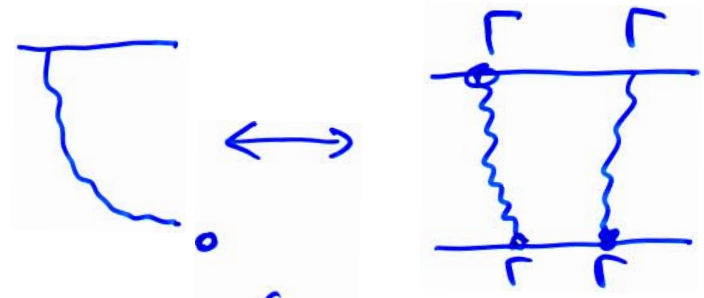
$$\sum_\lambda a^\lambda(k)\,\epsilon^{(\lambda)}_{--} = \frac{E}{m_p}\,\omega\,\frac{1}{k_\perp^2}$$

In gauge $\epsilon^{(\lambda)}_{\mu +} = 0$ and from $k^\mu \epsilon^{(\lambda)}_{\mu\nu} = 0$

$$\epsilon^{(\lambda)}_{--} = \epsilon^{(\lambda)}_{\perp\perp}\,\frac{(k^+)^2}{k_\perp^2}$$

$$dn^\perp(E,\omega,k_\perp) \sim \frac{(a^\lambda)^2}{\omega}\,d\omega\, d^2k_\perp \sim \left(\frac{E}{m_p}\right)^2 \frac{d\omega}{\omega^3}\, d^2k_\perp \qquad \frac{d\omega}{\omega}$$

$$dn^\perp \sim g_J^2\left(\frac{k_\perp E}{\omega}\right)^{2J}\left(\frac{\omega\, d\omega}{E^2}\right)\frac{d^2k}{k_\perp^4} \sim \frac{dx}{x^{2J+1}}\cdot\frac{d^2k_\perp}{k_\perp^{2(2-J)}}$$

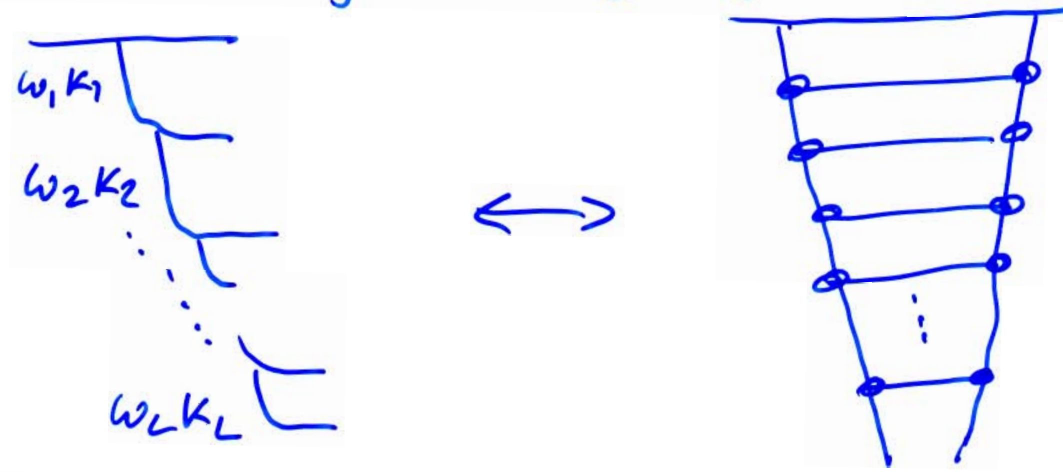


$$\sigma_{in}(E) \sim \int d\omega\, d^2k_\perp \cdot n(E,\omega,k_\perp)\, \hat{\sigma}(\omega,k_\perp) \sim E^2$$

$\downarrow$

Bare intercept $\alpha_{\psi}(0) = 3$

Cascading $\longleftrightarrow$ graviton ladder



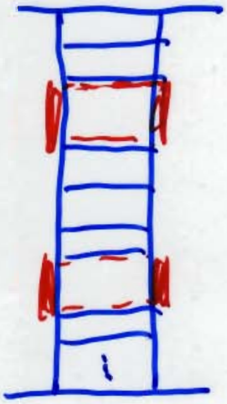
$$\int d\omega_1\, d^2k_1\, n(E,\omega_1,k_1) \int d\omega_2\, d^2k_2\, n(\omega_1,\omega_2,k_1-k_2) \dots$$

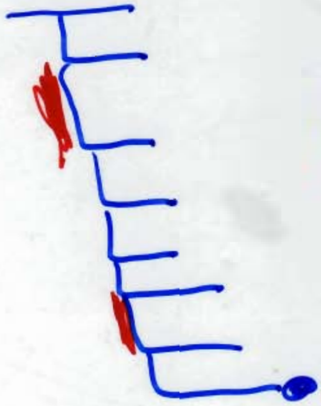
$$\dots \int d\omega_{L-1}\, d^2k_{L-1}\, n(\omega_{L-1},\omega_L,k_{L-1}-k_L) \sim \frac{\Delta^L}{(L-1)!}\left(\frac{E}{m_p}\right)^2 \frac{1}{\omega_L^3} \ln^{L-1}\left(\frac{E}{\omega_L}\right)$$

$$dn(E,\omega,k_\perp) \sim \left(\frac{E}{m_p}\right)^2 \left(\frac{E}{\omega}\right)^\Delta \frac{d\omega}{\omega^3}\, d^2k_\perp$$

$$\alpha_{\psi}(0) = 3 + \Delta \quad ; \quad \Delta = \frac{1}{m_p^2}\int d^2k_\perp\, \Gamma^2\left(\frac{k_\perp^2}{m_p^2}\right)$$

The infrared contributions to graviton ladder.





big $x_\perp$ steps

Soft WW spectra (one cell):

$$\frac{\partial n}{\partial\omega\,\partial x_\perp^2} \sim \left(\frac{E}{m_p}\right)^2 \frac{1}{\omega^3}\,\frac{1}{x_\perp^4}$$

$$\downarrow$$

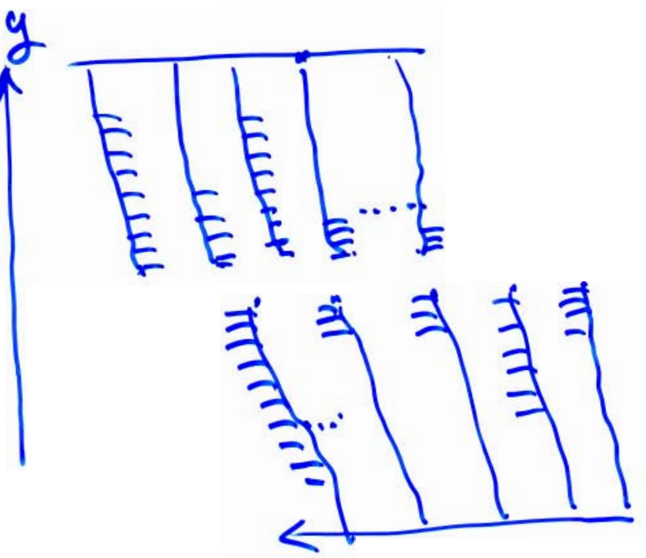
$$\sigma_{in}(s,b) \sim \left(\frac{s}{m_p^2}\right)^{2+\Delta} \frac{1}{(b\,m_p)^4}$$

$$\downarrow$$

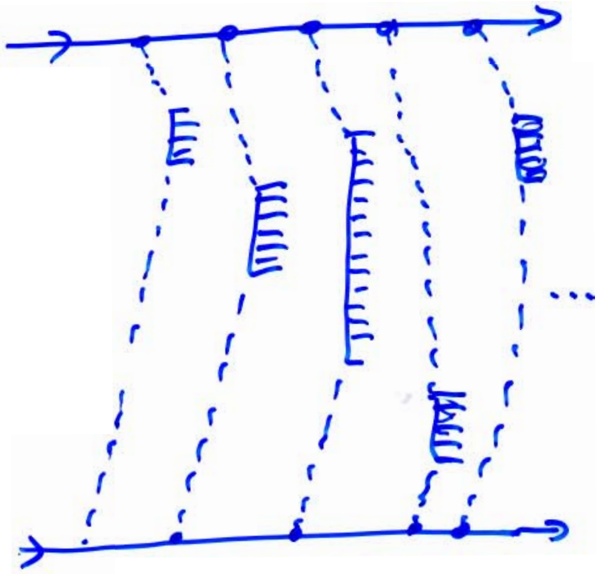
$$\sigma_{in}(s,b) \sim \left(\frac{s}{m_p^2}\right)^{2+\Delta}\left[\frac{a_0}{\ln s}\, e^{-b^2/4\alpha'\ln s} + \frac{a_1}{(b\,m_p)^4} + \frac{a_2 \ln b}{(b\,m_p)^6} + \dots\right]$$

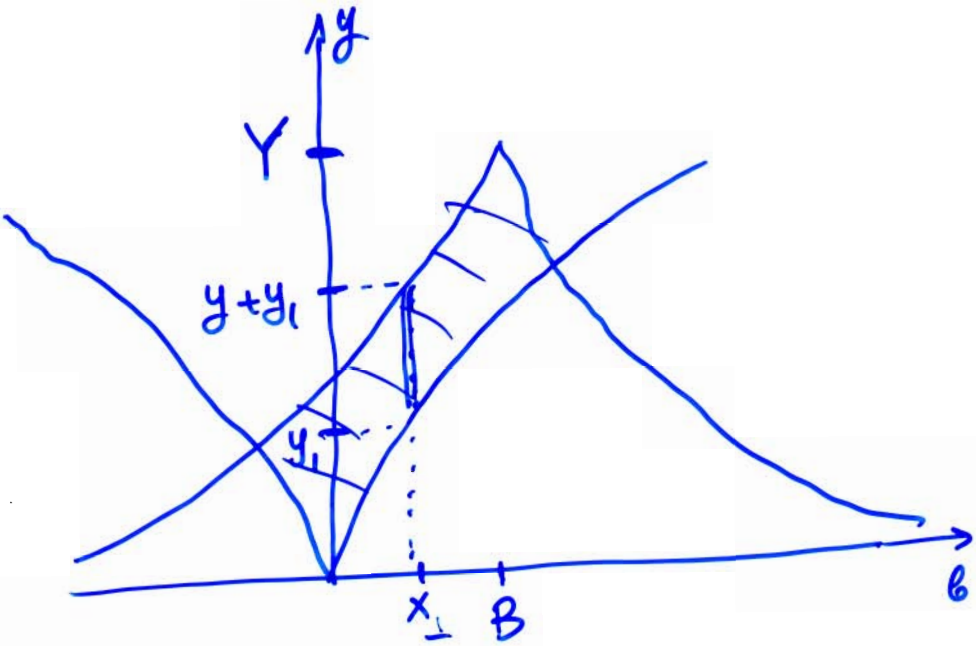
$$\mathrm{Im}\,A(s,q_\perp) \sim e^{\alpha(q_\perp)\ln s}\left[\tilde{a}_0 + \tilde{a}_1 q_\perp^2 \ln\frac{1}{q_\perp^2} + \tilde{a}_2\left(q_\perp^2 \ln\frac{1}{q_\perp^2}\right)^2 + \dots\right]$$

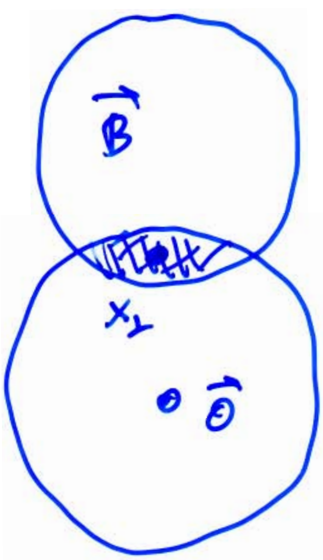
Colliding configurations $\leftrightarrow$ Feynman diagrams

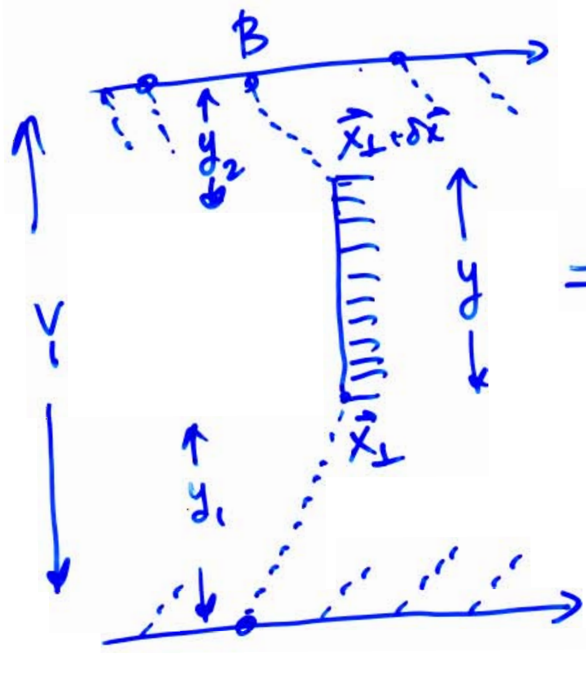


$\Rightarrow$









$$= Y - \ln\left(|x_\perp|^2 |\vec{B}-\vec{x}|^2 m_p^4\right) = \ln \frac{s}{m_p^6 |x_\perp|^2 |B-x_\perp|^2}$$

$$y_1 = \ln (x_\perp m_p)^2$$

$$y_2 = \ln\left[(B-x)^2 m_p^2\right]$$

The density of partons with various ω and $x_\perp$

Spectra:

$$dn = \left(\frac{E}{\omega}\right)^{2+\Delta} \frac{1}{(x_\perp m_p)^4} \left(m_p^2\, d^2 x_\perp\right) \frac{d\omega}{\omega} =$$

$$= e^{[2+\Delta](Y-y)-\varrho}\, dy\, d\varrho$$

$$Y = \ln E$$
$$y = \ln \omega$$
$$\varrho = \ln (x_\perp m_p)^2$$

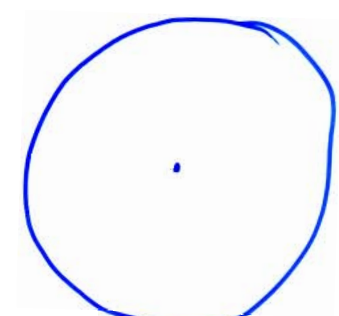
For partons with energy ω
the size of disk where density $n > m_p^{-2}$:

$$R_\perp^2(E,\omega) \sim m_p^{-2} \left(\frac{E}{\omega}\right)^{1+\Delta/2} \quad \to \quad R_\perp(E, m_p) \sim m_p^{-1} \sqrt{\frac{E}{m_p}}$$

$$\tilde{R}_\perp^{(max)}\left(E, \omega \sim 1/\tilde{R}\right) \sim m_p^{-1} \left(\frac{E}{m_p}\right)^{1+\frac{2}{2-\Delta}}$$

$$\downarrow$$

$$E/m_p^2$$



$$\delta x_\perp(\omega) \sim 1/\omega$$

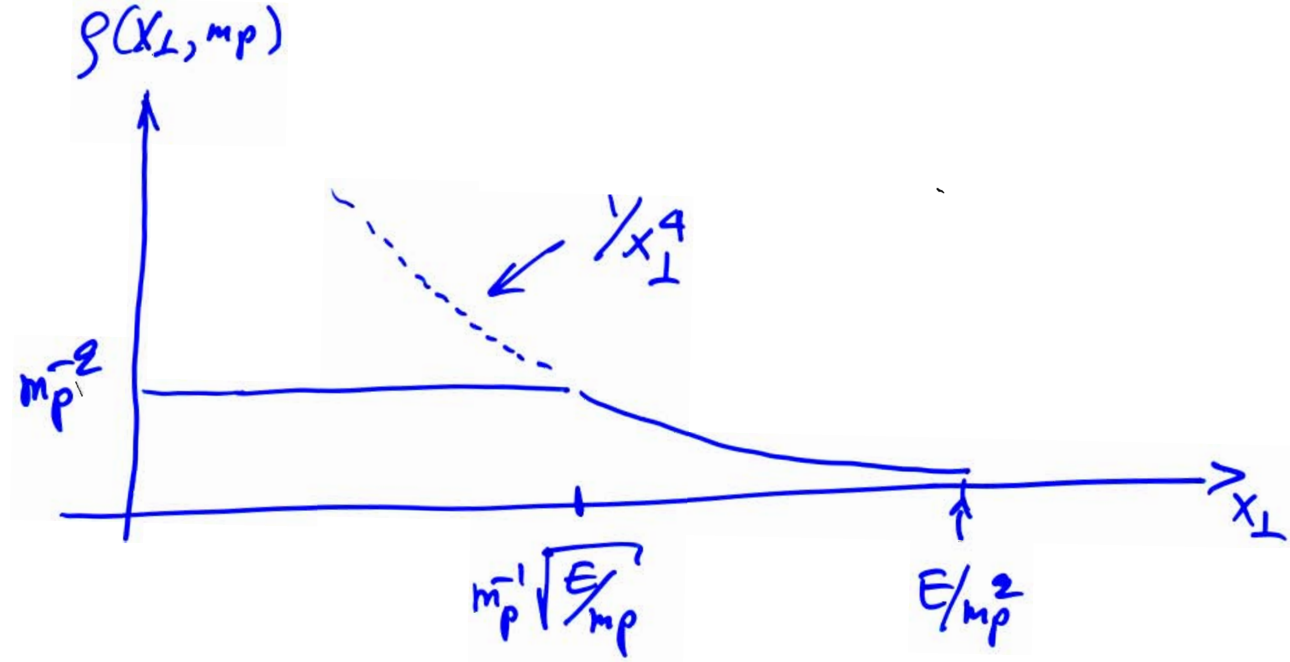
Unitarisation $\leftrightarrow$ parton density saturation

$$\sigma_{in}(s,b) \lesssim 1 \quad \longleftrightarrow \quad \max\left[n(E,\omega)\right] \lesssim m_\rho^{-2} \}$$

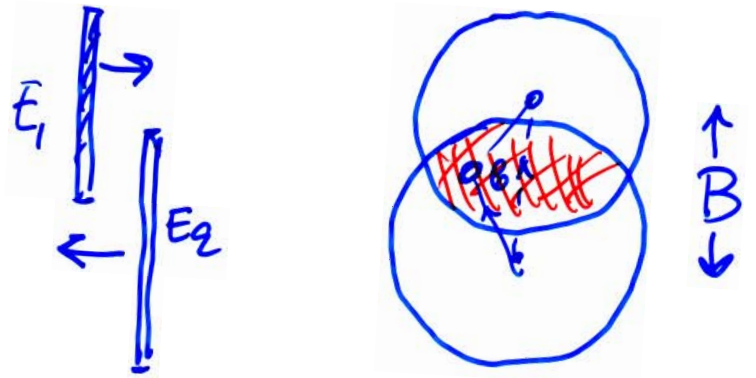
$$\sigma_{in}^{(0)}(s,b) \sim \left(\frac{s}{m_\rho^2}\right)^{2+\Delta} \frac{1}{(b m_\rho)^4}$$

$$\downarrow$$

$$\sigma_{in}(s,b) = 1 - e^{-2\sigma_{in}^{(0)}(s,b)}$$

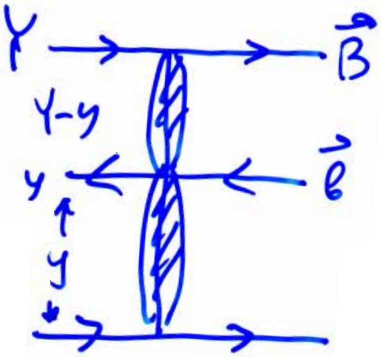


Two black disks interaction



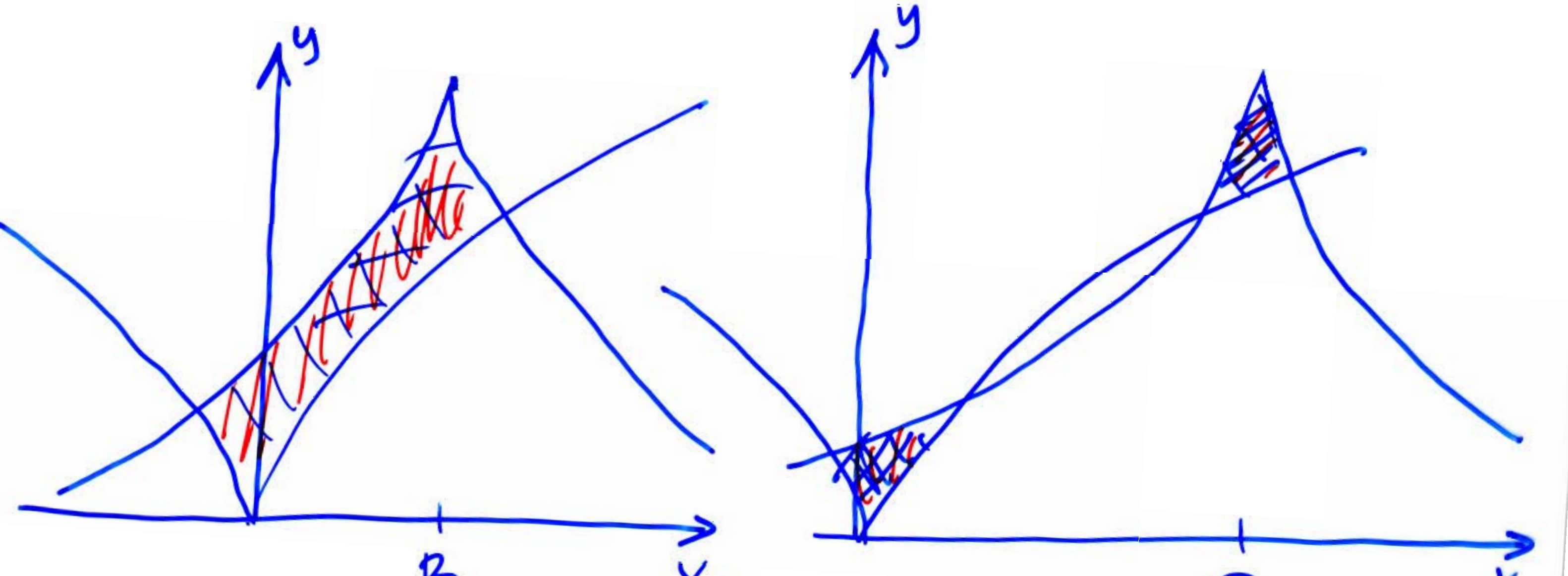
Inclusive cross-section for 'hard' graviton production

$$\rho = V \cdot D \cdot D$$

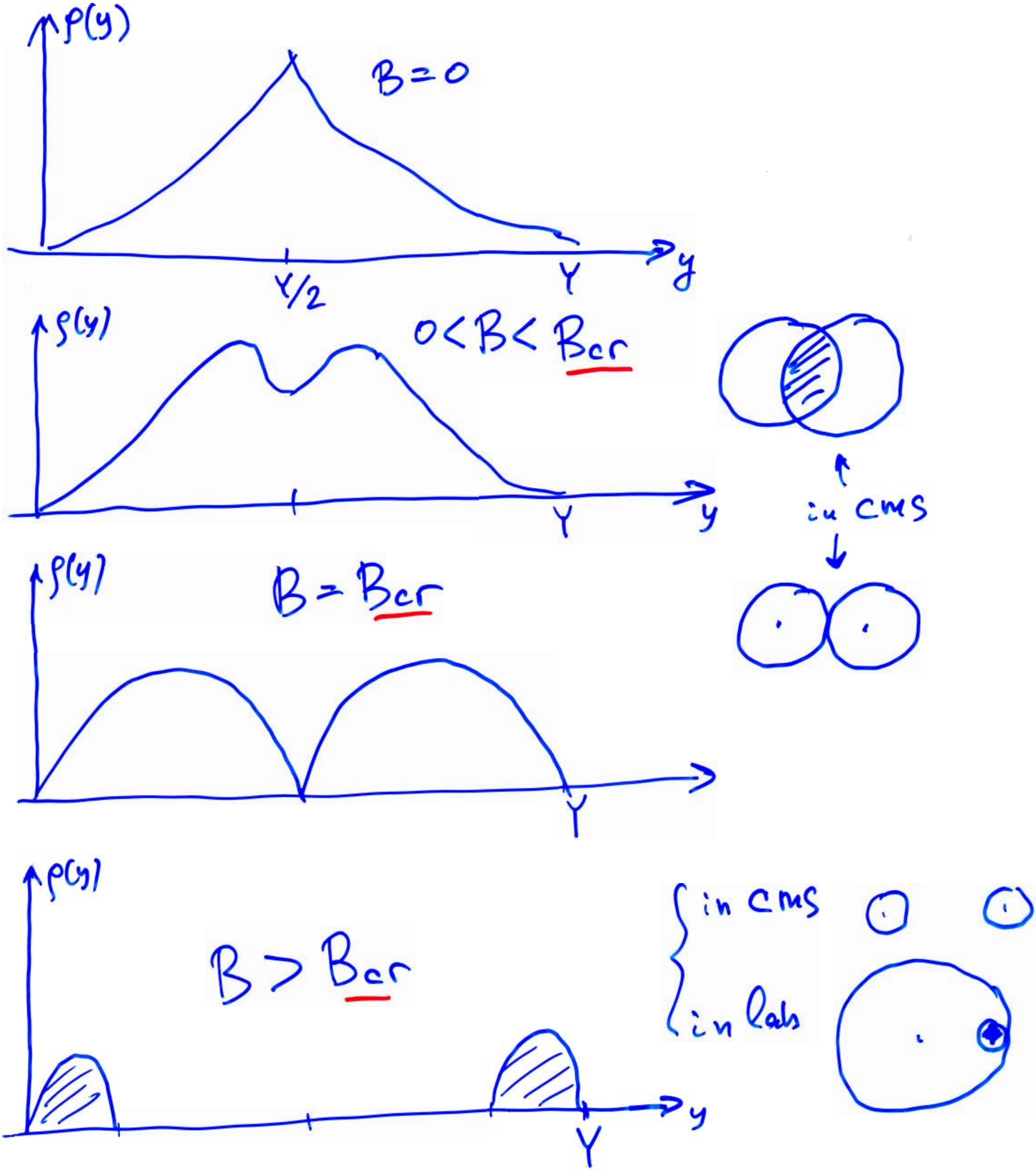


$$D(y,b) \simeq$$
$$\simeq i\,\theta\left(R_\perp^2(y) - b^2\right) + D^{soft}$$

$$R_\perp^2(y) = m_\rho^{-2} \exp\left[y\left(1 + \Delta/2\right)\right]$$



Inclusive cross-sections for various impact parameters B $\left[B_{cr} = 2R_\perp\left(\frac{\sqrt{s}}{2}\right) \sim m_\rho^{-1}\left(\frac{s}{m_\rho^2}\right)^{1/4} \right]$



Mean multiplicity of produced gravitons (with $k_\perp^2 \sim m_p^2$)

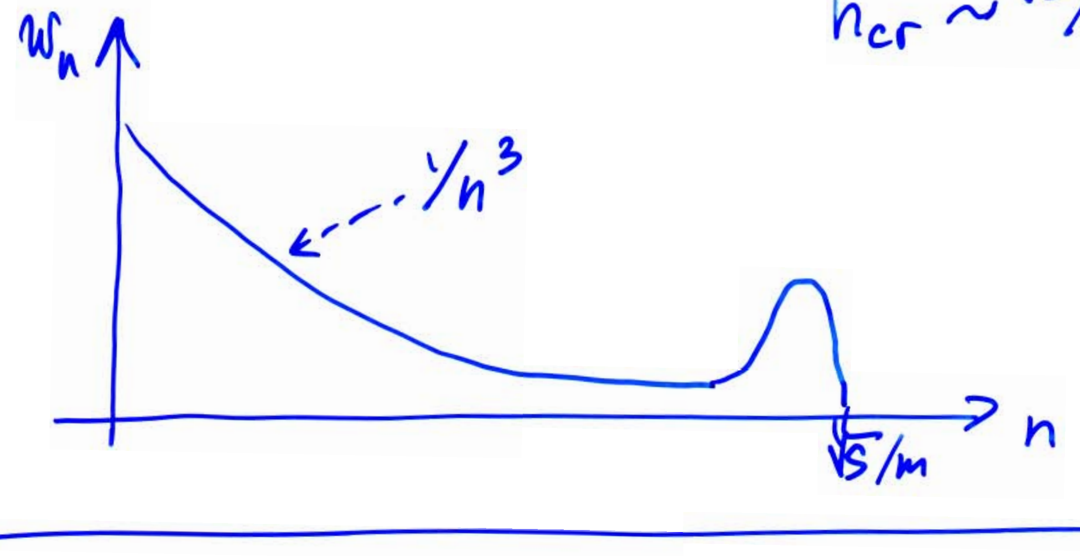
$$N(B,s) \simeq \begin{cases} \sim \dfrac{\sqrt{s}}{m_p} & \text{for } B \lesssim B_{cr} \\ \sim \dfrac{s}{m^4 B^2} & \text{for } B_{cr} < B < m_p^{-1}\sqrt{\dfrac{s}{m_p^2}} \end{cases}$$

$$B_{cr} \sim m_p^{-1} \left(s/m_p^2 \right)^{1/4}$$

Multiplicity distribution:

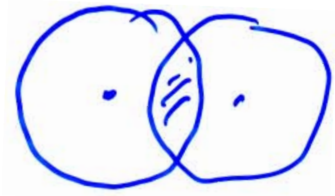
$$w_n(s) \sim \frac{1}{n^3} + \left(\frac{m_p^2}{s} \right) f\left(n \frac{m_p}{\sqrt{s}}\right) \Theta(n - n_{cr})$$

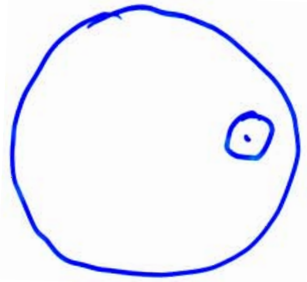
$$n_{cr} \sim \sqrt{s}/m_p$$

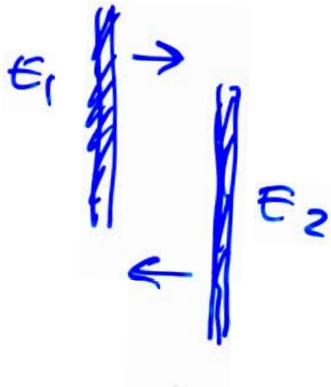


$$\tilde{\rho}(y) = \int \rho(Y, y, B, b)\, d^2b\, d^2B \sim \text{const}(y) \sim 1$$

Boost invariance of cross-sections



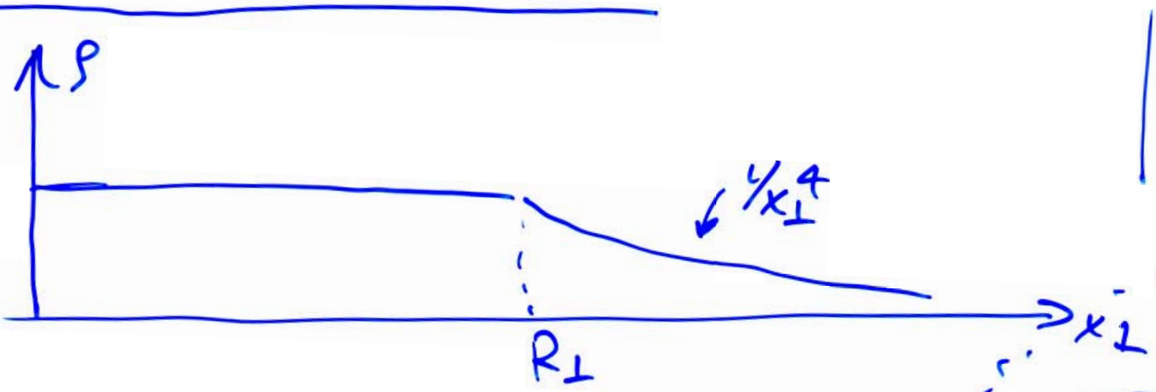


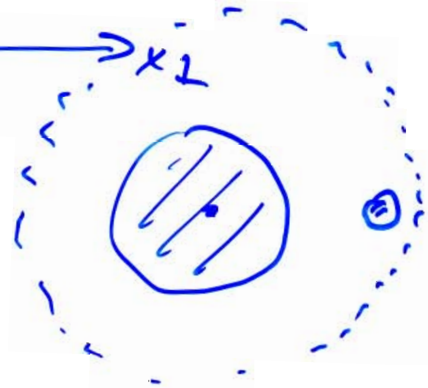


$$R_\perp(E, m_p) \sim m_p^{-1}\left(\frac{E}{m_p}\right)^{\frac{1}{2}+\frac{\Delta}{4}}$$

$E_1 \to \xi E_1\ , \ E_2 \to E_2/\xi$

$E_1 \cdot E_2$





$$\sigma_{in}(E_1, E_2) \sim$$

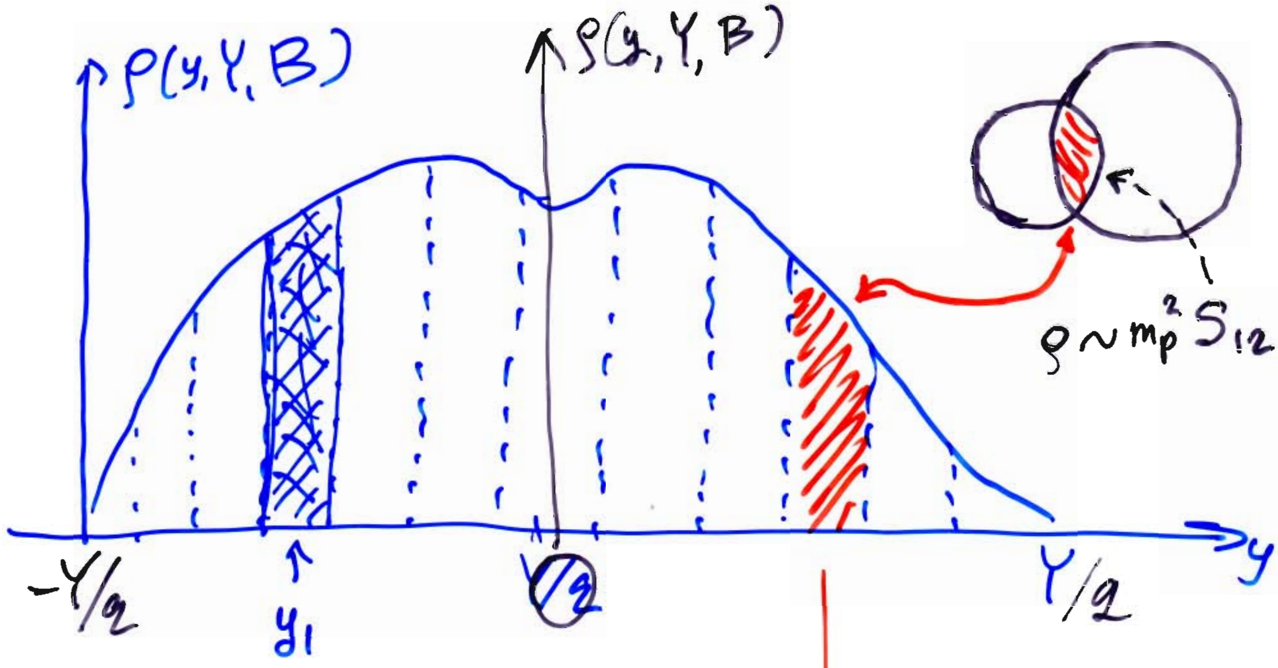
$$\sim m_p^{-2}\left[\left(\frac{E_1}{m_p}\right)^{1+\frac{3}{4}\Delta+\frac{1}{8}\Delta^2}\left(\frac{E_2}{m_p}\right)^{1+\frac{\Delta}{2}} + (E_1 \leftrightarrow E_2)\right]$$

is boost invariant only for $\Delta = 0$, $\Delta = -2$

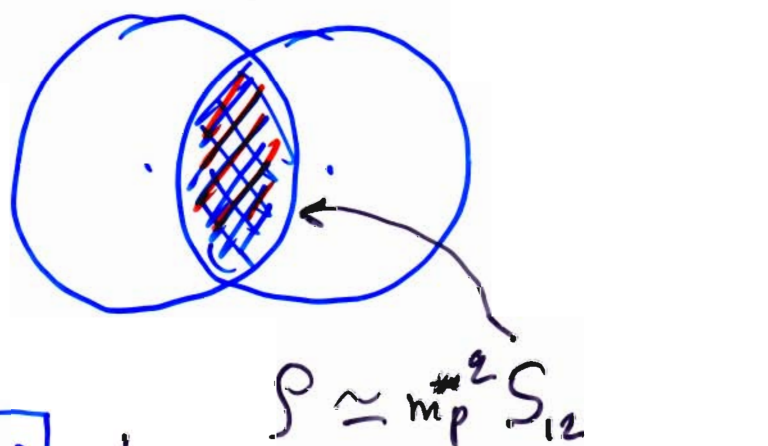
$$\alpha(0) = 3 + \Delta$$

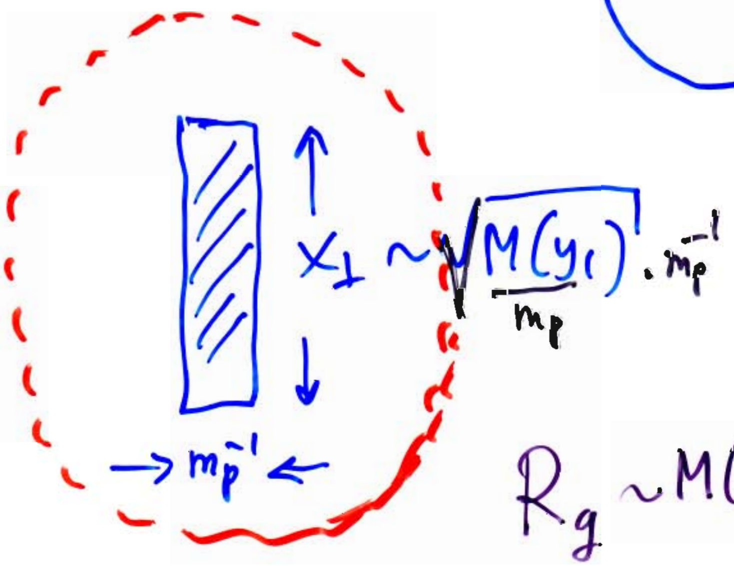
Final state interaction:

~~Possible~~ Large scale instability ~~and~~ $\Rightarrow$ B.H ~~production~~ creation

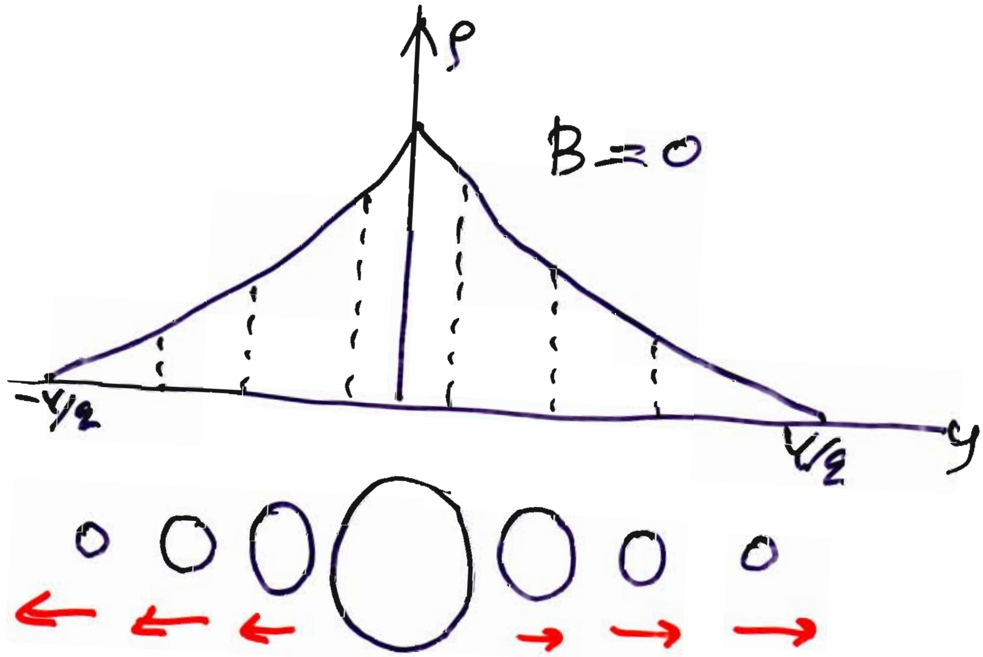


$$M(y_1) \sim \rho(y_1, Y, B) \cdot m_p$$

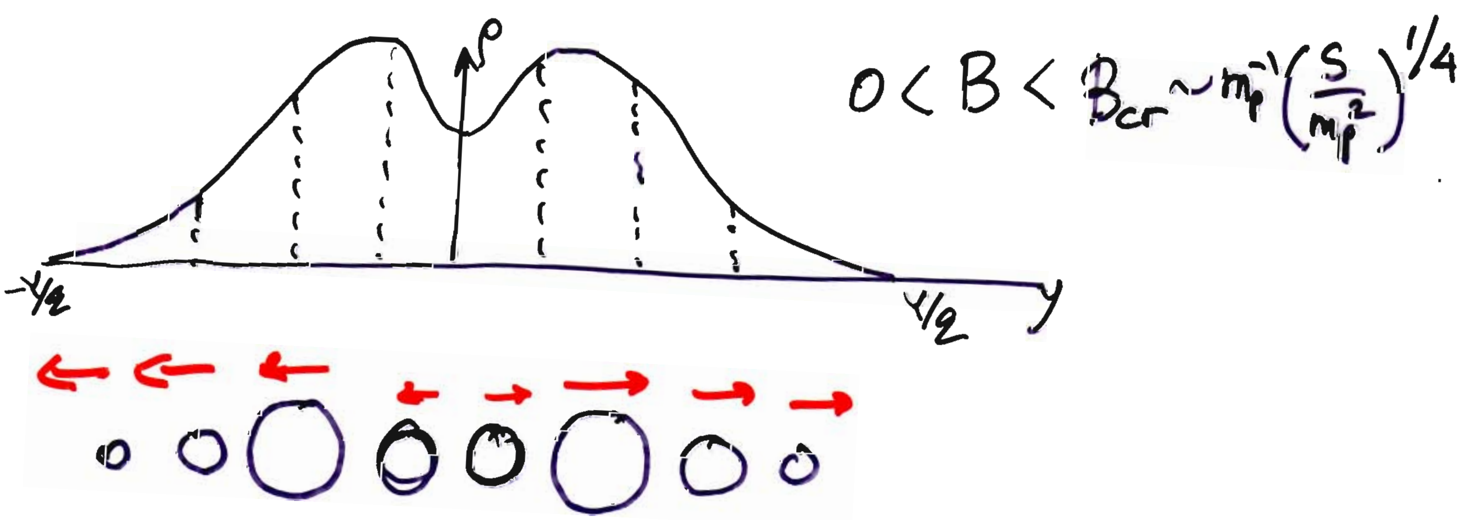




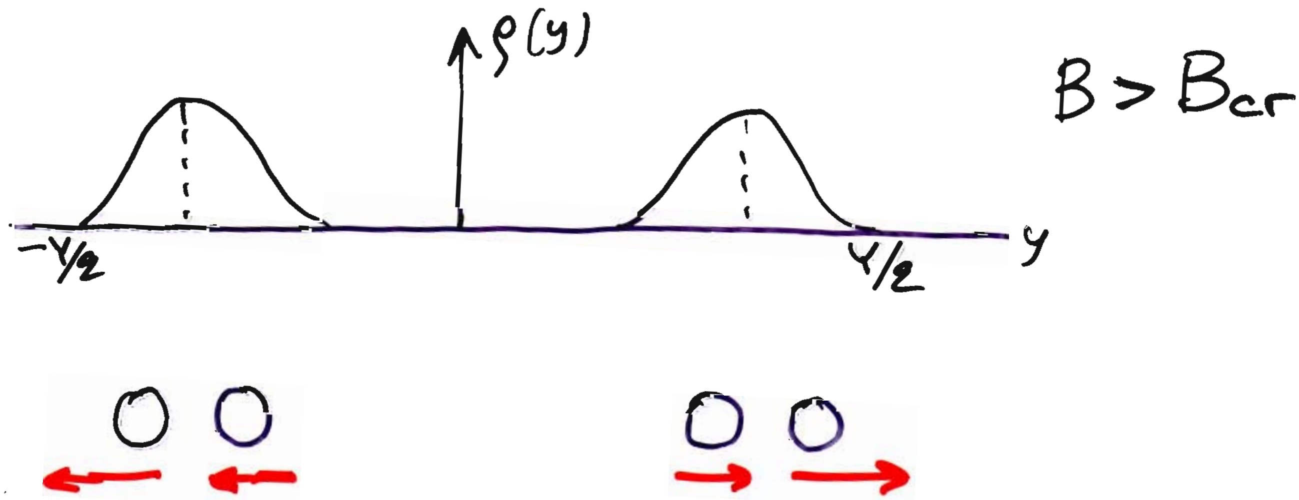
$$R_g \sim M(y_1)/m_p^2 \gg x_\perp, x^{\parallel}$$



ρ
B = 0
-Y/2
Y/2
y



ρ
$0 < B < B_{cr} \sim m_\rho^{-1}\left(\frac{s}{m_\rho^2}\right)^{1/4}$
-Y/2
Y/2
y



ρ(y)
$B > B_{cr}$
-Y/2
Y/2
y

Conclusion

① High energy gravitational interaction is (can be) in many respect similar to high-energy QCD interaction $\Longrightarrow$

$\Rightarrow$ Collision of black disks filled with strongly interacting partons

② Main difference:

a) Radiuses of disks grow $\sim \begin{cases} \sim \ln E & \text{in QCD} \\ \sim E^{1/2} & \text{in Gravity} \end{cases}$

$\downarrow$

a′) Inelastic cross-sections $\sigma_{in} \sim \begin{cases} \sim (\ln S)^2 - \text{QCD (Froissard limit)} \\ \sim S - \text{in Gravity} \end{cases}$

b) Interaction in final state:

[QCD] — only short range hadronisation

[Gravity] — long range attractive instability of final state leading to clastering of produced particles $\Longrightarrow$

$\Rightarrow$ Black holes creation